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An evidential dynamical model to predict the interference effect of categorization on decision making results

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ABSTRACT

Categorization is necessary for many decision making tasks. However, the categorization process may interfere the decision making result and bring about the disjunction fallacy. To predict the interference effect of categorization, some models based on quantum cognition theory have been proposed. In quantum dynamical models, like the quantum belief-action entanglement (BAE) model, actions and beliefs are deemed to be entangled. However, the entanglement degree is an artificially defined parameter. In this paper, a new evidential dynamical (ED) model based on Dempster–Shafer (D-S) evidence theory and quantum dynamical modelling is proposed. Considering that sometimes people hesitate to make a decision, it is reasonable to extend the action states by introducing an uncertain state. In an evidential framework, categorization can influence the uncertain state in actions. The interference effect is measured by handling the uncertain state while no extra parameter is defined artificially. The proposed model is applied to the classical categorization decision-making experiments. Compared with the existing models, the number of free parameters in the ED model is less than the classical quantum models, and the ED model is more rational and simpler than an evidential Markov model. The model application results and discussions show the correctness and effectiveness of the ED model. Not only the interference effect of categorization on decision making results is explained and predicted, but also an inspiring dynamical decision making framework is proposed in this paper. We believe that the proposed ED model will bring more opportunities and will result in more applications in the future.

1. Introduction

As an important and normal part of decision making process, categorization is widely involved in decision making tasks in reality [1–3]. Without precise categorization, many decisions can not be made. For instance, doctors must classify the tumor before doing the surgery; judges need to categorize the defendant before making a judgement; the commander need to categorize an unexpected aircraft before making a command. Additionally, categorization is also an important task in knowledge-based systems [4–6], which is the basis of decision making. However, lots of practical examples and experiments show that categorization may result in the disjunction fallacy, which violates the law of total probability [7,8]. In this case, some categorization-based decisions may be counterintuitive, which should be paid attention to.

Townsend et al. [9] proposed a categorization decision-making experiment, which is now a widely used paradigm for studying the disjunction effect of categorization. A Markov process can be used to model a random system that changes states according to a transition rule that only depends on the current state [10,11]. Unfortunately, however, the disjunction effect can not be predicted in a Markov model [9]. To address it, some quantum models have been proposed. Quantum probability theory is an effective tool to explain this phenomenon and predict the interference effect, which has been widely applied in the fields of cognition and decision making [12–14]. It is an effective approach to psychology and behavioral sciences [15–17]. Besides, the quantum framework has been widely applied in wide studies, including conceptual combination [18], cognition [19,20], reliability analysis [21], data fusion [22], optimization [23] and so on. Many paradoxes can be explained in a quantum framework, like the violation of the sure thing principle [24], the additive law of probability [25], the Ellsberg paradox paradox [26], and so on. It can also explain many irregular phenomena in classical theories, like order effect [27–29], disjunction fallacy [30–32], the prison dilemma [33], etc. Among them, the disjunction fallacy is one of the most popular problems. In a quantum model, the interference effect, a term in quantum mechanics, is borrowed to explain the disjunction fallacy. Many theories have been proposed to predict the interference

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effect, such as the quantum dynamical (QD) model [7,34,35], the quantum-like model [36–38], the quantum prospect decision theory [39–41] and the quantum-like Bayesian networks [42,43], etc. But most of the existing quantum models encounter a problem of introducing extra quantum-related parameters, which can't be determined clearly enough.

To predict the interference effect of categorization, in our previous work, we modeled the decision making process under an uncertain environment [44]. Decision making and optimization under uncertainty is normal in reality and has been heavily studied [45–47]. But it is still an open issue for uncertain information modeling and processing [48–50]. Many theories have been developed, such as fuzzy set theory [51,52], Dempster–Shafer (D-S) evidence theory [53,54], D number [55,56], Z number [57] and so on. Among them, D-S evidence theory is a powerful tool to handle the uncertainty [58–60]. It has been widely used in many applications, like risk analysis [61,62], pattern recognition [63–65], dependence assessment [66,67], multiple attribute decision making [68–70] and so on. In recent years, D-S theory has been widely combined with quantum theories [71–74]. An evidential Markov (EM) decision making model which combines D-S theory with Markov modelling to predict the disjunction effect was presented in previous work [44]. Nevertheless, the EM model is based on an assumption of holding tendency to certain decision under uncertainty, which is hard to be quantitatively proven.

In this paper, an evidential dynamical (ED) model based on D-S theory and quantum dynamical modelling is proposed to predict the interference effects of categorization on decision making results. Generally, a decision making process consists of a belief part and an action part. Numerous cases show that people may hesitate to make a certain decision during the decision making process. Hence, to address it, the action states are extended in an evidential framework by introducing an uncertain state, while the uncertainty in beliefs is represented as a superposition of certain states. The uncertain action state can be influenced by categorization which happens in beliefs. In a quantum theory, before making a final decision, human thoughts are seen as superposed waves that can interfere with each other. In the classical QD model, the interference process is driven by an extra parameter. In the quantum belief-action entanglement (BAE) model [8], this parameter is named as entanglement degree because the actions and beliefs are deemed to be entangled. The psychological function of entanglement is applied to coordinate beliefs and actions. Its usage is motivated by a need for explaining why people may be inclined to change their beliefs to be consistent with their own actions [75]. To the contrary, in the ED model, although the action states are extended, the number of free parameters decreases as no more parameter is defined artificially. Definitions of the remaining parameters are based on an optimization principle. Then the interference effect can be well predicted and measured by handling the uncertain state in actions. The ED model is applied to classical categorization decision-making experiments. ED model application results and their comparison with results obtained by means of using other approaches show that the model is an effective and efficient tool for predicting of the interference effects of categorization on decision making results.

The rest of the paper is organized as follows. In Section 2, the preliminaries of the basic theory employed will be briefly introduced. The background of the categorization decision-making experiment is illustrated in Section 3. Then our ED model is proposed to predict the interference effect and explain the experiment results in Section 4. The results of parameter determination, model application and sensitivity analysis are shown in Section 5. The ED model is compared with other three models in Section 6. Finally, Section 7 comes to the conclusion.

2. Preliminaries

2.1. Quantum dynamical model

The QD model first proposed by Busemeyer et al. in 2006 [35] is formulated as a random walk decision process. The evolution of complex valued probability amplitudes over time is described. The interference effect can be produced in a quantum model which is not possible in a classical Markov model. The QD model assumes that a participant has some potential to be in every state in the beginning. Thus the state is a superposition of all possible n states

$$|\psi\rangle = \psi_1|S_1\rangle + \psi_2|S_2\rangle + \cdots + \psi_n|S_n\rangle, \tag{1}$$

and the initial state corresponds to an amplitude distribution $\psi(0)$ represented by the $n \times 1$ matrix

$$\psi(0) = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix}. \tag{2}$$

During the decision making process, the state evolves across time obeying a Schr ö dinger equation:

$$\frac{d}{dt}\psi(t) = -i \cdot H \cdot \psi(t), \tag{3}$$

where H is a Hermitian matrix: $H^\dagger = H$, which has elements h_{ij} in row i and column j representing the instantaneous rate of change to $|i\rangle$ from $|j\rangle$. Eq. (3) has a matrix exponential solution:

$$\psi(t_2) = e^{-iHt} \cdot \psi(t_1) = U(t) \cdot \psi(t_1), \tag{4}$$

where matrix $U(t) = e^{-iHt}$ is a unitary matrix, which satisfies: $U(t)^\dagger U(t) = I$. It finally guarantees that $\psi(t)$ always has unit length. For $t = t_2 - t_1$, the transition probability is determined as:

$$T(t) = |U(t)|^2. \tag{5}$$

The element T_{ij} represents the probability of observing state i at time t_2 given that state j was observed at time t_1. Based on the above definition, the amplitude distribution of state evolves to $\psi(t)$ from the initial $\psi(0)$ across time t as Eq. (6):

$$\psi(t) = U(t) \cdot \psi(0), \tag{6}$$

which shows the dynamics in a decision making process.

2.2. Dempster–Shafer evidence theory

In Dempster–Shafer evidence theory, frame of discernment F is a fixed set of N mutually exclusive and exhaustive elements, symbolized by $\Theta = \{H_1, H_2, \ldots, H_N\}$. Let us denote $P(\Theta)$ as the power set composed of 2^N elements A of Θ:

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \ldots, \Theta\}$$

The support degree of an element is described by a mass function, which is defined as a mapping m from the power set to [0, 1], satisfying:

$$\sum_{A \subseteq P(\Theta)} m(A) = 1,$$
$$m(\emptyset) = 0, \tag{7}$$

where A is a subset of $P(\Theta)$, called the focal set. The mass function is also named as basic probability assignment (BPA) or basic belief assignment (BBA). From a mass function m, we can compute a belief function and a plausibility function, defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \ , A \subseteq P(\Theta) \tag{8}$$

Fig. 1. Example faces used in a categorization-decision experiment.

and

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) = 1 - Bel(A)\ , A \subseteq P(\Theta). \tag{9}$$

When all the focal sets of m are singletons, m is said to be *Bayesian; Bel* and *Pl* then degenerate into the same probability measure.

2.3. Pignistic probability transformation

Pignistic probability transformation (PPT) is one of the most classical method to transfer a mass function to a probability. The term "pignistic" proposed by Smets is originated from the word pignus, meaning 'bet' in Latin. The principle of insufficient reason is to distribute $m(X)$ equally among all elements of X. Given a frame of discernment F, PPT projects a mass function m to probability $BetP$ as follows [76]:

$$BetP(A) = \sum_{B \subseteq F} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \quad \forall A \subseteq F, \tag{10}$$

where $|B|$ is the cardinality of B, which denotes the number of elements in set B.

3. Categorization decision-making experiment

3.1. Experiment introduction

The categorization decision-making experiment paradigm was proposed by Townsend et al. [9] to study the interactions between categorization and decision making. It was designed to test Markov model initially. Subsequently, this paradigm was extended for quantum models as well by Busemeyer et al. [7]. The paradigm consists of two conditions, one is categorization decision-making (C-D) condition and the other one is decision alone (D alone) condition. In the C-D condition, in each trial, participants were shown pictures of faces, which vary along two dimensions: face width and lip thickness (like shown in Fig. 1). The following is the illustration of the experiment in Busemeyer et al. [7]. Participants were asked to categorize the face as a "good" (G) guy or a "bad" (B) guy and then make a decision to "attack" (A) or to "withdraw" (W). The faces can be roughly divided into two kinds: one kind is "narrow" faces with narrow width and thick lips; the other kind is "wide" faces with wide width and thin lips. The participates were informed that "narrow" faces had a 0.60 probability of belonging to "bad guy" population and "wide" faces had a 0.60 probability of belonging to "good guy" population. Participants were rewarded for attacking "bad guy" and withdrawing from "good guy". In the D alone condition, the participants were asked to make a decision directly without categorizing. The faces shown to participants were the same as those in the C-D condition. The whole experiment included a total of 26 participants, but each participant provided 51 observations for the C-D condition for a total of $26 \times 51 = 1326$ observations, while each person provided 17 observations for the D alone condition for a total of $17 \times 26 = 442$ observations.

Table 1
The results of the C-D condition and D alone condition.

Face type	$P(G)$	$P(A\|G)$	$P(B)$	$P(A\|B)$	P_T	$P(A)$	t
Wide	0.84	0.35	0.16	0.52	0.37	0.39	0.5733
Narrow	0.17	0.41	0.83	0.63	0.59	0.69	2.54

3.2. Experiment results

The experiment results are shown in Table 1. The column labeled $P(G)$ represents the probability of categorizing the face was a "good guy", the column labeled $P(A|G)$ represents the probability of attacking given that the face was categorized as a "good guy". The column labeled $P(B)$ represents the probability of categorizing the face as a "bad guy". The column labeled $P(A|B)$ represents the probability of attacking given that the face was categorized as a "bad guy". The column labeled P_T represents the total probability of attacking in the C-D condition which is computed as follows£º

$$P(A) = P(G) \cdot P(A|G) + P(B) \cdot P(A|B) \tag{11}$$

Additionally, the column labeled $P(A)$ represents the probability of attacking in the D alone condition. As shown in Table 1, deviation between P_T and $P(A)$ exists for both types. However, the prominent deviation occurs only for narrow type faces. It means that the interference effect is produced for narrow faces while the effect is weak for wide type faces. A paired t-test is used to test the significance of difference between P_T and $P(A)$ in [7]. The results shown in column t indicate that the interference effect is statistically significant for narrow faces, but not for wide faces. Therefore, we mainly aim to study the fallacy for narrow faces in this paper.

Besides, this paradigm has been used and analyzed in many other works. Literature of studying the categorization decision-making experiment and their results are shown below in Table 2. According to the law of total probability, the probability of attacking should be equal in two conditions. Hence, the categorization brings about the interference effect which influences the decision making results and breaks the law of total probability.

4. Proposed method

In this section, the ED model is applied to explain and predict the experiment results shown in last section. To begin with, the flow chart of our model is illustrated in Fig. 2.

Step 1. Representation of the state in an evidential framework

In a categorization decision-making experiment, an intact decision making process is consisted of beliefs and actions. The beliefs include categorizing the face as good (G) or bad (B). The uncertainty in beliefs is represented by a superposition of state G and B. The basic actions include a certain decision to withdraw (W) or to attack (A). In an evidential framework, the frame of discernment in actions is $\{A,\ W\}$. Hence the action states are extended by introducing an uncertain state $A \cup W$ (denoted as U), which represents that the decision maker's hesitation whether to attack or to withdraw. The uncertainty in actions is represented

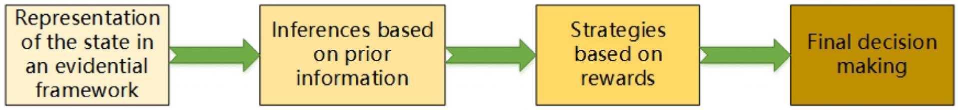

Fig. 2. The flow chart of the ED model.

Table 2
Results of categorization decision-making experiments.

Literature	Type	$P(G)$	$P(A\|G)$	$P(B)$	$P(A\|B)$	P_T	$P(A)$
Busemeyer et al. [7]	W	0.84	0.35	0.16	0.52	0.37	0.39
	N	0.17	0.41	0.83	0.63	0.59	0.69
Wang and Busemeyer Experiment 1 [8]	W	0.78	0.39	0.22	0.52	0.42	0.42
	N	0.21	0.41	0.79	0.58	0.54	0.59
Wang and Busemeyer Experiment 2 [8]	W	0.78	0.33	0.22	0.53	0.37	0.37
	N	0.24	0.37	0.76	0.61	0.55	0.60
Wang and Busemeyer Experiment 3(a) [8]	W	0.77	0.34	0.23	0.58	0.40	0.39
	N	0.24	0.33	0.76	0.66	0.58	0.62
Wang and Busemeyer Experiment 3(b) [8]	W	0.77	0.23	0.23	0.69	0.34	0.33
	N	0.25	0.26	0.75	0.75	0.63	0.64
Average	W	0.79	0.33	0.21	0.57	0.38	0.38
	N	0.22	0.36	0.78	0.65	0.58	0.63

1 In Busemeyer et al. [7], the classical experiment was replicated.
2 In Wang and Busemeyer [8], experiment 1 used a larger data set to replicate the classical experiment. Experiment 2 introduced a new X-D trial verse C-D trial and only the results of C-D trial are used here. In experiment 3(a), the reward for attacking bad people was a bit smaller than the reward in experiment 1 and 2. Whereas, in experiment 3(b), the reward for attacking bad people was a bit larger than the reward in experiment 1 and 2.

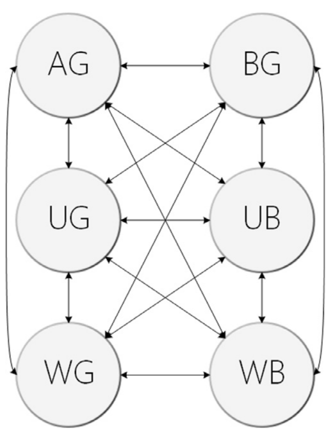

Fig. 3. Transition diagram in a categorization decision-making experiment.

by state U. Although categorization happens in the belief part, it can influence the action part by producing the interference effect. Hence, our goal is to predict it. The initial state involves six combination of beliefs and actions

$$\{|B_GA_A\rangle, |B_GA_U\rangle, |B_GA_W\rangle, |B_BA_A\rangle, |B_BA_U\rangle, |B_BA_W\rangle\},$$

where, for example, $|B_GA_A\rangle$ symbolizes the event in which the participant categorizes the face as good but he intends to attack. The model assumes that at the beginning of a trial, the participant has some potential to be in every circle in Fig. 3 which illustrates the possible transitions among the six states. Therefore the person's state is a superposition of the six basis states

$$\begin{aligned}|\psi\rangle = \psi_{AG}\cdot|B_GA_A\rangle + \psi_{UG}\cdot|B_GA_U\rangle + \psi_{WG}\cdot|B_GA_W\rangle + \psi_{AB}\cdot|B_BA_A\rangle \\ +\psi_{UB}\cdot|B_BA_U\rangle + \psi_{WB}\cdot|B_BA_W\rangle,\end{aligned} \tag{12}$$

and the initial state corresponds to an amplitude distribution represented by a 6×1 column matrix

$$\psi(0) = \begin{bmatrix}\psi_{AG}\\ \psi_{UG}\\ \psi_{WG}\\ \psi_{AB}\\ \psi_{UB}\\ \psi_{WB}\end{bmatrix}, \tag{13}$$

where, for example, $|\psi_{AG}|^2$ is the probability of observing state $|B_GA_A\rangle$ initially. The squared length of ψ must equal one, which satisfies: $\psi^{\dagger}\cdot\psi = 1$. Here we assume that the initial state is equally distributed.

Step 2. Inferences based on prior information

As the decision making process goes on, information about the player's beliefs changes the initial state at time $t = 0$ into a new state at time t_1. In the C-D condition, the categorization of faces is determined by participants. If the face is categorized as G, the amplitude distribution across states changes to

$$\psi(t_1) = \frac{1}{\sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2}}\begin{bmatrix}\psi_{AG}\\ \psi_{UG}\\ \psi_{WG}\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}\psi_G\\ \mathbf{0}\end{bmatrix}, \tag{14}$$

where $\sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2}$ is the initial probability of categorizing the face as G. The 3×1 matrix ψ_G is the conditional amplitude distribution across actions, which has a squared length equal to one. If the face is categorized as B, the amplitude distribution across states changes to

$$\psi(t_1) = \frac{1}{\sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2}}\begin{bmatrix}0\\ 0\\ 0\\ \psi_{AB}\\ \psi_{UB}\\ \psi_{WB}\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \psi_B\end{bmatrix}, \tag{15}$$

where $\sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2}$ is the initial probability of categorizing the face as B. The 3×1 matrix ψ_B is the conditional amplitude distribution across actions, which also has a squared length equal to one.

In the D alone condition, no new information about categorization is introduced. Hence the amplitude distribution across states keeps the same as the initial one.

$$\begin{aligned}\psi(t_1) = \psi(0) &= \begin{bmatrix}\sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2}\cdot\psi_G\\ \sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2}\cdot\psi_B\end{bmatrix}\\ &= \sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2}\begin{bmatrix}\psi_G\\ \mathbf{0}\end{bmatrix}\\ &\quad+\sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2}\begin{bmatrix}\mathbf{0}\\ \psi_B\end{bmatrix}.\end{aligned} \tag{16}$$

Eq. (16) expresses the initial state for the unknown condition as a superposition formed by a weighted sum of the amplitude distributions for the two known conditions.

Step 3. Strategies based on rewards

As actions are assumed to be taken based on rewards, the participants must evaluate the rewards in order to select an appropriate action, which changes the previous state at time t_1 into a new state $\psi(t_2)$ at time t_2. The evolution of the state during the time period corresponds to the thinking process which leads to a decision. Based on quantum dynamical modelling, the state evolution obeys a Schr ö dinger equation (Eq. (3)). A unitary matrix $U = e^{-iHt}$ is defined to satisfy the solution of Eq. (3), which is as follows:

$$\psi(t_2) = e^{-iHt} \cdot \psi(t_1), \tag{17}$$

where $\psi(t_2)$ is the amplitude distribution across states after evolution based on rewards. The Hamiltonian matrix H is defined as follows:

$$H = \begin{bmatrix} H_G & \mathbf{0} \\ \mathbf{0} & H_B \end{bmatrix}$$
$$H_G = \frac{1}{1+{h_G}^2}\begin{pmatrix} h_G & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -h_G \end{pmatrix},\ H_B = \frac{1}{1+{h_B}^2}\begin{pmatrix} h_B & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -h_B \end{pmatrix}. \tag{18}$$

The 3×3 Hamiltonian matrix H_G applies when the participant categorizes the face as G, and H_B applies when the participant categorizes the face as B. The parameter h_G is a function of the difference between the rewards for attacking relative to withdrawing given categorizing the face as G and the parameter h_B is a function of the difference between the reward for attacking relative to withdrawing given that the face is categorized as B. The Hamiltonian matrix transforms the state probabilities to favor attacking, withdrawing or being uncertain (hesitation), depending on the reward functions.

Based on the above, we can obtain the participant's state at time t_2. In the C-D condition, if the face is categorized as G, the state evolves to

$$\psi(t_2) = e^{-i \cdot H \cdot t} \cdot \psi(t_1) = \begin{bmatrix} e^{-i \cdot H_G \cdot t} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot H_B \cdot t} \end{bmatrix} \cdot \begin{bmatrix} \psi_G \\ \mathbf{0} \end{bmatrix} = e^{-i \cdot H_G \cdot t} \cdot \psi_G. \tag{19}$$

If the face is categorized as B, the state evolves to

$$\psi(t_2) = e^{-i \cdot H \cdot t} \cdot \psi(t_1) = \begin{bmatrix} e^{-i \cdot H_G \cdot t} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot H_B \cdot t} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} \\ \psi_B \end{bmatrix} = e^{-i \cdot H_B \cdot t} \cdot \psi_B. \tag{20}$$

In the D alone condition, the state evolves to

$$\begin{aligned} \psi(t_2) &= e^{-i \cdot H \cdot t} \cdot \psi(0) \\ &= \begin{bmatrix} e^{-i \cdot H_G \cdot t} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot H_B \cdot t} \end{bmatrix} \cdot \begin{bmatrix} \sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2} \cdot \psi_G \\ \sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2} \cdot \psi_B \end{bmatrix} \\ &= \sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2} \cdot e^{-i \cdot H_G \cdot t} \cdot \psi_G + \\ &\quad \sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2} \cdot e^{-i \cdot H_B \cdot t} \cdot \psi_B. \end{aligned} \tag{21}$$

Eq. (21) shows that state in time t_2 under unknown condition is still a superposition formed by a weighted sum of the amplitude distributions for the two known conditions

Step 4. Final decision making

Now the participant's state after evaluating the actions has been obtained. The last step is to calculate the mass function of an action state and do some necessary transformation. Each decision corresponds to a measurement of the state at time t_2. To pick out certain action state for each belief towards categorization, a measure matrix M is used:

$$M = \begin{pmatrix} M_G & \mathbf{0} \\ \mathbf{0} & M_B \end{pmatrix}, \tag{22}$$

where matrix M_G applies when the participant categorizes the face as G, and matrix M_B applies when the participant categorizes the face as B.

In the C-D condition, to obtain the mass function of attacking we set

$$M_{GA} = M_{BA} = diag\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{23}$$

The mass function of attacking given the face is categorized as G equals

$$\begin{aligned} m(A|G) &= \left\| M_A \cdot e^{-i \cdot t \cdot H} \cdot \psi(t_1) \right\|^2 \\ &= \left\| \begin{bmatrix} M_{GA} & \mathbf{0} \\ \mathbf{0} & M_{BA} \end{bmatrix} \cdot \begin{bmatrix} e^{-i \cdot t \cdot H_G} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot t \cdot H_B} \end{bmatrix} \cdot \begin{bmatrix} \psi_G \\ \mathbf{0} \end{bmatrix} \right\|^2 \\ &= \left\| M_{GA} \cdot e^{-i \cdot t \cdot H_G} \cdot \psi_G \right\|^2. \end{aligned} \tag{24}$$

The mass function of attacking given the face is categorized as B equals

$$\begin{aligned} m(A|B) &= \left\| M_A \cdot e^{-i \cdot t \cdot H} \cdot \psi(t_1) \right\|^2 \\ &= \left\| \begin{bmatrix} M_{GA} & \mathbf{0} \\ \mathbf{0} & M_{BA} \end{bmatrix} \cdot \begin{bmatrix} e^{-i \cdot t \cdot H_G} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot t \cdot H_B} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} \\ \psi_B \end{bmatrix} \right\|^2 \\ &= \left\| M_{BA} \cdot e^{-i \cdot t \cdot H_B} \cdot \psi_B \right\|^2. \end{aligned} \tag{25}$$

To obtain the mass function of being uncertain, we set

$$M_{GU} = M_{BU} = diag[0\ 1\ 0]. \tag{26}$$

The mass function of being uncertain given that face is categorized as G equals

$$\begin{aligned} m(U|G) &= \left\| M_U \cdot e^{-i \cdot t \cdot H} \cdot \psi(t_1) \right\|^2 \\ &= \left\| \begin{bmatrix} M_{GU} & \mathbf{0} \\ \mathbf{0} & M_{BU} \end{bmatrix} \cdot \begin{bmatrix} e^{-i \cdot t \cdot H_G} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot t \cdot H_B} \end{bmatrix} \cdot \begin{bmatrix} \psi_G \\ \mathbf{0} \end{bmatrix} \right\|^2 \\ &= \left\| M_{GU} \cdot e^{-i \cdot t \cdot H_G} \cdot \psi_G \right\|^2. \end{aligned} \tag{27}$$

The mass function of being uncertain given the face is categorized as B equals

$$\begin{aligned} m(U|B) &= \left\| M_U \cdot e^{-i \cdot t \cdot H} \cdot \psi(t_1) \right\|^2 \\ &= \left\| \begin{bmatrix} M_{GU} & \mathbf{0} \\ \mathbf{0} & M_{BU} \end{bmatrix} \cdot \begin{bmatrix} e^{-i \cdot t \cdot H_G} & \mathbf{0} \\ \mathbf{0} & e^{-i \cdot t \cdot H_B} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} \\ \psi_B \end{bmatrix} \right\|^2 \\ &= \left\| M_{BU} \cdot e^{-i \cdot t \cdot H_B} \cdot \psi_B \right\|^2. \end{aligned} \tag{28}$$

At the end of a trial, the participants must take a certain action whether to attack or to withdraw. As state U is introduced to represent the hesitation during a decision making process, it should be transformed into state A and W. In the categorization decision-making experiment, the interference effect for narrow faces ranges in a relatively small scope and it is positive. Hence, we follow the classical idea of PPT (Eq. (10)) to handle it (for situations with different interference effect, different methods can be adopted to distribute the amplitude of the uncertain states). The amplitude distribution of the uncertain state is assigned to two certain states equally. Hence, the total probability of attacking given that the face is categorized as G or B respectively equals

$$P(A|G) = \left\| \Psi(A|G) + \frac{1}{2}\Psi(U|G) \right\|^2 \tag{29}$$

$$P(A|B) = \left\| \Psi(A|B) + \frac{1}{2}\Psi(U|B) \right\|^2, \tag{30}$$

where, for example, $\Psi(A|G)$ is the conditional amplitude of attacking given the face is categorized as G, which has a squared length equal to $m(A|G)$. Once a probability, for example, $P(A|G)$ is

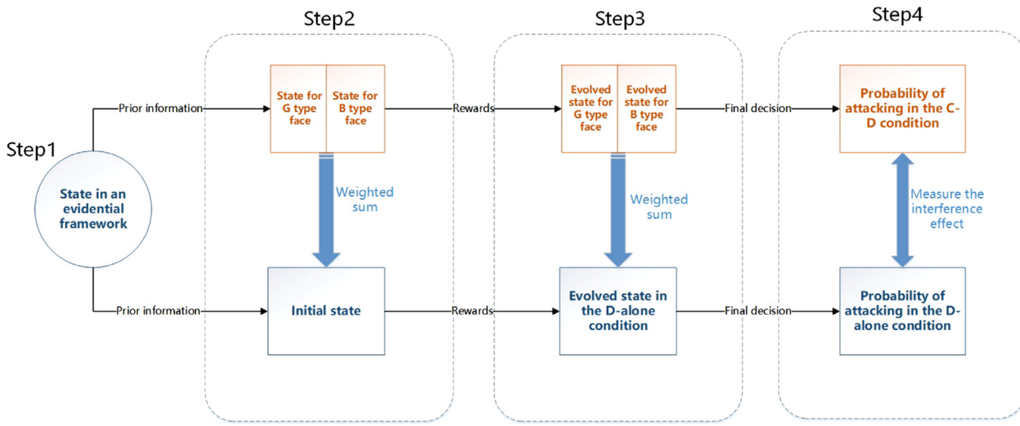

Fig. 4. The integral process of the ED model.

observed, its opposite probability $P(W|G)$ will be determined as $P(W|G) = 1 - P(A|G)$.

Then the total probability of attacking can be obtained:

$$\begin{aligned}P(A) &= P(G)\cdot P(A|G) + P(B)\cdot P(A|B)\\ &= \left(|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2\right)\cdot\left\|\Psi(A|G) + \frac{1}{2}\Psi(U|G)\right\|^2\\ &\quad +\left(|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2\right)\cdot\left\|\Psi(A|B) + \frac{1}{2}\Psi(U|B)\right\|^2.\end{aligned} \tag{31}$$

Inserting Eqs. (24)–(28) into Eq. (31) yields:

$$\begin{aligned}P(A) &= \left(|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2\right)\\ &\quad \cdot\left\|\left(M_{GA} + \frac{1}{2}M_{GU}\right)\cdot e^{-i\cdot t\cdot H_G}\cdot\psi_G\right\|^2\\ &\quad +\left(|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2\right)\\ &\quad \cdot\left\|\left(M_{BA} + \frac{1}{2}M_{BU}\right)\cdot e^{-i\cdot t\cdot H_B}\cdot\psi_B\right\|^2.\end{aligned} \tag{32}$$

In the D alone condition, without categorization, it is reasonable to assume that the mass functions $m(U|G)$ and $m(U|B)$ will not be produced. It means that the uncertain state will be picked out and transformed to two certain states at the same time, differing from the C-D condition. To achieve it, we set the measure matrix as

$$M_G = M_B = diag\begin{bmatrix}1 & \frac{1}{\sqrt{2}} & 0\end{bmatrix}. \tag{33}$$

Then the probability of attacking in the D alone condition equals

$$\begin{aligned}P(A) &= \left\|M\cdot e^{-i\cdot t\cdot H}\cdot\psi(0)\right\|^2\\ &= \left\|\begin{bmatrix}M_G & \mathbf{0}\\ \mathbf{0} & M_B\end{bmatrix}\cdot\begin{bmatrix}e^{-i\cdot t\cdot H_G} & \mathbf{0}\\ \mathbf{0} & e^{-i\cdot t\cdot H_B}\end{bmatrix}\right.\\ &\quad \left.\cdot\begin{bmatrix}\sqrt{|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2}\cdot\psi_G\\ \sqrt{|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2}\cdot\psi_B\end{bmatrix}\right\|^2\\ &= \left\|\left(|\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2\right)\cdot M_G\cdot e^{-i\cdot t\cdot H_G}\cdot\psi_G\right.\\ &\quad \left.+\left(|\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2\right)\cdot M_B\cdot e^{-i\cdot t\cdot H_B}\cdot\psi_B\right\|^2.\end{aligned} \tag{34}$$

Obviously the probabilities of attacking in Eqs. (32) and (34) are unequal. The difference value is exactly the predicted interference effect. In the C-D condition, the mass function of being uncertain is picked out and then transformed to two certain states separately and sequentially. In the D alone condition, however, no intermediate mass function $m(U|G)$ or $m(U|B)$ is produced as no categorization is made, and the uncertain state is picked out and transformed at the same time. In sum, the interference effect of categorization is predicted and measured by handling the uncertain state differently in two conditions. The integral process of the ED model is illustrated in Fig. 4. However, in this way only the interference effect for the narrow faces can be predicted, whose effect is significant. As for the wide faces, the similar method would also lead to a significant interference effect, which is incorrect obviously. To fit the observed result for wide faces, the uncertain state of the C-D and D-alone conditions should be handled equally as the C-D condition for narrow faces. Then no interference effect will be produced. Admittedly, needing further research, it is still an unsolved question why the hesitated people behave differently under the trials for narrow and wide faces.

5. Model result and analysis

5.1. Parameter determination

In the ED model, we set time process parameter $t = \frac{\pi}{2}$, whose value is set same as the one in literature [7,8]. Here, we summarize the two reasons as follows [7]: (1) $\frac{\pi}{2}$ corresponds to the task time (2 s) in practical experiments; (2) the final choice probabilities first reach their maximum at $t = \frac{\pi}{2}$. For example, from Eq. (29), we can obtain

$$P(A|G) = \frac{5}{12} + \frac{2h_G}{3\left(1 + {h_G}^2\right)}\sin^2(t). \tag{35}$$

Apparently, $P(A|G)$ comes to its maximum $\frac{5}{12} + \frac{2h_G}{3(1+{h_G}^2)}$ firstly at $\frac{\pi}{2}$. Besides, we could find that three free parameters exist in the ED model: the probability of categorizing a face as good or bad ($P(G) = |\psi_{AG}|^2 + |\psi_{UG}|^2 + |\psi_{WG}|^2$ or $P(B) = |\psi_{AB}|^2 + |\psi_{UB}|^2 + |\psi_{WB}|^2$), and two reward parameters (h_G and h_B). To predict the experiment results for narrow faces listed in Table 2, a total of six variables, namely $P(G)$, $P(A|G)$, $P(B)$, $P(A|B)$, P_T and $P(A)$, can be produced. To determine the aforementioned three free parameters, a basic optimization method is adopted by minimizing the standard error of six produced variables:

$$\begin{aligned}&\underset{P(G),h_G,h_B}{\arg\min}\sqrt{\sum_{i=1}^{6}(X_i - X_{i0})^2}\\ &X_1 = P(G), X_2 = P(A|G), X_3 = P(G),\\ &X_4 = P(A|B), X_5 = P_T, X_6 = P(A),\end{aligned} \tag{36}$$

where X_{i0} is the practical experiment result of each variable. By solving Eq. (36), the parameters for each experiment can be determined.

Table 3
The results of different models.

Literature		$P(G)$	$P(A\|G)$	$P(B)$	$P(A\|B)$	P_T	$P(A)$
Busemeyer et al. [7]	Obs	0.17	0.41	0.83	0.63	0.59	0.69
	ED	0.17	0.41	0.83	0.62	0.59	0.67
	EM	0.17	0.39	0.83	0.61	0.57	0.69
	Quantum BAE	0.17	0.41	0.83	0.66	0.62	0.68
	Markov BA	0.17	0.4	0.83	0.63	0.59	0.59
Wang and Busemeyer Experiment 1 [8]	Obs	0.21	0.41	0.79	0.58	0.54	0.59
	ED	0.21	0.41	0.79	0.56	0.52	0.61
	EM	0.21	0.42	0.79	0.58	0.55	0.6
	Quantum BAE	0.21	0.45	0.79	0.54	0.52	0.57
	Markov BA	0.21	0.39	0.79	0.6	0.55	0.55
Wang and Busemeyer Experiment 2 [8]	Obs	0.24	0.37	0.76	0.61	0.55	0.6
	ED	0.24	0.37	0.76	0.58	0.53	0.61
	EM	0.24	0.38	0.76	0.62	0.56	0.61
	Quantum BAE	0.21	0.33	0.79	0.68	0.61	0.63
	Markov BA	0.23	0.39	0.77	0.66	0.6	0.59
Wang and Busemeyer Experiment 3(a) [8]	Obs	0.24	0.33	0.76	0.66	0.58	0.62
	ED	0.24	0.33	0.76	0.62	0.55	0.63
	EM	0.25	0.34	0.75	0.66	0.58	0.64
	Quantum BAE	0.21	0.32	0.79	0.69	0.61	0.63
	Markov BA	0.23	0.47	0.77	0.55	0.53	0.53
Wang and Busemeyer Experiment 3(b) [8]	Obs	0.25	0.26	0.75	0.75	0.63	0.64
	ED	0.26	0.26	0.74	0.68	0.57	0.66
	EM	0.25	0.32	0.75	0.68	0.59	0.65
	Quantum BAE	0.21	0.33	0.79	0.68	0.61	0.63
	Markov BA	0.23	0.31	0.77	0.77	0.66	0.66
Average	Obs	0.22	0.36	0.78	0.65	0.58	0.63
	ED	0.22	0.35	0.78	0.61	0.55	0.63
	EM	0.23	0.37	0.77	0.63	0.57	0.64
	Quantum BAE	0.2	0.37	0.8	0.65	0.59	0.63
	Markov BA	0.21	0.39	0.79	0.64	0.59	0.58

5.2. Model result

With the basis of the ED model and determined parameters, a series of mass functions and probabilities could be obtained, as shown in Table 3 (second line in each block). It is easy to find that the model application results are very close to the observed results. Hence, we can conclude that the ED model is effective in predicting the interference effect.

5.3. Sensitivity analysis

Sensitivity analysis is a useful method to investigate the validity of simulation or a model. Here, we perform the sensitivity analysis to the ED model in five aspects. We observe the prediction of the averaged practical results (first line of 6th block in Table 3) when: (a) the variable $P_0(G)$ varies $\pm 5\%$ while other variables remain the same; (b) the variable $P_0(A|G)$ varies $\pm 5\%$ while other variables remain the same; (c) the variable $P_0(B)$ varies $\pm 5\%$ while other variables remain the same; (d) the variable $P_0(A|B)$ varies $\pm 5\%$ while other variables remain the same; (e) the variable $P_0(A)$ varies $\pm 5\%$ while other variables remain the same (we ignore P_{T0} here because it is calculated by other four variables). In Fig. 5 (a–e), the points in red line represent the predicted variables when the corresponding variable increases 5%. The points in pink line which are labeled with + represent the predicted variables in the practical situation. The points in blue line represent the predicted variables when the corresponding variable decreases 5%. The points labeled with * represent the practical results expect for the changing variable. When one variable changes, if the prediction results of other variables are closer to the practical points (labeled with *), the model is relatively more accurate. If the prediction results of other variables are closer to the points in pink line (labeled with +), the model is relatively more stable. Generally speaking, in the categorization decision-making experiment, the prediction can remain relatively accurate when each variable varies. The ED model is relatively less sensitive to $P_0(G)$ and $P_0(A|G)$ since their basic values are small. Expect for the changing variable, the other prediction results only vary slightly (see Fig. 5 (a) and (b)). Conversely, the ED model is relatively more sensitive to $P_0(B)$, $P_0(A|B)$ and $P_0(A)$ whose basic values are large. Not only the corresponding probability varies following the changing variable, other prediction results also vary clearly (see Fig. 5 (c–e)). Specifically, when the variables change drastically, a different method of dealing with amplitude distribution should be adopted since predicted interference effect is determined by the difference value of Eqs. (32) and (34). On the whole, the sensitivity analysis shows the validity of the model application results.

6. Model comparison

In the following, the comparisons among the ED model, a Markov belief-action (BA) model, a quantum belief-action entanglement (BAE) model and an evidential Markov (EM) model are performed. Townsend et al. [9] proposed a Markov BA model to do category-decision task. The model assumes that the categorization and decision-making are two parts in the chain, namely the categorization only depends on the face while the action only depends on the categorization (like in Fig. 6). In the C-D condition, the probability of attacking equals $P(G) \cdot P(A|G)$ given that the face is categorized as a good one and equals $P(B) \cdot P(A|B)$ given that the face is categorized as a bad one. In the D alone condition, the probability of attacking equals the probability of reaching a final state A by two different paths $P(A) = P(G) \cdot P(A|G) + P(B) \cdot P(A|B)$. Hence, the Markov BA model follows the law of total probability,

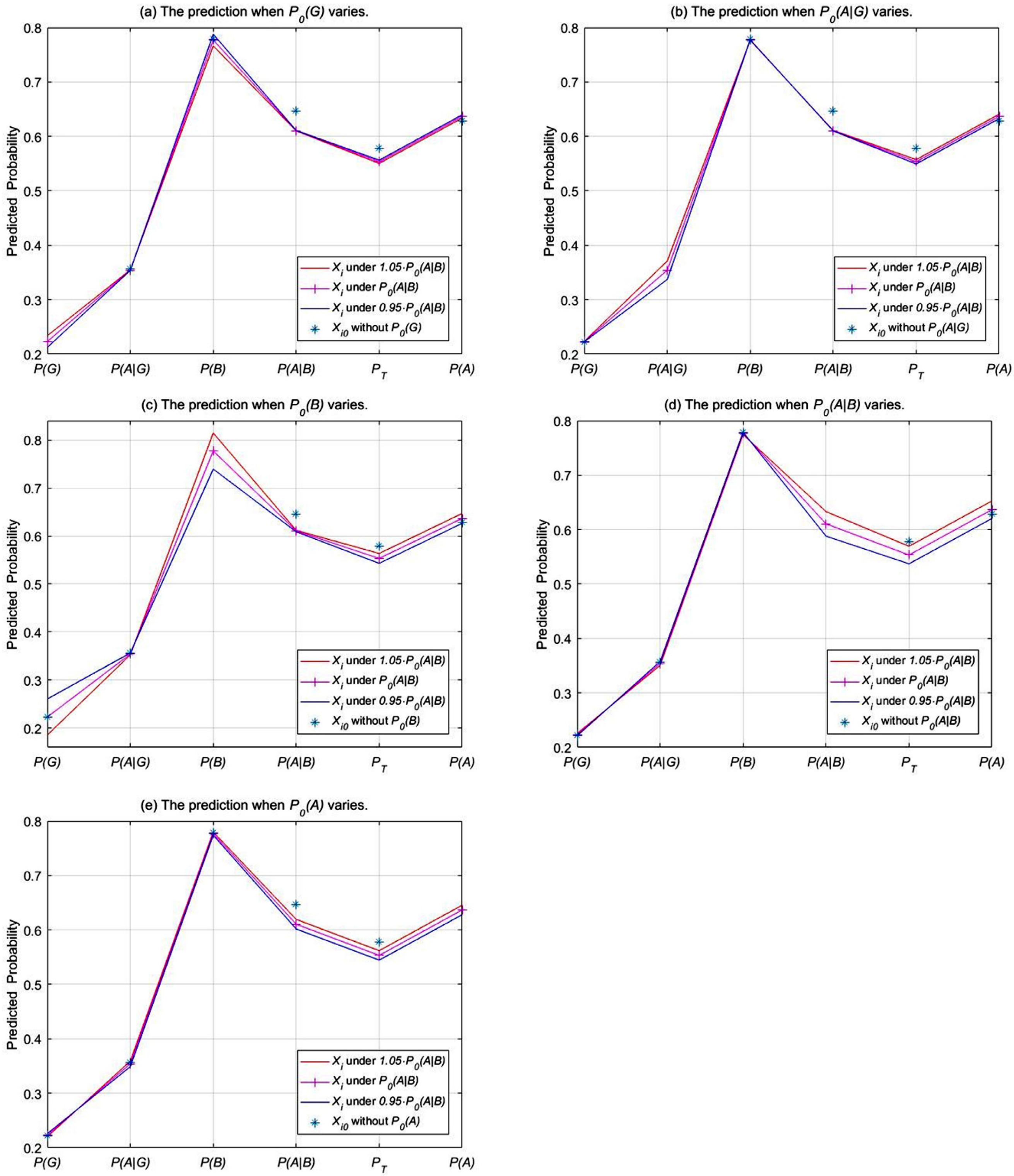

Fig. 5. The prediction when certain variable varies.

which means that no interference effect will be produced. There are four free parameters in the Markov BA model, including one probability, two payoff parameters and one belief transferring parameter (see the detail in [7]).

With the basis of quantum dynamical model, Wang and Busemeyer [8] proposed a quantum BAE model. It defines a free parameter c to describe the entanglement degree of beliefs and actions, which is the crucial factor for producing the interference effect. In the quantum BAE model, the unitary matrix in Eq. (4) is $e^{-i(H_1+H_2)t}$, where H_1 and H_2 are two 4×4 Hamiltonian matrixes. Just like in the ED model, H_1 serves as the payoffs which leads the belief transition.

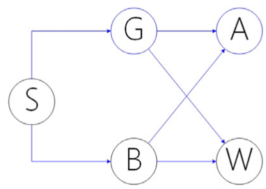

Fig. 6. The decision-making process of the Markov BA model.

$$H_1 = \begin{bmatrix} \frac{-h_G}{\sqrt{1+{h_G}^2}} & \frac{1}{\sqrt{1+{h_G}^2}} & 0 & 0 \\ \frac{1}{\sqrt{1+{h_G}^2}} & \frac{h_G}{\sqrt{1+{h_G}^2}} & 0 & 0 \\ 0 & 0 & \frac{h_B}{\sqrt{1+{h_B}^2}} & \frac{1}{\sqrt{1+{h_B}^2}} \\ 0 & 0 & \frac{1}{\sqrt{1+{h_B}^2}} & \frac{-h_B}{\sqrt{1+{h_B}^2}} \end{bmatrix} \tag{37}$$

H_2 rotates inferences for good face to match withdrawing action and rotates inferences for bad face to match attacking action. Action states G, W and belief states B, A are assumed to be entangled in some degree as decision makers tend to be consistent with their beliefs and actions, which leads to the interference effect.

$$H_2 = \frac{c}{\sqrt{2}} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{38}$$

Hence four free parameters are used in the quantum BAE model: one probability of categorization, two payoff parameters and one entanglement parameter.

An EM model, which is our previous work [44] combines the D-S theory with Markov modelling. Just like the ED model, the action states are extended in an evidential framework in the EM model. Then the uncertain state is handled by a data driven parameter named extra uncertainty degree, which is the key to produce the disjunction effect. The extra uncertainty degree is measured by a belief entropy, named Deng entropy. The key point to predict the interference effect is that participants are assumed to have a tendency to attack as self-defense under uncertain situations. The EM model also has three free parameters: one probability of categorization and two payoff parameters. However, the decision making process is still in a Markov framework, where the state must be certain in one time. The possible states are certain actions or hesitation. To the contrary, in a quantum framework, the state is allowed to be uncertain as the state is a superposition of several certain states. Without an assumption towards action tendency, the ED model is more simpler and reasonable since the uncertain state is handled based on the process of decision making, rather than driven by a determined uncertainty parameter.

The application results of the above three models are also shown in Table 3. As analyzed before, the interference effect can not be produced by the Markov model, but it can be predicted by the EM model, the ED model and the quantum BAE model. To compare the ability of predicting the interference effect of these models, the Bayesian information criterion (BIC) [77] is adopted, which is defined as follows:

$$BIC = -2\log(L) + k\log(n), \tag{39}$$

where L is a likelihood function, k is the number of parameters and n is the sample size (number of experiment participants). Generally speaking, the model with a lower BIC is preferred. Here, the likelihood function is defined as the accuracy of model prediction results. For the C-D condition and the D alone condition, the likelihood functions are as follows:

$$L_{C-D} = 1 - |P_T(A)_{obs} - P_T(A)_{model}|, \tag{40}$$

$$L_D = 1 - |P(A)_{obs} - P(A)_{model}|. \tag{41}$$

Table 4
The model selection based on BIC.

	Literature	L_{C-D}	L_D	k	n	BIC
Busemeyer et al. [7]	ED	1	0.98	3	26	8.95
	EM	0.98	1	3	26	8.95
	Quantum BAE	0.97	0.99	4	26	12.23
	Markov BA	1	0.9	4	26	13.7
Wang and Busemeyer Experiment 1 [8]	ED	0.98	0.98	3	169	19.30
	EM	0.99	0.99	3	169	16.32
	Quantum BAE	0.98	0.98	4	169	23.75
	Markov BA	0.99	0.96	4	169	25.29
Wang and Busemeyer Experiment 2 [8]	ED	0.98	0.99	3	286	22.25
	EM	0.99	0.99	3	286	19.73
	Quantum BAE	0.94	0.97	4	286	42.59
	Markov BA	0.95	0.99	4	286	34.89
Wang and Busemeyer Experiment 3(a) [8]	ED	0.97	0.99	3	129	17.2
	EM	1	0.98	3	129	14.93
	Quantum BAE	0.97	0.99	4	129	21.42
	Markov BA	0.95	0.91	4	129	33.2
Wang and Busemeyer Experiment 3(b) [8]	ED	0.94	0.98	3	137	22.59
	EM	0.96	0.99	3	137	23.12
	Quantum BAE	0.98	0.99	4	137	20.69
	Markov BA	0.97	0.98	4	137	23.12
Average	ED	0.97	1	3	149.4	17
	EM	0.99	0.99	3	149.4	15.65
	Quantum BAE	0.98	1	4	149.4	18.7
	Markov BA	0.99	0.95	4	149.4	25.36

Then the overall BIC of a model is the sum of two conditions here. It is evident from Table 4 that, although the original quantum model has relatively higher prediction accuracy, two of the evidential models perform better in BIC since they have less parameters. With regard to the comparison of two evidential models, the ED model is simpler and more rational. The crucial point to predict the interference effect in the case of using D alone condition without categorization in the ED model deals with preserving production of the intermediate mass functions. However, the EM model is based on an extra assumption and driven by an parameter measured by a belief entropy, which is named the extra uncertainty degree. By contrast, the ED model is more reasonable and convincing. It offers a new inspiring framework, which focuses on the decision making process, for studying human decision making and the categorization effect.

7. Conclusion

As an important task in knowledge-based systems, categorization is the basis of many decisions. However, some cases show that categorization may also result in disjunction fallacy, which breaks the law of total probability and is inexplainable in the classic probability theory. In this paper, to predict the disjunction effects (also called as the interference effect in quantum theory) of categorizing on decision making results, an evidential dynamical model is proposed. The ED model is based on Dempster-Shafer evidence theory and quantum dynamical modelling. To model the hesitation of decision makers, the action states are extended by introducing an uncertain state in an evidential framework. In the classical quantum BAE model, the interference effect is produced due to the entanglement of beliefs and actions. However, the entanglement degree is determined artificially. In the ED model, the interference effect of categorization is measured by handling the uncertain state in actions. Although an extended state seems to increase the complexity of the model, the ED model has less free parameters on the contrary. The model is applied to classical categorization decision-making experiments. The predicted results are relatively accurate and the comparisons with other models show the effectiveness and efficiency of our model. Not only does the ED model help understand the interference effect of categorization, but also offers an inspiring framework for human decision making modelling. The ED model here is applied in an experiment paradigm in this paper, but we believe that it will bring more opportunities and will result in more applications in knowledge-based systems, like a decision support system, especially when some counterintuitive knowledge exists. Admittedly, some open issues still exist in our model, like why no interference effect is produced for wide faces. In future research, we would like to design a practical experiment to verify our model. We are also going to apply the ED model in wider fields and excavate the uncertain state in a decision making process.

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