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Online retailers' platform-based Worry-Free-Shopping: Retailing strategy considering consumer valuation bias[☆]

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A R T I C L E I N F O

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A B S T R A C T

Platform-based Worry-Free Shopping (WFS) is an emerging retail service system developed by online platforms. Online retailers on such platforms can actively decide whether to provide WFS services for consumers. If WFS service is offered, a WFS logo will be displayed on the product information page by the platform. Although WFS can enhance consumers' willingness to pay, it incurs cost for online retailers and may cause more product returns due to valuation bias. This paper investigates when an online retailer on a platform should offer customers WFS, and it explores the detailed effects of WFS on the retailer, platform, customers, and society. We find that both the retailer and platform do not necessarily benefit from WFS. Although the platform can take a "hands-off" attitude towards the retailer in most cases, they may have conflicting interests while providing WFS, so we have developed a mechanism to coordinate their benefits. Our results explain why the platform develops such a WFS service system, and when it should incentivize the online retailer to offer WFS to customers. Furthermore, WFS always hurts customers when the valuation bias is small, while it may favor the society when the WFS cost is low. In extensions, we verify the robustness of our results and also get new insights. First, a small product salvage value motivates the platform to encourage the retailer to offer WFS. Second, though a larger proportion of overestimating customers always benefits the platform, it may hurt the retailer when it is less than a threshold. Finally, a larger heterogeneity in valuation increment allows the retailer to charge higher price and benefits the platform.

1. Introduction

E-commerce has experienced rapid growth in recent years (Yu et al., 2022). Forecasting shows that worldwide retail e-commerce sales will account for more than 23% of total retail sales, reaching $7.391 trillion in 2025,[1]. Despite booming e-commerce, evidence suggests that the customer conversion rate is relatively low (Chaudhuri et al., 2021). It is well recognized that the customer conversion rate is crucial for online retailers' financial performance, so online retailers often aim to enhance the customers' willingness to pay to attract consumers (Tong et al., 2022).

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[1] https://www.insiderintelligence.com/content/global-ecommerce-forecast-2022 accessed on 20 July 2022.

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As intermediaries between online retailers and customers, platforms are playing significant roles in enhancing customers' intention to buy. Platforms such as JD.com (one of the largest e-commerce platforms in China), have developed an online retail service system called Worry-Free-Shopping (WFS). The WFS service system contains various retail services. Retailers sells products on the platform and can actively decide whether to provide WFS services for customers. If an online retailer provides WFS service to customers, a WFS logo will be displayed on the product information page.[2] Other platforms, such as Tmall.com, pinduoduo.com, and ebay.com, have also developed similar WFS systems. For example, ebay.com develops a service system called "Shop with confidence", where consumers can enjoy services such as "eBay Money Back Guarantee"[3] and "Vehicle Purchase Protection".[4] Common wisdom suggests that WFS benefits both the online retailer and platform because it can enhance the consumers' willingness to pay and sales. In practice, only a retailer who meets the criteria set by the platform can apply to provide the WFS service. In this paper, we focus on discussing whether online retailers should apply to offer the service, if eligible.

We refer to valuation increment as the amount of increase in consumers' willingness to pay. There are two types of WFS services widely adopted in practice. The first type of services are fulfilled once products are ordered, regardless of whether customers keep or return the products later. For example, the WFS service called *deliver and install for free* ensures that customers can enjoy free delivery and installation when they purchase furniture or electric appliances from the retailers who provide this type of WFS. The cost of this type of WFS occurs once products are ordered, regardless of whether customers keep or return them. Besides, this type of service cannot be enjoyed after the transaction is over. This means that the effect of valuation increment no longer exists when customers decide whether to keep the product or return it, a situation we call *without Service Extension Effect* and label "without SEE". However, for the other type of WFS services, the effect of valuation increment still holds when customers decide whether to keep or return the product. We call this effect as *with Service Extension Effect* and denote this situation as "with SEE" for short (we examine this type of service in an extension). These two types of services together constitute the WFS service system we study in this paper.

Another point is that customers' perceived valuations of products while viewing the products online often do not match the realized valuations after receiving the products, i.e., valuation biases exist when customers shop online (Chen, 2011; Yu et al., 2017). Customers have difficulty in determining their true product valuations because they cannot physically experience the products before receiving them. Li et al. (2019) illustrates that customers cannot judge product quality when shopping online because an online store only describes a product through text or pictures. Xu and Duan (2020) and Sun et al. (2022) further explain that customers encounter uncertain product valuation due to the physical separation between products and consumers. Zhang and Tian (2021) also notes that in online campaigns such as crowdfunding, consumers cannot accurately estimate products because they can only try to understand the products through the information listed on the website.

Moreover, customers have heterogeneous valuation biases (Chen, 2011; Shulman et al., 2011). Some customers may overestimate the valuation of products, i.e., their initial valuation exceeds the realized valuation after they receive the products, while others may underestimate the product valuation, with their initial valuation being less than the realized valuation. When customers underestimate the product valuation, their willingness to pay will decrease, which favors neither the online retailer nor the platform; in this case, WFS serves as a tool to enhance the customers' willingness to pay and reduce the negative effect of the valuation bias for underestimating customers. WFS is also an emerging business model that helps align the interests of the retailer and platform; it stimulates sales for the retailer, which also benefits the platform via commission fees. To systematically examine WFS, we analyze scenarios WFS and No-WFS strategy (i.e., the retailer offers and does not offer WFS service for customers). Specifically, we address the following research questions: (1) What is the optimal WFS strategy for an online retailer? (2) How do the valuation bias, valuation increment, and WFS cost affect the optimal price and profits of the online retailer and platform? (3) Are the interests of the retailer and platform always aligned while providing WFS? If not, how can their benefits be coordinated? (4) Does the provision of WFS always benefit customers and society?

To study these problems, we develop an analytical model and derive the optimal prices and profits of the online retailer and platform in the cases of No-WFS and WFS strategy. We find that, despite the benefits, WFS also has two disadvantages. On the one hand, WFS can enhance the willingness to pay, which may lead to high expectations, and thus more products may be returned when the actual products do not meet such expectations. On the other hand, WFS will cause extra costs for the online retailer due to the extra service provided to customers. Considering these two aspects, it is unclear whether the online retailer should apply WFS to customers and how the retailer's decision will affect the platform. Given the possible benefits and costs, the overall effect of WFS service to the online retailer and platform is not necessarily positive. Furthermore, due to the seeming benefits brought by WFS, one may naturally think WFS is bound to benefit customers and society. However, our findings suggest that this is not always the case.

Our main findings are summarized as follows. First, if WFS cost is small, WFS strategy should be adopted by the online retailer when the valuation increment is large because it can increase sales and cover the negative effects brought by WFS cost. However, if WFS cost is large, though large valuation increment promotes sales, it generates more returns at the same time; in this case, No-WFS strategy should be adopted. Second, we find that the valuation increment from WFS can either positively or negatively influence the retailer's optimal profit due to the joint effects from the WFS cost. In contrast, larger valuation increment always benefits the platform. Third, the interests of the retailer and platform are not always aligned on the provision of WFS. This is because WFS

[2] https://rule.jd.com/rule/ruleDetail.action?ruleId=853909471615913984&type=0&btype=1 accessed on 20 July 2022.
[3] https://pages.ebay.com/ebay-money-back-guarantee/index.html, accessed on 20 July 2022.
[4] https://pages.ebay.com/motors/buy/purchase-protection/, accessed on 20 July 2022.

benefits the platform but hurts the retailer under some conditions; in this case, the platform can encourage the retailer to adopt WFS by offering subsidies to coordinate their benefits. Fourth, we find that the value of WFS to consumers and society is not always positive. Specifically, WFS hurts consumers when the valuation bias is small while it may benefit society when the WFS cost is low.

The paper's contributions are as follows. Theoretically, first, while many studies focus on value-added service from different aspects, few of them investigate it in a platform setting and analyze the effects of value-added service on the retailer and platform simultaneously. Second, our study combines value-added service with consumers' valuation biases, which is a common phenomenon in e-commerce but often overlooked by previous literature on value-added service. Practically, we investigate an emerging service system developed by platforms. We conclude that WFS does not necessarily benefit the retailer, platform, consumers, and society. We also find that though the platform in most cases can take a "hands-off" attitude towards the retailer, their interests may diverge on the provision of WFS under some conditions. Our analysis provides managerial implications for online retailers and platforms.

The remainder of the paper is structured as follows. We review the related literature in Section 2. Section 3 introduces the problem and Section 4 presents model analysis. Next, we discuss managerial implications in Section 5. Section 6 considers multiple extensions to check the robustness of the results and Section 7 concludes the paper with managerial implications and future research directions.

2. Literature review

Our work is primarily related to two streams of literature: (i) retail service and valuation increment; (ii) consumer valuation bias.

2.1. Retail service and valuation increment

This stream of literature focuses on two types of retail services and their roles in improving consumer valuation. The first type is the supporting services provided by websites based on information and communication technologies (ICT). Cenfetelli et al. (2008) reports that supporting services enhance the service quality and the customer's overall satisfaction for online shopping websites. More recently, some studies have proposed the concept of website service quality (Zhou et al., 2009; Shin et al., 2013). Wells et al. (2011) and Tandon et al. (2017) point out that website service quality can enhance consumers' satisfaction and strengthen their online purchase intention. Bolton et al. (2022) further points out that compared with in-store customers, consumers on websites weigh more heavily on cognitive and behavioral qualities during online service encounters.

Apart from platform-provided service, some studies focus on the retailing support service that is provided and fulfilled by retailers. Parasuraman et al. (1985, 1988) originally develops a service quality framework (known as SERVQUAL) with five dimensions: tangibles, reliability, responsiveness, assurance, and empathy, which is a known and commonly used measure.[5] Luo et al. (2012) suggests that service quality, website design, and product pricing dramatically mitigate the negative effects of high product uncertainty and low retailer visibility on customers' online shopping experiences. Ibrahim and Wang (2019) investigates the common topics about online retail services on social media through text analytics and points out that these topics on retail services drive retailers to improve their online services.

The aforementioned works suggest that both types of retail service can enhance consumer satisfaction and willingness to buy. Departing from the aforementioned studies, we develop an analytical model to conduct a comprehensive analysis of WFS by investigating the optimal WFS and pricing strategies of the retailer, as well as how WFS affects the online retailer, platform, consumers, and society.

Some works focus on retail services pricing and investments. Among them, the research on pricing with after-sale services has aroused interest of scholars. Li et al. (2014) investigates the situation where the retailer sells the product bundled with after-sales service to consumers in a fully competitive market. They find that outsourcing market motivates the manufacturer to reduce the wholesale price. There are also studies on warranty service (Tang et al., 2020; Zhang et al., 2020b). In forward and after-sales supply chains, Rezapour et al. (2016) finds that in price-sensitive markets, the firm's highest profit achieves at lower retail prices regardless of the warranty length. Cao et al. (2020) investigates the optimal trade-in and warranty period strategies under carbon tax policy and reveals that when trade-in services are offered for all products, the optimal retail prices of both the new and remanufactured products are higher than those when trade-in services are offered for only remanufactured products. Considering maintenance capacity limits, Zhu et al. (2022) studies the joint pricing and warranty strategy, and finds that OEMs should focus more on a static market with a high warranty length sensitivity but a dynamic market with low sensitivities of warranty length and price. Kirkizoğlu and Karaer (2022) investigates the after-sale service and warranty decisions of a monopolistic durable goods manufacturer and finds that the manufacturer will give away products for free and obtain all profit from after-sale service when the manufacturer can control the after-sale channel. The aforementioned literature investigates the traditional after-sale services in the absence of consumer behavior or online platforms, while our paper investigates an emerging platform-based WFS and incorporates consumer behavior.

Other studies examine the effects of service investment. Hou et al. (2018) concludes that competing retailers can benefit from the investments in delivery service when social network exists. Xue et al. (2020) studies the spillover effect of retail service investment on the timing preference of quantity decisions in a dual-channel supply chain. Based on firms' timing of pricing and service effort

[5] For more information about SERVQUAL, please refer to Ladhari (2009).

decisions, Liu et al. (2022) further considers three timing scheme strategies and finds that the wholesale price-led timing scheme strategies can achieve Pareto improvement. The former literature also focuses on the traditional retail service and does not consider valuation bias. Furthermore, Hong et al. (2020) studies value-added service and analyzes the optimal service investment by firms in a closed-loop supply chain with remanufacturing. However, they concentrate on the traditional supply chain without incorporating online platform and do not consider valuation bias, while our research studies a platform-based supply chain and valuation bias is considered in our paper. Indeed, some studies examine value-added service in a platform or e-commerce setting. For example, Liu et al. (2019) studies how the pricing decisions of a supply–demand matching platform are affected by threshold participating quantity of the providers, value-added service, and matching ability. This work focus on supply–demand matching platform and also does not consider valuation bias. Liu et al. (2021) investigates value-added service in e-commerce setting, but they focus on information sharing between manufacture and an e-tailer, and consumer bias is also not modeled. Zheng et al. (2022) studies supplementary service incorporating an online platform and concentrate on the optimal supplementary service strategies (i.e., third-party service or marketplace service) for the online retailer and online platform, but consumer bias is also not considered. In contrast, we incorporate value-added service and valuation bias simultaneously, which is overlooked in previous literature.

2.2. Consumer valuation bias

Our work is also related to studies on customer valuation bias. Customers' perceived valuation before purchasing often differs from the realized valuation after receiving the products. Dimoka et al. (2012) shows that buyers have difficulty in assessing how products perform in the future. Fu et al. (2016) notes that product returns occur due to transaction inconsistencies, e.g., received products do not match descriptions. Yang et al. (2022) further notes that it is difficult for consumers to obtain a "touch and feel" experience and evaluate the final products accurately. Considering random disturbance after using the products, Crapis et al. (2017) studies monopoly pricing with customers knowing the product quality via social learning (i.e., past experiences and other customers' reviews). Markopoulos and Clemons (2013) suggests that the variation instead of the expectation of buyer reviews is key to reducing consumer uncertainty about taste-related attributes. Choi and Leon (2020) investigates online product reviews to help reduce product uncertainty and facilitate purchases. Li et al. (2022) further points out that if consumers cannot get enough information from reviews of the current generation of products, to resolve uncertainty, they may examine online customer reviews for previous generations of products. This stream of literature mostly focuses on various ways (such as social learning and online reviews) to help consumers assess products. Unlike these studies, we study how WFS works when customers cannot accurately assess the product's value and how valuation bias affects the online retailer's optimal pricing strategy and the profits of the retailer and platform.

Many works investigate the effect of valuation bias on optimal decisions. Chen (2011) studies a capacity-constrained retailer's optimal selling strategy when jointly considering aggregate demand uncertainty and individual valuation uncertainty. Shulman et al. (2011) investigates the optimal pricing and restocking fee strategies when a duopoly facing consumers with heterogeneous tastes for products. Yu et al. (2017) investigates the retailer's optimal selling strategy when considering valuation bias. Although the aforementioned literature incorporates valuation bias in their model and find that it is indeed a non-negligible factor in decision-making, they do not consider platform-based value-added service in their setting. In addition, Yu et al. (2017) considers the scenario where all customers' valuation biases are equal. In contrast, however, following Shulman et al. (2011) and Chen (2011), we consider the more realistic case that the customers' valuation biases are heterogeneous.

2.3. Summary and research gaps

Through reviewing previous studies, we can conclude that our work contributes to the literature as follows. First, most literature on retail service focuses on traditional supply chain and does not incorporate consumer behavior or online platforms, while our paper investigates WFS services in an online platform setting and studies the role of consumer behavior in the retailer and platform's decisions. Second, most literature on valuation bias merely examines how itself influences players' decisions and does not investigate how it can interact with value-added service. Our work helps to address this gap by combining valuation bias with value-added service, and analyze their joint effects on the interests of the retailer and platform.

3. Problem description

Consider an online retailer selling products on a platform, which establishes a WFS system for the retailer who decides whether to provide WFS for customers. If the retailer ("he" for convenience) successfully applies to the platform to provide the WFS service, the WFS logo will be shown on the product's information page, indicating that WFS is provided by the retailer. When a customer ("she" for convenience) sees the logo, she will increase her product valuation by an amount denoted as a ($a > 0$). When purchasing online, customers face the same WFS service for one product hence receive the same information. Consistent with existing literature (Nelson, 1974; Yu et al., 2017), we assume customers have homogeneous valuation increment when they receive the same information. However, we also consider a situation where customers show heterogeneity in valuation increment in Section 6.4 and finds our main results hold. Note that the valuation increment varies across types of products. It is because one WFS service is often subject to similar type of products. For example, *deliver and install for free* service is mainly for furniture and fitness equipment while *exchange but no repair* service mainly for consumer electronics, such as earphones.

Recall that WFS service constitutes of two types of services, i.e., *without Service Extension Effect* (without SEE) and *with Service Extension Effect* (with SEE). For ease of illustration, we first focus on the situation without SEE in our base model and discuss the

Table 1
Summary of notation.

Symbol	Explanation
$\tilde{v}$	A consumer's prior valuation when she sees the products online; $\tilde{v} := v + \tilde{\varepsilon}$.
v	Each customer's true valuation for the product (known after purchase). $v \sim U\left[v_{\min}, v_{\max}\right]$, where $v_{\min}$ and $v_{\max}$ stand for the minimum and maximum valuations for the product.
$\tilde{\varepsilon}$	The consumers' valuation bias. $\tilde{\varepsilon} = \varepsilon$ implies a consumer overestimates, while $\tilde{\varepsilon} = -\varepsilon$ implies a consumer underestimates the true value of a product.
a	The valuation increment due to WFS.
K, $\hat{K}$, $\hat{K}_e$	The number of kept products in the cases of No-WFS strategy, WFS strategy without SEE, and WFS strategy with SEE, respectively.
R, $\hat{R}$, $\hat{R}_e$	The number of returned products in the cases of No-WFS strategy, WFS strategy without SEE, and WFS strategy with SEE, respectively.
π_R, $\hat{\pi}_R$, $\hat{\pi}_{R_e}$	The retailer's profit in the cases of No-WFS strategy, WFS strategy without SEE, and WFS strategy with SEE, respectively. Superscript '*' represents the corresponding optimal profit.
p^*, $\hat{p}^*$, $\hat{p}_e^*$	The optimal price in the cases of No-WFS strategy, WFS strategy without SEE, and WFS strategy with SEE, respectively.
β	The commission rate of platform; $0 < \beta < \frac{1}{2}$.
c	The acquisition cost of products.
s	The salvage value of products; $s \leq c$.
C	The fixed cost of providing WFS.
r	The customer's return cost for one product.
ϕ	The proportion of overestimating customers.

situation with SEE in a model extension. Despite the advantages brought by WFS, it does incur extra costs for the retailer. We denote the WFS cost as C; the product's acquisition cost as c (e.g., wholesale price paid to the manufacturer). In addition, product's salvage value is s ($s \leq c$). To focus on the effect of WFS, we first assume $s = c$ and will relax this assumption in an extension. If the retailer sets the product price at p, he will pay a commission fee βp to the platform, where β is the commission rate. In practice, β is typically unchangeable for a given category and is less than 50%, i.e., $0 < \beta < \frac{1}{2}$.

The customers are heterogeneous in their true product valuations. Each customer's true valuation for product is v, which can only be learned after receiving the products. We assume that v follows a uniform distribution on $\left[v_{\min}, v_{\max}\right]$, where $v_{\min}$ and $v_{\max}$ stand for the minimal and maximal valuations for the product, respectively. To avoid trivial cases and better focus on the value of WFS, we assume $v_{\min}$ is sufficiently small relative to $v_{\max}$ so that the retailer will not sell to all two types of customers, in which situation it is obviously non-optimal for the retailer (see the proof of Table C.1). The uniform distribution helps us clearly capture the heterogeneity of customers' valuations for the product and allows us to derive analytical solutions to investigate important properties. Similar setting has been widely adopted by previous work (Hong and Guo, 2019; Alan et al., 2019; Zhang et al., 2020a; Wan et al., 2020; Guo et al., 2020; Yu et al., 2020; Wang et al., 2021; Luo et al., 2022). Furthermore, We also check the robustness of our results by assuming customers' valuations follow a normal distribution in Section 6.5. We find that our results still qualitatively hold. For ease of exposition, we use $G(\cdot)$ to represent the distribution of v, and $g(\cdot)$ is the corresponding density function. When a customer visits the product's information page online, her perceived valuation of the product is generally not the same as her realized valuation after receiving the product, i.e., valuation bias occurs.

We define $\tilde{v} := v+\tilde{\varepsilon}$ as each consumer's prior valuation when a consumer finds the products online, and $\tilde{\varepsilon}$ denotes the consumer's valuation bias.[6] Note that each consumer's prior valuation consists of two parts: the true valuation v which can be learned by consumers only after they receive the products, and valuation bias ε which will disappear once consumers receive and experience the products. Buyers for whom $\tilde{\varepsilon}$ equals ε overestimate the true value of the product, and we call these *overestimating* customers. Conversely, those for whom $\tilde{\varepsilon}$ equals $-\varepsilon$ underestimate the true value of the product and are called *underestimating* customers. In our paper, we assume $v_{\max}$ is large enough (see the proof of Table C.1) to focus on the case that the retailer sells to both types of customers; if the retailer only sells to the overestimating customers, he has no incentives to provide WFS, which is not interesting. The proportion of overestimating customers is ϕ and that of underestimating customers is $1 - \phi$ ($0 < \phi < 1$). We first assume that over- and underestimating customers account for the same ratio, i.e., $\phi = \frac{1}{2}$, but we shall relax this in an extension.

Recall that WFS has a positive effect of a on the customers' willingness to pay for the product, i.e., WFS increases the customers' initial valuation for the product by a while purchasing. A retailer has no incentive to provide WFS with valuation increment exceeding the valuation bias. For example, a customer who underestimates the true valuation of the product may not buy it due to her low expected valuation. As such, the valuation increment brought by WFS enhances the customer's expected valuation and mitigates the negative effect of bias (i.e., the possibility that the customer does not purchase due to valuation bias). When $a = \varepsilon$, the possibility (or impact) is completely eliminated because an underestimating customer's perceived valuation is equal to her updated valuation after receiving the product. Thus, in equilibrium, the valuation should be increased by at most ε, i.e., $a \leq \varepsilon$. The notation used in this paper is summarized in Table 1.

[6] The valuation increment and valuation bias can be estimated through market investigation by retailers or third-party market research companies (Boyer and Hult, 2006; Guajardo and Cohen, 2018; Liu et al., 2021). For example, the retailer can estimate customers' valuation increments through social media or emails attached with questionnaires. To learn about the valuation bias, the retailer can repay visits after customers receive the products.

After receiving the product, the customer determines whether to keep the product. If she decides to return the product, a return cost will be incurred due to factors such as time, transportation, postal costs, and psychological burden. To highlight the effect of WFS, we assume that the return cost is constant for all consumers, which is consistent with previous literature such as Yu et al. (2017) and Su (2009).

4. Methodology and model analysis

We use the optimization theory to solve our model. We first analyze consumers' behavior and derive the amount of kept and returned products. Based on that, we use optimization method to figure out the optimal decisions. We then detect the value of WFS to the retailer, platform, consumers, and society. To verify the robustness of our results, we also extend our model to different scenarios. In the subsequent Section 4.1, we analyze the situation that the retailer does not provide WFS (No-WFS strategy), as the benchmark to highlight the value of WFS.

4.1. No-WFS strategy

Recall that $\tilde{v}$ is each consumer's prior valuation when a consumer views the product online. Only consumers with prior valuations greater than p, i.e., $\tilde{v} - p \geq 0$, will buy the product. Therefore, the demand for the product in the purchasing stage is: $\int_{p-\tilde{\varepsilon}}^{v_{\max}} g(v)dv = G(v_{\max}) - G(p-\tilde{\varepsilon})$. When customers receive the products, their true valuations, i.e. v, will be realized. The consumers with $v - p \geq -r$ will keep the products while others with $v - p < -r$ will return them. Therefore, the number of kept (K) and returned products (R) can be expressed as follows ($\bar{G}(\cdot) = 1 - G(\cdot)$):

$$K = \bar{G}(\max(p-\tilde{\varepsilon}, p-r)) = \begin{cases} \frac{v_{\max}-p}{v_{\max}-v_{\min}}, if\, \varepsilon \leq r \\ \frac{2(v_{\max}-p)+r-\varepsilon}{2(v_{\max}-v_{\min})}, if\, \varepsilon > r \end{cases} \tag{1}$$

$$R = \begin{cases} \bar{G}(p-\tilde{\varepsilon}) - \bar{G}(p-r),\ if\ p-\tilde{\varepsilon} \leq v < p-r \\ 0, otherwise \end{cases} = \begin{cases} 0, if\, \varepsilon \leq r \\ \frac{\varepsilon - r}{2(v_{\max}-v_{\min})}, if\, \varepsilon > r \end{cases} \tag{2}$$

Therefore, under No-WFS strategy, the profits of the retailer and platform are as follows:

$$\pi_R = ((1-\beta)p - c) \cdot K + (s-c) \cdot R \tag{3}$$

$$\pi_P = \beta p \cdot K \tag{4}$$

Solving the above model, we obtain the optimal pricing strategy and profits of the retailer and platform, as stated in Lemma 1.

Lemma 1. *Under No-WFS strategy, the optimal price and profits of the retailer and platform are as follows:*

(i) When $\varepsilon \leq r$, we have $p^* = \frac{1}{2}(v_{\max} + \frac{c}{1-\beta})$, $\pi_R^* = \frac{((1-\beta)v_{\max}-c)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, and $\pi_P^* = \frac{\beta\left[((1-\beta)v_{\max})^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(ii) When $\varepsilon > r$, we have $p^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(r-\varepsilon) + \frac{c}{1-\beta})$, $\pi_R^* = \frac{\left[(1-\beta)(v_{\max}+\frac{1}{2}(r-\varepsilon))-c\right]^2}{4(1-\beta)(v_{\max}-v_{\min})}$, and $\pi_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\frac{1}{2}(r-\varepsilon)))^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

All proofs are presented in Appendix D. Based on Lemma 1, we have the following proposition.

Proposition 1. *(i) When $\varepsilon \leq r$, ε has no effect on p^*, π_R^*, and π_P^*. (ii) When $\varepsilon > r$, we have p^*, π_R^*, and π_P^* decrease with ε.*

Proposition 1 indicates that the retailer's optimal pricing strategy is quite different under the two cases defined by whether the valuation bias exceeds the return cost. Specifically, if $\varepsilon \leq r$, the bias does not affect the retailer's optimal price, suggesting that the retailer does not need to consider valuation bias in setting a price when consumers can relatively assess the products accurately. Many standard products, such as bottled water, disposable paper cups, and software, which customers are familiar with, belong to this type of product. Otherwise, when the valuation bias is large, i.e., $\varepsilon > r$, the optimal price will decrease with ε, which reveals that the retailer should set a lower price to attract more customers to purchase despite a higher valuation bias. Consequently, the profits of the retailer and platform will decrease due to the lower price.

4.2. WFS strategy

We have analyzed the situation of the retailer not using WFS. In this section, we proceed to investigate the case that the retailer chooses to provide WFS (WFS strategy).

When the retailer provides WFS, each customer's valuation is higher compared to the basis of the initial valuation $\tilde{v}$, so the consumer's net utility is: $\tilde{v} + a - p$. Only consumers with $\tilde{v} + a - p \geq 0$ will buy the product. Therefore, the demand for the product at the time of purchasing is: $\int_{p-\tilde{\varepsilon}-a}^{v_{\max}} g(v)dv = G(v_{\max}) - G(p-\tilde{\varepsilon}-a)$. After each customer receives the product, her true valuation v will be realized, and the consumer's utility after receiving the product is $v - p$. Only consumers with $v - p \geq -r$ will keep the products, while the others with $v - p < -r$ will return them. Therefore, the amount of kept ($\hat{K}$) and returned products ($\hat{R}$) after purchasing in the case of WFS strategy can be expressed as follows:

$$\hat{K}\ :\ \bar{G}(\max(p-\tilde{\varepsilon}-a, p-r)) = \begin{cases} \frac{v_{\max}-p+a}{v_{\max}-v_{\min}}, if\, a \le r-\varepsilon \\ \frac{2(v_{\max}-p)+a-\varepsilon+r}{2(v_{\max}-v_{\min})}, if\, a > r-\varepsilon \end{cases} \tag{5}$$

$$\hat{R}\ :\ \begin{cases} \bar{G}(p-\tilde{\varepsilon}-a)-\bar{G}(p-r),\ if\ p-\tilde{\varepsilon}-a \le v < p-r \\ 0, otherwise \end{cases} = \begin{cases} 0, if\, a \le r-\varepsilon \\ \frac{a+\varepsilon-r}{2(v_{\max}-v_{\min})}, if\, a > r-\varepsilon \end{cases} \tag{6}$$

Therefore, the profits of the retailer and platform under WFS strategy can be expressed as follows:

$$\hat{\pi}_R = ((1-\beta)p - c - C)\cdot \hat{K} + (s-c-C)\cdot \hat{R} \tag{7}$$

$$\hat{\pi}_P = \beta p \cdot \hat{K} \tag{8}$$

We have the following Lemma 2.

Lemma 2. *Under WFS strategy, the retailer's optimal price and the retailer and platform's optimal profits are as follows:*

(i) When $0 \le \varepsilon \le \frac{r}{2}$, *or when* $\frac{r}{2} < \varepsilon \le r$ *and* $0 \le a \le r-\varepsilon$, *we have* $\hat{p}^* = \frac{1}{2}(v_{\max} + a + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^* = \frac{((1-\beta)(v_{\max}+a)-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, *and* $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+a))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(ii) When $\frac{r}{2} < \varepsilon \le r$ *and* $r-\varepsilon < a \le \varepsilon$, *or when* $\varepsilon > r$, *we have* $\hat{p}^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(a+r-\varepsilon) + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^* = \frac{\left[(1-\beta)(v_{\max}+\frac{1}{2}(a+r-\varepsilon))-c-C\right]^2}{4(1-\beta)(v_{\max}-v_{\min})} - C\cdot \frac{a+\varepsilon-r}{2(v_{\max}-v_{\min})}$, *and* $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\frac{1}{2}(a+r-\varepsilon)))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

Lemma 2 shows that the retailer should pay attention to the correlation between parameters ε, r, and a when making pricing decision if he adopts WFS. The following Proposition 2 reveals how the valuation bias affects the optimal price and profits of the retailer and platform.

Proposition 2. *Under WFS strategy, the effects of valuation bias on the optimal price and profits are as follows:*

(i) When $0 \le \varepsilon \le \frac{r}{2}$, *or when* $\frac{r}{2} < \varepsilon \le r$ *and* $0 \le a \le r-\varepsilon$, ε *has no effect on* $\hat{p}^*$, $\hat{\pi}_R^*$, *and* $\hat{\pi}_P^*$.

(ii) When $\frac{r}{2} < \varepsilon \le r$ *and* $r-\varepsilon < a \le \varepsilon$, *or when* $\varepsilon > r$, $\hat{p}^*$, $\hat{\pi}_R^*$, *and* $\hat{\pi}_P^*$ *decrease with* ε.

Proposition 2 provides results similar to Proposition 1 on how the valuation bias affects the optimal price and profits of the retailer and platform. However, the difference is that the optimal price and profits decrease with the valuation bias even when it is moderate (i.e., $\frac{r}{2} < \varepsilon \le r$), as long as the valuation increment is high (i.e., $a > r-\varepsilon$). This is because when the valuation increment is high, consumers with moderate valuation biases will be persuaded to buy the product. However, when valuation bias becomes larger, more customers will choose to return the product because they overestimated its value while purchasing. Therefore, the retailer should lower the price to motivate more consumers to keep, leading to lower profits for both the retailer and platform. This reminds both the retailer and platform that a higher valuation increment will make them worse off even when the valuation bias is moderate. As such, the retailer should set a lower price to induce more consumers to keep and achieve the optimal profitability.

In what follows, Proposition 3 further discusses how the valuation increment and WFS cost influence the optimal price and profits.

Proposition 3. *Under WFS strategy, the impact of the valuation increment and WFS cost on the optimal price and profits are as follows:*

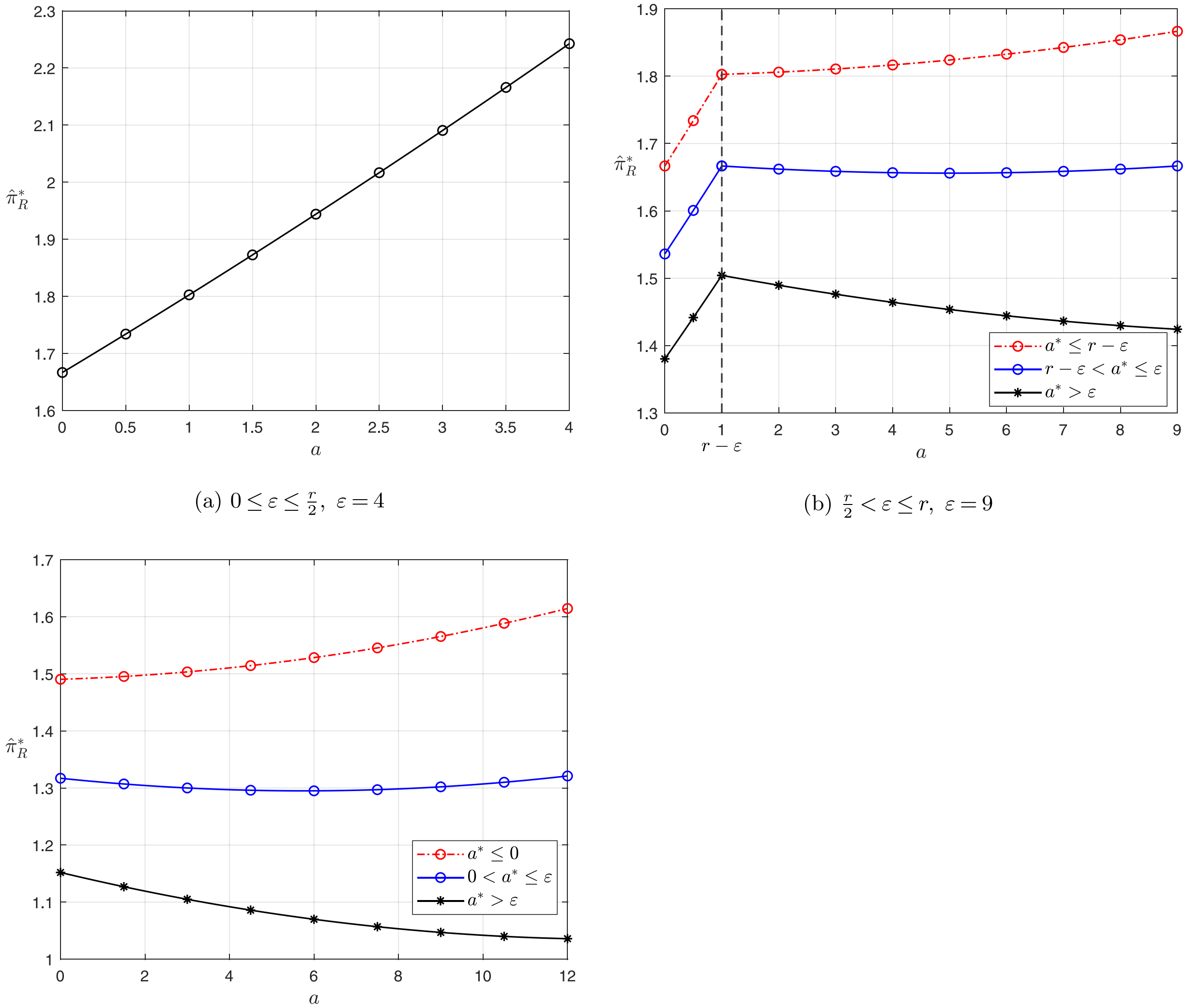
(i) $\hat{p}^*$ *increases with* a *and* C; $\hat{\pi}_P^*$ *increases with* a *but decreases with* C; *and* $\hat{\pi}_R^*$ *decreases with* C.

(ii) When $0 \le \varepsilon \le \frac{r}{2}$, *or when* $\frac{r}{2} < \varepsilon \le r$ *and* $0 \le a \le r-\varepsilon$, *we have that* $\hat{\pi}_R^*$ *increases with* a;

(iii) When $\frac{r}{2} < \varepsilon \le r$ *and* $r-\varepsilon < a \le \varepsilon$, *or when* $\varepsilon > r$, *there exists a threshold* $a^* = \frac{2c+6C}{1-\beta} - (2v_{\max} + r - \varepsilon)$ *with the following properties: If* $a^* \le \max(r-\varepsilon, 0)$, *then* $\hat{\pi}_R^*$ *increases with* a; *If* $\max(r-\varepsilon, 0) < a^* \le \varepsilon$, *then* $\hat{\pi}_R^*$ *decreases with* a *when* $a \in [\max(r-\varepsilon,0), a^*]$, *while* $\hat{\pi}_R^*$ *increases with* a *when* $a \in (a^*, \varepsilon]$; *If* $a^* > \varepsilon$, *then* $\hat{\pi}_R^*$ *decreases with* a.

As shown in Proposition 3, the retailer's optimal price increases with a and C. This result suggests that the retailer should set a higher price if customers care more about the WFS (i.e., if the valuation increment a is larger) or it is more costly to offer the WFS. Second, the platform's optimal profit increases with a because more customers will choose to purchase the products if the WFS is more appealing to them, thus benefiting the platform through the commission fee paid by the retailer. Third, the WFS cost negatively affects the profits of the retailer and platform, which happens because the higher price charged by the retailer (due to the higher WFS cost) causes fewer customers to purchase, thus hurting them both.

We now analyze the effect of the valuation increment on the retailer's optimal profit. Fig. 1 illustrates the results. Specifically, the retailer's optimal profit increases with the valuation increment under two conditions: (1) When the bias is small (Fig. 1(a)), or (2) when the bias is intermediate and the increment is small ($a \le r-\varepsilon$ in Fig. 1(b)). Together, these two conditions indicate that the valuation increment is relatively small, i.e., $a \le r-\varepsilon$ (when $0 \le \varepsilon \le \frac{r}{2}$, we have $a \le \varepsilon \le r-\varepsilon$). This means that the customers



(a) $0 \le \varepsilon \le \frac{r}{2},\ \varepsilon = 4$

(b) $\frac{r}{2} < \varepsilon \le r,\ \varepsilon = 9$

(c) $\varepsilon > r,\ \varepsilon = 12$

Fig. 1. The effect of valuation increment on the retailer's optimal profit. Note: (1) $\beta = 0.2$, $r = 10$, $c = 50$, $v_{\max} = 100$, $v_{\min} = 25$. (2) In (a), $C = 10$. In (b), when $a^* \le r - \varepsilon$, $C = 10$ and $a^* = -1$; when $r - \varepsilon < a^* \le \varepsilon$, $C = 10.8$ and $a^* = 5$; when $a^* > \varepsilon$, $C = 11.8$ and $a^* = 12.5$. In (c), when $a^* \le 0$, $C = 9.5$ and $a^* = -1.75$; when $0 < a^* \le \varepsilon$, $C = 10.5$ and $a^* = 5.75$; when $a^* > \varepsilon$, $C = 11.5$ and $a^* = 13.25$.

attracted by WFS to purchase the product are originally those with high valuations. As such, more customers will purchase with a higher valuation increment and will not return the products, which benefits the retailer.

In contrast, the effect of the valuation increment on the retailer's profit depends on the thresholds of the valuation increment under two other conditions: (1) When the valuation bias is moderate and the valuation increment is large ($a > r - \varepsilon$ in Fig. 1(b)), or (2) when the valuation bias is large (Fig. 1(c)). Note that these two conditions mean that the valuation increment is relatively large, i.e., $a > r - \varepsilon$ (when $\varepsilon > r$, we have $a \ge 0 > r - \varepsilon$). In this case, WFS is more attractive to customers, and those customers who purchase the products may have a relatively small true valuation (i.e., v) for the product, and they may choose to return after receiving products. In other words, when the valuation increment is relatively large, WFS has two opposing effects: the positive effect promotes sales, and the negative effect leads to more returns. We find a threshold that differentiates the effects of valuation increment on the retailer's optimal profit.

Note that the threshold of valuation increment increases with WFS cost ($\frac{\partial a^*}{\partial C} = \frac{6}{1-\beta} > 0$). Therefore, when the WFS cost is sufficiently small such that $a^* \le \max(r - \varepsilon, 0)$, then the retailer's optimal profit increases with the valuation increment. The reason is that the retailer can enjoy the benefits brought by valuation increment without paying high costs to provide WFS due to increased actual demand (i.e., the amount of kept products). When the WFS cost is intermediate such that $\max(r - \varepsilon, 0) < a^* \le \varepsilon$, the retailer's optimal profit first decreases then increases as valuation increment increases. Recall that the valuation increment has two opposing effects. Only when the valuation increment is large enough can the retailer's benefits cover the high cost of WFS due to the increased volume of both kept and returned products. Therefore, the retailer's optimal profit first decreases if the valuation increment is relatively small but increases if the valuation increment is relatively large. Lastly, when the WFS cost is large enough such that $a^* > \varepsilon$, the retailer is always worse off due to the high WFS cost.

Proposition 3 reveals that the retailer can set a higher price when the valuation increment and WFS cost are large, and the platform will always benefit from a large increment. However, the effects of the increment on the retailer's profit are strongly influenced by the WFS cost, which reminds the retailer that he should fully evaluate the WFS cost to detect the positive or negative effects caused by the valuation increment.

5. The value of WFS: Discussion and implications

In this section, we first elucidate the value of WFS to the retailer and platform, and then discuss its effect on consumers and society.

5.1. The value of WFS to the retailer and platform

Letting $\pi_i^* = \hat{\pi}_i^*(i = R, P)$, we can get the indifference curves for the retailer (C_R) and platform (C_P) in each case, which are summarized in Table C.1 in Appendix C. Based on those curves, we characterize the value of WFS for the retailer and platform as follows.

Proposition 4. *(i) If $(a, C) \in \Omega_1$ (or Ω_4 or Ω_7), the retailer should adopt WFS strategy, and this strategy will also benefit the platform.*
(ii) If $(a, C) \in \Omega_2$ (or Ω_5 or Ω_8), the retailer should adopt No-WFS strategy, but this strategy will hurt the platform.
(iii) If $(a, C) \in \Omega_3$ (or Ω_6 or Ω_9), the retailer should adopt No-WFS strategy, and this strategy will also benefit the platform.

Proposition 4 clarifies the value of WFS to both the retailer and platform. With the sample results given in Fig. 2, we can see the insights delivered by the results in Proposition 4. In region Ω_1 of Fig. 2(a), the retailer should provide WFS, which also benefits the platform. Relative to Ω_2 and Ω_3, Ω_1 means that, for the same WFS cost, the valuation increment is relatively large, which increases sales for the retailer. At the same time, the increased number of kept products also benefits the platform. Therefore, WFS benefits both the retailer and platform. In region Ω_2, if the retailer provides WFS, the platform benefits because the smaller valuation increment in region Ω_2 (relative to that in Ω_1) also increases the number of kept products. Unfortunately, WFS will hurt the retailer because the smaller valuation increment leads to a decrease in the number of kept products, so the benefit cannot cover the extra cost of providing WFS. In region Ω_3, the valuation increment is so small that it cannot affect the number of kept products sufficiently, though the retailer has to set a higher price to maximize his profit. Due to the fewer kept products, the platform's benefits are also damaged. Consequently, the provision of WFS hurts both the retailer and platform.

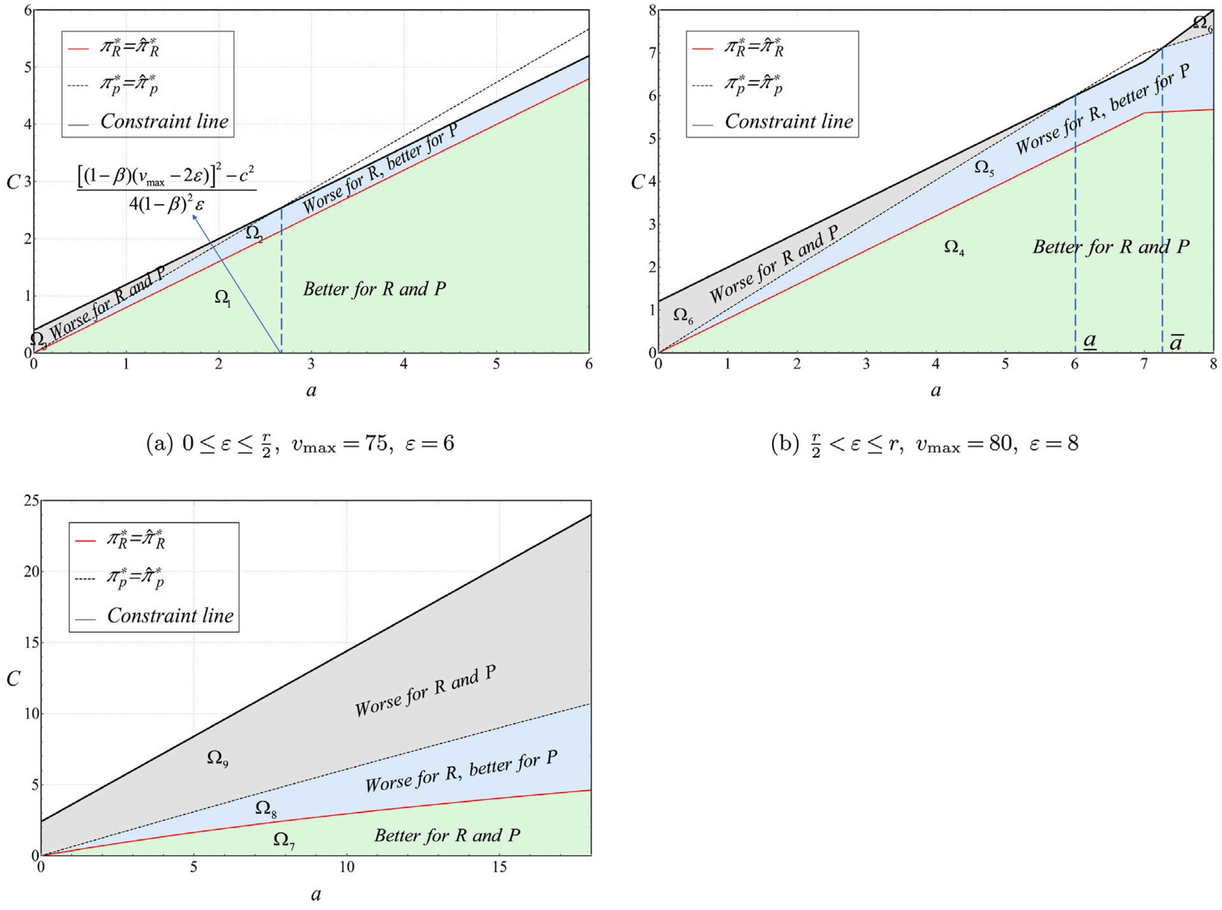
Proposition 4 provides the following managerial implications. First, for the same WFS cost, a tiny increase in valuation increment may benefit the platform while still hurting the retailer, i.e., the platform is more sensitive to valuation increment than the retailer when that increment is moderate. Also, for the same valuation increment, tiny increases in the WFS cost may hurt the retailer while remaining bearable for the platform, i.e., the platform is less sensitive to the WFS cost than the retailer when that cost is moderate. This indicates that the platform is more likely to benefit from WFS and has more incentives to encourage the retailer to adopt the WFS strategy, which may explain that, platforms, such as Tmall.com and JD.com, develop such WFS systems. In addition, platforms also encourage retailers to provide WFS by service illustration in practice. For instance, JD.com tells its retailers that WFS helps to promote consumption.[7]

Second, when the retailer offers WFS (which he does whenever doing so is profitable), the platform can definitely benefit from the retailer using WFS even with the platform taking no other action. This implies that in certain conditions (i.e., the retailer adopts WFS strategy), the platform can take a "hands-off" attitude towards the retailer since the retailer will rationally optimize the profit by himself, and his decision will benefit the platform as well. Third, consider the cases where both the valuation increment and WFS cost are moderate (here, we refer to regions Ω_2, Ω_5, and Ω_8). In such cases, the retailer prefers adopting No-WFS strategy but this strategy will hurt the platform, i.e., the retailer and platform have conflicting interests. This reminds us that under such conditions, the platform should encourage the retailer to adopt the WFS strategy so that the platform itself benefits. We will discuss this further in Section 5.3.

We next derive the conditions under which WFS is bound to be beneficial to the platform. The results are summarized in Corollary 1.

Corollary 1. *(i) When $\frac{((1-\beta)v_{\max})^2-c^2}{4(1-\beta)^2 v_{\max}} \le \varepsilon \le \frac{r}{2}$, if $\frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon} \le a \le \varepsilon$, then the WFS strategy always benefits the platform but may hurt the retailer.*
(ii) When $\frac{r}{2} < \varepsilon \le r$, if $r \ge \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ or $r \ge \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, then there always exists $\underline{a}$ and $\overline{a}$, such that when $a \in [\underline{a}, \overline{a}]$, the WFS strategy always benefits the platform but may hurt the retailer.

[7] See Chapter two on https://rule.jd.com/rule/ruleDetail.action?ruleId=853909471615913984&type=0&btype=1 accessed on 20 July 2022.



(a) $0 \leq \varepsilon \leq \frac{r}{2},\ v_{\max}=75,\ \varepsilon=6$

(b) $\frac{r}{2} < \varepsilon \leq r,\ v_{\max}=80,\ \varepsilon=8$

(c) $\varepsilon > r,\ v_{\max}=100,\ \varepsilon=18$

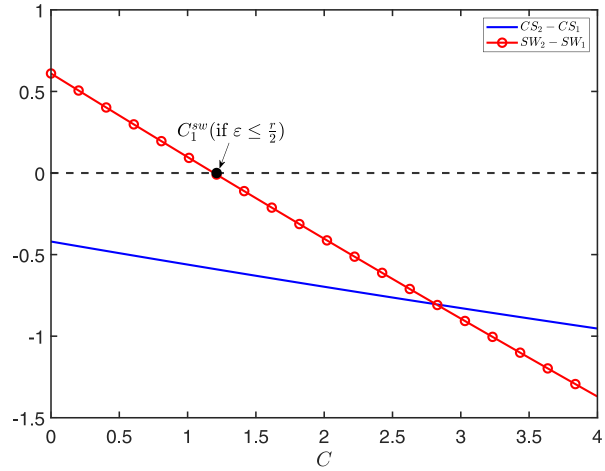
Fig. 2. The Value of WFS to the Retailer and Platform. Note: (1) $\beta = 0.2$, $r = 15$, $c = 50$, $v_{\min} = 25$. (2) We set different $v_{\max}$ when plotting Fig. 2(a), (b), (c) to highlight the results in Corollary 1. Our results hold for all settings of the parameters. (3) $\Omega_i (i = 1, 2, \cdots, 9)$ is defined in the proof of proposition Proposition 4 in Appendix D. (4) The indifference curves for the retailer (C_R) and platform (C_P), and constraint line (C_M) in each case are presented in Table C.1 in Appendix C.

(iii) When $\varepsilon > r$, the WFS strategy is always possible to hurt both the retailer and platform.

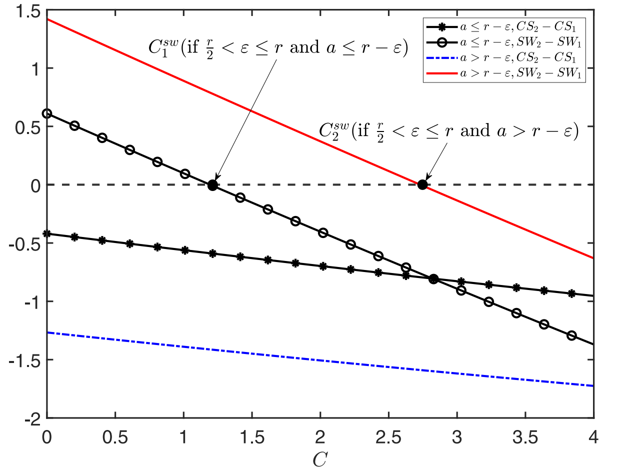
Corollary 1(i-ii) suggests that under some conditions, WFS is always beneficial to the platform even though it may hurt the retailer when the valuation bias is small (i.e., $\varepsilon \leq r$). In contrast, Corollary 1(iii) reveals that when the valuation bias is large (i.e., $\varepsilon > r$), the WFS strategy always has the possibility of hurting both the retailer and platform. Corollary 1 reminds the platform that it should have an especially cautious attitude towards developing WFS when the customer's realized valuation (i.e., v) for the product seriously deviates from the initially perceived valuation (i.e., $\tilde{v}$).

Corollary 1 has the following implications. First, if the valuation bias is relatively small, the platform can always benefit from WFS under some conditions, even though WFS may hurt the retailer. However, with a relative large valuation bias, the WFS strategy is always possible to hurt both the retailer and platform. Second, the platform should design its website to mitigate the consumers' valuation bias when customers read product information. In practice, various methods are used to give clear presentations to consumers, such as product reviews, product videos, live shows, and emerging 3D product introductions (both JD.com and Tmall.com have provided this way to present products[8]). These are effective methods to lessen the consumers' valuation biases for the products because they can significantly enhance consumer understanding of product information. Third, the conditions presented in Corollary 1 can also provide suggestions to the platform regarding whether and when it must be beneficial to encourage the retailer to adopt the WFS strategy.

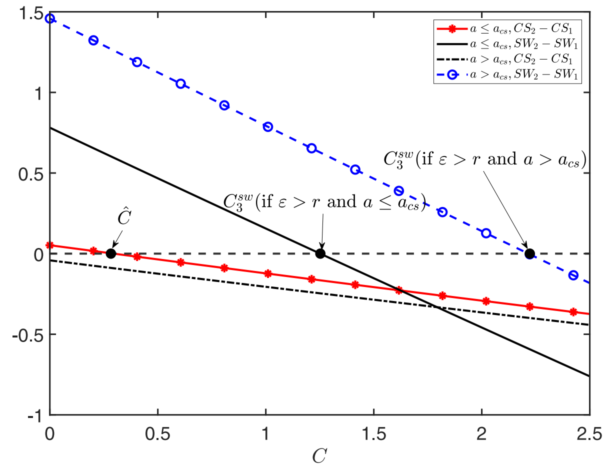
[8] see https://mtt.jd.com/index/video/detail?id=211761211 and http://www.pcpop.com/article/6211799.shtml accessed on 16 December 2021.



(a) $0 \le \varepsilon \le \frac{r}{2},\ \varepsilon = 6,\ r = 15,\ a = 3$



(b) $\frac{r}{2} < \varepsilon \le r,\ \varepsilon = 8,\ r = 15$



(c) $\varepsilon > r,\ \varepsilon = 18,\ r = 3$

Fig. 3. The Value of WFS to consumers and society. Note: (1) $v_{\max} = 100$, $v_{\min} = 25$, $\beta = 0.2$, $c = 50$. In (b), $a = 3$ when $a \le r - \varepsilon$; $a = 8$ when $a > r - \varepsilon$. In (c), $a = 5$ when $a \le a_{cs}$; $a = 10$ when $a > a_{cs}$. (2) $a_{cs} = \frac{2}{7}[7\varepsilon - 7r - 2v_{\max} + \frac{2c}{1-\beta}]$. (3) We use CS_2 (CS_1) and SW_2 (SW_1) to represent consumer surplus and social welfare when the retailer adopts WFS strategy (No-WFS strategy), respectively.

5.2. The value of WFS to consumers and society

In the previous section, we have investigated the value of WFS to the retailer and platform. It is also interesting to further analyze the value of WFS to consumers and society. Proposition 5 summarizes the results. Note that we use *CS* to denote the consumer surplus and *SW* to denote social welfare.

Proposition 5 (*Consumer Surplus and Social Welfare Change*). *compared to the case of No-WFS strategy, if the retailer adopts the WFS strategy, the following hold:*

(i) when $\varepsilon \le r$, WFS always hurts consumers; when $\varepsilon > r$, WFS benefits consumers if and only if $\hat{C} > \max(0, \underline{C}_L^3)$ and $C \le \hat{C}$.

(ii) WFS benefits society if and only if $C_i^{sw} > \max(0, \underline{C}_L^i)$ and $C < C_i^{sw}$, where $i = 1$ when $\varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r - \varepsilon$, $i = 2$ when $\frac{r}{2} < \varepsilon \le r$ and $a > r - \varepsilon$, and $i = 3$ when $\varepsilon > r$.

The thresholds used are summarized in Appendix A. Fig. 3 shows the results intuitively. Interestingly, Proposition 5 states that WFS does not necessarily benefit consumers and the society. Specifically, compared to the No-WFS strategy, Proposition 5(i) illustrates that when the valuation bias is small, *CS* decreases if the retailer provides WFS, which suggests that a small valuation bias will not benefit consumers (Fig. 3(a) and (b)). Secondly, when the valuation bias is large (Fig. 3(c)), WFS benefits consumers only when the valuation increment is small and WFS cost is low (i.e., $a \le a_{cs}$ and $C \le \hat{C}$). Note that the larger valuation bias

Table 2
The range of τ of each case.

	$\tau \in$
$0 \leq \varepsilon \leq \frac{r}{2}$	$\left[\frac{(C-(1-\beta)a)((1-\beta)(2v_{\max}+a)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})}, \frac{\beta(a(1-\beta)^2(2v_{\max}+a)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right]$
$\frac{r}{2} < \varepsilon \leq r$	$\begin{cases} \left[\frac{(C-(1-\beta)a)((1-\beta)(2v_{\max}+a)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})}, \frac{\beta(a(1-\beta)^2(2v_{\max}+a)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right], 0 \leq a \leq r-\varepsilon \\ \left[\frac{(C-\frac{(1-\beta)(a+r-\varepsilon)}{2})((1-\beta)(2v_{\max}+\frac{a+r-\varepsilon}{2})-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{(a+\varepsilon-r)C}{2(v_{\max}-v_{\min})}, \frac{\beta((1-\beta)^2(a+r-\varepsilon)(v_{\max}+\frac{a+r-\varepsilon}{4})-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right], r-\varepsilon < a \leq \varepsilon \end{cases}$
$\varepsilon > r$	$\left[\frac{(C-\frac{(1-\beta)a}{2})((1-\beta)(2v_{\max}+\frac{a+2(r-\varepsilon)}{2})-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{(a+\varepsilon-r)C}{2(v_{\max}-v_{\min})}, \frac{\beta(a(1-\beta)^2(v_{\max}+\frac{a+2(r-\varepsilon)}{4})-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right]$

generates more returns. If the valuation increment is larger, more customers will find the received products does not meet their expectations hence more products will be returned; in addition, higher WFS cost compels the retailer to charge a higher price. Therefore, consumers benefit from relatively small valuation increment and low WFS cost.

In contrast, though Proposition 5(i) shows that consumers will be hurt when the valuation bias is small even when WFS cost is low, Fig. 3 suggests that WFS benefit the society when the WFS cost is relatively low (e.g., $C \leq C_1^{sw}$ in Fig. 3(a)). Low WFS cost can lead to a lower price and more demands, so it is beneficial to all the three stakeholders. Therefore, the society will benefit from low WFS cost.

Proposition 5 generates the following implications. First, WFS always hurts customers in the case of a small valuation bias. In other words, WFS is meaningless for customers when the valuation bias is small, which highlights the fact that the value of WFS for customers does not exist in products for which the customers can accurately determine their valuations. Secondly, WFS is potentially useful to customers when there is a relatively large bias, which is consistent with the normal situation in online shopping for products such as clothes and shoes that probably require physical trials. However, we find that this is not always the case because customers may be hurt by a large valuation increment and high WFS cost. Finally, WFS with low cost should be encouraged since it is potentially beneficial to the society.

5.3. Mechanism design

In Section 5.1, we analyze the value of WFS to the retailer and platform. Through analyses, we find that in some situations (such as Ω_2 and Ω_5), although the retailer does not prefer WFS, it is beneficial to the platform. In other words, the interests of the retailer and platform diverge. In such a situation, the platform should encourage the retailer to adopt the WFS strategy so that both the retailer and platform can earn more profits with WFS. In this section, we design a mechanism that helps to coordinate the benefits of the retailer and platform. Note that coordination can only be achieved under the condition

$$\hat{\pi}_R^* + \hat{\pi}_P^* \geq \pi_R^* + \pi_P^* \tag{9}$$

We assume that the platform pays a subsidy τ to the retailer to encourage him to offer WFS, where τ must satisfy the condition that the profits of the retailer and platform increase when the retailer adopts WFS strategy. Namely, τ should be subject to:

$$\hat{\pi}_P^* - \tau \geq \pi_P^* \tag{10}$$

and

$$\hat{\pi}_R^* + \tau \geq \pi_R^* \tag{11}$$

We specify the range of τ in Lemma 3.

Lemma 3. *The ranges of subsidies the platform should pay to the retailer in different cases are presented in Table 2.*

Note that the exact value of τ can be achieved based on the bargaining power of the stakeholders. However, in our paper, we do not intend to discuss the exact value of τ. Instead, we focus on a more interesting question of how the interactions between the retailer and platform are affected by different parameters. We define the difference between the right and left boundary of τ as the negotiable space between the retailer and platform and denote this difference as S_p. Here, S_p measures the probability of a successful deal, and a high S_p implies both the retailer and platform have sufficient room for maneuvering, indicating a high probability of a successful deal (Note that if $S_p \leq 0$, the negotiation cannot proceed. Therefore, we focus on the case when $S_p > 0$). In the following Proposition 6, we present how the valuation increment and WFS cost affect S_p.

Proposition 6. *The negotiable space between the retailer and platform is affected by the WFS cost and valuation increment as follows:*

(i) S_p always decreases with C.

(ii) When $0 \leq \varepsilon \leq \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \leq r$ and $a \leq r - \varepsilon$, or when $\varepsilon > r$, S_p increases with a;

(iii) When $\frac{r}{2} < \varepsilon \leq r$ and $a > r - \varepsilon$, there exists a threshold $a_s = 6C - 2(v_{\max} - c) + \varepsilon - r$ such that S_p increases with a for $a_s < r - \varepsilon$; when $r - \varepsilon \leq a_s < \varepsilon$, S_p decreases with a for $a \in (r - \varepsilon, a_s]$, while S_p increases with a for $a \in (a_s, \varepsilon]$; S_p decreases with a for $a_s \geq \varepsilon$.

Proposition 6 discusses how the valuation increment and WFS cost affect the platform in different cases. First, we can see that the negotiable space decreases with the WFS cost, which indicates that a high WFS cost is unfavorable for the negotiation. Second, when the valuation bias is small, or when the valuation bias is moderate and the valuation increment is small, or when the valuation bias is large, the negotiable space increases with the increment, suggesting that WFS with a high increment benefits the negotiation in this case.

Furthermore, Proposition 6(iii) states how the effect of the valuation increment on the negotiable space is determined by the WFS cost in the form of a threshold (i.e., a_s). Note that the threshold increases with respect to the WFS cost. Therefore, with a sufficiently small WFS cost such that $a_s < r - \varepsilon$, a higher valuation increment would always benefit the negotiation. However, in the presence of a moderate WFS cost, a higher valuation increment benefits the negotiation only when it is larger than a threshold. In contrast, a higher valuation increment would reduce the probability of successful negotiation with a high WFS cost.

Proposition 6 reveals that both the retailer and platform prefer offering WFS with low cost because it will promote successful negotiation. However, their preferences for WFS with a high valuation increment are jointly affected by WFS cost.

In practice, a retailer may need to pay a joining fee to the platform (such as eBay.com) to obtain the opportunity of reminding consumers of a certain promotion. In this paper, it corresponds to the situation that the retailer applies to the platform for its products to be tagged with the WFS logo. In this case, the platform will charge a joining fee upon the retailer's application. In our mechanism design, the subsidy provided by the platform to the retailer can be considered as a reduction of the joining fee.

Our findings reveal that the valuation bias significantly affects the benefits of the retailer and platform. Its effects on stakeholders' interests are also studied in previous literature, such as Yu et al. (2017), Li et al. (2019), Sun et al. (2020) and Zhang and Tian (2021), while they do not combine platform-based value-added service with valuation bias. Especially, Yu et al. (2017) finds homogeneous valuation bias has significant effect on the retailer's optimal advance selling strategy in the presence of strategic consumers. Our key setting here is the heterogeneity in consumers' valuation biases and the market consists of under- and over-estimating customers. This setting is consistent with (Chen, 2011) and Shulman et al. (2011), who assume that consumers show heterogeneity in their biases. These two types of consumers inspire us to find the positive or negative effects of WFS on the benefits of the retailer and platform.

6. Extensions

In this section, we consider five extensions of our model: the effects of salvage value (s), market composition (ϕ), WFS with SEE (Service Extension Effect), normal distribution of consumers' valuations, and heterogeneity in valuation increment.

6.1. The effect of salvage value

In our main model analysis, we assumed that the product's salvage value equals the acquisition cost of products and the proportions of overestimating and underestimating customers are equal. In what follows, we will relax these two assumptions to check the robustness of our results and derive new insights.

6.1.1. The effect of salvage value on optimal decisions

In this section, we check the robustness of our results by investigating how the optimal decisions are affected by the salvage value and derive new insights.

Proposition 7. *When incorporating the salvage value and market composition, under WFS strategy, the impact of the valuation increment and WFS cost on the optimal price and profits are as follows:*

(i) $\hat{p}^*$ increases with a and C; $\hat{\pi}_P^*$ increases with a but decreases with C; and $\hat{\pi}_R^*$ decreases with C.

(ii) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r - \varepsilon$, we have that $\hat{\pi}_R^*$ increases with a;

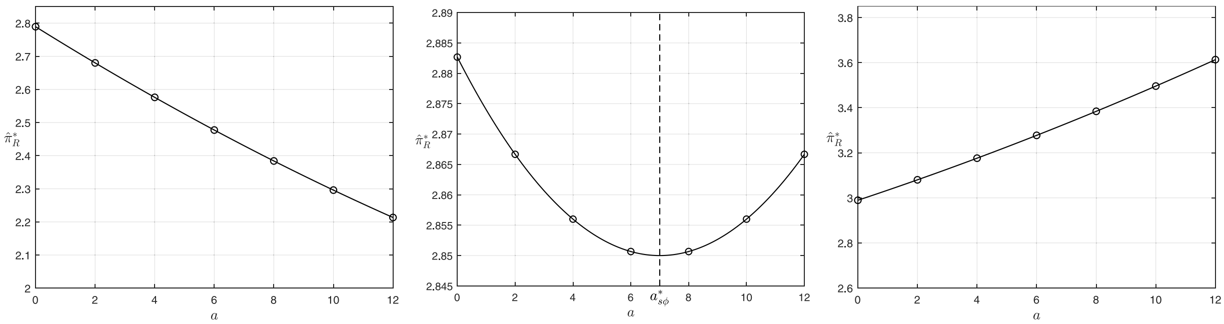
(iii) When $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon < a \le \varepsilon$, or when $\varepsilon > r$, there exists a threshold $a_{s\phi}^* = \frac{(1-\beta)(1-\phi)[(1-\phi)\varepsilon-\phi r-v_{\max}]-2s\phi+(1+\phi)(c+C)}{(1-\beta)(1-\phi)^2}$ with the following properties: If $a_{s\phi}^* \le \max(r-\varepsilon, 0)$, then $\hat{\pi}_R^*$ increases with a; If $\max(r-\varepsilon,0) < a_{s\phi}^* \le \varepsilon$, then $\hat{\pi}_R^*$ decreases with a when $a \in \left[\max(r-\varepsilon,0), a_{s\phi}^*\right]$, while $\hat{\pi}_R^*$ increases with a when $a \in \left(a_{s\phi}^*, \varepsilon\right]$; If $a_{s\phi}^* > \varepsilon$, then $\hat{\pi}_R^*$ decreases with a.

We can see from Proposition 7 that our results still hold qualitatively, compared with Proposition 3. In addition, we find that the threshold of $a_{s\phi}^*$ decreases with the salvage value (i.e., $\frac{\partial a_{s\phi}^*}{\partial s} = -\frac{2\phi}{(1-\beta)(1-\phi)^2} < 0$). To intuitively see this result, we plot Fig. 4 as an example (the case of $\varepsilon > r$). As Fig. 4 shows that, when the salvage value is small ($s = 30$), the retailer's optimal profit always decreases with a; when the salvage value is moderate ($s = 37$), that profit first decreases then increases with a; when the salvage value is large ($s = 45$), that profit always increases with a, which means that a large salvage value makes WFS strategy more favorable to the retailer.

Proposition 8. *When incorporating the salvage value and market composition, the negotiable space between the retailer and platform is affected by the WFS cost and valuation increment as follows:*

(i) $S_p^{s\phi}$ always decreases with C.

(ii) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r - \varepsilon$, or when $\varepsilon > r$, $S_p^{s\phi}$ increases with a;



(a) $s=30;\ a^*_{s\phi}=42$ (b) $s=37;\ a^*_{s\phi}=7$ (c) $s=45;\ a^*_{s\phi}=-33$

Fig. 4. The effect of salvage value on the threshold of valuation increment when $\varepsilon > r$. Note: $v_{\max}=100$, $v_{\min}=25$, $\beta=0.2$, $c=50$, $\varepsilon=12$, $r=10$, $C=2$.

(iii) When $\frac{r}{2}<\varepsilon\le r$ and $a>r-\varepsilon$, there exists a threshold $a_s^{s\phi}=\frac{c+\varepsilon-(2s+r-c+2\varepsilon)\phi+(r+\varepsilon)\phi^2-(1-\phi)v_{\max}+(1+\phi)C}{(1-\phi)^2}$ such that $S_p^{s\phi}$ increases with a for $a_s^{s\phi}<r-\varepsilon$; when $r-\varepsilon\le a_s^{s\phi}<\varepsilon$, $S_p^{s\phi}$ decreases with a for $a\in\left(r-\varepsilon,a_s^{s\phi}\right]$, while $S_p^{s\phi}$ increases with a for $a\in\left(a_s^{s\phi},\varepsilon\right]$; $S_p^{s\phi}$ decreases with a for $a_s^{s\phi}\ge\varepsilon$.

Proposition 8 verifies the robustness of our results in Proposition 6. In addition, Proposition 8(iii) suggests that the threshold of valuation increment decreases with the salvage value (i.e., $\frac{\partial a^*_{s\phi}}{\partial s}=-\frac{2\phi}{(1-\phi)^2}<0$), which indicates that a large salvage value facilitates successful negotiation.

6.1.2. The effect of salvage value on the retailer and platform

In this section, we first detect the effect of salvage value on the value of WFS to the retailer and platform.

Proposition 9. *The effect of salvage value on the retailer and platform is as follows:*

(i) In the cases of No-WFS and WFS strategy: the value of s does not affect the retailer's pricing strategy and the platform's profit.

(ii) Under No-WFS strategy: if the valuation bias is small, then s does not affect the retailer's profit, but when the valuation bias is large, the retailer's profit will increase with s.

(iii) Under WFS strategy: (a) if the valuation bias is small enough, or if valuation bias is moderate and valuation increment is small, then s has no effect on the retailer's profit; (b) if valuation bias is moderate and valuation increment is large, or if the valuation bias is large, then the retailer's profit will increase with s.

(iv) s has a greater impact on the retailer's profit when the WFS strategy is adopted than otherwise.

Proposition 9 reveals the effect of the salvage value on the value of WFS to the retailer and platform. Proposition 9(iv) indicates that if s is smaller, the WFS strategy is more likely to reduce the retailer's profit, which is because more products will be returned due to the high valuation increment. Figs. C.1 and C.2 in Appendix C present the results of Proposition 9(iv) intuitively. We can see from Figs. C.1 and C.2 that when $\varepsilon>\frac{r}{2}$, as s decreases, the WFS strategy is more likely to hurt the retailer while the platform's benefits remain the same. Also, as s decreases, the interests of the retailer and platform are more likely to diverge because the retailer is not willing to adopt WFS. This has two important implications. First, it reminds the retailer that when the salvage value is small, it may not be an optimal strategy for him to provide the WFS. Second, the platform should be aware that the retailer prefers not providing WFS when the salvage value is small and its benefits will be hurt by the retailer's strategy. These results give explanation for the fact that JD.com requires its online retailers of fresh produce to provide the WFS service called "compensate for the rotten products".[9] The retailer is not willing to adopt WFS strategy due to the low salvage value of fresh produce, which will hurt the platform's profit (as Proposition 9 shows). Therefore, the platform, as the rule-maker, optimally takes the enforcement measure to require retailers of fresh produce to provide that service.

6.1.3. The effect of salvage value on consumers and society

In this part, we discuss the effect of salvage value on the value of WFS to consumers and society. Since the salvage value does not affect consumers, here we only need to analyze how it affects social welfare. Proposition 10 summarizes the results.

Proposition 10 (*Social Welfare Change*). *(i) When $0\le\varepsilon\le\frac{r}{2}$, or when $\frac{r}{2}<\varepsilon\le r$ and $0\le a\le r-\varepsilon$, the salvage value has no effect on society;*

[9] See https://rule.jd.com/rule/ruleDetail.action?ruleId=787991265370312704&btype=1, accessed on 20 July 2022.

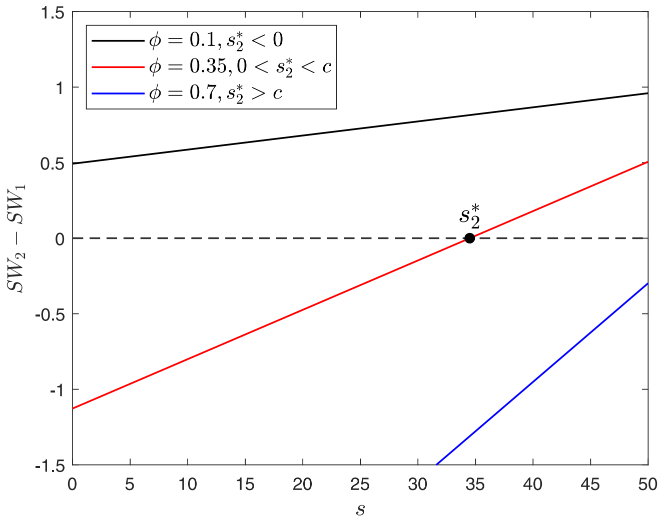


Fig. 5. The effect of salvage value on society when $\varepsilon > r$. Note: (1) $v_{\max} = 100$, $v_{\min} = 25$, $\beta = 0.2$, $c = 50$, $\varepsilon = 10$, $r = 5$, $a = 1$, $C = 1$. (2) Threshold s_2^* is affected by many factors (see Appendix A); in this figure, we focus on the effect of market composition, i.e., ϕ, on this threshold.

(ii) when $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon < a \le \varepsilon$ or when $\varepsilon > r$, compared to the case of No-WFS strategy, we have the following properties under WFS strategy:

(a) if $s_j^* \le 0$, WFS always benefits society; if $s_j^* \ge c$, WFS always hurts society; if $s_j^* \in (0, c)$, WFS hurts society when $s \in [0, s_j^*]$ while WFS benefits society when $s \in (s_j^*, c]$, where $j = 1$ when $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon \le a \le \varepsilon$, while $j = 2$ when $\varepsilon > r$.

The thresholds used in Proposition 10 are summarized in Appendix A. This proposition suggests that when the valuation bias is relatively small, or when the valuation bias is moderate but the valuation increment is small, the salvage value does not affect the social welfare because no customers will return their received products. However, when the valuation bias is moderate and valuation increment is large, or when the valuation bias is large, there exist thresholds s_j^* that differentiate the effects of salvage value on the society. To illustrate the results more clearly, as an example, we plot Fig. 5 that shows the effects of salvage value on society when $\varepsilon > r$. As Fig. 5 shows that, when ϕ is small ($\phi = 0.1$) such that $s_2^* < 0$, WFS strategy can always benefit the society regardless of the salvage value. When ϕ is large ($\phi = 0.7$) such that $s_2^* > c$, however, WFS strategy always hurts the society. When ϕ is moderate ($\phi = 0.35$) such that $0 < s_2^* < c$, only a larger salvage value (i.e., $s > s_2^*$) can make WFS strategy favor the society. This reminds us that, first, if there is a large proportion of overestimating customers in the market, WFS always hurts the society regardless of the salvage value; second, even though the proportion of overestimating customers is moderate, a small salvage value also makes WFS hurt the society.

6.2. The effect of market composition

In this section, we further discuss the effects of the market composition (ϕ) on the optimal decisions and benefits of stakeholders.

6.2.1. The effect of market composition on optimal decisions

In this section, we check the robustness of our results by investigating how the optimal decisions are affected by the market composition. We also derive new insights.

From Proposition 7, we find that the threshold of valuation increment (i.e., $a_{s\phi}^*$) is not monotonically influenced by ϕ. Due to its complexity, we use Fig. 6 (when $\varepsilon > r$) to see the property intuitively.

From Fig. 6, we can see that the effects of market composition (i.e., ϕ) on the threshold of retailer's optimal profit is moderated by the salvage value. Specifically, when the salvage value is relatively small ($s = 20$), $a_{s\phi}^*$ always increases with ϕ; while when the salvage value is relatively large ($s = 42$), $a_{s\phi}^*$ first decreases and then increases with ϕ. However, when ϕ is less than a threshold ($\phi < \phi_1$ or $\phi < \phi_3$; in this case, $a_{s\phi}^* < 0$), $\hat{\pi}_R^*$ always increases with a; when ϕ is moderate ($\phi_1 < \phi < \phi_2$ or $\phi_3 < \phi < \phi_4$; in this case, $a_{s\phi}^* \in (0, \varepsilon)$), $\hat{\pi}_R^*$ decreases with a when $a < a_{s\phi}^*$, and increases with a when $a > a_{s\phi}^*$; when ϕ is larger than a threshold ($\phi > \phi_2$ or $\phi > \phi_4$; in this case, $a_{s\phi}^* > \varepsilon$), $\hat{\pi}_R^*$ always decreases with a. The analysis above verifies the robustness of our results in Proposition 3, and reveals that a larger proportion of overestimating consumers makes WFS strategy less favorable to the retailer. Note that WFS can enhance consumers' willingness of pay. If the market consists of a large proportion of consumers who overestimate the product's value, it is not necessary to adopt WFS strategy and incurs extra costs.

Subsequently, we investigate the effects of market composition (i.e., ϕ) on the threshold of valuation increment in Proposition 8. Due to its complexity, we plot Fig. 7 to see the property intuitively.

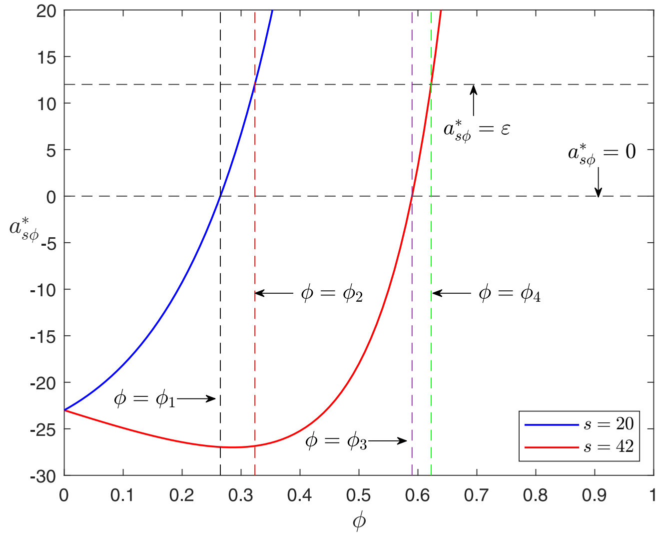


Fig. 6. The effect of market composition on the threshold of retailer's optimal profit when $\varepsilon > r$. Note: $v_{\max} = 100$, $v_{\min} = 25$, $\beta = 0.2$, $c = 50$, $\varepsilon = 12$, $r = 10$, $C = 2$.

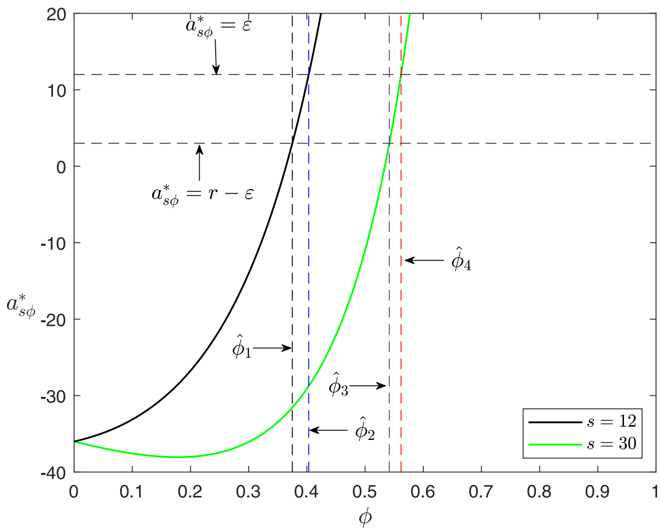


Fig. 7. The effect of market composition on the threshold of negotiable space when $\frac{r}{2} < \varepsilon \le r$ and $a > r - \varepsilon$. Note: $v_{\max} = 100$, $v_{\min} = 25$, $\beta = 0.2$, $c = 50$, $\varepsilon = 9$, $r = 10$, $C = 2$.

Similarly, Fig. 7 shows that the threshold of negotiable space is not monotonically influenced by ϕ (when $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$). Specifically, $a_s^{s\phi}$ always increases with ϕ if the salvage value is relatively small ($s = 12$), while it first decreases and then increases with ϕ if the salvage value is relatively large ($s = 30$). Fig. 7 verifies the robustness of results in Proposition 6 because, $S_p^{s\phi}$ always increases with ϕ if $\phi < \hat{\phi}_1$ or $\phi < \hat{\phi}_3$ (in this case, $a_s^{s\phi} < r - \varepsilon$); if ϕ is moderate (i.e., $\hat{\phi}_1 < \phi < \hat{\phi}_2$ or $\hat{\phi}_3 < \phi < \hat{\phi}_4$; in this case, $a_s^{s\phi} \in (r-\varepsilon, \varepsilon)$), $S_p^{s\phi}$ decreases with a when $a \in (r-\varepsilon, a_s^{s\phi})$, and increases with a when $a \in (a_s^{s\phi}, \varepsilon)$; $S_p^{s\phi}$ increases with a if $\phi > \hat{\phi}_2$ or $\phi > \hat{\phi}_4$ (in this case, $a_s^{s\phi} > \varepsilon$). This result indicates that a large proportion of overestimating consumers may reduce the probability of successful negotiation.

6.2.2. The effect of market composition on price and profits

In this part, we focus on the effect of market composition on pricing and profits.

Proposition 11. *(i) In the cases of No-WFS and WFS strategy: the retailer's optimal price and platform's profit increase with ϕ.*

(ii) Under No-WFS strategy: if $\varepsilon \le r$, then $\hat{\pi}_R^*$ increases with ϕ; if $\varepsilon > r$, there exists a threshold $\phi_1 = \frac{1}{\varepsilon+r}\left[\frac{(3c-2s)\varepsilon-(c-2s)r}{(1-\beta)(\varepsilon+r)} - v_{\max} + \varepsilon\right]$ such that: (a) when $\phi_1 < 0$, $\hat{\pi}_R^*$ increases with ϕ; (b) when $\phi_1 \in [0,1)$, $\hat{\pi}_R^*$ first decreases with ϕ in $(0,\phi_1]$ and then increases with ϕ in $(\phi_1, 1)$; (c) when $\phi_1 > 1$, $\hat{\pi}_R^*$ decreases with ϕ.

(iii) Under WFS strategy: if $0 \le \varepsilon \le \frac{r}{2}$, or if $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, we have that $\hat{\pi}_R^*$ increases with ϕ; if $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$, or if $\varepsilon > r$, there exists a threshold $\phi_2 = \frac{1}{\varepsilon+r-a}\left[\frac{(c-2s)(a-r)+(3c-2s)\varepsilon+(a-r+3\varepsilon)C}{(1-\beta)(\varepsilon+r-a)} - v_{\max} + \varepsilon - a\right]$ such that: (a) when $\phi_2 < 0$, $\hat{\pi}_R^*$ increases with ϕ; (b) when $\phi_2 \in [0,1)$, $\hat{\pi}_R^*$ first decreases with ϕ in $(0,\phi_2]$ and then increases with ϕ in $(\phi_2, 1)$; (c) when $\phi_2 > 1$, $\hat{\pi}_R^*$ decreases with ϕ.

As given in Proposition 11(i), the retailer's optimal price and the platform's profit always increase with ϕ, regardless of whether WFS is provided. When the proportion of overestimating customers becomes larger (larger ϕ), more customers desire to purchase so the retailer can set a higher price, and the platform can get more profits due to the increase of sales. Besides this, Proposition 11(ii–iii) shows that when the valuation bias and valuation increment are relatively small, the retailer's profit increases with ϕ. However, with large valuation bias or valuation increment, the effect of ϕ on the retailer's profit is not monotonic; it is affected by the valuation bias, WFS cost, and valuation increment through the thresholds presented above.

The implications from Proposition 11 are as follows. First, the platform can earn more when the proportion of overestimating customers in the market becomes larger. Note that the threshold of the proportion of overestimating customers increases with WFS cost. Therefore, a high proportion of overestimating customers will always benefit the retailer only when the WFS cost is small. Second, if the WFS cost is moderate, the retailer's profit could increase as long as the proportion of overestimating customers is larger than a threshold. Third, the retailer's profit will always be hurt with a sufficiently large WFS cost, even if there are many overestimating customers in the market. The result implies that excessive publicity of products (which leads to more overestimating customers) is not always encouraged, especially when the cost of providing WFS is high.

6.2.3. The effect of market composition on the retailer and platform

In this part, we detect the effect of market composition on the value of WFS to the retailer and platform. Since the graphical results are similar with Fig. 1, we present them in Appendix C as Figs. C.3–C.5.

Regardless of the market composition, we can get the similar results that are shown in Proposition 4, which examines the robustness of our results. In addition, we can further get some observations as market composition changes. Fig. C.3 shows that when valuation bias is very small, a larger ϕ makes WFS more likely benefit the platform while the retailer's benefits remain the same. Besides, the benefits of the retailer and platform are more likely to diverge as ϕ increases. This implies that in this case, the platform prefers a market with a larger proportion of overestimating customers and coordination of benefits between the retailer and platform is more urgently needed. Fig. C.4 presents that when valuation bias is moderate, as ϕ increases, WFS is more likely to hurt the retailer while the platform and retailer's benefits diverge more easily and coordination is greatly needed. In addition, with a large valuation bias, Fig. C.5 illustrates that a larger ϕ makes WFS strategy an unfavorable choice for both the retailer and platform so avoiding WFS tends to be a better choice for them; interestingly, with ϕ increases, their benefits first diverge then converge, which indicates that, as ϕ increases, relative to a high proportion of overestimating customers, the retailer and platform's benefits more likely need to be coordinated when that proportion is relatively low.

6.2.4. The effect of market composition on consumers and society

In this part, we detect the effects of market composition on consumers and society. Propositions 12 and 13 summarize the results.

Proposition 12 (*Consumer Surplus Change*)**.** *Compared to the case of No-WFS strategy, the following cases hold under WFS strategy:*

(i) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, the following properties hold:

(a) if $a = \frac{C}{3(1-\beta)}$, when $v_{\max} > \frac{2c+C}{2(1-\beta)}$, we have CS is increased; otherwise, CS is decreased; (b) if $a > \frac{C}{3(1-\beta)}$ ($a < \frac{C}{3(1-\beta)}$), there exists a threshold ϕ_{cs}^* such that: when $\phi_{cs}^* \le 0$ ($\phi_{cs}^* \ge 1$), CS is always decreased; when $\phi_{cs}^* \ge 1$ ($\phi_{cs}^* \le 0$), CS is always increased; when $\phi_{cs}^* \in (0,1)$, CS is increased (decreased) if $\phi \in (0,\phi_{cs}^*]$ while CS is decreased (increased) if $\phi \in (\phi_{cs}^*, 1)$.

(ii) When $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon \le a \le \varepsilon$, or when $\varepsilon > r$, (a) if $\sigma_{cs}^j \le 0$, CS is always decreased; (b) if $\sigma_{cs}^j > 0$, CS is increased if $\phi \in \left(\max(0,\underline{\phi}_{cs}^j), \min(\overline{\phi}_{cs}^j, 1)\right)$; otherwise, CS is decreased, where $j=1$ when $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$, while $j=2$ when $\varepsilon > r$.

The thresholds used in Proposition 12 are summarized in Appendix A. Similar to Proposition 5, Proposition 12 suggests that WFS does not necessarily benefit consumers. To see the results more clearly, as an example, we plot Fig. 8 that shows the effect of market composition on consumers when $\varepsilon > r$. Fig. 8 shows that, when valuation increment is small ($a = 5$) such that $\sigma_{cs}^2 > 0$, only when the proportion of overestimating consumers is small (i.e., $\phi < \overline{\phi}_{cs}^{2*}$) can WFS favor consumers; when valuation increment is moderate ($a = 7.2$) but still such that $\sigma_{cs}^2 > 0$, only when ϕ is moderate (i.e., $\phi \in (\underline{\phi}_{cs}^2, \overline{\phi}_{cs}^2)$) can consumers benefit from WFS; when valuation increment is large ($a = 10$) such that $\sigma_{cs}^2 < 0$, consumers will always be hurt if WFS strategy is adopted, regardless of the proportion of overestimating customers. These results present us with the following implications. First, a small proportion of overestimating consumers will make WFS benefit consumers if valuation increment is small. Second, if valuation increment is moderate, WFS strategy benefits consumers only when there is a moderate proportion of overestimating consumers in the market. Third, a large valuation increment can make WFS always hurt consumers regardless of the market composition.

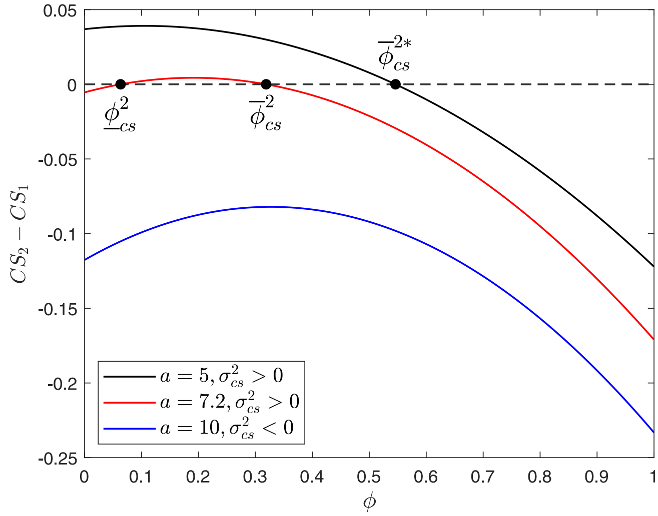


Fig. 8. The effect of market composition on consumers when $\varepsilon > r$. Note: (1) $v_{\max} = 115$, $v_{\min} = 25$, $\beta = 0.2$, $c = 65$, $\varepsilon = 15$, $r = 2$, $C = 0.1$. (2) Threshold σ_{cs}^2 is affected by many factors (see Appendix A); in this figure, we focus on the effect of valuation increment, i.e., a, on this threshold.

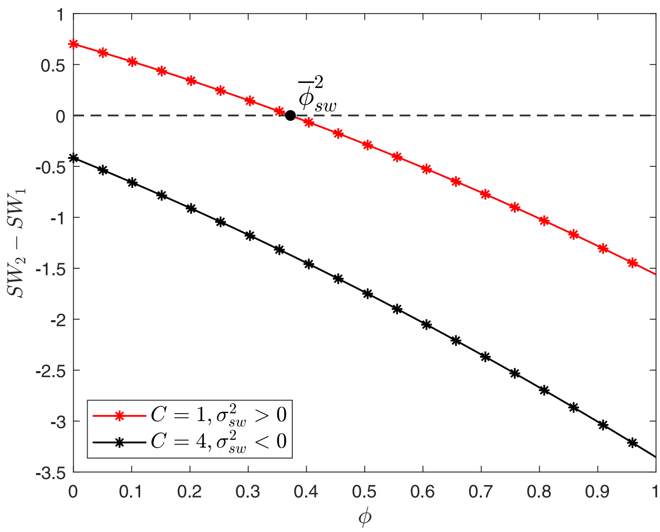


Fig. 9. The effect of market composition on society when $\varepsilon > r$. Note: (1) $v_{\max} = 115$, $v_{\min} = 25$, $\beta = 0.2$, $c = 65$, $\varepsilon = 15$, $r = 2$, $s = 50$. (2) Threshold σ_{sw}^2 is affected by many factors (see Appendix A); in this figure, we focus on the effect of WFS cost, i.e., C, on this threshold.

Proposition 13 (*Social Welfare Change*). *Compared to the case of No-WFS strategy, the following cases hold under WFS strategy:*

(i) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r - \varepsilon$, there exists a threshold ϕ_{sw}^* such that: when $\phi_{sw}^* \le 0$, SW is always decreased; when $\phi_{sw}^* \ge 1$, SW is always increased; when $\phi_{sw}^* \in (0,1)$, SW is increased if $\phi \in (0, \phi_{sw}^*]$ while SW is decreased if $\phi \in (\phi_{sw}^*, 1)$;

(ii) When $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon \le a \le \varepsilon$, or when $\varepsilon > r$, (a) if $\sigma_{sw}^j \le 0$, SW is always decreased; (b) if $\sigma_{sw}^j > 0$, SW is increased if $\phi \in \left(\max(0, \underline{\phi}_{sw}^j), \min(\overline{\phi}_{sw}^j, 1)\right)$; otherwise, SW is decreased, where $j = 1$ when $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon < a \le \varepsilon$, while $j = 2$ when $\varepsilon > r$.

The thresholds used in Proposition 13 can be found in Appendix A. Similar to Proposition 5, Proposition 13 also indicates that WFS may hurt society. To help us illustrate the implications more clearly, we present the result when $\varepsilon > r$ as an example in Fig. 9. As Fig. 9 shows that, if WFS cost is small ($C = 1$) such that $\sigma_{sw}^2 > 0$, only a small proportion of overestimating consumers (i.e., $\phi < \overline{\phi}_{sw}^2$) can favor the society; if WFS cost is large, however, WFS always hurts the society regardless of market composition. This reminds us that only a small proportion of overestimating consumers and WFS cost can benefit the society.

6.3. WFS with service extension effect

Recall that WFS services can be classified into two types: without SEE and with SEE. We have analyzed WFS services without SEE in the previous part of the paper. We now turn to analyze WFS services with SEE. WFS services with SEE benefit customers only if they keep the products. For example, the WFS service called *exchange but no repair*[10] refers to the service that, if the products have problems during use (e.g., faulty earphones), the retailer providing this WFS service will replace the defective product with a completely new one without any charge. The cost of this type of WFS service, however, occurs only if customers keep the products. Furthermore, this type of service can be enjoyed as long as the buyer encounters problems even though the transaction is over; in this case, the effect of valuation increment still matters in consumers' final decisions (i.e., whether to return or keep the products). We will show that our main results still qualitatively hold, and we also get some new insights.

6.3.1. Comparison between the cases with and without SEE

Compared to the case without SEE, two aspects differentiate the case with SEE. First, WFS with SEE still affects a consumer's utility after she receives the product (as she can enjoy WFS while keeping the product). Therefore, a consumer's utility after receiving the product is $v+a-p$ (if she receives the product, the true valuation can be realized, so $\tilde{\varepsilon}=0$). Only consumers with $v+a-p\geq -r$ will keep the products while the others, with $v+a-p<-r$, will return them. Second, different from the case without SEE (in which the WFS cost occurs whenever the product is ordered), the cost of the WFS service with SEE occurs only when the product is kept.

Based on these observations, we can express the online retailer and platform profits as follows:

$$\hat{\pi}_{R_e} = ((1-\beta)p-c-C_e)\cdot\hat{K}_e+(s-c)\cdot\hat{R}_e \tag{12}$$

$$\hat{\pi}_{P_e} = \beta p\cdot\hat{K}_e \tag{13}$$

Now we can determine the retailer's optimal pricing strategy and the retailer and platform's optimal profits, as expressed in Lemma 4.

Lemma 4 (*With SEE*)**.** *The optimal price and the optimal profits of the retailer and platform are as follows:*

(i) When $\varepsilon\leq r$, $\hat{p}_e^*=\frac{1}{2}(v_{\max}+a-(1-2\phi)\varepsilon+\frac{c+C_e}{1-\beta})$, $\hat{\pi}_{R_e}^*=\frac{((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C_e)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, *and* $\hat{\pi}_{P_e}^*=\frac{\beta\left[((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon))^2-(c+C_e)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(ii) When $\varepsilon>r$, $\hat{p}_e^*=\frac{1}{2}(v_{\max}+a+\phi r-(1-\phi)\varepsilon+\frac{c+C_e}{1-\beta})$, $\hat{\pi}_{R_e}^*=\frac{((1-\beta)(v_{\max}+a+\phi r-(1-\phi)\varepsilon)-c-C_e)^2}{4(1-\beta)(v_{\max}-v_{\min})}+(s-c)\frac{\phi(\varepsilon-r)}{(v_{\max}-v_{\min})}$, *and* $\hat{\pi}_{P_e}^*=\frac{\beta\left[((1-\beta)(v_{\max}+a+\phi r-(1-\phi)\varepsilon))^2-(c+C_e)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

We can see from Lemma 4 that the retailer's optimal price and platform's optimal profit always increases with the valuation increment, while the retailer and platform's optimal profits always decreases with the WFS cost; these results are the same with that in Proposition 3. However, different from Proposition 3, the following Proposition 14 tells us that a larger valuation increment always benefits the retailer.

Proposition 14. *Compared to the case without SEE, the retailer's optimal profit always increases with the valuation increment.*

Proposition 14 reveals that the influence of the valuation increment with SEE is different from that without SEE. Specifically, without SEE, the effects of valuation increment on the retailer's optimal profit is not monotonic, as shown in Proposition 3. However, with SEE, the retailer's profit always increases with the valuation increment. That happens because, with SEE, customers return products only due to valuation bias instead of the valuation increment, and a higher increment will increase sales without generating more returns.

6.3.2. The value of WFS with SEE to the retailer and platform

In this part, we further examine the value of WFS with SEE to the retailer and platform. We show that similar results can be seen compared to the case without SEE. As shown in Fig. 10, even with SEE, WFS is not always an optimal choice for the online retailer and platform. For example, in Ω_{12} and Ω_{15}, WFS will hurt both the retailer and platform. Otherwise, we can also find that the platform is more sensitive to the valuation increment and less sensitive to the WFS cost. In addition, We can also find that the benefits of the retailer and platform may diverge under some conditions, such as that represented by Ω_{11} and Ω_{14}. All these results are similar to the case without SEE, which testifies to the robustness of our results.

[10] The two services "exchange but no repair" and "warranty" are different. If a customer wants to enjoy "exchange but no repair" service, she/he first needs to return the faulty products (the return fee is paid by the retailer) and then the retailer will replace the defective product with a completely new one to her/him without any charge (see https://rule.tmall.com/tdetail-4758.htm?spm=a223k.10052707.0.0.7218496dfKUzKb&tag=self accessed on 16 December 2021.). In contrast, "warranty" service means that the retailer provides repair service without the need for consumers to return the products. Therefore, customers will get a completely new product under "exchange but no repair" service while they will get a repaired one under "warranty" service.

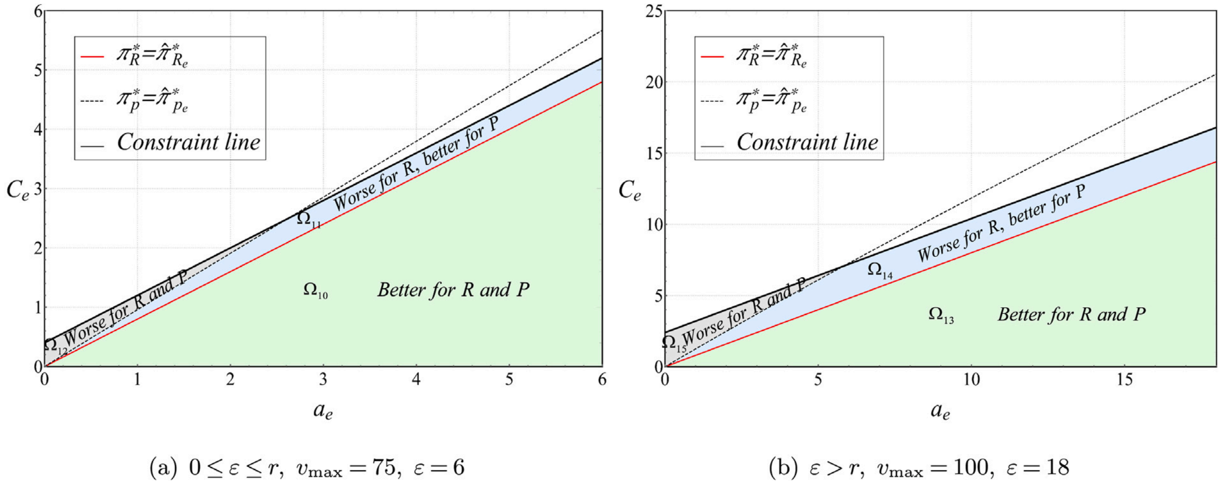


Fig. 10. The value of WFS to the retailer and platform (with SEE). Note: $\beta = 0.2$, $c = 50$, $s = 50$, $v_{\min} = 25$, $r = 15$, $\phi = 0.5$.

6.4. Heterogeneity in valuation increment

In our main model, we assume that customers' valuation increments are homogeneous. In this part, we detect the case when customers show homogeneity in valuation increment, besides customers' valuations and valuation biases.

We denote $\tilde{a}$ as consumers' valuation increments in this case. For tractability, we assume that a proportion η $(0 \le \eta \le 1)$ of customers have high valuation increments (i.e., $\tilde{a} = a_H$) while the remaining $1-\eta$ of them have low valuation increments (i.e., $\tilde{a} = a_L$) for WFS $(a_H > a_L)$. Then the number of kept and returned products can be expressed as follows:

$$\hat{K}^a = \bar{G}\left(\max\left(p - \tilde{\varepsilon} - \tilde{a}, p - r\right)\right) \tag{14}$$

$$\hat{R}^a = \max\left(\bar{G}\left(p - \tilde{\varepsilon} - \tilde{a}\right) - \bar{G}\left(p - r\right), 0\right) \tag{15}$$

Therefore, we can further express the retailer and platform's profits as follows:

$$\hat{\pi}_R^a = \left((1-\beta)p - c - C\right)\hat{K}^a + (s - c - C)\hat{R}^a \tag{16}$$

$$\hat{\pi}_P^a = \beta p \hat{K}^a \tag{17}$$

To detect how the extent of heterogeneity in valuation increment affects the benefits of the retailer and platform, we denote $\Delta_a = a_H - a_L$ as the extent of heterogeneity. larger Δ_a means larger extent of heterogeneity in valuation increment. Then we have the following proposition.

Lemma 5. *Considering heterogeneity in valuation increment, we have the optimal price and profits under WFS strategy as follows:*

(i)When $0 \le \varepsilon \le \frac{r}{2}$*, or when* $\frac{r}{2} < \varepsilon \le r$ *and* $\Delta_a \le r - \varepsilon - a_L$*, we have* $\hat{p}^{a*} = \frac{1}{2}(v_{\max} - (1-2\phi)\varepsilon + \eta\Delta_a + a_L + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^{a*} = \frac{(m_1-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, *and* $\hat{\pi}_P^{a*} = \frac{\beta[m_1{}^2-(c+C)^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$;

(ii)When $\frac{r}{2} < \varepsilon \le r$ *and* $0 \le r - \varepsilon - a_L < \Delta_a \le \varepsilon - a_L$*, we have* $\hat{p}^{a*} = \frac{1}{2}(v_{\max} - (1-2\phi)\varepsilon + (1-\phi\eta)a_L + (1-\phi)\eta\Delta_a + (r-\varepsilon)\eta\phi + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^{a*} = \frac{(m_2-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})} + (s-c-C)\eta\phi\frac{a_L+\Delta_a-r+\varepsilon}{v_{\max}-v_{\min}}$, *and* $\hat{\pi}_P^{a*} = \frac{\beta[m_2{}^2-(c+C)^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$;

(iii)When $\frac{r}{2} < \varepsilon \le r$ *and* $r - \varepsilon - a_L < 0 \le \Delta_a \le \varepsilon - a_L$*, or when* $\varepsilon > r$*, we have* $\hat{p}^{a*} = \frac{1}{2}(v_{\max} - \varepsilon + (\varepsilon + r)\phi + (1-\phi)(a_L + \eta\Delta_a) + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^{a*} = \frac{(m_3-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})} + (s-c-C)\phi\frac{(\eta\Delta_a+a_L-r+\varepsilon)}{v_{\max}-v_{\min}}$, *and* $\hat{\pi}_P^{a*} = \frac{\beta[m_3{}^2-(c+C)^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

First, from Lemma 5, we can get that the retailer's optimal price and the platform's optimal profit always increase with Δ_a. Second, when the valuation bias is small, or when it is moderate and Δ_a is small (less than $r - \varepsilon - a_L$), larger Δ_a also benefits the retailer; otherwise, however, the retailer's optimal profit may decrease with Δ_a.

We proceed to analyze the effect of heterogeneity in valuation increment on the profits of the retailer and platform. The results are shown in Fig. 11. As shown in Fig. 11, when considering the heterogeneity in valuation increment, we can show that the value of WFS on the retailer and platform is similar to that in our main model. Specifically, WFS strategy benefits both the retailer and platform in region Θ_1^a, favors the platform but hurts the retailer in Θ_2^a, and hurts both of them in Θ_3^a, which examines the robustness of our results. In addition, Fig. 11 also shows that, when the heterogeneity in valuation increment increases, for a constant moderate WFS cost, the retailer's optimal strategy switches from Θ_3^a to Θ_2^a, then to Θ_1^a; it indicates that the retailer has more incentives to adopt WFS strategy when the heterogeneity in valuation increment is lager.

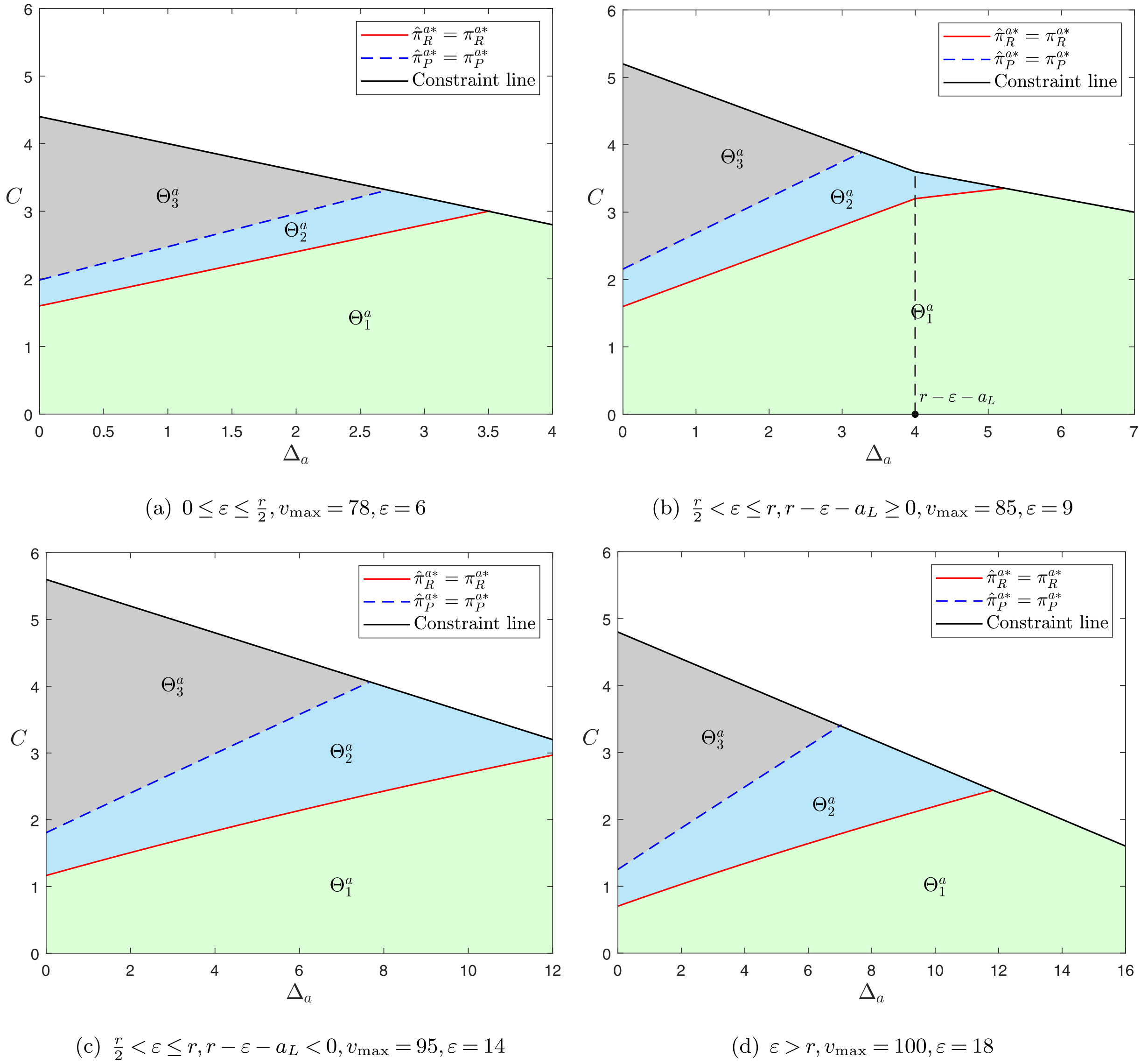


Fig. 11. The value of WFS to the retailer and platform (heterogeneity in valuation increment). Note: (1) $\beta = 0.2$, $r = 15$, $c = 50$, $s = 50$, $v_{\min} = 25$, $\phi = 0.5$, $\eta = 0.5$, $a_L = 2$. (2) For ease of exposition, we use Θ_i^a $(i = 1, 2, 3)$ to represent the following three situations: WFS benefits both the retailer and platform, WFS benefits the platform while hurts the retailer, and WFS hurts both the retailer and platform, when considering heterogeneity in valuation increment, respectively.

6.5. Normal distribution of consumers' valuations

In our main model, we assume that consumers' valuations follow a uniform distribution for tractability. In this section, we assume that consumers' true valuations follow a more general case, i.e., normal distribution to examine the robustness of our results.

We denote $f(\cdot)$ and $F(\cdot)$ as the probability density function and cumulative distribution function of consumers' true valuations. Then we obtain the number of kept and returned products when the retailer adopts No-WFS strategy are $K^n = \frac{F(v_{\max}) - F(\max(p-\tilde{\varepsilon}, p-r))}{F(v_{\max}) - F(v_{\min})}$ and $R^n = \frac{\max(F(p-r) - F(p-\tilde{\varepsilon}), 0)}{F(v_{\max}) - F(v_{\min})}$ (the superscript "n" represents the case that consumers' valuations follow a normal distribution). Therefore, the corresponding profits of the retailer and platform can be expressed as $\pi_R^n = ((1-\beta)p - c)K^n + (s-c)R^n$ and $\pi_P^n = \beta p K^n$.

Furthermore, we can derive the number of kept and returned products when the retailer adopts WFS strategy as $\hat{K}^n = \frac{F(v_{\max}) - F(\max(p-a-\tilde{\varepsilon}, p-r))}{F(v_{\max}) - F(v_{\min})}$ and $\hat{R}^n = \frac{\max(F(p-r) - F(p-a-\tilde{\varepsilon}), 0)}{F(v_{\max}) - F(v_{\min})}$. Then the retailer and platform's profits are $\hat{\pi}_R^n = ((1-\beta)p - c - C)\hat{K}^n + (s - c - C)\hat{R}^n$ and $\hat{\pi}_P^n = \beta p \hat{K}^n$.

However, due to the complexity of normal distribution, we cannot get closed-form solutions as before in the main model; instead, we use numerical simulations to check our main results. Figs. B.1–B.3 verify the robustness of our main conclusion about the optimal decisions of the retailer and the value of WFS to stakeholders (i.e., the retailer, platform, consumers, and society). Please refer to Appendix B for details.

7. Conclusion and future research

In summary, we study platform-based Worry-Free Shopping (WFS), an emerging service system developed by platforms (e.g., Tmall.com, JD.com, and pingduoduo.com) for online retailers to stimulate customer demand. A retailer can decide whether to provide WFS, at an extra cost, for customers. We develop a parsimonious analytical model to investigate the retailer's optimal WFS strategy and how the retailer's strategy affects the platform. We further detect the value of WFS to the online retailer, platform, consumers and society.

7.1. Managerial implications

Our work provides the following implications. First, if WFS cost is small, the retailer should offer WFS with a large valuation increment because it can cover the negative effects resulted from WFS cost by generating more sales. However, if WFS cost is large, No-WFS strategy should be adopted because a large valuation increment incurs more returns despite that it can promote sales. Besides, WFS strategy always has possibility to hurt the retailer and platform if the valuation bias is large, which suggests that appropriate measures (such as 3D presentation and live shows) should be taken to reduce valuation bias.

Second, the retailer should set a higher price if the valuation increment or WFS cost is large. However, a lower price should be set if the valuation bias is large. In addition, the retailer's optimal profit is not monotonically influenced by the valuation increment, which is moderated by WFS cost. In contrast, the platform always prefers a large valuation increment.

Third, interestingly, the interests of retailer and platform are not always aligned on the provision of WFS. Specifically, for a fixed WFS cost (or fixed valuation increment), if the valuation increment is low (or WFS cost is high), WFS strategy hurts both the retailer and platform; if the valuation increment is high (or WFS cost is low), WFS strategy benefits both the retailer and platform, which indicates that the platform can take a "hands-off" attitude towards the retailer since the retailer's offering WFS will also favor the platform. However, if both the valuation increment and WFS cost are moderate, the interests of retailer and platform diverge since WFS favors the platform while hurts the retailer. In this case, the platform should motivate the retailer to provide WFS (e.g., by offering a subsidy), where both their profits increase compared with No-WFS strategy. When negotiating on offering a subsidy, a larger WFS cost always reduces the probability of successful negotiation, while whether a large valuation increment promotes successful negotiation may be moderated by WFS cost.

Fourth, WFS does not necessarily benefit consumers and society. A small valuation bias makes WFS always hurt consumers and WFS with large cost is not beneficial to the society.

In extensions, our analysis suggests that when the salvage value is small, the retailer is not willing to adopt WFS strategy and the interest divergence between the retailer and platform expands, indicating that measures should be taken by the platform to coordinate their benefits. Besides, a large salvage value not only makes WFS strategy more favorable to the retailer, but also facilitates successful negotiation on offering a subsidy to motivate the retailer to offer WFS to consumers. Furthermore, a large proportion of overestimating customers can hurt the retailer due to the joint effects from WFS cost, suggesting that excessive publicity is not encouraged. Indeed, a larger proportion of overestimating consumers not only makes WFS less favorable to the retailer, but also may reduce the probability of successful negotiation. Lastly, large customers' heterogeneity in valuation increment always benefits the platform while may hurt the retailer.

7.2. Future research directions

We suggest some directions for future research. First, this paper assumes that the retailer has reached the platform's standard, so products will be tagged with the WFS logo so long as the retailer applies to provide WFS, and the logo will not be canceled during the selling period. However, in the real world, the platform will periodically assess whether the retailer's service level is sufficient and, if not, the WFS logo will be canceled. Therefore, a dynamic model with multi-period optimization could be interesting to study. Second, this paper only considers two players in the game: the retailer and the platform. However, it would also be interesting to take the manufacturer(s) into consideration and investigate the inventory problem. Third, we only consider one retailer selling products on the platform. It would be interesting to investigate the value of WFS when considering multiple competitive retailers and platforms. Fourth, incorporating product substitutability is also an inspiring way to extend our model. Last, we study WFS in a marketplace mode. However, in addition to marketplace mode, platforms (such as JD.com) may also operate in reselling mode and by themselves provide WFS services for customers. Therefore, it is also worthwhile to investigate WFS in that situation in future.

CRediT authorship contribution statement

Xiaolong Guo: Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Supervision, Funding acquisition, Project administration. **Qiang Zhou:** Conceptualization, Methodology, Formal analysis, Writing – original draft. **Junsong Bian:** Conceptualization, Methodology, Writing – review & editing, Supervision.

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Appendix A. Thresholds used in propositions and lemmas

The thresholds in Proposition 5. **(1)** $\hat{C} = \frac{1}{2}[(1-\beta)(2v_{\max} - a - r + \varepsilon) - 2c] - \frac{1}{2}\sqrt{[(1-\beta)(2v_{\max} - a - r + \varepsilon) - 2c]^2 + A_1}$, where $A_1 = a(1-\beta)[(1-\beta)(4v_{\max} + 7a + 14r - 14\varepsilon) - 4c]$.
(2) $C_1^{sw} = \frac{1}{(3-4\beta)}[a - 3c - 3a\beta + 4c\beta + 2a\beta^2 + (3-2\beta)(1-\beta)v_{\max} - \sqrt{[a - 3c - 3a\beta + 4c\beta + 2a\beta^2 + (3-2\beta)(1-\beta)v_{\max}]^2 + A_2}]$, where $A_2 = -(3-4\beta)[2a(1-\beta)^2 v_{\max} - a(1-\beta)(a + 2c - a\beta - 4c\beta)]$.
(3) $C_2^{sw} = \frac{1}{2(3-4\beta)}[a(1-\beta)(5-6\beta) + 3\varepsilon - 3r + \beta(5-2\beta)(r-\varepsilon) + (6-4\beta)(1-\beta)v_{\max} - 2(3-4\beta)c + \sqrt{[a(1-\beta)(5-6\beta) + 3\varepsilon - 3r + \beta(5-2\beta)(r-\varepsilon) + (6-4\beta)(1-\beta)v_{\max} - 2(3-4\beta)c]^2 + A_3}]$, where $A_3 = (3-4\beta)(1-\beta)((1-\beta)(5a^2 - 4(a+r-\varepsilon)v_{\max}) + (r-\varepsilon)(c(4-8\beta) - 11(1-\beta)(r-\varepsilon)) + 2a(5(1-\beta)(r-\varepsilon) + c(2-4\beta)))$.
(4) $C_3^{sw} = \frac{1}{2(3-4\beta)}[(1-\beta)(a(5-6\beta) + (6-4\beta)v_{\max}) - 3(r + 2c - \varepsilon) + \beta((5-2\beta)(r-\varepsilon) + 8c) + \sqrt{[(1-\beta)(a(5-6\beta) + (6-4\beta)v_{\max}) - 3(r + 2c - \varepsilon) + \beta((5-2\beta)(r-\varepsilon) + 8c)]^2 + A_4}]$, where $A_4 = a(1-\beta)(3-4\beta)((1-\beta)(5a - 4v_{\max}) + 2(1-\beta)(5r - 5\varepsilon) + 4c - 8c\beta)$.
(5) $\underline{C}_L^1 = (1-\beta)(a + 2\varepsilon + 2v_{\min} - v_{\max}) - c$; $\underline{C}_L^2 = \underline{C}_L^3 = (1-\beta)(\frac{1}{2}(3a - r + 5\varepsilon) + 2v_{\min} - v_{\max}) - c$.

The thresholds in Proposition 10. **(1)** $s_1^* = \frac{B_1 + B_2 + (-3+4\beta)C^2}{8(1-\beta)^2\phi(a-r+\varepsilon)}$, where $B_1 = (1-\beta)(a^2(1-\beta)(1-\phi)(1+3\phi) + (r-\varepsilon)\phi(c(-6+4\beta) - r(1-\beta)(4+3\phi) - (1-\beta)\varepsilon(9\phi - 10)) - 2a((1-\beta)(1+3\phi)(\varepsilon(1-\phi) - r\phi) + c(-1 - 3\phi + 2\beta(1+\phi)))) + 2(1-\beta)^2(-a + (a - r + \varepsilon)\phi)v_{\max}$ and $B_2 = -2c(3-4\beta) + 2(1-\beta)(-\varepsilon - r(3-2\beta)\phi + 5\varepsilon\phi - \beta\varepsilon(-2+6\phi) + a(1 + 3\phi - 2\beta(1+\phi)) + (3-2\beta)v_{\max})$;
(2) $s_2^* = \frac{B_3 + B_4 + (-3+4\beta)C^2}{8a(1-\beta)^2\phi}$, where $B_3 = 2ac(1-\beta)(1 + 3\phi - 2\beta(1+\phi)) + a(1-\beta)^2(a(1-\phi)(1+3\phi) - 2(1+3\phi)(\varepsilon(1-\phi) - r\phi) - 2(1-\phi)v_{\max})$ and $B_4 = 2c(-3+4\beta) + 2(1-\beta)(-\varepsilon + \beta\varepsilon(2-6\phi) - r(3-2\beta)\phi + 5\varepsilon\phi + a(1 + 3\phi - 2\beta(1+\phi)) + (3-2\beta)v_{\max})$.

The thresholds in Proposition 12. **(1)** $\phi_{cs}^* = \frac{1}{4(1-\beta)(3a(1-\beta)-C)\varepsilon}[a(1-\beta)(2w - (1-\beta)(3a - 6\varepsilon + 2v_{\max})) + 2((1-\beta)(a - \varepsilon - v_{\max}) + c)C + C^2]$;
(2) $\sigma_{cs}^1 = \frac{-A_5{}^2 + B_5(a-r-3\varepsilon)(a-r+\varepsilon)}{8(1-\beta)^2(a-r-3\varepsilon)(a-r+\varepsilon)(v_{\max}-v_{\min})}$, $\underline{\phi}_{cs}^1 = \frac{B_5}{(1-\beta)(A_5 - \sqrt{A_5{}^2 - B_5(a-r-3\varepsilon)(a-r+\varepsilon)})}$ and $\overline{\phi}_{cs}^1 = \frac{B_5}{(1-\beta)(A_5 + \sqrt{A_5{}^2 - B_5(a-r-3\varepsilon)(a-r+\varepsilon)})}$, where $A_5 = (1-\beta)((a-r)(-a+2r) + (2a+5r)\varepsilon - 3\varepsilon^2 - (a-r+\varepsilon)v_{\max}) + (a-r-\varepsilon)C + (a-r+\varepsilon)c$ and $B_5 = a(1-\beta)(2c - (1-\beta)(3a - 6\varepsilon)) - 2(1-\beta)v_{\max}(a(1-\beta) + C) + 2((1-\beta)(a-\varepsilon) + c)C + C^2$;
(3) $\sigma_{cs}^2 = \frac{-A_6{}^2 + aB_6(a-2(r+\varepsilon))}{8a(1-\beta)^2(a-2(r+\varepsilon))(v_{\max}-v_{\min})}$, $\underline{\phi}_{cs}^2 = \frac{B_6}{(1-\beta)(A_6 - \sqrt{A_6^2 - aB_6(a-2(r+\varepsilon))})}$ and $\overline{\phi}_{cs}^2 = \frac{B_6}{(1-\beta)(A_6 + \sqrt{A_6^2 - aB_6(a-2(r+\varepsilon))})}$, where $A_6 = -a((1-\beta)(v_{\max} - 3r + a - 2\varepsilon) - c) + (a - r - \varepsilon)C$ and $B_6 = a(1-\beta)(2c + (1-\beta)(6\varepsilon - 3a)) - 2(1-\beta)v_{\max}(a(1-\beta) + C) + 2((1-\beta)(a-\varepsilon) + c)C + C^2$.

The thresholds in Proposition 13. **(1)** $\phi_{sw}^* = \frac{1}{4(1-\beta)(a(1-\beta)+(1-2\beta)C)\varepsilon}[a(1-\beta)((1-\beta)(2v_{\max} + 2\varepsilon - a) - 2c + 4c\beta) + 2(-a + 3c + 3a\beta - 4c\beta - 2a\beta^2 + \varepsilon - 3\beta\varepsilon + 2\beta^2\varepsilon + (3-2\beta)(-1+\beta)v_{\max})C + (3-4\beta)C^2]$;
(2) $\sigma_{sw}^1 = \frac{1}{24(1-\beta)^2(v_{\max}-v_{\min})}[3B_7 - \frac{A_7{}^2}{(a-r-3\varepsilon)(a-r+\varepsilon)}]$, $\underline{\phi}_{sw}^1 = \frac{B_7}{(1-\beta)(A_7 - \sqrt{A_7{}^2 - 3B_7(a-r-3\varepsilon)(a-r+\varepsilon)})}$ and $\overline{\phi}_{sw}^1 = \frac{B_7}{(1-\beta)(A_5 + \sqrt{A_7{}^2 - 3B_7(a-r-3\varepsilon)(a-r+\varepsilon)})}$, where $A_7 = (r-a)(4s - (1-\beta)(2r+a) - 3c + 2(-2s+c)\beta) + ((1-\beta)(7r - 2a - 4s) + 3c - 2c\beta)\varepsilon - 5(1-\beta)\varepsilon^2 + (1-\beta)(a-r+\varepsilon)v_{\max} + ((r-a)(2\beta-3) + (5-6\beta)\varepsilon)C$ and $B_7 = a(1-\beta)((1-\beta)(2\varepsilon - a) - 2c + 4c\beta) - 2(a - 3c - 3a\beta + 4c\beta + 2a\beta^2 - \varepsilon + 3\beta\varepsilon - 2\beta^2\varepsilon)C + (3-4\beta)C^2 + 2(1-\beta)v_{\max}(a(1-\beta) - (3-2\beta)C)$;
(3) $\sigma_{sw}^2 = \frac{-A_8{}^2 + 3aB_8(a-2(r+\varepsilon))}{24a(1-\beta)^2(a-2(r+\varepsilon))(v_{\max}-v_{\min})}$, $\underline{\phi}_{sw}^2 = \frac{B_8}{(1-\beta)(A_8 - \sqrt{A_8{}^2 - 3aB_8(a-2(r+\varepsilon))})}$ and $\overline{\phi}_{sw}^2 = \frac{B_8}{(1-\beta)(A_8 + \sqrt{A_8{}^2 - 3aB_8(a-2(r+\varepsilon))})}$, where $A_8 = a((1-\beta)(a + r - 4s - 2\varepsilon + v_{\max}) + 3c - 2c\beta) + ((r-a)(2\beta-3) + (5-6\beta)\varepsilon)C$ and $B_8 = a(1-\beta)((1-\beta)(2\varepsilon - a) - 2c + 4c\beta) + 2(3c - 4c\beta - (1-\beta)(1-2\beta)(a-\varepsilon))C + (3-4\beta)C^2 + 2(1-\beta)v_{\max}(a(1-\beta) - (3-2\beta)C)$.

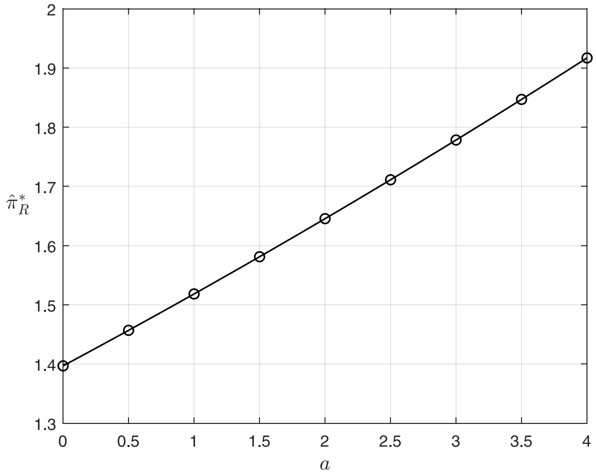
The thresholds in Lemma 5. **(1)** $m_1 = (1-\beta)(v_{\max} - (1-2\phi)\varepsilon + \eta\Delta_a + a_L)$;
(2) $m_2 = (1-\beta)(v_{\max} - (1-2\phi)\varepsilon + (1-\phi\eta)a_L + (1-\phi)\eta\Delta_a + (r-\varepsilon)\eta\phi)$;
(3) $m_3 = (1-\beta)(v_{\max} - \varepsilon + (\varepsilon + r)\phi + (1-\phi)(a_L + \eta\Delta_a))$.

Appendix B. Normal distribution of consumers' valuations

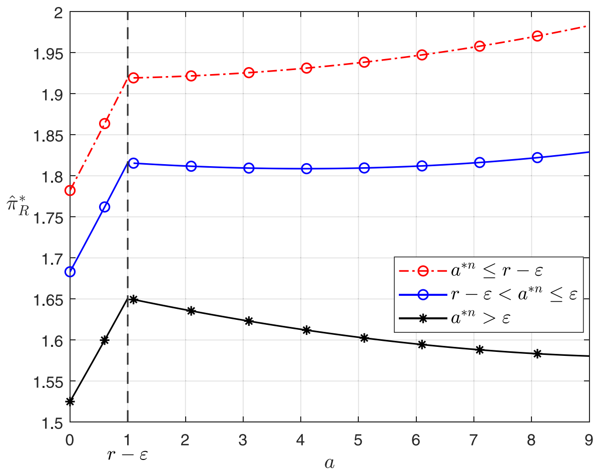
B.1. The effect of valuation increment on the retailer's profit

In this part, we detect the effect of valuation increment on the retailer's optimal profit when consumers' valuations follow a normal distribution. Assuming $v \sim N(62.5, 37.5^2)$, we plot the following Fig. B.1.

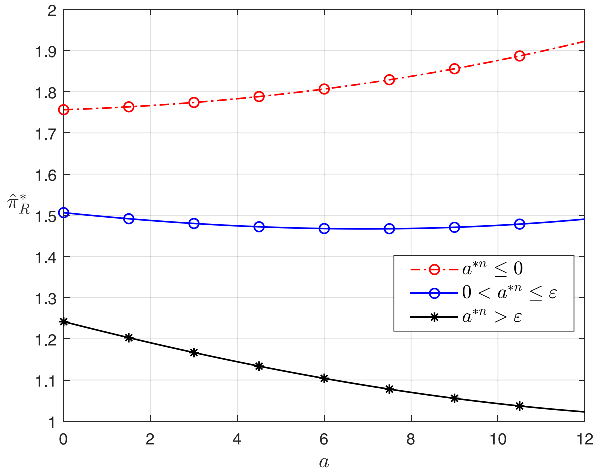
Fig. B.1 shows that when $0 \le \varepsilon \le \frac{r}{2}$ (Fig. B.1(a)), or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r - \varepsilon$ ($a \le r - \varepsilon$ in Fig. B.1(b)), the retailer's optimal profit increases with a; when $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon < a \le \varepsilon$ ($a > r - \varepsilon$ in Fig. B.1(b)), or when $\varepsilon > r$ (Fig. B.1), there exists a threshold a^{*n} that differentiates the effect of valuation increment, which shows the robustness of our results.



(a) $0 \leq \varepsilon \leq \frac{r}{2},\ \varepsilon = 4$



(b) $\frac{r}{2} < \varepsilon \leq r,\ \varepsilon = 9$



(c) $\varepsilon > r,\ \varepsilon = 12$

Fig. B.1. The effect of valuation increment on the retailer's optimal profit (normal distribution). Note: (1) $\beta = 0.2$, $r = 10$, $s = 50$, $c = 50$, $\phi = 0.5$, $v_{\max} = 100$, $v_{\min} = 25$. (2) In (a), $C = 10$. In (b), when $a^{*n} \leq r - \varepsilon$, $C = 8$; when $r - \varepsilon < a^{*n} \leq \varepsilon$, $C = 8.6$; when $a^{*n} > \varepsilon$, $C = 9.6$. In (c), when $a^{*n} \leq 0$, $C = 7$; when $0 < a^{*n} \leq \varepsilon$, $C = 8.4$; when $a^{*n} > \varepsilon$, $C = 10$.

B.2. The value of WFS to the retailer and platform

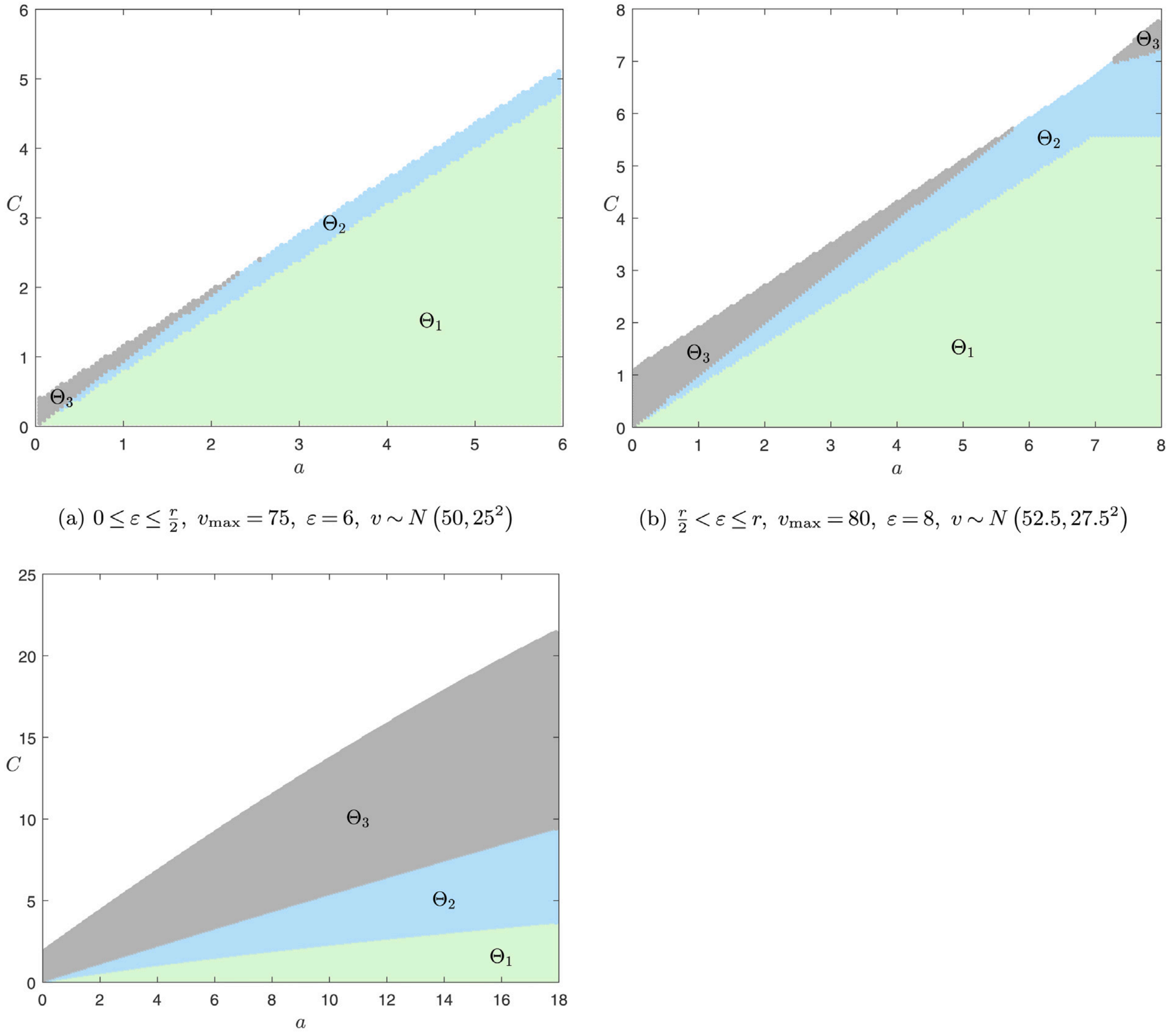
To compare to the results in our main model, we set the same parameters as those in Fig. 1. We assume consumers' true valuations follow a truncated normal distribution on $[v_{\min}, v_{\max}]$. Therefore, we set: $v \sim N\left(50, 25^2\right)$ if $0 \leq \varepsilon \leq \frac{r}{2}$; $v \sim N\left(52.5, 27.5^2\right)$ if $\frac{r}{2} < \varepsilon \leq r$; otherwise $v \sim N\left(62.5, 37.5^2\right)$. We plot the value of WFS to the retailer and platform in Fig. B.2.

From Fig. B.2, we can see that: WFS benefits both the retailer and platform in region Θ_1; in region Θ_2, while WFS is favorable to the retailer, it hurts the platform; in region Θ_3, WFS hurts both the retailer and platform. These results are qualitatively the same as those in our main analysis, which shows the robustness of our results.

B.3. The value of WFS to consumers and society

In this part, we verify the robustness of our results about the value of WFS to consumers and society when consumers' valuations follow a normal distribution, i.e., $v \sim N\left(62.5, 37.5^2\right)$. Fig. B.3 presents the results.

As shown in Fig. B.3, WFS benefits the society when the WFS cost is small; however, it will always hurt consumers when the valuation bias is small. When the valuation bias is larger, there exists thresholds a_{cs}^n and $\hat{C}_n$ such that WFS favors consumers only when $a \leq a_{cs}^n$ and $C \leq \hat{C}_n$. These results verify the robustness of the results in our main model.



(a) $0 \le \varepsilon \le \frac{r}{2},\ v_{\max}=75,\ \varepsilon=6,\ v \sim N\left(50, 25^2\right)$

(b) $\frac{r}{2} < \varepsilon \le r,\ v_{\max}=80,\ \varepsilon=8,\ v \sim N\left(52.5, 27.5^2\right)$

(c) $\varepsilon > r,\ v_{\max}=100,\ \varepsilon=18,\ v \sim N\left(62.5, 37.5^2\right)$

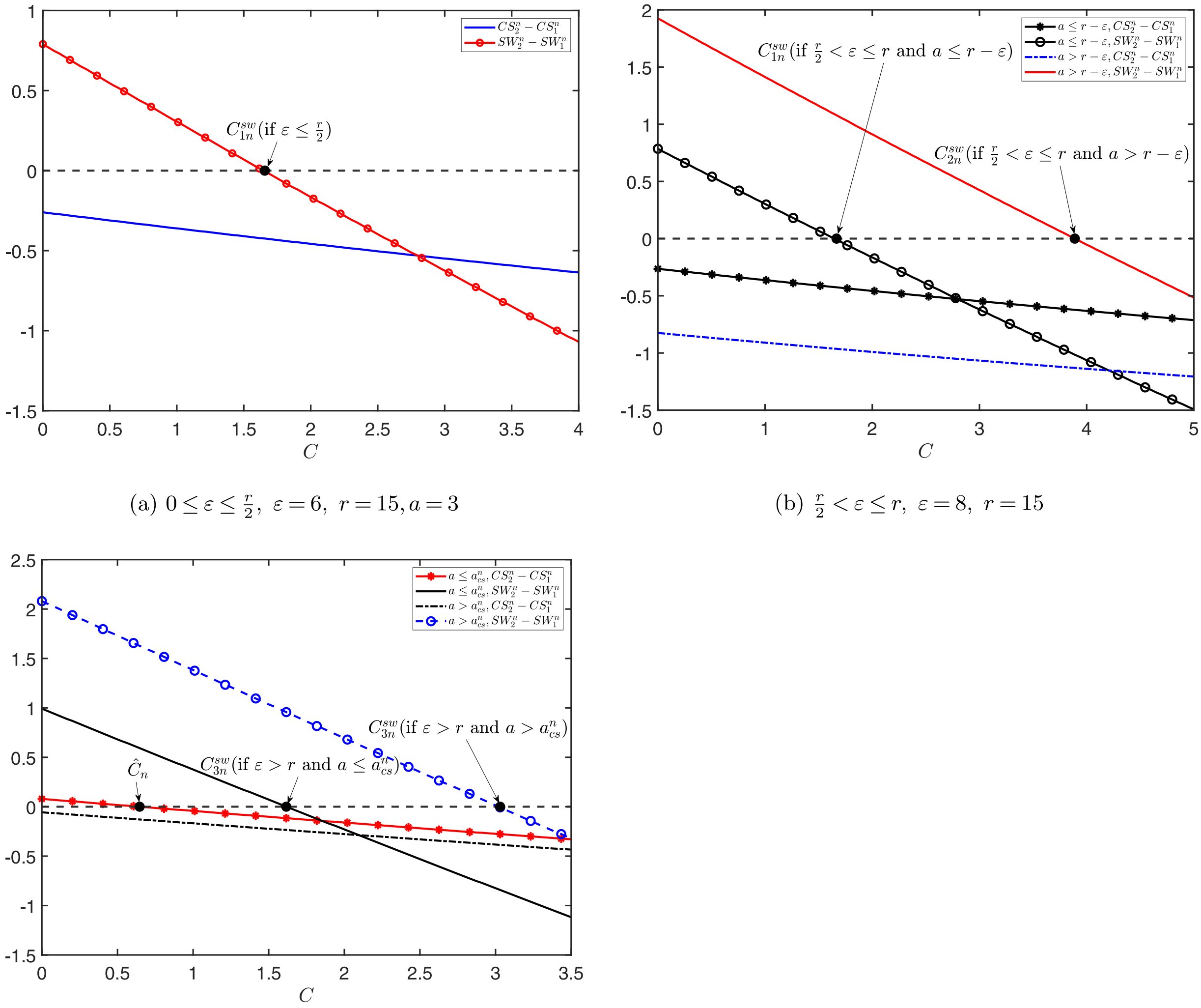
Fig. B.2. The Value of WFS to the Retailer and Platform (normal distribution). Note: (1) $\beta = 0.2$, $r = 15$, $s = 50$, $\phi = 0.5$, $c = 50$, $v_{\min} = 25$. (2) For ease of exposition, we use Θ_i $(i = 1, 2, 3)$ to represent the following three situations: WFS benefits both the retailer and platform, WFS benefits the platform while hurts the retailer, and WFS hurts both the retailer and platform, respectively.

Appendix C. Tables and figures

See Table C.1 and Figs. C.1–C.5.

Appendix D. Proofs

Proof of Lemma 1. When $\varepsilon \le r$, based on Eqs. (1) and (2), we have $K = \frac{1}{2}\bar{G}(p-\varepsilon) + \frac{1}{2}\bar{G}(p+\varepsilon) = \frac{v_{\max}-p}{v_{\max}-v_{\min}}$ and $R = 0$. Based on Eqs. (3) and (4), we have $\pi_R = \frac{((1-\beta)p-c)(v_{\max}-p)}{v_{\max}-v_{\min}}$ and $\pi_P = \frac{\beta p(v_{\max}-p)}{v_{\max}-v_{\min}}$. Solving $\frac{\partial \pi_R}{\partial p} = 0$, we have $p^* = \frac{1}{2}(v_{\max} + \frac{c}{1-\beta})$. Substituting $p^* = \frac{1}{2}(v_{\max} + \frac{c}{1-\beta})$ into π_R^* and π_P^*, we have $\pi_R^* = \frac{((1-\beta)v_{\max}-c)^2}{4(1-\beta)(v_{\max}-v_{\min})}$ and $\pi_P^* = \frac{\beta\left[((1-\beta)v_{\max})^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$; When $\varepsilon > r$, based on Eqs. (1) and (2), we have $K = \frac{1}{2}\bar{G}(p-r) + \frac{1}{2}\bar{G}(p+\varepsilon) = \frac{2(v_{\max}-p)+r-\varepsilon}{2(v_{\max}-v_{\min})}$ and $R = \frac{1}{2}\left[\bar{G}(p-\varepsilon) - \bar{G}(p-r)\right] = \frac{\varepsilon-r}{2(v_{\max}-v_{\min})}$. Based on Eqs. (3) and (4), we have $\pi_R^* = \frac{((1-\beta)p-c)(2(v_{\max}-p)+r-\varepsilon)}{2(v_{\max}-v_{\min})}$ and $\pi_P^* = \frac{\beta p(2(v_{\max}-p)+r-\varepsilon)}{2(v_{\max}-v_{\min})}$. Solving $\frac{\partial \pi_R^*}{\partial p} = 0$, we have $p^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(r-\varepsilon) + \frac{c}{1-\beta})$. Substituting



(a) $0 \le \varepsilon \le \frac{r}{2},\ \varepsilon = 6,\ r = 15, a = 3$

(b) $\frac{r}{2} < \varepsilon \le r,\ \varepsilon = 8,\ r = 15$

(c) $\varepsilon > r,\ \varepsilon = 18,\ r = 3$

Fig. B.3. The Value of WFS to Consumers and Society (normal distribution). Note: (1) $v_{\max} = 100$, $v_{\min} = 25$, $\beta = 0.2$, $c = 50$, $s = 50$, $\phi = 0.5$. (2) In (b), $a = 3$ when $a \le r - \varepsilon$; $a = 8$ when $a > r - \varepsilon$. In (c), $a = 7$ when $a \le a_{cs}^n$; $a = 15$ when $a > a_{cs}^n$. (3) We use CS_2^n (CS_1^n) and SW_2^n (SW_1^n) to represent consumer surplus and social welfare when the retailer adopts WFS strategy (No-WFS strategy), respectively.

$p^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(r-\varepsilon) + \frac{c}{1-\beta})$ into π_R and π_P, we have $\pi_R^* = \frac{\left[(1-\beta)(v_{\max}+\frac{1}{2}(r-\varepsilon))-c\right]^2}{4(1-\beta)(v_{\max}-v_{\min})}$ and $\pi_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\frac{1}{2}(r-\varepsilon)))^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$. This completes the proof of Lemma 1.

Proof of Proposition 1. The proof of Proposition 1 can be easily derived from Lemma 1 so we omit it.

Proof of Lemma 2. When $a \le r - \varepsilon$, based on Eqs. (5) and (6), we have $\hat{K} = \frac{1}{2}\bar{G}(p-(a+\varepsilon)) + \frac{1}{2}\bar{G}(p-(a-\varepsilon)) = \frac{v_{\max}-p+a}{v_{\max}-v_{\min}}$ and $\hat{R} = 0$. Based on Eqs. (7) and (8), we have $\hat{\pi}_R = \frac{((1-\beta)p-c-C)(v_{\max}-p+a)}{v_{\max}-v_{\min}}$ and $\hat{\pi}_P = \frac{\beta p(v_{\max}-p+a)}{v_{\max}-v_{\min}}$. Solving $\frac{\partial \hat{\pi}_R}{\partial p} = 0$, we have $\hat{p}^* = \frac{1}{2}(v_{\max} + a + \frac{c+C}{1-\beta})$, Substituting $\hat{p}^* = \frac{1}{2}(v_{\max} + a + \frac{c+C}{1-\beta})$ into $\hat{\pi}_R$ and $\hat{\pi}_P$, we have $\hat{\pi}_R^* = \frac{((1-\beta)(v_{\max}+a)-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})}$ and $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+a))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$; When $a > r - \varepsilon$, based on Eqs. (5) and (6), we have $\hat{K} = \frac{1}{2}\bar{G}(p-r) + \frac{1}{2}\bar{G}(p-(a-\varepsilon)) = \frac{(2(v_{\max}-p)+a-\varepsilon+r)}{2(v_{\max}-v_{\min})}$ and $\hat{R} = \frac{1}{2}\left(\bar{G}(p-(a+\varepsilon)) - \bar{G}(p-r)\right) = \frac{a+\varepsilon-r}{2(v_{\max}-v_{\min})}$. Based on Eqs. (7) and (8), we have $\hat{\pi}_R = \frac{((1-\beta)p-c-C)(2(v_{\max}-p)+a-\varepsilon+r)}{2(v_{\max}-v_{\min})} - C \cdot \frac{a+\varepsilon-r}{2(v_{\max}-v_{\min})}$ and $\hat{\pi}_P = \frac{\beta p(2(v_{\max}-p)+a-\varepsilon+r)}{2(v_{\max}-v_{\min})}$. Solving $\frac{\partial \hat{\pi}_R}{\partial p} = 0$, we have $\hat{p}^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(a+r-\varepsilon) + \frac{c+C}{1-\beta})$. Substituting $\hat{p}^* = \frac{1}{2}(v_{\max} + \frac{1}{2}(a+r-\varepsilon) + \frac{c+C}{1-\beta})$ into $\hat{\pi}_R$ and $\hat{\pi}_P$, we have

Table C.1
Indifference curves for the retailer (C_R) and platform (C_P) and market constraint line (C_M).

	C_R
$0 \le \varepsilon \le \frac{r}{2}$	$C_R^1 = (1-\beta)a$
$\frac{r}{2} < \varepsilon \le r$	$C_R^2 = \begin{cases} (1-\beta)a, 0 \le a \le r-\varepsilon \\ [k + (1-\beta)(a+\varepsilon-r)] - L_1, r-\varepsilon < a \le \varepsilon \end{cases}$
$\varepsilon > r$	$C_R^3 = [k + (1-\beta)(a+\varepsilon-r)] - L_2$
	C_P
$0 \le \varepsilon \le \frac{r}{2}$	$C_p^1 = -c + \sqrt{c^2 + \left[(1-\beta)\left(v_{\max}+a\right)\right]^2 - \left[(1-\beta)\,v_{\max}\right]^2}$
$\frac{r}{2} < \varepsilon \le r$	$C_p^2 = \begin{cases} -c + \sqrt{c^2 + \left[(1-\beta)\left(v_{\max}+a\right)\right]^2 - \left[(1-\beta)\,v_{\max}\right]^2}, 0 \le a \le r-\varepsilon \\ -c + \sqrt{c^2 + \left[(1-\beta)(v_{\max} + \frac{a+r-\varepsilon}{2})\right]^2 - \left[(1-\beta)v_{\max}\right]^2}, r-\varepsilon < a \le \varepsilon \end{cases}$
$\varepsilon > r$	$C_p^3 = -c + \sqrt{c^2 + \left[(1-\beta)(v_{\max} + \frac{a+r-\varepsilon}{2})\right]^2 - \left[(1-\beta)(v_{\max} + \frac{r-\varepsilon}{2})\right]^2}$
	C_M
$0 \le \varepsilon \le \frac{r}{2}$	$C_M^1 = (1-\beta)(v_{\max} + a - 2\varepsilon) - c$
$\frac{r}{2} < \varepsilon \le r$	$C_M^2 = \begin{cases} (1-\beta)(v_{\max} + a - 2\varepsilon) - c, 0 \le a \le r-\varepsilon \\ (1-\beta)(v_{\max} - \frac{1}{2}(a+r-\varepsilon) + 2(a-\varepsilon)) - c, r-\varepsilon < a \le \varepsilon \end{cases}$
$\varepsilon > r$	$C_M^3 = (1-\beta)(v_{\max} - \frac{1}{2}(a+r-\varepsilon) + 2(a-\varepsilon)) - c$

Note: $L_1 = \sqrt{[k + (1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2 - \left[(1-\beta)v_{\max} - c\right]^2\right\}}$,
$L_2 = \sqrt{[k + (1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2 - \left[(1-\beta)(v_{\max} + \frac{r-\varepsilon}{2}) - c\right]^2\right\}}$,
$k = (1-\beta)(v_{\max} + \frac{1}{2}(a+r-\varepsilon)) - c$.

$\hat{\pi}_R^* = \frac{\left[(1-\beta)(v_{\max}+\frac{1}{2}(a+r-\varepsilon))-c-C\right]^2}{4(1-\beta)(v_{\max}-v_{\min})} - C \cdot \frac{a+\varepsilon-r}{2(v_{\max}-v_{\min})}$ and $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\frac{1}{2}(a+r-\varepsilon)))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$. Note that $a \le \varepsilon$, we have, if $\varepsilon \le r-\varepsilon$, i.e., $0 \le \varepsilon \le \frac{r}{2}$, $a \le r-\varepsilon$ always holds; if $r-\varepsilon < \varepsilon \le r$, i.e., $\frac{r}{2} < \varepsilon \le r$, we get $0 \le a \le r-\varepsilon$ or $r-\varepsilon < a \le \varepsilon$; if $\varepsilon > r$, $a > r-\varepsilon$ always holds. Therefore, we have the classifications presented in Lemma 2. This completes the proof of Lemma 2.

Proof of Proposition 2. Using the range of parameters in the proof of Table C.1 and based on Lemma 2, we can easily derive the results in Proposition 2 so we omit it.

Proof of Proposition 3. The proof of Proposition 3(i) and Proposition 3(ii) can be easily derived from Lemma 2 so we omit them.

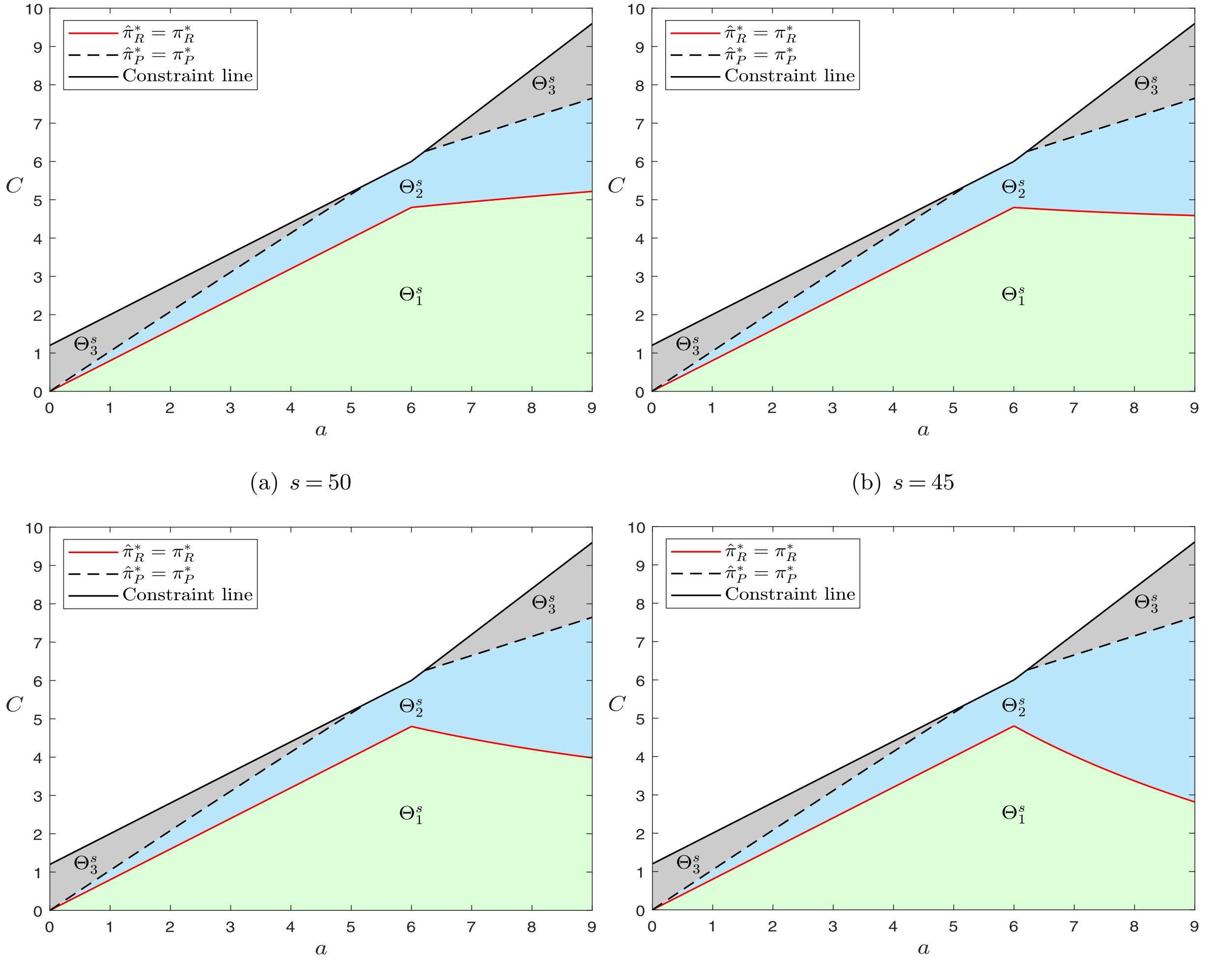
We now prove Proposition 3(iii). Solving $\frac{\partial \hat{\pi}_R^*}{\partial a} = 0$, we have $a^* = \frac{2c+6C}{1-\beta} - (2v_{\max} + r - \varepsilon)$. Because $\hat{\pi}_R^*$ is a convex function of a, so we have, if $a^* < \max(r-\varepsilon, 0)$, $\hat{\pi}_R^*$ increases with a; if $\max(r-\varepsilon,0) \le a^* < \varepsilon$, when $a \in (\max(r-\varepsilon,0), a^*]$, $\hat{\pi}_R^*$ decreases with a, while when $a \in (a^*, \varepsilon]$, $\hat{\pi}_R^*$ increases with a; if $a^* > \varepsilon$, $\hat{\pi}_R^*$ decreases with a. This completes the proof of Proposition 3(iii).

Proof of Table C.1. Because we focus on the interesting case that the retailer sells to both over- and under-estimating market, in the case of No-WFS strategy, we should make $v_{\max} - \varepsilon - p \ge 0$. Also, the retailer will optimally not sell to all two types of customers. Then we can get $v_{\min} - p + \varepsilon < 0$. Substituting p^* in Lemma 1 into $v_{\max} - \varepsilon - p \ge 0$ and $v_{\min} - p + \varepsilon < 0$, we can get market constraints as follows: when $\varepsilon \le r$, $v_{\max} \ge \frac{c}{1-\beta} + 2\varepsilon$ and $v_{\min} < \frac{1}{2}(v_{\max} + \frac{c}{1-\beta}) - \varepsilon$; when $\varepsilon > r$, $v_{\max} \ge \frac{c}{1-\beta} - \frac{1}{2}(\varepsilon - r) + 2\varepsilon$ and $v_{\min} < \frac{1}{2}(v_{\max} + \frac{c}{1-\beta} - \frac{1}{2}(5\varepsilon - r))$. In the case of WFS strategy, we should make $v_{\max} + a - \varepsilon - p \ge 0$ and $v_{\min} + \varepsilon + a - p < 0$. Substituting $\hat{p}^*$ in Lemma 2 into $v_{\max} + a - \varepsilon - p \ge 0$ and $v_{\min} + \varepsilon + a - p < 0$, we can get market constraints as follows: when $0 \le \varepsilon \le \frac{r}{2}$, $C_M^1 \in [\max(0, (1-\beta)(a + 2\varepsilon + 2v_{\min} - v_{\max}) - c), (1-\beta)(v_{\max} + a - 2\varepsilon) - c]$; when $\frac{r}{2} < \varepsilon \le r$, $C_M^2 \in \begin{cases} [\max(0, (1-\beta)(a + 2\varepsilon + 2v_{\min} - v_{\max}) - c), (1-\beta)(v_{\max} + a - 2\varepsilon) - c], 0 \le a \le r-\varepsilon \\ [\max(0, (1-\beta)(\frac{1}{2}(3a - r + 5\varepsilon) + 2v_{\min} - v_{\max}) - c), (1-\beta)(v_{\max} - \frac{r}{2} + \frac{3}{2}(a-\varepsilon)) - c], r-\varepsilon < a \le \varepsilon \end{cases}$; when $\varepsilon > r$, $C_M^3 \in [\max(0, (1-\beta)(\frac{1}{2}(3a - r + 5\varepsilon) + 2v_{\min} - v_{\max}) - c), (1-\beta)(v_{\max} - \frac{r}{2} + \frac{3}{2}(a-\varepsilon)) - c]$.

Therefore, we can get the market constraint lines (C_M) in Table C.1 when plotting Fig. 3.

Further, for ease of expose, when $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, let $\underline{C}_L^1 = (1-\beta)(a + 2\varepsilon + 2v_{\min} - v_{\max}) - c$ and $\overline{C}_L^1 = (1-\beta)(v_{\max} + a - 2\varepsilon) - c$; when $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$, or when $\varepsilon > r$, let $\underline{C}_L^2 = \underline{C}_L^3 = (1-\beta)(\frac{1}{2}(3a - r + 5\varepsilon) + 2v_{\min} - v_{\max}) - c$ and $\overline{C}_L^2 = \overline{C}_L^3 = (1-\beta)(v_{\max} - \frac{r}{2} + \frac{3}{2}(a-\varepsilon)) - c$. We can rewrite the market constraints as follows: when $0 \le \varepsilon \le \frac{r}{2}$, $C_M^1 \in [\max(0, \underline{C}_L^1), \overline{C}_L^1]$;

when $\frac{r}{2} < \varepsilon \le r$, $C_M^2 \in \begin{cases} [\max(0, \underline{C}_L^1), \overline{C}_L^1], 0 \le a \le r-\varepsilon \\ [\max(0, \underline{C}_L^2), \overline{C}_L^2], r-\varepsilon < a \le \varepsilon \end{cases}$; when $\varepsilon > r$, $C_M^3 \in [\max(0, \underline{C}_L^3), \overline{C}_L^3]$.

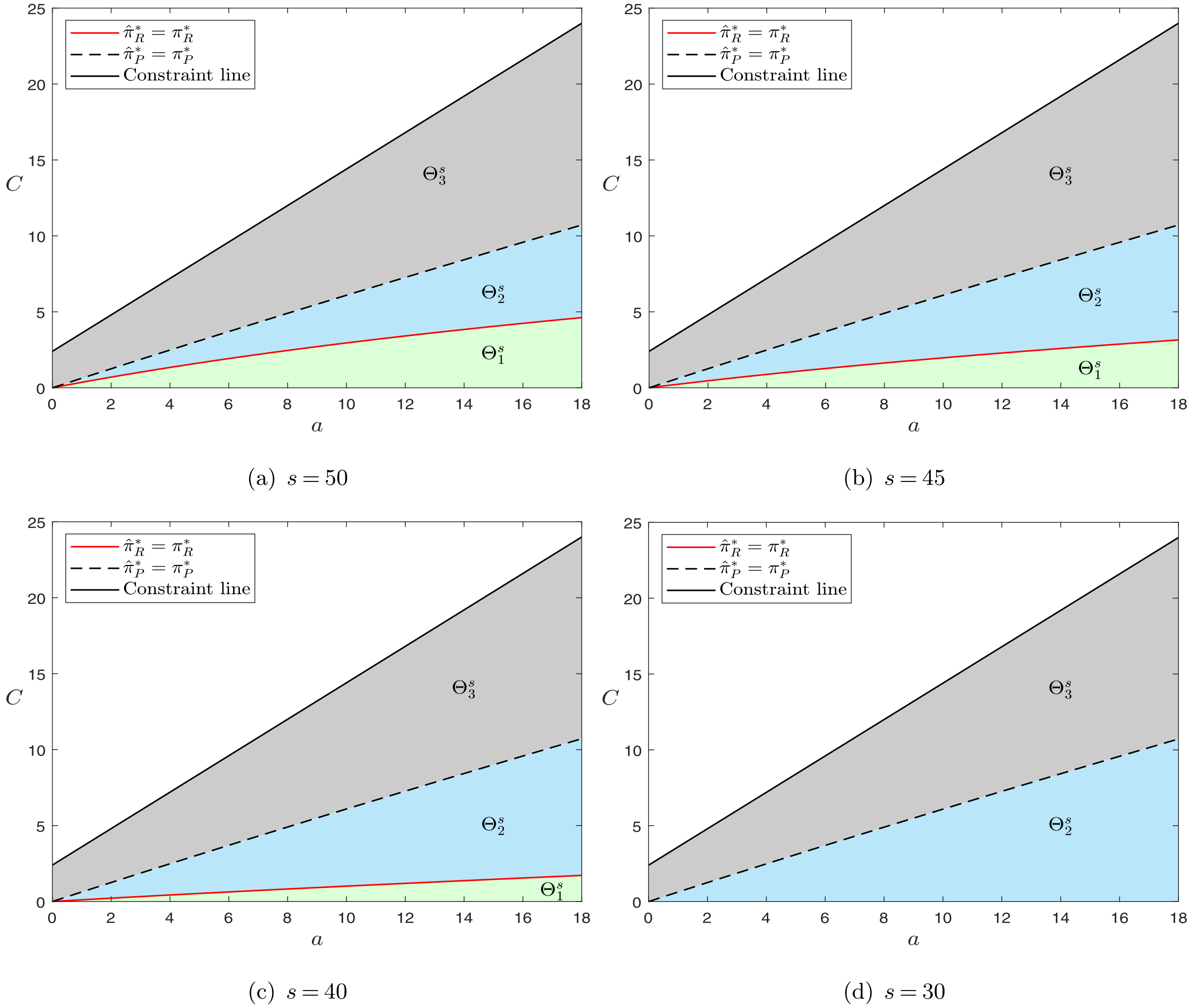


(a) $s = 50$ (b) $s = 45$ (c) $s = 40$ (d) $s = 30$

Fig. C.1. The effects of salvage value on the value of WFS ($\frac{r}{2} < \varepsilon \le r$). Note:(1) $\beta = 0.2$, $c = 50$, $v_{\max} = 82$, $v_{\min} = 25$, $\varepsilon = 9$, $r = 15$, $\phi = 0.5$. (2) We use Θ_i^s ($i = 1, 2, 3$) to represent cases that WFS benefits both the retailer and platform, WFS favors the platform while hurts the retailer, and WFS hurts both the retailer and platform, respectively. The meaning of Θ_i^s also applies to Fig. C.2.

We now prove the indifference curves for the retailer. Let $\pi_R^* = \hat{\pi}_R^*$, we can get the indifference curves for the retailer in each case. When $0 \le \varepsilon \le \frac{r}{2}$, we can get $C_R^1 = (1-\beta)a$ or $C_R^1 = (1-\beta)\left(2v_{\max} + a\right) - 2c$. If $C_R^1 = (1-\beta)\left(2v_{\max} + a\right) - 2c$, $C_R^1 - \overline{C}_L^1 = (1-\beta)\left(v_{\max} + 2\varepsilon\right) - c \ge (1-\beta)\left(\frac{c}{1-\beta} + 2\varepsilon + 2\varepsilon\right) - c = 4(1-\beta)\varepsilon \ge 0$ (using $v_{\max} \ge \frac{c}{1-\beta} + 2\varepsilon$), which shows that $C_R^1 = (1-\beta)\left(2v_{\max} + a\right) - 2c$ is out of the market constraint line and we should abandon it. Therefore, we have $C_R^1 = (1-\beta)a$. In a similar way, we can get that: when $\frac{r}{2} < \varepsilon \le r$, $C_R^2 = \begin{cases} (1-\beta)a, 0 \le a \le r-\varepsilon \\ [k + (1-\beta)(a+\varepsilon-r)] - \sqrt{[k + (1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2 - \left[(1-\beta)v_{\max} - c\right]^2\right\}}, r-\varepsilon < a \le \varepsilon \end{cases}$; when $\varepsilon > r$, $C_R^3 = [k + (1-\beta)(a+\varepsilon-r)] - \sqrt{[k + (1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2 - \left[(1-\beta)(v_{\max} + \frac{r-\varepsilon}{2}) - c\right]^2\right\}}$, where $k = (1-\beta)(v_{\max} + \frac{1}{2}(a + r - \varepsilon)) - c$.

We now prove the indifference curves for platform. Let $\pi_P^* = \hat{\pi}_P^*$, we can get the indifference curves for platform in each case. When $0 \le \varepsilon \le \frac{r}{2}$, we can get $C_p^1 = -c + \sqrt{c^2 + \left[(1-\beta)\left(v_{\max} + a\right)\right]^2 - \left[(1-\beta)v_{\max}\right]^2}$ or $C_p^1 = -c - \sqrt{c^2 + \left[(1-\beta)\left(v_{\max} + a\right)\right]^2 - \left[(1-\beta)v_{\max}\right]^2}$. Obviously, $C_p^1 = -c - \sqrt{c^2 + \left[(1-\beta)\left(v_{\max} + a\right)\right]^2 - \left[(1-\beta)v_{\max}\right]^2} \le 0$ and should be abandoned. Therefore, $C_p^1 = -c + \sqrt{c^2 + \left[(1-\beta)\left(v_{\max} + a\right)\right]^2 - \left[(1-\beta)v_{\max}\right]^2}$. In a similar way, when $\frac{r}{2} < \varepsilon \le r$, $C_p^2 =$



(a) $s = 50$ (b) $s = 45$

(c) $s = 40$ (d) $s = 30$

Fig. C.2. The effects of salvage value on the value of WFS ($\varepsilon > r$). Note: $\beta = 0.2$, $c = 50$, $v_{\max} = 100$, $v_{\min} = 25$, $\varepsilon = 18$, $r = 15$, $\phi = 0.5$.

$$\begin{cases} -c + \sqrt{c^2 + \left[(1-\beta)\left(v_{\max} + a\right)\right]^2 - \left[(1-\beta)\, v_{\max}\right]^2}, 0 \le a \le r - \varepsilon \\ -c + \sqrt{c^2 + \left[(1-\beta)(v_{\max} + \frac{a+r-\varepsilon}{2})\right]^2 - \left[(1-\beta) v_{\max}\right]^2}, r - \varepsilon < a \le \varepsilon \end{cases};$$

when $\varepsilon > r$, $C_p^3 = -c + \sqrt{c^2 + \left[(1-\beta)(v_{\max} + \frac{a+r-\varepsilon}{2})\right]^2 - \left[(1-\beta)(v_{\max} + \frac{r-\varepsilon}{2})\right]^2}$.

This completes the proof of Table C.1.

Proof of Proposition 4. By solving $\pi_i^* = \hat{\pi}_i^* (i = R, P)$, we can get the indifference curves for the retailer and platform in each case, which are presented in Table C.1. Before proving Proposition 4, we first define:

$$\begin{aligned}
&\Omega_1 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, \underline{C}_L^1) \le C \le C_R^1\right\}, \Omega_2 = \left\{(a, C) : 0 \le a \le \varepsilon, C_R^1 \le C \le \min(\overline{C}_L^1, C_P^1)\right\}, \\
&\Omega_3 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(C_P^1, \underline{C}_L^1) \le C \le \overline{C}_L^1\right\}, \Omega_4 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, \underline{C}_L^1, \underline{C}_L^2) \le C \le C_R^2\right\}, \\
&\Omega_5 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, C_R^2, \underline{C}_L^1, \underline{C}_L^2) \le C \le \min(C_P^2, \overline{C}_L^1, \overline{C}_L^2)\right\}, \\
&\Omega_6 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, C_P^2, \underline{C}_L^1, \underline{C}_L^2) \le C \le \min(\overline{C}_L^1, \overline{C}_L^2)\right\}, \\
&\Omega_7 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, \underline{C}_L^3) \le C \le C_R^3\right\}, \Omega_8 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(0, C_R^3, \underline{C}_L^3) \le C \le C_P^3\right\}, \\
&\Omega_9 = \left\{(a, C) : 0 \le a \le \varepsilon, \max(C_P^3, \underline{C}_L^3) \le C \le \overline{C}_L^3\right\}.
\end{aligned}$$

For ease of expose, we rewrite Proposition 4 in a form of three parts as follows:

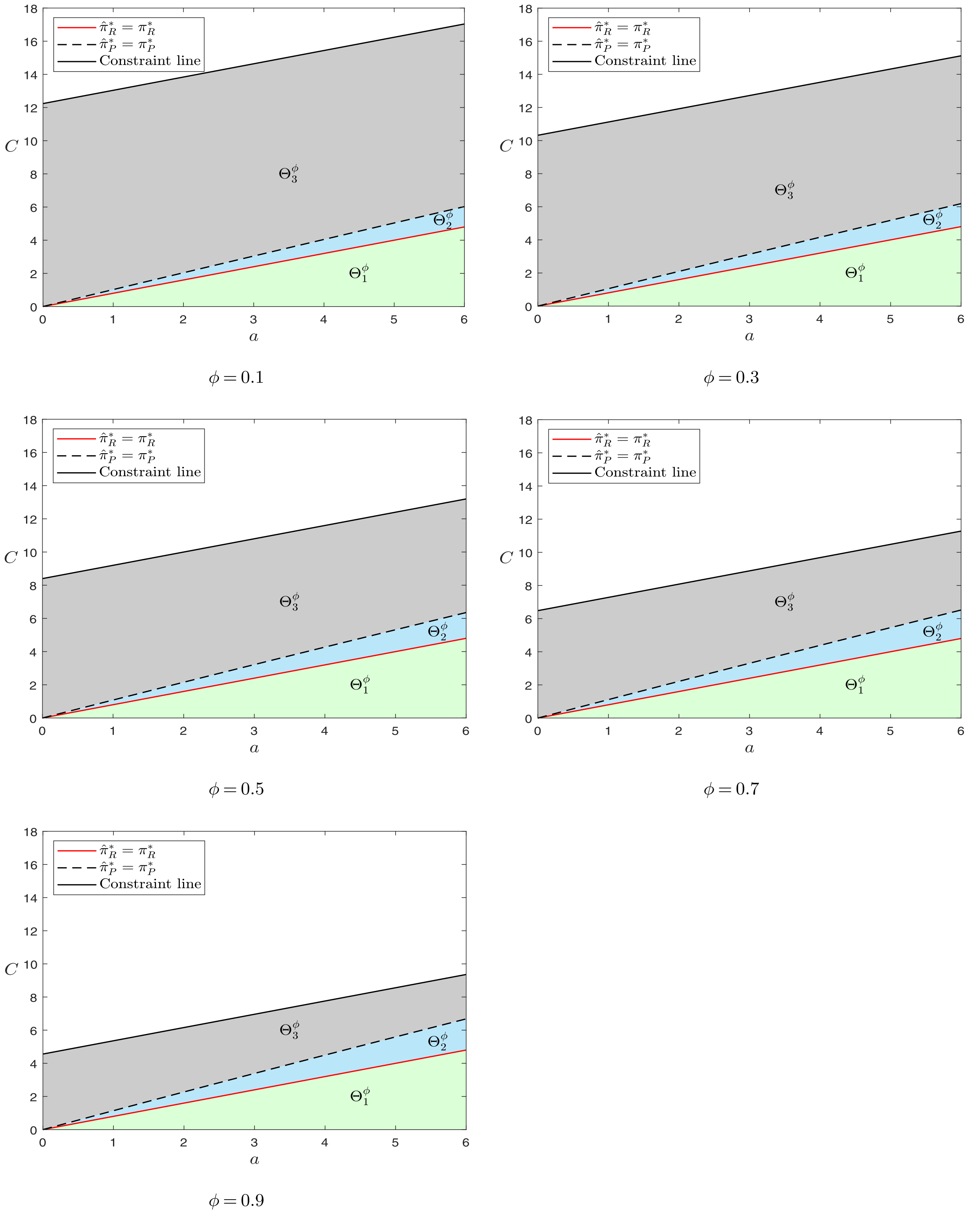


Fig. C.3. The effects of market composition on the value of WFS ($0 \le \varepsilon \le \frac{r}{2}$). Note:(1) $\beta = 0.2$, $c = 50$, $s = 50$, $v_{\max} = 85$, $v_{\min} = 25$, $\varepsilon = 6$, $r = 15$. (2) We use Θ_i^ϕ ($i = 1, 2, 3$) to represent cases that WFS benefits both the retailer and platform, WFS favors the platform while hurts the retailer, and WFS hurts both the retailer and platform, respectively. The meaning of Θ_i^ϕ also applies to Figs. C.4 and C.5.

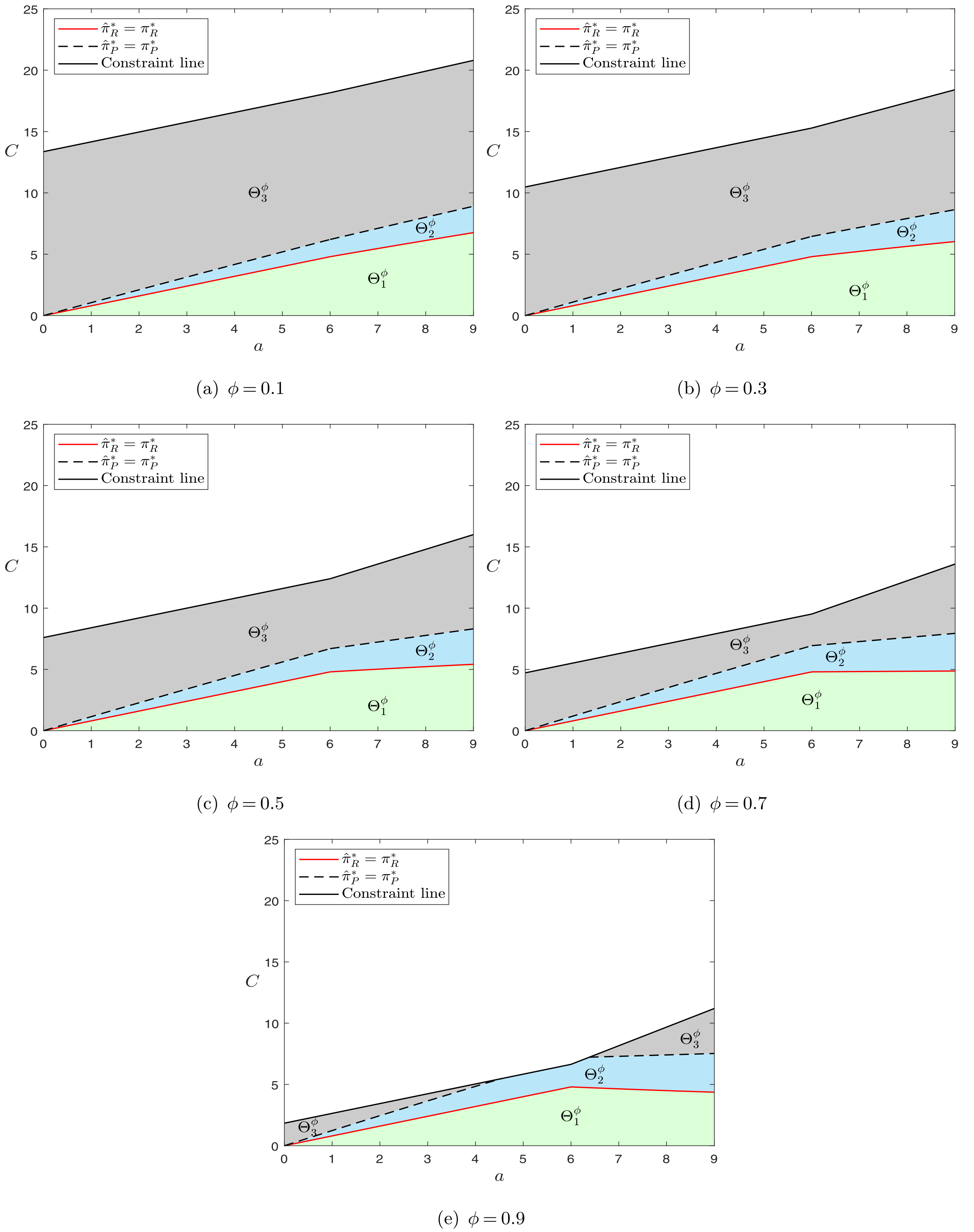


Fig. C.4. The effects of market composition on the value of WFS ($\frac{r}{2} < \varepsilon \leq r$). Note: $\beta = 0.2$, $c = 50$, $s = 50$, $v_{\max} = 90$, $v_{\min} = 25$, $\varepsilon = 9$, $r = 15$.

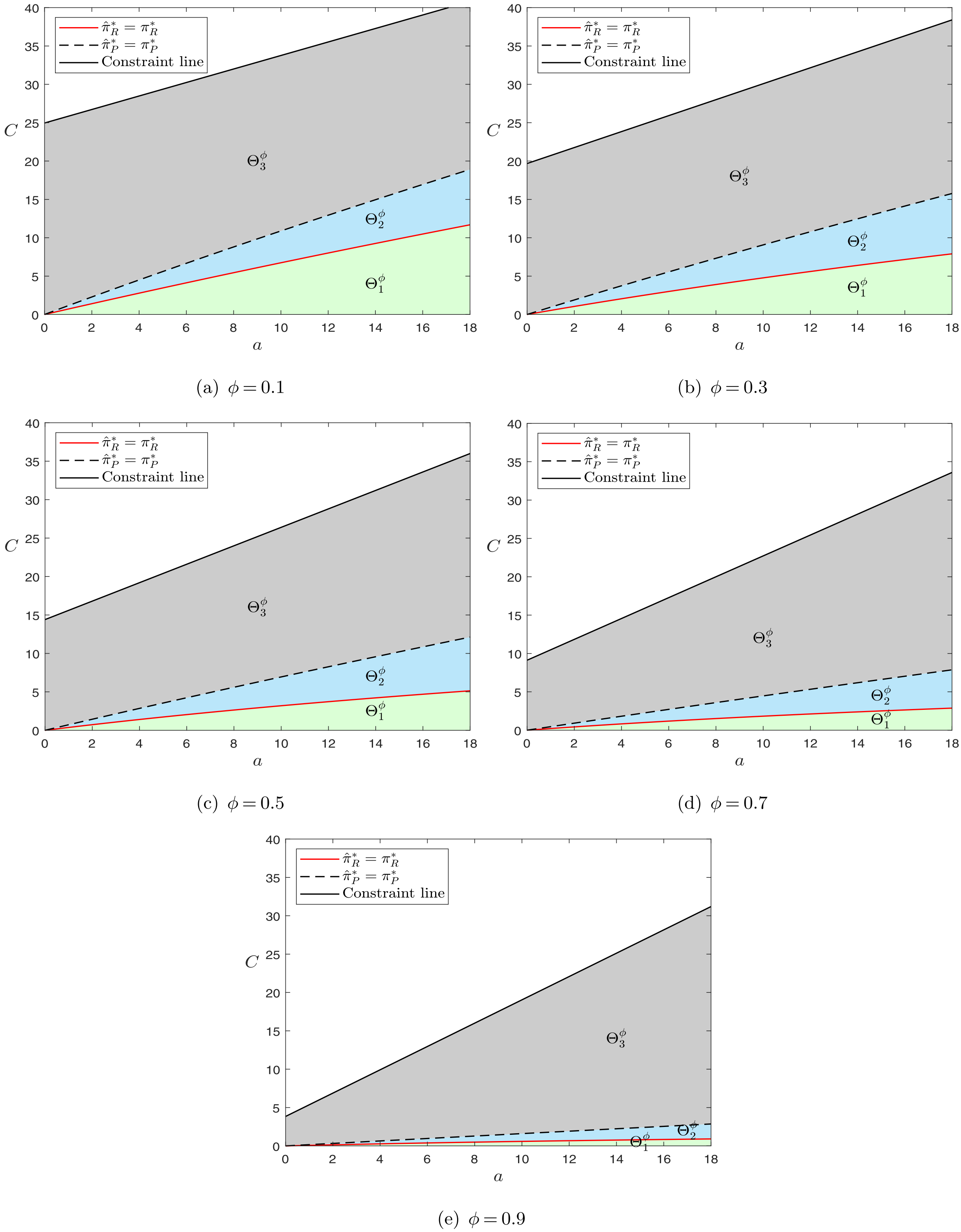


Fig. C.5. The effects of market composition on the value of WFS ($\epsilon > r$). Note: $\beta = 0.2$, $c = 50$, $s = 50$, $v_{\max} = 115$, $v_{\min} = 25$, $\epsilon = 18$, $r = 15$.

Proposition 4(i): if $(a, C) \in \Omega_1$, the retailer should adopt WFS strategy, and this strategy will also benefit the platform;if $(a, C) \in \Omega_2$, the retailer should adopt No-WFS strategy, but this strategy will hurt the platform; if $(a, C) \in \Omega_3$, the retailer should adopt No-WFS strategy, and this strategy will also benefit the platform.

Proposition 4(ii): if $(a, C) \in \Omega_4$, the retailer should adopt WFS strategy, and this strategy will also benefit the platform;if $(a, C) \in \Omega_5$, the retailer should adopt No-WFS strategy, but this strategy will hurt the platform; if $(a, C) \in \Omega_6$, the retailer should adopt No-WFS strategy, and this strategy will also benefit the platform.

Proposition 4(iii): if $(a, C) \in \Omega_7$, the retailer should adopt WFS strategy, and this strategy will also benefit the platform;if $(a, C) \in \Omega_8$, the retailer should adopt No-WFS strategy, but this strategy will hurt the platform; if $(a, C) \in \Omega_9$, the retailer should adopt No-WFS strategy, and this strategy will also benefit the platform.

We first prove Proposition 4(i). To prove Proposition 4(i), we need to prove that the following two conditions satisfy: $C_R^1 \le C_P^1$, $C_R^1 \le \overline{C}_L^1$, and $C_R^1 \ge \underline{C}_L^1$.

(1) We first prove $C_R^1 \le C_P^1$ holds. From Table C.1, we know that we equally prove $-c+\sqrt{c^2+\left[(1-\beta)(v_{\max}+a)\right]^2-\left[(1-\beta)v_{\max}\right]^2} \ge (1-\beta)a$. Through transformation, we equally prove $\sqrt{c^2+\left[(1-\beta)(v_{\max}+a)\right]^2-\left[(1-\beta)v_{\max}\right]^2} \ge c+(1-\beta)a$. Through transformation and simplification, we equally prove $2a(1-\beta)\left[(1-\beta)v_{\max}-c\right] \ge 0$. Using $v_{\max} \ge \frac{c}{1-\beta}+2\varepsilon$, we have $4(1-\beta)^2 a\varepsilon \ge 0$ always holds. Therefore, $C_R^1 \le C_P^1$ satisfies.

(2) We now prove $C_R^1 \le \overline{C}_L^1$ holds, that is, we should prove $(1-\beta)a \le (1-\beta)(v_{\max}+a-2\varepsilon)-c$. Through transformation, we equally prove $(1-\beta)(v_{\max}-2\varepsilon)-c \ge 0$. Using $v_{\max} \ge \frac{c}{1-\beta}+2\varepsilon$, we can get that $(1-\beta)(v_{\max}-2\varepsilon)-c \ge 0$ always holds. Therefore, $C_R^1 \le \overline{C}_L^1$ holds.

(3) We now prove $C_R^1 \ge \underline{C}_L^1$, i.e., $(1-\beta)a \ge (1-\beta)(a+2\varepsilon+2v_{\min}-v_{\max})-c$ holds. Through transformation, We equally prove $(1-\beta)(v_{\max}-2\varepsilon-2v_{\min})+c \ge 0$ holds. Using $v_{\min} < \frac{1}{2}(v_{\max}+\frac{c}{1-\beta})-\varepsilon$, we can get $(1-\beta)(v_{\max}-2\varepsilon-2v_{\min})+c \ge 0$ holds. Therefore, we have $C_R^1 \ge \underline{C}_L^1$.

This completes the proof of Proposition 4(i).

We now prove Proposition 4(ii). When $a \le r-\varepsilon$, the proof is the same as that in the proof of Proposition 4(i). When $a > r-\varepsilon$, we need to prove that the following two conditions satisfy: $C_R^2 \le C_P^2$ and $C_R^2 \le \overline{C}_L^2$.

(1) We first prove $C_R^2 \le C_P^2$ holds.

From Table C.1, we know that we should prove $-c+\sqrt{c^2+\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2} \ge [k+(1-\beta)(a+\varepsilon-r)]-\sqrt{[k+(1-\beta)(a+\varepsilon-r)]^2-\left\{k^2-\left[(1-\beta)v_{\max}-c\right]^2\right\}}$. We equally prove $\sqrt{[k+(1-\beta)(a+\varepsilon-r)]^2-\left\{k^2-\left[(1-\beta)v_{\max}-c\right]^2\right\}} \ge (1-\beta)(v_{\max}+\frac{1}{2}(a+r-\varepsilon))+(1-\beta)(a+\varepsilon-r)-\sqrt{c^2+\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2}$. By squaring both sides, we equally prove $[k+(1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2-\left[(1-\beta)v_{\max}-c\right]^2\right\} \ge [k+(1-\beta)(a+\varepsilon-r)]^2 - 2\left[k+(1-\beta)(a+\varepsilon-r)\right]\sqrt{c^2+\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2}+c^2+\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2$. Through transformation and simplification, we have $\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})+(1-\beta)(a+\varepsilon-r)\right]\sqrt{c^2+\left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2} \ge \left[(1-\beta)(v_{\max}+\frac{a+r-\varepsilon}{2})\right]^2-\left[(1-\beta)v_{\max}\right]^2+c(1-\beta)\left(v_{\max}+a+\varepsilon-r\right)$. By squaring both sides and after simplification, we equally prove $N\left(v_{\max}\right)=\frac{1}{4}(1-\beta)^2\{-4[(-1+\beta)v_{\max}{}^2+c\left(a+\varepsilon-r+v_{\max}\right)+\frac{1}{4}(1-\beta)\left(a-\varepsilon+r+2v_{\max}\right)^2]^2+\left(3a+\varepsilon-r+2v_{\max}\right)^2[c^2-(1-\beta)^2 v_{\max}{}^2+\frac{1}{4}(1-\beta)^2\left(a-\varepsilon+r+2v_{\max}\right)^2]\} \ge 0$. We can get that $\frac{\partial^2 N(v_{\max})}{\partial v_{\max}{}^2}=\frac{1}{2}(1-\beta)^3\left(a+r-\varepsilon\right)\left[(1-\beta)\left(9a+12v_{\max}-7\left(r-\varepsilon\right)\right)-8c\right]$. Obviously, $\frac{\partial^2 N(v_{\max})}{\partial v_{\max}{}^2}$ increases with $v_{\max}$ when $a \ge r-\varepsilon$ and $\varepsilon \le r$. Therefore, the minimal $\frac{\partial^2 N(v_{\max})}{\partial v_{\max}{}^2}$ is $\left.\frac{\partial^2 N(v_{\max})}{\partial v_{\max}{}^2}\right|_{v_{\max}=\frac{c}{1-\beta}+2\varepsilon}=\frac{1}{2}(1-\beta)^3\left(a+r-\varepsilon\right)\left[4c+(1-\beta)\left(2r+22\varepsilon\right)\right] \ge 0$. So $\frac{\partial N(v_{\max})}{\partial v_{\max}}=\frac{1}{4}(1-\beta)^2\{(1-\beta)^2\left(a+r-\varepsilon\right)\left(3a-r+\varepsilon+2v_{\max}\right)^2-8\left((1-\beta)\left(a+r-\varepsilon\right)+c\right)[(-1+\beta)v_{\max}{}^2+c\left(a+\varepsilon-r+v_{\max}\right)+\frac{1}{4}(1-\beta)\left(a-\varepsilon+r+2v_{\max}\right)^2]+4\left(3a+\varepsilon-r+2v_{\max}\right)[c^2-(1-\beta)^2 v_{\max}{}^2+\frac{1}{4}(1-\beta)^2\left(a-\varepsilon+r+2v_{\max}\right)^2]\}$ increases with $v_{\max}$. We then get the minimal $\frac{\partial N(v_{\max})}{\partial v_{\max}}$ is $\left.\frac{\partial N(v_{\max})}{\partial v_{\max}}\right|_{v_{\max}=\frac{c}{1-\beta}+2\varepsilon}=\frac{1}{2}(1-\beta)^3\left(a+r-\varepsilon\right)\left[4c\left(a+3\varepsilon-r\right)+(1-\beta)\left(5a^2-4ar-r^2+22a\varepsilon-12r\varepsilon+37\varepsilon^2\right)\right]$, which increases with a. Using $a \ge r-\varepsilon$, we have the minimal value of $\frac{\partial N(v_{\max})}{\partial v_{\max}}$ is $4(1-\beta)^3\left(r-\varepsilon\right)\varepsilon\left[2c+(1-\beta)\left(5\varepsilon+r\right)\right]$, which is greater than zero. Therefore, $N\left(v_{\max}\right)$ increases with $v_{\max}$. Then the minimal $N\left(v_{\max}\right)$ is $\left.N\left(v_{\max}\right)\right|_{v_{\max}=\frac{c}{1-\beta}+2\varepsilon}=\frac{1}{4}(1-\beta)^2\{-4[c(a-r+\frac{c}{1-\beta}+3\varepsilon)+\frac{1}{4}(1-\beta)(a+r+\frac{2c}{1-\beta}+3\varepsilon)^2-\frac{(c+2(1-\beta)\varepsilon)^2}{1-\beta}]^2+(-3a+r-5\varepsilon-\frac{2c}{1-\beta})^2[c^2-(c+2(1-\beta)\varepsilon)^2+\frac{1}{4}(a+r+2c-a\beta-r\beta+3\varepsilon-3\beta\varepsilon)^2]\}$. We need to prove the value of above expression is greater than zero. We denote the above expression as $M\left(a\right)$. We can get $\frac{\partial^2 M(a)}{\partial a^2}=(1-\beta)^2\left[2c^2+(1-\beta)^2\left(6a^2-r^2+4r\varepsilon+3a\left(9\varepsilon+r\right)\right)+c\left(1-\beta\right)\left(8\varepsilon+12a\right)+15\varepsilon^2-30\beta\varepsilon^2+15\beta^2\varepsilon^2\right]$, which increases with a. Therefore, the minimal value of $\frac{\partial^2 M(a)}{\partial a^2}$ is $\left.\frac{\partial^2 M(a)}{\partial a^2}\right|_{a=r-\varepsilon}=2(1-\beta)^2\left[c^2+4r^2(1-\beta)^2-2c\left(1-\beta\right)\varepsilon-3(1-\beta)^2\varepsilon^2+2r\left(1-\beta\right)\left(3c+4\left(1-\beta\right)\varepsilon\right)\right]$. Using $r \ge \varepsilon$, we have $\left.\frac{\partial^2 M(a)}{\partial a^2}\right|_{a=r-\varepsilon} \ge 2(1-\beta)^2\left[c^2+9(1-\beta)^2\varepsilon^2+4c\left(1-\beta\right)\varepsilon\right] \ge 0$. Then we can get that $\frac{\partial M(a)}{\partial a}$ increases with a. Using $a \ge r-\varepsilon$, we have the minimal $\frac{\partial M(a)}{\partial a}$ is $\left.\frac{\partial M(a)}{\partial a}\right|_{a=r-\varepsilon}=2(1-\beta)^2$

$\left[\varepsilon^3(1-\beta)^2+\varepsilon^2(1-\beta)(2c+3(1-\beta)\varepsilon)\right] \geq 0$, which reveals that $M(a)$ increases with a. So the minimal $M(a)$ is $M(a)|_{a=r-\varepsilon} = 4(1-\beta)^3(r-\varepsilon)\varepsilon^2[(1-\beta)(3\varepsilon+r)+2c] \geq 0$. Till now, we finally prove $C_R^2 \leq C_P^2$ holds when $a > r-\varepsilon$.

(2) We now prove $C_R^2 \leq \overline{C}_L^2$. We should prove $[k+(1-\beta)(a+\varepsilon-r)] - \sqrt{[k+(1-\beta)(a+\varepsilon-r)]^2 - \left\{k^2 - \left[(1-\beta)v_{\max}-c\right]^2\right\}} \leq (1-\beta)(v_{\max} - \frac{r}{2} + \frac{3}{2}(a-\varepsilon)) - c$. Through transformation and simplification, we equally prove $Q\left(v_{\max}\right) = [k+(1-\beta)(a+\varepsilon-r)]^2 - k^2 + \left[(1-\beta)v_{\max}-c\right]^2 - 4(1-\beta)^2\varepsilon^2 \geq 0$. $\frac{\partial Q(v_{\max})}{\partial v_{\max}} = 2(1-\beta)\left[(1-\beta)\left(v_{\max}+a+\varepsilon-r\right)-c\right]$. Using $v_{\max} \geq \frac{c}{1-\beta}+2\varepsilon$, we can get that $\frac{\partial Q(v_{\max})}{\partial v_{\max}} \geq 2(1-\beta)^2(a-r+3\varepsilon)$. Using $a > r-\varepsilon$, we can get that $\frac{\partial Q(v_{\max})}{\partial v_{\max}} \geq 2(1-\beta)^2(a-r+3\varepsilon) \geq 0$. Therefore, $C_R^2 \leq \overline{C}_L^2$ holds.

This completes the proof of Proposition 4(ii).

The proof of Proposition 4(iii) is similar to the proof of Proposition 4(i) and Proposition 4(ii), so we omit it. Till now, we finally prove Proposition 4.

Proof of Corollary 1. To explore the conditions where WFS always benefit the platform, we equally find the conditions where platform's indifference curves are over the upper bounds of the market constraint lines (presented in the proof of Table C.1). We first prove Corollary 1. Obviously, both $\overline{C}_L^1$ and C_p^1 increase with a. When $a=0$, we have $\overline{C}_L^1 = (1-\beta)(v_{\max}-2\varepsilon)-c \geq (1-\beta)(\frac{c}{1-\beta}+2\varepsilon-2\varepsilon)-c = 0$ and $C_p^1 = -c+\sqrt{c^2+\left[(1-\beta)v_{\max}\right]^2-\left[(1-\beta)v_{\max}\right]^2} = 0$. So, $\overline{C}_L^1 \geq C_p^1$ when $a=0$. Because $\overline{C}_L^1$ and C_p^1 increase with a and $\overline{C}_L^1 \geq C_p^1$ when $a=0$, only $\overline{C}_L^1 < C_p^1$ when $a=\varepsilon$ can the condition where WFS always benefits the platform appear. Solving $\overline{C}_L^1 < C_p^1$ when $a=\varepsilon$, we can get $\varepsilon > \frac{[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2 v_{\max}}$. Through solving $C_p^1 = \overline{C}_L^1$, we have $a = \frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon}$. Therefore, when $\frac{[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2 v_{\max}} \leq \varepsilon \leq \frac{r}{2}$, if $\frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon} \leq a \leq \varepsilon$, WFS always benefits the platform despite that it may hurt the retailer. This completes the proof of Corollary 1(i).

We now prove Corollary 1(ii). As illustrated in the proof of Corollary 1(i), we should find the conditions where platform's indifference curves are over the upper bounds of market constraint lines when $\frac{r}{2} < \varepsilon \leq r$. We can also prove that both $\overline{C}_L^2$ and C_p^2 increase with a. When $a \leq r-\varepsilon$, we can prove that $\overline{C}_L^2 \geq C_p^2$ when $a=0$, as illustrated in the proof of Corollary 1(i). Because $\overline{C}_L^2$ and C_p^2 increase with a and $\overline{C}_L^2 \geq C_p^2$ when $a=0$, only $C_p^2 > \overline{C}_L^2$ when $a=r-\varepsilon$ can the condition where WFS always benefits platform appear. Solving $C_p^2 \geq \overline{C}_L^2$ when $a=r-\varepsilon$, we can get $r \geq \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$. Then we can get that if $\frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon} \leq a \leq r-\varepsilon$, WFS always benefits platform. When $a > r-\varepsilon$, to guarantee that the condition where WFS always benefits platform appear, we should make $C_p^2 \geq \overline{C}_L^2$ when $a=\varepsilon$. Solving $C_p^2 \geq \overline{C}_L^2$ when $a=\varepsilon$, we can get that $r \geq \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$. Through solving $C_p^2 = \overline{C}_L^2$, we can get $a = \frac{-(1-\beta)(v_{\max}-r-2\varepsilon)+\sqrt{[(1-\beta)(v_{\max}-r-2\varepsilon)]^2-2\left[(1-\beta)^2(v_{\max}{}^2-2(r+\varepsilon)(v_{\max}-\varepsilon))-c^2\right]}}{2(1-\beta)}$. Then we can get that if $\frac{-(1-\beta)(v_{\max}-r-2\varepsilon)+\sqrt{[(1-\beta)(v_{\max}-r-2\varepsilon)]^2-2\left[(1-\beta)^2(v_{\max}{}^2-2(r+\varepsilon)(v_{\max}-\varepsilon))-c^2\right]}}{2(1-\beta)} \leq a \leq \varepsilon$, WFS always benefits platform. Through analyses above, we can get that: (1) if $r \geq \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ and $r \geq \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, we have WFS always benefits platform when $\frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon} \leq a \leq \varepsilon$; (2) if $r \geq \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ and $r < \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, we have WFS always benefits platform when $\frac{[(1-\beta)(v_{\max}-2\varepsilon)]^2-c^2}{4(1-\beta)^2\varepsilon} \leq a \leq \frac{-(1-\beta)(v_{\max}-r-2\varepsilon)+\sqrt{[(1-\beta)(v_{\max}-r-2\varepsilon)]^2-2\left[(1-\beta)^2(v_{\max}{}^2-2(r+\varepsilon)(v_{\max}-\varepsilon))-c^2\right]}}{2(1-\beta)}$; (3) if $r < \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ and $r \geq \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, we have WFS always benefits platform when $\frac{-(1-\beta)(v_{\max}-r-2\varepsilon)+\sqrt{[(1-\beta)(v_{\max}-r-2\varepsilon)]^2-2\left[(1-\beta)^2(v_{\max}{}^2-2(r+\varepsilon)(v_{\max}-\varepsilon))-c^2\right]}}{2(1-\beta)} \leq a \leq \varepsilon$; (4) however, if $r < \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ and $r < \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, we cannot derive that WFS always benefits platform. In other words, WFS service maybe hurt the platform.

In conclusion, we have if $r \geq \frac{8[(1-\beta)\varepsilon]^2+[(1-\beta)v_{\max}]^2-c^2}{4(1-\beta)^2\varepsilon} - v_{\max}$ or $r \geq \frac{[(1-\beta)v_{\max}]^2-c^2}{2(1-\beta)^2 v_{\max}}$, there always exist $\underline{a}$ and $\overline{a}$, such that when $a \in [\underline{a}, \overline{a}]$, WFS strategy will always benefit the platform despite that it may hurt the retailer. This completes the proof of Corollary 1(ii).

We now prove Corollary 1(iii). We can get that both $\overline{C}_L^3$ and C_p^3 increase with a. When $a=0$, we have $\overline{C}_L^3 = (1-\beta)(v_{\max}-\frac{r}{2}+\frac{3}{2}(a-\varepsilon))-c \geq (1-\beta)(\frac{c}{1-\beta}-\frac{1}{2}(\varepsilon-r)+2\varepsilon-\frac{r}{2}+\frac{3}{2}(a-\varepsilon))-c = \frac{3}{2}(1-\beta)a \geq 0$ and $C_p^3 = -c + \sqrt{c^2+\left[(1-\beta)\left(v_{\max}+\frac{1}{2}(r-\varepsilon)\right)\right]^2-\left[(1-\beta)\left(v_{\max}+\frac{1}{2}(r-\varepsilon)\right)\right]^2} = 0$. So we have $\overline{C}_L^3 \geq C_p^3$ when $a=0$. When $a=\varepsilon$, we have $\overline{C}_L^3 = (1-\beta)(v_{\max}-\frac{1}{2}r)-c$ and $C_p^3 = -c+\sqrt{c^2+\left[(1-\beta)\left(v_{\max}+\frac{1}{2}r\right)\right]^2-\left[(1-\beta)\left(v_{\max}+\frac{1}{2}(r-\varepsilon)\right)\right]^2}$. We further prove that $\overline{C}_L^3 \geq C_p^3$ when $a=\varepsilon$. Though transformation and simplification, we equally prove $\overline{C}_L^3 - C_p^3 = \left[(1-\beta)\left(v_{\max}-\frac{1}{2}r\right)\right]^2 - c^2 - \left[(1-\beta)\left(v_{\max}+\frac{1}{2}r\right)\right]^2 + \left[(1-\beta)\left(v_{\max}+\frac{1}{2}(r-\varepsilon)\right)\right]^2 \geq 0$. $\frac{\partial\left(\overline{C}_L^3-C_p^3\right)}{\partial v_{\max}} = 2(1-\beta)^2 v_{\max} - (1-\beta)^2(\varepsilon+r)$. Using $v_{\max} \geq \frac{c}{1-\beta} - \frac{1}{2}(\varepsilon-r)+2\varepsilon$, we can get that $\frac{\partial\left(\overline{C}_L^3-C_p^3\right)}{\partial v_{\max}} \geq (1-\beta)(2c+(1-\beta)(\epsilon-r))\epsilon \geq 0$ $(\epsilon > r)$. Therefore, $\overline{C}_L^3 \geq C_p^3$ when $\epsilon > r$. So $\overline{C}_L^3 \geq C_p^3$ holds in the range of a. In other words, WFS service is always possible to hurt both the platform and the retailer. This completes the proof of Corollary 1(iii).

In conclusion, we completes the proof of Corollary 1.

Proof of Proposition 5. Before we prove Proposition 5, we first derive consumer surplus and social welfare in the cases of No-WFS strategy and WFS strategy. Then we make differences between them to investigate the change of consumer surplus and social welfare.

We first calculate consumer surplus (denoted as CS_1) and social welfare (denoted as SW_1) in the case of No-WFS strategy.

(1) When $\epsilon \leq r$, CS_1 is the sum of overestimating and underestimating customers' utility, i.e., $CS_1 = \frac{1}{2}\int_{p-\epsilon}^{v_{\max}}(v-p)dG(v) + \frac{1}{2}\int_{p+\epsilon}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $p^* = \frac{1}{2}(v_{\max} + \frac{c}{1-\beta})$ into the above formulation, we have $CS_1 = \frac{[(1-\beta)v_{\max}-c]^2-4(1-\beta)^2\epsilon^2}{8(1-\beta)^2(v_{\max}-v_{\min})}$. Then, $SW_1 = \frac{[(1-\beta)v_{\max}-c]^2-4(1-\beta)^2\epsilon^2}{8(1-\beta)^2(v_{\max}-v_{\min})} + \frac{[(1-\beta)v_{\max}-c]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\beta\left[((1-\beta)v_{\max})^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(2) When $\epsilon > r$, $CS_1 = \frac{1}{2}\int_{p-r}^{v_{\max}}(v-p)dG(v) + \frac{1}{2}\int_{p-\epsilon}^{p-r}(-r)dG(v) + \frac{1}{2}\int_{p+\epsilon}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $p^* = \frac{1}{2}(v_{\max}+\frac{1}{2}(r-\epsilon)+\frac{c}{1-\beta})$ into the above formulation, we have $CS_1 = \frac{1}{8(v_{\max}-v_{\min})}\left[\left(v_{\max}+\frac{1}{2}\left(r-\epsilon+\frac{2c}{1-\beta}\right)\right)^2 + 2v_{\max}\left(\epsilon-r-\frac{2c}{1-\beta}\right) + 2r^2-4r\epsilon-2\epsilon^2\right]$. Therefore, $SW_1 = \frac{1}{8(v_{\max}-v_{\min})}\left[\left(v_{\max}+\frac{1}{2}\left(r-\epsilon+\frac{2c}{1-\beta}\right)\right)^2 + 2v_{\max}\left(\epsilon-r-\frac{2c}{1-\beta}\right)+2r^2-4r\epsilon-2\epsilon^2\right] + \frac{\left[(1-\beta)\left(v_{\max}+\frac{1}{2}(r-\epsilon)\right)-c\right]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\beta\left[\left((1-\beta)\left(v_{\max}+\frac{1}{2}(r-\epsilon)\right)\right)^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

We then calculate the consumer surplus (denoted as CS_2) and social welfare (denoted as SW_2) in the case of WFS strategy.

(1) when $0 \leq \epsilon \leq \frac{r}{2}$, or when $\frac{r}{2} < \epsilon \leq r$ and $0 \leq a \leq r-\epsilon$, $CS_2 = \frac{1}{2}\int_{p-\epsilon-a}^{v_{\max}}(v-p)dG(v) + \frac{1}{2}\int_{p+\epsilon-a}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $\hat{p}^* = \frac{1}{2}(v_{\max}+a+\frac{c+C}{1-\beta})$ into the above formulation, we have $CS_2 = \frac{1}{8(v_{\max}-v_{\min})}\left[\left(v_{\max}+a+\frac{c+C}{1-\beta}\right)^2 - \frac{4(c+C)v_{\max}}{1-\beta} - 4a^2 - 4\epsilon^2 - 4av_{\max}\right]$. Therefore, $SW_2 = \frac{1}{8(v_{\max}-v_{\min})}\left[\left(v_{\max}+a+\frac{c+C}{1-\beta}\right)^2 - \frac{4(c+C)v_{\max}}{1-\beta} - 4a^2-4\epsilon^2-4av_{\max}\right] + \frac{[(1-\beta)(v_{\max}+a)-c-C]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\beta\left[((1-\beta)(v_{\max}+a))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(2) When $\frac{r}{2} < \epsilon \leq r$ and $r-\epsilon < a \leq \epsilon$, or when $\epsilon > r$, $CS_2 = \frac{1}{2}\int_{p-r}^{v_{\max}}(v-p)dG(v) + \frac{1}{2}\int_{p-\epsilon-a}^{p-r}(-r)dG(v) + \frac{1}{2}\int_{p+\epsilon-a}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $\hat{p}^* = \frac{1}{2}(v_{\max}+\frac{1}{2}(a+r-\epsilon)+\frac{c+C}{1-\beta})$ into the above formulation, we have $CS_2 = \frac{1}{8(v_{\max}-v_{\min})}\left[\left(v_{\max}+\frac{1}{2}(a+r-\epsilon)+\frac{c+C}{1-\beta}\right)^2 - \frac{4(c+C)v_{\max}}{1-\beta} - 2a^2-4ar+2r^2+4a\epsilon-4r\epsilon-2\epsilon^2-2(a+r-\epsilon)v_{\max}\right]$. Therefore, we have $SW_2 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max}+\frac{1}{2}(a+r-\epsilon)+\frac{c+C}{1-\beta})^2 - \frac{4(c+C)v_{\max}}{1-\beta} - 2a^2-4ar+2r^2+4a\epsilon-4r\epsilon-2\epsilon^2-2(a+r-\epsilon)v_{\max}] + \frac{\left[(1-\beta)\left(v_{\max}+\frac{1}{2}(a+r-\epsilon)\right)-c-C\right]^2}{4(1-\beta)(v_{\max}-v_{\min})} - \frac{(a+\epsilon-r)C}{2(v_{\max}-v_{\min})} + \frac{\beta\left[\left((1-\beta)\left(v_{\max}+\frac{1}{2}(a+r-\epsilon)\right)\right)^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

Now we first prove Proposition 5(i).

When $0 \leq \epsilon \leq \frac{r}{2}$, or when $\frac{r}{2} < \epsilon \leq r$ and $0 \leq a \leq r-\epsilon$, $CS_2 - CS_1 = \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[C^2 - 2((1-\beta)v_{\max}-(a+c-a\beta))C + a(1-\beta)(2c-3(1-\beta)a) - 2a(1-\beta)^2v_{\max}]$. Solving $CS_2 - CS_1 = 0$, we get $\tilde{C}_1 = (1-\beta)(v_{\max}-a) - c - \sqrt{(1-\beta)^2(v_{\max}{}^2+4a^2)+c^2-2cv_{\max}}$ or $\tilde{C}_2 = (1-\beta)(v_{\max}-a) - c + \sqrt{(1-\beta)^2(v_{\max}{}^2+4a^2)+c^2-2cv_{\max}}$.

$\tilde{C}_2 - \overline{C}_L^1 = (1-\beta)(v_{\max}-a) - c + \sqrt{(1-\beta)^2(v_{\max}{}^2+4a^2)+c^2-2cv_{\max}} - ((1-\beta)(v_{\max}+a-2\epsilon)-c) = 2(1-\beta)(\epsilon-a) + \sqrt{(1-\beta)^2(v_{\max}{}^2+4a^2)+c^2-2cv_{\max}} > 0$ so $\tilde{C}_2$ is out of the market boundary. Furthermore, we have $\tilde{C}_1 \leq 0$. To prove that, we calculate $\left[(1-\beta)(v_{\max}-a)-c\right]^2 - \left(\sqrt{(1-\beta)^2(v_{\max}{}^2+4a^2)+c^2-2cv_{\max}}\right)^2 = a(1-\beta)\left[2c-3a(1-\beta)-2(1-\beta)v_{\max}\right]$. Since $v_{\max} \geq \frac{c}{1-\beta}+2\epsilon$, we have $a(1-\beta)\left[2c-3a(1-\beta)-2(1-\beta)v_{\max}\right] \leq -a(1-\beta)^2(3a+4\epsilon) \leq 0$. Therefore, we have $\tilde{C}_1 \leq 0$. Since $CS_2 - CS_1$ is a convex function of C, we have $CS_2 - CS_1 \leq 0$, i.e., CS decreases, in the field of definition(i.e., in the field of market boundary). We can prove that CS decreases with the similar way under the condition when $\frac{r}{2} < \epsilon \leq r$ and $r-\epsilon < a \leq \epsilon$.

When $\epsilon > r$, $CS_2 - CS_1 = \frac{1}{32(1-\beta)^2(v_{\max}-v_{\min})}[4C^2 - 4((1-\beta)(2v_{\max}-a-r+\epsilon)-2c)C - a(1-\beta)((1-\beta)(4v_{\max}+7a+14r-14\epsilon)-4c)]$. Solve $CS_2 - CS_1 = 0$, we have $\hat{C}_1 = \frac{1}{2}[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c] - \frac{1}{2}\sqrt{[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c]^2+A_1}$ and $\hat{C}_2 = \frac{1}{2}[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c]+\frac{1}{2}\sqrt{[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c]^2+A_1}$, where $A_1 = a(1-\beta)[(1-\beta)(4v_{\max}+7a+14r-14\epsilon)-4c]$. We now prove $\hat{C}_2$ is out of the market boundary. To prove that, through simplification, we equally prove $(1-\beta)(2v_{\max}-a-r+\epsilon)-2c-2[(1-\beta)(v_{\max}-\frac{1}{2}(a+r-\epsilon)+2(a-\epsilon))-c]+\sqrt{[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c]^2+A_1} = 4(1-\beta)(\epsilon-a)+\sqrt{[(1-\beta)(2v_{\max}-a-r+\epsilon)-2c]^2+A_1} > 0$. Because $\epsilon \geq a$, $4(1-\beta)(\epsilon-a) > 0$ always holds. Therefore we can get that $\hat{C}_2$ is out of the market boundary. i.e., $\hat{C}_2 > \underline{C}_L^3$. In addition, we cannot prove that $\hat{C}_1 > \max(0, \underline{C}_L^3)$ holds. Because $CS_2 - CS_1$ is a convex function of C and $\hat{C}_2 > \underline{C}_L^3$, if $\hat{C}_1 < \max(0, \underline{C}_L^3)$, then $CS_2 - CS_1 < 0$ always holds, i.e., CS decreases; if $\hat{C}_1 \geq \max(0, \underline{C}_L^3)$, then $CS_2 - CS_1 \geq 0$ when $C \in (\max(0, \underline{C}_L^3), \hat{C}_1]$ and $CS_2 - CS_1 \leq 0$ when $C \in (\hat{C}_1, \overline{C}_L^3]$. Let $\hat{C} = \hat{C}_1$, we have Proposition 5(i) holds. This completes the proof of Proposition 5(i).

We now prove Proposition 5(ii).

When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r-\varepsilon$, $SW_2 - SW_1 = \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[(3-4\beta)C^2 - 2(a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max})C + 2a(1-\beta)^2 v_{\max} - a(1-\beta)(a+2c-a\beta-4c\beta)]$. Solve $SW_2 - SW_1 = 0$, we can get $C_1 = \frac{1}{(3-4\beta)}[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max} - \sqrt{[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max}]^2 + A_2}]$ and $C_2 = \frac{1}{(3-4\beta)}[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max} + \sqrt{[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max}]^2 + A_2}]$, where $A_2 = -(3-4\beta)[2a(1-\beta)^2 v_{\max} - a(1-\beta)(a+2c-a\beta-4c\beta)]$. We first prove $C_2 > \overline{C}_L^1$. To prove that, through simplification, we equally prove $a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max} - (3-4\beta)[(1-\beta)(v_{\max}+a-2\varepsilon)-c] + \sqrt{[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max}]^2 + A_2} = 2(1-\beta)[a(-1+\beta)+(3-4\beta)\varepsilon+\beta v_{\max}] + \sqrt{[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max}]^2 + A_2} > 0$ holds. Using $v_{\max} \ge \frac{c}{1-\beta} + 2\varepsilon$, we have $2(1-\beta)[a(-1+\beta)+(3-4\beta)\varepsilon+\beta v_{\max}] \ge 2[-a(1-\beta)^2 + c\beta + (3-5\beta+2\beta^2)\varepsilon]$. Using $\varepsilon \ge a$, we have $2[-a(1-\beta)^2 + c\beta + (3-5\beta+2\beta^2)\varepsilon] \ge 2[c\beta + a(1-\beta)(2-\beta)] \ge 0$. Therefore, we have $C_2 > \overline{C}_L^1$ always holds. We next prove $C_1 > 0$ always holds. To prove that, we equally prove $-(3-4\beta)[2a(1-\beta)^2 v_{\max} - a(1-\beta)(a+2c-a\beta-4c\beta)] < 0$. Using $v_{\max} \ge \frac{c}{1-\beta} + 2\varepsilon$, we have $-(3-4\beta)[2a(1-\beta)^2 v_{\max} - a(1-\beta)(a+2c-a\beta-4c\beta)] < a(1-\beta)(3-4\beta)[a(1-\beta) - 4(c\beta+\varepsilon-\beta\varepsilon)]$. Using $\varepsilon \ge a$, we have $a(1-\beta)(3-4\beta)[a(1-\beta)-4(c\beta+\varepsilon-\beta\varepsilon)] \le -a(1-\beta)(3-4\beta)[3a(1-\beta)+4c\beta] \le 0$ always holds. Further, we need to prove that $C_1 \le \overline{C}_L^1$. To prove that, through simplification, we equally prove $a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max} - (3-4\beta)[(1-\beta)(v_{\max}+a-2\varepsilon)-c] < \sqrt{[a-3c-3a\beta+4c\beta+2a\beta^2+(3-2\beta)(1-\beta)v_{\max}]^2 + A_2}$. Making a difference after squaring both sides, we equally prove $f(v_{\max}) = -3(3-4\beta)(1-\beta)^2 v_{\max}^2 + 2(1-\beta)(3-4\beta)[c(3-2\beta)+4(1-\beta)\beta\varepsilon]v_{\max} + (3-4\beta)[-c^2(3-4\beta)+4(1-\beta)^2\varepsilon(-2a(1-\beta)+(3-4\beta)\varepsilon)] < 0$. Solve $\frac{\partial f(v_{\max})}{\partial v_{\max}} = 0$, we can get $v_{\max} = \frac{1}{3}\left[\frac{(3-2\beta)c}{1-\beta} + 4\beta\varepsilon\right]$. Since $\frac{1}{3}\left[\frac{(3-2\beta)c}{1-\beta} + 4\beta\varepsilon\right] - \left(\frac{c}{1-\beta} + 2\varepsilon\right) = -\frac{2(c\beta+(1-\beta)(3-2\beta)\varepsilon)}{3(1-\beta)} < 0$ and $f(v_{\max})$ is a concave function of $v_{\max}$, $f(v_{\max}) \le f\left(\frac{c}{1-\beta} + 2\varepsilon\right) = -8a(3-4\beta)(1-\beta)^3\varepsilon \le 0$ always holds. Therefore, $C_1 \le \overline{C}_L^1$. Because $SW_2 - SW_1 = 0$ is a convex function of C, we have when $C_1 > \max(0, \underline{C}_L^1)$, then $SW_2 - SW_1 \ge 0$ if $C \le C_1$ while $SW_2 - SW_1 < 0$ if $C > C_1$. Let $C_1^{sw} = C_1$, we get the results when $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r-\varepsilon$. The proofs of the cases under $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$, or $\varepsilon > r$ are similar to the above procedure so we omit them. This completes the proof of Proposition 5(ii). Then the proof of Proposition 5 is completed.

Proof of Lemma 3. From Lemmas 1 and 2, using equation $\hat{\pi}_P^* - \tau \ge \pi_P^*$ and $\pi_R^* + \tau \ge \hat{\pi}_R^*$, we can easily get the range of τ as shown in Lemma 3.

Proof of Proposition 6. Before we prove Proposition 6, we first derive S_p in each case. Based on Lemma 3, we can get S_p as follows: when $0 \le \varepsilon \le \frac{r}{2}$, $S_p = \frac{(1-2\beta)C^2 - 2\left((1-\beta)^2(v_{\max}+a)-(1-2\beta)c\right)C + a(1-\beta)^2(a-2c+2v_{\max})}{4(1-\beta)^2(v_{\max}-v_{\min})}$;

when $\frac{r}{2} < \varepsilon \le r$, $S_p = \begin{cases} \frac{(1-2\beta)C^2 - 2\left((1-\beta)^2(v_{\max}+a)-(1-2\beta)c\right)C + a(1-\beta)^2(a-2c+2v_{\max})}{4(1-\beta)^2(v_{\max}-v_{\min})}, 0 \le a \le r-\varepsilon \\ \frac{4(1-2\beta)C^2 - 4\left((1-\beta)^2(3a-r+\varepsilon)-2(1-2\beta)c\right)C + (1-\beta)^2\left((a+r-\varepsilon)(a+r-\varepsilon-4c)+4v_{\max}(a+r-\varepsilon-2C)\right)}{16(1-\beta)^2(v_{\max}-v_{\min})}, r-\varepsilon < a \le \varepsilon \end{cases}$; when $\varepsilon > r$, $S_p = \frac{4(1-2\beta)C^2 - 4\left((1-\beta)^2(3a-r+\varepsilon)-2(1-2\beta)c\right)C + (1-\beta)^2\left(a(a+2r-2\varepsilon-4c)+4v_{\max}(a-2C)\right)}{16(1-\beta)^2(v_{\max}-v_{\min})}$. Then we begin to prove Proposition 6.

We first prove Proposition 6(i). When $0 \le \varepsilon \le \frac{r}{2}$, $\frac{\partial S_p}{\partial C} = \frac{(1-2\beta)C - (1-\beta)^2(v_{\max}+a) + (1-2\beta)c}{2(1-\beta)^2(v_{\max}-v_{\min})}$. Using $C \le (1-\beta)(v_{\max}+a-2\varepsilon)-c$ (i.e., upper bound of market constraint; we will not repeat this hereafter.), we can get that $\frac{\partial S_p}{\partial C} \le -\frac{\beta(v_{\max}+a)+2(1-2\beta)\varepsilon}{2(1-\beta)(v_{\max}-v_{\min})} \le 0$. In a similar way, we can also get that $\frac{\partial S_p}{\partial C} \le 0$ when $\frac{r}{2} < \varepsilon \le r$ and when $\varepsilon > r$. Therefore, S_p decreases with C. This completes the proof of Proposition 6(i).

We now prove Proposition 6(ii). When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, $\frac{\partial S_p}{\partial a} = \frac{a-c+v_{\max}-C}{2(v_{\max}-v_{\min})}$. Using $C \le (1-\beta)(v_{\max}+a-2\varepsilon)-c$, we have $\frac{\partial S_p}{\partial a} \ge \frac{\beta(v_{\max}+a)+2(1-\beta)\varepsilon}{2(v_{\max}-v_{\min})} \ge 0$. When $\varepsilon > r$, $S_p(a)|_{a=0} = \frac{C((1-\beta)^2(r-\varepsilon-2v_{\max})+(1-2\beta)(2c+C))}{4(1-\beta)^2(v_{\max}-v_{\min})}$. Using $C < (1-\beta)(v_{\max}-\frac{r}{2}-\frac{3\varepsilon}{2})-c$ (at $a=0$), we have $(1-\beta)^2(r-\varepsilon-2v_{\max})+(1-2\beta)(2c+C) < \frac{1}{2}(r+2c-r\beta-4c\beta-5\varepsilon+(13-8\beta)\beta\varepsilon-2(1-\beta)v_{\max})$. Further, using $v_{\max} > \frac{c}{1-\beta} + \frac{r}{2} + \frac{3\varepsilon}{2}$, we have $\frac{1}{2}(r+2c-r\beta-4c\beta-5\varepsilon+(13-8\beta)\beta\varepsilon-2(1-\beta)v_{\max}) < -2(c\beta+2(1-\beta)^2\varepsilon) < 0$ holds. Because S_p is a convex function of a (i.e., $\frac{\partial^2 S_p^{s\phi}}{\partial a^2} = \frac{1}{8(v_{\max}-v_{\min})} > 0$), we have S_p ($S_p > 0$) increases with a holds. This completes the proof of Proposition 6(ii).

We now prove Proposition 6(iii). When $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$, $\frac{\partial S_p}{\partial a} = \frac{a+r-\varepsilon-2c+2v_{\max}-6C}{8(v_{\max}-v_{\min})}$. Solving $\frac{\partial S_p}{\partial a} = 0$, we can get $a_s = 6C - 2\left(v_{\max}-c\right) + \varepsilon - r$. As S_p is a convex function of a, so we have when $a_s < r-\varepsilon$, S_p increases with a; when $r-\varepsilon \le a_s < \varepsilon$, if $a \in \left(r-\varepsilon, a_s\right]$, S_p decreases with a, while when $a \in \left(a_s, \varepsilon\right]$, S_p increases with a; when $a_s \ge \varepsilon$, S_p decreases with a. This completes the proof of Proposition 6(iii).

The above completes the proof of Proposition 6.

Proof of Proposition 7. we first give the retailer's optimal strategy and the retailer and platform's optimal profit if WFS strategy is adopted when incorporating salvage value and market composition. Similar to the proof of Lemma 2, based on Eqs. (7), and Eqs. (8), we can get: under WFS strategy,

Table D.1
The range of $\tau_{s\phi}$ of each case.

	$\tau_{s\phi} \in$
$0 \le \varepsilon \le \frac{r}{2}$	$\left[\frac{(C-(1-\beta)a)((1-\beta)(2v_{\max}+a-2\varepsilon+4\varepsilon\phi)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})}, \frac{\beta(a(1-\beta)^2(2v_{\max}+a-2\varepsilon+4\varepsilon\phi)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right]$
$\frac{r}{2} < \varepsilon \le r$	$\begin{cases}\left[\frac{(C-(1-\beta)a)((1-\beta)(2v_{\max}+a-2\varepsilon+4\varepsilon\phi)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})}, \frac{\beta(a(1-\beta)^2(2v_{\max}+a-2\varepsilon+4\varepsilon\phi)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right], a \le r-\varepsilon \\ \left[\frac{(C-(1-\beta)(a-(a-r+\varepsilon)\phi))((1-\beta)(2v_{\max}-(a-r-3\varepsilon)\phi+a-2\varepsilon)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})} - \frac{\phi(s-c-C)(a+\varepsilon-r)}{(v_{\max}-v_{\min})}, \frac{\beta((1-\beta)^2(a-a\phi+(r-\varepsilon)\phi)(2v_{\max}+a-2\varepsilon-a\phi+r\phi+3\varepsilon\phi)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right], a > r-\varepsilon\end{cases}$
$\varepsilon > r$	$\left[\frac{(C-(1-\beta)(1-\phi)a)((1-\beta)(2v_{\max}+(a-2\varepsilon)(1-\phi)+2r\phi)-2c-C)}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\phi(a(c-s)+(a-r+\varepsilon)C)}{v_{\max}-v_{\min}}, \frac{\beta((1-\beta)^2(a-a\phi)(2v_{\max}+a-2\varepsilon-a\phi+2r\phi+2\varepsilon\phi)-2c\cdot C-C^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}\right]$

(i) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r-\varepsilon$, $\hat{p}^* = \frac{1}{2}(v_{\max} + a - (1-2\phi)\varepsilon + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^* = \frac{((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$;

(ii) When $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$, or when $\varepsilon > r$, $\hat{p}^* = \frac{1}{2}(v_{\max} + \phi r + (1-\phi)(a-\varepsilon) + \frac{c+C}{1-\beta})$, $\hat{\pi}_R^* = \frac{\left[(1-\beta)(v_{\max}+\phi r+(1-\phi)(a-\varepsilon))-c-C\right]^2}{4(1-\beta)(v_{\max}-v_{\min})} + (s-c-C)\cdot\frac{\phi(a+\varepsilon-r)}{(v_{\max}-v_{\min})}$, $\hat{\pi}_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\phi r+(1-\phi)(a-\varepsilon)))^2-(c+C)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$. Proposition 7(i) and (ii) can be easily derived from the above results so we omit the procedures.

We now prove Proposition 7(iii). Solving $\frac{\partial\hat{\pi}_R^*}{\partial a} = 0$, we have $a_{s\phi}^* = \frac{(1-\beta)(1-\phi)[(1-\phi)\varepsilon-\phi r-v_{\max}]-2s\phi+(1+\phi)(c+C)}{(1-\beta)(1-\phi)^2}$. Because $\hat{\pi}_R^*$ is a convex function of a, so we have, if $a_{s\phi}^* < \max(r-\varepsilon, 0)$, $\hat{\pi}_R^*$ increases with a; if $\max(r-\varepsilon,0) \le a_{s\phi}^* < \varepsilon$, when $a \in \left(\max(r-\varepsilon,0), a_{s\phi}^*\right]$, $\hat{\pi}_R^*$ decreases with a, while when $a \in \left(a_{s\phi}^*, \varepsilon\right]$, $\hat{\pi}_R^*$ increases with a; if $a_{s\phi}^* > \varepsilon$, $\hat{\pi}_R^*$ decreases with a. This completes the proof of Proposition 7(iii).

Proof of Proposition 8. Before we prove Proof of Proposition 8, we should first derive the ranges of subsidies. Based on Eqs. (10) and (11), we can derive and summarize them in Table D.1.

Denote the differences of the left and boundaries of $\tau_{s\phi}$ when incorporating the salvage value and market composition as $S_p^{s\phi}$. Then we can prove Proposition 8.

We first prove Proposition 8(i). When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, $\frac{\partial S_p^{s\phi}}{\partial C} = \frac{c-a(1-\beta)^2-2c\beta+(1-\beta)^2\varepsilon(1-2\phi)-(1-\beta)^2v_{\max}+(1-2\beta)C}{2(1-\beta)^2(v_{\max}-v_{\min})} < \frac{(1-\beta)(\varepsilon(\beta-4\phi+6\beta\phi)-a\beta-\beta v_{\max})}{2(1-\beta)^2(v_{\max}-v_{\min})}$ (using $C < (1-\beta)(v_{\max}+a-(1+2\phi)\varepsilon)-c$). Further, using $v_{\max} > \frac{c}{1-\beta} + (1+2\phi)\varepsilon$, we can get $(1-\beta)(\varepsilon(\beta-4\phi+6\beta\phi)-a\beta-\beta v_{\max}) < -c\beta - a(1-\beta)\beta - 4(1-\beta)^2\varepsilon\phi < 0$ holds. Therefore, we can get $\frac{\partial S_p^{s\phi}}{\partial C} < 0$ holds.

When $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$, we have $\frac{\partial S_p^{s\phi}}{\partial C} = \frac{(c+C)(1-2\beta)+(1-\beta)^2(\varepsilon+r\phi-3\varepsilon\phi-(1+\phi)a-v_{\max})}{2(1-\beta)^2(v_{\max}-v_{\min})} < \frac{(-1+\beta)(4\varepsilon\phi+a\beta(1+\phi)-\beta(\varepsilon+r\phi+5\varepsilon\phi)+\beta v_{\max})}{2(1-\beta)^2(v_{\max}-v_{\min})}$ (using $C < (1-\beta)(v_{\max}-\phi r+(1+\phi)(a-\varepsilon))-c$). Further, using $v_{\max} > \frac{c}{1-\beta}+(1+2\phi)\varepsilon$, we can get $(-1+\beta)(4\varepsilon\phi+a\beta(1+\phi)-\beta(\varepsilon+r\phi+5\varepsilon\phi)+\beta v_{\max}) < -c\beta + (1-\beta)(r\beta-4\varepsilon+3\beta\varepsilon)\phi - a(1-\beta)\beta(1+\phi)$. Because $r < 2\varepsilon$, we can get $-c\beta + (1-\beta)(r\beta-4\varepsilon+3\beta\varepsilon)\phi - a(1-\beta)\beta(1+\phi) < -c\beta + (-4+9\beta-5\beta^2)\varepsilon\phi + a(-1+\beta)\beta(1+\phi) < 0$. Therefore, we can get $\frac{\partial S_p^{s\phi}}{\partial C} < 0$ holds.

When $\varepsilon > r$, $\frac{\partial S_p^{s\phi}}{\partial C} = \frac{(c+C)(1-2\beta)+(1-\beta)^2(\varepsilon+r\phi-3\varepsilon\phi-(1+\phi)a-v_{\max})}{2(1-\beta)^2(v_{\max}-v_{\min})} < \frac{(-1+\beta)(4\varepsilon\phi+a\beta(1+\phi)-\beta(\varepsilon+r\phi+5\varepsilon\phi)+\beta v_{\max})}{2(1-\beta)^2(v_{\max}-v_{\min})}$ (using $C < (1-\beta)(v_{\max}-\phi r+(1+\phi)(a-\varepsilon))-c$). Further, using $v_{\max} > \frac{c}{1-\beta} + \phi r + (1+\phi)\varepsilon$, we can get $(-1+\beta)(4\varepsilon\phi + a\beta(1+\phi) - \beta(\varepsilon+r\phi+5\varepsilon\phi) + \beta v_{\max}) < -c\beta - 4(1-\beta)^2\varepsilon\phi - a(1-\beta)\beta(1+\phi) < 0$. Therefore, we can get $\frac{\partial S_p^{s\phi}}{\partial C} < 0$ holds. In conclusion, we have $\frac{\partial S_p^{s\phi}}{\partial C} < 0$ always holds. This completes the proof of Proposition 8(i).

We now prove Proposition 8(ii). When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, $\frac{\partial S_p^{s\phi}}{\partial a} = \frac{a-c-\varepsilon+2\varepsilon\phi+v_{\max}-C}{2(1-\beta)^2(v_{\max}-v_{\min})} > \frac{a\beta-\varepsilon(\beta-4\phi+2\beta\phi)+\beta v_{\max}}{2(1-\beta)^2(v_{\max}-v_{\min})}$ (using $C < (1-\beta)(v_{\max}+a-(1+2\phi)\varepsilon)-c$). Further, using $v_{\max} > \frac{c}{1-\beta}+(1+2\phi)\varepsilon$, we have $a\beta-\varepsilon(\beta-4\phi+2\beta\phi)+\beta v_{\max} > a\beta+\frac{c\beta}{1-\beta}+4\varepsilon\phi > 0$. Therefore, we can get $\frac{\partial S_p^{s\phi}}{\partial a} > 0$, i.e., $S_p^{s\phi}$ increases with a. When $\varepsilon > r$, $S_p^{s\phi}(a)|_{a=0} = \frac{C(2(c-2c\beta+(1-\beta)^2(\varepsilon+r\phi-3\varepsilon\phi))-2(1-\beta)^2q_{\max}+(1-2\beta)C)}{4(1-\beta)^2(v_{\max}-v_{\min})}$. Using $C < (1-\beta)(v_{\max}-\phi r+(1+\phi)(-\varepsilon))-c$ (at $a=0$), we have $2(c-2c\beta+(1-\beta)^2(\varepsilon+r\phi-3\varepsilon\phi))-2(1-\beta)^2v_{\max}+(1-2\beta)C < c-2c\beta+(1-\beta)(\varepsilon+r\phi+(-7+8\beta)\varepsilon\phi)-(1-\beta)v_{\max}$. Further, using $v_{\max} > \frac{c}{1-\beta}+\phi r+(1+\phi)\varepsilon$, we have $c-2c\beta+(1-\beta)(\varepsilon+r\phi+(-7+8\beta)\varepsilon\phi)-(1-\beta)v_{\max} < -2(c\beta+4(1-\beta)^2\varepsilon\phi) < 0$ holds. Because $S_p^{s\phi}$ is a convex function of a (i.e., $\frac{\partial^2 S_p^{s\phi}}{\partial a^2} = \frac{(1-\phi)^2}{2(v_{\max}-v_{\min})} > 0$), we have $S_p^{s\phi}$ ($S_p^{s\phi} > 0$) increases with a holds.

We now prove Proposition 8(iii). When $\frac{r}{2} < \varepsilon \le r$ and $a > r-\varepsilon$, by solving $\frac{\partial S_p^{s\phi}}{\partial a} = 0$, we can get $a_s^{s\phi} = \frac{c+\varepsilon-(2s+r-c+2\varepsilon)\phi+(r+\varepsilon)\phi^2-(1-\phi)v_{\max}+(1+\phi)C}{(1-\phi)^2}$. Because $S_p^{s\phi}$ is a convex function of a. If $a_s^{s\phi} < r-\varepsilon$, $S_p^{s\phi}$ increases with a; if $r-\varepsilon \le a_s^{s\phi} < \varepsilon$, $S_p^{s\phi}$ decreases with a if $a \le a_s^{s\phi}$, while increases with a if $a > a_s^{s\phi}$; if $a_s^{s\phi} > \varepsilon$, $S_p^{s\phi}$ increases with a. This completes the proof of Proposition 8(iii).

Proof of Proposition 9. In the proof of Proposition 7, we have derived the retailer's optimal price, and the optimal profits of the retailer and platform when WFS strategy is adopted. To prove Proposition 9, we should also give the retailer's optimal strategy and the retailer and platform's optimal profits in the cases of No-WFS strategy. Similar to the proof of Lemmas 1, based on Eqs. (3) and Eqs. (4), we can get:

Under No-WFS strategy,

(i) when $\varepsilon \le r$, $p^* = \frac{1}{2}(v_{\max} - (1-2\phi)\varepsilon + \frac{c}{1-\beta})$, $\pi_R^* = \frac{((1-\beta)(v_{\max}-(1-2\phi)\varepsilon)-c)^2}{4(1-\beta)(v_{\max}-v_{\min})}$, $\pi_P^* = \frac{\beta(((1-\beta)(v_{\max}-(1-2\phi)\varepsilon))^2-c^2)}{4(1-\beta)^2(v_{\max}-v_{\min})}$;

(ii) when $\varepsilon > r$, $p^* = \frac{1}{2}(v_{\max} + \phi r - (1-\phi)\varepsilon + \frac{c}{1-\beta})$, $\pi_R^* = \frac{[(1-\beta)(v_{\max}+\phi r-(1-\phi)\varepsilon)-c]^2}{4(1-\beta)(v_{\max}-v_{\min})} + (s-c)\frac{\phi(\varepsilon-r)}{v_{\max}-v_{\min}}$, $\pi_P^* = \frac{\beta\left[((1-\beta)(v_{\max}+\phi r-(1-\phi)\varepsilon))^2-c^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

Together with the optimal decisions and profits when WFS strategy is adopted in the proof of Proposition 7, Proposition 9 can be easily derived so we omit it.

Proof of Proposition 10. Before we prove Proposition 10, we first derive consumer surplus and social welfare in the cases of No-WFS strategy and WFS strategy. Then we make differences between them to investigate the change of consumer surplus and social welfare. we first derive the consumer surplus (CS_1) and social welfare SW_1 under No-WFS strategy.

(1) When $\varepsilon \le r$, $CS_1 = \phi\int_{p-\varepsilon}^{v_{\max}}(v-p)dG(v) + (1-\phi)\int_{p+\varepsilon}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $p^* = \frac{1}{2}(v_{\max} - (1-2\phi)\varepsilon + \frac{c}{1-\beta})$ into the above formulation, we can get $CS_1 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \frac{c}{1-\beta} - (1-2\phi)\varepsilon)^2 + 4\varepsilon(1-2\phi)v_{\max} - 4\varepsilon^2 - \frac{4cv_{\max}}{1-\beta}]$. Then $SW_1 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \frac{c}{1-\beta} - (1-2\phi)\varepsilon)^2 + 4\varepsilon(1-2\phi)v_{\max} - 4\varepsilon^2 - \frac{4cv_{\max}}{1-\beta}] + \frac{[(1-\beta)(v_{\max}-(1-2\phi)\varepsilon)-c]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\beta[((1-\beta)(v_{\max}-(1-2\phi)\varepsilon))^2-c^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$

(2) When $\varepsilon > r$, $CS_1 = \phi\int_{p-r}^{v_{\max}}(v-p)dG(v) + \phi\int_{p-\varepsilon}^{p-r}(-r)dG(v) + (1-\phi)\int_{p+\varepsilon}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $p^* = \frac{1}{2}(v_{\max} + \phi r - (1-\phi)\varepsilon + \frac{c}{1-\beta})$ into the above formulation, we have $CS_1 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \frac{c}{1-\beta} - \varepsilon + \phi r + \phi\varepsilon)^2 - 4\varepsilon^2 + 4r^2\phi - 8r\phi\varepsilon + 4\varepsilon^2\phi - \frac{4cv_{\max}}{1-\beta} + 4\varepsilon(1-\phi)v_{\max} - 4r\phi v_{\max}]$. Then, $SW_1 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \frac{c}{1-\beta} - \varepsilon + \phi r + \phi\varepsilon)^2 - 4\varepsilon^2 + 4r^2\phi - 8r\phi\varepsilon + 4\varepsilon^2\phi - \frac{4cv_{\max}}{1-\beta} + 4\varepsilon(1-\phi)v_{\max} - 4r\phi v_{\max}] + \frac{[(1-\beta)(v_{\max}+\phi r-(1-\phi)\varepsilon)-c]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\phi(s-c)(\varepsilon-r)}{v_{\max}-v_{\min}} + \frac{\beta[((1-\beta)(v_{\max}+\phi r-(1-\phi)\varepsilon))^2-c^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

We then calculate the consumer surplus (CS_2) and social welfare (SW_2) under WFS strategy.

(1) When $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $0 \le a \le r-\varepsilon$, $CS_2 = \phi\int_{p-\varepsilon-a}^{v_{\max}}(v-p)dG(v) + (1-\phi)\int_{p+\varepsilon-a}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $\hat{p}^* = \frac{1}{2}(v_{\max} + a - (1-2\phi)\varepsilon + \frac{c+C}{1-\beta})$ into the above formulation, we have $CS_2 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + a + \frac{c+C}{1-\beta} - (1-2\phi)\varepsilon)^2 - 4a^2 + 8a\varepsilon - 4\varepsilon^2 - 16a\phi\varepsilon - 4av_{\max} + 4(1-2\phi)\varepsilon v_{\max} - \frac{4(c+C)v_{\max}}{1-\beta}]$. Therefore, $SW_2 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + a + \frac{c+C}{1-\beta} - (1-2\phi)\varepsilon)^2 - 4a^2 + 8a\varepsilon - 4\varepsilon^2 - 16a\phi\varepsilon - 4av_{\max} + 4(1-2\phi)\varepsilon v_{\max} - \frac{4(c+C)v_{\max}}{1-\beta}] + \frac{[(1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{\beta[((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon))^2-(c+C)^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

(2) When $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$, or when $\varepsilon > r$, $CS_2 = \phi\int_{p-r}^{v_{\max}}(v-p)dG(v) + \phi\int_{p-\varepsilon-a}^{p-r}(-r)dG(v) + (1-\phi)\int_{p+\varepsilon-a}^{v_{\max}}(v-p)dG(v)$. By substituting the equilibrium price $\hat{p}^* = \frac{1}{2}(v_{\max} + \phi r + (1-\phi)(a-\varepsilon) + \frac{c+C}{1-\beta})$ into the above formulation, we have $CS_2 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \phi r + (a-\varepsilon)(1-\phi) + \frac{c+C}{1-\beta})^2 - 4\varepsilon^2 - 4(1-\phi)a^2 + 8a\varepsilon(1-\phi) - 8a\phi r + 4\phi r^2 - 8\phi\varepsilon r + 4\phi\varepsilon^2 - 4(a-\varepsilon)(1-\phi)v_{\max} - 4r\phi v_{\max} - \frac{4(c+C)v_{\max}}{1-\beta}]$. Therefore, we have $SW_2 = \frac{1}{8(v_{\max}-v_{\min})}[(v_{\max} + \phi r + (a-\varepsilon)(1-\phi) + \frac{c+C}{1-\beta})^2 - 4\varepsilon^2 - 4(1-\phi)a^2 + 8a\varepsilon(1-\phi) - 8a\phi r + 4\phi r^2 - 8\phi\varepsilon r + 4\phi\varepsilon^2 - 4(a-\varepsilon)(1-\phi)v_{\max} - 4r\phi v_{\max} - \frac{4(c+C)v_{\max}}{1-\beta}] + \frac{[(1-\beta)(v_{\max}+\phi r+(1-\phi)(a-\varepsilon))-c-C]^2}{4(1-\beta)(v_{\max}-v_{\min})} + \frac{(s-c-C)\phi(a+\varepsilon-r)}{v_{\max}-v_{\min}} + \frac{\beta[((1-\beta)(v_{\max}+\phi r+(1-\phi)(a-\varepsilon)))^2-(c+C)^2]}{4(1-\beta)^2(v_{\max}-v_{\min})}$.

Proposition 10(i) can be directly get from the above results so we omit it.

Now, we prove Proposition 10(ii). When $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$, $\frac{\partial(SW_2-SW_1)}{\partial s} = \frac{(a-r+\varepsilon)\phi}{v_{\max}-v_{\min}} > 0$. Therefore, $SW_2 - SW_1$ increases with s. Solving $SW_2 - SW_1 = 0$, we can get $s_1^* = \frac{B_1+B_2+(-3+4\beta)C^2}{8(1-\beta)^2\phi(a-r+\varepsilon)}$, where $B_1 = (1-\beta)(a^2(1-\beta)(1-\phi)(1+3\phi) + (r-\varepsilon)\phi(c(-6+4\beta) - r(1-\beta)(4+3\phi) - (1-\beta)\varepsilon(9\phi-10)) - 2a((1-\beta)(1+3\phi)(\varepsilon(1-\phi)-r\phi) + c(-1-3\phi+2\beta(1+\phi)))) + 2(1-\beta)^2(-a+(a-r+\varepsilon)\phi)v_{\max}$ and $B_2 = -2c(3-4\beta)+2(1-\beta)(-\varepsilon-r(3-2\beta)\phi+5\varepsilon\phi-\beta\varepsilon(-2+6\phi)+a(1+3\phi-2\beta(1+\phi))+(3-2\beta)v_{\max})$. Because SW_2-SW_1 increases with s, we can get: if $s_1^* \le 0$, $SW_2-SW_1 \ge 0$ always holds; if $s_1^* \ge c$, $SW_2-SW_1 \le 0$ always holds; if $s_1^* \in (0,c)$, $SW_2-SW_1 \le 0$ holds in $s \in [0, s_1^*]$ while $SW_2-SW_1 > 0$ holds in $s \in (s_1^*, c]$. When $\varepsilon > r$, the procedure is similar to that in the case when $\frac{r}{2} < \varepsilon \le r$ and $r-\varepsilon < a \le \varepsilon$ so we omit it. Note that $s_2^* = \frac{B_3+B_4+(-3+4\beta)C^2}{8a(1-\beta)^2\phi}$, where $B_3 = 2ac(1-\beta)(1+3\phi-2\beta(1+\phi))+a(1-\beta)^2(a(1-\phi)(1+3\phi)-2(1+3\phi)(\varepsilon(1-\phi)-r\phi)-2(1-\phi)v_{\max})$ and $B_4 = 2c(-3+4\beta)+2(1-\beta)(-\varepsilon+\beta\varepsilon(2-6\phi)-r(3-2\beta)\phi+5\varepsilon\phi+a(1+3\phi-2\beta(1+\phi))+(3-2\beta)v_{\max})$. This completes the proof of Proposition 10(ii). Therefore, we complete the proof of Proposition 10.

Proof of Proposition 11. Proposition 11(i) can be easily got from the retailer's optimal price and the retailer and platform's optimal profit in the proof of Proposition 9.

Now we prove Proposition 11(ii). Under No-WFS strategy, if $\varepsilon \le r$, $\frac{\partial\hat{\pi}_R^*}{\partial\phi} = \frac{((1-\beta)(v_{\max}-(1-2\phi)\varepsilon)-c)\varepsilon}{v_{\max}-v_{\min}}$. Using $v_{\max} \ge \frac{c}{1-\beta} + (1+2\phi)\varepsilon$, we can get $((1-\beta)(v_{\max}-(1-2\phi)\varepsilon)-c)\varepsilon \ge 4(1-\beta)\phi\varepsilon^2 \ge 0$, so $\hat{\pi}_R^*$ increases with ϕ. If $\varepsilon > r$, $\frac{\partial\hat{\pi}_R^*}{\partial\phi} = \frac{(\varepsilon+r)((1-\beta)(v_{\max}+\phi r-(1-\phi)\varepsilon)-c)+2(s-c)(\varepsilon-r)}{2(v_{\max}-v_{\min})}$. Solving $\frac{\partial\hat{\pi}_R^*}{\partial\phi} = 0$, we can get that $\phi_1 = \frac{1}{\varepsilon+r}\left[\frac{(3c-2s)\varepsilon-(c-2s)r}{(1-\beta)(\varepsilon+r)} - v_{\max} + \varepsilon\right]$. As $\hat{\pi}_R^*$ is a convex function of ϕ, so we have when $\phi_1 < 0$, $\hat{\pi}_R^*$ increases with ϕ; when $\phi_1 \in [0,1)$, $\hat{\pi}_R^*$ first decreases with ϕ in $(0,\phi_1]$ then increases with ϕ in $(\phi_1, 1)$; when $\phi_1 > 1$, $\hat{\pi}_R^*$ decreases with ϕ. This completes the proof of Proposition 11(ii).

We then prove the Proposition 11(iii). Under WFS strategy, if $0 \le \varepsilon \le \frac{r}{2}$, or when $\frac{r}{2} < \varepsilon \le r$ and $a \le r-\varepsilon$, $\frac{\partial\hat{\pi}_R^*}{\partial\phi} = \frac{((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C)\varepsilon}{v_{\max}-v_{\min}}$. Using $C \le (1-\beta)(v_{\max}+a-(1+2\phi)\varepsilon) - c$ (i.e., upper bound of market constraint.), we have $\frac{\partial \hat{\pi}_R^*}{\partial \phi} \geq \frac{4(1-\beta)\phi\epsilon^2}{v_{\max}-v_{\min}} \geq 0$. Therefore, $\frac{\partial \hat{\pi}_R^*}{\partial \phi}$ increases with ϕ. When $\frac{r}{2} < \epsilon \leq r$ and $a > r-\epsilon$, or when $\epsilon > r$, $\frac{\partial \hat{\pi}_R^*}{\partial \phi} = \frac{2(s-c-C)(a-r+\epsilon)-(a-r-\epsilon)\left((1-\beta)\left(v_{\max}+a-\phi(a-r-\epsilon)\right)-\epsilon\right)-c-C)}{2(v_{\max}-v_{\min})}$. Solving $\frac{\partial \hat{\pi}_R^*}{\partial \phi} = 0$, we can get that $\phi_2 = \frac{1}{\epsilon+r-a}\left[\frac{(c-2s)(a-r)+(3c-2s)\epsilon+(a-r+3\epsilon)C}{(1-\beta)(\epsilon+r-a)} - v_{\max} + \epsilon - a\right]$. As $\hat{\pi}_R^*$ is a convex function of ϕ, so we have when $\phi_2 < 0$, $\hat{\pi}_R^*$ increases with ϕ; when $\phi_2 \in [0,1)$, $\hat{\pi}_R^*$ first decreases with ϕ in $(0,\phi_2]$ then increases with ϕ in $(\phi_2, 1)$; when $\phi_2 > 1$, $\hat{\pi}_R^*$ decreases with ϕ. This completes the proof of Proposition 11(iii).

The above completes the proof of Proposition 11.

Proof of Proposition 12. We first prove Proposition 12(i). When $0 \leq \epsilon \leq \frac{r}{2}$, or when $\frac{r}{2} < \epsilon \leq r$ and $a \leq r-\epsilon$, $CS_2 - CS_1 = \frac{\epsilon(3a(-1+\beta)+C)}{2(1-\beta)(v_{\max}-v_{\min})}\phi + \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[a(1-\beta)(2c-(1-\beta)(2v_{\max}+3a-6\epsilon)) + 2((1-\beta)(a-\epsilon-v_{\max})+c)C + C^2]$.

If $a = \frac{C}{3(1-\beta)}$, $CS_2 - CS_1$ is not affected by ϕ. Substituting $a = \frac{C}{3(1-\beta)}$ into $CS_2 - CS_1$, we can get $CS_2 - CS_1 = \frac{4}{3}C(2c - 2(1-\beta)v_{\max} + C)$. If $v_{\max} > \frac{2c+C}{2(1-\beta)}$, $CS_2 - CS_1 > 0$; otherwise, $CS_2 - CS_1 \leq 0$. This completes the proof of Proposition 8(i)(a).

If $a > \frac{C}{3(1-\beta)}$, $\frac{\partial(CS_2-CS_1)}{\partial \phi} = \frac{\epsilon(-3a(1-\beta)+C)}{2(1-\beta)(v_{\max}-v_{\min})} < 0$. Then $CS_2 - CS_1$ decreases with ϕ. Solving $CS_2 - CS_1 = 0$, we can get $\phi_{cs}^* = \frac{1}{4(1-\beta)(3a(1-\beta)-C)\epsilon}[a(1-\beta)(2w-(1-\beta)(3a-6\epsilon+2v_{\max})) + 2((1-\beta)(a-\epsilon-v_{\max})+c)C + C^2]$. Because Then $CS_2 - CS_1$ decreases with ϕ, we have: when $\phi_{cs}^* \leq 0$, CS is always decreased; when $\phi_{cs}^* \geq 1$, CS is always increased; when $\phi_{cs}^* \in (0,1)$, CS is increased if $\phi \in (0, \phi_{cs}^*]$ while CS is decreased if $\phi \in (\phi_{cs}^*, 1)$.

If $a < \frac{C}{3(1-\beta)}$, $\frac{\partial(CS_2-CS_1)}{\partial \phi} > 0$ hence $CS_2 - CS_1$ increases with ϕ. Similar to the procedures above, we have: when $\phi_{cs}^* \geq 1$, CS is always decreased; when $\phi_{cs}^* \leq 0$, CS is always increased; when $\phi_{cs}^* \in (0,1)$, CS is decreased if $\phi \in (0, \phi_{cs}^*]$ while CS is increased if $\phi \in (\phi_{cs}^*, 1)$. This completes the proof of Proposition 12(i)(b). Therefore, the proof of Proposition 12(i) is completed. We next prove Proposition 12(ii).

When $\frac{r}{2} < \epsilon \leq r$ and $r-\epsilon < a \leq \epsilon$, $CS_2 - CS_1 = \frac{(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})}\phi^2 - \frac{1}{4(1-\beta)(v_{\max}-v_{\min})}((1-\beta)((a-r)(-a+2r)+(2a+5r)\epsilon-3\epsilon^2-(a-r+\epsilon)v_{\max}) + (a-r-\epsilon)C + (a-r+\epsilon)c)\phi + \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[a(1-\beta)(2c-(1-\beta)(3a-6\epsilon)) - 2(1-\beta)v_{\max}(a(1-\beta)+C) + 2((1-\beta)(a-\epsilon)+c)C + C^2] = \frac{(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})}\phi^2 - \frac{A_5}{4(1-\beta)(v_{\max}-v_{\min})}\phi + \frac{B_5}{8(1-\beta)^2(v_{\max}-v_{\min})}$, where $A_5 = (1-\beta)((a-r)(-a+2r)+(2a+5r)\epsilon-3\epsilon^2-(a-r+\epsilon)v_{\max}) + (a-r-\epsilon)C + (a-r+\epsilon)c$ and $B_5 = a(1-\beta)(2c-(1-\beta)(3a-6\epsilon)) - 2(1-\beta)v_{\max}(a(1-\beta)+C) + 2((1-\beta)(a-\epsilon)+c)C + C^2$. Note that $\frac{(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})} < 0$, $CS_2 - CS_1$ is a concave function of ϕ. By solving $\frac{\partial(CS_2-CS_1)}{\partial \phi} = 0$, we can get $\phi_1^* = \frac{A_5}{(1-\beta)(a-r-3\epsilon)(a-r+\epsilon)}$. Substituting ϕ_1^* into $CS_2 - CS_1$, we can get $\sigma_{cs}^1 = CS_2 - CS_1\Big|_{\phi=\phi_1^*} = \frac{-{A_5}^2 + B_5(a-r-3\epsilon)(a-r+\epsilon)}{8(1-\beta)^2(a-r-3\epsilon)(a-r+\epsilon)(v_{\max}-v_{\min})}$. If $\sigma_{cs}^1 \leq 0$, CS is always decreased. If $\sigma_{cs}^1 > 0$, by solving $CS_2 - CS_1 = 0$, we can get $\underline{\phi}_{cs}^1 = \frac{B_5}{(1-\beta)(A_5 - \sqrt{{A_5}^2 - B_5(a-r-3\epsilon)(a-r+\epsilon)})}$ and $\overline{\phi}_{cs}^1 = \frac{B_5}{(1-\beta)(A_5 + \sqrt{{A_5}^2 - B_5(a-r-3\epsilon)(a-r+\epsilon)})}$. In this case, we can get if $\phi \in \left(\max(0, \underline{\phi}_{cs}^1), \min(\overline{\phi}_{cs}^1, 1)\right)$, CS is increased; otherwise, CS is decreased.

When $\epsilon > r$, $CS_2 - CS_1 = \frac{a(a-2(r+\epsilon))}{8(v_{\max}-v_{\min})}\phi^2 - \frac{1}{4(1-\beta)(v_{\max}-v_{\min})}(a((1-\beta)(v_{\max}-3r+a-2\epsilon)-c) - (a-r-\epsilon)C)\phi + \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[a(1-\beta)(2c+(1-\beta)(6\epsilon-3a)) - 2(1-\beta)v_{\max}(a(1-\beta)+C) + 2((1-\beta)(a-\epsilon)+c)C + C^2] = \frac{a(a-2(r+\epsilon))}{8(v_{\max}-v_{\min})}\phi^2 - \frac{A_6}{4(1-\beta)(v_{\max}-v_{\min})}\phi + \frac{B_6}{8(1-\beta)^2(v_{\max}-v_{\min})}$, where $A_6 = -a((1-\beta)(v_{\max}-3r+a-2\epsilon)-c) + (a-r-\epsilon)C$ and $B_6 = a(1-\beta)(2c+(1-\beta)(6\epsilon-3a)) - 2(1-\beta)v_{\max}(a(1-\beta)+C) + 2((1-\beta)(a-\epsilon)+c)C + C^2$. Note that $\frac{a(a-2(r+\epsilon))}{8(v_{\max}-v_{\min})} < 0$, $CS_2 - CS_1$ is a concave function of ϕ. By solving $\frac{\partial(CS_2-CS)}{\partial \phi} = 0$, we can get that $\phi_2^* = \frac{A_6}{a(1-\beta)(a-2(r+\epsilon))}$. Substituting ϕ_2^* into $CS_2 - CS_1$, we can get $\sigma_{cs}^2 = CS_2 - CS_1\Big|_{\phi=\phi_2^*} = \frac{-A_6^2 + aB_6(a-2(r+\epsilon))}{8a(1-\beta)^2(a-2(r+\epsilon))(v_{\max}-v_{\min})}$. If $\sigma_{cs}^2 \leq 0$, we have $CS_2 - CS_1 \leq 0$, i.e., CS is always decreased. If $\sigma_{cs}^2 > 0$, solving $CS_2 - CS_1 = 0$, we can get $\underline{\phi}_{cs}^2 = \frac{B_6}{(1-\beta)(A_6 - \sqrt{A_6^2 - aB_6(a-2(r+\epsilon))})}$ and $\overline{\phi}_{cs}^2 = \frac{B_6}{(1-\beta)(A_6 + \sqrt{A_6^2 - aB_6(a-2(r+\epsilon))})}$. In this case, we can get if $\phi \in \left(\max(0, \underline{\phi}_{cs}^2), \min(\overline{\phi}_{cs}^2, 1)\right)$, CS is increased; otherwise, CS is decreased. Let $j = 1$ when $\frac{r}{2} < \epsilon \leq r$ and $r-\epsilon < a \leq \epsilon$ and $j = 2$ when $\epsilon > r$. Then we have the results presented in Proposition 12(ii). This completes the proof of Proposition 12(ii).

Therefore, the proof of Proposition 12 is completed.

Proof of Proposition 13. We first prove Proposition 13(i). When $0 \leq \epsilon \leq \frac{r}{2}$, or when $\frac{r}{2} < \epsilon \leq r$ and $0 \leq a \leq r-\epsilon$, $SW_2 - SW_1 = \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[-a(1-\beta)((1-\beta)(2(2\phi-1)\epsilon+a)+2c(1-2\beta)) - 2(c(-3+4\beta)+(1-3\beta+2\beta^2)(a+(2\phi-1)\epsilon))C + (3-4\beta)C^2 + 2(-1+\beta)v_{\max}(a(-1+\beta)+(3-2\beta)C)]$. $\frac{\partial(SW_2-SW_1)}{\partial \phi} = \frac{\epsilon(a-a\beta+(1-2\beta)C)}{2(-1+\beta)(v_{\max}-v_{\min})} < 0$, i.e., $SW_2 - SW_1$ decreases with ϕ. Solving $SW_2 - SW_1 = 0$, we can get $\phi_{sw}^* = \frac{1}{4(1-\beta)(a(1-\beta)+(1-2\beta)C)\epsilon}[a(1-\beta)((1-\beta)(2v_{\max}+2\epsilon-a)-2c+4c\beta) + 2(-a+3c+3a\beta-4c\beta-2a\beta^2+\epsilon-3\beta\epsilon+2\beta^2\epsilon+(3-2\beta)(-1+\beta)v_{\max})C + (3-4\beta)C^2]$. Because $SW_2 - SW_1$ decreases with ϕ, we have: when $\phi_{sw}^* \leq 0$, SW is always decreased; when $\phi_{sw}^* \geq 1$, SW is always increased; when $\phi_{sw}^* \in (0,1)$, SW is increased if $\phi \in (0, \phi_{sw}^*]$ while SW is decreased if $\phi \in (\phi_{sw}^*, 1)$. This completes the proof of Proposition 13(i). We then prove Proposition 13(ii).

When $\frac{r}{2} < \epsilon \leq r$ and $r-\epsilon < a \leq \epsilon$, $SW_2 - SW_1 = \frac{3(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})}\phi^2 - \frac{1}{4(1-\beta)(v_{\max}-v_{\min})}[(r-a)(4s-(1-\beta)(2r+a)-3c+2(-2s+c)\beta) + ((1-\beta)(7r-2a-4s)+3c-2c\beta)\epsilon - 5(1-\beta)\epsilon^2 + (1-\beta)(a-r+\epsilon)v_{\max} + ((r-a)(2\beta-3)+(5-6\beta)\epsilon)C]\phi + \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[a(1-\beta)((1-\beta)(2\epsilon-a)-2c+4c\beta) - 2(a-3c-3a\beta+4c\beta+2a\beta^2-\epsilon+3\beta\epsilon-2\beta^2\epsilon)C + (3-4\beta)C^2 + 2(1-\beta)v_{\max}(a(1-\beta)-(3-2\beta)C)] = \frac{3(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})}\phi^2 - \frac{A_7}{4(1-\beta)(v_{\max}-v_{\min})}\phi + \frac{B_7}{8(1-\beta)^2(v_{\max}-v_{\min})}$, where $A_7 = (r-a)(4s-(1-\beta)(2r+a)-3c+2(-2s+c)\beta) + ((1-\beta)(7r-2a-4s)+3c-2c\beta)\epsilon - 5(1-\beta)\epsilon^2 + (1-\beta)(a-r+\epsilon)v_{\max} + ((r-a)(2\beta-3)+(5-6\beta)\epsilon)C$ and $B_7 = a(1-\beta)((1-\beta)(2\epsilon-a)-2c+4c\beta) - 2(a-3c-3a\beta+4c\beta+2a\beta^2-\epsilon+3\beta\epsilon-2\beta^2\epsilon)C + (3-4\beta)C^2 + 2(1-\beta)v_{\max}(a(1-\beta)-(3-2\beta)C)$. Note that $\frac{3(a-r-3\epsilon)(a-r+\epsilon)}{8(v_{\max}-v_{\min})} < 0$, $SW_2 - SW_1$ is a concave function of ϕ. By solving $\frac{\partial(SW_2-SW_1)}{\partial \phi} = 0$, we can get $\phi_3^* = \frac{A_7}{3(1-\beta)(a-r-3\epsilon)(a-r+\epsilon)}$. Substituting ϕ_3^* into $SW_2 - SW_1$, we can get $\sigma_{sw}^1 = SW_2 - SW_1\big|_{\phi=\phi_3^*} = \frac{1}{24(1-\beta)^2(v_{\max}-v_{\min})}[3B_7 - \frac{{A_7}^2}{(a-r-3\varepsilon)(a-r+\varepsilon)}]$. If $\sigma_{sw}^1 \le 0$, we have $SW_2 - SW_1 \le 0$ always holds. Then $SW_2 - SW_1$ is always decreased. If $\sigma_{sw}^1 > 0$, by solving $SW_2 - SW_1 = 0$, we can get $\underline{\phi}_{sw}^1 = \frac{B_7}{(1-\beta)(A_7-\sqrt{{A_7}^2-3B_7(a-r-3\varepsilon)(a-r+\varepsilon)})}$ and $\overline{\phi}_{sw}^1 = \frac{B_7}{(1-\beta)(A_7+\sqrt{{A_7}^2-3B_7(a-r-3\varepsilon)(a-r+\varepsilon)})}$. In this case, we can get if $\phi \in \left(\max(0, \underline{\phi}_{sw}^1), \min(\overline{\phi}_{sw}^1, 1)\right)$, SW is increased; otherwise, SW is decreased.

When $\varepsilon > r$, $SW_2 - SW_1 = \frac{3a(a-2(r+\varepsilon))\phi^2}{8(v_{\max}-v_{\min})} - \frac{1}{4(1-\beta)(v_{\max}-v_{\min})}[a((1-\beta)(a+r-4s-2\varepsilon+v_{\max})+3c-2c\beta)+((r-a)(2\beta-3)+(5-6\beta)\varepsilon)C]\phi + \frac{1}{8(1-\beta)^2(v_{\max}-v_{\min})}[a(1-\beta)((1-\beta)(2\varepsilon-a)-2c+4c\beta)+2(3c-4c\beta-(1-\beta)(1-2\beta)(a-\varepsilon))C+(3-4\beta)C^2+2(1-\beta)v_{\max}(a(1-\beta)-(3-2\beta)C)] = \frac{3a(a-2(r+\varepsilon))\phi^2}{8(v_{\max}-v_{\min})} - \frac{A_8}{4(1-\beta)(v_{\max}-v_{\min})}\phi + \frac{B_8}{8(1-\beta)^2(v_{\max}-v_{\min})}$, where $A_8 = a((1-\beta)(a+r-4s-2\varepsilon+v_{\max})+3c-2c\beta)+((r-a)(2\beta-3)+(5-6\beta)\varepsilon)C$ and $B_8 = a(1-\beta)((1-\beta)(2\varepsilon-a)-2c+4c\beta)+2(3c-4c\beta-(1-\beta)(1-2\beta)(a-\varepsilon))C+(3-4\beta)C^2+2(1-\beta)v_{\max}(a(1-\beta)-(3-2\beta)C)$. Note that $\frac{3a(a-2(r+\varepsilon))}{8(v_{\max}-v_{\min})} < 0$, $SW_2 - SW_1$ is a concave function of ϕ. By solving $\frac{\partial(SW_2-SW_1)}{\partial\phi} = 0$, we can get $\phi_4^* = \frac{A_8}{3a(1-\beta)(a-2r-2\varepsilon)}$. Substituting ϕ_4^* into $SW_2 - SW_1$, we can get $\sigma_{sw}^2 = SW_2 - SW_1\big|_{\phi=\phi_4^*} = \frac{-{A_8}^2+3aB_8(a-2(r+\varepsilon))}{24a(1-\beta)^2(a-2(r+\varepsilon))(v_{\max}-v_{\min})}$. If $\sigma_{sw}^2 \le 0$, $SW_2 - SW_1 \le 0$, i.e., SW is decreased. If $\sigma_{sw}^2 > 0$, solving $SW_2 - SW_1 = 0$, we can get $\underline{\phi}_{sw}^2 = \frac{B_8}{(1-\beta)(A_8-\sqrt{{A_8}^2-3aB_8(a-2(r+\varepsilon))})}$ and $\overline{\phi}_{sw}^2 = \frac{B_8}{(1-\beta)(A_8+\sqrt{{A_8}^2-3aB_8(a-2(r+\varepsilon))})}$. In this case, we can get if $\phi \in \left(\max(0, \underline{\phi}_{sw}^2), \min(\overline{\phi}_{sw}^2, 1)\right)$, SW is increased; otherwise, SW is decreased. Let $j = 1$ when $\frac{r}{2} < \varepsilon \le r$ and $r - \varepsilon < a \le \varepsilon$ and $j = 2$ when $\varepsilon > r$. Then we have the results presented in Proposition 13 (ii). This completes the proof of Proposition 13(ii).

Therefore, the proof of Proposition 13 is completed.

Proof of Lemma 4. We denote K_e and R_e as the amount of kept and returned products. Then we have $K_e : \bar{G}(\max(p-\tilde{\varepsilon}-a, p-r-a))$ and $R_e : \begin{cases} \bar{G}(p-\tilde{\varepsilon}-a) - \bar{G}(p-r-a), if\, p-\tilde{\varepsilon}-a \le v_i < p-r-a \\ 0, otherwise \end{cases}$. When $\varepsilon \le r$, we have $K_e = \phi\bar{G}(p-\varepsilon-a) + (1-\phi)\bar{G}(p+\varepsilon-a) = \frac{v_{\max}-p+a-(1-2\phi)\varepsilon}{v_{\max}-v_{\min}}$ and $R_e = 0$. When $\varepsilon > r$, we have $K_e = \phi\bar{G}(p-a-r) + (1-\phi)\bar{G}(p+\varepsilon-a) = \frac{v_{\max}-p+a+\phi r-(1-\phi)\varepsilon}{v_{\max}-v_{\min}}$ and $R_e = \phi\left[\bar{G}(p-\varepsilon-a) - \bar{G}(p-a-r)\right] = \frac{\phi(\varepsilon-r)}{v_{\max}-v_{\max}}$. Based on Eqs. (12) and Eqs. (13), we have, when $\varepsilon \le r$, $\hat{\pi}_{R_e} = \frac{((1-\beta)p-c-C)(v_{\max}-p+a-(1-2\phi)\varepsilon)}{v_{\max}-v_{\min}}$ and $\hat{\pi}_{P_e} = \frac{\beta p(v_{\max}-p+a-(1-2\phi)\varepsilon)}{v_{\max}-v_{\min}}$; when $\varepsilon > r$, $\hat{\pi}_{R_e} = \frac{((1-\beta)p-c-C)(v_{\max}-p+a+\phi r-(1-\phi)\varepsilon)}{v_{\max}-v_{\min}} + (s-c)\frac{\phi(\varepsilon-r)}{v_{\max}-v_{\min}}$ and $\hat{\pi}_{P_e} = \frac{\beta p(v_{\max}-p+a+\phi r-(1-\phi)\varepsilon)}{v_{\max}-v_{\min}}$. When $\varepsilon \le r$, using first-order condition, we can get $\hat{p}_e^* = \frac{1}{2}(v_{\max} + a - (1-2\phi)\varepsilon + \frac{c+C_e}{1-\beta})$. Substituting $\hat{p}_e^*$ into $\hat{\pi}_{R_e}$ and $\hat{\pi}_{P_e}$, we can get $\hat{\pi}_{R_e}^* = \frac{((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C_e)^2}{4(1-\beta)(v_{\max}-v_{\min})}$ and $\hat{\pi}_{P_e}^* = \frac{\beta\left[((1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon))^2-(c+C_e)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$. In a similar way, we can get when $\varepsilon > r$, $\hat{p}_e^* = \frac{1}{2}(v_{\max}+a+\phi r-(1-\phi)\varepsilon+\frac{c+C_e}{1-\beta})$, $\hat{\pi}_{R_e}^* = \frac{((1-\beta)(v_{\max}+a+\phi r-(1-\phi)\varepsilon)-c-C_e)^2}{4(1-\beta)(v_{\max}-v_{\min})} + (s-c)\frac{\phi(\varepsilon-r)}{(v_{\max}-v_{\min})}$ and $\hat{\pi}_{P_e}^* = \frac{\beta\left[((1-\beta)(v_{\max}+a+\phi r-(1-\phi)\varepsilon))^2-(c+C_e)^2\right]}{4(1-\beta)^2(v_{\max}-v_{\min})}$. This completes the proof of Lemma 4.

Proof of Proposition 14. Before we prove Proposition 14, we first derive the market constraints to facilitate our proof. Similar to the proof of Table C.1. We can get that when $\varepsilon \le r$, $\max(0, (1-\beta)(2v_{\min}-v_{\max}+a+3\varepsilon-2\varepsilon\phi)-c) \le C_e \le (1-\beta)(v_{\max}+a-(1+2\phi)\varepsilon)-c$; when $\varepsilon > r$, $\max(0, (1-\beta)(2v_{\min}-v_{\max}+a+(3-\phi)\varepsilon-\phi r)-c) \le C_e \le (1-\beta)(v_{\max}+a-\phi r-(1+\phi)\varepsilon)-c$. When $\varepsilon \le r$, $\frac{\partial\hat{\pi}_{R_e}^*}{\partial a} = \frac{(1-\beta)(v_{\max}+a-(1-2\phi)\varepsilon)-c-C_e}{2(v_{\max}-v_{\min})}$. Using $C_e \le (1-\beta)\left(v_{\max}+a-(1+2\phi)\varepsilon\right)-c$, we have $\frac{\partial\hat{\pi}_{R_e}^*}{\partial a} \ge \frac{2(1-\beta)\phi\varepsilon}{v_{\max}-v_{\min}} \ge 0$; when $\varepsilon > r$, $\frac{\partial\hat{\pi}_{R_e}^*}{\partial a} = \frac{(1-\beta)(v_{\max}+a+\phi r-(1-\phi)\varepsilon)-c-C_e}{2(v_{\max}-v_{\min})}$. Using $C_e \le (1-\beta)\left(v_{\max}+a-\phi r-(1+\phi)\varepsilon\right)-c$, we have $\frac{\partial\hat{\pi}_{R_e}^*}{\partial a} \ge \frac{(1-\beta)\phi(\varepsilon+r)}{v_{\max}-v_{\min}} \ge 0$. Therefore, the retailer's optimal price always increases with a. This completes the proof of Proposition 14.

Proof of Lemma 5. Based on Eq. (14) and Eq. (15), we can get that when $a_L < a_H \le r-\varepsilon$, the number of kept and returned products is $\hat{K}^a = \phi(\eta\frac{v_{\max}-p+\varepsilon+a_H}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p+\varepsilon+a_L}{v_{\max}-v_{\min}})+(1-\phi)(\eta\frac{v_{\max}-p-\varepsilon+a_H}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p-\varepsilon+a_L}{v_{\max}-v_{\min}})$ and $\hat{R}^a = 0$. Then based on Eq. (16), we can get the retailer's profit function. By solving the first-order condition, we can get the optimal price is $\hat{p}^{a*} = \frac{1}{2}(v_{\max}-(1-2\phi)\varepsilon+\eta\Delta_a+a_L+\frac{c+C}{1-\beta})$. Substituting $\hat{p}^{a*}$ into Eq. (16) and (17), we can get the retailer and platform's optimal profits, which is presented in Lemma 5(i).

When $a_L < r-\varepsilon < a_H$, the number of kept and returned products is $\hat{K}^a = \phi(\eta\frac{v_{\max}-p+r}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p+\varepsilon+a_L}{v_{\max}-v_{\min}})+(1-\phi)(\eta\frac{v_{\max}-p-\varepsilon+a_H}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p-\varepsilon+a_L}{v_{\max}-v_{\min}})$ and $\hat{R}^a = \phi\eta\frac{a_H-r+\varepsilon}{v_{\max}-v_{\min}}$. Then based on Eq. (16), we can get the retailer's profit function. By solving the first-order condition, we can get the optimal price is $\hat{p}^{a*} = \frac{1}{2}(v_{\max} - (1-2\phi)\varepsilon + (1-\phi\eta)a_L + (1-\phi)\eta\Delta_a + (r-\varepsilon)\eta\phi + \frac{c+C}{1-\beta})$. Substituting $\hat{p}^{a*}$ into Eq. (16) and (17), we can get the retailer and platform's optimal profits, which is presented in Lemma 5(ii).

When $r-\varepsilon < a_L < a_H$, the number of kept and returned products is $\hat{K}^a = \phi\frac{v_{\max}-p+r}{v_{\max}-v_{\min}}+(1-\phi)(\eta\frac{v_{\max}-p-\varepsilon+a_H}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p-\varepsilon+a_L}{v_{\max}-v_{\min}})$ and $\hat{R}^a = \phi(\eta\frac{v_{\max}-p+\varepsilon+a_H}{v_{\max}-v_{\min}}+(1-\eta)\frac{v_{\max}-p+\varepsilon+a_L}{v_{\max}-v_{\min}}-\frac{v_{\max}-p+r}{v_{\max}-v_{\min}})$. Then we can get the retailer and platform's optimal profits, which is presented in Lemma 5(iii). Denote $\Delta_a = a_H - a_L$ and based on the relationship between ε and r, we can get the results in Lemma 5. This completes the proof of Lemma 5.

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