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# An SEI<sup>3</sup>R information propagation control algorithm with structural hole and high influential infected nodes in social networks



Qian Zhang, Xianyong Li\*, Yongquan Fan, Yajun Du

School of Computer and Software Engineering, Xihua University, Chengdu 610039, China

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# ABSTRACT

Information propagation and control have great significance to manage public opinion in social networks. This paper aims to establish a novel information propagation model and the corresponding control algorithm to display the processes of information evolution, propagation and control. Considering high influential nodes and structural hole nodes highly influence the decision-making choices of public users' opinions in the information propagation process, the structural hole node discovery and score (SHNDS) algorithm and high influential node discovery and score (HINDS) algorithm are proposed, respectively. According to different node status for a social network, the network nodes are divided into susceptible (S), latent (E), ordinary influential infected  $(I_n)$ , structural hole infected  $(I_s)$ , high influential infected  $(I_h)$  and removed nodes (R). Then, combining with the SHNDS and HINDS algorithms, a new SEI<sup>3</sup> R information propagation model and the corresponding control algorithm are proposed. Experimental results show that when the high influential nodes are regarded as the initial information propagation nodes, the information propagates to all nodes of their communities which contained the initial information propagation nodes. The information propagation ranges are wider than SEIR and SEI<sup>2</sup>R models. When the structural hole nodes are taken as initial information propagation nodes, the information propagates to the entire networks. When the structural hole nodes and high influential nodes are both seen as the initial information propagation nodes, information propagation speeds and ranges are faster and wider than the former two cases, respectively. By controlling high influential nodes, structural hole nodes, and both of them, the numbers of nodes received the information are reduced in turn. These indicate that information control effects are gradually improved.

# 1. Introduction

With the progress of society and the development of the Internet, more and more people share their lives on the Internet. People can freely express and exchange their opinions on social network platforms such as Weibo, Twitter and ins. If their opinions are positive, then they will positively influence in our society; vice versa. Therefore, it has a great social significance to effectively clarify information propagation laws and control information propagation trends in social networks.

Many scholars have studied information propagation laws based on the aspects of network topology (Gong and Wang, 2018; Lei et al., 2018), such as homogeneous networks (Jeon et al., 2011), heterogeneous networks (Peng et al., 2020; Zhu and Ma, 2018), scale-free networks (Ahn and Jeong, 2006), small world networks (Gandica et al., 2010) and community structures (Zhang and Li, 2017). Newman et al. (Clauset et al., 2004; Newman and Girvan, 2004) not only defined the community, but also proposed two community structure discovery methods in different size networks. Gong et al. (2020) thought that there were a large number of relatively independent communities

which created an "echo chamber" effect within the community in social networks. In the opinion evolution process, the structural hole users can reduce opinion differences. They also made entire networks reach a consensus positively. According to the position particularity of nodes in social networks, Yang et al. (2015) divided the agents into opinion leaders, structural holes and ordinary agents. For the structural holes, research results mainly focused on mining structural hole spanners. Lou and Tang (2013) introduced two algorithms: HIS and MaxD, to mine the top-k structural hole spanners. They found that the top 1% of structural hole nodes control 80% of information propagation between communities and 25% of the entire network. Rezvani et al. (2015) designed an effective and scalable structural hole discovery algorithm using hinge nodes and bounded inverse closeness centralities of nodes in networks. Xu et al. (2019) formulated the top-k structural hole spanner problem to identify k nodes in a network. If these nodes were removed, the information propagation range would be prevented in the network. Zhao et al. (2018a) found that opinion leaders promoted the information propagation range. They further

E-mail addresses: xian-yong@163.com, lixy@mail.xhu.edu.cn (X. Li).

Corresponding author.

influenced the information propagation trend for followers in networks. Based on the majority and expert impacts, Das et al. (2018) proposed an innovative model to capture the opinion dynamics. Fully considering network topological features, Sara and Hassan (2018) proposed a HybridRank algorithm to detect influential spreaders. They also simulated the spreading processes using the SIR spreading model in complex networks. Iannelli et al. (2018) studied line featuring random transport rates between arbitrarily distant sites using a metapopulation approach. Liu et al. (2019) separated network nodes into four groups. They built a novel compartmental model. Their model analyzed the properties of rumor information on mobile social networks. Consequently, information propagation models with different network characters are worth considering.

Some scholars have also studied the effects of user behavior characteristics, interaction rules and interest characteristics in the information propagation process of social networks. Combining the opportunity, social trust and game choice motivation, Wan et al. (2019) proposed a novel information propagation model in social networks. Alp and Oguducu (2018) speculated on interesting topics based on user information and tweet topics. They further predicted whether users are experts in certain topics. Considering the diffusive features, social features, temporal features, first-adopter features, target features and finger features, Li et al. (2018) forecasted whether users participated in information propagation in social networks. Considering the number of neighbors, influences of neighbors and clustering coefficients, Chen et al. (2013) proposed a novel ClusterRank algorithm. Their method identified influential nodes in large-scale directed networks. Experimental results showed that their algorithm outperformed PageRank, LeaderRank, etc. Ullah et al. (2021) proposed an effective distancebased centrality algorithm to identify influential nodes in complex networks. They evaluated the algorithm performance using the SIR epidemic model on several real-world networks. Their experimental results showed that their algorithm outperformed the existing methods. (Lawyer, 2015) considered expected forces according to spreading powers of all nodes and force distributions of infected nodes in social networks. Zhang et al. (2021b) proposed a network embedding-based community detection algorithm. They also designed an influence maximization method to identify influential nodes in social networks. Their experimental results showed that the proposed methods are superior to existing methods. Thus, it is necessary to carefully develop information propagation models based on different characteristics of information propagation in social networks.

Based on infectious disease models, scholars have proposed many information propagation models. Wei and Xue (2019) and Zhao et al. (2018b) established two different SEIR information propagation models. Using these models, they analyzed information propagation processes in networks. Wang and Li (2018) proposed an information propagation model based on individual cognition in social networks. They further analyzed the influence of individual cognition in the information propagation process. Integrating with user multi-dimensional features and a dynamic evolution game mechanism, Xiao et al. (2019) established an information propagation model to predict public opinion evolution laws. Liu et al. (2017) proposed an SEIR rumor propagation model. They further analyzed two immunization strategies in the rumor propagation process. Zhang et al. (2018) divided the infected nodes into independent nodes, internal nodes and cross-network nodes. They further proposed an SI<sup>3</sup>R model on coupled networks. Under this model, it did not consider latent nodes (E) that know information but do not propagate it and are still in a wait state. Based on the properties of malware in Internet, Liu and Zhong (2017) developed an SDIRS model. They further discussed information propagation mechanisms and control strategies. Zhang et al. (2021a) divided infectious nodes into two groups in social networks. They proposed an SE2IR model to characterize information propagation in social networks. They also quantified the model parameters based on the network features such as perceived values of users, social intervention intensity and information timeliness.

Shahri and Moud (2021) proposed a hybrid block-based neural network model for high-resolution landslide susceptibility map. The prediction rate, success rate and precision–recall curve are well consistent with real situations. Evidently, it is very important to establish information propagation models in social networks.

However, few results have considered social network factors including user persistence, social intervention intensity and information timeliness into information propagation models. Meanwhile, few researches have studied information propagation models by combining the disease model with high influential nodes and structural hole nodes in social networks. Considering different opinions of high influential nodes and structural nodes have significant impacts on other users' opinions, infectious nodes are divided into high influential nodes, structural hole nodes and ordinary nodes. Based on a community division algorithm, the structural hole node discovery and score algorithm and high influential node discovery and score algorithm are proposed, respectively. Then, an SEI<sup>3</sup>R model and the corresponding control algorithm are designed to analyze the information evolution, propagation and control laws in social networks. To quantify the proposed SEI<sup>3</sup>R model parameters, the weight of influence power, intensity of social intervention, user persistence and information timeliness of a topic are introduced, respectively. Finally, a large number of comparative experiments evaluate the effectiveness of the proposed model and algorithms on Email and Facebook networks. This article's contributions are summarized as follows.

- Based on the community division algorithm (Blondel et al., 2008; Newman, 2004), the high influential node discovery and score (HINDS) algorithm and the structural hole discovery and score (SHNDS) algorithm are respectively proposed to find out the high influential nodes and structural hole nodes in the network.
- According to network node divisions, i.e., susceptible nodes, latent nodes, high influential infected nodes, ordinary infected nodes, structural hole infected nodes and removed nodes, a novel SEI<sup>3</sup>R information propagation model is developed.
- On the basis of the HINDS, SHNDS and SEI<sup>3</sup>R model, an SEI<sup>3</sup>R information propagation control algorithm is presented.
- Some experiments verify the effectiveness of the proposed algorithm and control strategies on Email and Facebook networks.

The rest of paper is organized as follows. In Section 2, we carefully introduce the structural hole node discovery and score algorithm, high influential node discovery and score algorithm, SEI<sup>3</sup>R information propagation model and SEI<sup>3</sup>R information propagation control algorithm, respectively. In Section 3, extensive simulation experiments confirm the authenticity and effectiveness of the HINDS, SHNDS, SEI<sup>3</sup>R information propagation model and its corresponding control algorithm on two real datasets. Finally, a summarization of this work and some future work are given in Section 4.

# 2. Description of a novel $\mbox{SEI}^{3}\mbox{R}$ information propagation control algorithm

Let a social network be denoted by  $G=(V,\mathcal{E})$ , where  $V=\{v_1,v_2,\cdots,v_n\}$  is a set of all users, and  $\mathcal{E}$  is a set of relationships (edges) between users in the network. Considering centralities of nodes in different communities and their authorities in specific fields, the structural hole node discovery and score algorithm and high influential node discovery and score algorithm based on the community division algorithm (Blondel et al., 2008; Newman, 2004) are proposed on social networks. Then, we establish a novel SEI³R information propagation model. Finally, we further propose an SEI³R information propagation control algorithm.

# 2.1. Structural hole node discovery and score (SHNDS) algorithm

According to Girvan–Newman theory (Easley and Kleinberg, 2010), the betweenness of  $v \in V(G)$ , which reflects the role and influence of the node v in the network, is the proportion of the number of paths passing through the node v and the total number of shortest paths. Let  $\sigma_{su}(v)$  denote the number of the shortest paths from the node s to u via v. Let  $\sigma_{su}$  denote the number of the shortest paths from the node s to u, and let  $\delta_{su}(v) = \frac{\sigma_{su}(v)}{\sigma_{su}}$ . For arbitrary  $v \in V(G)$  and two communities  $C_i$  and  $C_i$ , the betweenness of the node v is defined as follows.

$$B_{s\to u}(v) = \sum_{s\in C_i, u\in C_j, s\neq v\neq u} \delta_{su}(v) = \sum_{s\in C_i, u\in C_j, s\neq v\neq u} \frac{\sigma_{su}(v)}{\sigma_{su}}, \tag{1}$$

Then, the structural hole score of v between  $C_i$  and  $C_j$ , denoted by  $H(v, C_{ij})$ , can be defined as:

$$H(v, C_{ij}) = \max_{C} \left\{ B_{s \to u}(v) \right\}. \tag{2}$$

Therefore, the structural hole node discovery and score (SHNDS) algorithm is presented as Algorithm 1. Using the SHNDS algorithm, the structural hole nodes and their scores between communities can be obtained in the network.

**Algorithm 1:** Structural hole node discovery and score (SHNDS) algorithm

```
Input: The social network G = \{V, \mathcal{E}\}
    Output: The structural hole nodes and their scores
    B[v] \leftarrow 0, v \in C_i (i = 1, 2...k)
 2 for s \in C_i do
         S \leftarrow \text{empty stack}
 3
 4
         P[w] \leftarrow emptylist, w \in V
         \sigma[t] \leftarrow 0, t \in C_i; \sigma[s] \leftarrow 1
 5
         d[t] \leftarrow -1, t \in C_i; d[t] \leftarrow 0
 6
         Q \leftarrow \text{empty queue}
 7
         enqueue s \rightarrow Q
 8
         while Q not empty do
 9
              dequeue v \leftarrow Q
10
              push v \rightarrow S
11
              foreach neighbor w of v do
12
                  if d[w] < 0 then
13
                       enqueue w \rightarrow Q
14
                       d[w] \leftarrow d[v] + 1
15
                  if d[w] = d[v] + 1 then
16
                       \sigma[w] \leftarrow \sigma[w] + \sigma[v]
17
                        append v \to P[w]
18
19
         \delta[v] \leftarrow 0, v \in V
         while S not empty do
20
              pop w \leftarrow S
21
             22
23
             if w \neq s then
24
                 B[w] \leftarrow B[w] + \delta[w]
25
          Output all structural hole nodes and their scores B_{s \to t}(v)
26
```

# 2.2. High influential node discovery and score (HINDS) algorithm

In social networks, high influential nodes are not only those with high centrality coefficients, but also those with high authorities under different topics. For example, the user u has a high centrality in a community, but u is not good at political knowledge. Although the centrality of a user v is less than the centrality of u, v still has higher authority in the political field. At this time, the importance of v should be fully considered in the political information propagation process.

Based on the structural hole node score, the node influence score of v in community  $C_i$ , denoted by  $I(v, C_i)$ , can be defined as follows.

$$I\left(v,C_{i}\right) = \max_{C_{i}} \left\{ I\left(v,C_{i}\right), \alpha_{i} I\left(v,C_{i}\right) + \beta_{C_{ij}} H(v,C_{ij}) \right\}$$

$$\tag{3}$$

where  $\alpha_i$  and  $\beta_{C_{ii}}$  are two tunable parameters.

To measure importance of network nodes, the influence weight  $PR_{v_i}$  of the node  $v_i$ , i.e., its possibility that the node influences other nodes in each community, is computed by the PageRank algorithm (Langville and Meyer, 2011). If a node is a high influential node, then its PageRank score is higher than the other node scores in a community. Similarly, the influence weight of a node is larger than those of other nodes. The PageRank value of the node  $v_i \in V$  can be calculated by:

$$PR_{v_i} = \frac{1-\alpha}{N} + \alpha \sum_{v_j \in N_i} \frac{PR_{v_j}}{NF_{v_i}}$$
(4)

where  $\alpha$  generally takes as 0.85 (Haveliwala, 2002), and N and  $NF_{v_i}$  are the numbers of all nodes and followings of  $v_i$ , respectively.

Based on the PageRank algorithm, the high influential node discovery and score (HINDS) algorithm can be proposed as follows. Based on the HINDS algorithm, all high influential nodes and their scores in each community are easily obtained in a network.

**Algorithm 2:** High influential node discovery and score (HINDS) algorithm

**Input**: The network  $G = (V, \mathcal{E})$ 

Output: High influential nodes and their scores

- 1 Divide the community into  $C_1, C_2, ..., C_k$  by the community division algorithm (Blondel et al., 2008; Newman, 2004)
- 2 Initialize the weight values of nodes in the community randomly
- 3 Calculate node contributions by MAPPER
- 4 Obtain PageRank values using REDUCER function and Eq.(4)
- 5 Obtain the new PageRank value  $PR_v$  for each node  $v \in V(G)$  after N iterations
- 6 Obtain  $H(v, C_{ij})$  for each node  $v \in V(G)$  using Algorithm 1
- foreach  $C_i$  do

```
\begin{array}{lll} \mathbf{8} & & \mathbf{foreach} \ v \in C_i \ \mathbf{do} \\ \mathbf{9} & & & \mathbf{calculate} \ \alpha_i I(v, C_i) + \beta_{C_{ij}} H(v, C_{ij}) \\ \mathbf{10} & & \mathbf{if} \ PR_v > \alpha_i PR_v + \beta_{C_{ij}} H(v, C_{ij}) \ \mathbf{then} \\ \mathbf{11} & & & \mathbf{I}(v, C_i) = PR_v \\ \mathbf{12} & & \mathbf{else} \\ \mathbf{13} & & & & \mathbf{I}(v, C_i) = \alpha_i PR_v + \beta_{C_{ij}} H(v, C_{ij}) \\ \mathbf{14} & & & \mathbf{Obtain} \ \mathbf{node} \ \mathbf{influence} \ \mathbf{score} \ I(v, C_i) \ \mathbf{for} \ \mathbf{each} \ \mathbf{node} \ v \ \mathbf{in} \ C_i \\ & & & & \mathbf{I}(v, C_i) = \alpha_i PR_v + \beta_{C_{ij}} H(v, C_{ij}) \\ \mathbf{15} & & \mathbf{Output} \ \mathbf{the} \ \mathbf{top} \ k \ \mathbf{high} \ \mathbf{influential} \ \mathbf{nodes} \ \mathbf{and} \ \mathbf{their} \ \mathbf{scores} \\ \end{array}
```

# 2.3. SEI<sup>3</sup>R information propagation model and its control algorithm

# 2.3.1. SEI<sup>3</sup>R information propagation model

Based on different network status of nodes, the nodes of a social network are divided into susceptible nodes, latent nodes, high influential infected nodes, ordinary infected nodes, structural hole infected nodes and removed nodes. These nodes are concretely defined as follows.

Susceptible nodes (*S*): the nodes that do not get the information of a topic in the social network.

Latent nodes (*E*): the nodes that get information but do not propagate it; and they are in a holding state.

Infected nodes (I): the nodes that know information and propagate it. Considering the network status of nodes and their propagation capacities in the social network, the infected nodes are further divided into ordinary influential infected nodes ( $I_n$ ), structural hole infected nodes ( $I_s$ ) and high influential infected nodes ( $I_h$ ). Specifically, ordinary influence infected nodes are the nodes that have ordinary influence and occupy a large number of nodes in the information propagation process.

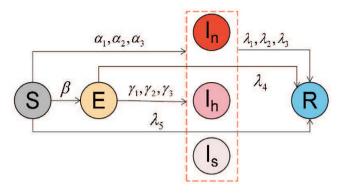


Fig. 1. State transitions for the six types of nodes in the social network.

Table 1
Descriptions of state transition parameters.

| Parameters  | Descriptions of the parameters               |  |  |  |  |
|-------------|--|--|--|--|--|
| A           | The rate that new users enter the network    |  |  |  |  |
| $\alpha_1$  | State transition probability from S to $I_n$ |  |  |  |  |
| $\alpha_2$  | State transition probability from S to $I_h$ |  |  |  |  |
| $\alpha_3$  | State transition probability from S to $I_s$ |  |  |  |  |
| β           | State transition probability from S to E     |  |  |  |  |
| $\gamma_1$  | State transition probability from E to $I_n$ |  |  |  |  |
| $\gamma_2$  | State transition probability from E to $I_h$ |  |  |  |  |
| $\gamma_3$  | State transition probability from E to $I_s$ |  |  |  |  |
| $\lambda_1$ | State transition probability from $I_n$ to R |  |  |  |  |
| $\lambda_2$ | State transition probability from $I_h$ to R |  |  |  |  |
| $\lambda_3$ | State transition probability from $I_s$ to R |  |  |  |  |
| $\lambda_4$ | State transition probability from E to R     |  |  |  |  |
| $\lambda_5$ | State transition probability from S to R     |  |  |  |  |

The structural hole nodes are bridge nodes connecting communities, which can be obtained from Algorithm 1. The high influential infected nodes are made up of the nodes that have high influence/relatively high social status and play an important role in the information propagation process. The high influential infected nodes can be got by Algorithm 2.

Removed nodes (*R*): the nodes that have received information but are not interested of it; and they do not propagate it.

Let S(t), E(t),  $I_h(t)$ ,  $I_n(t)$ ,  $I_s(t)$  and R(t) stand for the percentages of susceptible nodes, latent nodes, high influential infected nodes, ordinary influential infected nodes, structural hole infected nodes and removed nodes at time t in the social network, respectively. For simplicity, S(t), E(t),  $I_h(t)$ ,  $I_n(t)$ ,  $I_s(t)$  and R(t) are abbreviated as S, E,  $I_h$ ,  $I_n$ ,  $I_s$  and R, respectively. Obviously,  $S + E + I_h + I_n + I_s + R \equiv 1$ . Based on user status and features in the information propagation process, state transitions of the six types of nodes are shown in Fig. 1. Their state transition parameters are set as in Table 1. Then, the SEI³R information propagation model is established as following.

$$\begin{split} \frac{dS}{dt} &= A - \alpha_1 S I_n - \alpha_2 S I_h - \alpha_3 S I_s - \beta S E - \lambda_5 S, \\ \frac{dE}{dt} &= \beta S E - \left( \gamma_1 + \gamma_2 + \gamma_3 \right) E - \lambda_4 E, \\ \frac{dI_n}{dt} &= \alpha_1 S I_n + \gamma_1 E - \lambda_1 I_n, \\ \frac{dI_h}{dt} &= \alpha_2 S I_h + \gamma_2 E - \lambda_2 I_h, \\ \frac{dI_s}{dt} &= \alpha_3 S I_s + \gamma_3 E - \lambda_3 I_s, \\ \frac{dR}{dt} &= \lambda_1 I_n + \lambda_2 I_h + \lambda_3 I_s + \lambda_4 E + \lambda_5 S. \end{split}$$

$$(5)$$

# 2.3.2. Parameter analysis of the $SEI^3R$ information propagation model

For a node  $v \in V(G)$ , let d(v) denote the degree of v and let N(v) be the set of its neighbors. Based on influence characteristics of social networks, the weight of influence power, intensity of social intervention (Zhang et al., 2021b), user persistence and information timeliness of a topic are proposed as follows.

• Weight of influence power. Due to the special status of users in social networks, the mutual influence powers between two nodes are not the same. Therefore, the weight of influence power for a node  $v_i$  impacting on other node  $v_i$  can be defined as

$$\begin{split} W_{v_iv_j} &= \frac{\frac{d(v_i)}{\sum_{v \in N(v_j)} d(v)}}{\frac{d(v_i)}{\sum_{v \in N(v_j)} d(v)} + \frac{d(v_j)}{\sum_{v \in N(v_i)} d(v)}} \\ &= \frac{d(v_i) \sum_{v \in N(v_i)} d(v)}{d(v_i) \sum_{v \in N(v_i)} d(v) + d(v_j) \sum_{v \in N(v_j)} d(v)}. \end{split}$$

- Intensity of social intervention (Zhang et al., 2021b). Let  $\omega \in [0,1]$  and  $\eta$  represent the intensity of social intervention and the probability of information propagation, respectively. Therefore, the relationship concerning the intensity of social intervention and the propagation probability of a piece of information is represented as  $\eta(t) = e^{-\omega \times (t-T)}$ , where T is a control time.
- *User persistence*. Let  $\xi_i \in [0,1]$  be the persistence of a user  $v_i$ . The larger the value of  $\xi_i$  is, the more reliant the user is on its persistence value for information to choose a propagation behavior. Conversely, the smaller the value of  $\xi_i$  is, the more restrictive propagation behavior of users is by the intensity of social intervention.
- *Information timeliness of a topic*. Since different topics always have different information timeliness, the information timeliness of a topic can be set as  $\kappa$  (Wang and Wang, 2018) in a social network. When a topic propagates in the social network, the probability of a user u leaving the information propagation process at time t can be represented by  $P_u(t) = 1 e^{-\kappa t}$ .

Combining above influencing factors with the parameters of the SEI<sup>3</sup>R model, the state transition probabilities between the different types of nodes in the SEI<sup>3</sup>R model are specifically defined as following.

 $\alpha_1, \alpha_2, \alpha_3$ : Considering the impact of the user persistence and the information timeliness of a topic, the probabilities that a susceptible node  $v_i$  in S changes into an infected node in  $I_h, I_s, I_n$  are represented

$$\alpha_1, \alpha_2, \alpha_3 = \xi_i \times e^{-\kappa t}$$
.

 $\beta$ : Based on the weight of influence power and the user persistence, the probability that a susceptible node  $v_i$  in S transforms into a latent node is defined as:

$$\beta = \xi_i \times W_{v_i v_i},$$

where  $v_i$  is the node that influences the node  $v_i$ .

 $\gamma_1, \gamma_2, \gamma_3$ : In view of the user persistence, information timeliness and intensity of social intervention, the probabilities that a latent node  $v_i$  in E transfers to an infected node in  $I_h, I_s$  and  $I_n$  at time t are represented as:

$$\gamma_1, \gamma_2, \gamma_3 = \begin{cases} \xi_i + (1 - \xi_i) \times e^{-\kappa t}, & if \ t < T, \\ \xi_i + (1 - \xi_i) \times \eta(t), & otherwise. \end{cases}$$

 $\lambda_1, \lambda_2, \lambda_3$ : For an infected node  $v_i$  in  $I_h$ ,  $I_s$  and  $I_n$ , the probabilities that they transfer to removed nodes in R at time t are defined as:

$$\lambda_k = \begin{cases} \alpha_k - \xi_i + (1 - \xi_i) P_i(t), & if \ t < T, \\ \alpha_k - \xi_i + (1 - \xi_i) \times \eta(t), & otherwise. \end{cases}$$

where k = 1, 2, 3

 $\lambda_4$ : The probability that a latent node  $v_i$  in E transfers to a removed node in R at time t is presented as:

$$\lambda_4 = \begin{cases} \beta - \gamma_1 - \gamma_2 - \gamma_3 - \xi_i + (1 - \xi_i)P_i(t), & if \ t < T, \\ \beta - \gamma_1 - \gamma_2 - \gamma_3 - \xi_i + (1 - \xi_i) \times \eta(t), & otherwise. \end{cases}$$

 $\lambda_5$ : The probability that a susceptible node  $v_i$  in S transfers into a removed node of R at time t is regarded as:

$$\lambda_5 = 1 - \alpha_1 - \alpha_2 - \alpha_3 - \beta - e^{\kappa t}.$$

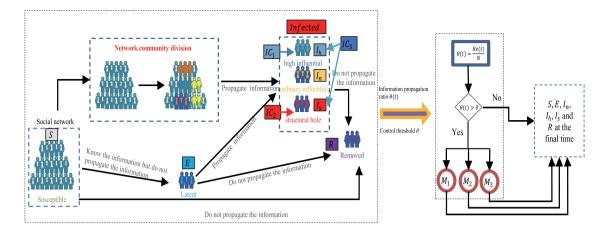


Fig. 2. The SEI<sup>3</sup>R information propagation control flowchart.

# 2.3.3. SEI<sup>3</sup>R information propagation control algorithm

In real social networks, considering that randomly selected nodes are regarded as initial propagation nodes, when a piece of information is propagated to high influential nodes or structural hole nodes, the information may quickly spread to whole networks. At this situation, the networks need to control the information propagation. So we randomly select nodes from high influential nodes and structural hole nodes as initial propagation nodes in the networks. In order to study the influences of high influential infected nodes ( $I_h$ ) and structural hole infected nodes ( $I_s$ ) in the information propagation process, there are three different initial conditions for the SEI<sup>3</sup>R information propagation:

 $IC_1$ : randomly select m nodes from the high influential nodes with the top k influential node scores by Algorithm 2 as information propagation nodes (infected nodes);

 $IC_2$ : randomly select m nodes from the structural hole nodes with the top k structural hole node scores by Algorithm 1 as information propagation nodes (infected nodes);

 $IC_3$ : randomly select m nodes from the nodes with the top k structural hole node scores and influential node scores by Algorithms 1 and 2 as information propagation nodes (infected nodes).

Let Re(t) be the number of nodes received the information at time t. Then we define the information propagation ratio of the number of nodes received information to the number of network nodes at time t, denoted  $\theta(t)$ , as follows.

$$\theta(t) = \frac{Re(t)}{N} \tag{6}$$

Obviously, Re(t) includes the numbers of high influential infected nodes, ordinary infected nodes, structural hole infected nodes, latent nodes and removed nodes at time t.

Suppose that network regulators are able to give a control threshold  $\theta$  according to the network state and information status at time t. If  $\theta(t) > \theta$ , then the network regulators could adopt three control strategies to control the information propagation:

 $M_1$ : Increasing the social intervention intensity  $\omega$  and reducing the probabilities received the information from high influential nodes;

 $M_2$ : Increasing the social interventions intensity  $\omega$  and reducing the probabilities received the information from structural hole nodes;

 $M_3:$  Increasing the social intervention intensity  $\omega$  and reducing the probabilities received the information from high influential nodes and structural hole nodes at the same time.

Under the three different initial conditions and three control strategies, the SEI<sup>3</sup>R information propagation control flowchart is shown in Fig. 2. Then, the SEI<sup>3</sup>R information propagation control algorithm is designed as follows.

In Algorithm 3, the inputs are the nodes and connection relationships of a network. The optimum topology is that it has an obvious

# **Algorithm 3:** The SEI<sup>3</sup>R information propagation control algorithm

**Input**: The social network G = (V, E)

Output: The information propagation states at the final time

- 1 Obtain community divisions  $C_1, C_2, ..., C_i$  by the community division algorithm (Blondel et al., 2008; Newman, 2004)
- 2 Obtain structural hole nodes using Algorithm 1
- 3 Obtain high influential nodes using Algorithm 2
- 4 Calculate the values of *S*, *E*, *I<sub>h</sub>*, *I<sub>n</sub>*, *I<sub>s</sub>* and *R* at time t by the differential equation system (5)
- 5 for the initial conditions  $IC_1$ , or  $IC_2$ , or  $IC_3$  do

```
if \theta(t) > \vartheta then
           if Control the information propagation within a community
7
               select the control strategy M_1, and compute S, E, I_h,
8
               I_n, I_s and R at the time t
           if Control the information propagation between communities
9
               select the control strategy M_2, and compute S, E, I_h,
10
               I_n, I_s and R at the time t
           if Quickly control the speed and range of information
11
           propagation in the network then
               select the control strategy M_3, and compute S, E, I_h,
12
               I_n, I_s and R at the time t
13
          Compute S, E, I_h, I_n, I_s and R at the time t
14
```

15 Output the information propagation states:  $S,\ E,\ I_h,\ I_n,\ I_s$  and R at the final time

community structure, and does not overlap the communities. This type of topology will improve the accuracies of network community division, and discoveries of structural hole nodes and high influential nodes. The modularity (Blondel et al., 2008; Newman, 2004) is adopted to evaluate the clustering effect. The greater the value of modularity is, the better the clustering effect is. The control threshold could be set by network regulators. When the information dissemination rate in the network exceeds, the control strategy can be selected different control strategies. When it does not exceed, any control strategy is not needed. When the percentages of four states' nodes i.e., S(t), E(t),  $I_h(t)$ ,  $I_n(t)$ ,  $I_s(t)$  and R(t), keep stable, the information propagation and control is stopping. If this condition is not satisfied, the information propagation with control strategies continues to work in networks.

Table 2

Description of two datasets.

| Data set         | Nodes | Edges | Modularities | Communities | Structural hole nodes |
|------------------|-------|-------|--------------|-------------|-----------------------|
| Email network    | 986   | 16686 | 0.47         | 7           | 83                    |
| Facebook network | 4039  | 88234 | 0.835        | 16          | 513                   |

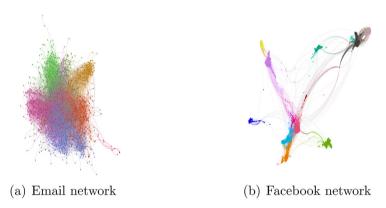


Fig. 3. The community divisions of the Email network and Facebook network. Different communities are marked by different colors. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

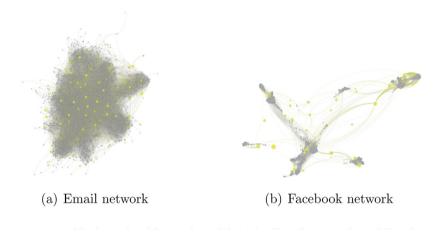


Fig. 4. Some structural hole nodes (the nodes with big sizes) in the Email network and Facebook network.

# 3. Experiment

To verify the validity and availability of the SEI<sup>3</sup>R information propagation model and the effectiveness of its control algorithm, we make experiments on two datasets: an Email network with documents under one topic and a Facebook network consisted of 'circles' (or 'friends lists') (see Table 2). In the Email dataset, each user is associated with a set of emails with sender IDs and recipient IDs. The Facebook dataset, which was collected from survey participants using the Facebook app, includes node features (profiles), circles, and ego networks. In Table 2, the modularities and the numbers of communities and structural hole nodes are computed by the community division algorithm (Blondel et al., 2008; Newman, 2004) and Algorithms 1. Throughout the processes of all experiments, the nodes that belong to some community will still belong to that community. From Fig. 3, we find that the communities of the Email network are denser than those of the Facebook network. From Fig. 4, it can be observed that most of the structural hole nodes are located between communities by Algorithm 1 in the two networks.

First, in order to verify the effectiveness of the proposed high influential node discovery and score (HINDS) algorithm, the following baseline methods are selected.

**DegreeCentrality (DC)** (Bonacich, 2008): This method calculates the centralities of nodes in a network. The nodes with top-k high centralities are selected as high influential nodes.

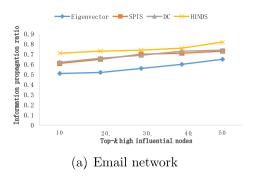
**SPIS** (Zhu et al., 2018): This method computes the shortest path increments of nodes in a social network. The nodes with the top-*k* shortest path increments are selected as high influential nodes.

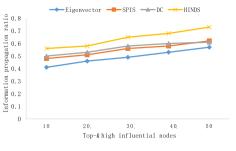
**Eigenvector** (Kitsak et al., 2010): For a node in a social network, its neighbor nodes and the number of neighbor nodes are very important. The nodes with top-k eigenvector scores are selected as the high influential nodes.

In Fig. 5, the information propagation ratios under different top-k high influential nodes are compared on the Email network and Facebook network, k=10, 20, 30, 40, 50. For the same value of k, the larger the information propagation ratio of a method is, the better the performance of the method is. The performance of the HINDS algorithm is the best, while the eigenvector method is the worst in the Email network and Facebook network. These show that the HINDS algorithm effectively discovers high influential nodes in the networks.

Second, to confirm the effectiveness of the proposed structural hole node discovery and score (SHNDS) algorithm, the following baseline methods are selected.

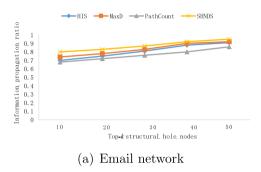
PathCount (Xu et al., 2017): For each node in a social network, the number of all shortest paths passing the node is set as the node

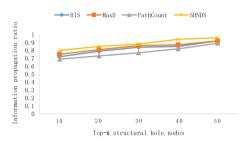




(b) Facebook network

Fig. 5. Comparisons of information propagation ratios under the DC, SPIS, Eigenvector and HINDS methods on the Email network and Facebook network.





(b) Facebook network

Fig. 6. Comparisons of information propagation ratios under the HIS, MaxD, PathCount and SHNDS methods on the Email network and Facebook network.

score. Then, the nodes with the top-k node scores are selected as top-k structural hole nodes.

**HIS** (Lou and Tang, 2013): Assuming that the community divisions of a social network are known, this method assigns each node a score in [0,1] as the possibility of a structural hole node on given community subsets. Then, the k nodes with the largest scores are selected as top-k structural hole nodes.

**MaxD** (Lou and Tang, 2013): Assuming that the community divisions of a social network are known, this method finds a set with k nodes using a greedy strategy such that deleting all nodes of the set will decrease the minimum cut of all communities in the network. Then, the nodes of the eligible set are selected as top-k structural hole nodes.

In Fig. 6, the information propagation ratios under different top-k structural hole nodes are compared on the Email network and Facebook network, k=10, 20, 30, 40, 50. For the same value of k, the larger the information propagation ratio of a method is, the better the performance of the method is. They show that the SHNDS algorithm is superior to the PathCount, HIS and MaxD methods on the Email network and Facebook network. The PathCount method obtains the worst performance on the two networks. It concludes that the proposed SHNDS algorithm can effectively obtain structural hole nodes in the networks.

Next, we select the traditional SEIR model (Wei and Xue, 2019) and SEI<sup>2</sup>R model (Zhang et al., 2018) as baseline methods. Then, we randomly select m nodes from the first k structural hole nodes and high influential nodes in any communities as the initial information propagation node to make information propagation experiments under the baseline models and SEI<sup>3</sup>R model. Comparing with their experimental results, we will verify the effectiveness of our SEI<sup>3</sup>R model in the two networks. For our experimental purposes, infected nodes are divided into high influential infected nodes and ordinary infected nodes in the SEI<sup>2</sup>R model. In the following, the colored nodes indicate that the nodes have received information, whereas the gray nodes indicate that the information has not been received. The simulation parameters are: A = 2,  $\alpha_1 = 0.5$ ,  $\alpha_2 = 0.3$ ,  $\alpha_3 = 0.2$ ,  $\beta = 0.2$ ,  $\gamma_1 = 0.3$ ,  $\gamma_2 = 0.2$ ,  $\gamma_3 = 0.2$ ,  $\lambda_1 = 0.6$ ,  $\lambda_2 = 0.4$ ,  $\lambda_3 = 0.3$ ,  $\lambda_4 = 0.2$ ,  $\lambda_5 = 0.3$ .

When the high influential nodes in the communities are taken as initial information propagation nodes under the SEIR, SEI<sup>2</sup>R and SEI<sup>3</sup>R models in the two networks, Figs. 7 and 8 show the range of these models at stable state. They show that the information propagates to all nodes of their communities contained the initial information propagation nodes, but the information propagation ability is limited in the entire network. Compared with SEIR and SEI<sup>2</sup>R models, the ability of information propagation under SEI<sup>3</sup>R model is significantly better than two baseline models. In the Facebook network, this information propagation phenomenon is very apparent because its communities are sparser than those of the Email network. Then, we can deduce that the high influential nodes directly impact on the information propagation within their communities of the networks.

When the structural hole nodes in the networks are regarded as the initial information propagation nodes, the information almost propagates to the entire networks (see Fig. 9). The reason is that the structural hole nodes connect to different communities in the networks. The information is easier to propagate to the entire networks. Therefore, the structural hole nodes play an important role in the process of information propagation between different communities in the networks.

When the high influential nodes and structural hole nodes are regarded as initial information propagation nodes at the same time, in order to clearly analyze the information propagation process, we take three different time snapshots of the Email network and Facebook network under three different time points for the same time range to observe the information propagation abilities in the networks (see Fig. 10). At this case, for T=30, the information propagates to a few number of nodes in all communities of the networks; for T=90, the majority of nodes in all communities get the information; for T=150, the information almost propagates to all nodes of the networks. It concludes that the speeds and ranges of the information propagations are really fast and wide in this situation. Compared with Figs. 7–9, the ranges (resp. speeds) of information propagations under this situation are obviously wider (resp. faster) than theirs. Thus, the high influential nodes and structural hole nodes jointly play an important role in the

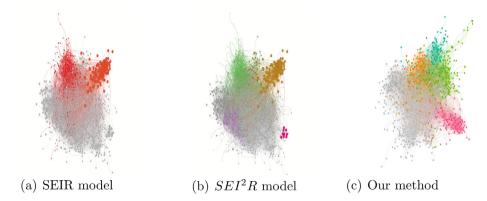


Fig. 7. Comparisons of information propagation abilities for SEIR model, SEI<sup>2</sup>R model and our model under the high influential nodes as the initial information propagation nodes in the Email network.

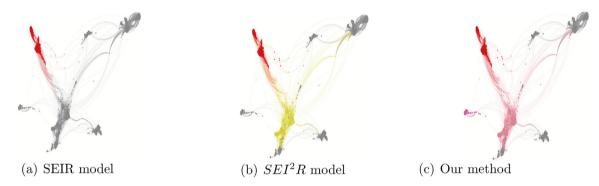


Fig. 8. Comparisons of information propagation abilities for SEIR model, SEI<sup>2</sup>R model and our model under the high influential nodes as the initial information propagation nodes in the Facebook network.

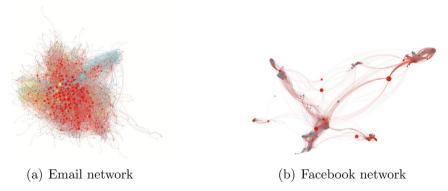


Fig. 9. Information propagation abilities for our model under the structural hole nodes as initial propagation nodes in the Email network and Facebook network. The larger the nodes are, the higher the structural hole node scores are in the networks.

process of information propagation in the networks. 10, compared with the high influential nodes as initial information propagation nodes, the structural hole nodes as the initial information propagation nodes make information propagate more wider in the entire networks. Whereas the high influential nodes are taken as the initial information propagation nodes, they have stronger information propagation abilities than other nodes in their communities. These nodes almost affect their entire communities in the networks. When the two types of nodes are jointly used as initial information propagation nodes at the same time, the speeds and ranges of information propagations in the networks are faster and wider than those under other types of the initial information propagation nodes. Therefore, for network regulators, if they need to control the information propagation speed and range in varying degrees, they may adopt three control strategies  $M_1, M_2$  and  $M_3$ , i.e., reducing the probabilities receiving information from the high influential nodes, structural hole nodes, and combination

of high influential nodes and structural hole nodes at the same time, respectively.

Now, we confirm the effectiveness of the SEI<sup>3</sup>R information control algorithm. Three control strategies  $M_1$ ,  $M_2$  and  $M_3$  will impact on the ranges of information propagations in the Email network and Facebook network. To facilitate the observation of the following experimental results, high influential nodes and structural hole nodes are taken as initial information propagation nodes in the networks.

In Fig. 11, the network regulators adopt the control strategy  $M_1$ , i.e., reducing the probability receiving the information from high influential nodes in the networks, the simulation parameters are A=2,  $\alpha_1=0.5$ ,  $\alpha_2=0.1$ ,  $\alpha_3=0.2$ ,  $\beta=0.2$ ,  $\gamma_1=0.3$ ,  $\gamma_2=0.1$ ,  $\gamma_3=0.2$ ,  $\lambda_1=0.6$ ,  $\lambda_2=0.6$ ,  $\lambda_3=0.3$ ,  $\lambda_4=0.2$  and  $\lambda_5=0.3$ . The numbers of nodes received the information in the entire networks are reduced in communities of the networks, but the information still propagates to the entire networks.

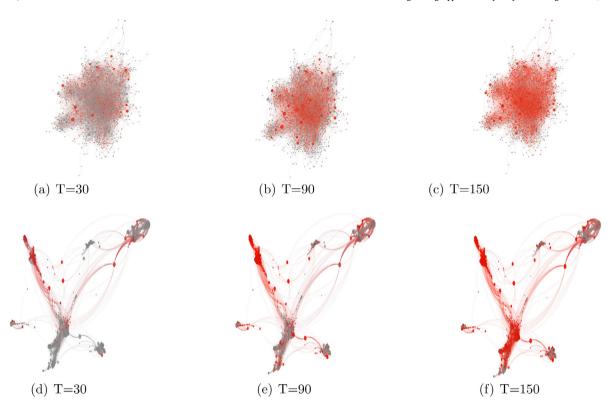


Fig. 10. Information propagation abilities for our model under the high influential nodes and structural hole nodes as the initial propagation nodes in the Email network and Facebook network.



Fig. 11. Information propagation abilities for our model with controlling high influential nodes.

In Fig. 12, when the network regulators adopt the control strategy  $M_2$ , i.e., reducing the probability receiving the information from the structural hole nodes in the networks, the simulation parameters are  $A=2,\alpha_1=0.5,\alpha_2=0.3,\alpha_3=0.1,\beta=0.2,\gamma_1=0.3,\gamma_2=0.2,\gamma_3=0.1,\lambda_1=0.6,\lambda_2=0.4,\lambda_3=0.5,\lambda_4=0.2,\lambda_5=0.3.$  Not only do the numbers of nodes received the information decrease, but also the ranges of information propagations are obviously scaled down in the networks.

In Fig. 13, when the network regulators adopt the control strategy  $M_3$ , i.e., reducing the probability receiving the information from high influential nodes and structural hole nodes at the same time in the networks, the simulation parameters are A=2,  $\alpha_1=0.5$ ,  $\alpha_2=0.1$ ,  $\alpha_3=0.1$ ,  $\beta=0.2$ ,  $\gamma_1=0.3$ ,  $\gamma_2=0.1$ ,  $\gamma_3=0.1$ ,  $\lambda_1=0.6$ ,  $\lambda_2=0.6$ ,  $\lambda_3=0.5$ ,  $\lambda_4=0.2$ ,  $\lambda_5=0.3$ . The numbers of nodes received the information are significantly lower than the numbers of the nodes received the information without control strategy in the processes of information propagations for the two networks. Under the same control strategy  $M_3$ , the ratio of nodes that did not receive the information in the Facebook network is distinctly higher than the ratio of the nodes that did not receive the information in the Email network. The reason

is that the communities of the Facebook network are sparser than the Email network. At this case, the ratios of the nodes received the information are eventually well below 50%. Thus, the control strategy  $M_3$  is extremely efficient for the Email network or Facebook network.

Finally, we use three evaluation indicators, i.e., Precision (P), Recall (R) and F1 value ( $F_1$ ), to assess the effectiveness of the SEI $^3R$  information control algorithm. For a piece of information, let TP present the number of positive control nodes that have known the information when any control strategy is not implemented, and that have not known the information after one control strategy is implemented under an information propagation model. Let FP stand for the number of negative control nodes that do not know the information when any control strategy is not implemented, but that have known the information after one control strategy is implemented. Let FN denote by the number of the nodes that have not known the information when any control strategy is not implemented, and that have still not known the information when one control strategy is implemented. Then, the values of P, R and  $F_1$  are defined as follows.

$$P = \frac{TP}{TP + FP}, R = \frac{TP}{TP + FN}, F_1 = \frac{2*P*R}{P+R}.$$





Fig. 12. Information propagation abilities for our model with controlling the structural hole nodes.

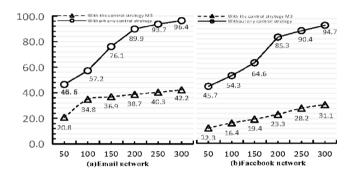


Fig. 13. Comparisons of information propagation ratios for our model without any control strategy and with controlling the high influential nodes and structural hole nodes at the same time  $(M_3)$  in the Email network and Facebook network.

Table 3
Performance comparisons of the SEIR, SEI<sup>2</sup>R and SEI<sup>3</sup>R models on the Email network.

| Indicator | SEIR model | SEI <sup>2</sup> R model | SEI <sup>3</sup> R model |       |       |
|-----------|------------|--------------------------|--------------------------|-------|-------|
|           |            |                          | $M_1$                    | $M_2$ | $M_3$ |
| P         | 0.894      | 0.923                    | 0.937                    | 0.957 | 0.962 |
| R         | 0.756      | 0.773                    | 0.810                    | 0.839 | 0.851 |
| $F_1$     | 0.819      | 0.841                    | 0.868                    | 0.894 | 0.903 |

In order to facilitate the comparison of evaluation indicators under SEIR and SEI²R models, the control strategy for the SEIR model is reducing the probabilities of infected nodes. For the SEI²R model, the control strategy is reducing the probabilities received the information from the high influential nodes and ordinary nodes. The results on Email network and Facebook network are shown in Tables 3 and 4, respectively. Compared with the results of the SEIR, SEI²R and SEI³R models on the Email network and Facebook network, the values of three evaluation indicators under the SEI³R model are the largest; the values under the SEI²R model are the second; and the values under the SEIR model are the lowest. They show that infected nodes play key roles in the processes of information propagations with different control strategies. When network regulators respectively adopt the  $M_3, M_2$  and  $M_1$  strategies, the performance of the corresponding SEI³R information propagation control models decreases in turn.

Based on above analysis of the experiments, it concludes that, in a social network, if the network regulators need to control the information propagation within the communities, they can select the control strategy  $M_1$ ; if they need to control the information propagation between communities, they can select the control strategy  $M_2$ ; and if they need to quickly control the speed and range of information propagation, they need to select the control strategy  $M_3$  in the network.

 $\label{thm:comparisons} \textbf{Table 4} \\ \text{Performance comparisons of the SEIR, SEI$^2R and SEI$^3R models on the Facebook network.} \\$ 

| Indicator | SEIR model | SEI <sup>2</sup> R model | SEI <sup>3</sup> R model |       |       |
|-----------|------------|--------------------------|--------------------------|-------|-------|
|           |            |                          | $M_1$                    | $M_2$ | $M_3$ |
| P         | 0.869      | 0.911                    | 0.954                    | 0.978 | 0.986 |
| R         | 0.684      | 0.715                    | 0.817                    | 0.896 | 0.923 |
| $F_1$     | 0.765      | 0.801                    | 0.880                    | 0.907 | 0.953 |

# 4. Conclusion

This article studies the information propagation process in social networks based on the infectious disease model. According to the nodes' status in social networks, the infected nodes are divided into high influential infected nodes, structural hole infected nodes and ordinary influence infected nodes. Based on such considerations, this paper first proposes the high influential node discovery and score algorithm and structural hole node discovery and score algorithm. These methods effectively obtain the high influential nodes and structural hole nodes. Then, to depict the information evolution, propagation and control processes in social networks, the paper further builds the SEI<sup>3</sup>R model and the SEI<sup>3</sup>R information propagation control algorithm. To measure the parameters of the proposed SEI<sup>3</sup>R model, the weight of influence power, intensity of social intervention, user persistence and information timeliness of a topic are proposed. By simulation experiments in the Email network and Facebook network, we find that high influential nodes have strong influence on nodes within the communities. The structural hole nodes, which act as bridges between communities, make the information propagate in different communities in the networks. Therefore, if we control structure hole nodes in the process of the information propagation, we can effectively control the propagation range; if we control high influential nodes, we can control the information propagation within the community; and if we control structural hole nodes and high influential nodes at the same time, we can quickly control the information propagation speed and range in the entire networks. Finally, to verify the effectiveness of the proposed model, three evaluation indicators including Precision, Recall and  $F_1$  are adopted. Experimental results show that the performance of SEI<sup>3</sup>R model and corresponding control algorithm are superior to other baseline models. However, other sensitivity analysis methods (Asheghi et al., 2020), such as CAM, EM, RC, and PaD, are also worth considering in different information propagation control models. As the text information and post content information of users are not considered in our model and algorithm, our next research work is fusing the two information of users with different information propagation models to analyze the information evolution, propagation and control laws in social networks.

#### CRediT authorship contribution statement

Qian Zhang: Conceptualization, Software, Writing – original draft, Visualization. Xianyong Li: Writing – review & editing, Conceptualization, Methodology, Supervision, Project administration, Funding acquisition. Yongquan Fan: Software, Validation, Formal analysis, Investigation. Yajun Du: Conceptualization, Project administration, Funding acquisition.

# Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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