Recurrent Hierarchical Topic-Guided RNN for Language Generation

Dandan Guo [1] **Bo Chen** [1] **Ruiying Lu** [1] **Mingyuan Zhou** [2]

Abstract

To simultaneously capture syntax and global semantics from a text corpus, we propose a new larger-context recurrent neural network (RNN) based language model, which extracts recurrent hierarchical semantic structure via a dynamic deep topic model to guide natural language generation. Moving beyond a conventional RNN-based language model that ignores long-range word dependencies and sentence order, the proposed model captures not only intra-sentence word dependencies, but also temporal transitions between sentences and inter-sentence topic dependencies. For inference, we develop a hybrid of stochastic-gradient Markov chain Monte Carlo and recurrent autoencoding variational Bayes. Experimental results on a variety of real-world text corpora demonstrate that the proposed model not only outperforms larger-context RNN-based language models, but also learns interpretable recurrent multilayer topics and generates diverse sentences and paragraphs that are syntactically correct and semantically coherent.

1. Introduction

Both topic and language models are widely used for text analysis. Topic models, such as latent Dirichlet allocation (LDA) (Blei et al., 2003; Griffiths & Steyvers, 2004; Hoffman et al., 2013) and its nonparametric Bayesian generalizations (Teh et al., 2006; Zhou & Carin, 2015), are well suited for extracting document-level word concurrence patterns into latent topics from a text corpus. Their modeling power has been further enhanced by introducing multilayer deep representation (Srivastava et al., 2013; Mnih & Gregor, 2014; Gan et al., 2015; Zhou et al., 2016; Zhao et al., 2018; Zhang et al., 2018). While having semantically meaningful

[1]National Laboratory of Radar Signal Processing, Xidian University, Xi'an, China. [2]McCombs School of Business, The University of Texas at Austin, Austin, TX 78712, USA. Correspondence to: Bo Chen <bchen@mail.xidian.edu.cn>.

latent representation, they typically treat each document as a bag of words (BoW), ignoring word order (Griffiths et al., 2004; Wallach, 2006). To take the word order into consideration, Wang et al. (2019a) introduce a customized convolutional operator and probabilistic pooling into a topic model, which successfully captures local dependencies and forms phrase-level topics but has limited ability in modeling sequential dependencies and generating word sequences.

Language models have become key components of various natural language processing tasks, such as text summarization (Rush et al., 2015; Gehrmann et al., 2018), speech recognition (Mikolov et al., 2010; Graves et al., 2013), machine translation (Sutskever et al., 2014; Cho et al., 2014), and image captioning (Vinyals et al., 2015; Mao et al., 2015; Xu et al., 2015; Gan et al., 2017; Rennie et al., 2017; Fan et al., 2020). The primary purpose of a language model is to capture the distribution of a word sequence, commonly with a recurrent neural network (RNN) (Mikolov et al., 2011; Graves, 2013) or a Transformer-based model (Vaswani et al., 2017; Dai et al., 2019; Devlin et al., 2019; Radford et al., 2018; 2019). In this paper, utilizing a deep dynamic model for sequentially observed count vectors and introducing a recurrent variational inference network, we focus on improving RNN-based language models that often have much fewer parameters and are easier to perform end-to-end training.

While RNN-based language models do not ignore word order, they often assume that the sentences of a document are independent of each other. This simplifies the modeling task to independently assigning probabilities to individual sentences, ignoring their order and document context (Tian & Cho, 2016). Such language models may consequently fail to capture the long-range dependencies and global semantic meaning of a document (Dieng et al., 2017; Wang et al., 2018). While a naive solution to explore richer contextual information is to concatenate all previous sentences into a single "sentence" and use it as the input of an RNN-based language model, in practice, the length of that sentence is limited given the constraint of memory and computation resource. Even if making the length very long, this naive solution rarely works well enough to satisfactorily address the long-standing research problem of capturing long-range dependencies, motivating a variety of more sophisticated methods to improve existing language models (Dieng et al., 2017; Lau et al., 2017; Wang et al., 2018; 2019b; Dai et al.,

2019). Moreover, such a solution often clearly enlarges the model size, increasing the difficulty of optimization and risk of overfitting (Dieng et al., 2017). Finding better ways to model long-range dependencies in language modeling is therefore an open research challenge. To relax the sentence independence assumption in language modeling, Tian & Cho (2016) propose larger-context language models that model the context of a sentence by representing its preceding sentences as either a single or a sequence of BoW vectors, which are then fed directly into the sentence modeling RNN.

Since topic models are well suited for capturing long-range dependencies, an alternative approach attracting significant recent interest is leveraging topic models to improve RNN-based language models. Mikolov & Zweig (2012) use pre-trained topic model features as an additional input to the RNN hidden states and/or output. Dieng et al. (2017) and Ahn et al. (2017) combine the predicted word distributions, given by both a topic model and a language model, under variational autoencoder (Kingma & Welling, 2013). Lau et al. (2017) introduce an attention based convolutional neural network to extract semantic topics, which are used to extend the RNN cell. Wang et al. (2018) learn the global semantic coherence of a document via a neural topic model and use the learned latent topics to build a mixture-of-experts language model. Wang et al. (2019b) further specify a Gaussian mixture model as the prior of the latent code in variational autoencoder, where each mixture component corresponds to a topic.

While clearly improving the performance of the end task, these existing topic-guided methods still have clear limitations. For example, they only utilize shallow topic models with only a single stochastic hidden layer for data generation. Note several neural topic models use deep neural networks to construct their variational encoders, but still use shallow generative models as decoders (Miao et al., 2017; Srivastava & Sutton, 2017). Another key limitation lies in ignoring the sentence order, as each document is treated as a bag of sentences. Thus once the topic weight vector learned from the document context is given, the task is often reduced to independently assigning probabilities to individual sentences (Lau et al., 2017; Wang et al., 2018; 2019b).

In this paper, as depicted in Fig. 1, we propose to use recurrent gamma belief network (rGBN) to guide a stacked RNN for language modeling. We refer to the model as rGBN-RNN, which integrates rGBN (Guo et al., 2018), a deep recurrent topic model, and stacked RNN (Graves, 2013; Chung et al., 2017), a neural language model, into a novel larger-context RNN-based language model. It simultaneously learns a deep recurrent topic model, extracting document-level multi-layer word concurrence patterns and sequential topic weight vectors for sentences, and an expressive language model, capturing both short- and long-

range word sequential dependencies. For inference, we equip rGBN-RNN (decoder) with a novel recurrent variational inference network (encoder), and train it end-to-end by maximizing an evidence lower bound (ELBO). Different from the stacked RNN based language model in Chung et al. (2017), which relies on three types of customized training operations (UPDATE, COPY, FLUSH) to extract multi-scale structures, the language model in rGBN-RNN learns such structures purely in a data driven manner, under the guidance of the temporally and hierarchically connected stochastic layers of rGBN. Note while both rGBN and stacked-RNN are existing methods, integrating them as a larger-context language model involves non-trivial efforts, as we need to not only carefully design how to connect the recurrent hierarchical stochastic layers of rGBN with the deterministic ones of stacked-RNN, but also design a suitable recurrent variational inference network.

The effectiveness of rGBN-RNN as a new larger-context language model is demonstrated both quantitatively, with perplexity and BLEU scores, and qualitatively, with interpretable latent structures and randomly generated sentences and paragraphs. Notably, moving beyond a usual RNN-based language model that generates individual sentences, the proposed rGBN-RNN can generate a paragraph consisting of a sequence of semantically coherent sentences.

2. Recurrent Hierarchical Topic-Guided Language Model

Denote a document of J sentences as $\mathcal{D} = (S_1, S_2, \ldots, S_J)$, where $S_j = (y_{j,1}, \ldots, y_{j,T_j})$ consists of T_j words from a vocabulary of size V. Conventional statistical language models often only focus on the word sequence within a sentence. Assuming that the sentences of a document are independent of each other, they often define

$$
\begin{aligned}
P(\mathcal{D}) &\approx \prod_{j=1}^{J} P(S_j) \\
&= \prod_{j=1}^{J} \left[p(y_{j,1}) \prod_{t=2}^{T_j} p(y_{j,t} \mid y_{j,<t}) \right].
\end{aligned}
$$

RNN-based neural language models define the conditional probability of each word $y_{j,t}$ given all the previous words $y_{j,<t}$ within the sentence S_j, through the softmax function of a hidden state $\boldsymbol{h}_{j,t}$, as

$$
\begin{aligned}
p(y_{j,t} \mid y_{j,<t}) &= p(y_{j,t} \mid \boldsymbol{h}_{j,t}), \\
\boldsymbol{h}_{j,t} &= f(\boldsymbol{h}_{j,<t}, y_{j,t-1}),
\end{aligned} \tag{1}
$$

where $f(\cdot)$ is a non-linear function typically defined as an RNN cell, such as long short-term memory (LSTM) (Hochreiter & Schmidhuber, 1997) and gated recurrent unit (GRU) (Cho et al., 2014).

These RNN-based language models are typically applied only at the word level, without exploiting the document

context, and hence often fail to capture long-range dependencies. While Dieng et al. (2017), Lau et al. (2017), and Wang et al. (2018; 2019b) remedy this issue by guiding the language model with a topic model, they still treat a document as a bag of sentences, ignoring sentence order, and lack the ability to extract hierarchical and recurrent topic structures.

We introduce rGBN-RNN, as depicted in Fig. 1 (a), as a new larger-context language model. It consists of two key components: (i) a hierarchical recurrent topic model (rGBN), and (ii) a stacked RNN-based language model. We use rGBN to capture both global semantics across documents and long-range inter-sentence dependencies within a document, and use the language model to learn the local syntactic relationships between the words within a sentence. Similar to Lau et al. (2017) and Wang et al. (2018), we represent a document as a sequence of sentence-context pairs as $(\{S_1, \boldsymbol{d}_1\}, \dots, \{S_J, \boldsymbol{d}_J\})$, where $\boldsymbol{d}_j \in \mathbb{Z}_+^{V_c}$ summarizes the document excluding S_j, specifically $(S_1, ..., S_{j-1}, S_{j+1}, ..., S_J)$, into a BoW count vector, with V_c denoting the size of the vocabulary excluding stop words. During testing, we redefine $\boldsymbol{d}_j$ as the BoW vector summarizing only the preceding sentences, *i.e.*, $S_{1:j-1}$, which will be further clarified when presenting experimental results. Note a naive way to utilize sentence order is to treat each sentence as a document, use a dynamic topic model (Blei & Lafferty, 2006) to capture the temporal dependencies of the latent topic-weight vectors, each of which is fed to the RNN to model the word sequence of its corresponding sentence. However, the sentences are often too short to be well modeled by a topic model. In our setting, as $\boldsymbol{d}_j$ summarizes the document-level context of S_j, it is in general sufficiently long for topic modeling.

2.1. Hierarchical Recurrent Topic Model

As shown in Fig. 1 (a), to model the time-varying sentence-context count vectors $\boldsymbol{d}_j$ in document $\mathcal{D}$, the generative process of the rGBN component, from the top to bottom layers, is expressed as

$$\boldsymbol{\theta}_j^L \sim \text{Gam}\left(\boldsymbol{\Pi}^L \boldsymbol{\theta}_{j-1}^L,\ \tau_0\right), \cdots,$$
$$\boldsymbol{\theta}_j^l \sim \text{Gam}\left(\boldsymbol{\Phi}^{l+1} \boldsymbol{\theta}_j^{l+1} + \boldsymbol{\Pi}^l \boldsymbol{\theta}_{j-1}^l,\ \tau_0\right), \cdots,$$
$$\boldsymbol{\theta}_j^1 \sim \text{Gam}\left(\boldsymbol{\Phi}^2 \boldsymbol{\theta}_j^2 + \boldsymbol{\Pi}^1 \boldsymbol{\theta}_{j-1}^1,\ \tau_0\right),$$
$$\boldsymbol{d}_j \sim \text{Pois}\left(\boldsymbol{\Phi}^1 \boldsymbol{\theta}_j^1\right), \quad (2)$$

where $\boldsymbol{\theta}_j^l \in \mathbb{R}_+^{K_l}$ denotes the gamma distributed topic weight vector of sentence j at layer $l \in \{1, \dots, L\}$, $\boldsymbol{\Pi}^l \in \mathbb{R}_+^{K_l \times K_l}$ the transition matrix of layer l that captures cross-topic temporal dependencies, $\boldsymbol{\Phi}^l \in \mathbb{R}_+^{K_{l-1} \times K_l}$ the loading matrix at layer l, K_l the number of topics of layer l, and $\tau_0 \in \mathbb{R}_+$ a scaling hyperparameter. At $j = 1$, $\boldsymbol{\theta}_1^l \sim \text{Gam}\left(\boldsymbol{\Phi}^{l+1} \boldsymbol{\theta}_1^{l+1}, \tau_0\right)$ for $l = 1, \dots, L-1$

and $\boldsymbol{\theta}_1^L \sim \text{Gam}\left(\nu, \tau_0\right)$, where $\nu = \mathbf{1}_{K_L}$. Following Guo et al. (2018) and Zhou et al. (2015), the Dirichlet priors are placed on the columns of $\boldsymbol{\Pi}^l$ and $\boldsymbol{\Phi}^l$, *i.e.*, $\boldsymbol{\pi}_k^l$ and $\boldsymbol{\phi}_k^l$, which not only makes the latent representation more identifiable and interpretable, but also facilitates inference. The count vector $\boldsymbol{d}_j$ can be factorized into the product of $\boldsymbol{\Phi}^1$ and $\boldsymbol{\theta}_j^1$ under the Poisson likelihood. The shape parameters of $\boldsymbol{\theta}_j^l \in \mathbb{R}_+^{K_l}$ can be factorized into the sum of $\boldsymbol{\Phi}^{l+1} \boldsymbol{\theta}_j^{l+1}$, capturing inter-layer hierarchical dependence, and $\boldsymbol{\Pi}^l \boldsymbol{\theta}_{j-1}^l$, capturing intra-layer temporal dependence. rGBN not only captures the document-level word occurrence patterns inside the training text corpus, but also the sequential dependencies of the sentences inside a document. Note ignoring the recurrent structure, rGBN will reduce to the gamma belief network (GBN) of Zhou et al. (2016), which can be reformulated as a multi-stochastic-layer deep generalization of LDA (Cong et al., 2017a); if setting the number of stochastic hidden layer as $L = 1$, GBN reduces to Poisson factor analysis (Zhou et al., 2012; Zhou & Carin, 2015) . If ignoring its hierarchical structure (*i.e.*, $L = 1$), rGBN reduces to Poisson–gamma dynamical systems of Schein et al. (2016) that generalizes the gamma Markov chain of Acharya et al. (2015) by adding latent state transitions. We refer to the rGBN-RNN without its recurrent structure as GBN-RNN, which no longer models sequential sentence dependencies; see Appendix A for more details.

2.2. Language Model

Different from a conventional RNN-based language model, which predicts the next word only using the preceding words within the sentence, we integrate the hierarchical recurrent topic weight vectors $\boldsymbol{\theta}_j^l$ into the language model to predict the word sequence in the jth sentence. Our proposed language model is built upon the stacked RNN proposed in Graves (2013) and Chung et al. (2017), but with the help of rGBN, it no longer requires specialized training heuristics to extract multi-scale latent structures. As shown in Fig. 1 (b), to generate $y_{j,t}$, the tth token of sentence j in a document, we construct the hidden states $\boldsymbol{h}_{j,t}^l$ of the language model, from the bottom to top layers, as

$$\boldsymbol{h}_{j,t}^l = \begin{cases} \text{LSTM}_{\text{word}}^l \left(\boldsymbol{h}_{j,t-1}^l, \boldsymbol{W_e}\left[x_{j,t}\right]\right), & \text{if } l = 1, \\ \text{LSTM}_{\text{word}}^l \left(\boldsymbol{h}_{j,t-1}^l, \boldsymbol{a}_{j,t}^{l-1}\right), & \text{if } 1 < l \leq L, \end{cases} \quad (3)$$

where $\text{LSTM}_{\text{word}}^l$ denotes the word-level LSTM at layer l, $\boldsymbol{W_e}$ are word embeddings to be learned, and $x_{j,t} = y_{j,t-1}$. Note $\boldsymbol{a}_{j,t}^l$ denotes the coupling vector, which combines the temporal topic weight vectors $\boldsymbol{\theta}_j^l$ and hidden output of the word-level LSTM $\boldsymbol{h}_{j,t}^l$ at each time step t. Following Lau et al. (2017), we realize $\boldsymbol{a}_{j,t}^l = g^l\left(\boldsymbol{h}_{j,t}^l, \boldsymbol{\theta}_j^l\right)$ with a gating unit similar to a GRU (Cho et al., 2014), described as

$$\boldsymbol{a}_{j,t}^l = \left(1 - \boldsymbol{z}_{j,t}^l\right) \odot \boldsymbol{h}_{j,t}^l + \boldsymbol{z}_{j,t}^l \odot \hat{\boldsymbol{h}}_{j,t}^l, \quad (4)$$
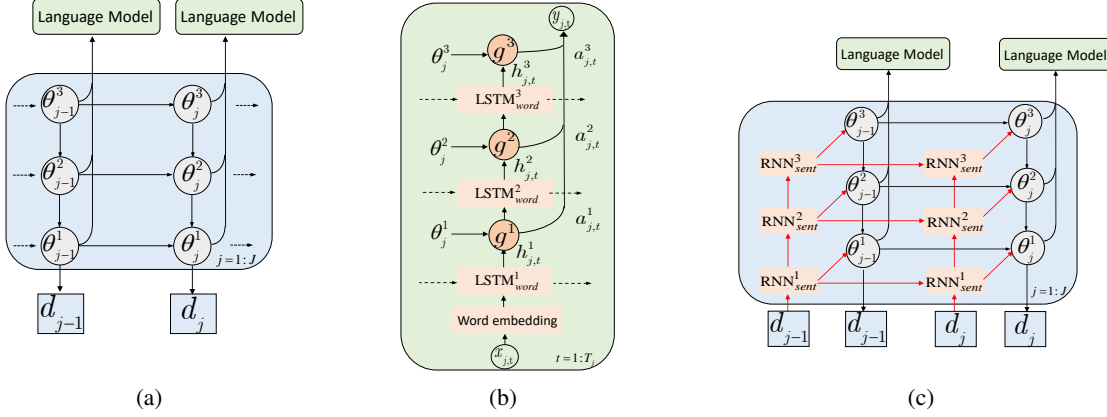
Figure 1. (a) The generative model of a three-hidden-layer rGBN-RNN, where the bottom part is the deep recurrent topic model (rGBN), document contexts of consecutive sentences are used as observed data, and upper is the language model. (b) Overview of the language model component, where input $x_{j,t}$ denotes the tth word in jth sentence of a document, $x_{j,t} = y_{j,t-1}$, $h_{j,t}^l$ is the hidden state of the stacked RNN at time step t, and θ_j^l is the topic weight vector of sentence j at layer l. (c) The overall architecture of the proposed model, including the decoder (rGBN and language model) and encoder (recurrent variational inference network), where the red arrows denote the inference of latent topic weight vectors and the black arrows denote the data generation.

where

$$z_{j,t}^l = \sigma\left(\mathbf{W}_z^l \theta_j^l + \mathbf{U}_z^l h_{j,t}^l + \mathbf{b}_z^l\right),$$
$$r_{j,t}^l = \sigma\left(\mathbf{W}_r^l \theta_j^l + \mathbf{U}_r^l h_{j,t}^l + \mathbf{b}_r^l\right),$$
$$\hat{h}_{j,t}^l = \tanh\left(\mathbf{W}_h^l \theta_j^l + \mathbf{U}_h^l \left(r_{j,t}^l \odot h_{j,t}^l\right) + \mathbf{b}_h^l\right).$$

Denote $a_{j,t}^{1:L}$ as the concatenation of $a_{j,t}^l$ across all layers and $\mathbf{W}_o$ as a weight matrix with V rows; different from (1), the conditional probability of $y_{j,t}$ becomes

$$p\left(y_{j,t} \,|\, y_{j,<t}, \theta_j^{1:L}\right) = \text{softmax}\left(\mathbf{W}_o a_{j,t}^{1:L}\right). \quad (5)$$

There are two main reasons for combining all the latent representations $a_{j,t}^{1:L}$ for language modeling. First, the latent representations exhibit different statistical properties at different stochastic layers of rGBN-RNN, and hence are combined together to enhance their representation power. Second, having "skip connections" from all hidden layers to the output one makes it easier to train the proposed network, reducing the number of processing steps between the bottom of the network and the top and hence mitigating the "vanishing gradient" problem (Graves, 2013).

To sum up, as depicted in Fig. 1 (a), the topic weight vector θ_j^l of sentence j quantifies the topic usage of its document context d_j at layer l. It is further used as an additional feature of the language model to guide the word generation inside sentence j, as shown in Fig. 1 (b). It is clear that rGBN-RNN has two temporal structures: a deep recurrent topic model to extract the temporal topic weight vectors from the sequential document contexts, and a language model to estimate the probability of each sentence given its corresponding hierarchical topic weight vector. Characterizing the word-sentence-document hierarchy to incorporate both intra- and inter-sentence information, rGBN-RNN

learns more coherent and interpretable topics and increases the generative power of the language model. Distinct from existing topic-guided language models, the temporally related hierarchical topics of rGBN exhibit different statistical properties across layers, which helps better guide language model to improve its language generation ability.

2.3. Model Likelihood and Inference

For rGBN-RNN, given $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^L$, the marginal likelihood of the sequence of sentence-context pairs $(\{s_1, d_1\}, \ldots, \{s_J, d_J\})$ of document $\mathcal{D}$ is defined as

$$P\left(\mathcal{D} \,|\, \{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^L\right) = \int \prod_{j=1}^J p\left(d_j \,|\, \boldsymbol{\Phi}^1 \theta_j^1\right)$$
$$\left[\prod_{t=1}^{T_j} p\left(y_{j,t} \,|\, y_{j,<t}, \theta_j^{1:L}\right)\right]\left[\prod_{l=1}^L p\left(\theta_j^l \,|\, e_j^l, \tau_0\right)\right] d\theta_{1:J}^{1:L}, \quad (6)$$

where $e_j^l := \boldsymbol{\Phi}^{l+1}\theta_j^{l+1} + \boldsymbol{\Pi}^l \theta_{j-1}^l$. The inference task is to learn the parameters of both the topic model and language model components. One naive solution is to alternate the training between these two components in each iteration: First, the topic model is trained using a sampling based iterative algorithm provided in Guo et al. (2018); Second, the language model is trained with maximum likelihood estimation under a standard cross-entropy loss. While this naive solution can utilize readily available inference algorithms for both rGBN and the language model, it may suffer from stability and convergence issues. Moreover, the need to perform a sampling based iterative algorithm for rGBN inside each iteration limits the scalability of the model for both training and testing.

To this end, we introduce a recurrent variational inference network (encoder) to learn the latent temporal topic weight vectors $\theta_{1:J}^{1:L}$. Denoting $Q = \prod_{j=1}^J \prod_{l=1}^L q(\theta_j^l \,|\, d_j)$, an

ELBO of the log marginal likelihood shown in (6) can be constructed as

$$L = \sum_{j=1}^{J} \sum_{l=1}^{L} \mathbb{E}_Q \left[\ln p\left(\boldsymbol{d}_j \mid \boldsymbol{\Phi}^1 \boldsymbol{\theta}_j^1 \right) + \sum_{t=1}^{T_j} \ln p\left(y_{j,t} \mid y_{j,<t}, \boldsymbol{\theta}_j^{1:L} \right) \right]$$
$$- \sum_{j=1}^{J} \sum_{l=1}^{L} \mathbb{E}_Q \left[\ln \frac{q(\boldsymbol{\theta}_j^l \mid \boldsymbol{d}_{\leq j})}{p(\boldsymbol{\theta}_j^l \mid \boldsymbol{e}_j^l, \tau_0)} \right], \tag{7}$$

which unites both the terms primarily responsible for training the recurrent hierarchical topic model component, and terms for training the RNN language model component. Similar to Zhang et al. (2018), we define $q(\boldsymbol{\theta}_j^l \mid \boldsymbol{d}_j) =$ Weibull$(\boldsymbol{k}_j^l, \boldsymbol{\lambda}_j^l)$, a random sample from which can be obtained by transforming standard uniform noises $\boldsymbol{\epsilon}_j^l$ as

$$\boldsymbol{\theta}_j^l = \boldsymbol{\lambda}_j^l \left(-\ln(1 - \boldsymbol{\epsilon}_j^l) \right)^{1/\boldsymbol{k}_j^l}. \tag{8}$$

To capture the temporal dependencies between the topic weight vectors, both $\boldsymbol{k}_j^l$ and $\boldsymbol{\lambda}_j^l$, from the bottom to top layers, can be expressed as

$$\boldsymbol{h}_j^{s,l} = \text{RNN}_{\text{sent}}^l \left(\boldsymbol{h}_{j-1}^{s,l}, \boldsymbol{h}_j^{s,l-1} \right),$$
$$\boldsymbol{k}_j^l = f_{\boldsymbol{k}}^l(\boldsymbol{h}_j^{s,l}), \quad \boldsymbol{\lambda}_j^l = f_{\boldsymbol{\lambda}}^l(\boldsymbol{h}_j^{s,l}), \tag{9}$$

where $\boldsymbol{h}_j^{s,0} = \boldsymbol{d}_j$, $\boldsymbol{h}_0^{s,l} = 0$, $\text{RNN}_{\text{sent}}^l$ denotes the sentence-level recurrent encoder at layer l implemented with a basic RNN cell, capturing the sequential relationship between sentences within a document, $\boldsymbol{h}_j^{s,l}$ denotes the hidden state of $\text{RNN}_{\text{sent}}^l$, and superscript s in $\boldsymbol{h}_j^{s,l}$ denotes "sentence-level RNN" used to distinguish the hidden state of language model in (3) . Note both $f_{\boldsymbol{k}}^l$ and $f_{\boldsymbol{\lambda}}^l$ are nonlinear functions mapping state $\boldsymbol{h}_j^{s,l}$ to the parameters of $\boldsymbol{\theta}_j^l$, implemented with $f(\boldsymbol{x}) = \ln(1 + \exp(\mathbf{W}\boldsymbol{x} + \boldsymbol{b}))$.

Rather than finding a point estimate of the global parameters $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^{L}$ of the rGBN, we adopt a hybrid inference algorithm by combining TLASGR-MCMC described in Cong et al. (2017a) and Zhang et al. (2018) and our proposed recurrent variational inference network. In other words, the global parameters $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^{L}$ can be sampled with TLASGR-MCMC, while the parameters of the language model and recurrent variational inference network, denoted by $\boldsymbol{\Omega}$, can be updated via stochastic gradient descent (SGD) by maximizing the ELBO in (7). We describe a hybrid variational/sampling inference for rGBN-RNN in Algorithm 1 and provide more details about sampling $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^{L}$ with TLASGR-MCMC in Appendix B. We defer the details on model complexity to Appendix D.

To sum up, as shown in Fig. 1 (c), the proposed rGBN-RNN works with a recurrent variational autoencoder inference framework, which takes the document context of the jth sentence within a document as input and learns hierarchical topic weight vectors $\boldsymbol{\theta}_j^{1:L}$ that evolve sequentially with j. The learned topic vectors in different layer are then used to reconstruct the document context input and as an additional feature for the language model to generate the jth sentence.

Algorithm 1 Hybrid TLASGR-MCMC and recurrent autoencoding variational inference for rGBN-RNN.

Set mini-batch size m and the number of layer L
Initialize encoder and neural language model parameters $\boldsymbol{\Omega}$, and topic model parameters $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^{L}$.
for $iter = 1, 2, \cdots$ **do**
    Randomly select a mini-batch of m documents consisting of J sentences to form a subset $\mathbf{X} = \{\boldsymbol{d}_{i,1:J}, \boldsymbol{s}_{i,1:J}\}_{i=1}^{m}$;
    Draw random noise $\{\boldsymbol{\epsilon}_{i,j}^l\}_{i=1,j=1,l=1}^{m,J,L}$ from uniform distribution;
    Calculate $\nabla_{\boldsymbol{\Omega}} L\left(\boldsymbol{\Omega}, \boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l; \mathbf{X}, \boldsymbol{\epsilon}_{i,j}^l\right)$ according to (7), and update $\boldsymbol{\Omega}$; Sample $\boldsymbol{\theta}_{i,j}^l$ from (8) and (9) via $\boldsymbol{\Omega}$ to update $\{\boldsymbol{\Pi}^l\}_{l=1}^{L}$ and $\{\boldsymbol{\Phi}^l\}_{l=1}^{L}$, as described in Appendix B;
end for

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3. Experimental Results

We consider three publicly available corpora, including APNEWS, IMDB, and BNC. The links, preprocessing steps, and summary statistics for them are deferred to Appendix C. We consider a recurrent variational inference network for rGBN-RNN to infer $\boldsymbol{\theta}_j^l$, as shown in Fig. 1 (c), whose number of hidden units in (9) are set the same as the number of topics at the corresponding layer. Following Lau et al. (2017), word embeddings are pre-trained 300-dimension word2vec Google News vectors (https://code.google.com/archive/p/word2vec/). Dropout with a rate of 0.4 is used to the input of the stacked-RNN at each layer, i.e., $\boldsymbol{a}_{j,t}^l$ or $\mathbf{W}_e[x_{j,t}]$ in (3). The gradients are clipped if the norm of the parameter vector exceeds 5. We use the Adam optimizer (Kingma & Ba, 2015) with learning rate 10^{-3}. The length of an input sentence is fixed to 30. We set the mini-batch size as 8, number of training epochs as 5, and τ_0 as 1. Python (TensorFlow) code is provided at https://github.com/Dan123dan/rGBN-RNN

3.1. Quantitative Comparison

Perplexity: For fair comparison, we use standard language model perplexity as the evaluation metric. We consider the following baselines: *1)* A standard LSTM language model (Hochreiter & Schmidhuber, 1997); *2)* LCLM (Tian & Cho, 2016), a larger-context language model that incorporates context from preceding sentences, which are treated as a bag of words; *3)* A standard LSTM language model incorporating the topic information of a separately trained LDA (LDA+LSTM); *4)* Topic-RNN (Dieng et al., 2017), a hybrid model rescoring the prediction of the next word by incorporating the topic information through a linear transformation; *5)* TDLM (Lau et al., 2017), a joint learning framework that learns a convolution based topic model and a language model simultaneously; *6)* TCNLM (Wang et al., 2018), which extracts the global semantic coherence of a document via a neural topic model, with the probability of each learned latent topic further adopted to build

a mixture-of-experts language model; *7)* TGVAE (Wang et al., 2019b), combining a variational auto-encoder based neural sequence model with a neural topic model; *8)* GBN-RNN, a simplified rGBN-RNN that removes the recurrent structure of its rGBN component; *9)* rGBN-RNN-flipped, which is an additional architectural variation of the proposed rGBN-RNN that modifies $\boldsymbol{\theta}_j^3$ and $\boldsymbol{\theta}_j^1$ shown in Fig. 1(b) by swapping their locations; *10)* Transformer-XL (Dai et al., 2019), which enables learning dependency beyond a fixed length by introducing a recurrence mechanism and a novel position encoding scheme into the Transformer architecture; *11)* GPT-2 (Radford et al., 2019), which can be realized by a generative pre-training of a Transformer-based language model on a diverse set of unlabeled text, followed by discriminative fine-tuning on each specific dataset.

For rGBN-RNN, to ensure the information about the words in the jth sentence to be predicted is not leaking through the sequential document context vectors at the testing stage, the input $\boldsymbol{d}_j$ in (9) only summarizes the preceding sentences $S_{<j}$. For GBN-RNN, following TDLM (Lau et al., 2017) and TCNLM (Wang et al., 2018), all the sentences in a document, excluding the one being predicted, are used to obtain the BoW document context. As shown in Table 1, rGBN-RNN outperforms all RNN-based baselines, and the trend of improvement continues as its number of layers increases, indicating the effectiveness of incorporating recurrent hierarchical topic information into language generation. rGBN-RNN consistently outperforms GBN-RNN, suggesting the benefits of exploiting the sequential dependencies of the sentence-contexts for language modeling.

In Table 1, we further compare the number of parameters between various language models, where we follow the convention to ignore the word embedding layers. The number of parameters for some models are not reported, as we could not find sufficient information from their corresponding papers or code to provide accurate estimations. When used for language generation at the testing stage, rGBN-RNN no longer needs its topics $\{\boldsymbol{\Phi}^l\}$, whose parameters are hence not counted. Note the number of parameters of the topic model component is often dominated by that of the language model component. Table 1 suggests rGBN-RNN, with its hierarchical and temporal topical guidance, achieves better performance with fewer parameters than comparable RNN-based language models.

Note that for language modeling, there has been significant recent interest in replacing RNNs with the Transformer (Vaswani et al., 2017), which consists of stacked multi-head attention modules, and its variants (Dai et al., 2019; Devlin et al., 2019; Radford et al., 2018; 2019). For comparison, we also report the performance of GPT-2 and Transformer-XL, two Transformer-based models. Although shown in Table 1, GPT-2 can obtain better performance than our pro-
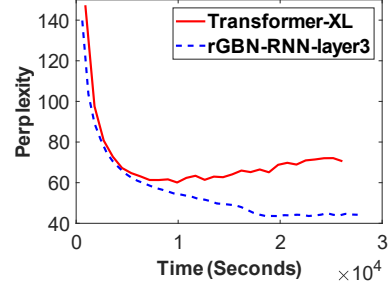

Figure 2. Comparison of Transformer-XL and rGBN-RNN on the test perplexity as a function of training time on APNEWS.

posed models, GPT-2 has significantly more parameters and requires a huge text corpus for pre-training. For example, GPT-2 with 12L (Radford et al., 2019) has 117M parameters, while the proposed rGBN-RNN with three hidden layers has as few as 7.3M parameters for language modeling. Moreover, without pre-training, we have tried training the GPT-2 directly with the APNEWS corpus on one machine with 4 NVIDIA RTX 2080 Ti GPUs: even after running 24 hours, the perplexity stays above 600 and does not show a clear trend of improvement as the time progresses. Therefore, we only display in Fig. 2 how Transformer-XL and rGBN-RNN behave during training, by showing the test perplexity of APNEWS documents. It is clear that rGBN-RNN is able to fit the data well, while Transformer-XL behaves well during the early stage of training, it shows a clear trend of overfitting as the training progresses further, possibly because it has an overly large number of model parameters, making it prone to overfitting and hence difficult to generalize.

From a structural point of view, we consider the proposed rGBN-RNN as complementary to rather than competing with Transformer based language models, and consider replacing RNN with Transformer to construct a GBN or rGBN guided Transformer as a promising future extension.

BLEU: Following Wang et al. (2019b), we use test-BLEU to evaluate the quality of generated sentences with a set of real test sentences as the reference, and self-BLEU to evaluate the diversity of the generated sentences (Zhu et al., 2018). Given the global parameters of the deep recurrent topic model (rGBN) and language model, we can generate the sentences by following the data generation process of rGBN-RNN: we first generate topic weight vector $\boldsymbol{\theta}_j^L$ randomly and then downward propagate it through rGBN as in (2) to generate $\boldsymbol{\theta}_j^{<L}$. By assimilating the generated topic weight vectors to the hidden states of the language model in each layer, as depicted in (3), we generate a corresponding sentence, where we start from a zero hidden state at each layer in the language model, and sample words sequentially until the end-of-the-sentence symbol is generated. The BLEU scores of various methods are shown in Fig. 3, using the benchmark tool in Texygen (Zhu et al., 2018); We show below BLEU-3 and BLEU-4 for BNC and defer the

Table 1. Comparison of perplexity on three different datasets and the number of parameters when used for language generation.

Model	LSTM Size	#LM Param	Topic Size	#TM Param	#All Param	Perplexity		
						APNEWS	IMDB	BNC
LCLM (Tian & Cho, 2016)	600	—	—	—	—	54.18	67.78	96.50
	900-900	—	—	—	—	50.63	67.86	87.77
LDA+LSTM	600	2.16M	100	0M	2.16M	55.52	69.64	96.50
	900-900	9.72M	100	0M	9.72M	50.75	63.04	87.77
TopicRNN (Dieng et al., 2017)	600	4M	100	4M	4M	54.54	67.83	93.57
	900-900	4M	100	4M	4M	50.24	61.59	84.62
TDLM (Lau et al., 2017)	600	3.33M	100	0.019M	3.35M	52.75	63.45	85.99
	900-900	13.36M	100	0.019M	13.38M	48.97	59.04	81.83
TCNLM (Wang et al., 2018)	600	—	100	—	—	52.63	62.64	86.44
	900-900	—	100	—	—	47.81	56.38	80.14
TGVAE (Wang et al., 2019b)	600	—	50	—	—	48.73	57.11	87.86
basic-LSTM (Hochreiter & Schmidhuber, 1997)	600	2.16M	—	—	2.16M	64.13	72.14	102.89
	900-900	10.80M	—	—	10.80M	58.89	66.47	94.23
	900-900-900	17.28M	—	—	17.28M	60.13	65.16	95.73
GBN-RNN	600	3.4M	100	0.02M	3.42M	47.42	57.01	86.39
	600-512	6.5M	100-80	0.04M	6.54M	44.64	55.42	82.95
	600-512-256	7.2M	100-80-50	0.05M	7.25M	44.35	54.53	80.25
rGBN-RNN	600	3.4M	100	0.03M	3.43M	46.35	55.76	81.94
	600-512	6.5M	100-80	0.06M	6.56M	43.26	53.82	80.25
	600-512-256	7.2M	100-80-50	0.07M	7.27M	**42.71**	**51.36**	**79.13**
rGBN-RNN-flipped	600-512-256	7.2M	100-80-50	0.07M	7.27M	43.55	53.28	81.12
Transformer-XL (Dai et al., 2019)	—	151M	—	—	151M	58.73	60.11	97.14
Pretrained GPT-2 (Radford et al., 2019)	—	117M	—	—	117M	**35.78**	**44.71**	**46.04**
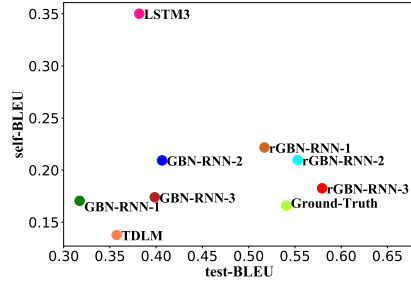
(a) BLEU-3
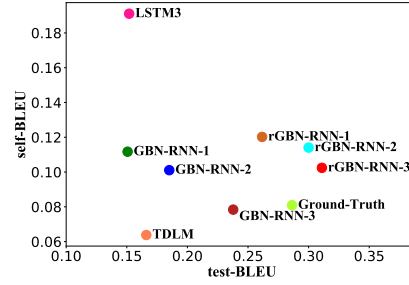
(b) BLEU-4

Figure 3. BLEU scores of different methods for BNC. BLEU scores towards the lower right corner are preferred.
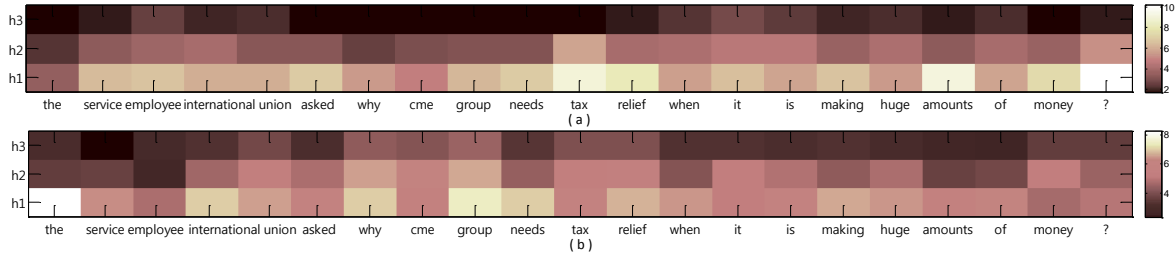
Figure 4. Visualizing the L_2 norms of the hidden states of rGBN-RNN and GBN-RNN, shown in the top and bottom rows, respectively.

analogous plots for IMDB and APNEWS to Appendices E and F, respectively. Note we set the validation dataset as the ground-truth. For all datasets, it is clear that rGBN-RNN yields both higher test-BLEU and lower self-BLEU scores than related methods do, indicating the stacked-RNN based language model in rGBN-RNN generalizes well and does not suffer from mode collapse (*i.e.*, low diversity).

3.2. Qualitative Analysis

Hierarchical structure of language model: In Fig. 4, we visualize the hierarchical multi-scale structures learned with the language model of rGBN-RNN and that of GBN-RNN, by visualizing the L_2-norm of the hidden states in each layer, while reading a sentence from the APNEWS validation set

48 generated sentence:
looming monday , the assembly added a proposal to balance the budget medicaid plan for raising the rate to $ 142 million , whereas years later , to $ 200 million .

48
budget lawmakers gov. revenue vote proposal community legislation

35
democratic taxes proposed future state spending association administration

35 generated sentence:
the council on a group of republican senators and the governor 's joint public defenders , brandon baltimore and other state and local officials , have campaigned in chants of defending and speaking with the defense have been very confident .

57 generated sentence:
the last of the four companies and the mississippi inter national speedway was voted to accept the proposal .

57
lawmakers pay proposal legislation credit session meeting gambling

68
rate rose prices income trade crisis supply management

60
budget gov. revenue vote costs mayor california conservative

60 generated sentence:
adrian on thursday issued an officer a news release saying the two groups will take more than $ 40,000 for contacts with the private nonprofit .
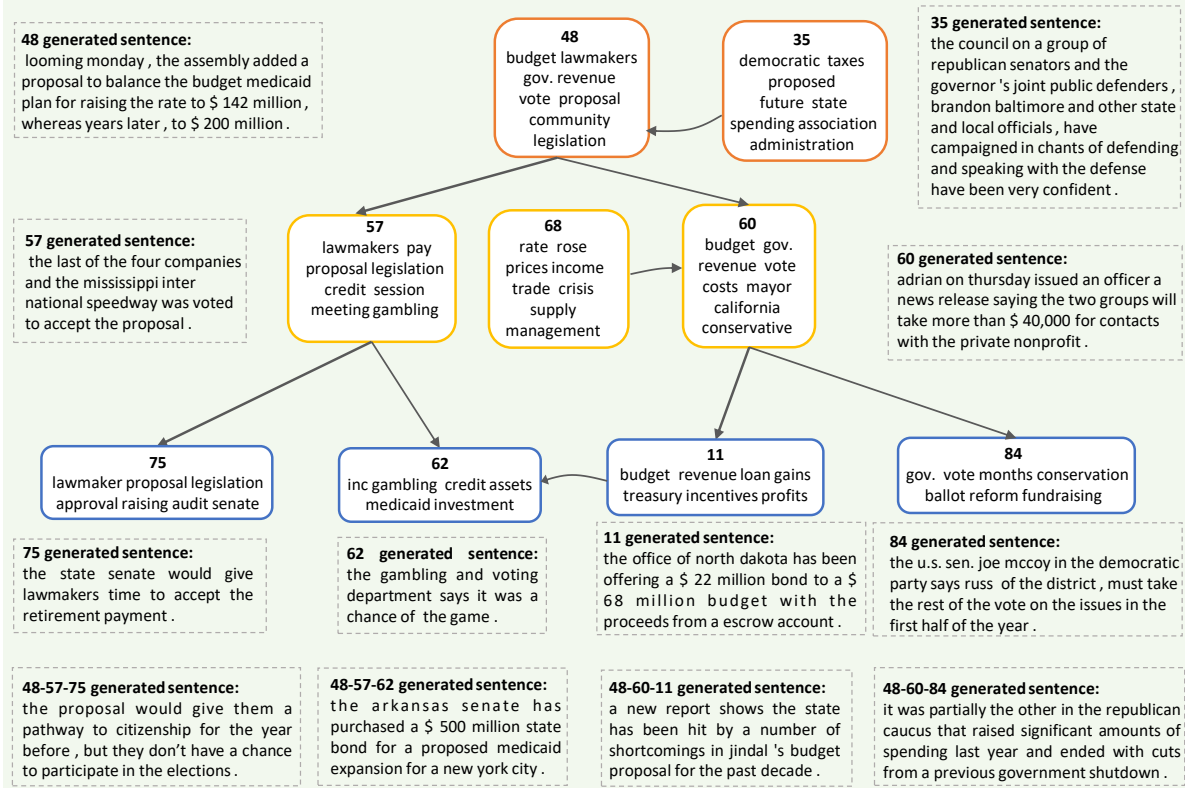
75
lawmaker proposal legislation approval raising audit senate

62
inc gambling credit assets medicaid investment

11
budget revenue loan gains treasury incentives profits

84
gov. vote months conservation ballot reform fundraising

75 generated sentence:
the state senate would give lawmakers time to accept the retirement payment .

62 generated sentence:
the gambling and voting department says it was a chance of the game .

11 generated sentence:
the office of north dakota has been offering a $ 22 million bond to a $ 68 million budget with the proceeds from a escrow account .

84 generated sentence:
the u.s. sen. joe mccoy in the democratic party says russ of the district , must take the rest of the vote on the issues in the first half of the year .

48-57-75 generated sentence:
the proposal would give them a pathway to citizenship for the year before , but they don't have a chance to participate in the elections .

48-57-62 generated sentence:
the arkansas senate has purchased a $ 500 million state bond for a proposed medicaid expansion for a new york city .

48-60-11 generated sentence:
a new report shows the state has been hit by a number of shortcomings in jindal 's budget proposal for the past decade .

48-60-84 generated sentence:
it was partially the other in the republican caucus that raised significant amounts of spending last year and ended with cuts from a previous government shutdown .

Figure 5. Example topics and their hierarchical and temporal connections inferred by a three-hidden-layer rGBN-RNN from the APNEWS corpus, and the generated sentences under topic guidance. Top words of each topic at layers 3, 2, and 1 are shown in orange, yellow, and blue boxes, respectively, and each sentence is shown in a dotted line box labeled with the corresponding topic index. Sentences generated with a combination of topics at different layers are shown at the bottom. See the Appendix for analogous plots for both IMDB and BNC.

as "*the service employee international union asked why cme group needs tax relief when it is making huge amounts of money?*" As shown in Fig. 4 (a), in the bottom hidden layer (h1), the L_2 norm sequence varies quickly from word to word, except within short phrases such as "service employee," "international union," and "tax relief," suggesting layer h1 is in charge of capturing short-term local dependencies. By contrast, in the top hidden layer (h3), the L_2 norm sequence varies slowly and exhibits semantic/syntactic meaningful long segments, such as "service employee international union," "asked why cme group needs tax relief," "when it is," and "making huge amounts of," suggesting that layer h3 is in charge of capturing long-range dependencies. Therefore, the language model in rGBN-RNN can allow more specific information to transmit through lower layers, while allowing more general higher level information to transmit through higher layers. Our proposed model have the ability to learn hierarchical structure of the sequence, despite without designing the multiscale RNNs on purpose like Chung et al. (2017). We also visualize the language model of GBN-RNN in Fig. 4 (b); with much less smoothly time-evolved deeper layers, GBN-RNN fails to utilize its stacked RNN structure as effectively as rGBN-RNN does.

This suggests that the language model is better trained in rGBN-RNN than in GBN-RNN for capturing long-range temporal dependencies, which helps explain why rGBN-RNN exhibits clearly boosted BLEU scores in comparison to GBN-RNN.

Sentence generation under topic guidance: Given the learned rGBN-RNN, we can sample the sentences both conditioning on a single topic of a certain layer and on a combination of the topics from different layers. Shown in the dotted-line boxes in Fig. 5, most of the generated sentences conditioned on a single topic or a combination of topics are highly related to the given topics in terms of their semantical meanings but not necessarily in key words, indicating the language model is successfully guided by the recurrent hierarchical topics. These observations suggest that rGBN-RNN has successfully captured syntax and global semantics simultaneously for natural language generation. Similar to Fig. 5, we also provide hierarchical topics and corresponding generated sentences for both IMDB and BNC in Appendix G. Besides, in Appendix H, we provide additional example topic hierarchies and generated sentences given different topics.

Document
- the senate sponsor (...) , a house committee last week removed photo ids issued by public colleges and universities from the measure sponsored by republican rep. susan lynn , who said she agreed with the change . the house approved the bill on a 65-30 vote on monday evening . but republican sen. bill ketron in a statement noted that the upper chamber overwhelmingly rejected efforts to take student ids out of the bill when it passed 21-8 earlier this month . ketron said he would take the bill to conference committee if needed .

Generated Sentences with GBN-RNN
- if the house and senate agree , it will be the first time they 'll have to seek their first meeting .
- the proposal would also give lawmakers with more money to protect public safety , he said .

Generated Sentences with rGBN-RNN
- the proposal , which was introduced in the house on a vote on wednesday , has already passed the senate floor to the house .
- the city commission voted last week to approve the law , which would have allowed the council to approve the new bill .

Generated temporal Sentences with rGBN-RNN (Paragraph)
- senate president pro tem joe scarnati said the governor 's office has never resolved the deadline for a vote in the house . the proposal is a new measure version of the bill to enact a senate committee to approve the emergency manager 's emergency license . the house gave the bill to six weeks of testimony , but the vote now goes to the full house for consideration . jackson signed his paperwork wednesday with the legislature .the proposal would also give lawmakers with more money to protect public safety , he said . "a spokesman for the federal department of public safety says it has been selected for a special meeting for the state senate to investigate his proposed law . a new state house committee has voted to approve a measure to let idaho join a national plan to ban private school systems at public schools . the campaign also launched a website at the university of california , irvine , which are studying the current proposal .

Figure 6. An example of generated sentences and paragraph conditioned on a document from APNEWS (green denotes novel words, blue the key words in document and generated sentences.) See the Appendix for analogous plots for both IMDB and BNC.

Hierarchical topics: We present an example topic hierarchy inferred by a three-layer rGBN-RNN from APNEWS. In Fig. 5, we select a large-weighted topic at the top hidden layer and move down the network to include any lower-layer topics connected to their ancestors with sufficiently large weights. Horizontal arrows link temporally related topics at the same layer, while top-down arrows link hierarchically related topics across layers. For example, topic 48 of layer 3 on "budget, lawmakers, gov., revenue" is related not only in hierarchy to topic 57 on "lawmakers, pay, proposal, legislation" and topic 60 of the lower layer on "budget, gov., revenue, vote, costs, mayor," but also in time to topic 35 of the same layer on "democratic, taxes, proposed, future, state." Highly interpretable hierarchical relationships between the topics at different layers, and temporal relationships between the topics at the same layer are captured by rGBN-RNN, and the topics are often quite specific semantically at the bottom layer while becoming increasingly more general when moving upwards.

Sentence/paragraph generation conditioning on a paragraph: Given the GBN-RNN and rGBN-RNN learned on APNEWS, we further present the generated sentences conditioning on a paragraph, as shown in Fig. 6. We provide analogous plots to Fig. 6 for both IMDB and BNC in Appendix I. To randomly generate sentences, we encode the paragraph into a hierarchical latent representation and then feed it into the stacked-RNN. Besides, we can generate a paragraph with rGBN-RNN, using its recurrent inference network to encode the paragraph into a dynamic hierarchical latent representation, which is fed into the language model to predict the word sequence in each sentence of the input paragraph. It is clear that both the proposed GBN-RNN and

rGBN-RNN can successfully capture the key textual information of the input paragraph, and generate diverse realistic sentences. Interestingly, the proposed rGBN-RNN can generate semantically coherent paragraphs, incorporating contextual information both within and beyond the sentences. Note that with the topics that extract the document-level word concurrence patterns, our proposed models can generate semantically-related words, which may not exist in the original document.

4. Conclusion

We propose a recurrent gamma belief network (rGBN) guided RNN-based language modeling framework, a novel method to jointly learn a neural language model and a deep recurrent topic model. For scalable inference, we develop hybrid stochastic gradient MCMC and recurrent autoencoding variational inference, allowing efficient end-to-end training. Experiments conducted on real world corpora demonstrate that the proposed models outperform a variety of shallow-topic-model-guided RNN-based language models, and effectively generate the sentences from the designated multi-level topics or noise, while inferring interpretable hierarchical latent topic structures of documents and hierarchical multiscale structures of sequences. For future work, we plan to extend the proposed models to specific natural language processing tasks, such as machine translation, image paragraph captioning, and text summarization. Another promising extension is to replace the stacked-RNN in GBN-RNN or rGBN-RNN with Transformer, *i.e.*, constructing a GBN or rGBN guided Transformer as a new larger-context neural language model.

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A. GBN-RNN

GBN-RNN: $\{y_{1:T}, \boldsymbol{d}\}$ denotes a sentence-context pair, where $\boldsymbol{d} \in \mathbb{Z}_+^{V_c}$ represents the document-level context as a word frequency count vector, the vth element of which counts the number of times the vth word in the vocabulary appears in the document excluding sentence $y_{1:T}$. The hierarchical model of an L-hidden-layer GBN, from top to bottom, is expressed as

$$\boldsymbol{\theta}^L \sim \text{Gam}\left(\boldsymbol{r}, c^{L+1}\right), \ldots, \boldsymbol{\theta}^l \sim \text{Gam}\left(\boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1}, c^{l+1}\right), \ldots,$$
$$\boldsymbol{\theta}^1 \sim \text{Gam}\left(\boldsymbol{\Phi}^2\boldsymbol{\theta}^2, c^2\right), \ \boldsymbol{d} \sim \text{Pois}\left(\boldsymbol{\Phi}^1\boldsymbol{\theta}^1\right). \tag{10}$$

The stacked-RNN based language model described in (3) is also used in GBN-RNN.

Statistical inference: To infer GBN-RNN, we consider a hybrid of stochastic gradient MCMC (Welling & Teh, 2011; Patterson & Teh, 2013; Li et al., 2015; Ma et al., 2015; Cong et al., 2017a), used for the GBN topics ϕ_k^l, and auto-encoding variational inference (Kingma & Welling, 2013; Rezende et al., 2014), used for the parameters of both the inference network (encoder) and RNN. More specifically, GBN-RNN generalizes Weibull hybrid auto-encoding inference (WHAI) of Zhang et al. (2018): it uses a deterministic-downward-stochastic-upward inference network to encode the bag-of-words representation of $\boldsymbol{d}$ into the latent topic-weight variables $\boldsymbol{\theta}^l$ across all hidden layers, which are fed into not only GBN to reconstruct $\boldsymbol{d}$, but also a stacked RNN in language model, as shown in (3), to predict the word sequence in $y_{1:T}$. The topics ϕ_k^l can be sampled with topic-layer-adaptive stochastic gradient Riemannian (TLASGR) MCMC, whose details can be found in Cong et al. (2017a); Zhang et al. (2018), omitted here for brevity. Given the sampled topics ϕ_k^l, the joint marginal likelihood of $\{y_{1:T}, \boldsymbol{d}\}$ is defined as

$$p\left(y_{1:T}, \boldsymbol{d} \,|\, \{\boldsymbol{\Phi}^l\}_l\right) = \int p\left(\boldsymbol{d} \,|\, \boldsymbol{\Phi}^1\boldsymbol{\theta}^1\right) \left[\prod_{t=1}^T p\left(y_t \,|\, y_{1:t-1}, \boldsymbol{\theta}^{1:L}\right)\right] \left[\prod_{l=1}^L p\left(\boldsymbol{\theta}^l \,|\, \boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1}\right)\right] d\boldsymbol{\theta}^{1:L}. \tag{11}$$

For efficient inference, an inference network as $Q = \prod_{l=1}^L q(\boldsymbol{\theta}^l \,|\, \boldsymbol{d}, \boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1})$ is used to provide an ELBO of the log joint marginal likelihood as

$$L(y_{1:T}, \boldsymbol{d}) = \mathbb{E}_Q\left[\ln p\left(\boldsymbol{d} \,|\, \boldsymbol{\Phi}^1\boldsymbol{\theta}^1\right) + \sum_{t=1}^T \ln p\left(y_t \,|\, y_{1:t-1}, \boldsymbol{\theta}^{1:L}\right)\right] - \sum_{l=1}^L \mathbb{E}_Q\left[\ln \frac{q\left(\boldsymbol{\theta}^l \,|\, \boldsymbol{d}, \boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1}\right)}{p\left(\boldsymbol{\theta}^l \,|\, \boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1}\right)}\right] \tag{12}$$

and the training is performed by maximizing $\mathbb{E}_{p_{\text{data}}(y_{1:T}, \boldsymbol{d})}[L(y_{1:T}, \boldsymbol{d})]$; following Zhang et al. (2018), we define $q(\boldsymbol{\theta}^l \,|\, \boldsymbol{d}, \boldsymbol{\Phi}^{l+1}, \boldsymbol{\theta}^{l+1}) = \text{Weibull}(\boldsymbol{k}^l + \boldsymbol{\Phi}^{l+1}\boldsymbol{\theta}^{l+1}, \boldsymbol{\lambda}^l)$, where both $\boldsymbol{k}^l$ and $\boldsymbol{\lambda}^l$ are deterministically transformed from $\boldsymbol{d}$ using neural networks. Distinct from a usual variational auto-encoder whose inference network has a pure bottom-up structure, the inference network here has a determistic-upward–stoachstic-downward ladder structure (Zhang et al., 2018).

B. TLASGR-MCMC for rGBN-RNN

To allow for scalable inference, we apply the TLASGR-MCMC algorithm (Cong et al., 2017a; Zhang et al., 2018; Guo et al., 2018), which can be used to sample simplex-constrained global parameters (Cong et al., 2017b) in a mini-batch based manner. It improves its sampling efficiency via the use of the Fisher information matrix (FIM) (Girolami & Calderhead, 2011), with adaptive step-sizes for the latent factors and transition matrices of different layers. In this section, we discuss how to update the global parameters $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^L$ of rGBN in detail and give a complete one in Algorithm in 1.

Sample the auxiliary counts: This step is about the "backward" and "upward" pass. Let us denote $Z_{\cdot kj}^l = \sum_{k_l=1}^{K_l} Z_{k_l kj}^l$, $Z_{\cdot k, J+1}^l = 0$, and $x_{kj}^{(1,1)} = d_{vj}$, where $\boldsymbol{d}_j = \{d_{1j}, .., d_{vj}, .., d_{V_c j}\}$ is the same as in (2). Working backward for $j = J, ..., 1$ and upward for $l = 1, ..., L$, we draw

$$\left(A_{k1j}^l, ..., A_{kK_l j}^l\right) \sim \text{Multi}\left(x_{kj}^{(l,l)}; \frac{\phi_{k1}^l \theta_{1j}^l}{\sum_{k_l=1}^{K_l} \phi_{kk_l}^l \theta_{k_l j}^l}, ..., \frac{\phi_{kK_l}^l \theta_{K_l j}^l}{\sum_{k_l=1}^{K_l} \phi_{kk_l}^l \theta_{k_l j}^l}\right), \tag{13}$$

$$x_{kj}^{l+1} \sim \text{CRT}\left[A_{\cdot kj}^l + Z_{\cdot k, j+1}^l, \tau_0\left(\sum_{k_{l+1}=1}^{K_{l+1}} \phi_{kk_{l+1}}^{l+1} \theta_{k_{l+1} j}^{l+1} + \sum_{k_l=1}^{K_l} \pi_{kk_l}^l \theta_{k_1, j-1}^l\right)\right]. \tag{14}$$

Note that via the deep structure, the latent counts x_{kj}^{l+1} will be influenced by the effects from both time $j-1$ at layer l and time j at layer $l+1$. With $p_1 := \sum_{k_l=1}^{K_l} \pi_{kk_l}^l \theta_{k_l j-1}^l$ and $p_2 := \sum_{k_{l+1}=1}^{K_{l+1}} \phi_{kk_{l+1}}^{l+1} \theta_{k_{l+1} j}^{l+1}$, we can sample the latent counts at

layer l and $l+1$ by

$$\left(x_{kj}^{l+1,l}, x_{kj}^{l+1,l+1}\right) \sim \text{Multi}\left(x_{kj}^{l+1}, p_1/(p_1+p_2), p_2/(p_1+p_2)\right), \tag{15}$$

and then draw

$$\left(Z_{k1j}^l, ..., Z_{kK_lj}^l\right) \sim \text{Multi}\left(x_{kj}^{l+1,l}; \frac{\pi_{k1}^l \theta_{1,j-1}^l}{\sum_{k_l=1}^{K_l} \pi_{kk_l}^l \theta_{k_l,j-1}^l}, ..., \frac{\pi_{kK_l}^l \theta_{K_l,j-1}^l}{\sum_{k_l=1}^{K_l} \pi_{kk_l}^l \theta_{k_l,j-1}^{(l)}}\right). \tag{16}$$

In rGBN, the prior and the likelihood of $\{\boldsymbol{\Phi}^l\}_{l=1}^L$ is very similar to $\{\boldsymbol{\Pi}^l\}_{l=1}^L$, so we also apply the TLASGR-MCMC sampling algorithm on both of them conditioned on the auxiliary counts.

Sample the hierarchical components $\{\boldsymbol{\Phi}^l\}_{l=1}^L$: For ϕ_k^l, the kth column of the loading matrix $\boldsymbol{\Phi}^l$ of layer l, its sampling can be efficiently realized as

$$\begin{aligned}
\left(\phi_k^l\right)_{n+1} = &\left[\left(\phi_k^l\right)_n + \frac{\varepsilon_n}{P_k^l}\left[\left(\rho \tilde{\boldsymbol{A}}_{:k.}^l + \eta_0^l\right) - \left(\rho \tilde{A}_{\cdot k.}^l + K_{l-1}\eta_0^l\right)\left(\phi_k^l\right)_n\right]\right. \\
&\left.+ \mathcal{N}\left(0, \frac{2\varepsilon_n}{P_k^l}\left[\text{diag}(\phi_k^l)_n - (\phi_k^l)_n(\phi_k^l)_n^T\right]\right)\right]_{\angle},
\end{aligned} \tag{17}$$

where P_k^l is calculated using the estimated FIM, $\tilde{A}_{k_lj.}^l = \sum_{j=1}^J A_{k_lkj}^l$, $\tilde{\boldsymbol{A}}_{:k.}^l = \{\tilde{A}_{1j.}^l, \cdots, \tilde{A}_{K_lj.}^l\}^T$ and $\tilde{A}_{\cdot k.}^l = \sum_{k_l=1}^{k_l} \tilde{A}_{k_lj.}^l$, $A_{k_lkj}^l$ comes from the augmented latent counts A^l in (13), η_0^l denote the prior of ϕ_k^l, and $[\cdot]_{\angle}$ denotes the simplex constraint.

Sample the transmission matrix $\{\boldsymbol{\Pi}^l\}_{l=1}^L$: For $\boldsymbol{\pi}_k^l$, the kth column of the transition matrix $\boldsymbol{\Pi}^l$ of layer l, its sampling can be efficiently realized as

$$\begin{aligned}
\left(\boldsymbol{\pi}_k^l\right)_{n+1} = &\left[\left(\boldsymbol{\pi}_k^l\right)_n + \frac{\varepsilon_n}{M_k^l}\left[\left(\rho \tilde{\boldsymbol{Z}}_{:k.}^l + \boldsymbol{\eta}_{:k}^l\right) - \left(\rho \tilde{Z}_{\cdot k.}^l + \eta_{\cdot k}^l\right)\left(\boldsymbol{\pi}_k^l\right)_n\right]\right. \\
&\left.+ \mathcal{N}\left(0, \frac{2\varepsilon_n}{M_k^l}\left[\text{diag}(\boldsymbol{\pi}_k^l)_n - (\boldsymbol{\pi}_k^l)_n(\boldsymbol{\pi}_k^l)_n^T\right]\right)\right]_{\angle},
\end{aligned} \tag{18}$$

where M_k^l is calculated using the estimated FIM, $\tilde{Z}_{k_lj.}^l = \sum_{j=1}^J Z_{k_lkj}^l$, $\tilde{\boldsymbol{Z}}_{:k.}^l = \{\tilde{Z}_{1j.}^l, \cdots, \tilde{Z}_{K_lj.}^l\}^T$ and $\tilde{Z}_{\cdot k.}^l = \sum_{k_l=1}^{k_l} \tilde{Z}_{k_lj.}^l$, $Z_{k_lkj}^l$ comes from the augmented latent counts Z^l in (16), and $[.]_{\angle}$ denotes a simplex constraint, and $\boldsymbol{\eta}_{:k}^l$ denotes the prior of $\boldsymbol{\pi}_k^l$, more details about TLASGR-MCMC for our proposed model can be found in Cong et al. (2017a).

C. Datasets

We consider three publicly available corpora[1]. APNEWS is a collection of Associated Press news articles from 2009 to 2016, IMDB is a set of movie reviews collected by Maas et al. (2011), and BNC is the written portion of the British National Corpus (Consortium, 2007). Following the preprocessing steps in Lau et al. (2017), we tokenize words and sentences using Stanford CoreNLP (Klein & Manning, 2003), lowercase all word tokens, and filter out word tokens that occur less than 10 times. For the topic model, we additionally exclude stopwords[2] and the top 0.1% most frequent words. All these corpora are partitioned into training, validation, and testing sets, whose summary statistics are provided in Table 2 of the Appendix.

D. Complexity of rGBN-RNN

The proposed rGBN-RNN consists of both language model and topic model components. For the topic model component, there are the global parameters of rGBN (decoder), including $\{\boldsymbol{\Phi}^l, \boldsymbol{\Pi}^l\}_{l=1}^L$ in (2), and the parameters of the recurrent variational inference network (encoder), consisting of $\text{RNN}_{\text{sent}}^l$, $f_{\boldsymbol{k}}^l$, and $f_{\boldsymbol{\lambda}}^l$ in (9). The language model component is parameterized by $\text{LSTM}_{\text{word}}^l$ in (3) and the coupling vectors g^l described in (4). We summarize in Table 3 the complexity of rGBN-RNN (ignoring all bias terms), where V denotes the vocabulary size of the language model, E the dimension

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[1] https://ibm.ent.box.com/s/ls61p8ovc1y87w45oa02zink2zl7l6z4
[2] We use Mallet's stopword list: https://github.com/mimno/Mallet/tree/master/stoplists

Table 2. Summary statistics for the datasets.

Dataset	Vocubalry		Training			Validation			Testing		
	LM	TM	Docs	Sents	Tokens	Docs	Sents	Tokens	Docs	Sents	Tokens
APNEWS	34231	32169	50K	0.8M	15M	2K	33K	0.6M	2K	32K	0.6M
IMDB	36009	34925	75K	1.1M	20M	12.5K	0.18M	0.3M	12.5K	0.18M	0.3M
BNC	43703	41552	15K	1M	18M	1K	57K	1M	1K	66K	1M

Table 3. Complexity of the three-layer rGBN-RNN.

Component	Language Model		Topic Model				
Param	$\text{LSTM}^l_{\text{word}}$ in (3)	g^l in (4)	Φ^l in (2)	Π^l in (2)	$\text{RNN}^l_{\text{sent}}$ in (9)	f^l_k in (9)	f^l_λ in (9)
Layer1	$O(4 \times (E + H^w_1) \times H^w_1)$	$O(3 \times (K_1 + H^w_1) \times H^w_1)$	$O(V_c \times K_1)$	$O(K_1 \times K_1)$	$O((V_c + H^s_1) \times H^s_1)$	$O(H^s_1)$	$O(K_1 \times H^s_1)$
Layer2	$O(4 \times (H^w_1 + H^w_2) \times H^w_2)$	$O(3 \times (K_2 + H^w_2) \times H^w_2)$	$O(K_1 \times K_2)$	$O(K_2 \times K_2)$	$O((H^s_1 + H^s_2) \times H^s_2)$	$O(H^s_2)$	$O(K_2 \times H^s_2)$
Layer3	$O(4 \times (H^w_2 + H^w_3) \times H^w_3)$	$O(3 \times (K_3 + H^w_3) \times H^w_3)$	$O(K_2 \times K_3)$	$O(K_3 \times K_3)$	$O((H^s_2 + H^s_3) \times H^s_3)$	$O(H^s_3)$	$O(K_3 \times H^s_3)$

of word embedding vectors, V_c the size of the vocabulary of the topic model that excludes stop words, H^w_l the number of hidden units of the word-level LSTM at layer l (stacked-RNN language model), H^s_l the number of hidden units of the sentence-level RNN at layer l (recurrent variational inference network), and K_l the number of topics at layer l. Comparison of the number of parameters between various language models is provided in Table 1.

E. BLEU scores for IMDB

Figure 7. BLEU scores of different methods for IMDB. x-axis denotes test-BLEU, and y-axis self-BLEU. Left panel is BLEU-3 and right is BLEU-4, and a better BLEU score would fall within the lower right corner, where black point represents mean value and circles with different colors denote the elliptical surface of probability of BLEU in a two-dimensional space.

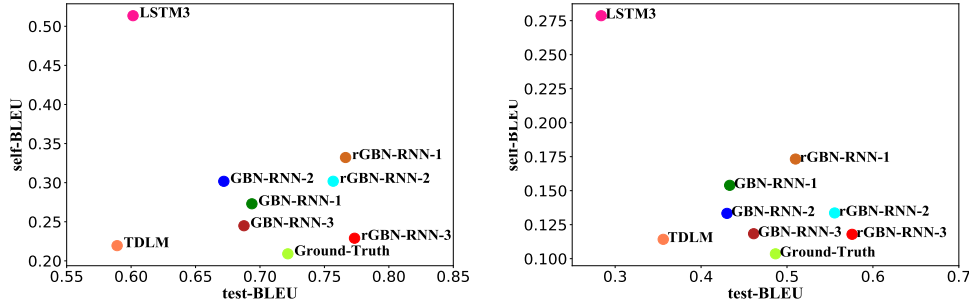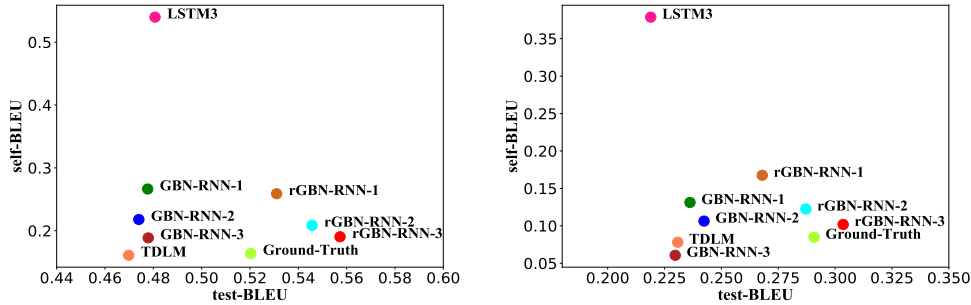# F. BLEU scores for APNEWS

Figure 8. BLEU scores of different methods for APNEWS. x-axis denotes test-BLEU, and y-axis self-BLEU. Left panel is BLEU-3 and right is BLEU-4, and a better BLEU score would fall within the lower right corner, where black point represents mean value and circles with different colors denote the elliptical surface of probability of BLEU in a two-dimensional space.
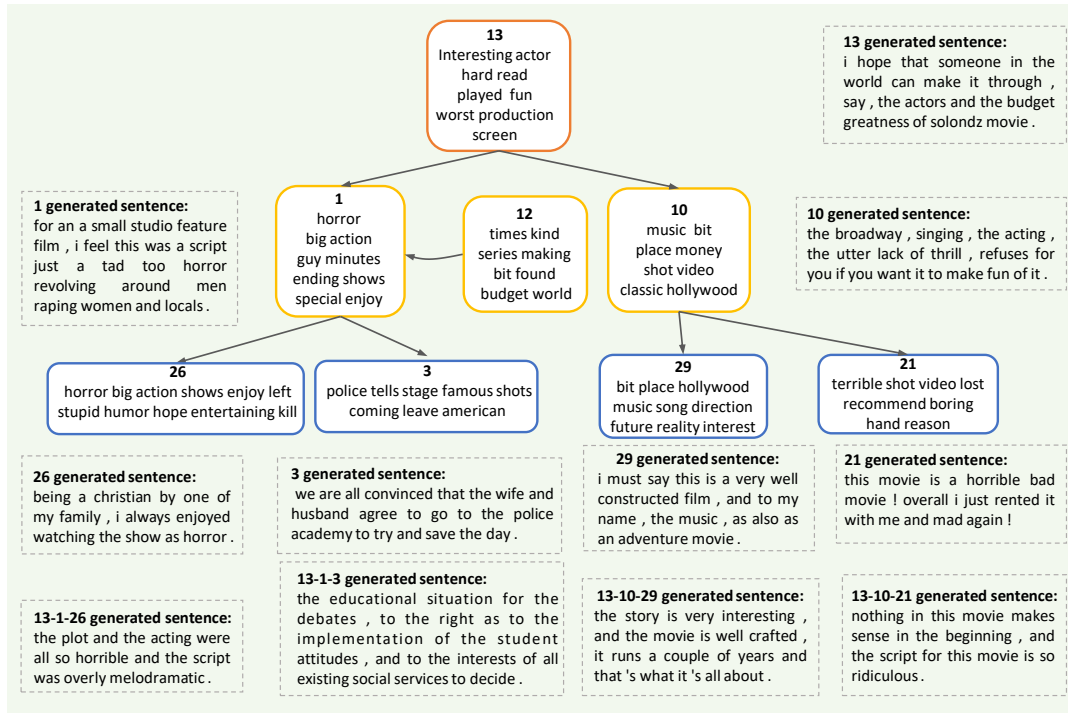
G. Additional experimental results on IMDB and BNC

13
Interesting actor
hard read
played fun
worst production
screen

13 generated sentence:
i hope that someone in the world can make it through , say , the actors and the budget greatness of solondz movie .

1
horror
big action
guy minutes
ending shows
special enjoy

12
times kind
series making
bit found
budget world

10
music bit
place money
shot video
classic hollywood

10 generated sentence:
the broadway , singing , the acting , the utter lack of thrill , refuses for you if you want it to make fun of it .

1 generated sentence:
for an a small studio feature film , i feel this was a script just a tad too horror revolving around men raping women and locals .

26
horror big action shows enjoy left stupid humor hope entertaining kill

3
police tells stage famous shots coming leave american

29
bit place hollywood music song direction future reality interest

21
terrible shot video lost recommend boring hand reason

26 generated sentence:
being a christian by one of my family , i always enjoyed watching the show as horror .

3 generated sentence:
we are all convinced that the wife and husband agree to go to the police academy to try and save the day .

29 generated sentence:
i must say this is a very well constructed film , and to my name , the music , as also as an adventure movie .

21 generated sentence:
this movie is a horrible bad movie ! overall i just rented it with me and mad again !

13-1-26 generated sentence:
the plot and the acting were all so horrible and the script was overly melodramatic .

13-1-3 generated sentence:
the educational situation for the debates , to the right as to the implementation of the student attitudes , and to the interests of all existing social services to decide .

13-10-29 generated sentence:
the story is very interesting , and the movie is well crafted , it runs a couple of years and that 's what it 's all about .

13-10-21 generated sentence:
nothing in this movie makes sense in the beginning , and the script for this movie is so ridiculous .

Figure 9. Analogous plot to Fig. 5 for the IMDB corpus.

19
tax service
royal industry
air secretary
bsia church
agreements

15
society members
education formal
appraisal curriculum
questionnaire
classroom minister

15 generated sentence:
the advance of the educational survey on agreement highlighted these principles and many of the other major obstacles to effective compliance .

8 generated sentence:
a good example of the poor would be lodge with the educated counsellor if the lea have no regional governors .

8
schools education
welfare central
young curriculum
class formal

46
cent form general
included forces section
single environment
personal meeting

73
association
society formal authorities
contributions professional

73 generated sentence:
how we get to know what the need is for the <unk> to consider in considering that our right to think of the crucial outcome which we were to give account of the public and civil society .

7
appraisal questionnaire curriculum schools formal decisions training primary

86
members education welfare central minister create reform benefits reform development

21
society authorities contributions formal election list voters professional alliance

28
association bill determination details decision british, france

7 generated sentence:
the school is considering what the society thinks it is .

86 generated sentence:
on the reforms such as the extent to which the administration might have been divided by the recent enactment of the alarm it had entrusted staff at all politicians to allow a legal order on it .

21 generated sentence:
as a civil cabinet for the present processing authority at the national assembly , described the reforms as not an opportunity to compel the sovereign to appreciate .

28 generated sentence:
the government was to continue to attempt to establish a meeting with france , but it in turn of its motivation must be lost away .

15-8-7 generated sentence:
the school was found into the property exchanges for further subsistence changes , or the existence of a deduction of added discontent with the prime minister .

15-8-86 generated sentence:
the educational situation for the debates , to the right as to the implementation of the student attitudes , and to the interests of all existing social services to decide .

15-73-21 generated sentence:
this election is said to have been contested of the high islam that dragged the government on the peril of the old janata alliance and generally seen one-sixth of voters in the european .

15-73-28 generated sentence:
the second part of the aid is to demonstrate that the practical strategy of the association is to be negotiated by the general secretary of the government .
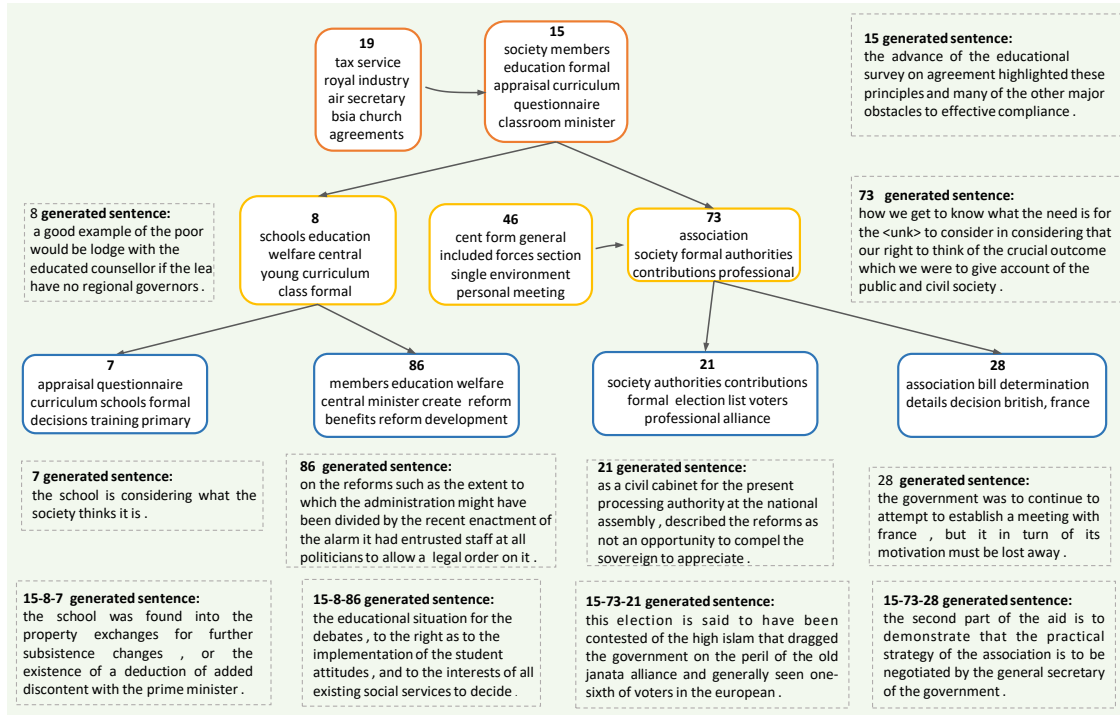
Figure 10. Analogous plot to Fig. 5 for the BNC corpus.

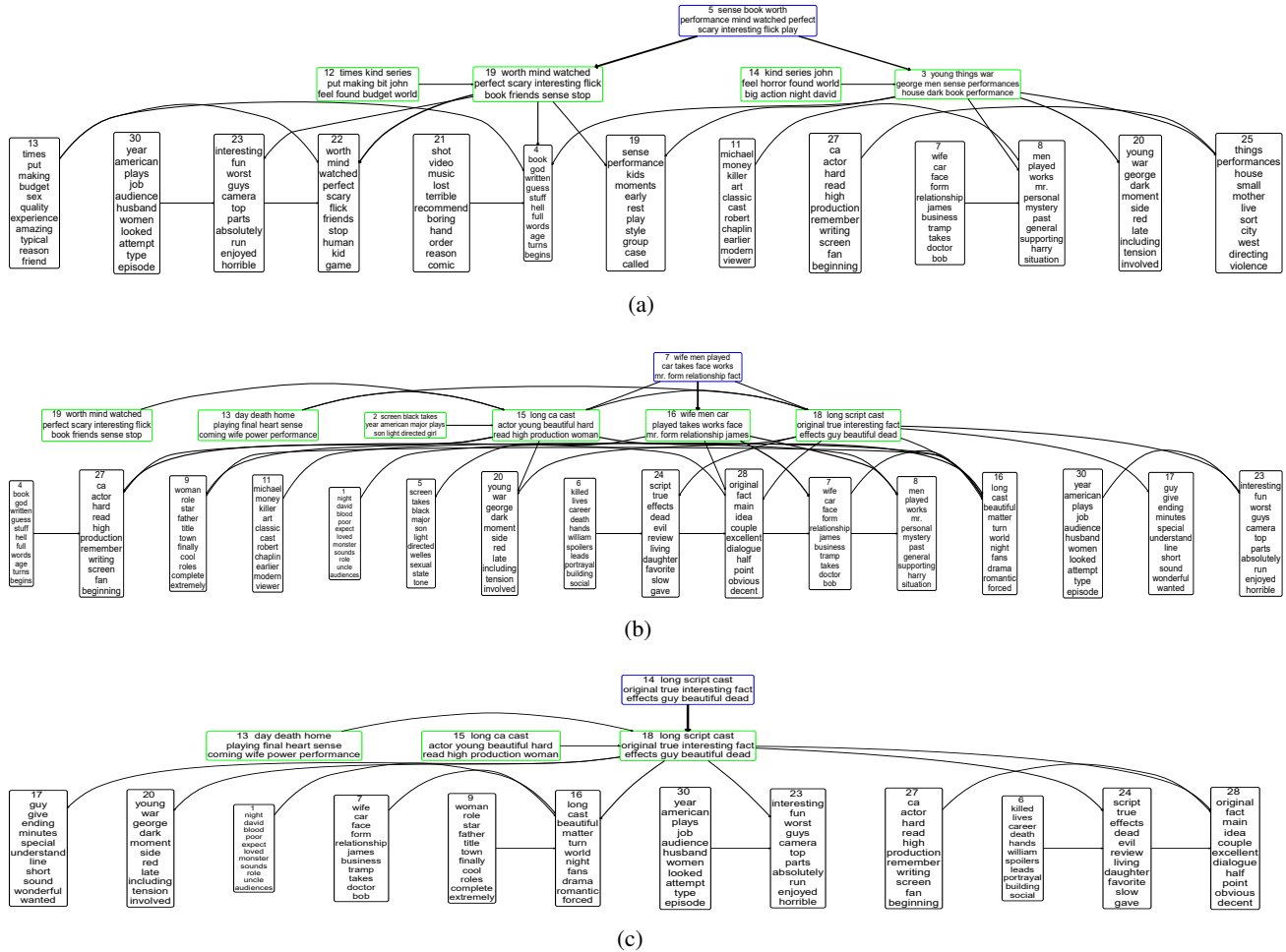## H. Additional example topic hierarchies and generated sentences.

Figure 11. Example topics and their hierarchical and temporal connections inferred by a three-hidden-layer rGBN-RNN from the IMDB corpus. Top words of each topic at layers 3, 2, and 1 are shown in blue, green, and black boxes respectively. Shown in (a)-(c) are the 5th, 7th and 14th nodes of the top layer, respectively.

Generated sentences conditioned on topic 5 at layer 3: *(a) i love this movie , i strongly recommend it , just watch it with friends and laugh . (b) i was seriously shocked with hogan 's performance. (c) the performances are terrible , the storyline is non-existing , the directing (if you can call it that) is horrible , and the acting is horrible .*

Generated sentences conditioned on topic 3 at layer 2: *(a) her performance is a bit afro looking and they have a very neat southern accent and the movie was well cast as in previous movies , was so much more beautiful especially when woody allen made this film . (b) without their personal history or the fact that he would like to kill his american adoptive parents in their own homes , it all wo n't make sense . (c) this is one of those romantic comedies where we have some good action and good acting .*

Generated sentences conditioned on topic 20 at layer 1: *(a) the new story was very touching , real , a new perspective of what it is like to be a back to war . (b) the movie is ok in my eyes , i know you will find it too scary ... but i kind of got a good movie from a somewhat dark sense . (c) at the same time i believe that the film was made just a couple of years earlier , the members of the u.s. government were almost always bad .*

Generated sentences conditioned on a combination of topics 5, 19 and 22 at layer 3, 2, 1: *(a) the show is well worth watching , i feel it is the best movie i have ever seen . (b) the movie is amazing , i thought the acting was great , and the movie was well worth the rental . (c) it would have been much better if it was a science fiction movie with a little more humor and some more action .*
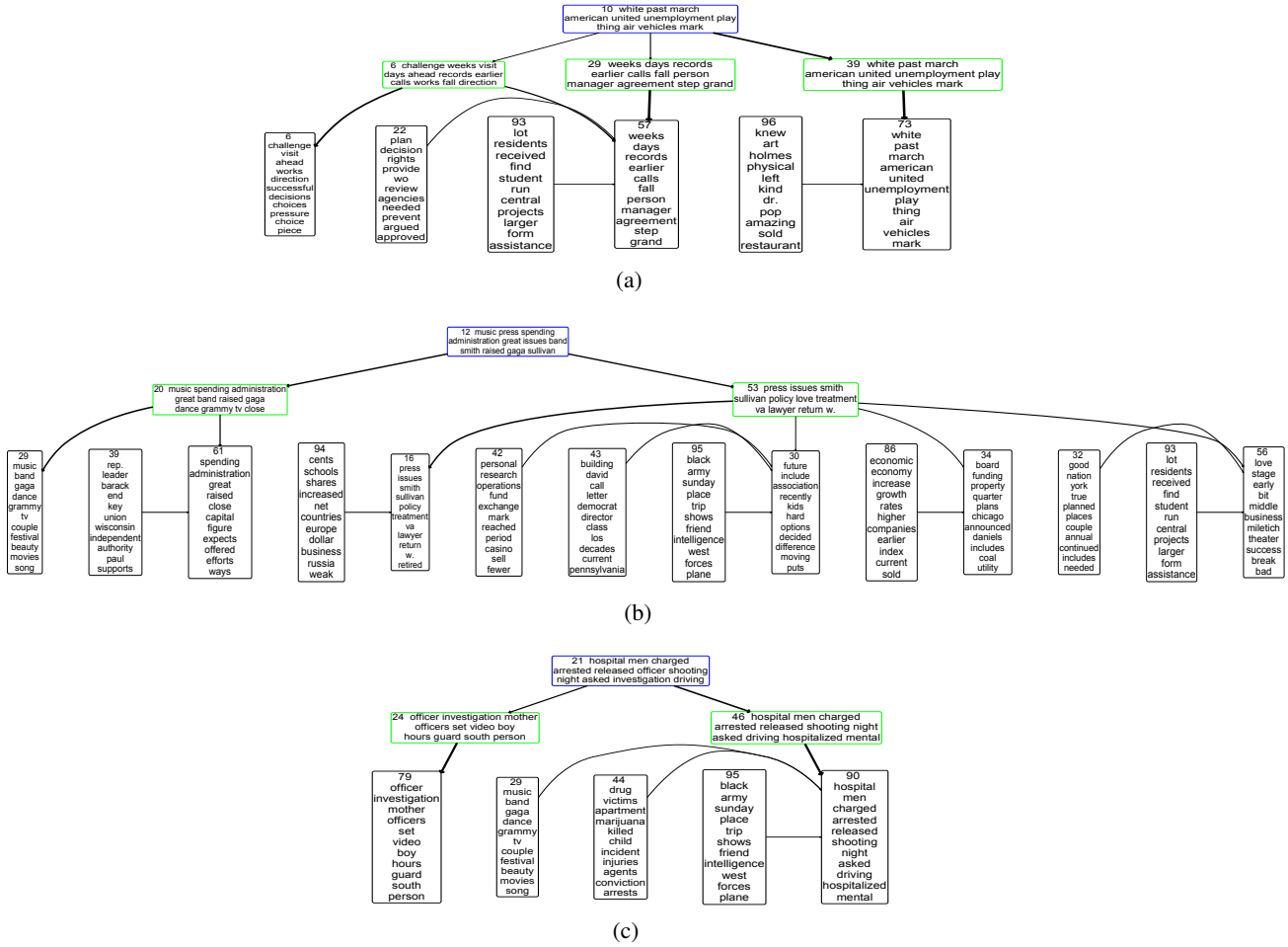
Figure 12. Example topics and their hierarchical and temporal connections inferred by a three-hidden-layer rGBN-RNN from the APNEWS corpus. Top words of each topic at layers 3, 2, and 1 are shown in blue, green, and black boxes respectively. Shown in (a)-(c) are the 10th, 12th and 21th nodes of the top layer, respectively.

Generated sentences conditioned on topic 12 at layer 3: *(a) they 're planning to attend a concert hall held by the rev. jesse. (b) approved by the standard free press , it will generate water for their own offices . (c) the christie administration will not give him an opinion if the of the state has issued its name .*

Generated sentences conditioned on topic 53 at layer 2: *(a) . the national park service said the maine department of law will hold a agreement on the <unk> law for the first time . (b) the detroit free press reports the city asked former winston-salem public schools commission chairman <unk> <unk> to take the seat . (c) earlier this month , the state police issued several orders to the fbi and send a <unk> team to the sheriff 's office .*

Generated sentences conditioned on topic 29 at layer 1: *(a) but it was and a few months later , the music had performed at the <unk> theatre in its <unk> . (b) the festival draws hundreds of thousands of viewers , a tourist year by a member of the oxford state team . (c) the university made the first " the most exciting , very beautiful " album followed by the 1996 " the sky " includes a <unk> version .*

Generated sentences conditioned on a combination of topics 21, 46 and 44 at layer 3, 2, 1: *(a) police say the suspect was taken to a hospital for treatment . (b) the man was arrested after police say he was driving in a car in north mississippi , which was the first <unk> to be used in the shootout . (c) police said wednesday that the victims ' deaths are not believed to be gang affiliation .*
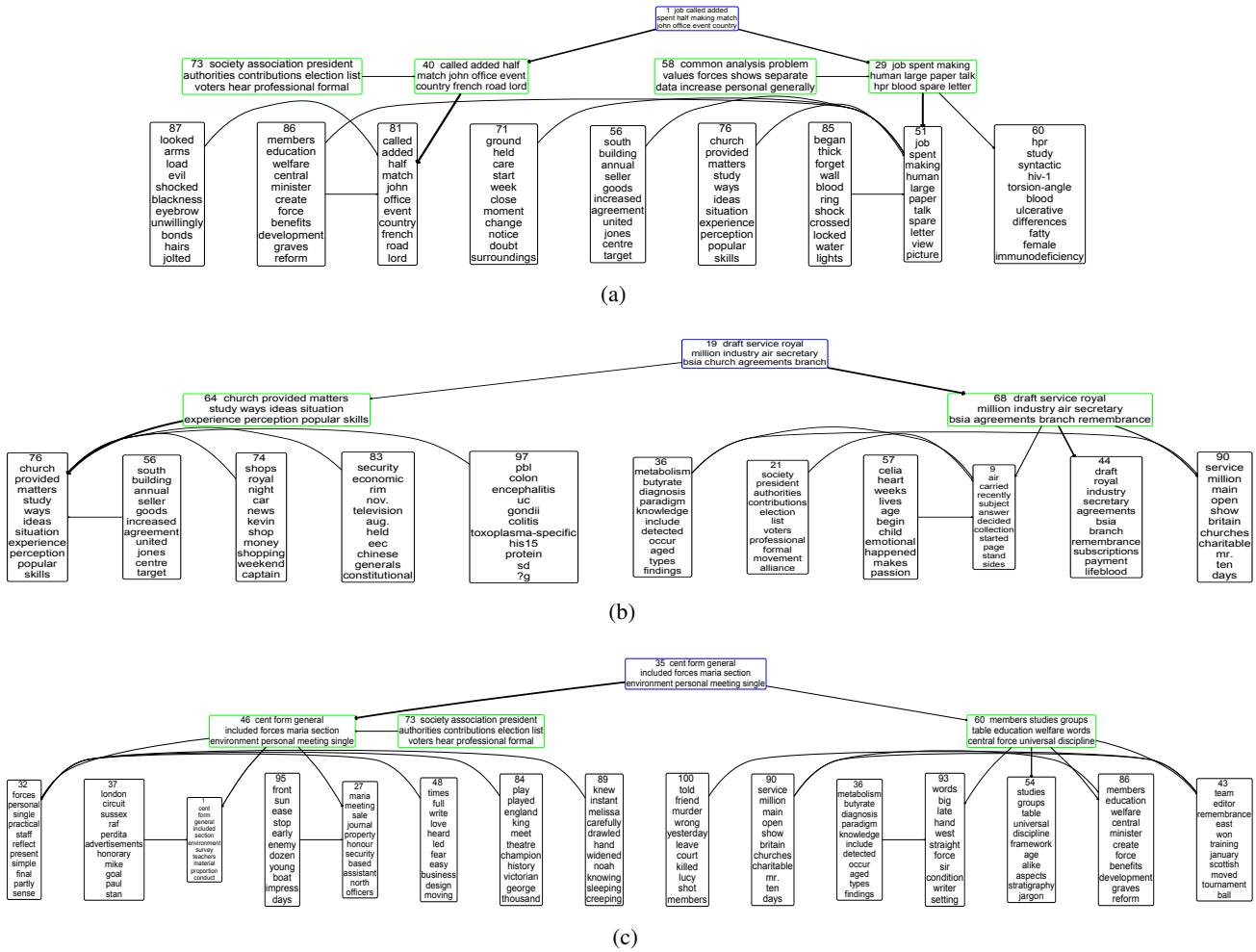
Figure 13. Example topics and their hierarchical and temporal connections inferred by a three-hidden-layer rGBN-RNN from the BNC corpus. Top words of each topic at layers 3, 2, and 1 are shown in blue, green, and black boxes respectively. Shown in (a)-(c) are the 15th, 19th and 35th nodes of the top layer, respectively.

Generated sentences conditioned on topic 1 at layer 3: *(a) the fourth should be in the obligation of the enforcement officer for proof where a supplementary liability of pension funds has not been advocated. (b) approved by the standard free press , it will generate water for their own offices . (c) the court , has recently agreed to participate in the investigation to allow the justices to succeed to be responsible for the full remit of the submissions .*

Generated sentences conditioned on topic 73 at layer 2: *(a) another , which was the period of message from a federal of the new presidential opposition to the new constitution , was to change his autonomy rather than through the different strategies . (b) the president is to bring out the best of all and the most widely understood state of affairs in the country . (c) the icrc has announced that the <unk> could not be accepted on the factors outlined in the societies ' choice .*

Generated sentences conditioned on topic 76 at layer 1: *(a) the church of st clement danes – the great continent , the southern of the realm , a treaty give such assistance to brother. (b) the legality of the political instability that followed by the profession has been criticised for the necessity for the study of individuals and friends . (c) the character of the monarchy is dominated by a panoramic style which includes and attempts to limit the genre to his/her ideas . '*

Generated sentences conditioned on a combination of topics 19, 68 and 44 at layer 3, 2, 1: *(a) again the royal air force in the middle of the war was now part of the struggle by the indian resistance . (b) the mailing list for another example of the british aerospace industry shows a, exclusive catalogue to enable object to be changed to steam . (c) the great britain will continue to sumbit to the thinking and nature of the reciprocal international economic agreement.*

I. Additional examples of generated sentences / paragraph conditioning on a paragraph.

Document

○ splitting up the disintegration of any relationship is painful . but the problems which follow may not only be emotional ones , and this is where a solicitor can help . divorce or separation mean thinking about arrangements for children ; maintenance ; and the division of property .these are all areas in which a solicitor offers professional expertise and practical advice -- as well as support and comfort . an unmarried couple splitting up can also face legal problems . if they did not take legal advice before buying property together , its division could be complicated . there may also be difficulties about parental responsibility for their children . a solicitor will be able to advise on all these matters .

- -

Generated Sentences with GBN-RNN

○ divorce which can not be completion of couple, or on behalf of the disaster .

○ despite divorced , and also to the right to rely on anything at the magistrates ' courts .

- -

Generated Sentences with rGBN-RNN

○ children 's divorced parents were to be allowed to provide the best for that to happen .

○ parents to protect themselves from the existing community and to a mutual trust in the home in the child 's mind.

- -

Generated temporal Sentences with rGBN-RNN (Paragraph)

○ parents or fathers were denied to care for and respect for both the self-employed and his wife . for the first time , since the fact that many parents and children were earning their lives during the war , the health and safety act in the <unk> household . for this reason , a mutual relationship to voluntary arrangements for which the rights of its members are subject to : compensation ! parents in the <unk> should be allowed to test the impact of liability on the community life fund . Both the parents and housing , should be able to provide good teaching and care for any elderly parent community . the children live in australia , organised by social services and housing associations . divorce and remarriage are just as important as was due to the ill and the whole of the damage or the reservation . it is to be hoped that , more than that , it must be necessary to discuss the analyses of family needs .

Figure 14. An example of generated sentences and paragraph conditioned on a document from BNC (green denotes novel words, blue the key words in document and generated sentences.)

Document

○ this is one of the best and moodiest vampire tales ever ! i love this movie really . the character are great , even the locations and the story . indeed the picture is n't a big budget production , but it is absolutely worth seeing . ok there are some faults (especially the names of the castle and the locations) in this movie , but such mistakes are typically and are almost in every horror movie . the scenery fits perfect to the story and is close to reality , i can say that honest , because i visit them once when i was in romania in my vacations . in my opinion this is the best part of the subspecies series .

- -

Generated Sentences with GBN-RNN

○ it was one of the most powerful films i 've ever seen .

○ okay , i would say special up about it, but if you were a true horror fanatic , you wo n't be disappointed .

- -

Generated Sentences with rGBN-RNN

○ this is one of the more refreshingly horrible films you will treasure, great music and even the scenes.

○ this is a wonderful movie, worth to look in and the plot is very gold and some of the characters are true to their characters .

- -

Generated temporal Sentences with rGBN-RNN (Paragraph)

○ i 'm a fan of vampire or zombie movies and i was never bored. this film is well worth a look for years over. the effect and scale are enough to refresh the horror movie. the story between vampire and werewolf runs through the story, and vampire wants to become the werewolf . i am very glad to see that the vampires in this movie have the hunger to kill the zombies. what i like about this film is the cast , their performances , the plausibility , and its relationship. the ending of about vampire is very beautiful and i guess that is all that would make the movie so great for me .

Figure 15. An example of generated sentences and paragraph conditioned on a document from IMDB (green denotes novel words, blue the key words in document and generated sentences.)