Date:

NAME : Ibadullah Shaikh
ROLL NO : 19K-0259
COURSE : Discrete Structures
SECTION : BCS-3G
SUBMITTED TO : Sir Fahad Samad
DATE :
DAY :

ASSIGNMENT No: 01

Q: No 01

(a) Proposition, and true.
(b) Proposition, but false.
(c) Proposition, and true.
(d) Proposition, but false.
(e) Not a proposition.
(f) Not a proposition.

Q: No 02

	RAM	ROM	Camera
Phone 1	256MB	32 GB	8 MP
Phone 2	288 MB	64 GB	4 MP
Phone 3	128 MB	32 GB	5 MP

(a) True, statement P.
(b) True, statement P∨Q
∴P: Phone C has higher resolution than Phone B
Q: C has more ROM than B

Date:

(c) False , Statement $P \wedge Q \wedge R$

P: B has more RAM than A

Q: B has more ROM than A

R: B has more resolution than A.

(d) False , Statement $(P \wedge Q) \rightarrow R$

P: B has more RoM than C

Q: B has more RAM than C

R: B has more resolution than C

(e) False , statement: $P \leftrightarrow Q$

P: A has more RAM than B.

Q: B has more RAM than A.

Q no 03:

	Annual	Net
Acme Computer	138 billion	8 billion
Nadir Software	87 billion	5 billion
Quixote Media	111 billion	13 billion.

(a) False , Statement P

P: Quixote has largest Annual revenue.

(b) True , statement $P \wedge Q$

P: Nadir has lowest net profit.

Q: Acme has largest annual revenue.

(c) True , statement $P \vee Q$

P: Acme has largest net profit

Q: Quixote has largest net profit.

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(d) True; statement : $P \rightarrow Q$
P: Quixote has smallest net Profit.
Q: Acme has largest annual revenue.

(e) True: statement $P \leftrightarrow Q$
P: Nadir has smallest net profit.
Q: Acme has largest annual revenue.

Q no: 4

(a) $P \rightarrow Q$
If you have flu then you will miss the final examination.

(b) $\neg Q \leftrightarrow r$
It is not the case that you will miss the final examination iff you pass the course.

(c) $Q \rightarrow \neg r$
If you will miss the final examination the you will not pass the course.

(d) $P \vee Q \vee r$
You have the flu or you miss the final examination or you pass the course.

(e) $(P \rightarrow \neg r) \vee (q \rightarrow \neg r)$
If you have flu then you will not pass the course or if you miss the final examination then you will not pass the course

(f) $(P \wedge Q) \vee (\neg Q \wedge R)$
You have flu and you will miss the final examination or you donot miss the final examination and you will pass the course.

Date:

Qno: 07

(a)

(i) q if p

(ii) q when p

(iii) if p, q

(iv) q unless $\neg p$

(v) a necessary condition for p is q.

English

(i) I will go for a walk in the woods, then If it is sunny tomorrow.

(ii) I will go for a walk in the wood, when it is sunny tomorrow.

(iii) If it is sunny tommorrow, I will go for a walk in the woods.

(iv) I will go for a walk in the woods unless it is not sunny tomorrow.

(v) A necessary condition for, it is sunny tommorrow is I will go for walk in the woods.

(b)

(i) Converse: If I will go for a walk in the woods, then it is sunny tommorrow.

(ii) Contrapositive: It is not the case that I will go for a walk in the woods, then it's not the sunny tomorrow.

(iii) Inverse: If it is not sunny tommorrow, then I will not go for a walk in the woods.

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(c) (a) Inverse of Inverse: If it is sunny tommorrow, then I will go for walk in the woods.

(b) Inverse of converse: If I will not go for a walk in the woods, then It will not be sunny tommorrow.

(c) Inverse of contrapositive: If I go for a walk in the woods then it will be sunny tommorrow.

Qno: 08

(a) Jan is not rich or happy.

(b) Carlos will not bicycle and not run tommorrow.

(c) The fan is not slow and it is not very hot.

(d) Akram is not unfit or saleem is not injured.

Q: 09

(a) OR is used in exclusive sense.

(b) OR is used in inclusive sense.

(c) OR is used in inclusive sense.

(d) OR is used in inclusive sense.

Date:

Q:10

(a) $(P \wedge (\neg(\neg P \vee Q))) \vee (P \wedge Q) \equiv P$

Solution:

$(P \wedge (\neg(\neg P \vee q))) \vee (P \wedge q)$

$(P \wedge (P \wedge \neg Q)) \vee (P \wedge Q)$

$((P \wedge Q) \vee (P \vee \neg Q)) \wedge ((P \wedge Q) \vee (P))$

$(P \wedge Q) \vee (P \vee \neg Q) \wedge (P)$

$(P \wedge T) \wedge P$

$P \wedge P$

$P \equiv P$ Proved.

(b) $\neg(P \leftrightarrow Q) \equiv (P \leftrightarrow \neg Q)$

$\neg(P \leftrightarrow Q)$

$\neg[(P \wedge Q) \vee (\neg P \wedge \neg Q)]$

$\neg(P \wedge Q) \wedge \neg(\neg P \wedge \neg Q)$

$\neg P \vee \neg Q \wedge (P \vee Q)$

$[\neg P \wedge (P \vee Q) \vee \neg Q \wedge (P \vee Q)]$

$[(\neg P \wedge P) \vee (\neg P \wedge Q)] \vee [(\neg Q \wedge P) \vee (\neg q \vee q)]$

$(\neg P \vee q) \vee (\neg q \wedge P)$

$P \leftrightarrow \neg Q$ Proved.

(c) $\neg P \leftrightarrow q \equiv P \longleftrightarrow \neg Q$

$\neg P \longleftrightarrow Q$

$(\neg P \rightarrow Q) \wedge (q \rightarrow P)$

$(P \vee Q) \wedge (\neg q \vee \neg P)$

$(q \vee P) \wedge (\neg P \vee \neg Q)$

$(\neg q \rightarrow p) \wedge (P \rightarrow \neg q)$

$p \longleftrightarrow \neg Q$

Proved.

Date:

(d) $(P \wedge Q) \rightarrow (P \rightarrow q) \equiv T$

$(P \wedge Q) \rightarrow (P \rightarrow q)$

$\neg (P \wedge Q) \vee (P \rightarrow q)$

$\neg (P \wedge Q) \vee (\neg P \vee q)$

$(\neg P \vee \neg Q) \vee (\neg P \vee q)$

$(\neg P) \vee (\neg Q \vee (\neg P \vee q))$

$(\neg P) \vee ((\neg P \vee q) \vee \neg Q)$

$(\neg P) \vee (\neg P \vee (q \vee \neg q))$

$(\neg P) \vee (\neg P \vee T)$

$(\neg P) \vee T$

$T \equiv T$ Hence proved.

(e) $\neg (P \vee \neg (P \wedge q)) \equiv F$

$\neg (P \vee \neg (P \wedge q))$

$\neg P \wedge \neg (\neg P \wedge q))$

$(\neg P \wedge P) \wedge q$

$F \wedge q$

$F \equiv F$ Hence proved.

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Q: 11

(a) $(p \to r) \land (q \to r)$ and $(p \lor Q) \to r$

P	Q	R	$P \to r$	$q \to r$	$(P \to r) \land (q \to r)$	$P \lor Q$	$(P \lor Q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

hence proved.

(b) $(P \to Q) \lor (P \to r)$ and $P \to (Q \lor R)$

P	Q	R	$P \to Q$	$P \to R$	$(P \to Q) \lor (P \to R)$	$(Q \lor R)$	$P \to (Q \lor R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Hence Proved.

(c) $(P \to Q) \to (r \to s)$ and $(P \to r) \to (q \to s)$

P	Q	R	S	$P \to Q$	$R \to S$	$(P \to Q) \to (R \to S$	$R \to R$	$Q \to S$	$(P \to r) \to (Q \to S$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F
T	T	F	T	T	T	T	F	T	T
T	T	F	F	T	T	T	F	F	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

Not equal.

Date:

Q:12

(a) False
(b) True
(c) True
(d) True
(e) False
(f) True

Q:13

(a) False
(b) False
(c) True
(d) True

Q:14

(a) $\forall x\, f(x, \text{Bob})$
(b) $\forall x\, f(\text{Alice}, y)$
(c) $\forall x\, \exists y\, f(x, y)$
(d) $\neg \exists x\, \forall y\, f(x, y)$
(e) $\forall y\, \exists x\, f(x, y)$

Q:15

(a) $\exists x\, (P(x) \wedge Q(x))$
(b) $\exists x\, (P(x) \wedge \neg Q(x))$
(c) $\forall x\, (P(x) \vee Q(x))$
(d) $\forall x\, (\neg P(x) \vee \neg Q(x))$

Date:

Q16:

(a) There is an student in your class who has sent an e-mail to some student in your class.

(b) There is an student in your class who has sent an e-mail to all students in your class.

(c) Every student in your class has sent an e-mail to someone student in your class.

(d) There is an student in your class who has recieved an email from every student of your class.

(e) Every student in your class has recieved an e-mail from someone student in your class.

(f) Every student in your class has sent an e-mail to every student in your class.

Q:17:-

(a) There is a student who has taken class of at-least one computer Science course at your school.

(b) There is a student who has taken class of all computer science courses at your school.

(c) Every student has taken class of at least one computer science course at your school.

(d) At least one computer science course at your school has been taken by every student.

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(e) All the classes of computer science courses at your school, has been taken by at least one student.

(f) Every student has taken classes of all computer science courses at your school.

Q:18

(a) P: Alice is a mathematics major.
Q: Alice is a Computer Science major
$P \rightarrow (P \vee Q)$
therefore, Addition rule of Inference is used.

(b) P: Jerry is a maths major.
Q: Jerry is a CS major.
$(P \wedge Q) \rightarrow P$
Therefore, Simplification rule of Inference is used.

(c) P: It is rainy , Q: The pool will be closed.
$(P \wedge (P \rightarrow Q)) \rightarrow Q$
Therefore, Modus Ponens rule of Inferenence is used.

(d) P: It snows today . Q: The university will close.
$(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$
Therefore. Modus Tollens Inference rule is used.

(e) P: I go swimming, Q: I will stay in the sun too long.
R: I will sunburn
$(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$
Therefore, Hypothetical syllogism rule of Inference is used.

Date:

Q: 19

(a) P: Today is tuesday.
Q: I have test in maths.
R: My Economics professor is sick.
S: I have a test in Economics.

(i) $P \to (Q \vee S)$
(ii) $R \to \neg S$
(iii) $Q \wedge R$

$\therefore Q$ (valid)

(b) P: Ali is a lawyer
Q: He is ambitious
R: Ali is an early riser
S: He doesnot like chocolates.

(i) $P \to Q$
(ii) $R \to \neg S$.
(iii) $Q \to R$

$P \to Q$	$P \to R$
$Q \to R$	$R \to \neg S$
$P \to R$ — (i)	$P \to \neg S$ — (ii)

$\therefore P \to \neg S$ = Then if Ali is a lawyer then he does not like chocolates. (valid).

Date:

Q: 20 $U=\{1,2,3,4,5,6,7,8\}$, $A=\{1,2,4,5\}$, $B=\{2,3,5,6\}$, $C=\{4,5,6,7\}$

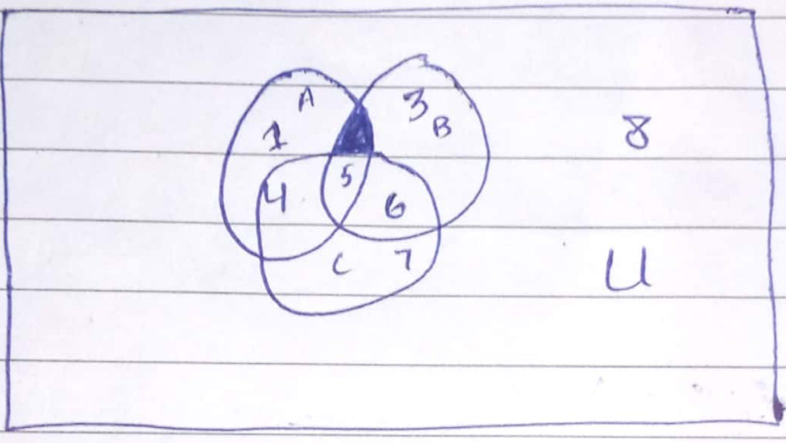
(a) $(A\cap B)\cap \bar{C}$

$A\cap B=\{1,2,4,5\}\cap\{2,3,5,6\}$

$A\cap B=\{2,5\}$

$(A\cap B)\cap \bar{C}=\{2,5\}\cap\{1,2,3,8\}$

$(A\cap B)\cap \bar{C}=\{2\}$

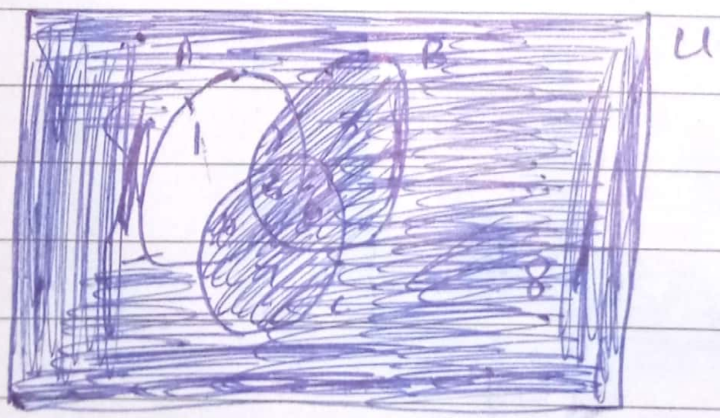


(2) $\bar{A}\cup(B\cup C)$

$B\cup C=\{2,3,4,5,6,7\}$

$\bar{A}\cup(B\cup C)=\{3,6,7,8\}\cup\{2,3,4,5,6,7\}$

$\bar{A}\cup(B\cup C)=\{2,3,4,5,6,7,8\}$



Date:

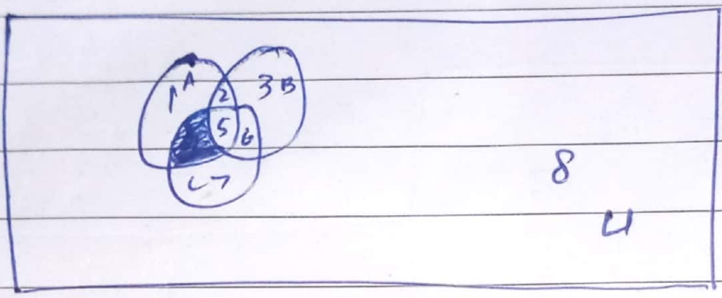
(c) $(A - B) \cap C$

$(A - B) = \{1, 2, 4, 5\} - \{2, 3, 5, 6\}$

$(A - B) = \{1, 4\}$

$(A - B) \cap C = \{1, 4\} \cap \{4, 5, 6, 7\}$

$(A - B) \cap C = \{4\}$



(d) $(A \cap \bar{B}) \cup \bar{C}$ -

$(A \cap \bar{B}) = \{1, 2, 4, 5\} \cap \{1, 4, 7, 8\}$

$(A \cap \bar{B}) = \{1, 4\}$

$(A \cap \bar{B}) \cup \bar{C} = \{1, 4\} \cup \{1, 2, 3, 8\}$

$(A \cap \bar{B}) \cup \bar{C} = \{1, 2, 3, 4, 8\}$

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Date:

Q:21

(a) $(A-(A\cap B))\cap(B-(A\cap B))=\phi$

Solution:-

$\because (A-B)=A\cap\bar{B}$

$=(A\cap\overline{(A\cap B)})\cap(B\cap\overline{(A\cap B)})=\phi$

$=(A\cap B)\cap(\overline{(A\cap B)}\cap\overline{(A\cap B)})=\phi$ $\quad\because$ Associative Law

$=(A\cap B)\cap(\overline{A\cap B})=\phi$ $\quad\because (A\cap A)=A$

$=\phi=\phi$ $\quad\because (A\cap\bar{A})=\phi$

Hence Proved.

(b) $(A-B)\cup(A\cap B)=A$

Solution:-

$(A\cap\bar{B})\cup(A\cap B)$ $\quad\because A-B=A\cap\bar{B}$

$A\cap(\bar{B}\cup B)$ $\quad\because$ Distributive law

$A\cap U$ $\quad\because$ Complement laws.

$A=A$ $\quad\because$ Identity

Hence Proved.

(c) $(A-B)-C=(A-C)-B$

Solution.

$A-B=A\cap\bar{B}$

$A\cap B\cap\bar{C}\equiv$ ~~$A\cap\bar{C}$~~ $A\cap\bar{C}\cap\bar{B}$

By associative law:

$A\cap\bar{C}\cap B\equiv A\cap\bar{C}\cap\bar{B}$

Proved.

Date:

(d) $\overline{(\bar{B} \cup (\bar{B} - A))} = B$

Sol:-

$= \overline{(\bar{B} \cup (\bar{B} \cap \bar{A})} = B$

$= (B \cap \overline{(\bar{B} \cap \bar{A})} = B$

$= (B \cap (B \cup A) = B$

$= \quad B = B$

Hence Proved.

Q: 22

(a)

Total Apples $= 100 \Rightarrow n(A)$

Apples worm $n(W) = 20$

Apples bruises $n(B) = 15$

Apples with both worms and bruises: $n(W \cap B) = 10$

Find apple with worms or bruises:

$n(W \cup B) = n(W) + n(B) - n(W \cap B) = \quad 20 + 15 - 10 = 25$

apples that can be sold:- $n(A) - n(W \cup B) = 100 - 25$

$\Rightarrow$ 75 Apples can be sold.

(b)

Total Students: $n(T) = 1000$

CS students: $n(CS) = 350$

Software students $n(SE) = 450$

Both CS and SE $= n(CS \cap SE) = 100$

Either of them $n(SE \cup CS) = n(CS) + n(SE) - n(CS - SE)$

$= 450 + 350 - 100 = 700$

Neither of them $n(T) - n(CS \cup SE) = 1000 - 700 = 300$.

Date:

(c)

78 Mixed Berry → MB
32 Irish cream → IC
57 Tiramisu → T
13 mixed Berry and Irish cream ⇒ MBIC
21 Irish cream and Tiramisu → ICT
16 Tiramisu and mixed Berry. → TMB
5 All flavours. → All
14 No any flavour → NO

Total = MB + IC + T − MBIC − ICT − TMB + All + NO
= 78 + 32 + 57 − 13 − 21 − 16 + 5 + 14

$\boxed{\text{Total} = 136}$

(d) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ Using set builder notation.

Solution:

$\{x:y;\ x \in A \wedge (y \in B \cap C)\}$

$\{x:y;\ x \in A \wedge (y \in B) \wedge (y \in C))\}$

$\{x:y: [(x \in A) \wedge (x \in A)] \wedge [(y \in B) \wedge (y \in C)]\}$.

$\{x:y: (x \in A) \wedge (y \in B) \wedge (x \in A) \wedge (y \in C)\}$

$\{x:y: (x \in A) \wedge (y \in B)\} \cap \{x:y: (x \in A) \wedge (y \in C)\}$

$= (A \times B) \cap (A \times C)$