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Leeture 2

Finish Appendix section 5. (if needed)

Appendix section 6

Solving equations:

$3x - 10 = 2 \rightarrow 3x = 12$

$x = 4$

$x^3 - 4x = 0 \rightarrow x(x^2 - 4) = 0$

$x(x-2)(x+2) = 0$

$x = 0, 2, -2$

Be careful with dividing by zero!

$x(x^2 - 4) = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2, -2$

Missing Solution

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Solving an equation that has an absolute value.

$|4x+3| = 15 \rightarrow 4x+3 = 15 \rightarrow x = 3$

$4x+3 = -15 \rightarrow x = \frac{-9}{2}$

$|2x+1| - 3 = 4 \rightarrow 2x+1 = 7 \rightarrow x = 3$

$2x+1 = -7 \rightarrow x = -4$

Factoring quadratic equations

$x^2 - 6x + 8 = 0 \qquad (x-4)(x-2) = 0$

$x = 4, 2$

$2x^2 - x - 3 = 0 \rightarrow (2x \quad)(x \quad) = 0$

$(2x - 3)(x + 1) = 0$

$x = -1, 3/2$

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Completing the square:

$x^2 - 4x + 9 = 0$ Can't factor.

$x^2 - 4x = +9$

$x^2 - 4x + 4 = +9 + 4$

$(x-2)^2 = 13$

$x - 2 = \pm\sqrt{13}$

$x = 2 \pm \sqrt{13}$

try on wolfram

$2x^2 - 6x - 10 = 0$

$2(x^2 - 3x - 5) = 0$

$x^2 - 3x - 5 = 0$

$x^2 - 3x = 5$

$x^2 - 3x + \frac{9}{4} = 5 + \frac{9}{4}$

$\Rightarrow (x - 3/2)^2 = \frac{29}{4}$

$x = 3/2 \pm \sqrt{29/4}$

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Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

always works!

$x^2 + 4 = 0$

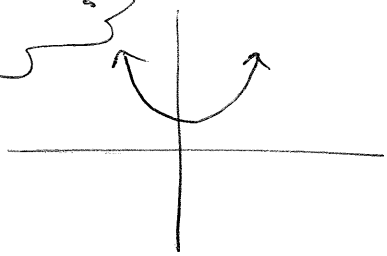
$a = 1$
$c = 4$
$b = 0$

$$x = \frac{-0 \pm \sqrt{0 - 4(1)(4)}}{2}$$

$$x = \frac{\pm \sqrt{-16}}{2}$$

no real answer.

Why? what does this mean?



$x^2 - 4 = 0$

$a = 1$
$b = 0$
$c = -4$

$$x = \frac{-0 \pm \sqrt{0 - 4(1)(-4)}}{2}$$

$$x = \frac{\pm \sqrt{16}}{2} = \frac{\pm 4}{2} = 2, -2$$

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Appendix section 7

Imaginary numbers:

$i^2 = -1$

Real numbers on a line

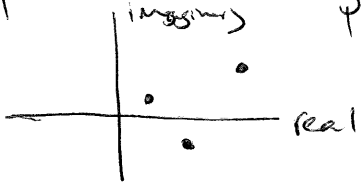
Complex numbers

$4 + 3i$

$1 + 2i$

$3 - i$

Complex numbers on a plane.



Adding Complex numbers

$$(4 + 3i) + (3 + 2i) = 7 + 5i$$

Subtracting Complex numbers

$$(4 + 3i) - (3 + 2i) = 1 + i$$

Multiplying Complex numbers (foil)

$$(4 + 3i)(3 + 2i) = 12 + 8i + 9i + 6i^2$$

$$= 12 + 17i - 6$$

$$= 6 + 17i$$

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The <u>Conjugate</u> of a Complex number.

→ Change the sign of the imaginary part.

- the conjugate of $4+3i$ is $4-3i$
- the conjugate of $2-i$ is $2+i$

adding, subtracting, multiplying are easy when in <u>standard form</u> $a+bi$.

the way to "fix" $\frac{1}{3+2i}$ is to multiply top and bottom by the conjugate.

$$\frac{1}{3+2i}\cdot\frac{(3-2i)}{(3-2i)} = \frac{3-2i}{9-6i+6i+4i^2} = \frac{3-2i}{5} = \frac{3}{5}-\frac{2}{5}i$$

Powers of i

$i^0 = 1$ $\quad i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$= i$

remainder 1

$= -1$ remainder 2 | remainder 0 $= 1$ | remainder 3 $= -i$

$i^{20} = 1$

$i^{31} = -i$

$i^{101} = i$

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Square root of a negative number.

$\sqrt{-1} = i$ $\qquad$ $\sqrt{-5} = \sqrt{5}\,i$

$\sqrt{-4} = 2i$

$\sqrt{-9} = 3i$

So now $x^2 + 4 = 0$ has answers.

$a = 1$
$b = 0$
$c = 4$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(4)}}{2}$$

$$= \frac{\pm \sqrt{-16}}{2} = \frac{\pm 4i}{2}$$

$$= \pm 2i$$

$$x = 2i, -2i$$

You can check your answers!