Statistical Computing HW 2

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```
set.seed(1234)
```

Problem 3.10)

Question

Use simulation to approximate $cov(U, e^U)$ where U is uniform on $(0, 1)$. Compare your approximation with the exact answer.

Answer

1) Derivation

Using the definition of covariance, we have:

$$cov(U, e^U) = E(Ue^U) - E(U)E(e^U)$$

No other derivation is needed.

2) Algorithm

1) Generate u from $U(0, 1)$
2) Evaluate Ue^U, U, and e^U as three separate values, store values
3) Repeat (1) and (2) 10,000 times
4) Find the average of each of the the 10,000 values, then find $E(Ue^U) - E(U)E(e^U)$

3) Simulation

```
# Initialize dataframe, 3 columns and 10,000 rows
df <- data.frame(UeU=rep(0,10000),
                 U=rep(0,10000),
                 eU=rep(0,10000))
# For loop to calcuate Ue^U, U, and e^U, and store values
for(i in 1:10000){
  U <- runif(1)
  df[i,1] <- U*exp(U)
  df[i,2] <- U
  df[i,3] <- exp(U)
}

# Find column means, then perform final calculation
covariance <- mean(df$UeU) - mean(df$U)*mean(df$eU)
covariance
```

```
## [1] 0.1392725
```

The estimated covariance is 0.1392725.

4) Analytical Result

The exact value of the covariance is:

$$
\begin{aligned}
cov(U, e^U) &= E(Ue^U) - E(U)E(e^U) \\
&= \int_0^1 Ue^U \, du - \left(\int_0^1 U \, du \right) \left(\int_0^1 e^U \, du \right) \\
&= \left[Ue^U - e^U \right]_0^1 - \left[\frac{1}{2} U^2 \right]_0^1 * \left[e^U \right]_0^1 \\
&= (e - e) - (0 - 1) - (1/2 - 0)(e - 1) \\
&= (3/2) - e/2 \\
&\approx 0.1408591
\end{aligned}
$$