Date: _____

Name : Ibadullah Shaikh
Roll No : 19k-0259
Section : BCS-3G
Course : Discrete Structures Assignment:02
Submitted to : Dr. Fahad Samad
Date : 27/10/2020
Day : Wednesday

Q:1:

(a) $P \rightarrow (P \vee Q)$ ∴ Addition

(b) $(P \wedge Q) \rightarrow P$ ∴ Simplification

(c) $(P \rightarrow Q) \rightarrow Q$ ∴ Modus Ponens

(d) $(P \rightarrow Q) \rightarrow \neg Q$ ∴ Modus Tollens

(e) $(P \rightarrow Q) \rightarrow (Q \rightarrow R)$

$\therefore P \rightarrow R$ ∴ Hypothetical Syllogism

Date:

Q:2

(a) P(n): I take the n day off.
Q(n): It rains on n day.
R(n): It snows on n day.

$P(n) \rightarrow (Q(n) \vee R(n))$

$P(Tues) \vee P(Thurs)$

$\neg (Q(Tues) \vee R(Tues))$

$\neg R(n) \rightarrow Thurs$

$P(Tues) \rightarrow (Q(tues) \vee R(tues))$

$P(Thurs) \rightarrow Q(Thurs) \vee R(Thurs)$

$\neg P(Thurs)$

$P(Thurs)$

$Q(thurs) \vee R(Thurs)$

$Q(Thurs)$

(b) P: I ate spicy foods
Q: I have strange dreams
R: There is thunder while I sleep

$P \rightarrow Q$ — (i)

$R \rightarrow Q$ — (ii)

Q — (iii)

i & iii $\Rightarrow \neg P$

ii & iii $\Rightarrow \neg R$

I didn't take spicy foods, There was no thunder while I sleep.

Date:

c) P: I am clever
Q: I am Lucky
R: I'll win the lottery

$P \vee Q$ — (1)
$\neg Q$ — (2)
$\neg P \rightarrow R$ — (3)

$\therefore$ I am clever

d) P(n): Every Computer Science Major.
Q(n): has a personal computer.

$\forall n (P(n) \rightarrow Q(n))$
$\neg Q(Ralph)$
$Q(Ann)$
$P(Ralph) \rightarrow Q(Ralph)$
$\neg Q(Ralph)$
$\neg P(Ralph)$
$\therefore$ Ralph is not a CS major.

Date:

(e) P(x): x is good for corporation
Q(x): x is good for US
R(x): x is good for me
S: you by lots of stuff

$P \to q$ -(i)

$q \to r$ -(ii)

$S \to p$ -(3)

~~$p \to r$ -(4)~~

=> from i and (ii)

$P \to r$ -(iv)

=> from (iii) and (iv)

$S \to r$

$S \to q$

∴ what is good for corporation is good for me.
∴ you buy lots of stuff is good for me.
∴ you buy lots of stuff is good for united states.

(f) P: is a radents
q: gnaw their feed

$P \to q$

P(mice)

$\neg P$ (Bats)

$\neg q$ (Rabbits)

P(Rabbits) $\to$ q(Rabbits)

$\neg q$ (Rabbits)

$\neg p$ (Rabbits)

P(mice) $\to$ q(mice)

P(mice)

∴ Rabbits are not q(mice)
∴ Mice gnaw their feed.

Date:

Q:3 $(p \wedge t) \rightarrow (R \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$ and $\neg s$

$\therefore$ conclusion:- $q \rightarrow r$

$(p \wedge t) \rightarrow (R \vee s)$ — i

$q \rightarrow (u \rightarrow t)$ — ii

$u \rightarrow p$ — iii

$\neg s$ — iv

(i) $q \rightarrow u$

(ii) $u \rightarrow p$ $\Rightarrow$ $q \rightarrow p$

$q \rightarrow p$ $\quad$ $p \rightarrow (R \vee s)$ — (i)

$q \rightarrow R \vee s$

$R \vee s$

$\neg s$ — iv

R

$q \rightarrow R$ proved.

Q:04

(a) let $P(x)$ = x has red convertible

$Q(x)$ = x has got a speeding ticket

$\forall x (P(x) \rightarrow Q(x))$

$P(\text{Linda})$

$\exists x (Q(x))$

$P(\text{Linda}) \rightarrow Q(\text{Linda})$ Modus Tonem.

$P(\text{Linda})$

$\therefore Q(\text{Linda})$

$\therefore$ Linda has got a speeding ticket

Date:

(b) $\forall x\,(P(x) \to Q(x))$

$\forall x\,(P(x))$ Modus Ponens

$\forall x\; Q(x)$

(c) $P(x)$: x movie is produced by John

$Q(x)$: x movie is wonderful

$\forall x\,(P(x) \to Q(x))$

P (coal Miners) Modus Ponens

Q (coal Miners)

(d) $P(x)$: x has been to France

$Q(x)$: x has visited Louvre

$\exists x\,(P(x))$

$\forall x\,(P(x) \to Q(x))$ Modus tonens

$\exists x\,(P(x))$

(f) $P \to (q \vee R)$

$S \to \neg R$

$P \wedge S \to S$

q

$P \to (q \vee R)$

$P \to q$

$\therefore P \to$ conclusion

(g) $P \to q$

$R \to \neg S$ $\Rightarrow$ $q \to \neg S$

$q \to R$

$P \to q$

$P \to \neg S$

$P \to \neg S$ proved

Date:

Q no 5:

(i) $(A - (A \cap B)) \cap (B - (A \cap B)) = \phi$

Sol:-

$(A \cap \overline{(A \cap B)}) \cap (B \cap \overline{(A \cap B)})$

$(A \cap (\bar{A} \cup \bar{B}) \cap (B \cap (\bar{A} \cup \bar{B}))$

$(A \cap \bar{A}) \cup (A \cap \bar{B}) \cap [(\bar{A} \cap B) \cap (B \cap \bar{B})]$

$(A \cap \bar{B}) \cap (\bar{A} \cap B)$

$(A \cap \bar{A}) \cap (B \cap \bar{B}) = \phi$

$\phi = \phi$ proved.

(ii) $(A - B) \cup (A \cap B) = A$

$(A \cap \bar{B}) \cup (A \cap B)$

$A \cup (\bar{B} \cap B)$

$A = A$ Proved.

(iii) $(A - B) - C = (A - C) - B$

$(A \cap \bar{B}) - C = (A \cap \bar{C}) \cap \bar{B}$

$(A \cap \bar{B}) \cap \bar{C} =$

$(A \cap \bar{C}) \cap \bar{B} = (A \cap \bar{C}) \cap \bar{B}$

Proved

Date:

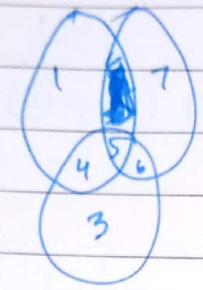
Q:6

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(a) $(A \cap B) = \{2, 5\}$

$\bar{E} = \{1, 2, 3, 8\}$

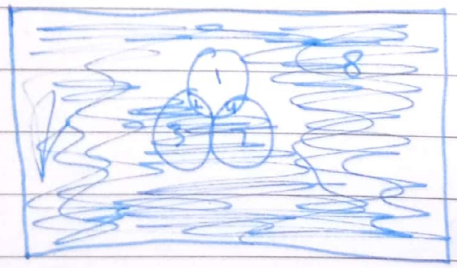
$= \{2\}$.



(b) $B \cup C = \{2, 3, 4, 5, 6, 7\}$

$\bar{A} = \{2, 3, 6, 7, 8\}$

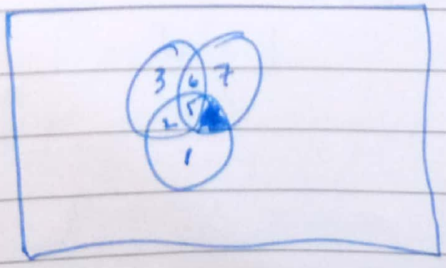
$\bar{A} \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8\}$



(c) $(A - B) \cap C$

$A - B = \{1, 4\}$

$(A - B) \cap C = \{4\}$.



Date:

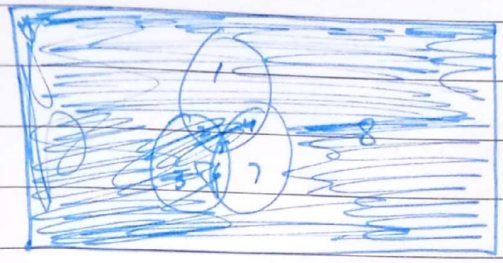
(d) $(A \cap \bar{B}) \cup \bar{C}$

$\bar{B} = \{1, 4, 7, 8\}$

$A \cap \bar{B} = \{1, 4\}$

$\bar{C} = \{1, 2, 3, 8\}$

$(A \cap \bar{B}) \cup \bar{C} = \{1, 2, 3, 4, 8\}$



Q:7

(a) Domain : {a, b, c, d}

Co-domain: {a, b, c, d}

Range: {a, b, c, d}

~~Subje~~ Surjective type

(b) Domain : {a, b, c, d}

Co-domain: {a, b, c, d}

Range: {b, c, d}

neither type

(c) Domain: {a, b, c, d}

Co-domain : {a, b, c, d}

Range : {d, b, c}

neither type

Date:

(d) Domain: $\{a, b, c, d\}$
Co-Domain: $\{a, b, c, d\}$
Range: $\{a, b, c, d\}$

Surjective type

Q: No 8

$f(a) = 2a + 3$
$g(a) = 3a + 2$

$f(g(a)) = 2(3a+2) + 3$
$= 6a + 7$

$g(f(a)) = 3(2a+3) + 2$
$= 6a + 11$

∴ Function f and g both are one to one, bijectiv and invertible at the same time.

Q: 9

(a) $f(b) = c$ $f(y) = n$
$g(a) = b$ $g(z) = y$

$f(g(z)) = f(g(z)) = f(y) = n$

fog function is onto.

Date:

(b) $f(x) = f(y)$

$g(x) = g(y)$

$fog(g(a)) = f(g(b))$

$g(a) = g(b)$

$a = b$

fog function is one to one.

(c) If fog is bijective the g is onto and surjective and f is only one to one or injective.

∴ one to one = injective.

Q:10

(a) It is onto for every valve of m and n.

(b) It is onto and one to one.

(c) It is only onto

(d) It is onto and function is many to one.