

Lecture 3 January 13th

Finish Appendix Section 7 if needed.

Appendix Section 8 word problems

1) Simple interest: (the interest is not compounding)

$$\underline{I} = P r t$$

interest ↑ principal ↑ time ↓
 ↑ interest rate

ex Deposit \$1000 (principal) = ?

$$\text{interest} = 6\% = r \quad \rightarrow I = \$1000 \cdot .06 \cdot 2$$

$$\text{time} = 2 \text{ years} = t \quad = 1000 \cdot .12 \\ = 120$$

The interest is \$120

ex

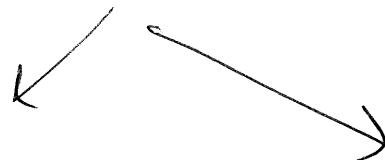
Borrow \$500 for 6 Months, interest is 10% per year. $I = 500 \cdot .10 \cdot \frac{1}{2}$
 $= 25 \text{ dollars}$

ex Borrow \$500 for 6 months. $I = 500 \cdot .10 \cdot 6$
 interest is 10% per month. $= 300 \text{ dollars}$

- Blending problem (omit for now)
- Uniform motion.

Moving at a constant speed. (rate)

$$\text{Distance} = \text{rate} \cdot \text{time}$$



$$500? = 50 \text{ mph} \cdot 10 \text{ minutes}$$

Wrong

$$500 \text{ miles} = 50 \text{ mph} \cdot 10 \text{ hours.}$$

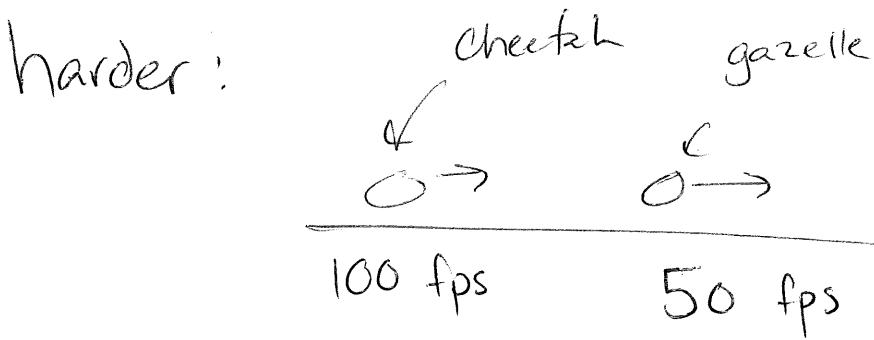
right ✓

Easy: Orlando is 140 miles away.
How long does the drive take if
driving at 70 mph?

$$140 = 70 \cdot t \rightarrow t = 2 \text{ hours.}$$

Hard:

Jack leaves Orlando driving 70 mph.
Jill leaves Jax driving 60 mph.
When and where do they meet?



the gazelle has a ~~100 second~~^{100 feet} head start. How long + far does the cheetah run?

caught at time t .



$$d = rt$$

$$d = 100(t) \quad \text{cheetah}$$

$$(d - 100) = 50(t) \quad \text{gazelle}$$

$$d = 50t + 100$$

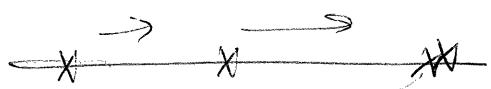
set equal to each other.

$$100(t) = 50(t) + 100$$

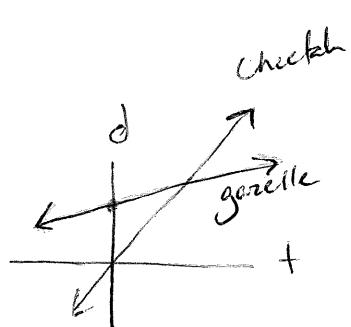
$$50(t) = 100$$

$$t = 2 \text{ seconds.}$$

Check answer,
makes sense!



2 seconds
200 feet.



Example. Working together on
a job.

- Jack takes 2 hours to paint a room and Jill takes 2 hours to paint a room.
How about if they work together?
= 1 hour.

Jack takes 4 hours, Jill takes 2 hours, now what?

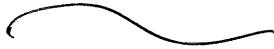
In one hour, Jack has painted $\frac{1}{4}$
Jill has painted $\frac{1}{2}$.

Together, $\frac{3}{4}$ in one hour.

How many hours until done?

$$\frac{1}{\frac{3}{4}} = \frac{4}{3} \text{ hours } 1:20.$$

See 11.1



See 11.1

systems of
linear equations.

ex) Drinks are \$2

Snacks are \$3 $\rightarrow 2x + 3y = 30$
made \$30

lets say ~~10~~¹² items sold total.

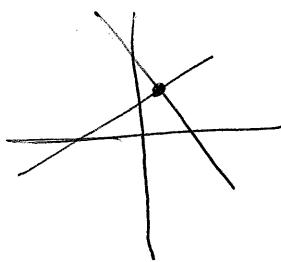
How many of each?

$2x + 3y = 30$ is there an
 $x + y = \del{10}$ x, y that makes
this work?

$$x=6, y=6$$

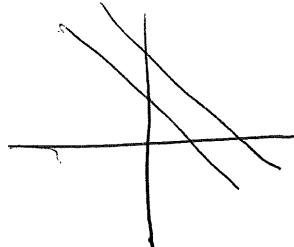
this is a system
of equations.

there are three scenarios:



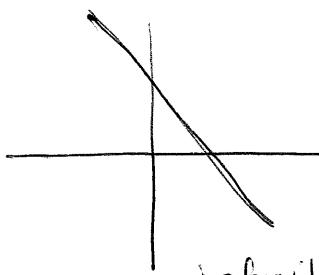
One answer
intersection

$$\begin{aligned} 2x + 3y &= 30 \\ x + y &= 12 \end{aligned}$$



No
answer
intersection

$$\begin{aligned} x + y &= 3 \\ x + y &= 5 \end{aligned}$$



infinite
answers
intersections.

$$\begin{aligned} x + y &= 6 \\ 2x + 3y &= 12 \end{aligned}$$

There are 2 ways to
Solve a system:

$$2x + 3y = 30$$

$$x + y = 12 \leftarrow \text{Solve for } y.$$

Substitution

$$y = -x + 12.$$

Substitute into the other.

$$2x + 3(-x + 12) = 30$$

$$2x - 3x + 36 = 30$$

$$-x = -6 \rightarrow x = 6.$$

$$6 + y = 12 \rightarrow y = 12.$$

Elimination

$$2x + 3y = 30$$

$$x + y = 12$$

$$2x + 3y = 30$$

$$-2x - 2y = -24$$

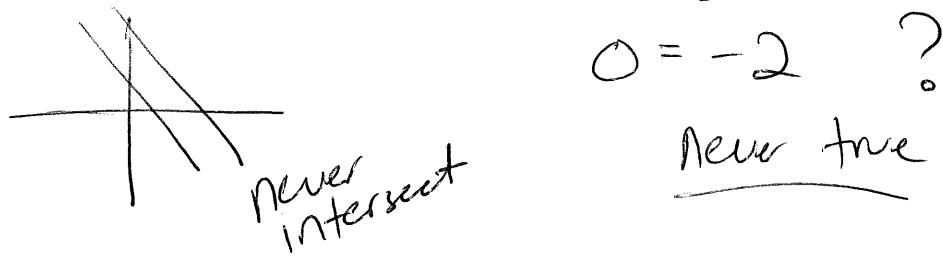
$$y = 6$$

$$\rightarrow x = 6$$

Those were examples of 1 solution.

No solution example:

$$\begin{array}{l} 2x + y = 5 \\ 4x + 2y = 8 \end{array} \rightarrow \begin{array}{l} -4x - 2y = -10 \\ 4x + 2y = 8 \end{array}$$

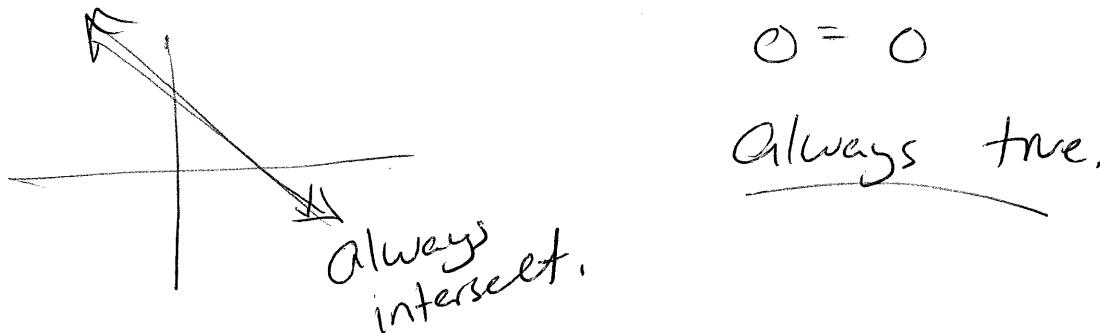


$$0 = -2 ?$$

Never true

Infinite solution example:

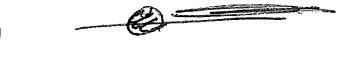
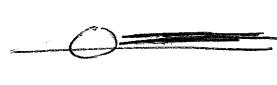
$$\begin{array}{l} 2x + y = 5 \\ 4x + 2y = 10 \end{array} \rightarrow \begin{array}{l} -4x - 2y = -10 \\ 4x + 2y = 10 \end{array}$$



$$0 = 0$$

Always true.

A9) interval notation + inequalities.

<u>interval</u>	<u>inequality</u>	<u>graph / shading</u>
$(-1, 2)$	$-1 < x < 2$	
$[-1, 2]$	$-1 \leq x \leq 2$	
$[-1, 2)$	$-1 \leq x < 2$	
$(-1, 2]$	$-1 < x \leq 2$	
$[-1, \infty)$	$-1 \leq x \quad x \geq -1$	
$(-1, \infty)$	$-1 < x \quad x > -1$	
$(-\infty, 2]$	$x \leq 2$	
$(-\infty, 2)$	$x < 2$	
$(-\infty, \infty)$	$x = \text{anything}$	

infinity is a number?

$(2, -1)$ wrong or right?

Rules

- $a^2 \geq 0$

- $6 > 4$ divide by negative 2,
 \downarrow \downarrow multiply by negative 1.

$$-3 < -2 \quad -6 < -4$$

* Always be careful about
dividing by a negative.

$$x^2 < x \rightarrow x < 1 \quad ? \quad \underline{\text{No.}}$$

Ex $-4x + 7 \geq 2x - 3$ same as
equation.

~~Ans~~ $2x \geq -10$

$$x \geq -5$$

Ex $-5 < 3x - 2 < 1$ means "and."

$$-5 < 3x - 2 \quad \text{and} \quad 3x - 2 < 1$$

$$-3 < 3x$$

$$-1 < x \quad \text{and}$$

$$3x < +3$$

$$x < 1$$

Ex $\frac{1}{4x-1} > 0$ ~~Recall~~ What does this mean about $4x-1$?

$$4x-1 > 0$$

$$x > \frac{1}{4}$$

Inequalities combined with absolute values.

$$|x| < 5$$



$$\frac{(-\infty, -5) \cup (5, \infty)}{-5 \quad 5}$$

$$(-5, 5)$$

$$|x| > 5$$

$$\frac{(-\infty, -5) \cup (5, \infty)}{\text{---}}$$

$$(-\infty, -5) \cup (5, \infty)$$

Rules

$$|x| < a \rightarrow -a < x < a$$

$-a < x$ and $x < a$

$$|x| > a \rightarrow x > a \text{ or } x < -a$$

Example

$$|2x + 4| \leq 3$$

$$-3 \leq 2x + 4 \leq 3$$

$$\downarrow -3 \leq 2x + 4 \quad \underline{\text{and}} \quad 2x + 4 \leq 3$$

$$x \leq -\frac{7}{2} \quad \text{and} \quad x \leq -\frac{1}{4}$$

3 ways
to write
answer.

$$\begin{array}{c} \text{---} \\ -\frac{7}{2} \quad -\frac{1}{4} \end{array} \quad (-\frac{7}{2}, -\frac{1}{4})$$

$$-\frac{7}{2} < x < -\frac{1}{4}$$

Example $|2x - 5| > 3$

$$2x - 5 > 3 \quad \underline{\text{or}}$$

$$2x - 5 < -3$$

$$2x > 8$$

$$2x < 2$$

$$x > 4 \quad \text{or} \quad x < 1$$

$$x < 1$$

$$\begin{array}{ccc} \text{~~~~~} & \text{~~~~~} \\ | & 4 \\ \text{~~~~~} & \text{~~~~~} \end{array}$$
$$(-\infty, 1) \cup (4, \infty)$$

$$\{x \mid x < 1 \text{ or } x > 4\}$$