Measurement and Data Driven Modeling

Assignment Report

Trade-offs in FRF Measurements

Objective

This assignment aims to explore and demonstrate the practical trade-offs between:

1. Frequency resolution
2. Measurement time
3. Signal-to-Noise Ratio (SNR)

in the context of measuring Frequency Response Functions (FRFs). This understanding is crucial when designing excitation signals for system identification.

Background

Measuring accurate FRFs requires balancing three competing factors:

1. **Frequency Resolution**: Higher resolution allows more frequency lines to be distinguished within a given bandwidth.
2. **Measurement Time**: Longer signals allow more averaging, reducing random noise.
3. **SNR**: The clarity with which excited frequency components emerge from the noise floor.

This trade-off is fundamental and unavoidable in any practical measurement setup. The trade-offs explored in this assignment assume:

- The RMS amplitude of the excitation signal is fixed to $1\,V$.
- The frequency band of interest lies between 5 Hz and 10 Hz.

Multisine Design Summary

The table below summarizes six example multisines designed to illustrate the trade-offs under different constraints:

Table 1: Example Multisine Designs for Each Trade-off Case

Trade-off Case	Excited Frequencies (Hz)	Period (s)	Notes
Fixed Resolution 1	5 to 10 Hz (30 lines)	6 s	Balanced power
Fixed Resolution 2	5 to 10 Hz (30 lines)	12 s	Higher SNR
Fixed Time 1	5 to 10 Hz (30 lines)	6 s	Baseline
Fixed Time 2	5 to 10 Hz (60 lines)	6 s	Finer resolution
Fixed SNR 1	5 to 10 Hz (30 lines)	6 s	Target SNR 40 dB
Fixed SNR 2	5 to 10 Hz (60 lines)	12 s	Same SNR, more lines

Results and Explanation

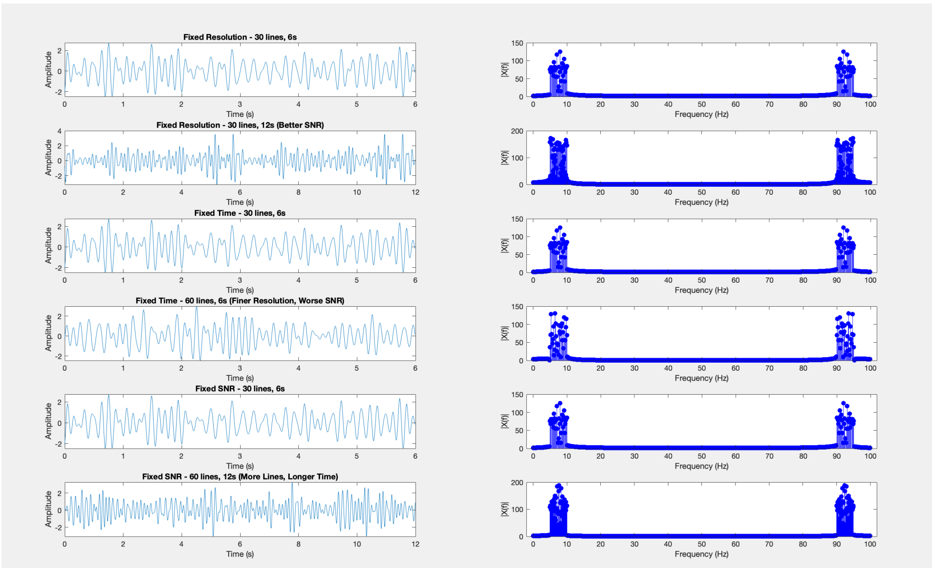

Figure 1: Time and Frequency Domain Plots for Different Multisine Designs Illustrating Trade-offs

Overview

Figure 1 displays the time domain and frequency domain representations of all six multisines. The left column shows the time-domain signals, while the right column shows the corresponding magnitudes of the Discrete Fourier Transform (DFT).

Analysis of Trade-offs

Trade-off 1: Fixed Frequency Resolution

The first two rows represent cases where 30 equally spaced frequencies between 5 Hz and 10 Hz are excited.

- Row 1 uses a 6-second signal.
- Row 2 uses a 12-second signal.

The longer duration reduces noise variance, improving the SNR. This can be seen from the cleaner peaks in the frequency spectrum of Row 2 compared to Row 1.

Trade-off 2: Fixed Measurement Time

The third and fourth rows correspond to a fixed 6-second measurement duration.

- Row 3 excites 30 frequencies.
- Row 4 excites 60 frequencies, doubling the resolution.

Since the total power (RMS fixed to 1V) must be spread over more frequencies in Row 4, the SNR per frequency decreases. This leads to less distinct peaks in the frequency domain.

Trade-off 3: Fixed SNR

The fifth and sixth rows maintain a constant SNR.

- Row 5 excites 30 frequencies over 6 seconds.
- Row 6 excites 60 frequencies, requiring the measurement duration to increase to 12 seconds to maintain SNR.

This illustrates that improving resolution (more lines) requires a proportional increase in measurement time to maintain SNR.

Summary of Observations

This experiment confirms the fundamental triangle of FRF trade-offs:

- Increasing measurement time allows better SNR at constant resolution.
- Increasing frequency resolution reduces SNR if time is fixed.
- Maintaining SNR while increasing resolution requires longer measurement time.

The ability to choose optimal parameters depends on the specific constraints of the experiment (available time, acceptable noise, required resolution).

MATLAB Code Implementation

The MATLAB code used to generate these results is included below:

```
% Tradeoff in FRF Measurement - MATLAB Assignment
clc; clear; close all;

fs = 100;              % Sampling frequency (Hz)
RMS_des = 1;           % Desired RMS value
f_start = 5;           % Start frequency (Hz)
f_end = 10;            % End frequency (Hz)

% Trade-off 1: Fixed Frequency Resolution
N1 = fs * 6; % 6s
frequencies1 = linspace(f_start, f_end, 30);
x1 = generate_multisine(N1, frequencies1, fs, RMS_des);

N2 = fs * 12; % 12s
x2 = generate_multisine(N2, frequencies1, fs, RMS_des);

% Trade-off 2: Fixed Measurement Time
x3 = x1; % Same as Fixed Resolution 1
frequencies4 = linspace(f_start, f_end, 60);
x4 = generate_multisine(N1, frequencies4, fs, RMS_des);

% Trade-off 3: Fixed SNR
x5 = x1;
x6 = generate_multisine(N2, frequencies4, fs, RMS_des);

% Function to generate multisine
```

```
function x = generate_multisine(N, frequencies, fs,
   RMS_des)
    t = (0:N-1) / fs;
    x = zeros(1, N);
    for f = frequencies
        phase = 2*pi*rand();
        x = x + cos(2*pi*f*t + phase);
    end
    x = x * (RMS_des / rms(x));
end
```

Conclusion

This assignment provides a hands-on demonstration of how measurement time, frequency resolution, and SNR are interrelated in FRF measurements. The design of an optimal multisine excitation depends on the specific measurement goals whether the priority is fast measurement, high resolution, or high SNR. Understanding these trade-offs is essential for practical system identification and experimental design.