

Optimal sensor placement method for wastewater treatment plants based on discrete multi-objective state transition algorithm

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ABSTRACT

Parameters monitoring is essential to maintain the stability and efficiency of the wastewater treatment process, which has spurred ubiquitous installation of sensors in wastewater treatment plants (WWTPs). As the rich process data of WWTPs is not effectively transformed into actionable knowledge for system optimization due to improper sensor installation, the sensor placement scheme needs to be optimized. In this paper, a weighted sensor placement optimization model based on sensor cost, information richness and reliability is established to transform the sensor optimization problem to a nonlinear mathematical programming problem. Then a discrete multi-objective state transition algorithm is proposed to find the Pareto optimal solutions. Finally, an evaluation strategy is designed to select the most suitable solution for industrial application. The results of simulation experiments on three different WWTPs demonstrate the validity and superiority of the proposed method, increasing the degree of variable observability and measurement redundancy while keeping the sensor cost at a low level.

1. Introduction

Currently, the contradiction between water shortage and water pollution is becoming increasingly prominent (Dutta et al., 2021). Water pollution caused by organic contaminants, such as oils, dyes, and proteins has become a serious ecological issue (Yousefi et al., 2019, 2021). Municipal wastewater treatment plants (WWTPs) is a strategic initiative to realize comprehensive utilization and virtuous cycle of water resources, and have been built up to treat domestic sewage, industrial wastewater, and rainfall runoff. For the sake of safety and reliability of WWTPs, parameters monitoring is important, which has spurred ubiquitous application of various kinds of sensors to provide technical foundation.

Although the emergence of acceptable computing, low-priced sensor equipment, and fast and efficient communications has stimulated the deployment of sensor networks in WWTPs (Corominas et al., 2018), there remains two challenges. On the one hand, wastewater treatment process is a typical distributed parameter system with infinite degrees of freedom. Due to the economic and technical constraints, it is unrealistic to fully cover the whole plant by industrial detection. On the other hand, in spite of the data-rich feature of the plants, limited actionable knowledge useful for system optimization is obtained resulting from improper placement of sensors. Consequently, it is urgent to optimize

the installation locations, types, and quantity of the sensors in WWTPs.

Because of the dynamic characteristics of process parameters such as flow rate, dissolved oxygen concentration, and pollutants concentrations, non-linear relationships between the variables of interest, interference of environmental factors such as temperature and pH (Alwan, 2012), and the networked nature of systems, optimal sensor placement in water treatment system is quite a complicated issue. Some scholars have conducted research on this issue. E.g., Mukherjee designed a best sensor network with minimum pollutants influence considering demand, location uncertainty and response of the sensors (Mukherjee et al., 2017). Cardoso presented a method of Pareto front disposal, and selected the best solution automatically by means of distance (Cardoso et al., 2021). Le proposed a stepwise experimental design method to optimize the sensor layout of the WWTP, improving the accuracy of process measurement data while ensuring the sensor cost (Le et al., 2018). Rehman proved that rationally arranging the position of dissolved oxygen sensors in large bioreactors in WWTPs can improve the mixing effect, ensure the effluent quality and reduce the aeration cost (Rehman et al., 2015). Khorshidi proposed an information theoretic approach to determine the best possible potential locations of sensors, which enhanced the decision space and warranted more accurate and robust results (Khorshidi et al., 2018). Villez proposed a sensor placement method for WWTPs using the discrete deterministic optimization

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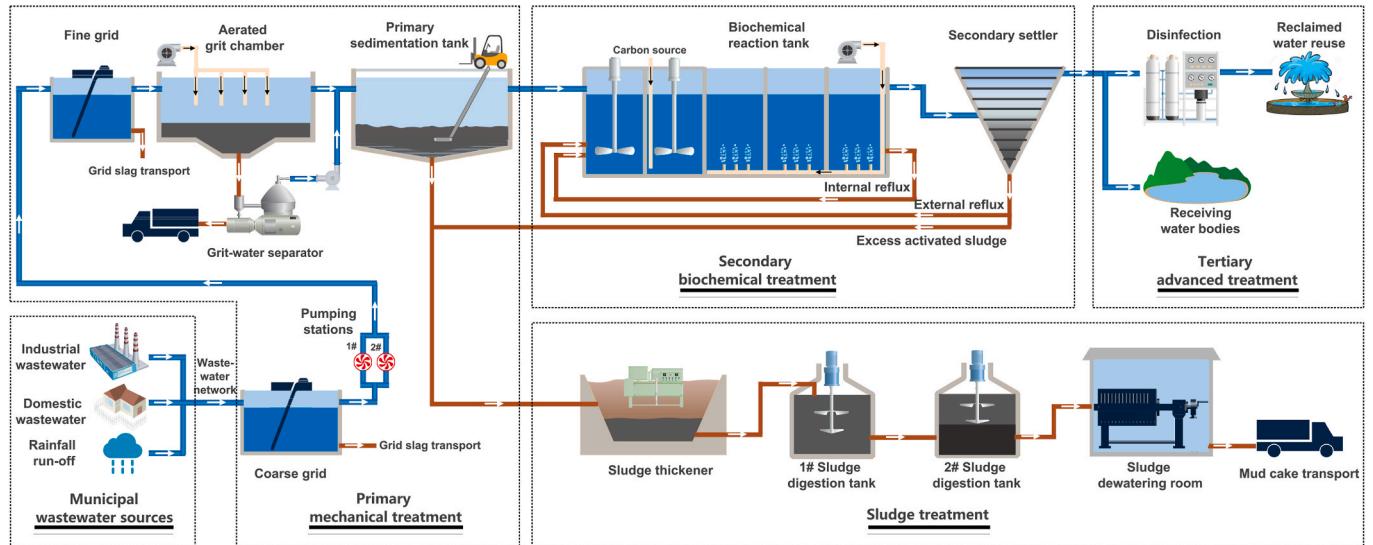


Fig. 1. Flow chart of municipal wastewater treatment process.

method with multi-objectives (Villez et al., 2016). These researches realized the optimal trade-off between production cost and revenue about the optimization problems of different types of sensors, but they neglected the difference in the importance of sensor function at different locations, leading to extreme flexibility and increasing the computational complexity.

Generally speaking, the sensor placement optimization problem in WWTPs can be converted to a discrete multi-objective optimization problem. For one thing, deterministic algorithms such as branch and bound algorithm (Saghand et al., 2019), dynamic programming (Zeng et al., 2019), filled function method (Zhang et al., 2017), and mixed integer nonlinear programming (Moazeni and Khazaei, 2021) are adopted to solve the optimization problem. Nevertheless, the time complexity of these approaches exhibits exponential characteristics. For another, some stochastic algorithms, such as particle swarm optimization (PSO) (Phoemphon et al., 2020), genetic algorithm (GA) (Asadi and McPhedran, 2021), ant colony optimization (ACO) (Sun et al., 2019), and simulated annealing (SA) (Li et al., 2021), are also widely applied to discrete multi-objective optimization problems, so as to get optimal solutions in a reasonable amount of time. Yet these methods may fail to yield stable solutions or deviate from the high-quality solutions because of the improper setting of complex parameters. Therefore, the purpose of this research is to find a new method that can quickly and effectively obtain the global optimal solution of the flow rate sensor placement optimization problem in the WWTP.

The main contributions and novelties of this study are listed as follows. (1) A discrete weighted flow rate sensor placement optimization model considering the importance of flows in different units is established, ensuring operating cost, data quality and reliability of the WWTP; (2) A discrete multi-objective state transition algorithm (discrete MOSTA) is proposed to find optimal solutions of the sensor optimization problems; (3) An evaluation strategy based on the corrosion degree and maintenance cost is designed to select the most suitable solution for further industrial application. The validity and superiority of the proposed method is demonstrated by applying to three different WWTPs.

The rest of this paper is organized in accordance with the following scheme. In Section 2, the typical municipal wastewater treatment process is introduced, and a discrete weighted flow rate sensor placement optimization model is established. In Section 3, a discrete multi-objective state transition algorithm (discrete MOSTA) is proposed for sensor optimization as well as an evaluation strategy for decision making. Then in Section 4 the experimental results of three different WWTPs

are compared and analyzed. Section 5 concludes this paper and provides some potential future research directions and challenges.

2. Modeling and analysis for sensor placement

This section briefly introduces the typical municipal wastewater treatment process, and leads to the necessity of optimizing sensor placement. Then a discrete weighted flow rate sensor placement model considering the importance of flows in different units of the WWTP is established, offering a model basis for the subsequent research.

2.1. Process analysis

The municipal wastewater treatment process is a complex physical and biochemical reaction process consisting of serially connected units. The activated sludge method is one of the most widely used municipal wastewater treatment methods, whose typical flow chart is shown in Fig. 1.

The whole process includes three levels, namely primary mechanical treatment, secondary biochemical treatment, and tertiary advanced treatment. Firstly, the grid, grit chamber and primary sedimentation tank in the primary treatment process are used to remove large suspended impurities and particle sands. Next, in the secondary treatment process, the biochemical reaction tank is the core procedure where wastewater is mixed with the active sludge containing microorganisms, stirred and aerated. The organic contaminants are degraded via the biological coagulation, adsorption and oxidation of the microorganisms. Then the treated water flows into the secondary settler for solid-liquid separation. The upper water enters the next step. While part of the lower resulting sludge flows back to the anoxic zone to ensure the continuous biochemical reactions, and the rest is mixed with the sludge from primary treatment process and transported out after concentration, digestion and dehydration. Finally, the effluent from the upper layer of the secondary settler is reused after tertiary advanced treatment by activated carbon ion exchange, electrodialysis or ultraviolet disinfection.

The municipal wastewater treatment process is a dynamic non-stationary process and flow rate is one of the fluctuating factors. If flow rate is too high, the treatment time will be short, leading to insufficient degradation of pollutants and substandard effluent quality. Conversely, if flow rate is too low, the retention time of wastewater will be too long, reducing the treating efficiency and resulting in waste of chemicals and electric energy. Hence, it is significant to optimize the

flow rate sensor placement scheme to monitor flow rates and guarantee the efficient and stable operation of the WWTP.

2.2. Optimization model for sensor placement

2.2.1. Definitions of optimization objectives

Based on the above analysis and literature (Villez et al., 2013), the objectives of sensor placement optimization problem in this paper are sensor cost, the degree of variable observability, and the degree of measurement redundancy, defined as follows:

- (1) Sensor cost: Sensor cost here means an operational cost of sensor ownership and installation, mainly depending on the type, number, and univalence of the sensors installed in the plant.
- (2) Variable observability: A structurally observable variable needs to tally with one of the following requirements: (i) the variable is measured directly, or (ii) a unique value of the variable is deducible from other available measurements by means of mathematical representations of the measured process.
- (3) Measurement redundancy: A measurement of the variable is regarded as structurally redundant in case of the measured variable remains observable after removing the measurement from the available measurements. Because of the practical issues present in the WWTP, the reliability of sensors will decrease gradually, it is significant to have some level of measurement redundancy in the sensor network to detect sensor faults on-line and improve global reliability of the plants.

As the sensor cost is the expenditure of the plant, and the degree of variable observability and measurement redundancy is the benefit it brings, which characterizes the information richness and reliability obtained during the operation of the plant, the optimal sensor placement scheme of the WWTP aims to achieve: (i) the minimum sensor cost, (ii) the maximum degree of variable observability, and (iii) the maximum degree of measurement redundancy.

2.2.2. Graphical representation and labeling

To evaluate the degree of variable observability and measurement redundancy for a given sensor placement scheme, the studied WWTP can be represented as a topological graph G , based on the graph theory (Sun et al., 2021). The flow junction or unit process (reactor or settler) in the plant is represented by a node visualized as a circle, and the flow is represented as an edge visualized as a directed arc. The number of nodes and edges is noted as m and n , respectively. To ensure the integrity of G , each flow should connect two nodes. So, an environment node is introduced to connect the system boundary points with external environment, making G mathematically consistent.

In the general case, each flow is assumed to be a feasible candidate for sensor placement. Any particular selection of sensors for all candidate locations is defined as a sensor placement scheme, noted as \mathbf{x} (dimensions $n \times k$), where n and k denote the number of flows and variable types, respectively. And all available sensor placement schemes constitute the solution set X . In this paper, only flow rate of the plant is considered so that k equals to 1. And the overall number of available sensor locations equals to n . Therefore, a sensor placement scheme is denoted as $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$. The observability vector δ_O (dimensions $1 \times n$) is a Boolean vector used to symbolize the presence (1) or absence (0) of the sensor on a particular candidate location. As with δ_O , the redundancy vector δ_R (dimensions $1 \times n$) is used to symbolize the redundancy (1) or nonredundancy (0) of an installed sensor. (Note that if there is no sensor on the location, it is marked as 0.)

The observability vector δ_O and redundancy vector δ_R are calculated as functions of topological graph G and sensor placement scheme \mathbf{x} , based on the GENOBS and GENRED algorithms that are valid labeling algorithms to mark observability and redundancy (Kretsovalis and Mah, 1988). The functions are noted as L_O and L_R , then δ_O and δ_R are denoted

Table 1
Labeling results of observability and redundancy.

	Measured	Deducible	Observability	Redundancy
Case I	No	No	0	0
Case II	No	Yes	1	0
Case III	Yes	No	1	0
Case IV	Yes	Yes	1	1

as follows:

$$\delta_O = L_O(G, \mathbf{x}) = \text{GENOBS}(G, \mathbf{x}) \quad (1)$$

$$\delta_R = L_R(G, \mathbf{x}) = \text{GENRED}(G, \mathbf{x}) \quad (2)$$

Based on the labeling algorithms, four labeling rules are defined below. The labeling results of variable observability and measurement redundancy are shown in Table 1, where 1 means observable or redundant, while 0 otherwise.

- (1) An unmeasured variable that is not deducible is unobservable and nonredundant.
- (2) An unmeasured variable that is deducible is observable but nonredundant.
- (3) A measured variable that is not deducible is observable but nonredundant.
- (4) A measured variable that is deducible is observable and redundant.

2.2.3. Multi-objective optimization model for sensor placement

The purpose of flow rate sensor placement optimization in WWTPs is minimizing sensor cost and maximizing the degree of variable observability and measurement redundancy simultaneously. The sensor cost function f_C is equal to the product of the sensor placement scheme \mathbf{x} and cost weight vector \mathbf{w}_C for all sensor candidate locations, defined as follows:

$$f_C(\mathbf{x}) = \mathbf{w}_C \cdot \mathbf{x} = \sum_{i=1}^n (w_{C,i} \cdot x_i) \quad (3)$$

Similarly, the variable observability objective f_O is equal to the product of complementary vector of δ_O and observability weight vector \mathbf{w}_O , and the measurement redundancy objective f_R is equal to the product of complementary vector of δ_R and redundancy weight vector \mathbf{w}_R , defined as follows:

$$f_O(\mathbf{x}) = \mathbf{w}_O \cdot [1 - \delta_O(G, \mathbf{x})] = \sum_{i=1}^n (w_{O,i} \cdot (1 - \delta_{O,i})) \quad (4)$$

$$f_R(\mathbf{x}) = \mathbf{w}_R \cdot [1 - \delta_R(G, \mathbf{x})] = \sum_{i=1}^n (w_{R,i} \cdot (1 - \delta_{R,i})) \quad (5)$$

By minimizing (4) and (5), the maximal degree of variable observability and measurement redundancy can be achieved. Therefore, the optimization model of flow rate sensor placement optimization problem in this paper is formulated as follows:

$$\begin{aligned} \min \quad & \mathbf{F}(\mathbf{x}) = [f_C(\mathbf{x}), f_O(\mathbf{x}), f_R(\mathbf{x})] \\ \text{s.t.} \quad & \mathbf{x} = [x_1, x_2, \dots, x_n] \\ & f_C(\mathbf{x}), f_O(\mathbf{x}), f_R(\mathbf{x}) \in [0, n] \\ & D_i = \{0, 1\}, \quad i = 1, 2, \dots, n \end{aligned} \quad (6)$$

where $f_C(\mathbf{x})$, $f_O(\mathbf{x})$, and $f_R(\mathbf{x})$ are three conflicting objective functions to be optimized, \mathbf{x} is the decision variable consisting of n design variables, and D_i is the available value set of x_i .

Refer to the effect study of objectives on decision variables in Alwan (2015), from (6) one can conclude that the sensor cost objective function value f_C has a positive effect on the decision variable \mathbf{x} . There is a linear positive correlation between the sensor cost objective function value and the 1-norm of \mathbf{x} . But the variable observability objective function value f_O and measurement redundancy objective function value f_R have complex nonlinear relationships with the 1-norm of \mathbf{x} , represented as

equations (4) and (5), respectively.

2.2.4. Weighting factors design for sensor placement

Weighting factors are usually postulated to be equal for the observability and redundancy objectives, respectively, leading to extreme flexibility and abundant Pareto optimal solutions and increasing the complexity of selecting the final optimal solution. Moreover, considering the actual operation of the wastewater treatment plant, the importance of observability and redundancy of flow rate in different units is actually different. Therefore, it is necessary to design the weighting factors of the flows from in terms of variable observability and measurement redundancy.

The analytic hierarchy process (AHP) has been widely used in multi-attribute decision-making problems, which requires human to draw out subjective preference in the form of accurate numbers based on comparisons of each pair of alternatives or attributes. For example, AHP based geospatial aquaculture planning brings out more regions (Jayanthi et al., 2020), and the ranking lists of hotels and restaurants is obtained by AHP (Fang and Partovi, 2021). Whereas, the conventional AHP may have difficulties in quantifying the preference of the decision-makers due to the inexact and indefinite appraisals. Therefore, Ho and Singh combined fuzzy set theory with AHP and used fuzzy numbers to cope with ambiguous linguistic terms representing human appraisals, with application to synthesising wastewater treatment process and supplier selection (Ho et al., 2021) or optimizing weights of water quality index (Singh et al., 2021). To design proper weighting factors for flow rate sensor placement, the triangular fuzzy AHP (TFAHP) method is investigated in this paper, whose steps are summarized in Algorithm 1 (Table Algorithm 1) (Wang, 2021).

Algorithm 1

Pseudo-code for the TFAHP method

- Step1:** Determine the goal, criteria and alternatives, and then establish a three-layer hierarchical structure as shown in Fig. 2;
- Step2:** Construct the triangular fuzzy multiplicative preference relations (TFMPRs) of the criteria and flows, respectively, according to the analysis of the characteristics of the WWTP;
- Step3:** Check the acceptability of the TFMPRs constructed in Step 2. If they are acceptable, then go to Step 4; otherwise, go to Step 9 and alter the elements in the TFMPRs;
- Step4:** Derive an optimal fuzzy weight vector from each acceptable TFMPR;
- Step5:** Aggregate the local fuzzy weight vectors into global fuzzy weights as per a triangular fuzzy weighting operator;
- Step6:** Establish a possibility matrix from the global fuzzy weights;
- Step7:** Define the ranking indices of the global fuzzy weights based on the possibility matrix;
- Step8:** Derive the final global crisp weights of all the flows in accordance with the ranking indices;
- Step9:** End.

2.2.5. Significance of the sensor placement model

General research focuses on identifying fault data using a given set of sensors installed in the WWTPs, but little attention is paid to how to improve fault detection and identification by optimizing the sensor placement. Therefore, in this paper, a method is developed to obtain optimal sensor placements in terms of cost, information richness and reliability. In the sensor placement model, the sensor cost objective means the expense of sensor ownership and installation, the variable observability objective measures the information volume of process parameters, and the measurement redundancy objective measures the reliability of process measurements. Besides, the weighting factors of flows are designed to reduce computational complexity. The optimal sensor placement is beneficial to reduce cost, keep the reactions continuous and ensure the effluent up to standard, thereby improving the environmental friendliness and economic benefits of the WWTPs.

Since the objectives in the sensor placement model are set based on

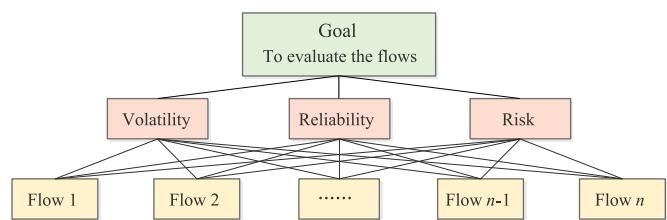


Fig. 2. Hierarchical structure of the flows importance evaluation problem.

structured criteria and then are not affected by the climatic components for various geographical settings (Schaffer-Smith et al., 2020), or cloud bursts and other hydrologic uncertainties (Foorginezad et al., 2021), the model can be used to provide a more cost-efficient reference scheme for the upgrade of existing plants or the optimized design of new plants all over the world. And the data collected from the sensors, in the future research, can be used to improve the ability to reduce random errors, i.e. improve noise reduction and estimation accuracy, and the redundant process data can be used for fault detection.

3. Discrete multi-objective optimization approach based on state transition algorithm

In this section, a discrete multi-objective state transition algorithm is proposed to solve the sensor optimization problem formulated in Section 2. Then an evaluation strategy is designed to select the most suitable solution for industrial application from the obtained Pareto optimal solutions.

3.1. Discrete multi-objective state transition algorithm

State transition algorithm (STA) is an intelligent optimization algorithm based on structural learning theory (Zhou et al., 2018, 2020). The core idea of STA is regarding the solution of the optimization problem as a state, and the process of finding the optimal solution by iterative updating as a state transition process (Han et al., 2017). The state space expression in modern control theory is used as a unified framework for generating candidate solutions, and then the state transformation operators are designed based on this framework to find the global optimal solution. Based on the basic STA, a novel discrete multi-objective state transition algorithm (discrete MOSTA) is proposed to obtain the optimal sensor placement schemes in WWTPs efficiently (Zhou et al., 2019). The basic framework of generating a new solution in discrete MOSTA is defined as (7):

$$\begin{cases} \mathbf{x}_{k+1} = A_k(\mathbf{x}_k) \oplus B_k(\mathbf{u}_k) \\ y_{k+1} = f(\mathbf{x}_{k+1}) \end{cases} \quad (7)$$

where $\mathbf{x}_k, \mathbf{x}_{k+1} \in \mathbb{Z}^n$ are the current state and the state of next generation, respectively, equivalent to the solutions of the optimization problem; \mathbf{u}_k is a function of the current state and historical states; A_k and B_k are state transformation operators, usually in the form of state transition matrixes; \oplus is an operation connecting two states; f is the evaluation function, and y_{k+1} is the function value at \mathbf{x}_{k+1} .

In discrete MOSTA, four state transformation operators composed of swap, shift, symmetry, and substitute transformation operator are used to permute current solution to generate candidate solutions, defined as follows:

(1) Swap transformation operator

$$\mathbf{x}_{k+1} = A_k^{swap} \mathbf{x}_k \quad (8)$$

where $A_k^{swap} \in \mathbb{Z}^{n \times n}$ is a random Boolean matrix with the function of exchanging two random position elements in the current state, which is called the swap permutation matrix. For instance,

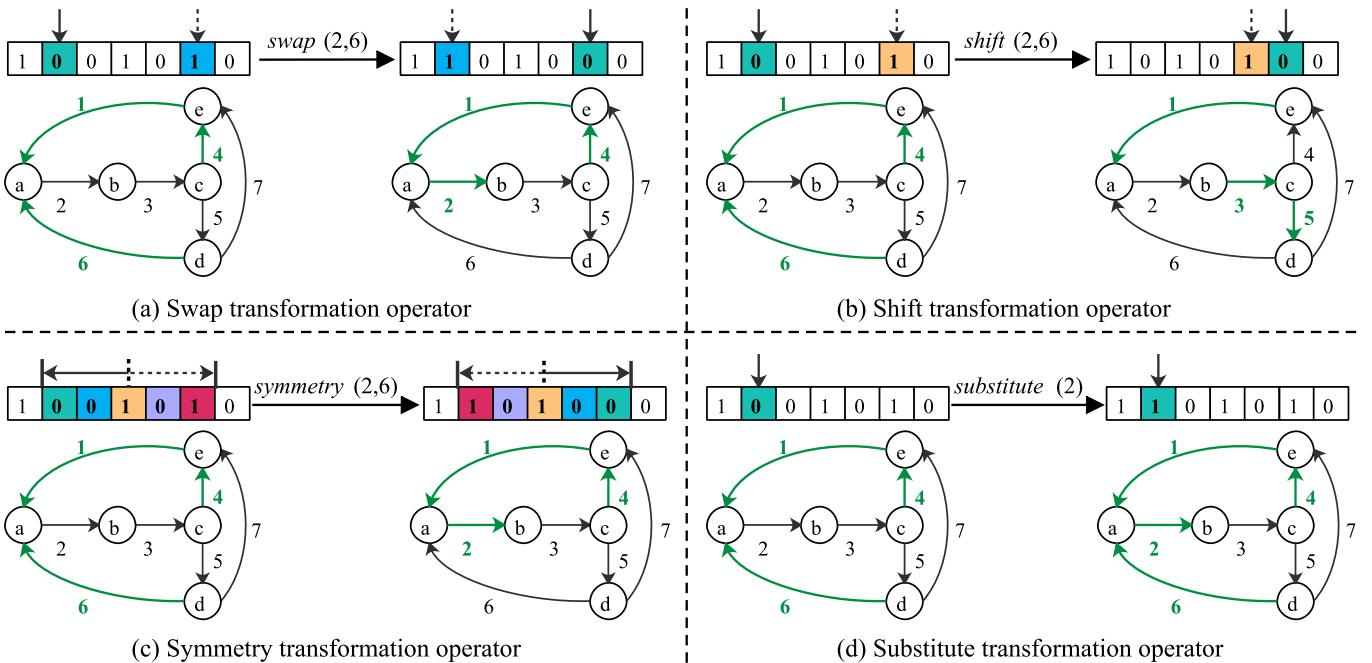


Fig. 3. Illustrations and applications of four state transformation operators.

position 2 and position 6 in \mathbf{x}_k are selected to be exchanged, and the illustration of swap transformation operator and its application in a simple WWTP are shown in Fig. 3(a). (Note that the numbers and arrows in bold and green in the graph representations indicate that flow rate sensors are installed in the corresponding flows, while others are not.)

(2) Shift transformation operator

$$\mathbf{x}_{k+1} = A_k^{\text{shift}} \mathbf{x}_k \quad (9)$$

where $A_k^{\text{shift}} \in \mathbb{Z}^{n \times n}$ is a random Boolean matrix with the function of moving a random position element behind another in the current state, which is called the shift permutation matrix. To make it more clearly, position 2 is selected to be shifted behind position 6, as described in Fig. 3(b).

(3) Symmetry transformation operator

$$\mathbf{x}_{k+1} = A_k^{\text{sym}} \mathbf{x}_k \quad (10)$$

where $A_k^{\text{sym}} \in \mathbb{Z}^{n \times n}$ is a random Boolean matrix with the function of inverting all elements between two random positions in the current state, which is called the symmetry permutation matrix. The illustration of symmetry transformation operator and its application is shown in Fig. 3(c).

(4) Substitute transformation operator

$$\mathbf{x}_{k+1} = A_k^{\text{sub}} \mathbf{x}_k + B_k^{\text{sub}} \mathbf{u}_k \quad (11)$$

where the sum of A_k^{sub} , and $B_k^{\text{sub}} \in \mathbb{Z}^{n \times n}$ is a random Boolean matrix with the function of replacing the value of a random position element in the current state, which is called the substitute permutation matrix. Fig. 3(d) gives the illustration of substitute transformation operator and its application.

Candidate solutions are generated by the state transformation operators defined in (8)-(11), and then the following update strategies are introduced to select optimal solutions for flow rate sensor placement problem efficiently.

(1) Efficient nondominated sorting strategy

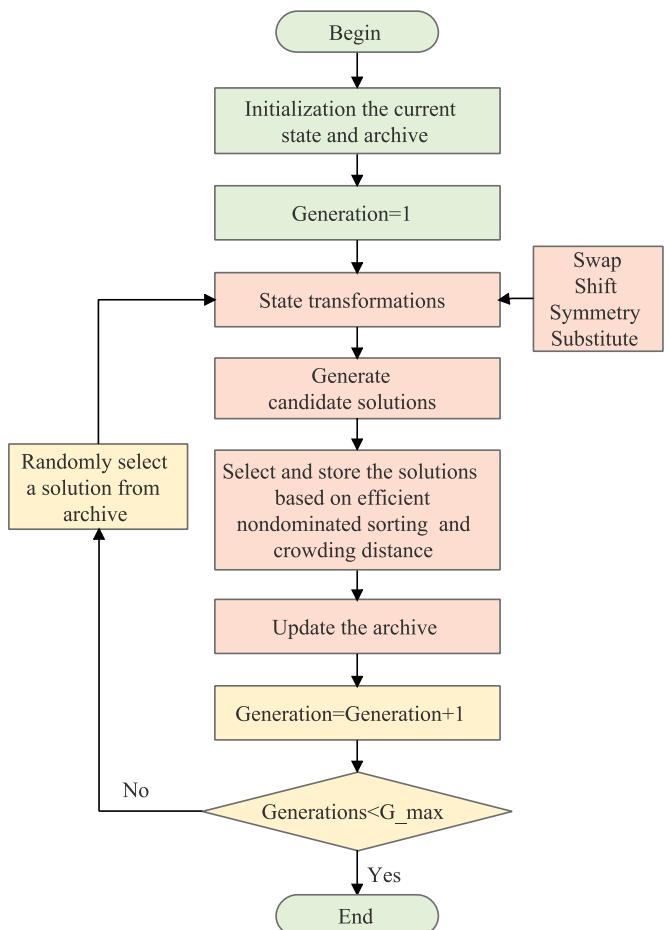


Fig. 4. The flow chart of discrete MOSTA.

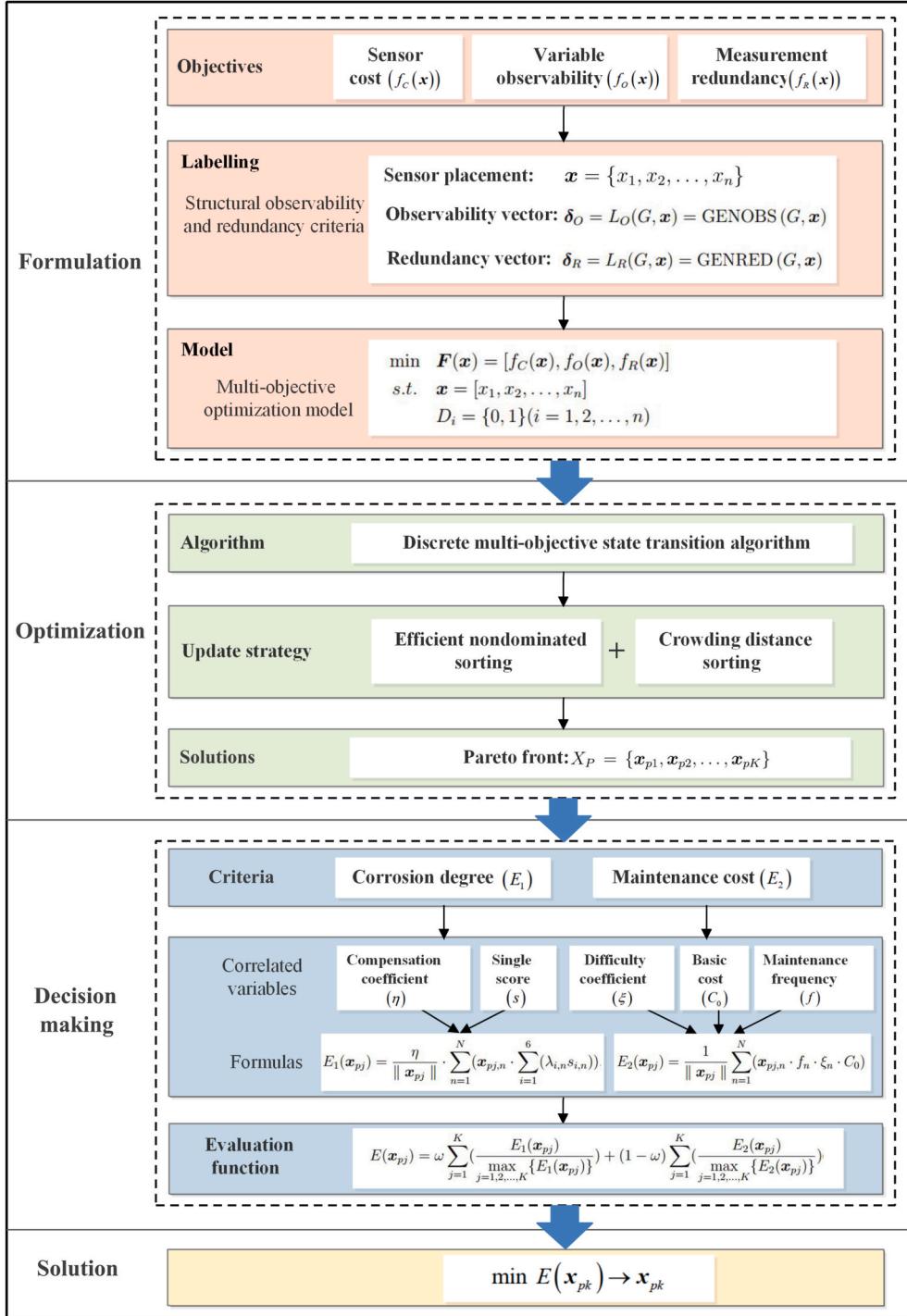


Fig. 5. Framework of the flow rate sensor placement optimization process.

Nondominated sorting is widely used to select solutions for multi-objective optimization problems, where solutions are designated different fronts based on the dominance relationship. Because of determining the front of all solutions on the same front as a whole, the computational expense of the typical nondominated sorting approach is relatively high, especially for large populations. Hence, an efficient nondominated sorting strategy is introduced where the front of each candidate solution is determined separately until all fronts are identified (Zhang et al., 2014). The merit of this strategy is that a solution to be assigned only needs to be compared with solutions that have been assigned to a certain front, which greatly reduces repeated comparisons

and ameliorates computational efficiency.

(2) Crowding distance diversity maintenance strategy

To make the solutions in the Pareto front well-distributed, a diversity maintenance strategy based on the crowding distance is introduced to filter out the intensive candidate solutions (Han et al., 2020). The crowding distance D_l of the candidate solution x_l is defined as follows:

$$D_l = \sum_{m=1}^M \frac{f_m(x_{l+1}) - f_m(x_{l-1})}{f_m^{\max} - f_m^{\min}} \quad (12)$$

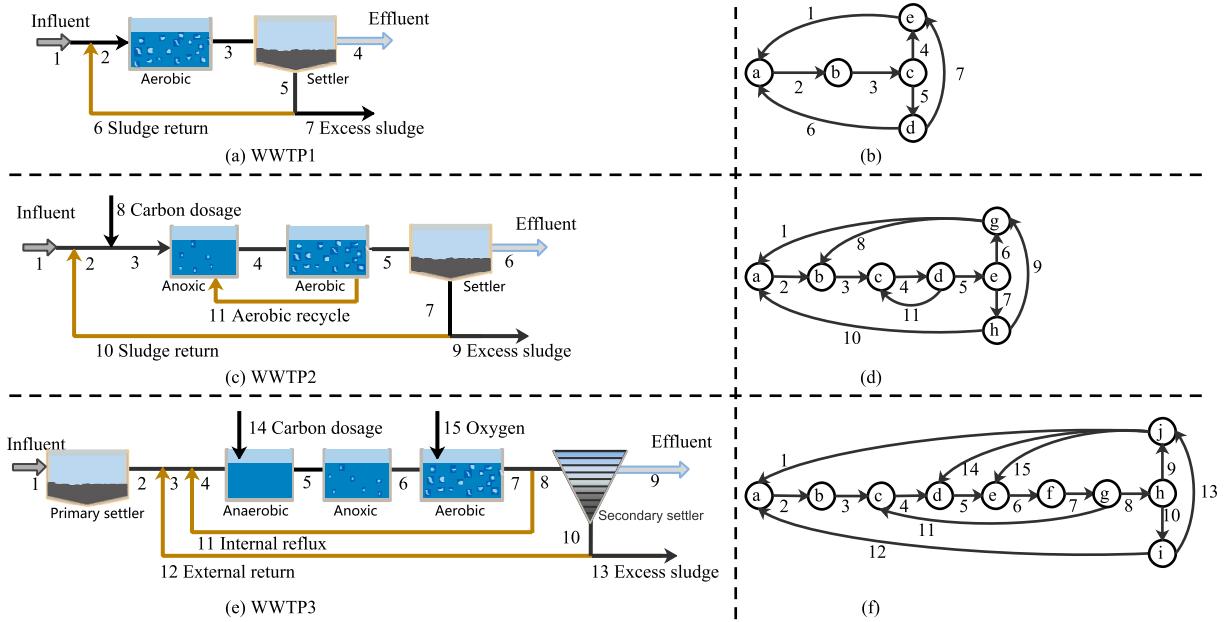


Fig. 6. Schemes (left) and graphical representations (right) of the WWTPs.

where M is the number of objective functions, l is the number of the candidate solution in a front, f_m^{\max} and f_m^{\min} are the maximum value and minimum value of function f_m , respectively.

On the basis of the efficient nondominated sorting strategy and crowding distance diversity maintenance strategy, candidate solutions in different fronts with smaller front number are selected, and candidate solutions in same front with larger crowding distance are selected.

The flow chart of discrete MOSTA used to solve the sensor placement optimization problem in this paper is shown in Fig. 4. Firstly, the current state and archive are initialized, and candidate solutions are generated through four state transformation operators. Secondly, the optimal solutions are selected and stored in the archive by the efficient non-dominated sorting strategy and crowding distance diversity maintenance strategy. Then the archive is updated and a solution is selected to repeat the above operations until the termination condition is reached. Finally, the optimal solutions of the optimization problem is composed of the solutions stored in the archive.

3.2. Evaluation strategy

The discrete MOSTA for multi-objective optimization problems provides a set of Pareto optimal solutions, represented as $X_P = \{x_{p1}, x_{p2}, \dots, x_{pK}\}$. For reasons of selecting the most suitable solution for industrial application, a Pareto optimal solutions evaluation strategy based on the average corrosion degree and average maintenance cost is designed in this section. The evaluation strategy includes two separate parts: (1) The average corrosion degree of the flows equipped with sensors in different processing units; (2) The average maintenance cost of the flow rate sensors installed in the WWTP.

In the first part of the evaluation strategy, a corrosion status evaluation system is formulated to evaluate the corrosion degrees of flows in different processing units (Foorginezhad et al., 2021). As the flow rate sensors have long been subjected to various forms of corrosion, such as the flow wear of solid particles, erosion of chemical substances and corrosion of microorganism, and the corrosion mechanisms are extremely complex, the operation status and service life of sensors are affected negatively.

In the corrosion status evaluation system, six types of corrosion are considered, consisting of acid corrosion, alkali corrosion, organic corrosion, inorganic corrosion, microbiologically induced corrosion,

and mechanical shock corrosion. To quantify the corrosion effect, each corrosion type is assigned a score s_i ($i = 1, 2, \dots, 6$), an integer between 0 and 3. The score of 0 means impact-free, while 1, 2, and 3 signify low impact, medium impact, and high impact, respectively. The larger s_i is, the greater the effect of the corrosion type on sensors is. Moreover, according to the superposition effect of different corrosion types, which means the sum of corrosion effect of single corrosion type is less than the comprehensive effect of all corrosion types, a corrosion effect compensation coefficient is defined as η . Then the calculation function of the average corrosion degree for X_P is defined as (13):

$$E_1(x_{pj}) = \frac{\eta}{\|x_{pj}\|} \sum_{n=1}^N \left(x_{pj,n} \cdot \sum_{i=1}^6 (\lambda_{i,n} s_{i,n}) \right), \quad j = 1, 2, \dots, K \quad (13)$$

where $\|x_{pj}\|$ is the number of installed sensors in solution x_{pj} , N is the total number of flows in the WWTP, n is the flow index, λ is the weight vector of six corrosion types. The higher the final score, the stronger the corrosion effect of the flow on the installed sensor.

In the second part of the evaluation strategy, the average maintenance cost of flow rate sensors of each optimal solution is computed. Maintenance costs of sensors have drawn great attention of many operators, due to that the maintenance operation is directly related to the service life of the sensors and the accuracy of the measured data (Abbasi et al., 2021). Sensors maintained in sound conditions can ensure the authenticity and sustainability of measured data, so as to reduce excess data processing and provide more accurate reference data for process monitoring and exception elimination.

The routine maintenance operations of flow rate sensors mainly include three forms: cleaning, calibration, and repair. Because the materials and structures of different types of sensors are not completely the same, the corrosion status in the same wastewater environment is different, as well as the maintenance cost. Therefore, the basic maintenance cost of the flow rate sensor studied in this paper is defined as C_0 . As the flow rate sensors are installed in different units of the WWTP, complexity of the sensor maintenance operations is different as well. Then a maintenance difficulty coefficient (ξ) is defined to measure the operation complexity. In addition, the maintenance frequency (f) of a flow rate sensor is defined, which is determined by the flow velocity, turbidity, temperature, pH, etc. Thus, the calculation function of the average maintenance cost for X_P is defined as (14):

$$E_2(\mathbf{x}_{pj}) = \frac{1}{\|\mathbf{x}_{pj}\|} \sum_{n=1}^N (\mathbf{x}_{pj,n} \cdot f_n \cdot \xi_n \cdot C_0), \quad j = 1, 2, \dots, K \quad (14)$$

As the two partial calculation functions are obtained, the holistic evaluation function is given in a normalized form as (15). And the solution possessing minimum evaluation value is selected to be the final best solution.

$$\begin{aligned} \min E(\mathbf{x}_{pj}) = & \omega \sum_{j=1}^K \left(\frac{E_1(\mathbf{x}_{pj})}{\max_{j=1,2,\dots,K} \{E_1(\mathbf{x}_{pj})\}} \right) \\ & + (1 - \omega) \sum_{j=1}^K \left(\frac{E_2(\mathbf{x}_{pj})}{\max_{j=1,2,\dots,K} \{E_2(\mathbf{x}_{pj})\}} \right) \end{aligned} \quad (15)$$

where ω is a weighting factor, adjusted based on the plant configuration and operating experience.

3.3. Sensor placement optimization framework

A comprehensive study of the weighted flow rate sensor placement model and the discrete MOSTA gives rise to the optimization framework of the flow rate sensor placement problem in the WWTP, as shown in Fig. 5.

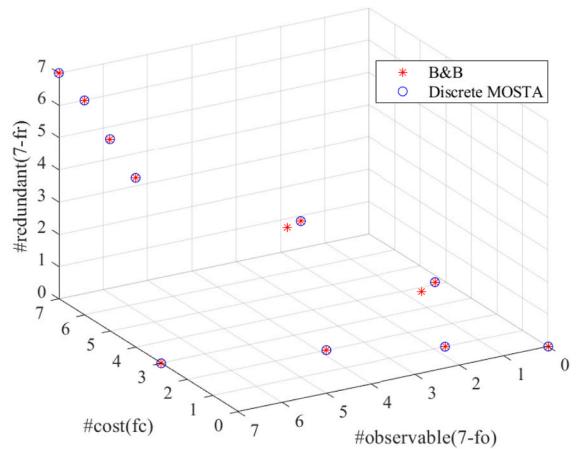
Based on the three objectives defined in the flow rate sensor placement problem, structural labeling method and weighting factor design method are adopted to establish the weighted discrete multi-objective optimization model. To minimize the sensor cost, and maximize the degree of variable observability and measurement redundancy simultaneously, a novel discrete MOSTA is proposed to search for Pareto optimal solutions. To select the most suitable solution for further industrial application, an evaluation strategy comprised of two separate parts is designed: the first part is calculating the average corrosion degree of the flows equipped with sensors in different processing units, and the second part is calculating the average maintenance cost of the flow rate sensors installed in the WWTP. Finally, the solution with minimum evaluation value is selected to be the final most suitable solution.

4. Experiments and discussion

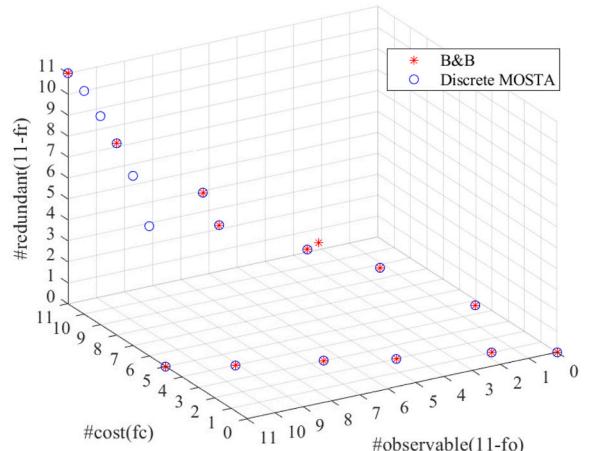
In this section, to demonstrate the practicality of the proposed weighted flow rate sensor placement model and discrete MOSTA for flow rate sensor optimization problem in WWTPs, experiments are carried out in three WWTPs with different configurations shown in Fig. 6: (i) WWTP1 - a simple organics removing plant (Villez et al., 2016), (ii) WWTP2 - an organics and nitrogen removing plant, and (iii) WWTP3 - a complex biological nitrogen and phosphorus removing plant. In Fig. 6, on the left side are the schemes of the studied WWTPs, and on the right side are the corresponding graphical representations obtained on the basis of the graph theory. In the graphical representations, each flow junction or unit process is represented by a node visualized as a circle and each flow between nodes is represented as an edge visualized as an arc. An environment node (node e in Fig. 6(b), and node g in Fig. 6(d), node j in Fig. 6(f)) is introduced to connect the system boundary points with external environment and thereby represents the mass balances over the whole plant.

4.1. Performance analysis of the discrete MOSTA

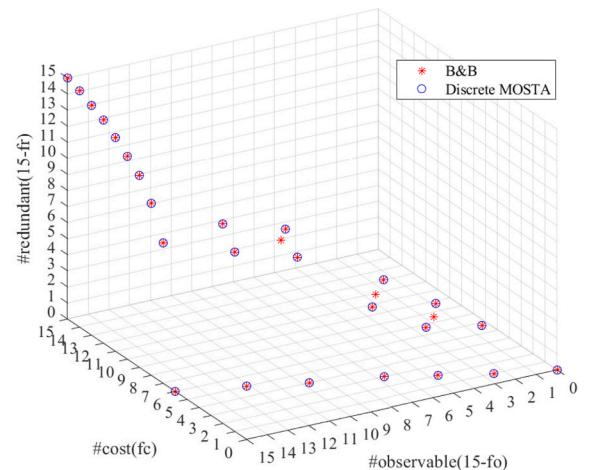
In order to analyze the performance of the proposed discrete MOSTA, the branch-and-bound algorithm (B&B) (Villez et al., 2016) is applied to optimize the flow rate sensor placement of the same WWTPs for comparison, respectively. After consulting the previous literature, the branch-and-bound algorithm is a typical multi-objective combinatorial optimization method that have been widely applied in many fields, such as sensor placement problem (Villez et al., 2020) and the knapsack problem with conflicts (Coniglio et al., 2021).



(a) WWTP1



(b) WWTP2



(c) WWTP3

Fig. 7. Pareto-fronts obtained by discrete MOSTA and B&B.

Table 2

Comparison results of the B&B and discrete MOSTA.

Plant	Problem Dimension	Algorithm	<i>S</i>	Time/(s)
WWTP1	7	B&B	1.201	4.231
		Discrete MOSTA	1.098	8.721
WWTP2	11	B&B	1.247	42.408
		Discrete MOSTA	0.780	35.156
WWTP3	15	B&B	1.001	2418.833
		Discrete MOSTA	0.784	578.667

For the three studied WWTP3 in Fig. 6, the Pareto-fronts obtained by discrete MOSTA and B&B are shown in Fig. 7. The solutions obtained by the two algorithms are not completely coincident. Specifically, in Fig. 7(a), the number of Pareto optimal solutions obtained by discrete MOSTA is less than that of B&B, and the values of partial solutions are different. In Fig. 7(b), the number of solutions obtained by discrete MOSTA is more than that of B&B on the whole. And in Fig. 7(c), the solutions obtained by discrete MOSTA is similar to that of B&B, but some densely distributed solutions are partially filtered out. As it is hard to justify the Pareto-fronts obtained by different methods, let alone evaluate their merits and demerits. Therefore, a performance parameter - the spacing metric (*S*) is introduced to assess the distribution and convergence of the obtained Pareto-fronts (Han et al., 2020). For the Pareto-front $X_P = \{x_{p1}, x_{p2}, \dots, x_{pK}\}$, the spacing metric (*S*) is defined as follows:

$$S(X_P) = \sqrt{\frac{1}{K-1} \sum_{v=1}^K (S_{average} - S_v)^2} \quad (16)$$

$$\text{where } S_v = \min_{v \neq v'} \left\{ \sum_{m=1}^M |f_m(x_v) - f_m(x_{v'})| \right\} \quad (17)$$

$$S_{average} = \frac{1}{K-1} \sum_{v=1}^K S_v \quad (18)$$

The distribution and convergence of the non-dominated solutions change for the better in pace with the diminution of *S*.

In Table 2, the discrete MOSTA and B&B are implemented 20 times independently. The average value of *S* of discrete MOSTA is smaller than that of B&B, indicating that discrete MOSTA performs better than B&B. The quantitative analysis proves that discrete MOSTA offers a more efficient and better-distributed Pareto-front for the flow rate sensor optimization problem.

As for the computational complexity of discrete MOSTA and B&B, analysis is based on the method in Curry and Dagli (2014). Assuming that the two optimization algorithms are to be performed on a *N* dimensional problem, of which the population size and objective number are *SE* and *M*, respectively. Then the time complexity of discrete MOSTA is $O(M \cdot SE \cdot \sqrt{SE})$ in the best case and $O(M \cdot SE^2)$ in the worst case. While the time complexity of B&B is $O(2^N)$, which increases exponentially as the dimension increases and even leads to dimensional disaster. The comparison of the running time of discrete MOSTA and B&B is listed in the last column of Table 2, which shows that discrete MOSTA has a comprehensive time superiority over B&B as the dimensionality of the optimization problem increases. In summary, therefore, discrete MOSTA obtains a Pareto-front with higher accuracy and better diversity than B&B, and has absolute time superiority in the flow rate sensor optimization problems with high dimensionality.

4.2. Analysis of the decision making results

Fig. 8 shows the sensor cost, the degree of variable observability and measurement redundancy of the Pareto optimal solutions obtained by the discrete MOSTA. The evaluation values obtained by the evaluation strategy designed in Section 3 are displayed by colors of the points. Referring to the practical plant configurations and operating experience, the weighting factor in the holistic evaluation function is set to be $\omega =$

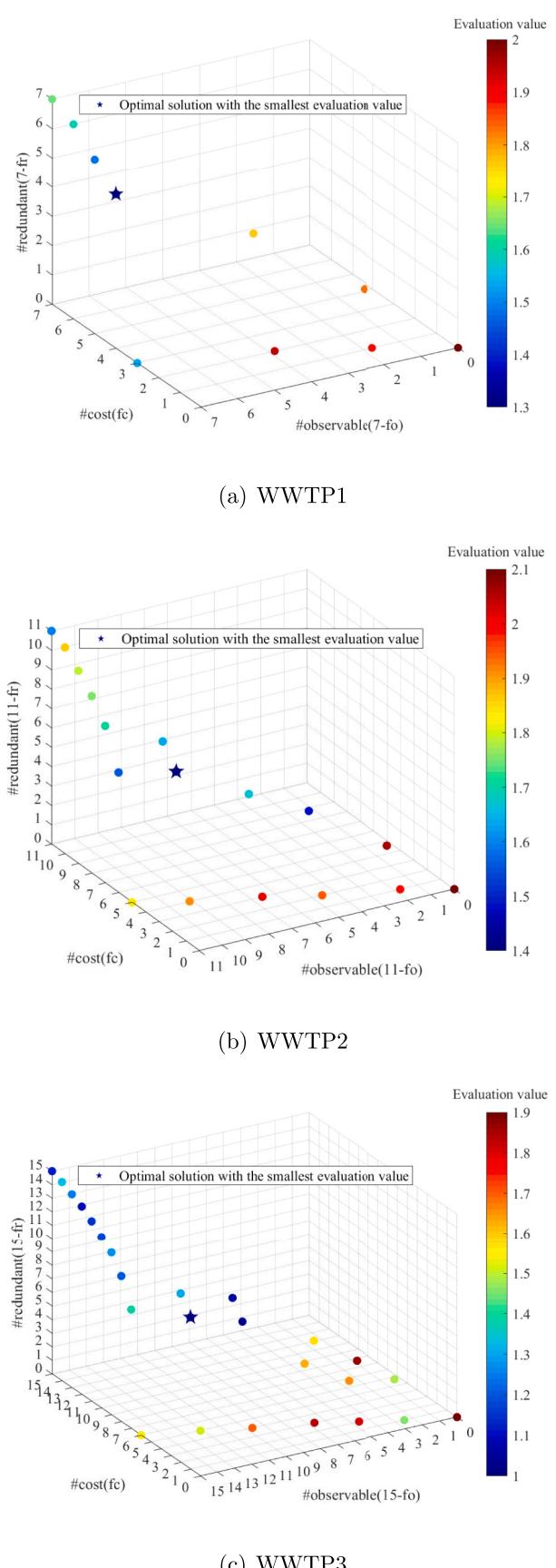


Fig. 8. Optimal solutions with evaluation strategy of three WWTPs.

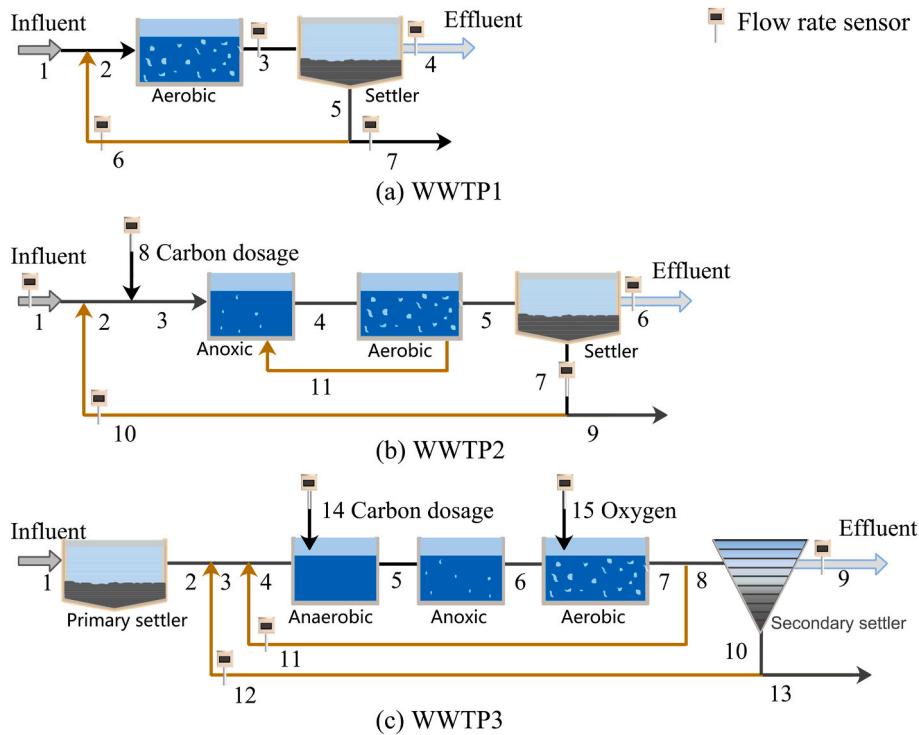


Fig. 9. Final optimal flow rate sensor placement schemes.

0.5. The solution possessing the minimum evaluation value is selected to be the final most suitable solution for further industrial application, which is marked as a blue solid pentagram (*). To display the final most suitable solution more intuitively, a graphical depiction of the selected solutions of the three WWTPs is shown in Fig. 9.

Aiming at the two challenges in sensor placement optimization problem in the wastewater treatment process mentioned in the introduction, the method proposed in this paper solves the problem successfully. Fig. 9(a),(b),(c) illustrate the final optimal flow rate sensor placement schemes of the three WWTPs. As shown in Fig. 9(a), there are 4 flow rate sensors installed in WWTP1, but we can get information of all the 7 flows with the redundant objective value of 5.25 simultaneously, realizing the goal of obtaining the most information and the greatest reliability with the least sensor cost. Similarly, the information objective value is 9.1 and the redundant objective value is 6.2 with 5 flow rate sensors installed in WWTP2, and the information objective value is 12.12 and the redundant objective value is 7.842 with 6 flow rate sensors installed in WWTP3.

5. Conclusions

This paper mainly studied the optimization problem of flow rate sensor placement in the municipal wastewater treatment plants. Based on the graph theory and structural observability and redundancy criteria, a graphical system model of the WWTP is constructed. A weighting factor design method combining the triangular fuzzy analytic hierarchy process and industrial conditions is introduced to measure the importance of flows in the different processing units, transforming the optimal flow rate sensor placement problem into a discrete multi-objective optimization problem. Then a novel discrete multi-objective state transition algorithm is proposed to efficiently obtain the optimal trade-off solution set. And an evaluation strategy comprised of the average corrosion degree and average maintenance cost is designed to select the most suitable solution for further industrial application. After comparative analysis, the discrete multi-objective state transition algorithm is proved to possess better comprehensive performance in regard

to the Pareto solution set and computational complexity compared with the branch-and-bound algorithm. At last the experimental results of three different WWTPs demonstrated that the flow rate sensors arranged according to the selected optimal placement schemes realized the goal of obtaining the most information and the greatest reliability with the least sensor cost, ensuring the data quality, operating costs and reliability of the WWTP are maintained at optimum levels.

In this paper, we mainly focus on the optimization placement of flow rate sensors in the whole process of WWTPs. Apart from the information about hydraulic flow rates, the information about other extensive variables (such as pH) and intensive variables (such as temperature and concentration of wastewater contaminants) is also crucial to the operation of the plant. Therefore, the optimal placement considering many different types of sensors in the WWTP can be further studied in the future. In this case, however, the mathematical equations used for estimation and labeling of variable observability and measurement redundancy should not only consider linear flow balance equations, but also include bilinear mass balance equations result from the product of the corresponding volume flowrate and component concentration. Although some researches have solved this optimization problem with bilinear equations, there is still a long and difficult way to solve it quickly and effectively. Beyond this, for the time being, the available results and applications are limited to academic examples, we are looking forward to applying the optimization methods to more complex and realistic wastewater treatment plant configurations.

Credit author statement

Wenting Li: Conceptualization, Methodology, Software, Investigation, Writing - Original Draft. **Jie Han:** Software, Validation, Writing - Review & Editing. **Yonggang Li:** Project administration, Funding acquisition. **Fengxue Zhang:** Visualization, Writing - Review & Editing. **Xiaojun Zhou:** Writing - Review & Editing. **Chunhua Yang:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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