AE3212-11

Flight Dynamics Assignment

	AE3212-I	Flight Test	AE3212-II
Week 3.1	*Lectures*		*Structures assignment*
Week 3.2	*Lectures*		
Week 3.3	*Lectures*		
Week 3.4	*Lectures*	*Test flights*	*Flight Dynamics assignment (incl. synthesis)*
Week 3.5	*Lectures*	*Test flights*	
Week 3.6	*Lectures*	*Test flights*	
Week 3.7	*Lectures*		

Check BrightSpace!

Admission to the flight test

- Good progress in SVV Structures Assignment
- Currently taking AE3212-I FD
- Not flown flight test before
- Sign the attendance list!

Trouble registering

<u>Not on attendance list / unable to sign in</u>

- Add your name at the end of the list, we'll get back to you

<u>Can't / don't want to fly</u>

- Email Wouter van der Wal: W.vanderWal@tudelft.nl
  you will be added to a group

Incomplete groups will be arranged *later today*

Sign up!

- testflight.tudelft.nl
- See also link on BrightSpace
- Open since Thursday, Feb 27
- SVV-group = flight group

			Escury,F.A.G.	
Tuesday **17/03/2020**	Dekkers,M.G. Kroon,R. Beumer,M.C. Hoogendoorn,H.J. Nagels,T. Keesom,T.B.	van Dalsum,M. Gaffarel,J.Y.P. de Boer,S. Smits,R. van Ede,M.E. El-Kebir,H.	Kanaar,C. Desiderio,M. Campolucci,P. Rodriguez Plaza,E. Trávník,M. Martinez Ruts,M.	Laohamethanee,M.K. Aung Thu Tun, Verkammen,M.H.G. Francotte,T. Kethiri,S.R. Bourgois,J.F.
Wednesday **18/03/2020**	Dijkman,G. Cürgül,Y. Pockelé,J.S. Eftekhar,S. Garcia de Quevedo Suero,B. Riha,B.	Burdo,I.Y. Tunjov,V. van Dalen,S.R. Ummels,R. Vink,T.L.A. Jeon,J.	De hulsters,K. Burr,Z.I. van Veen,A.S.Y. Janssen,I. Tagliacarne,F. van Huffelen,M.C.	Gijsberts,P.R. Sarrazin,N.P.G. de Lange,K.Y.T.Y. Hwang,J. Capus,L. Wen,S.
Thursday **19/03/2020**	Manieri,M. Laar,Y.A. Wegener,M. Sträter,L.P.J. Van Meenen,A.A. Appels,J.P.	Vrouwes,R.M. Middendorp,L.M. Jankowski,M.R. Daugulis,E. Rebosolan,M. Sambath,B.	van Rijthoven,S. Bislip,K.A. Overbosch,E.S.J. Kalsbeek,M.J. Rietveld,M.J. Enting,M.F.G.	Vemulapalli,D. Cacais Pires,D.F. Sakhida,K. Jahjah,N.M. Fazaeli,M.R.B. Hamza,S.M.
Friday **20/03/2020**	3 Tuomaala,A.A.E. Cummins,A.L.G. Floris,M.	Powis,H.M. Sara Boby, Sabino Lopes Vilela Tuna,A. Yoganarasimhan,M.H. Corte Vargas,F. Ramírez Montero,M.	Gonzalez Martinez,P. Silvagni,P. Chatterjee,A. Magri,F. Chung,Y.S. Storme,R.C.G.	1 Dam,F. van Maarschalkerweerd,T.B.G. Schön,P.S. van Deursen,V.D. Steenhuizen,V.C.
Monday **23/03/2020**	Dziarnowska,W. Géczi,M. Origer,S. van Marion,F. Stebbins Dahl,N.P. Jahilo,E.	6		

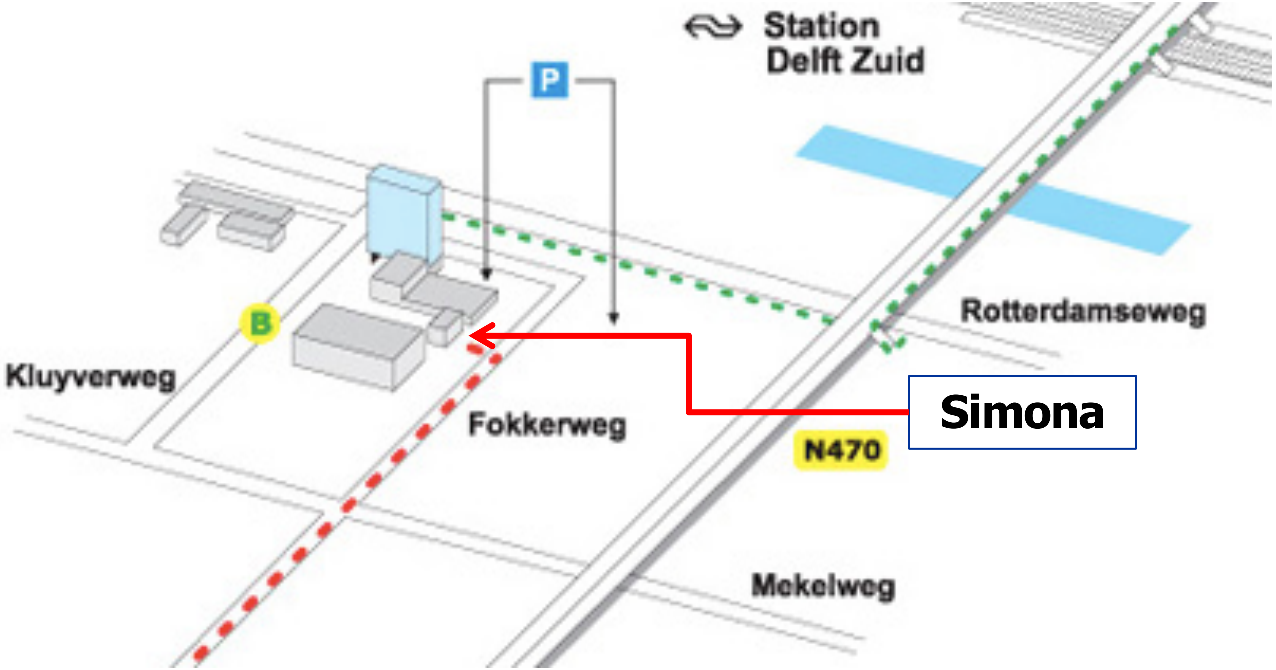
Flight Schedule

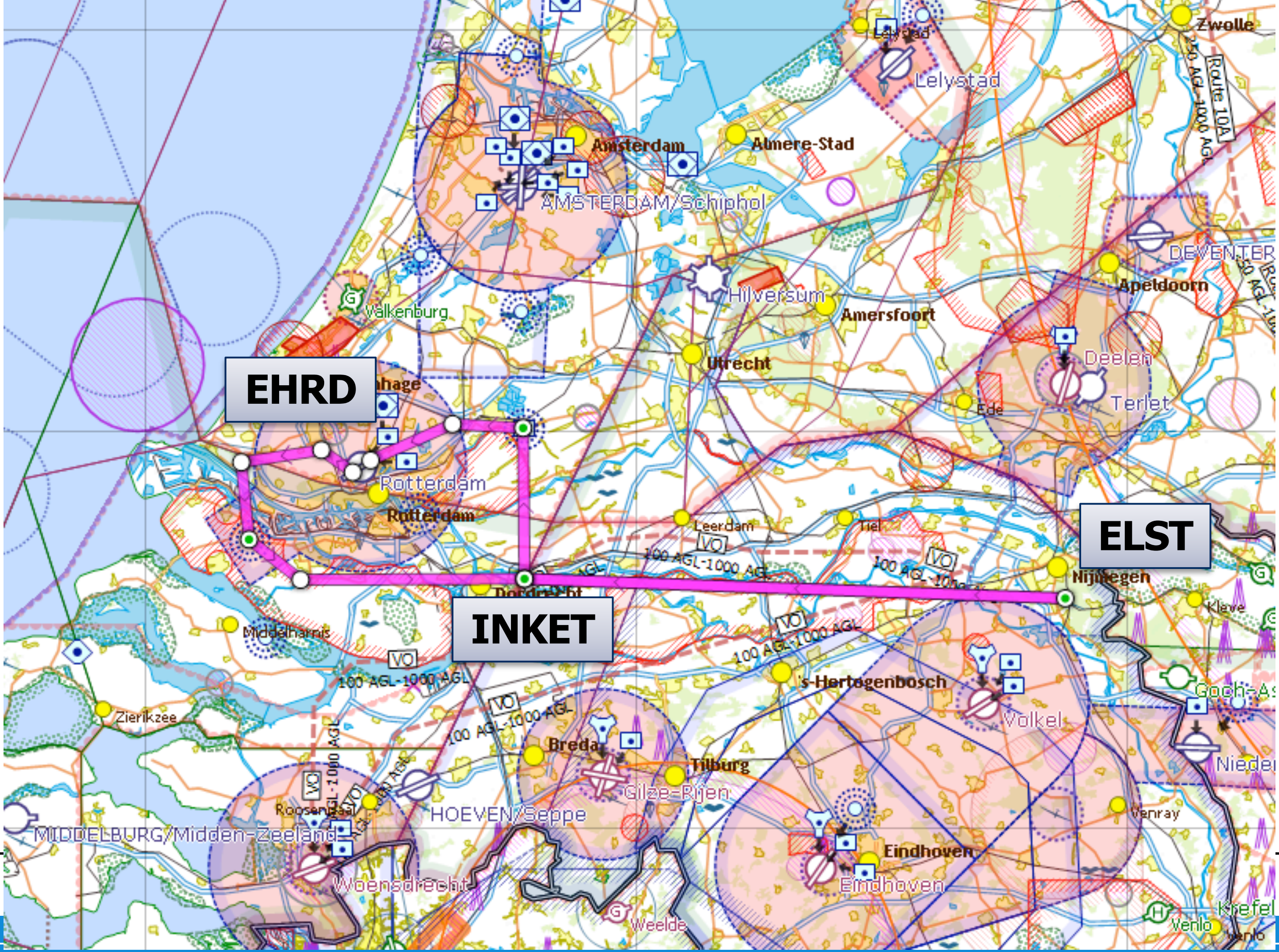
	1st flight	2nd flight	3rd flight	4th flight
mandatory report+check-in Report / Info **06-21203049**	Report 07.45-08.00 in person Check in at 07.45	Report 09.15-09.45 in person Check in at 09.30	Report 09.15-09.45 by telephone Check in at 12.00	Report 09.15-09.45 by telephone Check in at 13.45
Check-in closed, Transportation to Airport	08.00	09.45	12.15	14.00
Start briefing	08.45	10.30	13.00	14.45
Test flight	09.30 – 10.45	11.15 – 12.30	13.45 – 15.00	15.30 – 16.45

BRING YOUR PASSPORT !

Report at room sim 0.11 (Simona building)

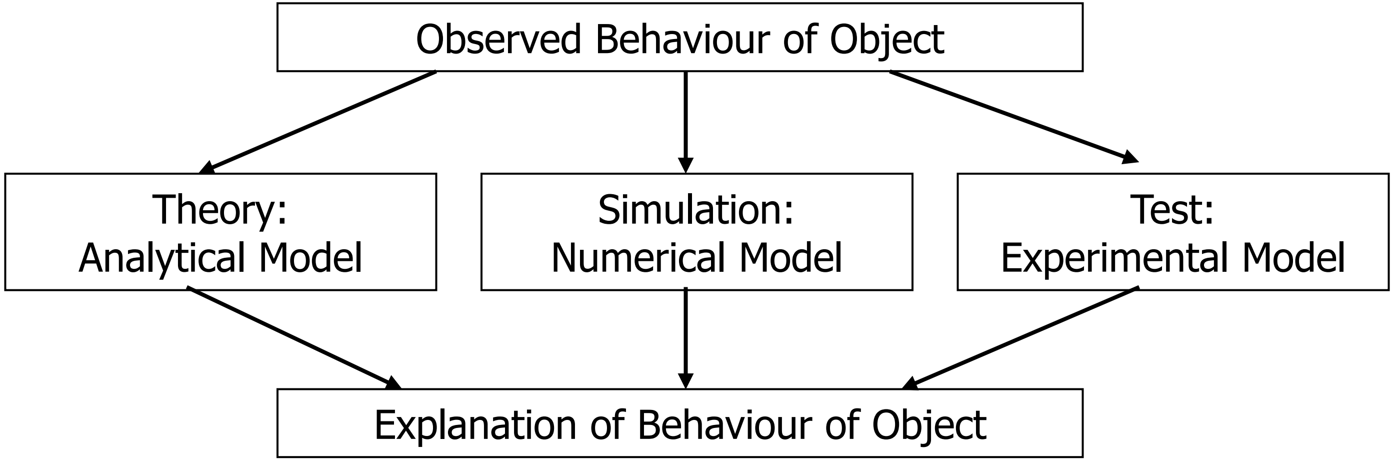
Go to lecture room G, follow signs from there




EHRD
INKET
ELST
Amsterdam
AMSTERDAM/Schiphol
Almere-Stad
Lelystad
Zwolle
Hilversum
Amersfoort
Utrecht
Apeldoorn
Deelen
Terlet
Ede
Valkenburg
Rotterdam
Leerdam
Tiel
Nijmegen
Kleve
Middelharnis
Zierikzee
's-Hertogenbosch
Volkel
Breda
Tilburg
Gilze-Rijen
HOEVEN/Seppe
MIDDELBURG/Midden-Zeeland
Woensdrecht
Eindhoven
Venray
Venlo
Weelde
100 AGL-1000 AGL
VO

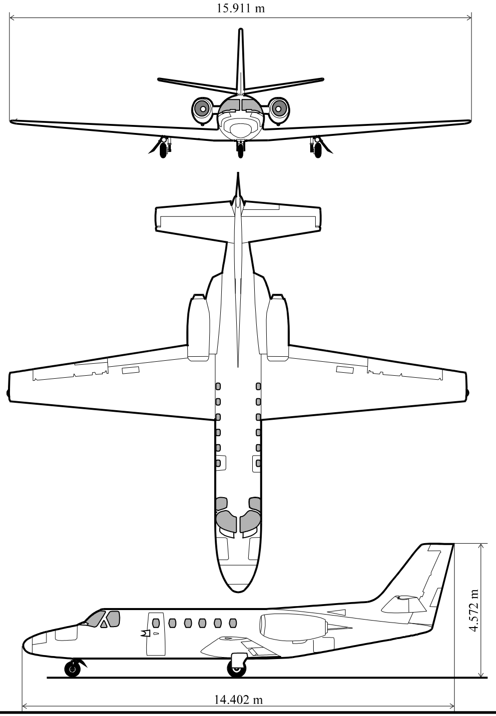
Simulation, Verification and Validation

Theoretical analysis, computer simulation and measuring or testing are used to evaluate, verify and validate observed performance or failure of aerospace vehicles and phenomena



AE3212-II Flight Dynamics Assignment

- New aircraft design
- Limited data available
- Predict dynamic behavior
- Adjust design to improve behavior



Model development

$$C_{m\alpha} = 1.435$$
$$G_m = 7dB$$
$$ROC = 4500\,ft/\min$$

PREDICT

$$C_{m\alpha} = \cancel{1.435}\quad 1.327$$
$$G_m = \cancel{7dB}\quad 5$$
$$ROC = \cancel{4500}\,ft/\min\quad 4250$$

VALIDATE

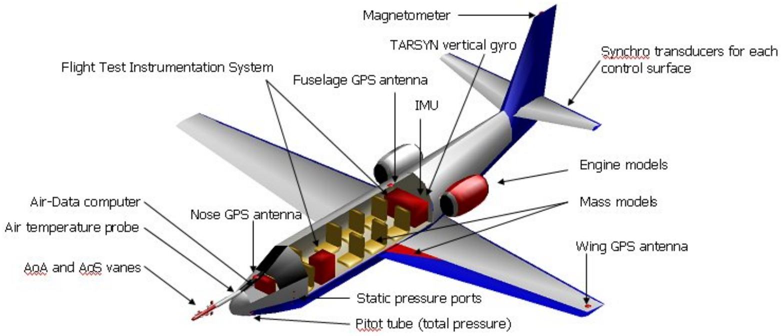
Model validation test method

- Build a mathematical model
- Predict performance and behavior
- Test the model
- Adjust the mathematical model

Flight Simulation purposes

- Fly before you build
- Testing dangerous conditions
- Training simulators
- Future modifications

Flight Test Data



<u>Stationary measurements series 1</u>

- To determine a number of aerodynamic coefficients

<u>Stationary measurements series 2</u>

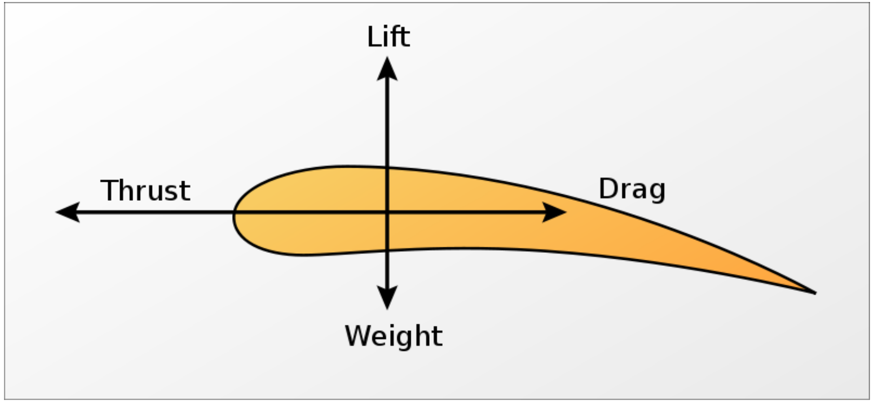
- To determine a number of stability derivatives
- To determine longitudinal stability

<u>Dynamic measurements</u>

- To determine Eigenmotion characteristics
- To validate simulation results

First measurement series

- Steady, horizontal flight
- Constant altitude, varying thrust
- Data for speedrange $V_S \rightarrow V_{MO}$
- One configuration:
  - Gear up, flaps up



Aerodynamic coefficients

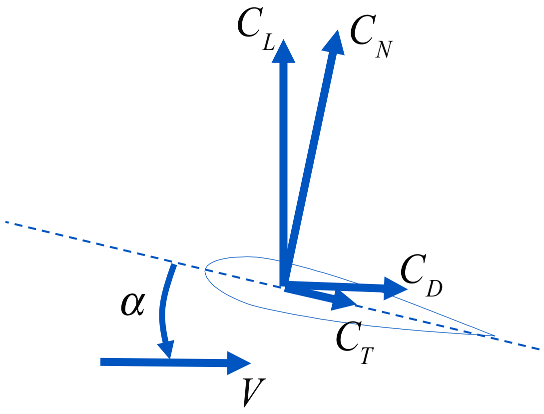
Steady, horizontal flight

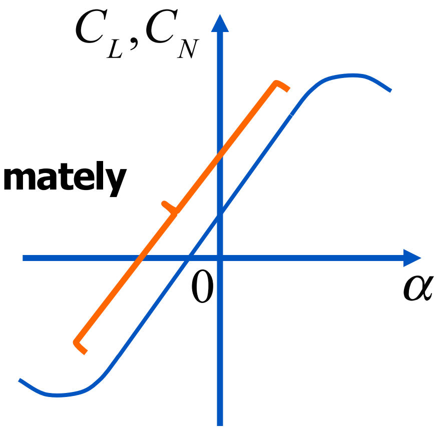
Vertical equilibrium:

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = C_{L_\alpha}(\alpha - \alpha_0) \approx C_{N_\alpha}(\alpha - \alpha_0)$$

→ We can easily measure h, V_{IAS}, α, $Wfuel_{used}$ and T (temperature)….

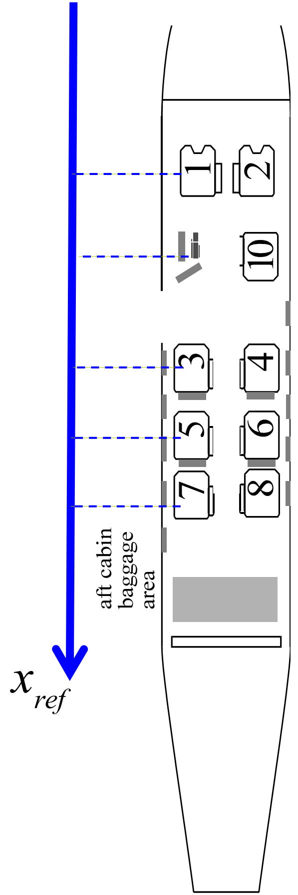
→ and determine ρ, V_{TAS} and W





payload computations			
crew and pax	*[inches]*	*mass [lbs]*	*moment [lbsinches]*
seat 1	131		
seat 2	131		
seat 3	214		
seat 4	214		
seat 5	251		
seat 6	251		
seat 7	288		
seat 8	288		
seat 10	170		
baggage			
nose	74		
aft cabin	321		
	338		
payload			

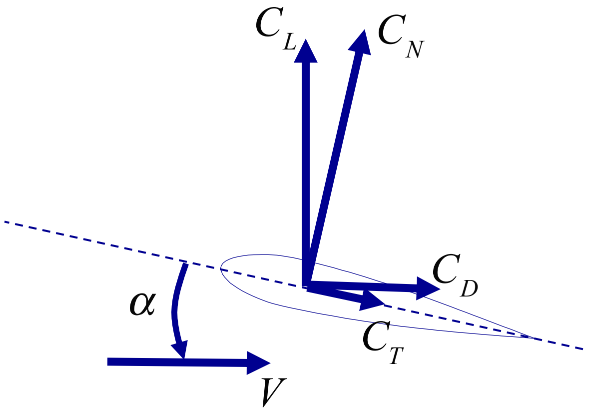
mass and balance computations		
item	*mass [lbs]*	*moment [lbsinches]*
basic empty mass $x_{cg_{datum}}$ at BEM = _____		
payload		
zero fuel mass $x_{cg_{datum}}$ at ZFM = _____		
fuel load		
ramp mass $x_{cg_{datum}}$ at RM = _____		



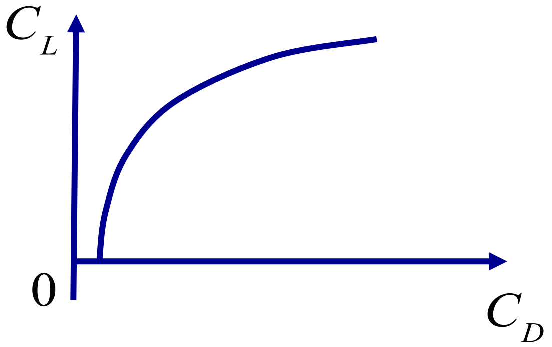
Aerodynamic coefficients

Steady, horizontal flight:

Horizontal equilibrium:



$$C_D = \frac{T}{\frac{1}{2}\rho V^2 S} = C_{D_0} + \frac{C_L{}^2}{\pi A e}$$

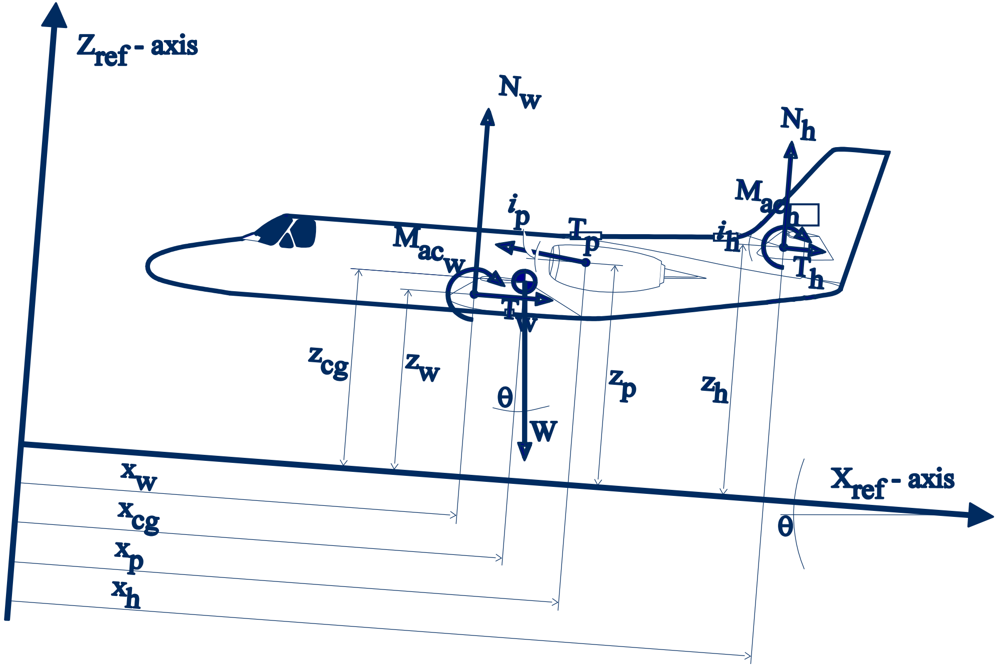


→ T, V and e can be determined from flight test data

Second measurement series

- Quasi-steady, horizontal flight
- Contstant thrust, varying altitude
- Small speedrange around V_{TR}
- One configuration
  - Gear up, flaps up

Forces and moments in symmetric flight



$$M = C_m \tfrac{1}{2} \rho V^2 S \bar{c}$$

Moment equilibrium

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_f}}\delta_f + C_{m_{T_c}}T_c + C_{m_{\mathrm{lg}}}\Big|_{\mathrm{lg\ down}} = 0$$

Static stability for: $C_{m_\alpha} = \dfrac{dC_m}{d\alpha} < 0$

Normal pitch control: $C_{m_\delta} = \dfrac{dC_m}{d\delta_e} < 0$

Elevator trim curve $\delta_e - \alpha$

Equilibrium, landing gear up, flaps up

$$C_m = C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_\delta}\delta_e + C_{m_{\delta_{cf}}}\delta_{cf} + C_{m_{T_c}}T_c + C_{m_{lg}}\Big|_{\text{lg down}} = 0$$

Rewrite:

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + C_{m_\alpha}\left(\alpha - \alpha_0\right) + C_{m_{Tc}}T_c\right\} = f(\alpha)$$

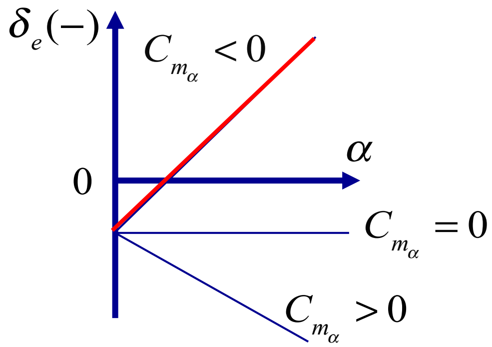
Slope:

$$\frac{d\delta_e}{d\alpha} = -\frac{1}{C_{m_\delta}}C_{m_\alpha}$$

$\ominus$

$$\boxed{\frac{d\delta_e}{d\alpha} < 0}$$

Static stability



Test condition

Equilibrium, landing gear up, flaps up, constant thrust

$$C_m = C_{m_0} + C_{m_\alpha}(\alpha - \alpha_0) + C_{m_\delta}\delta_e + C_{m_{\delta_{cf}}}\delta_{cf} + C_{m_{T_c}} T_c + C_{m_{lg}}\Big|_{lg\ down} = 0$$

Rewrite:

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + C_{m_\alpha}(\alpha - \alpha_0) + C_{m_{Tc}} T_c\right\} = f(\alpha)$$

Using Force Equilibrium W=N:

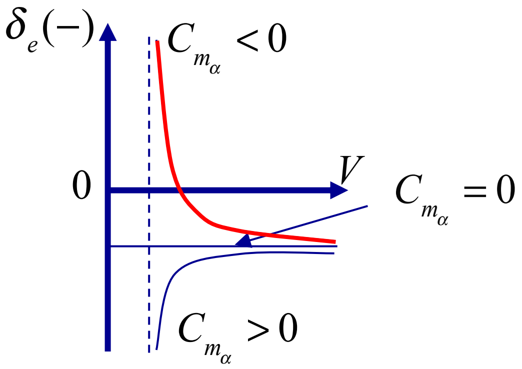
$$C_N \approx C_{N_\alpha}(\alpha - \alpha_0) \approx \frac{W}{\frac{1}{2}\rho V^2 S} \Rightarrow (\alpha - \alpha_0) = \frac{1}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S}$$

Elevator trim curve $\delta_e - V$

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + \frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + C_{m_{T_c}} T_c\right\} = f(V)$$

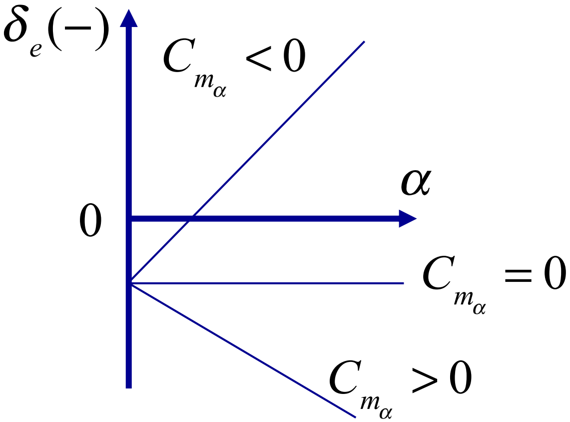
Slope:

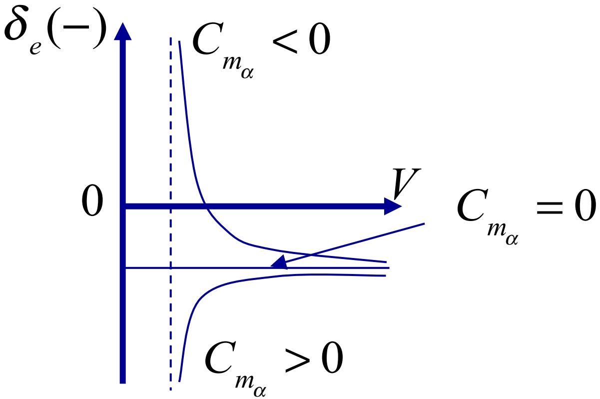
$$\frac{d\delta_e}{dV} = \frac{4W}{\rho V^3 S}\frac{1}{C_{m_\delta}}\frac{C_{m_\alpha}}{C_{N_\alpha}}$$



Static stabilty ($C_{m_\alpha} < 0$ **) for:** $\dfrac{d\delta_{e_{eq}}}{dV} > 0$

Elevator trim curve





Static stabilty for: $\dfrac{d\delta_{e_{eq}}}{d\alpha} < 0 \qquad \dfrac{d\delta_{e_{eq}}}{dV} > 0$

→ We can easily measure *V*,*α* and *δ_e*

Elevator Trim Curve Measurements

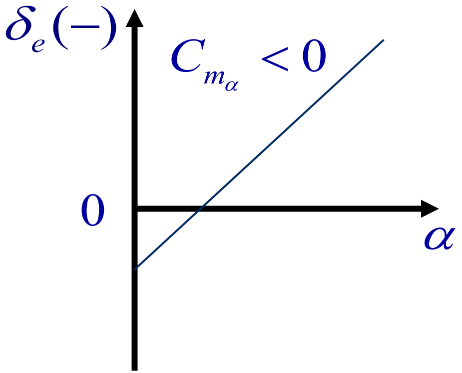
Quasi-steady, horizontal flight

Moment equilibrium:

Stable if: $\dfrac{dC_m}{d\alpha} = C_{m_\alpha} < 0$

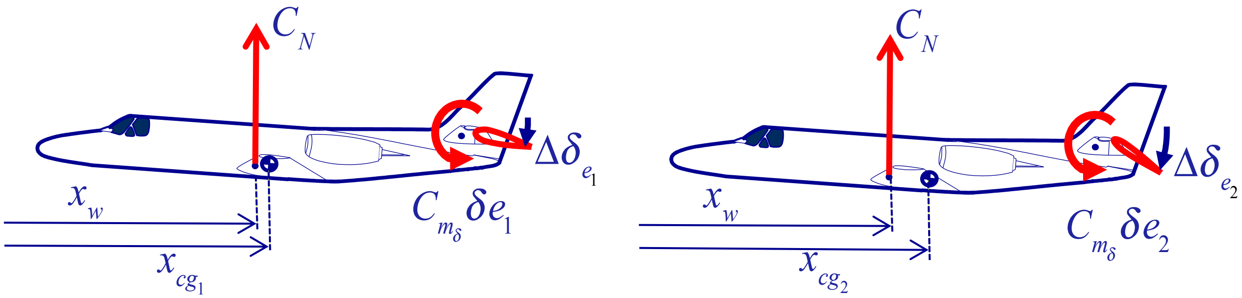
Slope: $\dfrac{d\delta_e}{d\alpha} = -\dfrac{1}{C_{m_\delta}} C_{m_\alpha}$

Long. stabilty: $C_{m_\alpha} = -\dfrac{d\delta_e}{d\alpha} C_{m_\delta}$



Determining elevator effectiveness C_{m_δ}

$$C_m = C_{m_0} + C_{m_\alpha}\Big|_{\delta_e=0}(\alpha-\alpha_0) + C_{m_\delta}\delta_e + C_{m_{\delta_f}}\delta_f + C_{m_{T_c}}T_c + C_{m_{lg}}\Big|_{\text{lg down}} = 0$$



$$\left.\begin{aligned}\Delta C_m &= C_{m_\delta}\cdot\Delta\delta_e\\ \Delta C_m &= C_N\frac{x_{cg2}-x_{cg1}}{\bar{c}}\end{aligned}\right\}\quad C_{m_\delta} = -\frac{1}{\Delta\delta_e}C_N\frac{\Delta x_{cg}}{\bar{c}}$$

with $C_N = \dfrac{W}{\frac{1}{2}\rho V^2 S}$

Data reduction

Test conditions

- Uncontrollable variables — air temperature, density
- Adjustable variables — mass, cg
- Controllable variables — altitude, airspeed, angle of attack

In order to compare results data must be adjusted with respect to standard conditions

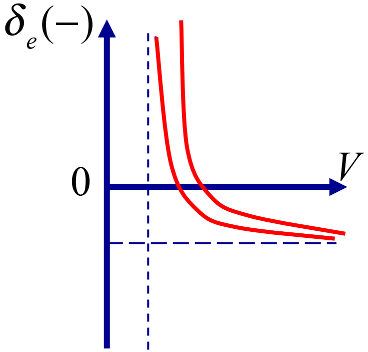
Reduced Airspeed

- We can determine V_c from V_i, using calibration data
- By converting V_c to the equivalent airspeed V_e , the atmospheric variables are reduced to ISA values
- V_e is the airspeed that gives the same dynamic pressure at sea level ISA, as the true airspeed V_t at altitude

$$q = \tfrac{1}{2}\rho V_t^2 = \tfrac{1}{2}\rho_0 V_e^2 \approx \tfrac{1}{2}\rho_0 V_c^2$$

Reduced elevator trim curve $\delta_e^* - \tilde{V}_e$

$$\delta_{e_{eq}} = -\frac{1}{C_{m_\delta}}\left\{C_{m_0} + \frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + C_{m_{T_c}} T_c\right\}$$



Data points for V = 120 m/s:

	δ_e
m = 6200 kg	-0.4
m = 4742 kg	-0.6

Reduced EAS: $\tilde{V}_e = V_e\sqrt{\frac{W_s}{W}}$

Building the mathematical model

- Based on flight dynamics theory
- Use parameters derived from flight test data
- Verify the model by using known data points
- Validate the model against dynamic flight test data
- Predict dynamic behavior

State Space Representation

General Form:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x	**- state vector**	**A**	**- state matrix**
y	**- output vector**	**B**	**- input matrix**
u	**- input vector**	**C**	**- output matrix**
		D	**- direct matrix**

State Space Representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- very compact notation
- we can apply linear algebra
- computers can easily work with matrices

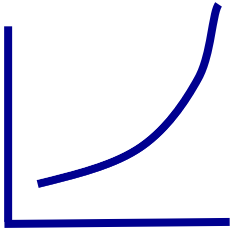
Solution of the equations

$$\dot{x} = Ax$$

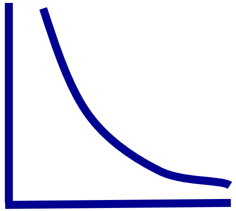
General solution:

$$\overline{x}(t) = ce^{\lambda t}\overline{v}$$

System behavior (λ is real):



$$\lambda > 0$$



$$\lambda < 0$$

Eigenvalues and stability

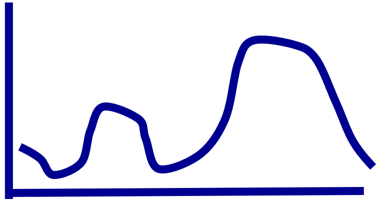
$$\overline{x}(t) = ce^{\lambda t}\overline{v}$$

Complex eigenvalues:

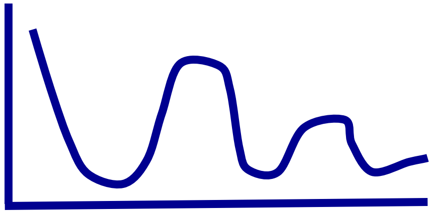
$$\lambda = a + ib$$

$$\Rightarrow \overline{x}(t) = ce^{(a+ib)t}\overline{v} = ce^{at}(\cos b + i\sin b)\overline{v}$$

System behavior real part:



$$a > 0$$

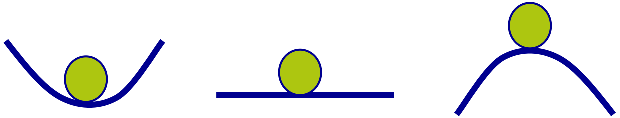


$$a < 0$$

Eigenvalues and stability

$$Ax = \lambda x$$

Positive real part	undamped
Negative real part	damped
Imaginary part	oscilatory



Longitudinal EOM

Linearized, homogenous, deviation equations for symmetric motions:

$$\begin{bmatrix} C_{X_u} - 2\mu_c D_c & C_{X_\alpha} & C_{Z_0} & 0 \\ C_{Z_u} & C_{Z_\alpha} + \left(C_{Z_{\dot{\alpha}}} - 2\mu_c\right) D_c & -C_{X_0} & C_{Z_q} + 2\mu_c \\ 0 & 0 & -D_c & 1 \\ C_{m_u} & C_{m_\alpha} + C_{m_{\dot{\alpha}}} D_c & 0 & C_{m_q} - 2\mu_c K_Y^2 D_c \end{bmatrix} \begin{bmatrix} \hat{u} \\ \alpha \\ \theta \\ \frac{q\bar{c}}{V} \end{bmatrix} = \underline{0}$$

- Eigenvalues describe stability of short period and phugoid

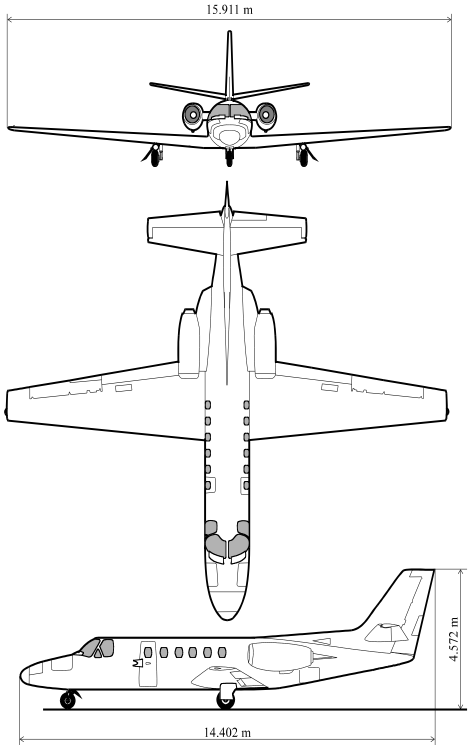
Lateral EOM

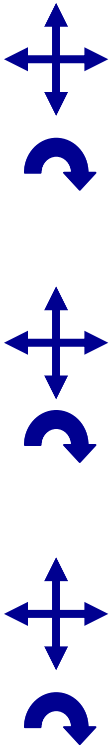
Linearized, homogenous, deviation equations for asymmetric motions:

$$\begin{bmatrix} C_{Y_\beta} + \left(C_{Y_{\dot\beta}} - 2\mu_b\right) D_b & C_L & C_{Y_p} & C_{Y_r} - 4\mu_b \\ 0 & -\frac{1}{2}D_b & 1 & 0 \\ C_{\ell_\beta} & 0 & C_{\ell_p} - 4\mu_b K_X^2 D_b & C_{\ell_r} + 4\mu_b K_{XZ} D_b \\ C_{n_\beta} + C_{n_{\dot\beta}} D_b & 0 & C_{n_p} + 4\mu_b K_{XZ} D_b & C_{n_r} - 4\mu_b K_Z^2 D_b \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ \frac{pb}{2V} \\ \frac{rb}{2V} \end{bmatrix} = \underline{0}$$

- Eigenvalues describe stability of aperiodic roll, spiral and Dutch roll
- Derivation will be treated in week 3.5

Symmetric / asymmetric eigenmotions





Asymmetric

forces and moments: Y, L, N

angular rates: p, r

angles: β, φ

Symmetric

forces and moments: X,Z,M

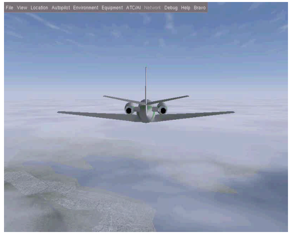
angular rates: q

angles: α, θ

Eigenmotions

Symmetric

- **Short period**
- **Phugoid**

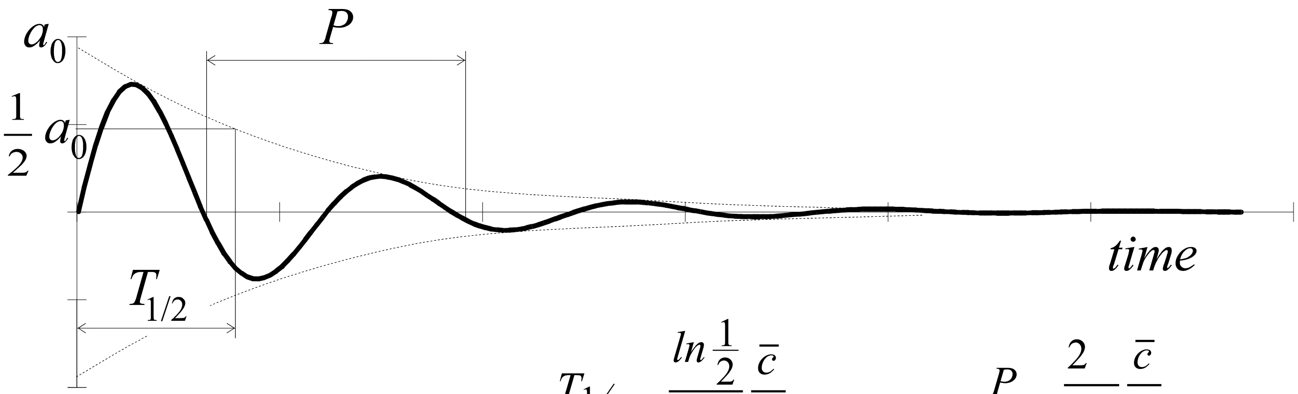


Asymmetric

- **A-periodic roll**
- **Spiral**
- **Dutch roll**

Validating the model

- Compare model properties with observed behaviour

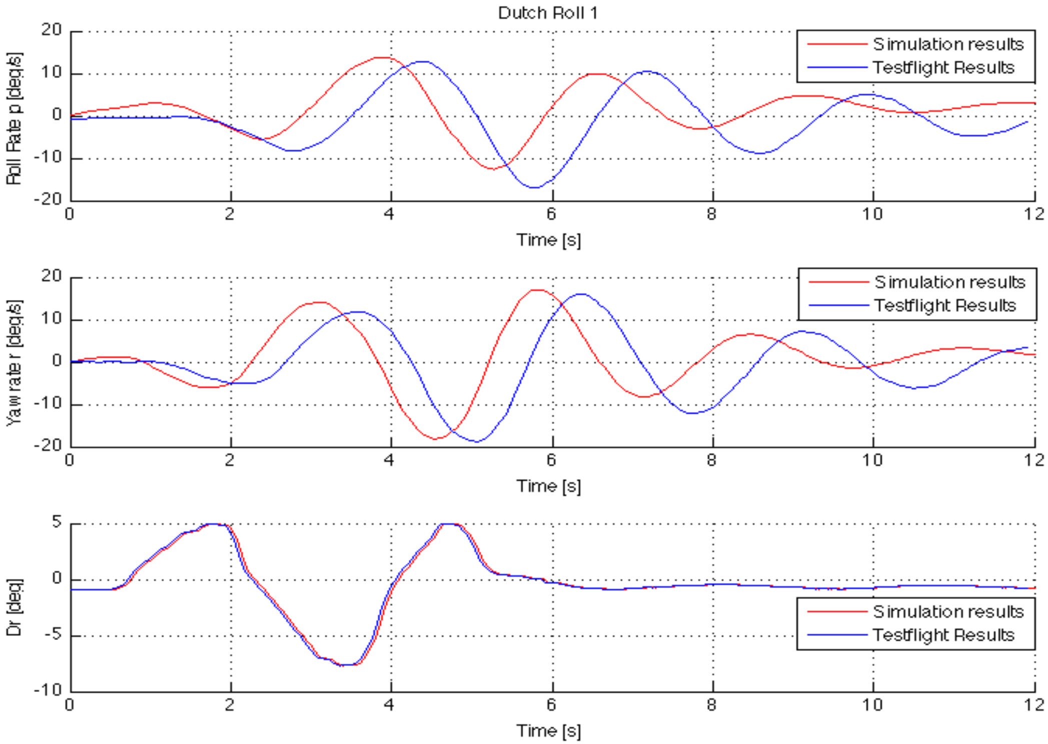


$$T_{1/2} \qquad \frac{\ln\frac{1}{2}}{c}\,\frac{\bar{c}}{V}$$

$$P \qquad \frac{2}{c}\,\frac{\bar{c}}{V}$$

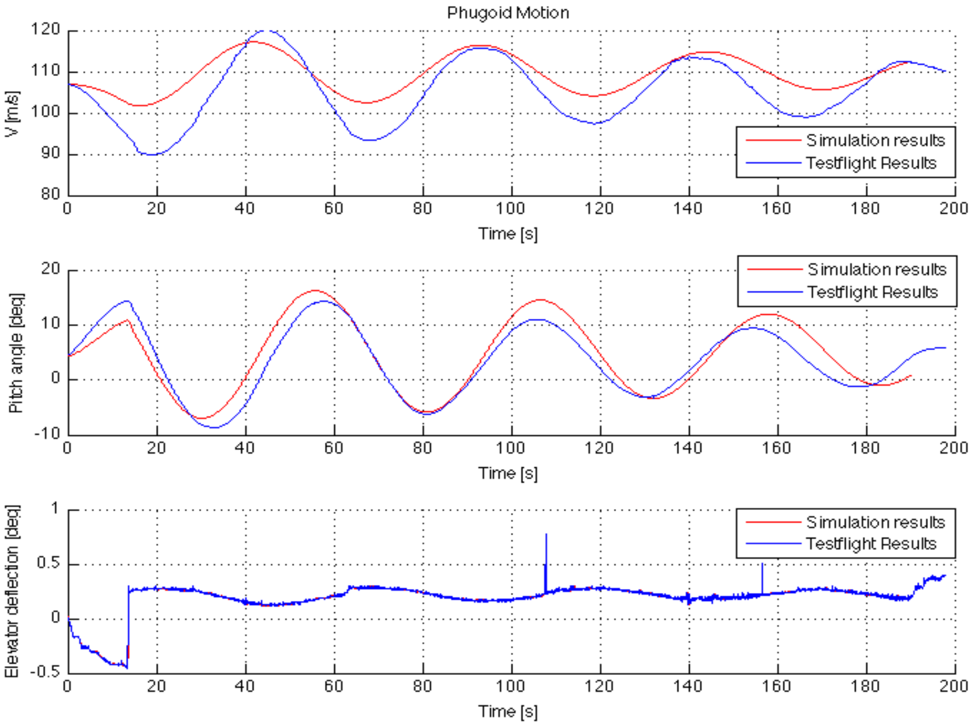
Validating the model

- Compare predicted behaviour to test data



Improving the model

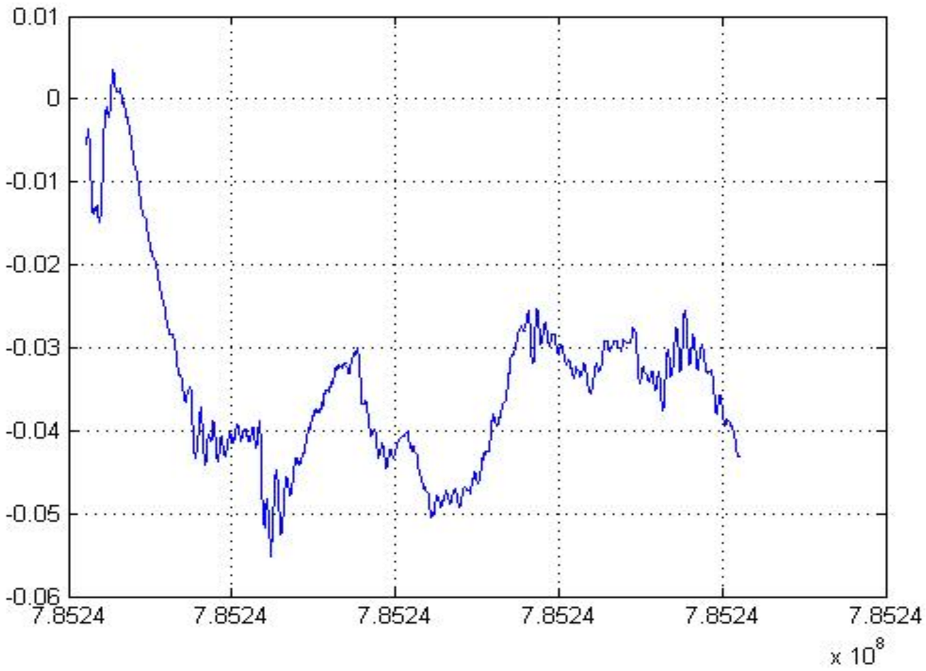
- Change model parameters to get a better match



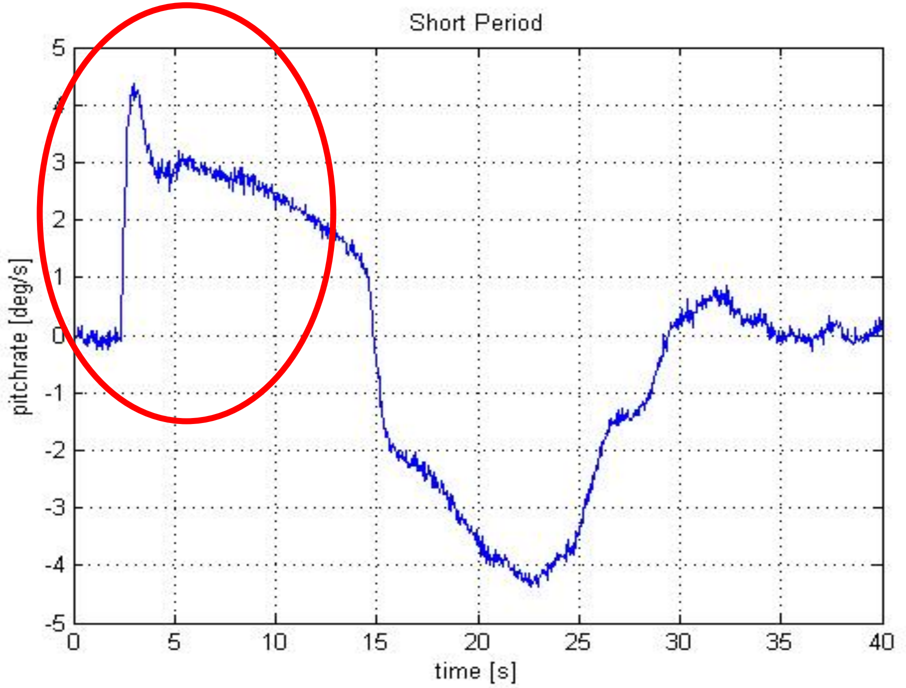
Pointers for the report

- Correct usage of units
- Use only relevant measurements
- Significant digits

How not to present your data



How not to present your data



Last points

- Sign the attendance list before you leave!
- Questions about
  - Flight test: Alexander or Hans
  - SVV groups: Wouter
- Sign up for a flight
- **Don't forget your passport**
- Have fun!