Test 2 Review Problems

Equations and things to memorize/know…

- Vertex of a parabola
- Quadratic formula
- How to get the equation of a line if given two points
- Basic familiarity with the functions from the Excel spreadsheet, how the shifts up, down, left, right and stretches work.
- When can a horizontal/slant asymptote exist? (in other words, when should we bother using long division and when can we skip this step.
- Holes versus vertical asymptotes
- Polynomial long division with no mistakes!
- Know when you can check your answers, and make sure to do so in those cases!

1) Graph the following piece-wise function, and do parts a) b) and c).

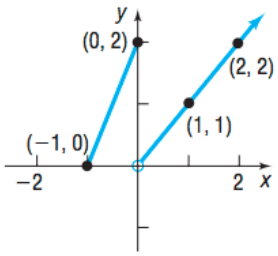
$$\text{If } f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$$

find: (a) $f(-2)$ (b) $f(0)$ (c) $f(2)$

2) Find the domain, range, intercepts. Is the graph continuous? Sketch a graph.

$$f(x) = \begin{cases} 2x + 5 & \text{if } -3 \le x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$$

3) Write the equation for the piecewise function:



4)

Parking at O'Hare International Airport The short-term (no more than 24 hours) parking fee F (in dollars) for parking x hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 3 & \text{if } 0 < x \le 3 \\ 5\,\text{int}(x + 1) + 1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \le x \le 24 \end{cases}$$

Determine the fee for parking in the short-term parking garage for

(a) 2 hours (b) 7 hours (c) 15 hours
(d) 8 hours and 24 minutes

Source: *O'Hare International Airport*

5)

If $f(x) = \begin{cases} 2x - 4 & \text{if } -1 \le x \le 2 \\ x^3 - 2 & \text{if } 2 < x \le 3 \end{cases}$

find: (a) $f(0)$ (b) $f(1)$ (c) $f(2)$ (d) $f(3)$

6)

If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = -f(x)$?

(a) $(6, 3)$ (b) $(6, -3)$
(c) $(3, -6)$ (d) $(-3, 6)$

7)

If $(4, 2)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(2x)$?

(a) $(4, 1)$ (b) $(8, 2)$
(c) $(2, 2)$ (d) $(4, 4)$

8)

write the function whose graph is the graph of $y = x^3$*, but is:*

(4 separate questions, write an equation for each one:)

Shifted to the right 4 units

Shifted up 4 units

Reflected about the y-axis

Vertically stretched by a factor of 4

9)

Suppose that the x-intercepts of the graph of $y = f(x)$ are -8 and 1.

(a) What are the x-intercepts of the graph of $y = f(x + 4)$?
(b) What are the x-intercepts of the graph of $y = f(x - 3)$?
(c) What are the x-intercepts of the graph of $y = 2f(x)$?
(d) What are the x-intercepts of the graph of $y = f(-x)$?

10)

A function g is defined by

$$g(x) = \frac{A}{x} + \frac{8}{x^2}$$

If $g(-1) = 0$, find A.

11) Determine if the following "x y" table is for line, or if belongs to a function that isn't a line:

x	$y = f(x)$
-2	-8
-1	-3
0	0
1	1
2	0

12)

Suppose that $f(x) = 3x + 5$ and $g(x) = -2x + 15$.

(a) Solve $f(x) = 0$.
(b) Solve $f(x) < 0$.
(c) Solve $f(x) = g(x)$.
(d) Solve $f(x) \geq g(x)$.
(e) Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.

13)

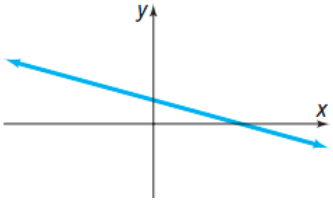
Long Distance A phone company offers a domestic long distance package by charging \$5 plus \$0.05 per minute.

(a) Write a linear model that relates the cost C, in dollars, of talking x minutes.
(b) What is the cost of talking 105 minutes? 180 minutes?

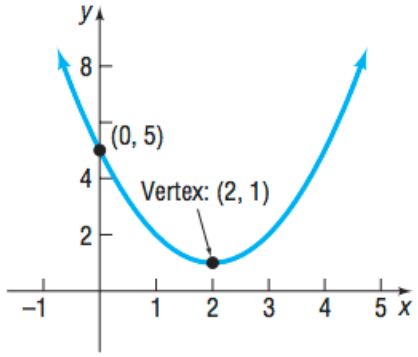
14)

Which of the following functions might have the graph shown? (More than one answer is possible.)

(a) $f(x) = 3x + 1$
(b) $g(x) = -2x + 3$
(c) $H(x) = 3$
(d) $F(x) = -4x - 1$
(e) $G(x) = -\frac{2}{3}x + 3$



15) Find the quadratic function that has this graph: (Think about what shift up and right you see, the y intercept you see, etc.)



16)

Suppose that $f(x) = x^2 + 4x - 21$

(a) What is the vertex of f?

(b) What are the x-intercepts of the graph of f?

17)

Find the point on the line $y = x + 1$ that is closest to the point $(4, 1)$.

(there are two ways to solve this, one is creative and the other uses the distance formula, which you're going to try and minimize)

18)

Maximizing Revenue The John Deere company has found that the revenue, in dollars, from sales of riding mowers is a function of the unit price p, in dollars, that it charges. If the revenue R is

$$R(p) = -\frac{1}{2}p^2 + 1900p$$

what unit price p should be charged to maximize revenue? What is the maximum revenue?

19)

Find a quadratic function whose x-intercepts are -4 and 2 and whose range is $[-18, \infty)$.

20) Find the x and y intercepts, the maximum or minimum (the vertex), the domain and range, and on what intervals the function is increasing/decreasing.

$f(x) = 2x^2 + 5x + 3$

21)

Enclosing the Most Area with a Fence A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

22) Solve the inequality (remember this is very easy if you draw it!

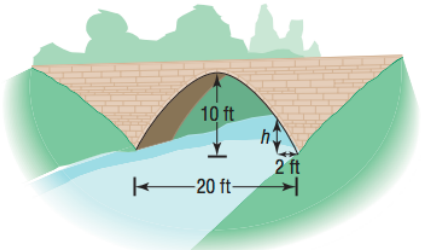
$4x^2 + 9 < 6x$

23) Same as the previous question.

$6(x^2 - 1) > 5x$

24) (Parabolic arch literally means the underside of the bridge is the shape of a parabola)

Parabolic Arch Bridge A horizontal bridge is in the shape of a parabolic arch. Given the information shown in the figure, what is the height h of the arch 2 feet from shore?



25)

Architecture A special window in the shape of a rectangle with semicircles at each end is to be constructed so that the outside dimensions are 100 feet in length. See the illustration. Find the dimensions of the rectangle that maximizes its area.

26)

For the function f defined by $f(x) = x^2 - 4x + 1$, find:

(a) $f(2)$

(b) $f(x) + f(2)$

(c) $f(-x)$

(d) $-f(x)$

(e) $f(x + 2)$

(f) $\dfrac{f(x + h) - f(x)}{h}$ $h \neq 0$

27) Can functions cross horizontal asymptotes, vertical asymptotes, and/or slant asymptotes?

28) Find the asymptotes of this function and draw a rough sketch:

$$R(x) = \frac{6x^2 + 7x - 5}{3x + 5}$$

29)

Can the graph of a rational function have both a horizontal and an oblique asymptote? Explain.

30) Find the asymptotes of this function and draw a rough sketch:

$$R(x) = \frac{x + 1}{x(x + 4)}$$

31) Find the asymptotes of this function and draw a rough sketch:

$$f(x) = 2x + \frac{9}{x}$$

32)

Cost of a Can A can in the shape of a right circular cylinder is required to have a volume of 500 cubic centimeters. The top and bottom are made of material that costs 6¢ per square centimeter, while the sides are made of material that costs 4¢ per square centimeter.

(a) Express the total cost C of the material as a function of the radius r of the cylinder. (Refer to Figure 43.)

(b) Graph $C = C(r)$. For what value of r is the cost C a minimum?

33)

Demand Equation The price p (in dollars) and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{10}x + 150,$$

(a) Express the revenue R as a function of x.

(b) What is the revenue if 100 units are sold?

(c) What quantity x maximizes revenue? What is the maximum revenue?

(d) What price should the company charge to maximize revenue?

34)

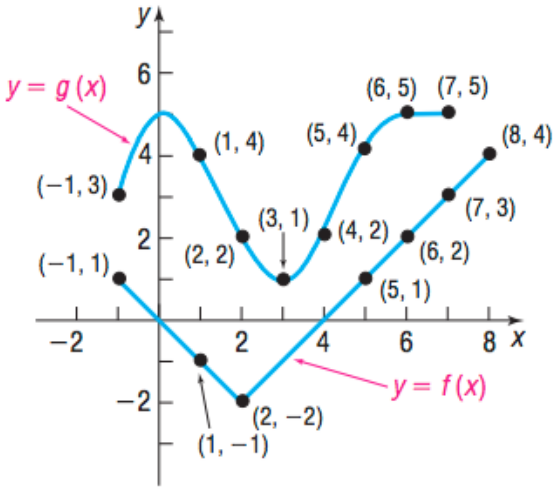
True or False The domain of the composite function $(f \circ g)(x)$ is the same as the domain of $g(x)$.

35)

evaluate each expression using the graphs of $y = f(x)$ *and* $y = g(x)$ *shown in the figure.*

(a) $(g \circ f)(1)$ (b) $(g \circ f)(5)$

(c) $(f \circ g)(0)$ (d) $(f \circ g)(2)$



36) Do parts a) b) c) and d) for the following three questions:

Find:

(a) $(f \circ g)(4)$ *(b)* $(g \circ f)(2)$ *(c)* $(f \circ f)(1)$ *(d)* $(g \circ g)(0)$

$f(x) = \sqrt{x + 1}; \quad g(x) = 3x$

$f(x) = |x - 2|; \quad g(x) = \dfrac{3}{x^2 + 2}$

$f(x) = x^{3/2}; \quad g(x) = \dfrac{2}{x + 1}$

37)

If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 68.

38) Verify that these two functions are inverses of each other:

$f(x) = \dfrac{x - 5}{2x + 3}; \quad g(x) = \dfrac{3x + 5}{1 - 2x}$

39) Find the inverse of:

$$f(x) = \frac{4}{x+2}$$

40) Find the inverse of:

$$f(x) = \frac{2x-3}{x+4}$$

41) What transformations have occurred in the following functions, compared to $f(x) = e^x$?

$$f(x) = e^{x+2}$$

$$f(x) = e^x - 1$$

$$f(x) = e^{-x}$$

42) Solve the following equations:

$9^{-x+15} = 27^x$

$5^{x^2+8} = 125^{2x}$

43) Solve the following two questions:

If $2^x = 3$, what does 4^{-x} equal?

If $5^{-x} = 3$, what does 5^{3x} equal?

44)

Suppose that $f(x) = 3^x$.

(a) What is $f(4)$? What point is on the graph of f?

(b) If $f(x) = \frac{1}{9}$, what is x? What point is on the graph of f?

45)

find $f \circ g, g \circ f, f \circ f$, and $g \circ g$ for each pair of functions. State the domain of each composite function.

$$f(x) = \sqrt{3x}; \quad g(x) = 1 + x + x^2$$