Statistical Computing HW 4

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Problem 4.3)

Give an efficient algorithm to simulate the value of a random variable X such that

$$\begin{aligned} P\{X = 1\} &= 0.3 \\ P\{X = 2\} &= 0.2 \\ P\{X = 3\} &= 0.35 \\ P\{X = 4\} &= 0.15 \end{aligned}$$

Answer

Derivation

The CDF of the distribution is:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.3 & 1 \leq x < 2 \\ 0.5 & 2 \leq x < 3 \\ 0.85 & 3 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

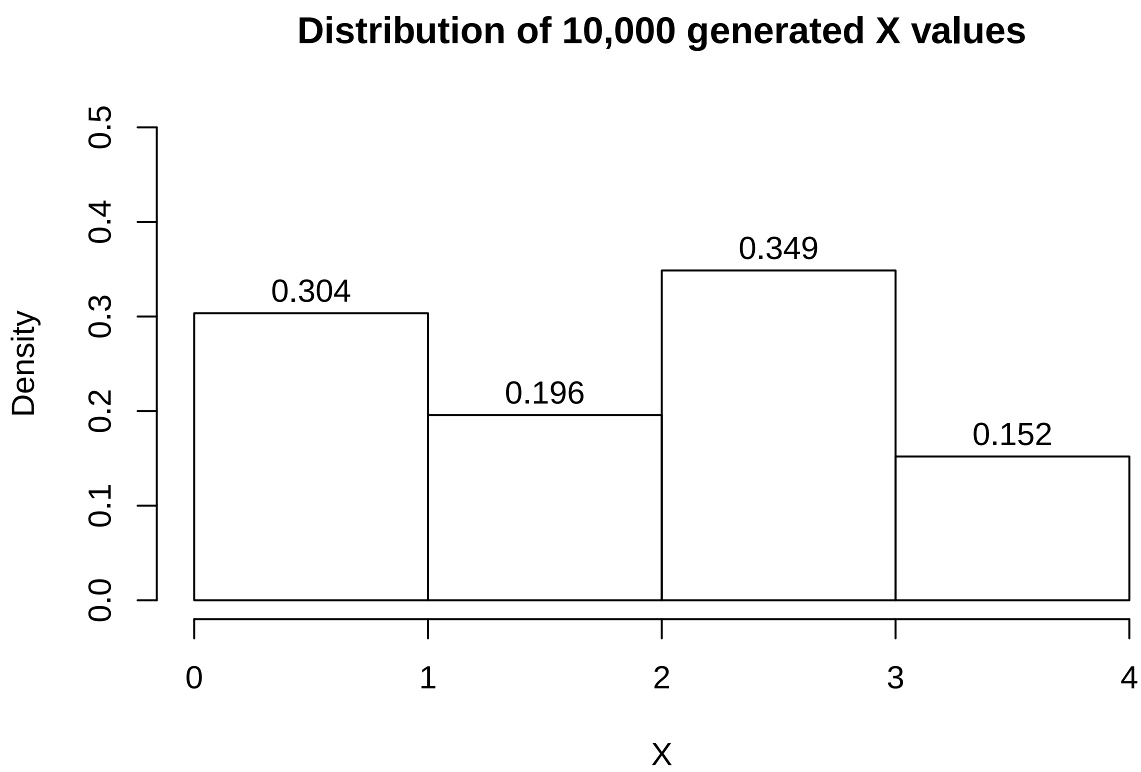
The can be used along with a standard uniform random variable to generate values of the random variable X.

Algorithm

1. Generate a number u from the standard uniform distribution
2. Set $x = F^{-1}(u) + 1$
3. Repeat n times

Program

Below is a histogram using the derivation and algorithm. 10,000 values were generated. The distribution is quite good.



Problem 4.14a)

Let X be a binomial random variable with parameters n and p. Suppose that we want to generate a random variable Y whose probability mass function is the same as the conditional mass function of X given that $X \geq k$, for some $k \leq n$. Let $\alpha = P\{X \geq k\}$ and suppose that the value of α has been computed. Give the inverse transform method for generating Y.

Answer

Derivation

Algorithm

Program

Analytical Result

Problem 4.15)

Give a method for simulating X, having the probability mass function $p_j,\ j = 5, 6, 7, ..., 14$, where

$$p_j = \begin{cases} 0.11 & \text{when } j \text{ is odd and } 5 \leq j \leq 13 \\ 0.09 & \text{when } j \text{ is even and } 6 \leq j \leq 14 \end{cases}$$

Use the text's random number sequence to generate X.

Answer

Derivation

Algorithm

Program

Analytical Result

Problem 5.10)

A casualty insurance company has 1000 policyholders, each of whom will independently present a claim in the next month with probability 0.05. Assuming that the amounts of the claims made are independent exponential random variables with mean \$800, use simulation to estimate the probability that the sum of these claims exceeds \$50,000.

Answer

Derivation

In order to generate random values from an exponential distribution with mean 800, we'll use the fact that the CDF of this distribution is

$$F(x) = 1 - e^{-x/800}$$

Given that $F(x)$ has a standard uniform distribution, then:

$$\begin{aligned} u &= 1 - e^{-x/800} \\ e^{-x/800} &= 1 - u \\ \frac{-x}{800} &= ln(1-u) \\ x &= (-800)ln(u) \end{aligned}$$

Note that the random variables U and $1-U$ have the same distribution if U is standard uniform.

Also, since each policyholder has a 5% chance of making a claim, then the number of claims made out of 1000 has a binomial distribution with $n = 1000$ and $p = 0.05$. The random variable we are interested in is then

$$S = X_1 + X_2 + X_3 + ... + X_N$$

where the X_i are *iid* exponential with mean 800 and N is binomial with $n = 1000$ and $p = 0.05$. For simplicity in the program, values from the binomial distribution will be done using an R function, but the exponential values will be generated using values from a uniform distribution according to the derivation above.

Algorithm

- For each trial:
  - Generate a value n from Binomial(1000, 0.05)
  - Generate n values $u_1, u_2, ..., u_n$ from U(0,1)

    - Calculate each of the n claim amounts x by evaluating $x_i = (-800)ln(u_i)$
    - Find the sum of the n claims for that trial

- After the sum of each trial is calculated, then determine how many are greater than $50,000

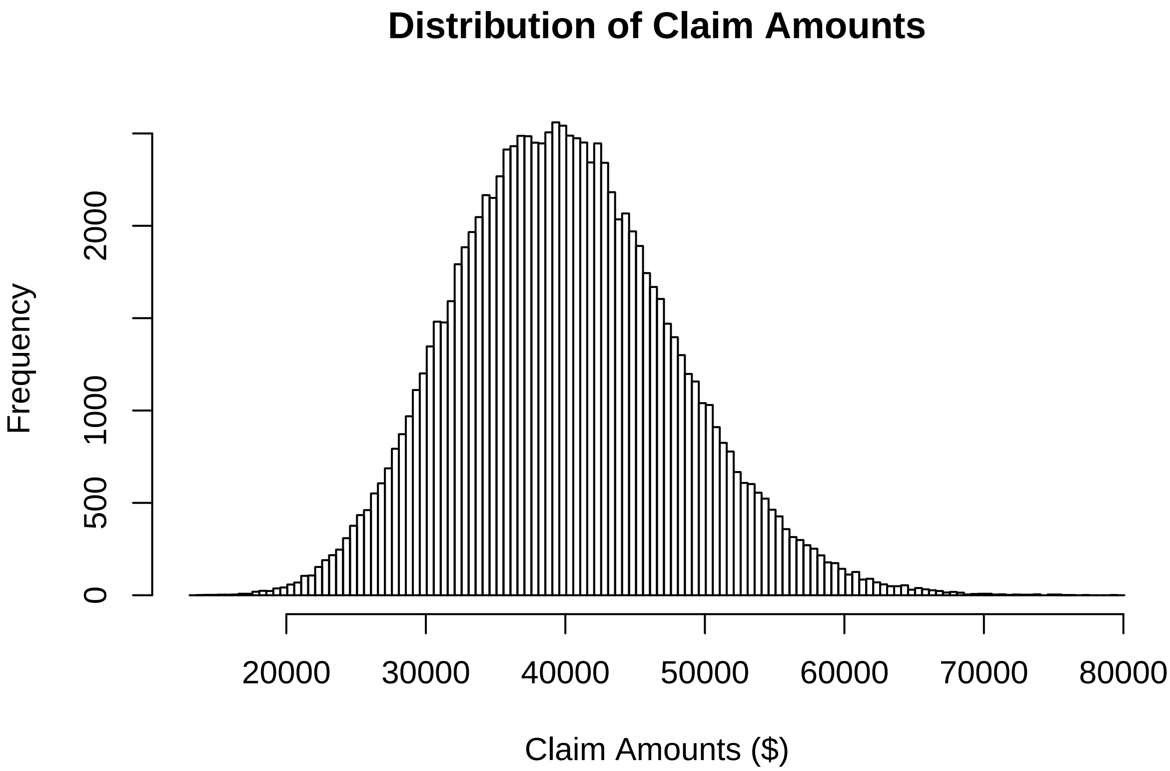
The R code appears different than this algorithm, but in practice this is what is happening. The R language benefits from avoiding explicitly writing *for* loops, and instead using other methods like *apply*, *lapply*, and *sapply*.

Program

The program uses the *generate_claims()* function to generate n claims from an exponential distribution, where n has a binomial distribution. 100,000 trials are run, and then the sum of the claims from each trial is calculated.

```
# Set seed, set N = number of simulations
set.seed(1234)
N = 10^5
# Returns variable number claims from exponential distribution
generate_claims <- function(vector, holders = 1000, chance = 0.05, beta = 800){
  # Number of claims that will occur is binomial(holders, chance)
  num_of_claims <- rbinom(1, holders, chance)
  # Generate claim amounts from exponential distribution for each claimant
  claims <- -beta * log(runif(num_of_claims))
  # Return vector of claims
  return(claims)
}
# Initialize claims list
claims <- rep(list(rep(0,100)), N)
# Simulate N times
claims <- lapply(claims, generate_claims)
# Find the sum for each trial
distribution <- sapply(claims, sum)
```

Below is a histogram of the results. The distribution is a sum of exponential random variables and thus is a continuous distribution. It appears to be slightly right tailed with a mean around $40,000.



The following R command returns the estimated probability that the sum of the claims is above $50,000:

```
mean(distribution>50000)
```

```
## [1] 0.10661
```

Out of 100,000 trials, approximately 0.10661 trials resulted in an outcome where the total value of the claims exceeded $50,000.

Analytical Result

find out how to do this analytically