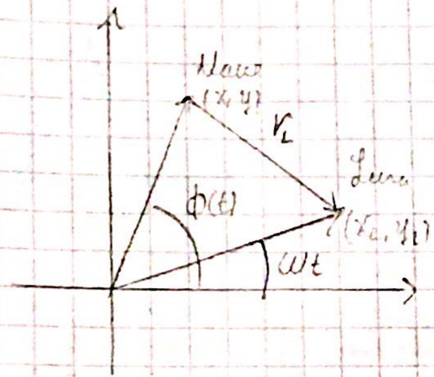
Calculo de trayectoria de nave de exploración lunar.

c.) Muestre que la distancia Nave Luna está dado por:

$$r_L(r,\phi,t) = \sqrt{r(t)^2 + d^2 - 2r(t)\cdot d\cdot \cos(\phi - \omega t)}$$



$$r_L = \sqrt{(x_L - x)^2 + (y_L - y)^2}$$

$$r_L = \sqrt{x_L^2 - 2x_L x + x^2 + y_L^2 - 2yy_L + y^2}$$

$$r_L = \sqrt{\underbrace{(x_L^2 + y_L^2)}_{d^2} + \underbrace{(x^2 + y^2)}_{r(t)^2} - 2(x_L x + y y_L)}$$

d^2: distancia tierra luna

$r(t)^2$: distancia tierra cohete

$x = r(t)\cdot\cos(\phi(t))$,

$y = r(t)\cdot\sin(\phi(t))$,

$x_L = d\cdot\cos(\omega t)$

$y_L = d\cdot\sin(\omega t)$.

$$\Rightarrow r_L = \sqrt{r(t)^2 + d^2 - 2\left(d\,r(t)\cdot\cos(\phi(t))\cos(\omega t) + d\,r(t)\cdot\sin(\phi(t))\cdot\sin(\omega t)\right)}$$

$$r_L = \sqrt{r(t)^2 + d^2 - 2d\,r(t)\cdot\left(\cos(\phi(t))\cdot\cos(\omega t) + \sin(\phi(t))\cdot\sin(\omega t)\right)}$$

$$\therefore\ r_L = \sqrt{r(t)^2 + d^2 - 2d\,r(t)\cdot\cos(\phi(t) - \omega t)}\ \square$$

d.) Encuentre el Hamiltoniano de la nave.

$\mathcal{L} = K - U$ | $K \rightarrow$ Energía Cinética ; $U \rightarrow$ Energía Potencial

$\hookrightarrow$ Lagrangiano.

Momento lineal   Momento angular

$\Rightarrow H = P_r \cdot \dot{r} + P_\phi \cdot \dot{\phi} - \mathcal{L}$ | $P_r = m \cdot \dot{r}$, $P_\phi = m \cdot r^2 \cdot \dot{\phi}$

$$P_r \cdot \dot{r} = (m \cdot \dot{r}) \cdot \dot{r} = m \cdot \dot{r}^2 = \frac{m^2 \cdot \dot{r}^2}{m} = \frac{P_r^2}{m}$$

$$P_\phi \cdot \dot{\phi} = (m r^2 \dot{\phi}) \cdot \dot{\phi} = m r^2 \dot{\phi}^2 = \frac{m^2 r^4 \dot{\phi}^2}{m r^2} = \frac{P_\phi^2}{m r^2}$$

$K = \frac{1}{2} m \cdot (\dot{x}^2 + \dot{y}^2)$ | $x = r \cdot \operatorname{Cos}(\phi(t))$; $y = r \cdot \operatorname{Sin}(\phi(t))$.

$\dot{x} = \dot{r} \cdot \operatorname{Cos}(\phi(t)) - r \cdot \operatorname{Sin}(\phi(t)) \cdot \dot{\phi}(t)$; $\dot{y} = \dot{r} \cdot \operatorname{Sin}(\phi(t)) + r \cdot \operatorname{Cos}(\phi(t)) \cdot \dot{\phi}(t)$.

$$\dot{x}^2 = \dot{r}^2 \operatorname{Cos}^2(\phi(t)) - 2 \dot{r} r \cdot \operatorname{Sin}(\phi(t)) \cdot \operatorname{Cos}(\phi(t)) \cdot \dot{\phi}(t) + r^2 \cdot \operatorname{Sin}^2(\phi(t)) \cdot \dot{\phi}(t)^2$$

$$\dot{y}^2 = \dot{r}^2 \operatorname{Sin}^2(\phi(t)) + 2 \dot{r} r \cdot \operatorname{Sin}(\phi(t)) \operatorname{Cos}(\phi(t)) \cdot \dot{\phi}(t) + r^2 \cdot \operatorname{Cos}^2(\phi(t)) \cdot \dot{\phi}(t)^2 \quad +$$

$$\dot{x}^2 + \dot{y}^2 = \dot{r}^2 (\operatorname{Sin}^2(\phi(t)) + \operatorname{Cos}^2(\phi(t))) + r^2 \cdot \dot{\phi}(t)^2 (\operatorname{Sin}^2(\phi(t)) + \operatorname{Cos}^2(\phi(t)))$$

$$= \dot{r}^2 + r^2 \cdot \dot{\phi}(t)^2$$

$$\Rightarrow K = \frac{1}{2} m \cdot \dot{r}^2 + \frac{1}{2} r^2 \cdot \dot{\phi}(t)^2 = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2 m r^2}$$

$U = U_T + U_L$ | U_T = Potencial respecto a la tierra

U_L = Potencial respecto a la luna

$$\Rightarrow U = -G \cdot \frac{m \cdot M_T}{r} - G \cdot \frac{m \cdot M_L}{r_L(r, \phi, t)}$$

$$\Rightarrow H = \frac{P_r^2}{m} + \frac{P_\phi^2}{mr^2} - \frac{P_r^2}{2m} - \frac{P_\phi^2}{2mr^2} - G\cdot\frac{m M_T}{r} - G\cdot\frac{m\cdot M_L}{r_L(r,\phi,t)}$$

$$\therefore\ H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - G\cdot\frac{m M_T}{r} - G\cdot\frac{m\cdot M_L}{r_L(r,\phi,t)} \ \square$$

e) Deduzca las ecuaciones del movimiento a partir del Hamiltoniano.

$$\dot r = \frac{\partial H}{\partial P_r} = \frac{2P_r}{2m} = \frac{P_r}{m},$$

$$\dot\phi = \frac{\partial H}{\partial P_\phi} = \frac{2\cdot P_\phi}{2mr^2} = \frac{P_\phi}{mr^2},$$

$$\dot P_r = -\frac{\partial H}{\partial r} = -\left(\frac{-P_\phi^2}{mr^3} + G\cdot\frac{m M_T}{r^2} + \frac{G\cdot m\cdot M_L}{r_L(r,\phi,t)^2}\cdot\frac{\partial r_L}{\partial r}\right)$$

$$\frac{\partial r_L}{\partial r} = \frac{\partial}{\partial r}\left[\sqrt{r(t)^2+d^2-2d\cdot r(t)\cdot Cos(\phi(t)-\omega t)}\right]$$

$$\frac{\partial r_L}{\partial r} = \frac{2(r(t)-d\cdot Cos(\phi(t)-\omega t))}{2\sqrt{r(t)^2+d^2-2d\, r(t)\cdot Cos(\phi(t)-\omega t)}} = \frac{r(t)-d\cdot Cos(\phi(t)-\omega t)}{r_L}$$

$$\Longrightarrow \dot P_r = -\frac{\partial H}{\partial r} = \frac{P_\phi^2}{mr^3} - \frac{G\cdot m M_T}{r^2} - \frac{G\, m M_L}{r_L(r,\phi,t)^3}\cdot\left[r(t)-d\cdot Cos(\phi(t)-\omega t)\right] \ \square$$

$$\dot P_\phi = -\frac{\partial H}{\partial\phi} = +\left(\frac{G\cdot m M_L}{r_L(r,\phi,t)^2}\cdot\frac{\partial r_L}{\partial\phi}\right).$$

$$\frac{\partial r_L}{\partial\phi} = \frac{\partial}{\partial\phi}\left[\sqrt{r(t)^2+d^2+2d\, r(t)\cdot Cos(\phi(t)-\omega t)}\right]$$

$$= \frac{-2d\cdot r(t)\cdot Sin(\phi-\omega t)}{2\sqrt{r(t)^2+d^2+2d r(t)\cdot Cos(\phi(t)-\omega t)}} = \frac{d\cdot r\cdot Sin(\phi-\omega t)}{r_L\cdot(\phi,r,t)}$$

$$\Rightarrow \dot P_\phi = \frac{-\partial H}{\partial\phi} = \frac{-G\cdot m M_L}{r_L(r,\phi,t)^3}\cdot\left[d\cdot r\cdot Sin(\phi-\omega t)\right] \ \square$$

5.) Deduzca las ecuaciones de movimiento con el siguiente cambio de variables:

$$\tilde{r} = \frac{r}{d}, \quad \phi = \phi, \quad \tilde{P}_r = P_r/md, \quad \tilde{P}_\phi = P_\phi/md^2$$

$$\dot{\tilde{r}} = \frac{d}{dt}\left(\frac{r}{d}\right) = \frac{\dot{r}}{d} = \frac{P_r}{md} = \tilde{P}_r$$

$$\dot{\phi} = \frac{P_\phi}{mr^2} = \frac{\tilde{P}_\phi \, md^2}{mr^2} = \frac{\tilde{P}_\phi}{\tilde{r}^2}$$

$$\dot{\tilde{P}}_r = \frac{d}{dt}\left(\frac{P_r}{md}\right) = \frac{1}{md}\dot{P}_r = \frac{1}{md}\left(\frac{P_\phi^2}{mr^3} - \frac{GmM_1}{r^2} - \frac{GmM_2}{r_2^3}\left(r - d\cos(\phi - \omega t)\right)\right)$$

$$\checkmark \ \frac{1}{md}\frac{P_\phi^2}{mr^3} = \frac{P_\phi^2}{m^2 d r^3} = \frac{\tilde{P}_\phi^2 \cancel{m^2} d^4}{\cancel{m^2} d r^3} = \tilde{P}_\phi^2 \frac{d^3}{r^3} = \frac{\tilde{P}_\phi^2}{\tilde{r}^3}$$

$$\checkmark \left(\frac{-GmM_1}{r^2} - \frac{GmM_2}{r_2^3}\left(r - d\cos(\phi - \omega t)\right)\right)\cdot\frac{1}{md}$$

$$= \frac{-\Delta}{md}\left(\frac{md^3}{r^2} + \frac{m\mu d^3}{r_2^3}\left(r - d\cos(\phi - \omega t)\right)\right)$$

$$= -\Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu d^2}{r_2^3}\left(r - d\cos(\phi - \omega t)\right)\right)$$

$$= -\Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu d^3}{r_2^3}\left(\tilde{r} - \cos(\phi - \omega t)\right)\right) \Big\} \, \alpha$$

$$¿\, r_1^3/d^3 \,? = \left(\frac{1}{d}\sqrt{r^2 + d^2 - 2dr\cos(\phi - \omega t)}\right)$$

$$= \sqrt{\tilde{r}^2 + 1 - 2\tilde{r}\cos(\phi - \omega t)} = \tilde{r}'$$

$$\Rightarrow \alpha = -\Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}\left(\tilde{r} - \cos(\phi - \omega t)\right)\right)$$

$$\Rightarrow \dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \Delta\cdot\left(\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}\left(\tilde{r} - \cos(\phi - \omega t)\right)\right)$$

$$\dot{\tilde{P}}_\phi = \frac{d}{dt}\left(\frac{P_\phi}{md^2}\right) = \frac{1}{md^2}\cdot\dot{P}_\phi = \frac{-GmM_L}{md^2 r_L^3}\left(d\,r\sin(\phi - \omega t)\right)$$

$$= \frac{-GM_L}{d^2 r_L^3}\cdot d\,r\cdot\sin(\phi - \omega t) = \frac{-G\cdot M_L}{r_L^3}\cdot\tilde{r}\cdot\sin(\phi - \omega t)$$

$$= -\Delta\ \mu\cdot\frac{d^3}{r_L^3}\cdot\tilde{r}\cdot\sin(\phi - \omega t) = -\frac{\Delta\mu\tilde{r}}{\tilde{r}_L'^3}\cdot\sin(\phi - \omega t)\ \square$$

g.) Antes de simular el vuelo, encuentre expresiones para los momentos canónicos iniciales.

$$\tilde{P}_r^0 = \frac{P_r^0}{md} = \frac{1}{d}\cdot\frac{dr}{dt} = \frac{1}{d}\cdot\frac{d}{dt}\left[\sqrt{x^2+y^2}\right] = \frac{2x\cdot\dot{x} + 2y\cdot\dot{y}}{2rd}$$

$$= \frac{x\cdot\dot{x} + y\cdot\dot{y}}{rd} = \frac{x\cdot v_0\cos\theta + y\cdot v_0\sin\theta}{r\cdot d}$$

$$= \frac{v_0\cdot r\cos\phi\cdot\cos\theta + v_0\, r\sin\phi\cdot\sin\theta}{rd} = \frac{v_0}{d}\cdot\cos(\theta - \phi) = \tilde{v}_0\cdot\cos(\theta - \phi).$$

$$\tilde{P}_\phi^0 = \frac{P_\phi}{md^2} = \frac{mr^2\dot{\phi}}{md^2} = \tilde{r}^2\cdot\frac{d}{dt}\arctan\left(\frac{y}{x}\right) = \frac{\tilde{r}^2}{1+(y^2/x^2)}\cdot\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$= \left(\frac{\dot{y}\cdot x - y\,\dot{x}}{x^2}\right)\cdot\frac{x^2\cdot\tilde{r}^2}{x^2+y^2} = \frac{\tilde{r}^2}{r^2}\left(\dot{y}x - y\dot{x}\right)$$

$$= \frac{\tilde{r}^2}{r^2}\left(r \cdot \cos(\phi) \cdot v_0 \cdot \sin(\theta) - r \sin(\phi) \cdot v_0 \cdot \cos\theta \right)$$

$$= \frac{\tilde{r}^2}{r} \cdot \sin(\theta - \phi) \cdot v_0 = \frac{\tilde{r}^2 v_0}{d \cdot \tilde{r}} \sin(\theta - \phi) = \tilde{r}_0 \cdot \tilde{v}_0 \cdot \sin(\theta - \phi) \quad \square$$