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Become familiar with wolframalpha!

Appendix Section 1.

Set - a well defined collection of distinct objects.

example:
- the set of even numbers is
$$\{\ldots -4, -2, 0, 2, 4, 6, \ldots\}$$
- the "set" of "cool things" is not a set, because it's up for debate what's in the set and whats not.

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- the set $\{2, 2, 4, 6\}$ Should not have the number 2 repeated. Should be written $\{2, 4, 6\}$.

Set builder notation.

A shortcut way of writing huge (or infinite) sets.

ex $\{x \mid x \text{ is a color}\} = \{\text{red}, \text{blue}, \text{white}, \ldots\}$

the objects.

the instruction, criteria.

example: $\{p \mid p \text{ is a prime number}\}$

$= \{2, 3, 5, 7, 11 \ldots\}$

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Example $\{2n \mid n = 1, 2, 3, 4, \ldots\}$

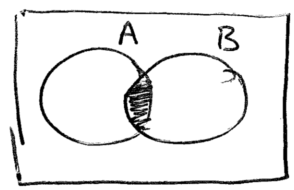
$= \{2, 4, 6, 8, 10, \ldots\}$.

- intersection of two sets.

the symbol used is "$\cap$".

$A \cap B$ is read "A intersection B".

it means, what do the two sets have in common. Visualize →



$\{$red, white, blue$\} \cap \{$white, yellow$\}$

$= \{$white$\}$.

- Union of two sets.

The symbol used is "$\cup$".

$A \cup B$ is read "A union B".

it means merge the two sets into one, and no repetition!

$\{$red, white, blue$\} \cup \{$white, yellow$\}$

$= \{$red, white, blue, yellow$\}$

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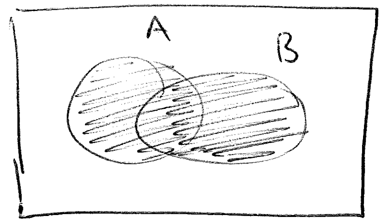
Visualization → 

- <u>Subset of a set.</u>

A subset is like a mini set.

A subgroup.

if the set is {red, white, blue},

then:

are subsets

{red, white}

{red}

{white, blue}

{red, white, blue, green}

{red, green}

{black}

are not subsets.

the symbol used is "⊂"

$\{red, white\} \subset \{red, white, blue\}$

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Sets of numbers

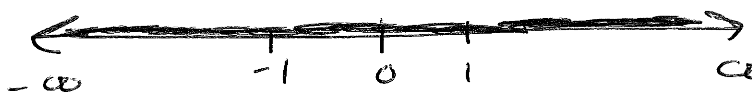
$\mathbb{N}$ = Natural numbers : $\{0, 1, 2, 3, 4, 5, \ldots\}$

$\mathbb{Z}$ = integers : $\{\ldots -4, -3, -2, -1, 0, 1, 2 \ldots\}$

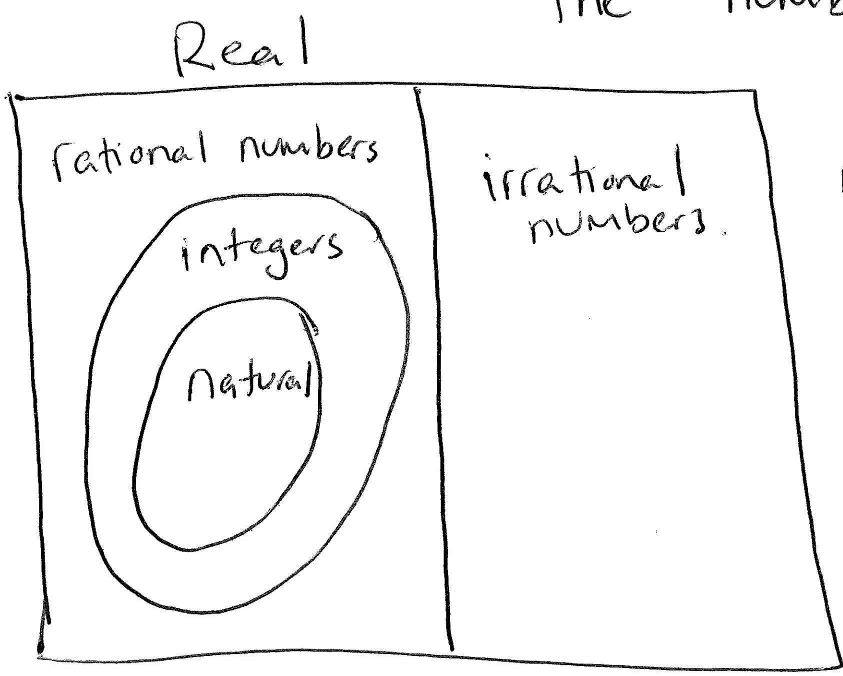
$\mathbb{Q}$ = rational : $\{a/b \mid a \text{ and } b \text{ are integers, } b \neq 0\}$

irrational : numbers cannot be written as fractions.
π, e, $\sqrt{2}$, $\sqrt{3}$, etc.

$\mathbb{R}$ = real numbers :



all numbers on the number line.



natural $\subset$ integer

natural is a subset of integers.

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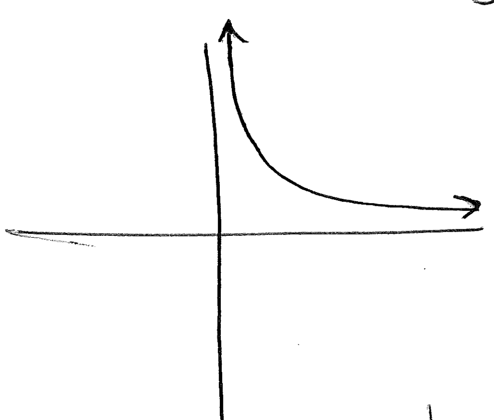
Domain:

set of numbers that can be plugged into an equation or expression.

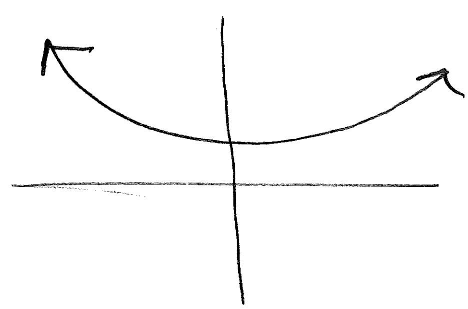
$2x$ anything can be plugged in. domain is all real numbers.

$1/x$ Cannot plug in zero. domain is $\{x \mid x \text{ is real but not zero}\}$

Also can be thought of how far left and right the graph goes.



Domain = $\{x \mid x > 0\}$



Domain = all real numbers.

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Exponent Rules

Rules	Examples
$a^0 = 1$	$2^0 = 1 \quad 5^0 = 1$
$a^{-n} = 1/a^n$	$2^{-3} = 1/2^3$
$a^n = 1/a^{-n}$	$2^3 = 1/2^{-3}$
$a^m a^n = a^{m+n}$	$2^2 2^3 = 2^5$
$(a^m)^n = a^{mn}$	$(2^2)^3 = 2^6$
$(ab)^n = a^n b^n$	$(2 \cdot 3)^2 = 2^2 3^2$
$\cancel{(a+b)^n = a^n + b^n}$	$\cancel{(2+3)^2 = 2^2 + 3^2}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^4}{2^2} = 2^{4-2} = 2^2$
$(a/b)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

* know about absolute values.

* order of operations

P . E . $\underbrace{\text{M D}}$. $\underbrace{\text{A S}}$

parentheses exponents multiplication division addition subtraction.

$(-4)^2 = 64$

$-4^2 = -64$

$-4^{-2} = -\frac{1}{64}$

Appendix section 3

polynomials: functions of the form:

$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0$

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Examples:

Polynomials	not polynomials
$8x$	$8x + 1/x$
$4x^2+3$	$4x^{1/2} + 2$
$8x^3+2x^2+1$	$4x^{-2}+1$
4	

The exponents have to be whole numbers. no fractions or negatives.

The largest exponent is the ~~g~~ degree of the polynomial.

$8x^3+2x^2+1$ has degree 3.

Review "foiling": $(x+2)(x-3) = x^2+2x-3x-6$
$= x^2-x-6$

Factoring: $2x+4 = 2(x+2)$

$x^2+x = x(x+1)$

$x^2+4x-12 = (x+6)(x-2)$

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Factoring Rules. (Memorize these.)

- Difference of squares.

$$x^2 - a^2 = (x+a)(x-a)$$
$$x^2 - 4 = (x+2)(x-2)$$

- Difference of cubes

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$
$$x^3 - 8 = (x-2)(x^2 + 2x + 4)$$

- Sum of cubes

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$
$$x^3 + 8 = (x+2)(x^2 - 2x + 4)$$

Dividing polynomials.

First, remember:

$$
\begin{array}{r}
21 \\
15 \overline{\smash{)}\,320} \\
-\ 30 \\
\hline
20 \\
-\ 15 \\
\hline
5
\end{array}
$$

$$\frac{320}{15} = 21 + \frac{5}{15}$$

$$
\begin{array}{rl}
 & 3x + 4 \\
x^2+1 \, \big) & 3x^3 + 4x^2 + x + 7 \\
- & 3x^3 + 0x^2 + 3x \\
\hline
 & 0 + 4x^2 - 2x + 7 \\
- & 4x^2 + 0x + 4 \\
\hline
 & 0 - 2x + 3 \leftarrow \text{Remainder.}
\end{array}
$$

Answer:

$$\frac{3x^3+4x^2+x+7}{x^2+1} = 3x+4+\frac{-2x+3}{x^2+1}$$

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$$
\begin{array}{r|l}
 & 3x - 11 \\
\hline
x^2 + 3x + 3 & 3x^3 - 2x^2 + 4x - 3 \\
 & -(3x^3 + 9x^2 + 9x) \\
\hline
 & 0 - 11x^2 - 5x - 3 \\
 & -(-11x^2 - 33x - 33) \\
\hline
 & 0 + 28x + 30
\end{array}
$$

← Remainder

answer:

$$\frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3} = 3x - 11 + \frac{28x + 30}{x^2 + 3x + 3}$$

Appendix Section 5

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Review fraction rules:

Numbers	Variables
$\dfrac{\frac{4}{5}}{\frac{1}{3}} = \dfrac{4}{5} \cdot \dfrac{3}{1}$	$\dfrac{\frac{x+2}{x-3}}{\frac{x+3}{x-1}} = \dfrac{x+2}{x-3} \cdot \dfrac{x-1}{x+3}$
$\dfrac{1}{3} + \dfrac{1}{3} = \dfrac{2}{3}$	$\dfrac{x+2}{x-1} + \dfrac{x+3}{x-1} = \dfrac{x+2+x+3}{x-1}$
$\dfrac{1}{3} \cdot \dfrac{1}{3} = \dfrac{1}{9}$	$\dfrac{x-1}{x+3} \cdot \dfrac{x-2}{x+3} = \dfrac{(x-1)(x-2)}{(x+3)^2}$
$\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3}{2(3)} + \dfrac{2}{3(2)}$ $= \dfrac{5}{6}$	$\dfrac{x+2}{x-1} + \dfrac{x-3}{x-2} = \dfrac{(x+2)(x-2)}{(x-1)(x-2)} + \dfrac{(x-3)(x-1)}{(x-2)(x-1)}$ $= \dfrac{(x+2)(x-2)+(x-3)(x-1)}{(x-1)(x-2)}$
$\dfrac{1}{2} \cdot \dfrac{1}{3} = \dfrac{1}{6}$	$\dfrac{2}{x-1} \cdot \dfrac{x-3}{x-2} = \dfrac{2(x-3)}{(x-1)(x-2)}$