



Target Market Optimal Coverage Algorithm Based on Heat Diffusion Model

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Abstract. The maximization of personalized influence is a branch of maximizing the influence of social networks, and the goal is to target specific social network users and mine the set of initial impact diffusion users that have made the most impact. However, most of the existing methods are based on the IC model and the LT model. The prediction of the impact of these two models on the nodes depends on the Monte Carlo simulation. In order to avoid Monte Carlo simulation time and simulate real life more, this paper introduces the heat diffusion model into the problem of maximizing the influence of personalization. The heat diffusion process was used to simulate the diffusion process of information influence. And the thermal energy was applied to measure the impact on the target users, and cluster candidate users. The cluster center as a seed node was proposed to spread information and maximizing the impact on specific users. The comparison experiments on real social networks show that the personalized maximization algorithm based on the thermal diffusion model has better time performance and diffusion effect than the traditional diffusion model.

Keywords: Social network · Influence maximization · Heat diffusion · Clustering

1 Introduction

In recent years, with the rapid development of information technology, the emergence of network information and social media had further changed the way people communicate with each other.

The problem of maximizing social network impact is to find the optimal set of users in a given social network, and the seed users initiate information diffusion and try to spread the information to as many users as possible. The problem originated from a viral marketing strategy in the marketing field. When a company promotes a new product or service, it usually provides a free product or service for a small number of trail users. It is hoped that these users will recommend the company's products to their friends. In this way by expanding the popularity of the product, is the so-called word-of-mouth effect [1–3]. Scholars have conducted in-depth research on the problem of maximizing influence and proposed CELF algorithm [4], New Greedy algorithm [5] and CELF++ algorithm [6], Degree Discount algorithm [6], LDGA algorithm [7] and simulated annealing algorithm [8].

However, in many cases, the traditional influence maximization problem cannot meet the needs of practical problems [9, 10]. For example, if a company wants to promote its own high-end sports car, if it uses traditional influence solutions, it will waste lots of resources to influence the people who cannot afford or are not interested. Before product information is passed on to potential customers, customers may be lost to competitors due to the consideration of irrelevant users. Nowadays, companies can predict which products the users are interested in from the information data they have mastered, and concentrate marketing resources on these target consumers through target marketing so that a small number of resources can be used to achieve the desired result. In reality, the target users of the company are often a user group or a plurality of different types of groups, so the problem of maximizing the impact of the research target coverage is more practical.

Since the traditional propagation model required a large number of Monte Carlo simulations to better simulate the propagation of information in real life, the time performance of the algorithm is reduced. To avoid this problem, this paper introduces a heat propagation model that uses heat flow to simulate information diffusion in social networks.

This paper proposes an algorithm for continuously updating cluster centers based on the heat propagation model. Firstly, K cluster centers are randomly selected, and the similarity between nodes is defined. Then the target nodes are clustered, and the nodes with the most influence on the cluster are found from each cluster, and the clusters are continuously updated. The center can maximize the total impact. Since the nodes that can have a greater influence on the target market will be in a certain range of subspace of the target node, the breadth-first search method can be used to obtain the subspace to improve the performance of the algorithm.

The main contributions of this paper are:

- (1) We introduce the heat diffusion model into the study of the problem of maximizing the impact of personalization.
- (2) We propose a clustering algorithm based on the thermal propagation model is proposed to solve the problem of maximizing the impact of the target group. The similarity and total cost of the clustering algorithm are defined and optimized.
- (3) We conduct comparative experiments in real social networks, analyze and verify the effectiveness of the proposed algorithm.

Section 2 of this paper introduces related work; Sect. 3 introduces preparatory knowledge; Sect. 4 introduces the known influence maximization algorithm of the target population based on the thermal propagation model and its optimization algorithm; Sect. 5 introduces the experimental analysis of the algorithm; Sect. 6 is the conclusion.

2 Related Work

The problem of maximizing the impact is to select k initial users to maximize their range of propagation. Kempe et al. [11] modeled the process of influence propagation as a discrete stochastic process, introducing an independent cascade model (IC) and a

linear threshold model (LT). They proved that the problem of maximizing impact is NP-hard, and proposed a classic greedy algorithm, but the implementation efficiency is low. Therefore, in order to improve the performance of the algorithm, Jung et al. [12] proposed the IRIE algorithm based on the traditional model, first estimated the influence ranking of each node, and then estimated the gain influence of the node by the linear method, and got good results.

With the rapid development of networks and technologies, a large amount of data information has become a “wealth”. In order to achieve a better impact intensity and less time consumption, the problem of maximizing individualized influence was proposed in 2013. However, most of the algorithms are based on linear threshold models or independent cascade models. In order to make the simulation results closer to reality, it is necessary to increase the number of Monte Carlo simulations, improve the seed quality and reduce the efficiency of the algorithm.

In 2013, Ma et al. [13] proposed a social network marketing model called heat diffusion model, which applied the theory of physics—heat diffusion to describe the influence of communication in social networks. This method avoids the Monte Carlo simulation process. In 2014, Doo et al. [14, 15] proposed an excitation algorithm based on the heat diffusion model. In 2017, Yang et al. [16] proposed a target hot greedy algorithm for a specific node based on the heat diffusion model. The algorithm can quickly calculate the influence of a particular target, but its algorithm is still based on traditional greedy ideas.

Search for the best subsets by using clustering, because homogeneous users are more likely to form a whole. Zhang et al. [17] proposed a maximum coverage algorithm for information oriented to the target market. Based on the independent cascade model, the algorithm performs cluster analysis on users and finds out the representative users of each cluster. Finally, this algorithm can get high-quality seed nodes, and the better the effect is as the number of nodes increases.

We introduced the heat propagation model to remove the time cost of a Monte Carlo simulation in the traditional model and propose a set coverage algorithm for a specific user group. This method uses the clustering method to continuously update the cluster center. When the central node is optimally affected, the seed set can be obtained.

3 Preliminary Knowledge

3.1 Heat Propagation Model

In nature, the transfer of heat from a relatively high temperature position on the medium to a lower temperature position is a physical phenomenon. Also in social networks, information is also transmitted by users who are affected earlier to users who have not yet been affected. Users with higher levels of influence are disseminated to users with lower levels of influence [13]. Many research scholars have proposed a variety of thermal propagation models. In order to be practical, we use a heat propagation model based on directed network graphs.

The social network is represented by a directed graph $G = (V, E)$, where V represents a set of nodes, and E represents a set of directed edges between nodes. The heat

propagation model is used to simulate the information dissemination of social networks.

3.2 The Principle of Heat Propagation

Definition (Thermal Conductivity). The heat transferred by a unit temperature gradient per unit time through a unit heat transfer surface, also known as the thermal conductivity. It can reflect the ability of the physical medium to conduct heat, expressed as α [13].

For the node v_i in the network, the propagation process of the heat propagation model is as follows:

1. At time t_0 , the heat of node v_i is denoted as $h_i(0)$; at time t , the heat possessed by v_i is denoted as $h_i(t)$. The heat of all nodes in the network at time t is represented by a vector as follows:

$$h(t) = [h_1(t), h_2(t), h_3(t), h_4(t), \dots, h_n(t)]^T \quad (1)$$

2. If v_j points to the edge e_{ji} of v_i , at time t , node i receives the heat from v_j in Δt time as: $RH = (i, j, t, \Delta t)$. Where RH is proportional to the heat of Δt and v_j ; if there is no edge between v_i and v_j , then RH is equal to zero.

The heat that v_i can get from neighboring nodes is calculated as follows:

$$RH = \alpha \cdot \Delta t \cdot \sum_{j: v_j \in N^+(v_i)} \frac{h_j(t)}{d_j} \quad (2)$$

$(N^+(v_i))$ represents the set of edges where v_i is connected to v_j .

3. The heat that v_i flows out to its neighbor nodes is represented by $DH = (i, t, \Delta t)$. If there is no edge starting from v_i , the heat of v_i does not propagate outward. DH is proportional to Δt and proportional to the heat of v_i ; The formula for calculating DH is as follows:

$$DH = \tau_i \cdot \alpha \cdot \Delta t \cdot h_i(t) \quad (3)$$

τ_i is a sign of the heat output of v_i , indicating whether node v_i outputs heat outward. When d_i (d_i is the degree of the node i) is greater than 0, τ_i is 1, v_i heat can be transmitted to its successor through the interconnected edges, the total heat transferred is $\alpha \cdot \Delta t \cdot h_i(t)$; When d_i is equal to 0, τ_i is 0, indicating that v_i does not point to the network side of other nodes, and its heat cannot be transferred outward.

4. After Δt , the heat change of node v_i is $h_i(t + \Delta t) - h_i(t)$:

$$h_i(t + \Delta t) - h_i(t) = RH - DH = \alpha \cdot \left[\sum_{v_j \in N(v_i)} \frac{h_j(t)}{d_j} - \tau_i h_i(t) \right] \cdot \Delta t \quad (4)$$

Organize the various styles:

$$h(t) = e^{\alpha \cdot t \cdot H} \cdot h(0) = \left(1 + \alpha \cdot t \cdot H + \frac{\alpha^2 \cdot t^2}{2!} H^2 + \frac{\alpha^3 \cdot t^3}{3!} H^3 + \dots \right) \cdot h(0) \quad (5)$$

$$H_{ij} = \begin{cases} 1/d_j, & (v_j, v_i) \in E, \\ -1, & i = j, d_j > 0 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Where H is an n -order matrix and e is a natural constant.

Figure 1 is an example. Observing the change process of the holding heat of each node from 0 to 10, the thermal conductivity α is 0.15, and the initial heat of the node v_1 is 10 in the figure.

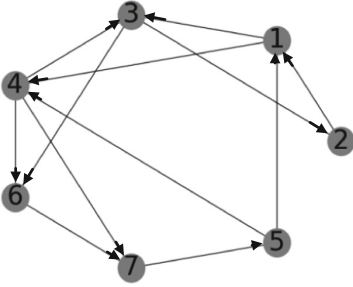


Fig. 1. Social network diagram

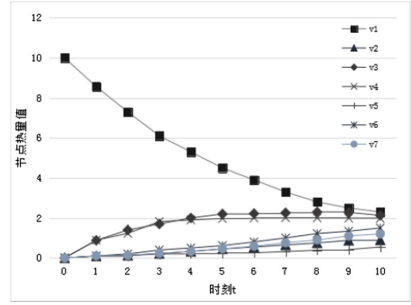


Fig. 2. Change in heat held by each node over time

The initial heat distribution of the network is expressed as:

$$h(0) = [10, 0, 0, 0, 0, 0, 0]^T$$

According to formula (5), each element of the main diagonal of matrix H takes a value of -1 , two sides with v_1 as the starting point, d is 2, and the end point distribution is v_3, v_4 , so H_{31} and H_{41} are both $1/2$. In the same way, the matrix H is finally obtained as follows:

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & -1 & 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & -1 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & -1 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1/2 & 1/3 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1/3 & 0 & 1 & -1 \end{bmatrix} \quad (7)$$

According to formula (6), the change of heat held by each node with time can be calculated, as shown in Fig. 2:

As show in the figure, as time passes, heat flows from the node v_1 to the other nodes, and eventually tends to balance when the time t is 10.

4 TMOCA Algorithm

Studies have shown that similar users tend to be homogenous, these users may choose to establish a friendly relationship with people with the same interests, or may choose friends' preferences for some reasons. These users will form a more aggregated structure, so cluster analysis is used to explore influential nodes. This section proposes the Target Market Optimal Coverage Algorithm (TMOCA) based on the thermal propagation model and its optimization, algorithm.

4.1 TMOCA Basic Algorithm

We clustering each node in a social network. Since there is relatively independent information coverage between clusters, all nodes that can produce the greatest influence can be found in each cluster, and these nodes can be used as seed nodes. In this clustering algorithm, k nodes are randomly selected as the center of each cluster, and then other nodes are assigned to clusters with higher similarity. In each cluster, the distance between the nodes should be calculated. Since this paper is based on the heat propagation model, the characteristics of the heat propagation model can be used to find the heat that the target node propagates to the cluster nodes. We can get the influence of the target node to the cluster node, and the magnitude of the distance between the nodes is determined by the magnitude of the influence. If a target node has the largest amount of heat for a cluster center, then divide the node into this cluster. Use the formula to define the distance between nodes:

$$Heat(v, S_i) = h_{Si(t)} \quad (8)$$

In the above formula, S_i is the i -th cluster center, and $h_{Si(t)}$ is the heat of the clustering S_i of the target node v at time t . Firstly, the target node i is taken out from the target node set in turn, one at a time, the heat Q_0 is given to it, and the heat is propagated under the heat propagation model, thereby obtaining the heat of the node i to the k cluster centers. Add node i to the cluster that gets its most heat. If a node has the same impact on multiple clusters, the influence on the node is also added to other clusters.

Each node is assigned, then clusters are calculated, the total impact generated by each cluster center is obtained, and then the clustering scheme with the best total impact is found.

The total impact of formalizing the definition of clustering is:

$$Inf(S) = \sum_i \sum_{v \in S} Heat(S_i, v), \quad i = 1, 2, \dots, k. \quad (9)$$

Replace the cluster center with random nodes and calculate the total influence of the cluster until the best cluster center set is found. Then the k cluster centers are the seed nodes.

Algorithm 1: Target Market Optimal Coverage Algorithm(TMOCA)

Input: Directed graph $G(V, E)$, Target set T , Thermal conductivity α , Seed size k , The initial heat is Q_0 , Propagation time t

Output: Seed set s

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1. Take  $S \subseteq G[V]$  and  $|S|=k$ 
2. randomly select  $k$  cluster centers
3. FOR  $v$  in  $T$  do
4.    $h_v(0)=Q_0$ ;
5.   FOR  $S_i$  in  $S$  do
6.     compute  $Heat(v, S_i)$  (use equation (7))
7.     Incorporate  $v$  into the cluster  $S_i$  of the maximum frequency
8.   End
9. End
10. Calculate the overall effect of species  $Inf(S)$  (use equation (7))
11. FOR  $S_i$  in  $S$  do
12.    $h_{S_i}(0)=Q_0$ ;
13.   compute  $h(t)$  (use equation (4))
14.   compute  $Inf(S)$  (use equation (8))
15. End
16. FOR  $S_i$  in  $S$  do
17.   FOR  $j \in V$ 
18.     IF  $Inf(S - S_i + j) > Inf(S)$  then
19.        $S$ .remove( $S_i$ );
20.        $S$ .append( $j$ );
21.   End;
22. End
23. End
24. UNTIL  $Inf(S)$  no longer changes
25. RETURN  $S$ 

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Complexity Analysis: The clustering analysis in Algorithm 1 occupies the main time, the number of cycles is n , k is the size of the cluster, and $|V|$ is the number of network nodes, and the complexity of the algorithm is $O(n k |V|)$.

4.2 TMCOA Algorithm Improvement

Algorithm 1 finds k seed nodes from the entire social network as cluster centers, and the time overhead is large. The target node is known. In order to reduce the search of the node independent of the target node and improve the performance of the algorithm,

this section proposes an improved algorithm, which uses the breadth-first traversal method to narrow the search space.

In Fig. 1, the node v_1 is the initial node, the thermal conductivity α is 0.2, and the time t is fixed. The heat held by the different nodes at the time t can be obtained (Fig. 2). It is not difficult to find that the target node closer to the seed node is at the time t , the target node has the largest amount of heat; the farther away from the target node, the heat will be lost or flow elsewhere, and the influence will be weak. Therefore, the distance between the node that has a large influence on the target node and the target node is within a certain range.

We use the breadth-first traversal to narrow down the search space to improve the algorithm. The specific process is as follows: starting from the target node, from near to far, the access and the target node are in the same path and the path length is 1, 2, 3. Each node of ... is counted by the search step, and a limited set of steps is obtained to obtain a set of nodes having a certain influence on the target node.

Then we used the Algorithm 1 to computed this subs.

Complexity Analysis: The target seed set is much smaller than the reduced set of all users, so cluster analysis and selection of the optimal seed set still consume a lot of time, the algorithm complexity is $O(nk|U|)$, where n is the number of cycles, k is the set size, and $|U|$ is the number of nodes in the reduced subgraph.

5 Experimental Comparison and Analysis

In this section, the design experiments verify the proposed algorithm performance. We observe the influence range of the proposed algorithm and other algorithms on the target node set and the time-solving efficiency of different algorithms.

5.1 Experimental Data

This paper selects Wiki-Vote, com-Youtube and Ca-GrQc data sets to verify the performance of the proposed algorithm. Table 1 shows the nodes and sides information of the three data sets. The social networks represented by these three data sets all have the characteristics of a complex network.

Table 1. Experimental data set description.

Datasets	WiKi-Vote	Com-Youtube	Ca-GrQc
Nodes	7115	1134890	5242
Sides	103689	2987624	28980

Since the data set only contains relationships between nodes, the experiment randomly extracts the target nodes from each data set. The simulation experiments were carried out on different data sets, and the seed sets generated by different algorithms and the range of influence were recorded.

5.2 Parameter Settings

- (1) The number of seed nodes k set the number of seed nodes from 1 to 15, and observe the change in the intensity of the affected nodes as the seed nodes increase.
- (2) The higher the thermal conductivity α . According to the literature [12] and the multiple experiments selected α value of 0.15.
- (3) For the experimental propagation unit time Δt , influence the final time t of propagation, and the unit time Δt takes a value of 0.5. The propagation result in the heat propagation model is related to the duration of the influence propagation. According to the literature [12] and the experimental measurement setting, the final time of the influence propagation is $50 \Delta t$, so that the heat can be highly diffused.
- (4) The breadth traverses the number of steps. Step takes values 3 and 6 respectively for comparison.
- (5) The target seed set is randomly selected from 1000.

5.3 Analysis of Experimental Results

The target market optimal coverage algorithm (TMOCA) based on the heat propagation model proposed in this paper is compared with the following algorithm:

- (1) THGA [16], a single-objective influence maximization algorithm based on heat propagation model.
- (2) The KCC algorithm [17] is based on the target market information coverage maximization algorithm of the independent cascade model.
- (3) The IRIE algorithm, based on the literature [12], the algorithm can calculate the influence ranking of each node, and uses a linear method to estimate the gain influence of the node, thereby selecting the seed node set. The time performance and seed quality of the algorithm are both good.
- (4) TMOCA+ algorithm, the optimal algorithm for target market optimal coverage algorithm based on heat propagation model proposed in this paper.

In order to calculate the influence range of the seed node set by different algorithms, the Monte Carlo simulation information propagation process is used in this paper. Each node set is simulated for 2000 times, and then the average value is taken as the estimated value of the node set influence propagation value. Figure 3 compares Fig. 4 with the influence range of different algorithms in two different datasets. The x-axis is the number of seed nodes and the y-axis is the influence range.

Figures 3, 4 and 5 shows that the performance of the algorithms on different data sets is slightly different due to the different internal structure of the data set, but they tend to rise. For our given target group, when the number of seeds reaches 15, the growth tends to be flat, so we choose the number of seed nodes from 1 to 15 to compare the performance of different algorithms.

The KCC algorithm has little difference with the algorithm proposed in this paper when the number of seed nodes is increasing. This is because the KCC algorithm also adopts the clustering method to select the seed set. The performance of the THGA

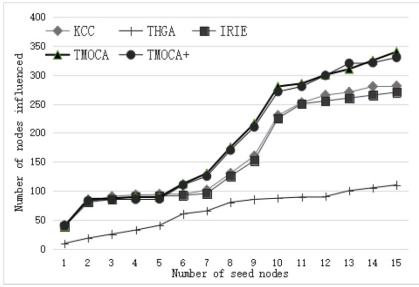


Fig. 3. Influence spread on WiKi-Vote

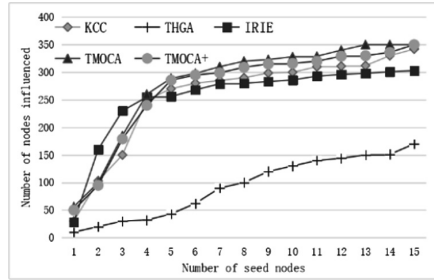


Fig. 4. Influence spread on com-YouTube

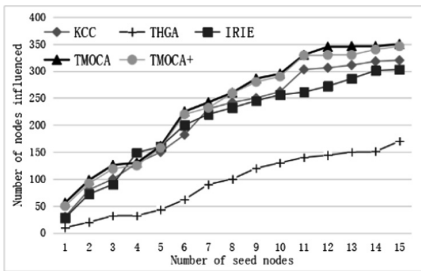


Fig. 5. Influence spread on Ca-GrQc

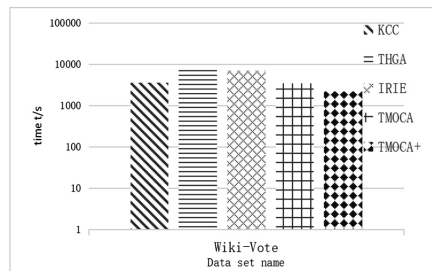


Fig. 6. Running time on Wiki-Vote

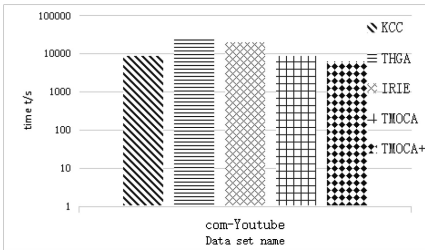


Fig. 7. Running time on com-YouTube

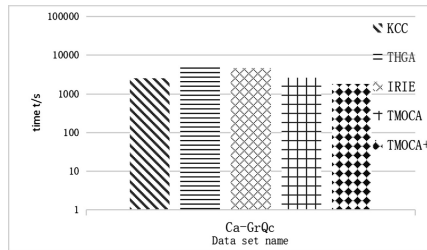


Fig. 8. Running time on Ca-GrQc

algorithm on the three data sets is not special. Well, because the THGA algorithm affects the single target, the performance of the algorithm can't be faster with the increasing seed set; the IRIE algorithm is relatively poor, probably because IRIE uses a linear method to estimate when a node joins Interference with the influence of other nodes after the seed node set. However, when it is affected by a given target node, it will be interfered by the unrelated nodes, which will reduce its performance. For any selected target node, the TMOCA algorithm proposed in this paper can get good results. This is because the algorithm is generated by clustering. Seed nodes are the most influential nodes in a certain area. As the number of selected seed nodes increases, the number of clusters increases, which leads to an increasing range of influence

propagation. The optimization algorithm does not produce a larger range of influence, and the performance of the selected seed set is similar.

Figures 6, 7 and 8 shows the running time of different algorithms when 10 seed nodes are selected. Since the seed node is 10 in Figs. 3, 4 and 5, the effect of each algorithm tends to be stable. Therefore, select the seed node in this experiment. It is more representative of 10. It is obvious that the running time in Fig. 7 is long, and it may be because Fig. 7 is an experiment of each algorithm on the com-YouTube. The data set node is more related to the other two data sets, and the nodes are closely related to each other, resulting in an operation. The amount is too large. Comparing the five algorithms, it is obvious that the THGA algorithm takes the longest time because the target greedy algorithm is aimed at maximizing the single-objective influence. In this experiment, multiple loops are used to achieve the purpose of influencing the target group, resulting in its operation. The time is relatively high; the KCC algorithm runs slightly longer because the algorithm requires multiple Monte Carlo simulations to ensure the accuracy of the seed set; IRIE takes slightly longer than the KCC algorithm, possibly due to its To seed selection. The TMOCA algorithm proposed in this paper has a slightly better influence on the propagation time than the KCC algorithm, but its optimization algorithm running time can reach a reasonable time requirement.

6 Conclusion

In this paper, when the target user group is known, we want to select a certain number of seed users in the social network to disseminate information, so that the propagation impact is optimal. This paper introduces the heat propagation model to simulate the propagation of information in social networks, so that the influence of nodes can be directly calculated, avoiding the high time consumption in the Monte Carlo simulation. In this paper, the TMOCA algorithm is proposed, and the clustering method is used to find k cluster centers with the greatest influence on the target users as seed nodes. In order to improve the performance of the algorithm, an optimization algorithm (TMOCA+) is proposed, and the breadth-first traversal method is used to narrow the social network map. Experiments on real data sets verify that the proposed algorithm can cover the target users to the greatest extent and has better time performance.

Acknowledgment. This work was supported in part by the Education Department of Heilongjiang Province (12531498).

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