

**EXAMPLE 8****Simplifying a Complex Rational Expression**

$$\text{Simplify: } \frac{\frac{x^2}{x-4} + 2}{\frac{2x-2}{x} - 1} \quad x \neq 0, 2, 4$$

**Solution** We will use Option 1.

$$\begin{aligned} \frac{\frac{x^2}{x-4} + 2}{\frac{2x-2}{x} - 1} &= \frac{\frac{x^2}{x-4} + \frac{2(x-4)}{x-4}}{\frac{2x-2}{x} - \frac{x}{x}} = \frac{\frac{x^2 + 2x - 8}{x-4}}{\frac{2x-2-x}{x}} \\ &= \frac{\frac{(x+4)(x-2)}{x-4}}{\frac{x-2}{x}} = \frac{(x+4)(x-2)}{x-4} \cdot \frac{x}{x-2} \\ &= \frac{x(x+4)}{x-4} \end{aligned}$$

**Now Work** PROBLEM 33**A.5 Assess Your Understanding****Concepts and Vocabulary**

- When the numerator and denominator of a rational expression contain no common factors (except 1 and  $-1$ ), the rational expression is \_\_\_\_\_.
- LCM is an abbreviation for \_\_\_\_\_.
- True or False** The rational expression  $\frac{2x^3 - 4x}{x - 2}$  is reduced to lowest terms.
- True or False** The LCM of  $2x^3 + 6x^2$  and  $6x^4 + 4x^3$  is  $4x^3(x + 1)$ .
- Choose the statement that is not true. Assume  $b \neq 0, c \neq 0$ , and  $d \neq 0$  as necessary.
  - $\frac{ac}{bc} = \frac{a}{b}$
  - $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
  - $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$
  - $\frac{b}{c} = \frac{ac}{bd}$

- Choose the rational expression that simplifies to  $-1$ .
  - $\frac{a-b}{b-a}$
  - $\frac{a-b}{a-b}$
  - $\frac{a+b}{a-b}$
  - $\frac{b-a}{b+a}$

**Skill Building**

In Problems 7–14, reduce each rational expression to lowest terms.

7.  $\frac{3x+9}{x^2-9}$

8.  $\frac{4x^2+8x}{12x+24}$

9.  $\frac{x^2-2x}{3x-6}$

10.  $\frac{15x^2+24x}{3x^2}$

11.  $\frac{24x^2}{12x^2-6x}$

12.  $\frac{x^2+4x+4}{x^2-4}$

13.  $\frac{y^2-25}{2y^2-8y-10}$

14.  $\frac{3y^2-y-2}{3y^2+5y+2}$

In Problems 15–36, perform the indicated operation and simplify the result. Leave your answer in factored form.

15.  $\frac{3x+6}{5x^2} \cdot \frac{x}{x^2-4}$

16.  $\frac{3}{2x} \cdot \frac{x^2}{6x+10}$

17.  $\frac{4x^2}{x^2-16} \cdot \frac{x^3-64}{2x}$

18.  $\frac{12}{x^2+x} \cdot \frac{x^3+1}{4x-2}$

19.  $\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}}$

20.  $\frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}}$

21.  $\frac{\frac{4-x}{4+x}}{\frac{x^2-16}{x^2-16}}$

22.  $\frac{\frac{3+x}{3-x}}{\frac{x^2-9}{9x^3}}$

23.  $\frac{x^2}{2x-3} - \frac{4}{2x-3}$

24.  $\frac{3x^2}{2x-1} - \frac{9}{2x-1}$

25.  $\frac{x}{x^2-4} + \frac{1}{x}$

26.  $\frac{x-1}{x^3} + \frac{x}{x^2+1}$

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27.  $\frac{x}{x^2 - 7x + 6} - \frac{x}{x^2 - 2x - 24}$

28.  $\frac{x}{x-3} - \frac{x+1}{x^2 + 5x - 24}$

29.  $\frac{4x}{x^2 - 4} - \frac{2}{x^2 + x - 6}$

30.  $\frac{3x}{x-1} - \frac{x-4}{x^2 - 2x + 1}$

31.  $\frac{3}{(x-1)^2(x+1)} + \frac{2}{(x-1)(x+1)^2}$

32.  $\frac{2}{(x+2)^2(x-1)} - \frac{6}{(x+2)(x-1)^2}$

33.  $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$

34.  $\frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}}$

35.  $\frac{\frac{x-2}{x+2} + \frac{x-1}{x+1}}{\frac{x}{x+1} - \frac{2x-3}{x}}$

36.  $\frac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}}$

**Applications and Extensions**

In Problems 37–44, expressions that occur in calculus are given. Reduce each expression to lowest terms.

37.  $\frac{(2x+3) \cdot 3 - (3x-5) \cdot 2}{(3x-5)^2}$

38.  $\frac{(4x+1) \cdot 5 - (5x-2) \cdot 4}{(5x-2)^2}$

39.  $\frac{x \cdot 2x - (x^2 + 1) \cdot 1}{(x^2 + 1)^2}$

40.  $\frac{x \cdot 2x - (x^2 - 4) \cdot 1}{(x^2 - 4)^2}$

41.  $\frac{(3x+1) \cdot 2x - x^2 \cdot 3}{(3x+1)^2}$

42.  $\frac{(2x-5) \cdot 3x^2 - x^3 \cdot 2}{(2x-5)^2}$

43.  $\frac{(x^2 + 1) \cdot 3 - (3x + 4) \cdot 2x}{(x^2 + 1)^2}$

44.  $\frac{(x^2 + 9) \cdot 2 - (2x - 5) \cdot 2x}{(x^2 + 9)^2}$

**45. The Lensmaker's Equation** The focal length  $f$  of a lens with index of refraction  $n$  is

$$\frac{1}{f} = (n-1) \left[ \frac{1}{R_1} + \frac{1}{R_2} \right]$$

where  $R_1$  and  $R_2$  are the radii of curvature of the front and back surfaces of the lens. Express  $f$  as a rational expression. Evaluate the rational expression for  $n = 1.5$ ,  $R_1 = 0.1$  meter, and  $R_2 = 0.2$  meter.

**46. Electrical Circuits** An electrical circuit contains three resistors connected in parallel. If the resistance of each is  $R_1$ ,  $R_2$ , and  $R_3$  ohms, respectively, their combined resistance  $R$  is given by the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Express  $R$  as a rational expression. Evaluate  $R$  for  $R_1 = 5$  ohms,  $R_2 = 4$  ohms, and  $R_3 = 10$  ohms.

**Explaining Concepts: Discussion and Writing**

47. The following expressions are called **continued fractions**:

$$1 + \frac{1}{x}, \quad 1 + \frac{1}{1 + \frac{1}{x}}, \quad 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}, \quad 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}}, \quad \dots$$

Each simplifies to an expression of the form

$$\frac{ax+b}{bx+c}$$

Trace the successive values of  $a$ ,  $b$ , and  $c$  as you "continue" the fraction. Can you discover the patterns that these values follow? Go to the library and research Fibonacci numbers. Write a report on your findings.

48. Explain to a fellow student when you would use the LCM method to add two rational expressions. Give two examples of adding two rational expressions, one in which you use the LCM and the other in which you do not.

49. Which of the two methods given in the text for simplifying complex rational expressions do you prefer? Write a brief paragraph stating the reasons for your choice.



## Concepts and Vocabulary

5. An equation that is satisfied for every choice of the variable for which both sides are defined is called a(n) \_\_\_\_\_.
6. **True or False** The solution of the equation  $3x - 8 = 0$  is  $\frac{3}{8}$ .
7. **True or False** Some equations have no solution.
8. To solve the equation  $x^2 + 5x = 0$  by completing the square, you would \_\_\_\_\_ the number \_\_\_\_\_ to both sides.
9. The quantity  $b^2 - 4ac$  is called the \_\_\_\_\_ of a quadratic equation. If it is \_\_\_\_\_, the equation has no real solution.
10. **True or False** Quadratic equations always have two real solutions.

## Skill Building

In Problems 15–80, solve each equation.

15.  $3x = 21$

16.  $3x = -24$

17.  $5x + 15 = 0$

18.  $3x + 18 = 0$

19.  $2x - 3 = 5$

20.  $3x + 4 = -8$

21.  $\frac{1}{3}x = \frac{5}{12}$

22.  $\frac{2}{3}x = \frac{9}{2}$

23.  $6 - x = 2x + 9$

24.  $3 - 2x = 2 - x$

25.  $2(3 + 2x) = 3(x - 4)$

26.  $3(2 - x) = 2x - 1$

27.  $8x - (2x + 1) = 3x - 10$

28.  $5 - (2x - 1) = 10$

29.  $\frac{1}{2}x - 4 = \frac{3}{4}$

30.  $1 - \frac{1}{2}x = 5$

31.  $0.9t = 0.4 + 0.1t$

32.  $0.9t = 1 + t$

33.  $\frac{2}{y} + \frac{4}{y} = 3$

34.  $\frac{4}{y} - 5 = \frac{5}{2y}$

35.  $(x + 7)(x - 1) = (x + 1)^2$

36.  $(x + 2)(x - 3) = (x - 3)^2$

37.  $z(z^2 + 1) = 3 + z^3$

38.  $w(4 - w^2) = 8 - w^3$

39.  $x^2 = 9x$

40.  $x^3 = x^2$

41.  $t^3 - 9t^2 = 0$

42.  $4z^3 - 8z^2 = 0$

43.  $\frac{3}{2x - 3} = \frac{2}{x + 5}$

44.  $\frac{-2}{x + 4} = \frac{-3}{x + 1}$

45.  $(x + 2)(3x) = (x + 2)(6)$

46.  $(x - 5)(2x) = (x - 5)(4)$

47.  $\frac{2}{x - 2} = \frac{3}{x + 5} + \frac{10}{(x + 5)(x - 2)}$

48.  $\frac{1}{2x + 3} + \frac{1}{x - 1} = \frac{1}{(2x + 3)(x - 1)}$

49.  $|2x| = 6$

50.  $|3x| = 12$

51.  $|2x + 3| = 5$

52.  $|3x - 1| = 2$

53.  $|1 - 4t| = 5$

54.  $|1 - 2z| = 3$

55.  $|-2x| = 8$

56.  $|-x| = 1$

57.  $|-2|x = 4$

58.  $|3|x = 9$

59.  $|x - 2| = -\frac{1}{2}$

60.  $|2 - x| = -1$

61.  $|x^2 - 4| = 0$

62.  $|x^2 - 9| = 0$

63.  $|x^2 - 2x| = 3$

64.  $|x^2 + x| = 12$

65.  $|x^2 + x - 1| = 1$

66.  $|x^2 + 3x - 2| = 2$

67.  $x^2 = 4x$

68.  $x^2 = -8x$

69.  $z^2 + 4z - 12 = 0$

70.  $v^2 + 7v + 12 = 0$

71.  $2x^2 - 5x - 3 = 0$

72.  $3x^2 + 5x + 2 = 0$

73.  $x(x - 7) + 12 = 0$

74.  $x(x + 1) = 12$

75.  $4x^2 + 9 = 12x$

76.  $25x^2 + 16 = 40x$

77.  $6x - 5 = \frac{6}{x}$

78.  $x + \frac{12}{x} = 7$

79.  $\frac{4(x - 2)}{x - 3} + \frac{3}{x} = \frac{-3}{x(x - 3)}$

80.  $\frac{5}{x + 4} = 4 + \frac{3}{x - 2}$

In Problems 81–86, solve each equation by the Square Root Method.

81.  $x^2 = 25$

82.  $x^2 = 36$

83.  $(x - 1)^2 = 4$

84.  $(x + 2)^2 = 1$

85.  $(2y + 3)^2 = 9$

86.  $(3x - 2)^2 = 4$

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In Problems 87–92, solve each equation by completing the square.

87.  $x^2 + 4x = 21$

88.  $x^2 - 6x = 13$

89.  $x^2 - \frac{1}{2}x - \frac{3}{16} = 0$

90.  $x^2 + \frac{2}{3}x - \frac{1}{3} = 0$

91.  $3x^2 + x - \frac{1}{2} = 0$

92.  $2x^2 - 3x - 1 = 0$

In Problems 93–104, find the real solutions, if any, of each equation. Use the quadratic formula.

93.  $x^2 - 4x + 2 = 0$

94.  $x^2 + 4x + 2 = 0$

95.  $x^2 - 5x - 1 = 0$

96.  $x^2 + 5x + 3 = 0$

97.  $2x^2 - 5x + 3 = 0$

98.  $2x^2 + 5x + 3 = 0$

99.  $4y^2 - y + 2 = 0$

100.  $4t^2 + t + 1 = 0$

101.  $4x^2 = 1 - 2x$

102.  $2x^2 = 1 - 2x$

103.  $x^2 + \sqrt{3}x - 3 = 0$

104.  $x^2 + \sqrt{2}x - 2 = 0$

In Problems 105–110, use the discriminant to determine whether each quadratic equation has two unequal real solutions, a repeated real solution, or no real solution without solving the equation.

105.  $x^2 - 5x + 7 = 0$

106.  $x^2 + 5x + 7 = 0$

107.  $9x^2 - 30x + 25 = 0$

108.  $25x^2 - 20x + 4 = 0$

109.  $3x^2 + 5x - 8 = 0$

110.  $2x^2 - 3x - 4 = 0$

## Applications and Extensions

In Problems 111–116, solve each equation. The letters  $a$ ,  $b$ , and  $c$  are constants.

111.  $ax - b = c$ ,  $a \neq 0$

112.  $1 - ax = b$ ,  $a \neq 0$

113.  $\frac{x}{a} + \frac{x}{b} = c$ ,  $a \neq 0, b \neq 0, a \neq -b$

114.  $\frac{a}{x} + \frac{b}{x} = c$ ,  $c \neq 0$

115.  $\frac{1}{x-a} + \frac{1}{x+a} = \frac{2}{x-1}$

116.  $\frac{b+c}{x+a} = \frac{b-c}{x-a}$ ,  $c \neq 0, a \neq 0$

Problems 117–122 list some formulas that occur in applications. Solve each formula for the indicated variable.

117. **Electricity**  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$  for  $R$

118. **Finance**  $A = P(1 + rt)$  for  $r$

119. **Mechanics**  $F = \frac{mv^2}{R}$  for  $R$

120. **Chemistry**  $PV = nRT$  for  $T$

121. **Mathematics**  $S = \frac{a}{1-r}$  for  $r$

122. **Mechanics**  $v = -gt + v_0$  for  $t$

123. Show that the sum of the roots of a quadratic equation is  $-\frac{b}{a}$ .

127. Show that the real solutions of the equation  $ax^2 + bx + c = 0$  are the negatives of the real solutions of the equation  $ax^2 - bx + c = 0$ . Assume that  $b^2 - 4ac \geq 0$ .

124. Show that the product of the roots of a quadratic equation is  $\frac{c}{a}$ .

128. Show that the real solutions of the equation  $ax^2 + bx + c = 0$  are the reciprocals of the real solutions of the equation  $cx^2 + bx + a = 0$ . Assume that  $b^2 - 4ac \geq 0$ .

125. Find  $k$  such that the equation  $kx^2 + x + k = 0$  has a repeated real solution.

126. Find  $k$  such that the equation  $x^2 - kx + 4 = 0$  has a repeated real solution.

## Explaining Concepts: Discussion and Writing

129. Which of the following pairs of equations are equivalent?

Explain.

(a)  $x^2 = 9$ ;  $x = 3$

(b)  $x = \sqrt{9}$ ;  $x = 3$

(c)  $(x-1)(x-2) = (x-1)^2$ ;  $x-2 = x-1$

130. The equation

$$\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$$

has no solution, yet when we go through the process of solving it, we obtain  $x = -3$ . Write a brief paragraph to explain what causes this to happen.