National University
of computer and emerging sciences

Foundation of Advancement
of Science and Technology

Divergence, Curl and Gradient

[Assignment # 02]

PRESENTED BY:

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COURSE NAME: APPLIED PHYSICS

COURSE INSTRUCTOR: MADAM RABIA

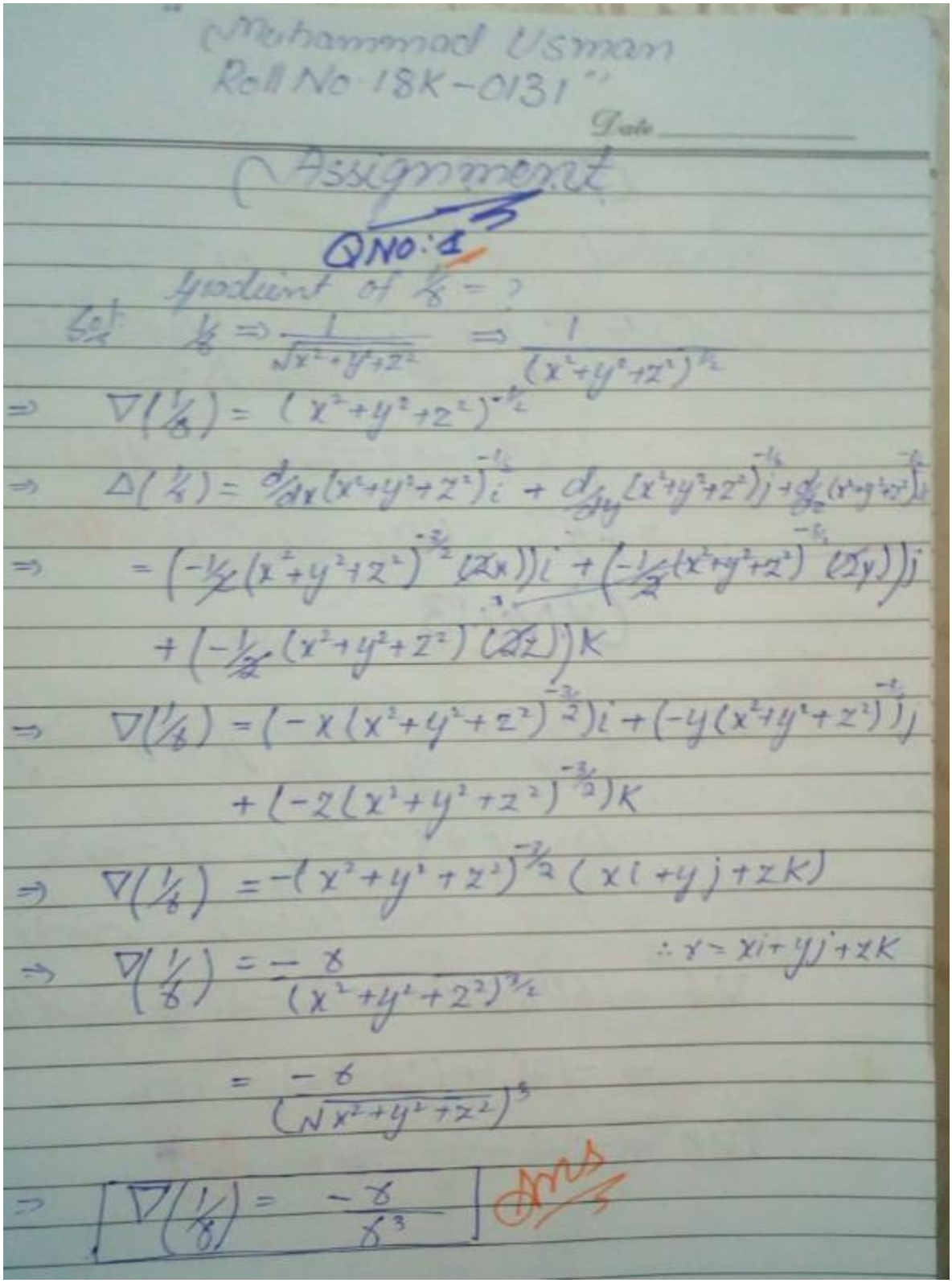
Questions

1. Find the gradient of $1/r$, where $\mathbf{r} = x\,\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $|r| = \Gamma x^2+y^2+z^2$
2. Find the gradient of $f(x,y,z) = xy^2 - yz$
3. If $f(x,y,z) = 3yx^2 - y^3z^{2,}$ find gradient of f at the point (1, -2, -1).
4. Find the divergence of $G(x,y,z) = 3x^2\boldsymbol{i} + 2zy\boldsymbol{j}$.
5. Find the divergence of $\mathbf{r}/r^3$ where, $r = |r|$ and $\mathbf{r} = x\,\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$.
6. Find the curl of $F(x,y,z) = x^2\boldsymbol{i} + xyz\boldsymbol{j} + z\boldsymbol{k}$ at the point (2, 1, -2).
7. Find the divergence of
   (a) $F = y^3\boldsymbol{i} + xyj$ (b) $F = 3x^2\boldsymbol{i} - 6xy\boldsymbol{j}$ (c) $G = x^2\,\boldsymbol{i} + 2z\boldsymbol{j} + y\boldsymbol{k}$
   (d) $G = (4y/x^2)\,\boldsymbol{i} + (Siny)\boldsymbol{j} + 3\boldsymbol{k}$
8. Find the curl of (a)$F = x\,\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ (b) $F = y^3\,\boldsymbol{i} + xy\boldsymbol{j} - z\boldsymbol{k}$ (c) $F = x^2\,\boldsymbol{i} + 2z\boldsymbol{j} - y\boldsymbol{k}$

Ans :

1. $-\mathbf{r}/r^3$
2. $y^2i + (2xy - z)j - yk$
3. $-12i - 9j - 16k$
4. $6x + 2z$
5. Zero
6. $-2i - 2k$
7. (a) x (b) 0 (c) 2x (d) $-8y/x^3 + Cosy$
8. (a) 0 (b) $(y-3y^2)k$ (c) $-3i$

Solutions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Assignment
QNO: 1
Gradient of 1/r = ?
Let 1/r ⇒ 1/√(x²+y²+z²) ⇒ 1/(x²+y²+z²)^½
∇(1/r) = (x²+y²+z²)^-½
∴ r = xi + yj + zk
∇(1/r) = -r/r³
∇(1/r) = -r/(x²+y²+z²)^3/2
= -r/(√(x²+y²+z²))³
∇(1/r) = -r/r³

Date

QNO: 2

Sol: Gradient $f(x,y,z) = xy^2 - yz$

$$\nabla(f) = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)(xy^2 - yz)$$

$$= y^2 i + (2xy - z)j + (-y)k$$

$$= \boxed{y^2 i + (2xy - z)j - yk}$$ Ans

QNO: 3

Sol:

$$\nabla \cdot f \Rightarrow \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)(3x^2y - y^3z^2)$$

$$\Rightarrow (6xy)i + (3x^2 - 3y^2z^2)j - 2y^3z$$

$\Rightarrow$ Put Point $(x,y,z) = (1,-2,-1)$

$$\nabla \cdot f \Rightarrow (6(1)(-2))i + (3(1)^2 - 3(-2)^2(-1))j - 2(-2)^3(-1)k$$

$$\Rightarrow -12i + (3 - 12)j - 16k$$

$$\boxed{\nabla f \Rightarrow -12i - 9j - 16k}$$ Ans

Date

Q NO: 4

Divergence of $G(x,y,z) = 3x^2 i + 2yx j$

Sol:

$$\nabla \cdot G = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)\left(3x^2 i + 2yx j\right)$$

$$\boxed{\nabla \cdot G = (6x + 2z)} \quad \text{Ans}$$

Q NO: 5

Divergence of $\frac{r}{r^3}$

Sol:

$$\Rightarrow \nabla \cdot \frac{r}{r^3} = \frac{x\,i}{(\sqrt{x^2+y^2+z^2})^3} + \frac{y\,j}{(\sqrt{x^2+y^2+z^2})^3} + \frac{z\,k}{(\sqrt{x^2+y^2+z^2})^3}$$

$$\Rightarrow \nabla \cdot \frac{r}{r^3} = \frac{\partial}{\partial x} x.(x^2+y^2+z^2)^{-\frac{3}{2}} + \frac{\partial}{\partial y}(x^2+y^2+z^2)^{-\frac{3}{2}} + \frac{\partial}{\partial z}(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\Rightarrow = x\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2x) + (x^2+y^2+z^2)^{-\frac{3}{2}}$$
$$y\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2y) + (x^2+y^2+z^2)^{-\frac{3}{2}}$$
$$z\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2z) + (x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\Rightarrow \nabla \cdot \left(\frac{r}{r^3}\right) = -3(x^2+y^2+z^2)^{-\frac{5}{2}}(x^2+y^2+z^2) + 3(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$= -3r^{-5} \cdot r^2 + 3r^{-3}$$

$$= -3r^{-3} + 3r^{-3}$$

$$\Rightarrow \boxed{\nabla \cdot \frac{r}{r^3} = 0}$$

Q NO: 6

Find Curl of $F(x,y,z) = x^2 i + xyzj + zk$ at Point $(2, 1, -2)$

Sol:

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xyz & z \end{vmatrix}$$

$$i(0 - xy) - j(0) + k(yz)$$

$$-xyi + yzk$$

Point $= (2, 1, -2)$

$$(-2)(1)i + (1)(-2)k$$

$$-2i - 2k$$

Q NO: 7

Date

Find the Divergence of:

a) $F = y^3 i + xy\, j$

$$\nabla . F = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(y^3 i + xy\, j)$$

$$\Rightarrow 0 + x + 0$$

$$\boxed{\nabla . F = x}$$

b) $F = 3x^2 i - 6xy\, j$

$$\nabla . F = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(3x^2 i - 6xy\, j)$$

$$= 6x - 6x + 0$$

$$\boxed{\nabla . F = 0}$$

c) $G = x^2 i + 2z\, j + y\, k$

$$\nabla . G = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(x^2 i + 2z\, j + y\, k)$$

$$\Rightarrow 2x + 0 + 0$$

$$\boxed{\nabla . G \Rightarrow 2x}$$

Nice

Date

QNO: 7

a) $G = \left(\frac{4y}{x^2}\right)i + (\sin y)j + 3k$

Sol:

$$\nabla \cdot G = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\left(\frac{4y}{x^2}i + \sin y\, j\right)$$

$$= 4y\left(\frac{-2}{x^3}\right) + \cos y$$

$$\boxed{\nabla \cdot G = -\frac{8y}{x^3} + \cos y}$$

Q NO: 8

Date

Find the Curl

a) $F = xi + yj + zk$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= i(0-0) - j(0) + k(0)$$

$$\boxed{\nabla \times F = 0}$$

b) $F = y^3 i + xy j - z k$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3 & xy & -z \end{vmatrix}$$

$$= i(0-0) - j(0) + (y - 3y^2)k$$

$$\boxed{\nabla \times F = (y - 3y^2)k}$$

c) $F = x^2 i + 2z j - y k$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2z & -y \end{vmatrix}$$

$$= i(-1-2) - j(0) + k(0)$$

$$\boxed{\nabla \times F = -3i}$$