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Survey paper

A retrospective study on handwritten mathematical symbols and expressions: Classification and recognition

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ABSTRACT

Context: Many scientific and technical literature documents contain MSs and MEs that are more challenging to be recognized by computers than plain text. The recognition of HMSE becomes not only an ambitious task but a motivating research area covering concepts of computer vision, pattern recognition, feature extraction, and artificial intelligence.
Objective: The objective is to perform an extensive state of the art on the techniques and methods used for recognizing and classifying HMSE. The authors endeavor to bring out all significant findings in sub-processes, representation models, algorithms, tools, datasets, and comparative analysis of the accuracy of the recognition models.
Method: The current research implements the standard SLR method based on a comprehensive set of 120 articles published in 21 leading journals and 39 premier conferences and workshops.
Results: Existing literature about recognition techniques and models is classified broadly into three categories; AI technique (65%) is majorly implemented in the selected studies. The prominent sub-process 'segmentation' (52%) is mostly used. The box and tree are the prevailing representation models. The popular datasets are recognized as CROHME 2014 and CROHME 2016, used by 60% of the selected studies. Masaki Nakagawa, C. Viard Guardin, Richard Zanibbi, and Harold Mouchere are the most noticed authors in ME recognition.
Conclusion: The reviewers call for increased awareness of the potential benefits of existing and emerging recognition techniques and identify the need to develop a more accurate and semantic-based recognition model. Recommendations are given for future research.

1. Introduction

Being the universal language of science, mathematics witnesses its importance by also being an essential part of most of the scientific and technical literature. This literature contains ample notes of MSEs that are not easily recognizable as the plain text. Moreover, mathematical notations are part of visual language. Visual language is defined as a communication system using visual elements, like graphics (Chang, 1986). It can be comprehended by its two or three-dimensional graphics rather than the linear text (Kremer, 1998). Due to its inherent visual elements, the recognition of MSE becomes a challenging task. Moreover, MEs, a 2-Dimensional structure of the math symbols (Wang et al., 2020), could disseminate knowledge in all essential technical and scientific literature. Therefore the identification and realization of these mathematical notations become a natural essence of great practical importance.

1.1. Role of recognition in information transformation

Most of the scientific and engineering publications contain MSEs. Apart from reducing and diminishing the efforts required for framing and creating the technical documents, the recognition of handwritten mathematics could also be helpful in transforming existing handwritten documents into electronic format by facilitating the easy transfer between machines whenever needed. Therefore, HMR is one of the fundamental forces that drive the information transformation between humans and machines. As MEs appear enormously in scientific documents, thus there is a need to acquire utilities for recognition of mathematical content to transfer such forms into electronic format. Handwriting input is one of the easiest known ways of input that provides an efficient and appropriate mechanism to input mathematical text into a computer for storage or sharing with others, underlining the necessity of effective mathematical recognition software.

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1.2. Recognition process and problem

The problem of ME recognition has been a subject of intensive research for over 40 years. Thus, the study focuses on recognizing HMSE that can be considered the easiest method to input the typical math notations directly into the systems. Also, it involves the identification of the handwritten input of these notations. Another approach is extracting the numerical data from the pdf documents by scanning; however, the former process is more rudimental than the latter. Thus, recognition of HMSE has become an exciting and stimulating research area of pattern recognition with unlimited real-world implications. For recognition and identification of HMSE, the objective is to produce a machine that could take the math equations and symbols as vector input and output the exact identification of symbols and expressions. As the cardinal issue is all due to the handwriting and its associated variations, several authors attempted to segregate and understand these underlying heuristics yet proved to produce insignificant results. When applied to the system, ML approaches were successfully able to give an unparalleled and preeminent solution (Zanibbi and Blostein, 2012). The current study is endeavoring to review the performance of all the applied recognition models, techniques and understand the works done in this field to date, with all chronicles of algorithms, datasets, tools, and representative structures used in the process.

1.3. Highlights of the study

- The study presents an elaborative and all-inclusive knowledge around the research theories, technologies, tools, datasets, algorithms along the summarizations of metadata trend analysis that gravitate with HMSER. Additionally, the sub-process involved in the recognition has been exclusively addressed.
- This report is the first-ever systematic literature review representing all crisp and fine information about the vast and emerging field of HMSER.
- This analysis work provides an updated and mature state of the art of all recognition procedures, challenges, and methodologies that are being applied recently.
- This review is a complete stem to stern analysis investigating the recognition trends and categorized the techniques to their associated approaches. Moreover, the review targets extracting several representation models used for recognition.
- The uniqueness of this work is ensured by systematically planning every phase, with comprehensive documentation of every involved step with a clear vision to focus and extract out the information that could direct the future research.

1.4. Contribution of the study

- This review contributes towards the representation of interesting extractions about MER. This report offers statistically and graphically more insights into the aspects of the mathematical recognition process, emphasizing neoteric recognition models and approaches.
- The factor that makes this review distinct and different is applying SLR and SLM concepts. SLR and the benefiting steps of SLM that focus on extracting internal and external properties have been deployed in the review process.
- Adding to the significance of this study, we consider the value contribution of each research question. The formulation of every research question has crisp objectives, and factual findings reported in this review after the dense analysis are reported.
- The identification, extraction, and then categorization of the recognition techniques of recent times is one of the remarkable contributions of this study. The current recognition approaches have been highlighted evidential of their frequency of occurrences, and all the supporting concepts have been excitingly depicted through graphical representations.
- The informative facts and findings provided in the conclusion will benefit the readers and practitioners for understanding the entire research history on MER in the gist. Complementing the readability of this study, the summary points and tables are vividly displayed.

1.5. Paper organization

This review identifies different key areas of research on recognition methods and models (Karasu and Altan, 2019), discusses the concepts and processes involved in the recognition procedure, and specific stages involved, and extracts out significant findings related to the current state of the art. On assessing the present state of the art in recognition of HMSE research, it was realized that there is a lack of SLR. The reviewers in the current study aligned the existing research based on an extensive review systematically and reported the research gaps for further investigation. The structure of this article's remaining section is as follows: Section 2 covers the motivating energies that drive our research in this direction. Section 3 introduces the background, definitions, and basic concepts based on the recognition of HMSE. A glossary of used acronyms is described in Appendix A. Section 4 elucidates the research questions, a research method deployed for selecting and reviewing the gathered data material, and unveils the analysis framework. Section 5 presents the results, findings, and implications of the SLR. Section 6 consists of concluded remarks that focus on the meta-analysis of the study. Section 7 concludes and provides recommendations for further study in the area of recognition of HMSE. Appendices B.1 and B.2 have been appended at last to depict the quality assessment process followed in selecting the studies.

2. Research motivation

Mathematics is one of the core subjects, which can be considered the "center of gravity" for any emerging technique, tool, and method. So, the appearance of the associated mathematical formulations in math equations, expressions, and symbols is indispensable. Undoubtedly, they describe the problems and theories in scientific subjects like physics, maths, and many other congruent areas. Thus, these are incessantly deployed. The attention of the research in this field was drifted by the research article of Anderson (Anderson, 1967), which is known to be a flagman in this research arena. Ever since then, the volume of research in the field is increased by large. The recognition works of the domain embarked on and are still in considerable progress as the ML and DL methods have almost emerged and are producing exceptional results (Altan et al., 2021; Karasu et al., 2020). Following, we list the factors that have inspired this review work:

Scarcity in Research Review and Surveys: Considering the amount of review of this topic is scarcer, the literature witnesses very few research reviews. There are two landmark literature surveys in the field of MSEs by researchers (Chan and Yeung, 2000a,b; Zanibbi and Blostein, 2012). It has led to a need for critical evaluation and integration of the available research, in particular the need for an SLR.

Significant rift in the history of surveys on HMSE: The last significant review on this domain was ever since the year 2012. The void of 8 years to now corroborates the implementation of so many diverse recognition models, methods, algorithms, and approaches that are needed to be compiled thoroughly. This review concentrates on shelling out the exigent integrated concepts involved in the recognition processes. It presents a current developing picture of the hype and growth in the domain of recognition methods for HMSE.

Advent of various advanced recognition technologies: The current times' witnessed the emergence of various ML, DL, and other hybrid approaches that have been extensively deployed for recognition and classification tasks. The recent technologies are captivating the interest of the researchers' community, and thus the research work on this domain embarks on the hype.

3. Background

As mentioned earlier, the initial research reported in the literature of recognition of math content was by Anderson (Anderson, 1967), where the author worked on syntax-directed recognition of printed 2D MEs. With the pace of time and advancement of technology, researchers' interest and the requirement shifted from MEs to HMSE. The emergence of new technologies, digital tools, and devices accelerated this divergence as devices like digital pens, tablets, smartphones, etc., supported a natural and convenient method to input the data to the machines. But this was thoroughly dependent on the fact of how well are the recognition models and the recognition systems. While digitizing the knowledge in the form of math text, there are two main actions. One is to input the mathematical notations in the systems. The other is to recognize the inputted text. Though the former is all so well endorsed due to advancements in these digital devices, the latter still holds its fulcrum on accuracy achieved by the recognition methods and systems. The past four decades of research in this area are evolving to improve and enhance the accuracy of recognition methods. At the onset of this review, the authors endeavor to portray the background by defining and illustrating the preliminaries to enhance the understanding of this compiled review work.

3.1. Classification of MSEs

There have been different parameters observed while compiling the literature studies on this domain of HMSE, and it has been noticed that the various research works are evidence of classifying the MSEs based on varying parameters.

3.1.1. Based on modes of inputs

The kind of input fed to the system is also one of the essential factors that signify MSEs. It is actually according to handwritten information. Handwritten data can be inputted into machines by two modes, namely online and offline mode (Mouchère et al., 2016; Davila et al., 2014; Álvaro et al., 2014a; Alvaro et al., 2014b; Dong and Liu, 2017; Ramadhan et al., 2016; Nazemi et al., 2019; Mouchere et al., 2014; Julca-Aguilar and Hirata, 2018; Hu and Zanibbi, 2013). So, based on the parameter of input, the authors define the HMSE as online and offline HMSEs. The formal definitions are presented below:

- **Offline HMSE**

The offline primarily focuses on the MSE written or in printed form and is inputted in the form of scanned data. In other words, it is a static representation of data in the form of printed material, and this data, i.e., MSE, is once written on paper and then scanned for input, constituting the necessary procedure in offline mode of input. It thereby involves offline recognition where the MSEs are written and are in the form of images and bitmaps. The offline MSE recognition adduces the conversion of handwritten or printed MSEs expressed regarding images into editable symbols (Álvaro et al., 2016). Every symbol from ME from offline sources is treated as a group of black pixels. It involves the grouping of these pixels to recognize the overall expression. The input image in offline mode does not contain the needed information in the form of geometric and temporal information. As the offline expression contains limited information, so, therefore, their recognition accuracy rate is comparatively less than the online handwritten symbols and expressions. Thereby also making this mode of input less accessible and relatively less used in the recognition process.

- **Online HMSE**

The online HMSE deals with MSE; these MSE can be directly drawn or written on a contact-sensitive surface gadget like a data tablet, touch screen, PDA, electronic whiteboards, etc. It is a dynamic representation of data (Tapia and Rojas, 2003), as it simulates simultaneous interaction with the connected system and displays results on monitors. The online case also deals with a Spatio-temporal representation of the input (Plamondon and Srihari, 2000). In the recognition process, each symbol of ME is recognized on the basis of strokes. A stroke can be defined as a sequence of points between a pen-down to a pen-lift. Awal et al. (2010a,b,c). The stroke is a basic unit and assumed to belong only to one symbol. However, any symbol to be written can be drawn using one or more strokes, which are not essentially sequential. That implies that the strokes needed to form a symbol can be random and not necessarily follow sequence order. Existing works do not support the latter hypothesis since interspersed symbols enhance the complexity of the segmentation process described in Section 3.2.2. Most of the considered current works consider that all strokes are present before starting the segmentation and recognition process, along with a thorough consideration of the global context (Awal et al., 2014; Álvaro et al., 2016).

3.1.2. Based on the occurrence and positioning of mathematical text

According to Madisetty (Madisetty et al., 2020), MEs in scientific documents can be classified into two categories: (i) Isolated or Independent MEs and (ii) Inline MEs

- **Isolated MEs**

Isolated MEs are the type of expressions that are always written outside of the text portion of the document. They possess a specific layout or style, which is found in scientific documents. It is simple to identify and recognize such kinds of MEs using OCR techniques with predefined features.

- **Inline MEs**

Inline MEs are mixed with regular text, and there is no specific layout or pattern for identifying inline MEs embedded with the plain data. It is complicated and challenging to identify these types of expressions. Inline ME detection can be helpful in many ways. For instance, it can be beneficial for IR for better indexing and other retrieval tasks. It also minimizes the errors in sentence parsing. It further leads to advancement in scientific text mining as mathematical concepts often present in such documents.

3.2. Phases involved in the recognition process

There are two or three phases involved in the recognition system. In the research community, distinct researchers have endeavored to design and implement different recognition processes in phases or sub-processes. Recognition systems and procedures for HMSE can be effectuated using computer vision, pattern recognition, ML, and other grammar-based approaches and techniques. Various technologies and methods can be used for the recognition process, so the phases in each case could vary to an extent.

The research work of Chan and Yeung (2000a,b), Garain and Chaudhuri (2003), Shan et al. (2019), Mohan and Lu (2015), and many other researchers includes two phases: symbol recognition and structural analysis in the recognition procedures. The problems involved in the recognition phases can be solved sequentially (Zanibbi et al., 2002) or globally (Álvaro et al., 2016). However, many studies (Álvaro et al., 2011, 2014a; Alvaro et al., 2014b; Le et al., 2014; Chan and Yeung, 2000a,b; Julca-Aguilar et al., 2015; Hu and Zanibbi, 2016a; Le and Nakagawa, 2015; Awal et al., 2010c; Sain et al., 2010; Lods et al., 2019; Wang and Shan, 2020; Hirata and Honda, 2011; Yamamoto et al., 2006; Muñoz, 2010; Toyozumi et al., 2004) employed the recognition process in three major stages: symbol segmentation, symbol recognition, and structural analysis.

All the involved stages, or the significant steps involved in the recognition process of HMSE, are illustrated below:

3.2.1. Preprocessing

The objective of preprocessing is to discard irrelevant information (duplicated points, wild points, hooks, noise, etc.) that can negatively affect the recognition (Huang et al., 2007). It is the foremost step in the recognition procedure that gravitates to remove noise or extraneous data from the input. As already discussed, the input can be offline (scanned image) or online (directly handwritten). For the offline input image, the preprocessing step is a necessary step for document analysis (Plamondon and Srihari, 2000) that involves clearing the image for further processing and outputs the grayscale image (Drsouza and Mascarenhas, 2018). Some of the common operations performed under this step are normalization (Golubitsky et al., 2010; Clark et al., 2013; Deepu et al., 2004; Chajri and Bouikhalene, 2016; Simistira et al., 2015; Le et al., 2019b), binarization (Shinde et al., 2018; Drsouza and Mascarenhas, 2018; Chan, 2020; Celar et al., 2015), thinning (Ahmed et al., 2004), smoothing (Ramsay, 2000; Álvaro et al., 2016; Hu and Zanibbi, 2016a; Gharde, 2012), removal of duplicate points, hooks (Davila et al., 2014; Reddy et al., 2012; Hu and Zanibbi, 2011), noise (Průša and Hlaváč, 2007; Petersen, 2020), and resampling (Thimbleby, 2004; Hu and Zanibbi, 2011; Tapia and Rojas, 2003; Mouchere et al., 2013; Chan and Yeung, 2000a,b; Clark et al., 2013) that can be applied on the kind of inputted discrete mathematical text.

3.2.2. Segmentation

A good segmentation is the key point of good recognition and interpretation (Awal et al., 2010c). It is one of the most crucial steps in the entire recognition process, where the process of segregation is applied to the pre-processed MEs. The objective of this process is to refine the inputted ME in a way that individual characters and symbols could be passed as inputs to the further stages of the recognition process. It also involves segmenting the entire document or input into text, graphics, and mathematics (Thimbleby, 2004). In the case of MSs, the segmentation procedures incorporate the grouping of individual strokes that belongs to the same symbol (Zhang et al., 2016). This set of input strokes is divided into hypothetical symbols, where each stroke represents the series of coordinates from pen/touch-down to pen/touch-up (Le, 2017). The segmentation problem has been tackled by connected computing components (Álvaro et al., 2014a; Alvaro et al., 2014b; Chajri and Bouikhalene, 2016; Hu and Zanibbi, 2016a; Zanibbi and Yuan, 2011; Tian et al., 2006), applying the projection profile cutting method (Nazemi et al., 2019; Okamoto et al., 2001; Okamoto and Miyazawa, 1992; Okamoto, 1991; Tian et al., 2006) or more sophisticated techniques (Chen and Okada, 2001).

There exist some methods which have been proposed to resolve the classical segmentation problem, such as HMM (Garain and Chaudhuri, 2003; Hu et al., 2012; Hu and Zanibbi, 2011; Reddy et al., 2012), Elastic Matching (Chan and Yeung, 1998; Simistira et al., 2012), or Support Vector Machines (Ronald et al., 2013; Le et al., 2014; Lin et al., 2012; Malon et al., 2008; Simistira et al., 2015). Moreover, to perform hybrid classification and to improve recognition results, some of these proposed methods combine online as well as offline information (Álvaro et al., 2014a; Alvaro et al., 2014b; Awal et al., 2010a; Keshari and Watt, 2007; Nguyen et al., 2019). The segmentation problem is not an easy task, given that some MSs are composed of multiple components ($i, j, =, :$) (Álvaro et al., 2012). Also, image degradation causes regular symbols to be split into several segments. Image degradation also adds noise and creates the appearance of touching characters, which is a challenging problem.

3.2.3. Symbol recognition

The symbol recognition problem is a more classic pattern recognition problem (Awal et al., 2009). It is a fundamental step, which causes most of the ambiguities, and a human has to disambiguate it using the whole grammatical structure of the expression (Yamamoto et al., 2006). The task of labeling the symbols (Awal et al., 2010) and allocating them proper symbol class is performed in this phase (Zhang et al., 2018a,b, 2016). It associates a label to each found symbol in a segmentation phase. The structural analysis then evaluates the relationships between the symbols and uses grammar to propose a valid interpretation of the ME. This phase also involves the segmentation of the input strokes into mathematical characters and symbol classification of the hypotheses (Winkler, 1994; Álvaro et al., 2012). Several techniques like HMM (Koschinski et al., 1995; Winkler et al., 1995; Winkler, 1996; Winkler and Lang, 1997; Hu and Zanibbi, 2011), SVM classifier (Keshari and Watt, 2007), template matching algorithm (Lee and Wang, 1995), neural networks (Dimitriadis and López Coronado, 1995; Ha et al., 1995; Yan, 2019; Yogatama et al., 2019) traditional template matching (Nakayama, 1989; Okamoto, 1991) are used in this phase.

Many symbol recognition techniques work under the assumption that the symbols have already been isolated from each other (Chan and Yeung, 2000a,b), where the segmentation process is considered separately. However, in many works, researchers combine the segmentation task of strokes and symbols of HMSE in this phase of symbol recognition. Thus, the metrics used to evaluate the symbol recognition can be symbol segmentation rate and symbol recognition rate (Álvaro et al., 2012).

3.2.4. Structural analysis

Structural analysis in HMEs focuses on interpreting the recognized symbols using geometrical information such as relative sizes and positions of the symbols (Vuong et al., 2008). It deals with finding out the structure of the expression according to the arrangement of the symbols (Álvaro et al., 2012).

In this phase, the spatial relations among the recognized symbols are determined (Zhang et al., 2018a,b,c), and the structure of ME is analyzed by using grammatical rules (Celik and Yanikoglu, 2011) and parsing the recognized symbols to determine the most likely interpretation as a ME. In short, it involves the identification of relations between symbols and a coherent, integrated interpretation (Julca-Aguilar et al., 2020). The structure analysis process is driven by the results obtained during symbol recognition (Winkler et al., 1995) used for structural analysis of MEs. There are several techniques and methods that have been implemented on operator dominance (Chang, 1970; Zanibbi et al., 2002), HMMs (Kosmala and Rigoll, 1998; Garain and Chaudhuri, 2003; Reddy et al., 2012; Wang et al., 2020), cutting pixel projection profile (Okamoto, 1991; Okamoto et al., 2001; Kumar et al., 2019), the identification of symbols of the dominant baseline (Matsakis, 1999; Tapia and Rojas, 2004; Le and Nakagawa, 2016b), and penalty graph minimization (Eto and Suzuki, 2001; Hu and Zanibbi, 2016a), etc. Though many processing methods and techniques are deployed for structural analysis, they share common characteristics. For the input to be given for the structural analysis phase, there is a need for full ME with all symbols correctly recognized (Vuong et al., 2008).

3.3. Issues involved in the recognition process of HMSE

The major issues and several challenges are spotted in the recognition task of MSEs. These issues and challenges are enough, and a sufficient listing of these can be maintained. The primary reason for the profuse number of these issues is mostly because of the inherent properties of the HMSE. The varying writing styles and ME formats accrue to the level of challenge in the recognition process (Wu et al., 2020). Another factor that inflates the issues is the two-dimensional structure and spatial relationship among the symbols used in the ME. The list of the pivotal issues involved in the recognition process are mentioned as follows:

Distinct style of writing by different authors and writers makes the task of interpretation and recognition of the expressions more difficult as the expressions use special symbolic notations which are even hard to be understood by even the human eye.

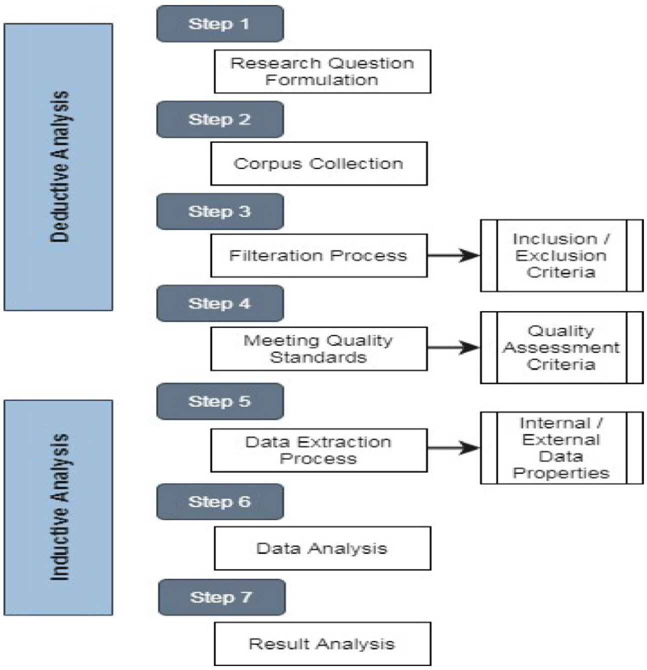

Fig. 1. Research process.

a. *Varied spatial relationships* among the symbols make the recognition take more challenging and necessitates the need for extensive training of the recognizer.
b. *The mathematical content's multidisciplinary nature* is the root of significant ambiguities caused by the HMSE as every symbol, sign, and notation holds different interpretations in different contexts.
c. *No formal definitions* and lack of the same dialects for mathematical text representation cause an issue while defining the grammar and generation of production rules.
d. *Two-dimensional nature* of the HMSE is a significant issue that makes the recognition procedures more complex and uncertain.

Let us consider the expression "x^2x", where 2 represents a superscript or denotes a power of x, and in another case, "x_2x", where 2 represents the subscript to predict a variable. However, in "x2x", a multiplication x by 2 is represented (Chan and Yeung, 2000a,b).

4. Review protocol

The review protocol intends to elucidate the comprehensive layout and set an arrayed framework plan related to tasks ranging from data collection to result in the analysis, as shown in Fig. 1. Fig. 1 depicts the entire task flow of the review process, beginning with the research question formulation, corpus collection, and study filtration process being carried out as the part of the deductive analysis and data extraction process performed to fetch results discussion process comes under our inductive analysis.

The most significant part of the data extraction process is the exceptional focus on the studies' internal and external properties while considering them for answering the formulated research questions. To ensure the rigorousness and repeatability of this review and to reduce researcher bias, the authors elaborately developed the review protocol by frequently holding group discussion meetings on protocol design. As the flow of the review is aligned under the set of rules for SLR (Kitchenham et al., 2009; Kitchenham, 2012; Brereton et al., 2007), the entire format of the survey plan is designed in a way to present this review in a crisp and veracious manner that could be suggestive for the novice researchers in this field with integral, and necessitous details regarding the recognition work done in this domain. This survey helps the researchers and the readers disseminating the details of the recognition processes and methods, categorize them based on principle methodology, and also delved into covering an extra mile by providing all the essentials needed for the beginning of the research on this topic. The authors have kept to concept discovery process all crystalline, depicting each step in the review process, how the study is done, and how it will be helpful.

4.1. Planning the review process

The purpose, as well as the reason for the recognition systems related to HMSE, begins with the planning of the survey. The techniques related to approach the target; inclusion as well as exclusion criteria. A review protocol and the review process are critical for an SLR (Wen et al., 2012). In the following Sections 4.2–4.5, the details of the review process aligned with the defined review protocol are presented.

4.2. Research questions

The core intent for formulating research questions is to help the readers know what this systematic review intends to answer. As the main objective of this study is to investigate the current state of the research pertaining to recognition in HMSE, thus the format of questions is tailored in a way that all fundamental concepts and core pivots are comprehensively covered. Specifically, in this research, the authors aim to investigate the existing literature to characterize and synthesize available contributions concerning the investigated research topics, research types, research methods, contribution types, and research gaps from the perspective of both scholars and practitioners in the context of recognition related to HMSE. The primary research question driving this research and reflecting our objective is:

What is the current state of the research pertinent to recognition works in HMSE?

To answer this question, five questions are posed. Table 1 shows five questions along with sub-questions.

4.3. Inclusion/ exclusion criteria

At this point in the process, it is imperative to determine that forgathered literature is relevant and propitious for review. While the focus of each study may vary depending on the overall purpose, it becomes crucial to set decisive parameters to select and extract the most substantial and pertinent literature, which is more concentrated on the research area of recognition of symbols and expressions of mathematical text. Defining this exigent refining process, the following factors are considered while selecting/inclusion of a study in the research articles' candidate group. At the same time, some parameters are settled for excluding several studies. The listing of both inclusion and exclusion parameters is given in Table 2.

4.4. Quality assessment criteria

QA of selected studies is used initially as the basis for weighting the retrieved quantitative data in the meta-analysis (Higgins et al., 2019), which is a critical data synthesis strategy. In pursuance of the inclusion/exclusion criterion used to find the relevant papers, QA is performed on the remaining studies. The authors used QA results to align the interpretation of review findings and indicate the strength of inferences more towards the review's objectivity. The summary of the selected QA questions is presented in (Appendix B). In short, to boost the inclusion–exclusion results, the authors performed QA, which assured the rigorousness, credibility, and relevance of the selected studies.

Table 1
Research Questions.

	Research Questions	Rationale
5.1.	Intensity of research work done	
	What is the intensity of research activity on HMSER?	To identify and analyze the research activities in the area of HMSER as per the time domain.
	What is the frequency of research work on HMSER undertaken on different continents?	To make a comparative analysis between the works done by different countries and continents and highlight the active countries performing research on HMSER.
	Who are the leading researchers directing the research investigation on HMSER?	To extract out the area experts and researchers who had contributed well to HMSER and motivate the research in this direction.
	What are influential journals for papers analyzing the recognition works on HMSE?	To guide the researchers and fresher in this domain for new projects and for providing better referencing sources.
5.2.	Techniques for Recognition of HMSE	
	What are the approaches and techniques used for HMSE recognition?	To identify, analyze, and organize the technologies implemented for HMSER.
	What is the current state of art, limitations, and future scope of the techniques for recognition of HMSE?	Involves the classification of HMSER and highlights the growth in each category of the technique along with their limitations.
5.3.	Dataset used for Recognition process	
	What are the datasets used and available for HMSE recognition?	To assist the observational studies on HMSER and facilitate the research by assuring the availability of corpus for better analysis.
	Is the size of the dataset affect the accuracy of the recognition system?	To compare the size of the dataset and corresponding accuracies rate.
5.4.	Stages of the recognition process	
	What are the sub-processes involved in the recognition processes?	To deduce the research activity steps in the HMSER process.
	How frequently is work done using the sub-processes identified?	To determine the prevalence of the research concentration on phases/ sub-stages involved in the recognition process.
5.5.	Representation methods, algorithms, and tools used	
	What are the representation models used in the recognition process?	To identify and substantiate the representation structures used in the process of recognition of HMSE.
	What are the algorithms used in the recognition process?	To trace out the achievable algorithms that can be used for HMSER.
	What are the tools that are available for HMSER?	To recognize and vindicate the tools that can be helpful for future researchers for the research considerations.

Table 2
Inclusion/ Exclusion Criteria.

Inclusion Criteria	Exclusion Criteria
Full-text studies are included.	No full text available.
Studies' language considered: English.	Study in a language other than English.
Study with their primary objective: recognition of HMSE.	The study is related to handwritten math symbols without any context of recognition
The study targeting the improvement in current recognition systems was contained.	Studies not relevant to the research questions.
The study mainly focuses on a stage or sub-process in the recognition systems was considered.	Theoretical and concept-based study with no context of experimentation.
Study with implementation and experimental analysis is included.	The study that addresses the math symbols and expressions from different languages (e.g., Arabic, Chinese)
Only the journal version will be included for a study with both the conference version and journal version.	Studies concentrating on factors (issues or review-based) other than recognition are excluded.
For duplicate publications of the same study, only the most complete and newest ones are included.	

4.5. Data extraction

The data properties and the data extraction procedure are elaborated in this section.

Extraction of optimal answers to the research questions is an integral phenomenon that acquires critical investigation and supervision that implicates a defined and perpetual screening process. The authors formulated a data extraction form for this purpose.

The attributes of Table 3 have been exemplified below:

- The initial two columns are the attributes depicting the data property identifier and name.
- The third column refers to the cardinality of the relationship between an individual primary study and data property. In contrast, the fourth column determines the traceability between an individual data property and the formulated research questions.
- The authors identified that the data properties and categorized them as per their extraction methods. The kinds of data properties are defined below:

External Data Properties: The external properties revolve around the extraction of pretty generic information and be named generic attributes. These properties of the articles can be extracted without the full view of the context and findings of the research study. These attributes need fewer efforts and can be thoroughly extracted by analyzing the considered article explicitly. The details are presented in Table 3.

Internal Data Properties: These comprise the attributes which can be identified only after an extensive review of the entire context of the research study. Unlike external properties, internal details demand more effort and time. Also, a decent amount of prerequisite tasks is needed to extract these data attributes finely and precisely. The details are presented in Table 4.

4.6. Data extraction procedure

Extracting data from the research articles is not just an extensive exercise, but it acquires a stimulating undermining process that necessitates many undertakings and prudent investigations. The task of

Table 3
External Data Properties.

Id	Data property	Cardinality	Research question
DPId1.	Publication Year	1:1	RQ1
DPId2.	Publication Channel	1:1	RQ1
DPId3.	Author Name	1:1	RQ1
DPId4.	Country(Authorship)	1:*	RQ1
DPId5.	Continent(Authorship)	1:*	RQ1
DPId6.	Institute(Authorship)	1:*	RQ1

Table 4
Internal Data Properties.

Id	Data property	Cardinality	Research question
DPId7.	Recognition Technique (RT)	1:*	RQ2
DPId8.	Category of RT	1:*	RQ2
DPId9.	Dataset	1:*	RQ3
DPId10.	Dataset size	1:*	RQ3
DPId11.	Accuracy metric	1:*	RQ7
DPId12.	Sub-process identification	1:*	RQ4
DPId13.	Representation model	1:*	RQ5
DPId14.	Algorithm	1:*	RQ5
DPId15.	Tool used	1:1	RQ5

determining the data properties header is like a short preview. The exciting part of the story is underpinned in this extraction procedure, where every title calls for exhilarated observational skills while reviewing the selected studies. All the aforementioned properties were settled as attributes, and a sheet of tabular extraction form was generated. The data properties were assigned to the columns, and each row contained the selected study name to be investigated. The chosen studies' titles populated the first column.

The extracted value of data property was divulged, corresponding to each selected research study after an attentive and thorough review process. One of the authors accomplished this first phase of data extraction while the other author rigorously cross-verified the contents of the data extraction form. There were two data extraction forms concocted so that the first form integrates the generic details (external properties), and the other assimilates internal information. The second author proposed the creation of separate data extraction forms for HMSEs. So, after data extraction, all the different sheets need to be assembled, extracting the results as per the requisite of the formulated research questions could be performed. The data extraction sheets were verified and finalized after several rounds of meetings and profound discussions.

4.7. Data analysis and interpretation

An exceptional prototypical characteristic of this review is the fact that the research questions were formulated in a way that each question served as a squeezer for the extraction of the best and needed stack of information on the topic of HMSER. The very first question of the study elicited descriptive statistics and quantitative descriptions of frequencies, which was mainly used to analyze the meta-data crux of the chosen articles concerning research intensity, frequently investigating researchers and listing the dominant research channels for publications. As the objective was limited to quantitative analysis, so the rest of the questions series was devised following qualitative analysis. The remaining research questions supported the primary essence of this review's objectivity and were mostly inclined to extract details of recognition techniques used. All the questions directed the data analysis and interpretation aspect of the study quite legitimately and in a motivating scheme.

5. Results and discussions

The preeminent objective of this study is to collate, relate, and organize the recognition-oriented research tasks for HMSE that have been carried out in the past four decades. The complete exploration of this review demos and unveils a coherent expedition about how the several techniques have kept emerging and been used sustainably for HMSER. This entire process of reviewing has been performed to deduce the details from the selected studies as per the defined research questions in Table 1. Thus, in this section, the authors describe the sublimated results obtained from the chosen set of studies for each research question.

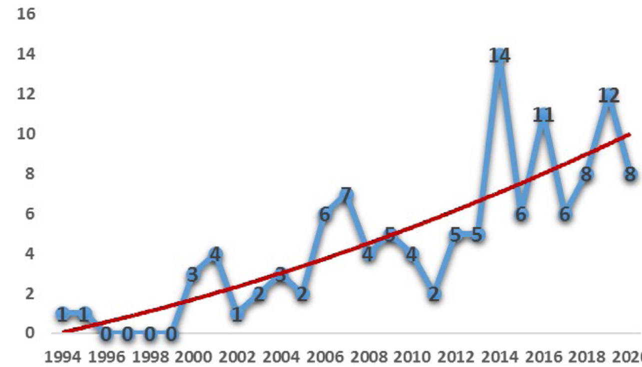

Fig. 2. Count of publications year-wise.

5.1. Intensity of research work

This research question is modeled to identify and analyze the intensity of research activities associated with the current literature on HMSE. This question summarizes the gathered work and presents the exact quantification and precise depiction of publication trends and other affiliated information. The intensity of research is portrayed in a way that all aspects ranging from frequency of year-wise publications, authorship information, publication channels, publication titles, and keywords have been explicitly used as evidential parameters for illustrating the research trends and the intensity accurately.

5.1.1. What is the intensity of research publication per year on HMSER?

It becomes veritably essential to notice and substantiate the research trends as it is the source that shapes and cynosures the interests of the researchers in this domain. Observing the research trends annually also brings out the consequential points and reasons that lead to the observed research trends in the chosen scope. Fig. 2 represents the research trends on the grounds of research publications per year. The selected set of studies in this review passed several selection procedures and then, resulting in a total count of 120 studies. However, the actual number that depicts the amount of research carried since the 1994s is far more than the chosen studies selected by this review. The authors have strived to highlight the evidence of association in all possible aspects, bringing out the facts and results in good agreement with the previous literature.

The research activity till the year 2000 depicts a linear graph trend, with a considerably fewer number of publications. Even after the year 2000, the average research activities continued to hold a relatively low value of approximately four papers per year. The year 2011 proved remarkable as it proliferated the number of research activities and produced a decent average that is approximately double the previously obtained value, i.e., eight papers per year. The incremented average in the number of publications evacuated a space for interrogation of the reasons behind this inflated progress. The researchers contemplated and legitimized the underlying factors and found that the CROHME series of competitions launched in 2011 has accelerated the magnitude of the research activities associated with HMSER, distinctly demonstrating the value of the included research articles the significant year 2011. It enlightens the point that the CROHME series has proved to the point

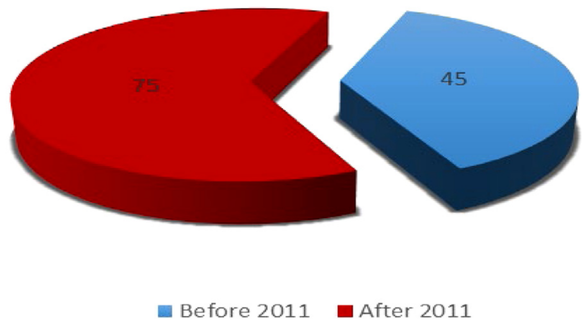

Fig. 3. Count of studies as to the year 2011.

of escalation of the research activities pertinent to HMSER. Out of 120 chosen studies, 75 studies are published after 2011, resulting in a major proportion of total review studies (63%) shown in Fig. 3.

Summary of findings: From the beginning till the year 2011, the research activities were rare in number. After that, the period considered from 2011–present time witnesses increased research interests, owing to the advent of CROHME competitions, which became an absolute reason for the cause and catalyzing event in the research history HMSER.

5.1.2. What is the frequency of research work on HMSER undertaken on different continents and countries?

The geographical distribution of publications allied to HMSER over the years is presented in Fig. 4. It exhibits the amount of concentration of research carried in a country in a particular year. Fig. 5 depicts the active continents where the research on recognizing mathematical text is being carried out. The major countries generating papers about HMSER over the years were China, Japan, USA, and Canada. For the analysis of the country-wise distribution of articles, the country of origin of the first author was taken into consideration (Zawacki-richter et al., 2009) as the first author is the one who undertakes most of the research and considered to be the major contributor during research writing (Levitt and Thelwall, 2009; Nicholas et al., 2017; Kumar and Mutanga, 2018). The dominating country that contributed best to the research scenario on the theme is China, followed by Japan and USA. A nominating country in the list known for average participation is India. The implication of findings is also evidential of the fact that the continent with strengthening research and active interest in this scope is Asia. The graph of the leading country with a total included research publication of 21 studies shows a linear and exponential rise. It can be observed from Fig. 4 that research publications are proportionally rising with time as China has produced five effective research studies in the present year 2020 and that too in limited considered time till July 2020. The pro-active continents and countries diligently involved in research on HMSER are vivaciously represented in Table 5.

It is also vital to highlight that we cannot make a valid conclusion concerning 2020 as our search covers only part of that year. Even though the number of publications on HMSER changed over time, the research activity in this area persists in aggrandizing. Its evolution has also showcased a stable growth, particularly during the last two decades.

Summary of findings: The most active continent witnessing significant works on HMSER in Asia and the country with ever-hyping progress of research in this area is China.

5.1.3. Who are leading researchers directing the research investigation on HMSER?

To characterize who researches HMSER, the authors analyzed the authorship information of selected primary studies. The authors collected and scrutinized the contribution of the individual researcher to the area of research and developed the four groups of authors as per the observed frequency of articles in the selected suite for this review. The first group has depicted in Fig. 6 illustrates the names of those authors who have contributed around two or three influential papers in this dominion.

In contrast, the second group holds the researcher's list with publication count equals four, five, or six. The third category of researchers is those who had value participation more than six. Likewise, the fourth group depicts the authorship information of the leading researchers in this research domain. On investigating the affiliations of prominent researchers, the active universities and institutions working on this research subject came into the observation. The following list of active universities was generated. List of active universities working on HMSER

- University of Nantes, Nantes, France[1]
- Tokyo University Of Agriculture and Technology, Tokyo, Japan[2]
- Queens University, Ontario, Canada[3]
- University of Science and Technology, Hefei, Anhui, China[4]
- Freie University, Berlin, Germany[5]
- Kyoto University, Kyoto, Japan[6]
- Aston Business School, Birmingham, United Kingdom[7]
- Kyushu University, Fukuoka, Japan[8]
- Drexel University, Philadelphia, United States[9]

Summary of Findings: The prominent researchers in this field are Richard Zannibi, C. Viard Guardin, and Masaki Nakagawa. Eventually, all three leading researchers are Professors at different prestigious institutions. Thus, the corresponding universities associated with them are Queens University, University of Nantes, and Tokyo University of Agriculture and Technology. It also highlights that the research in HMSER is led by academicians, and different universities in Japan are proactively working on the HMSER.

5.1.4. What are influential journals and conferences for papers analyzing the recognition works on HMSE?

On analyzing the publications channels and investigating the associating shreds of evidence in regard, it was found that a large number of significant publications are channelized and published in conference proceedings. It becomes irresistible to ignore the influence of the CROHME series, which is a conference and has competitive proceedings. CROHME series of competitions began in 2011, when the first version of this event, CROHME-1, was held at ICDAR 2011. The start of this competition has not just enhanced the attention of researchers in the field of HMSER, but it has optimized and amplified the number of research publications over the years.

The presented Fig. 7 depicts the contribution of eight leading journals on this subject. Pattern Recognition, IJDAR, and Pattern Recognition letters have an equal contribution to the selected set of papers for

1 https://english.univ-nantes.fr/.
2 https://www.tuat.ac.jp/en/.
3 https://www.queensu.ca/.
4 https://www.ustm.ac.in/.
5 https://www.fu-berlin.de/en/index.html.
6 https://www.kyoto-u.ac.jp/en.
7 https://www.aston.ac.uk/bss/aston-business-school.
8 https://www.kyushu-u.ac.jp/en/.
9 https://drexel.edu/.

Table 5
Active Continents and Countries in HMER.

Country	Continent	Reference
Brazil	South America	(Julca-Aguilar et al., 2014; Fontenele Marques Junior et al., 2019; Julca-Aguilar et al., 2020, 2015; Álvaro et al., 2014a; Alvaro et al., 2014b)
Canada	North America	(MacLean and Labahn, 2015; MacLean et al., 2013; MacLean and Labahn, 2010; Ahmed et al., 2004; Zanibbi et al., 2001; Zhang et al., 2005; Keshari and Watt, 2007)
China	Asia	(Tian et al., 2007; Wan et al., 2019; Shi and Soong, 2008; Fang and Zhang, 2020; Wu et al., 2020; Nguyen et al., 2020; Wang and Shan, 2020; Zhang et al., 2019; Dai et al., 2019; Zhang et al., 2018a,b,c, 2017a,c; Hu et al., 2014; Lin et al., 2012; Qi et al., 2009; Shi et al., 2007; Tian et al., 2006; He et al., 2016; Fang et al., 2019; Fu et al., 2020; Aly et al., 2008)
England	European	(Baker et al., 2010)
France	Western Europe	(Medjkoune et al., 2011; Zhang et al., 2017b, 2016; Awal et al., 2014, 2010c,b, 2009)
Germany	Western Europe	(Tapia and Rojas, 2004, 2003)
India	Asia	(Garain et al., 2004; Drsouza and Mascarenhas, 2018; Shinde et al., 2018; Bharambe, 2015; Bage et al., 2010; Gharde et al., 2013; Garain, 2009; Ramteke and Mehrotra, 2006; Kumar et al., 2019; Abirami and Jaganathan, 2019)
Ireland	Western Europe	(Huang et al., 2007; Fitzgerald et al., 2007; Genoe et al., 2006; Nazemi et al., 2019)
Japan	Asia	(Iwatsuki et al., 2017; Le et al., 2019a,b; Ung et al., 2018; Phan et al., 2018; Le and Nakagawa, 2017a,b; Dai Nguyen et al., 2016; Le and Nakagawa, 2016a, 2015; Aly et al., 2009; Malon et al., 2008; Toyozumi et al., 2004; Eto and Suzuki, 2001; Kanahori et al., 2000; Suzuki, 2000; Le, 2020; Khuong et al., 2019)
Spain	Western	(Dimitriadis and López Coronado, 1995; Álvaro et al., 2016, 2012, 2011)
USA	North America	(Hu and Zanibbi, 2016a; Zhu et al., 2013; Hu and Zanibbi, 2016b; Mohan and Lu, 2015; Pillay, 2014a; Jain and Zanibbi, 2017; Hu et al., 2012; Vuong et al., 2008; LaViola and Zeleznik, 2007; Zanibbi et al., 2002; Hu and Zanibbi, 2011; Davila et al., 2014)
Vietnam	Asia	(Le et al., 2019a,b)

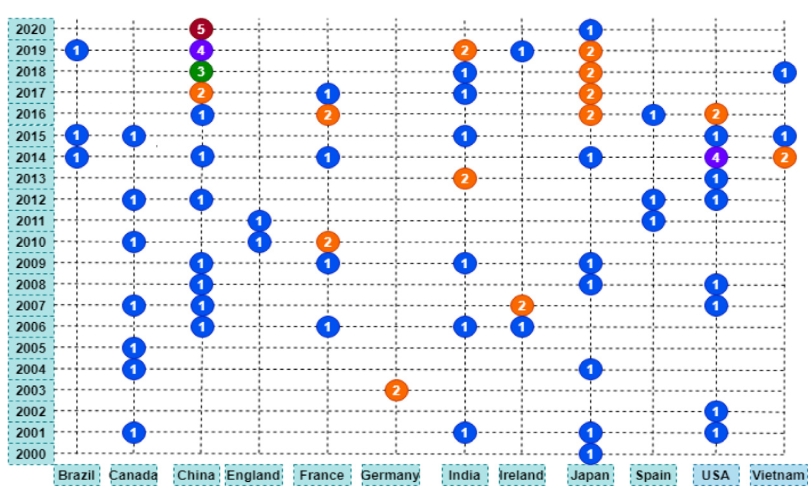

Fig. 4. Geographical distribution of paper over the years.

this review. Hence, it becomes subtle to specify a single name in terms of dominant journals as the trio has an equivalent share to the collected pile of papers. On examining and contemplating the conferences and their corresponding count of selected papers (Fig. 8), it can be observed that the ICDAR has the maximum count of the research publications.

Summary of Findings (RQ1): The prominent publication channel of this domain is the conference. The renowned conference in this area is ICDAR, which conducts the CROHME competition series. The leading journals affiliated to our field are Pattern Recognition, IJDAR, and Pattern Recognition Letters.

5.2. Techniques for recognition of HMSE

The rationale for this question is all associated with the core motive of this review analysis. The primary objective of this review is to recapitulate the recognition techniques used in HMSER and elucidate the entire shift of trends in recognition steps with the variation and emerging recognition technologies. Here in this section, the authors have strived to present the whole walkabout of the research trends used for HMSER to exhibit the preview from a brand new scenario, as illustrated below.

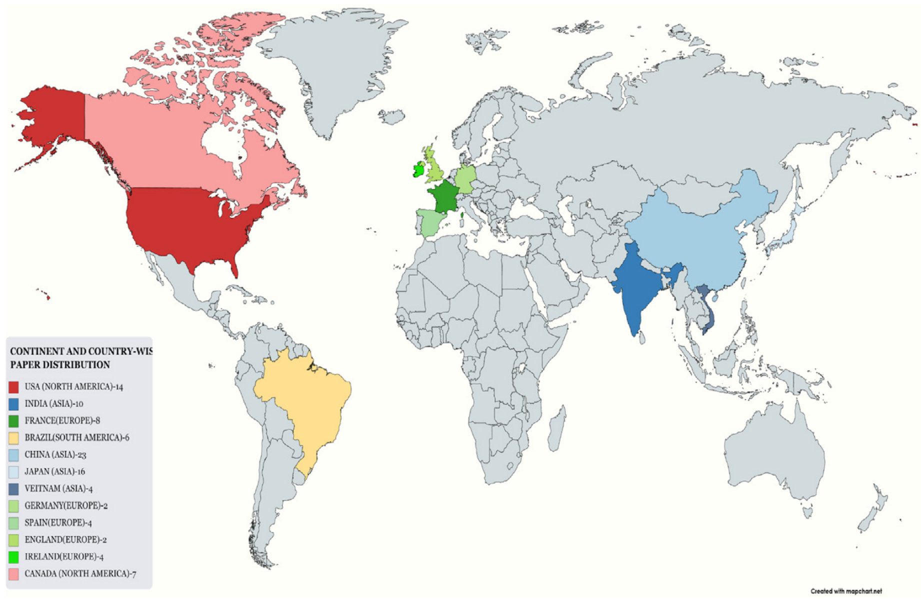

Fig. 5. Continent and country wise publication record.

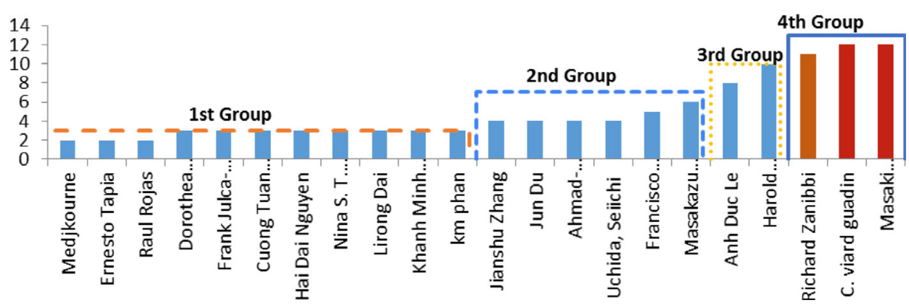

Fig. 6. Authorship information and frequency of the research work on HMSER.

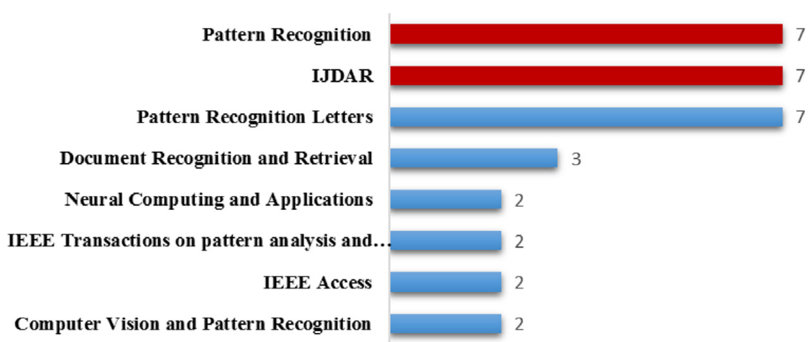

Fig. 7. Journals information and the corresponding count of selected publications.

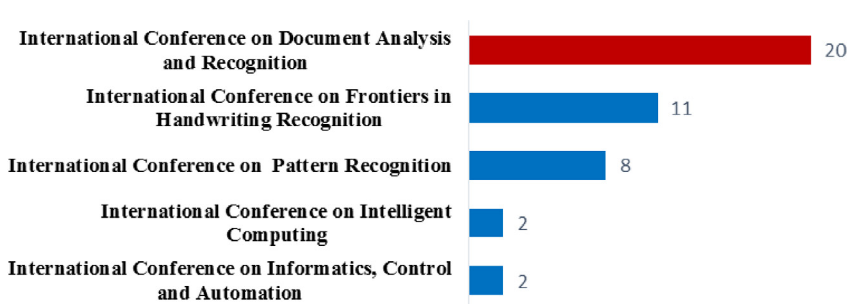

Fig. 8. Conferences and the corresponding count of selected publications.

Table 6
CROHME Dataset Analysis.

Authors	Technique	Accuracy Measure	CROHME 2011	CROHME 2012	CROHME 2013	CROHME 2014	CROHME 2016	CROHME 2019
(Julca-Aguilar et al., 2014)	Neural Network	Recognition rate(%)	X	X	84.38	X	X	X
(Hu et al., 2012)	HMM+LL(1) PARSER	Structural Recognition(%) Symbol Segmentation Rate (%) Symbol Recognition Rate (%) Exp Recognition Rate(%)	41.69 47.35 87.00 13.26	X	X	X	X	X
(Simistira et al., 2012)	Elastic matching distance	Structural Recognition(%) Symbol Segmentation Rate (%) Symbol Recognition Rate (%) Exp Recognition Rate(%)	X	75.90 86.67 81.76 9.94	X	X	X	X
(Mohan and Lu, 2013)	HMM+SVM	Accuracy(%)	77	X	X	X	X	X
(MacLean et al., 2013)	Grammar-based approach	Stroke Recognition% Symbol Recognition% Expression Recognition%	71.73 84.09 85.99 32.04	X	X	X	X	X
(Álvaro et al., 2014a; Alvaro et al., 2014b)	two-dimensional stochastic context-free grammar (2D-SCFG) and HMM	Stroke Classification Rate(%) Symbol Segmentation Rate(%) Symbol Recognition rate for well-segmented symbols (%) Expression Recognition Rate (%)	78.38 87.82 92.56 19.83	X	X	X	X	X
(Awal et al., 2014)	Global Learning Approach, global classifier, and a Gaussian structural model.	Segmentation rate(%) Recognition rate(%) Expression Rate(%)	87.04 92.47 47.41	X	X	X	X	X
(Le et al., 2014)	SVM+SCFG+CYK SVM	Symbol segmentation(%) Segmentation+ Classification Rate(%) Relational Tree(%) Expression Recognition(%)	X	X	81.95 67.06 49.12 20.72	X	X	X
(Álvaro et al., 2014a; Alvaro et al., 2014b)	RNN Recurrent Neural Network	Online Feature Recognition rate (%) Offline Feature Recognition rate (%)	X	X	82.5 84.1	X	X	X
(Simistira et al., 2014)	SVM/ANN	Recognition Rate(ANN)% Recognition Rate (SVM)%	X	96.27 98.16	X	X	X	X
(Jan and Daniel Průša, 2014)	STATISTICAL APPROACH	Error rate(%)	X	3.9	2.7	X	X	X
(Julca-Aguilar et al., 2015)	Graph grammar	Recognition Rate(%) (Recall and precision)% Segmentation Rate Classification Rate Tree Relation	X	X	21.61 75.70, 83.41 62.63, 69.00 44.67, 49.73	X	X	X
(Mohan and Lu, 2015)	Convolutional Neural Network(CNN)	Accuracy	90	X	X	X	X	X
(Álvaro et al., 2016)	2D PCFG	Recall and Precision (%)	X	X	92.0 90.7	X	X	X
(Dai Nguyen et al., 2016)	Deep neural network(DMCN) offline+ (BLSTM)online	N-Best Recognition Rate (%)	X	87.37	91.28	X	X	X
(Le and Nakagawa, 2016a)	SCFG + CYK algorithm	Expression Recognition Rate (%) Symbol Segmentation Rate (%) Segmentation +Classification Rate (%) Relational tree (for structure analysis)(%)	X	X	20.72 81.95 67.06 49.12	X	X	X
(Hu and Zanibbi, 2016b)	CYK parsing with 2-dimensional Stochastic Context-Free Grammars (SCFG)	Segmentation Rate(recall) Symbol Relationship Classification Rate	X	94.87 97.64	X	92.41 96.55	X	X

(*continued on next page*)

5.2.1. *What are the approaches and techniques used for HMSE recognition?*

Inspired by the previous works, the authors observed that the techniques used for recognition are majorly driven by two kinds, one of which represented a recognition approach thoroughly based on grammar-driven methods. The other had artificial intelligence and computer vision techniques as its foothold. And the different concomitant strategies can be categorized under these two significant classification headers, as depicted in Fig. 9.

- **Grammar-Driven Recognition Approach**

There are three main intermediate steps in the recognition procedure of HMSE, namely segmentation, symbol recognition, and

Table 6 (*continued*).

Authors	Technique	Accuracy Measure	CROHME 2011	CROHME 2012	CROHME 2013	CROHME 2014	CROHME 2016	CROHME 2019
(Zhang et al., 2016)	BLSTM	Segmentation Recognition Recognition class	X	X	X	92.77 85.17	X	X
(Le et al., 2016)	x–y cut	Symbol segmentation(%) Segmentation+ Classification Rate(%) Relational Tree(%) Expression Recognition(%)	X	X	X	89.85 83.21 71.55 35.80	X	X
(Hu and Zanibbi, 2016a)	Line-of-Sight (LOS)	F-measure%	X	X	X	92.43	X	X
(Ramadhan et al., 2016)	CNN	Accuracy(%)	X	X	X	87.72	X	X
(Le and Nakagawa, 2017a,b)	CNN	Recognition rate(%)	X	X	X	35.19%	X	X
(Zhang et al., 2017c)	GRU+RNN+	Correct Expression Recognition Rate(%)	X	X	X	52.43	X	X
(Zhang et al., 2017a)	CNN	Expression Recognition Rate (%)	X	X	X	44.42	X	X
(Zhang et al., 2017b)	BLSTM	Expression Recognition Rate (%)	X	X	X	21.59	X	X
(Zhang et al., 2018b)	CNN	Recognition rate(%)	X	X	X	52.8	50.1	X
(Zhang et al., 2018a)	Neural Network Based	Expression Recognition Rate(%)	X	X	X	61.16	57.02	X
(Zhang et al., 2018b)	Dense Net (based on densely connected convolutional blocks)	Expression Recognition Rate(%)	X	X	X	52.8	50.1	X
(Zhang et al., 2018c)	Based on RNN, namely BLSTM	(Recall and Precision)% Segmentation Rate Segmentation+Classification Rate Relational Trees(for structural analysis)	X	X	X	95.52 91.81 89.55 85.60 78.08 74.64	95.64 91.49 89.84 85.90 77.23 74.08	X
(Le et al., 2019a)	CNN	Recognition rate	X	X	X	48.78	45.60	X
(Le et al., 2019b)	CNN BLSTM	Expected Rate of stroke order normalization(%) Expression Recognition rate(%)	X	X	X	95.89 37.63	X	X
(Khuong et al., 2019)	LaTeX or MathML script	Two sample t-test	X	X	X	X	$t = -1.87$ p-value 0.07>0.05	X
(Fang et al., 2019)	CNN	Recognition rate(%)	X	X	X	91.72	93.12	X
(Fang and Zhang, 2020)	CNN	Recognition rate(%)	X	X	89.55	92.44	92.96	X
(Nguyen et al., 2020)	CNN/ K means ++ Clustering	Purity for classification(%)	X	X	X	X	0.99	0.96
(Wu et al., 2020)	With paired adversarial learning	Expression Rate(%)	X	X	X	54.87	57.89	X
(Nguyen et al., 2020)	CNN	Clustering purity					0.99	0.96
(Wang and Shan, 2020)	CNN	Expression Rate(%)	X	X	X	49.459	46.062	X
(Julca-Aguilar et al., 2020)	Graph grammar-based framework	(Recall and Precision)% Segmentation Rate Classification Rate Relations Recognition Rate	X	X	X	X	95.32 96.84 89.39 90.81 76.68 79.93	X
(Le, 2020)	Dual loss attention model	Expression Recognition Rates	X	X	X	49.85	47.34	X

structural analysis. It has been observed that many research investigations as (Chou, 1989; Chan and Yeung, 2000a,b), used grammar driven approach for performing structural analysis. The researchers preferred this approach based on predefined grammar production rules and framework as it is a customary way to define and solve the recognition problem. The authors have retrograded the research investigation in a natural alignment with this context to stipulate the evidential math grammar-based approaches from the gathered literature sources. The existing literature also witnesses several distinct implications of grammar-based strategies, which can be further classified into six different categories, as depicted in Fig. 9.

Probabilistic Context-Free Grammar approach is implemented by authors (Álvaro et al., 2016; Yamamoto et al., 2006). The authors (Yamamoto et al., 2006) have used the form of the CYK to implement parsing bi-dimensional PCFG to limit the symbols and relationships, the writing order must be followed. Probability functions are defined based on region, and its representation is known as "hidden writing area". The proposal presented by Álvaro et al. (2016) has many probabilistic models used to extract the information throughout the process of recognition.

Graph grammar proffers a powerful formalism to elucidate structural manipulations of multi-dimensional data. The history of graph grammars methods for graphics and other symbolic recognition have been proposed by different authors (Fahmy and Blostein, 1993; Baumann,

Table 7
Self-Created Datasets Analysis.

<table>
<tr><th>Author-Year</th><th>Number of writers/users</th><th>Description of database</th><th>Accuracy rate</th></tr>
<tr><td>(Lee and Lee, 1994)</td><td>No details of writers mentioned</td><td>The training data consisted of 105 MEs, including 1904 symbols. After training, 50 expressions used to test the system</td><td>Recognition rate 83.06%</td></tr>
<tr><td>(Dimitriadis and López Coronado, 1995)</td><td>The chosen users were engineering graduate and under-graduate students and wrote on different paper types attached to the digitizing tablet.</td><td>The dataset was constructed using alphanumeric text and MEs. Around 100–150 symbols were considered and handwritten by a set of users</td><td>• Detection of correct words: 97.47–98.87∼o.
• Detection of erroneous words: 70.52–89.39∼o.
• Average of correct detection: 89.10%–96.40%.</td></tr>
<tr><td>(Xuejun et al., 1997)</td><td>No details of users mentioned</td><td>The dataset included 369 symbols and 166 898 recordings. Training set = 1,34, 804 recordings Validation set = 15,161 recordings Test set=17,012 recordings Symbols = 369 were tested.</td><td>TOP1 error of less than 17.5% and a TOP3 error of 4.0%.</td></tr>
<tr><td>(Kanahori et al., 2000)</td><td>Thirty writers</td><td>The training set was created using 30 writes, and each one wrote all alphabets and numerical three times. Each writer wrote 20 characters.</td><td>The rate of current results
<table>
<tr><th></th><th>Capital</th><th>Small</th><th>Numeric</th><th>Appear</th><th>Not appear</th><th>Total</th></tr>
<tr><td>1^{st} candidate</td><td>96.5%</td><td>91.7%</td><td>94.5%</td><td>90.9%</td><td>93.4%</td><td>93.8%</td></tr>
<tr><td>Third cndidate</td><td>97.8%</td><td>97.1%</td><td>97.3%</td><td>97.2%</td><td>96.6%</td><td>97.4%</td></tr>
</table>
Recognition rate: 97.4%</td></tr>
<tr><td>(Chan and Yeung, 2000a,b)</td><td>Multiple writers</td><td>The author takes four domains of expression: Elementary algebra Trigonometric functions Geometry Indefinite integrals. Small, medium and large sizes are taken in each domain. Total = 60 expressions.</td><td>Parsing time has been compared The time required for recognizing the structures of different expressions with hierarchical decomposition parsing
<table>
<tr><th colspan="4">The time required for hierarchical decomposition parsing (in seconds)</th></tr>
<tr><td></td><td colspan="3">Small Size</td></tr>
<tr><td>Expression Domain</td><td>Min</td><td>Median</td><td>Max</td></tr>
<tr><td>Elementary algebra</td><td>0.02</td><td>0.03</td><td>0.05</td></tr>
<tr><td>Trigonometric functions</td><td>0.02</td><td>0.02</td><td>0.05</td></tr>
<tr><td>Geometry</td><td>0.02</td><td>0.03</td><td>0.05</td></tr>
<tr><td>Indefinite integrals</td><td>0.02</td><td>0.05</td><td>0.05</td></tr>
<tr><td colspan="4">Median Size</td></tr>
<tr><td>Expression Domain</td><td>Min</td><td>Median</td><td>Max</td></tr>
<tr><td>Elementary algebra</td><td>0.05</td><td>0.07</td><td>0.08</td></tr>
<tr><td>Trigonometric functions</td><td>0.05</td><td>0.07</td><td>0.07</td></tr>
<tr><td>Geometry</td><td>0.05</td><td>0.08</td><td>0.1</td></tr>
<tr><td>Indefinite integrals</td><td>0.07</td><td>0.08</td><td>0.08</td></tr>
<tr><td colspan="4">Large Size</td></tr>
<tr><td>Expression Domain</td><td>Min</td><td>Median</td><td>Max</td></tr>
<tr><td>Elementary algebra</td><td>0.08</td><td>0.15</td><td>0.25</td></tr>
<tr><td>Trigonometric functions</td><td>0.08</td><td>0.1</td><td>0.15</td></tr>
<tr><td>Geometry</td><td>0.1</td><td>0.12</td><td>0.15</td></tr>
<tr><td>Indefinite integrals</td><td>0.1</td><td>0.16</td><td>0.17</td></tr>
</table></td></tr>
<tr><td>(Suzuki, 2000)</td><td>No details of users mentioned</td><td>The dataset in the experimentation is not defined. A data tablet was used to draw the strokes, which was further used in handwritten mathematical formulas.</td><td>The authors achieved an 80% character recognition rate and a 92% recognition rate for the mathematical structures. At present, the best recognition rate achieved is 69%.</td></tr>
<tr><td>(Zanibbi et al., 2001)</td><td>No details of users mentioned</td><td>The author used FFES system to generate test expressions with the help of a data tablet and mouse.</td><td>Not specified</td></tr>
</table>

(*continued on next page*)

1995; Fahmy and Blostein, 1992; Han and Zhu, 2005), but applications of graph grammar methods are restricted to particular languages which lead to difficulty in generalization. Out of all methods, Bunke (Bunke, 1982) proposed one of the first methods, which objectifies recognizing the circuit diagrams and flowcharts. As graph grammar defines the structure and semantics, the author has presented a control diagram demonstrating how the production rules of graph grammar can be implemented to produce a graphic. In the research work of Lavirotte and Pottier (1998), an intermediate combinatorial structure was introduced between the recognized symbols concerning the position in the plane and the formula tree. The main two problems of eliminating ambiguities between grammar rules and the construction of the graph were solved. For the recognition of numerical symbols, graph grammar favored constructing better syntactic and semantic rules for parsing. There are much significant research works in the field of HMSER that implement the graph grammar methods.

The researchers (Celik and Yanikoglu, 2011) proposed a method that deploys a probabilistic CFG grammar that guides in finding mathematically accurate interpretations and assigns probabilities to each possible interpretation of a particular ME. The authors (Julca-Aguilar et al., 2020) translated the recognition problem into a graph parsing problem. Given the set of strokes (input data), parse tree representation is the best interpretation. The graph parsing algorithm produces various meanings that can be graded according to a global cost function that takes into account the probability of symbols and structures. Under this method, it is easy to relax the stroke ordering constraint, allowing interspersed symbols instead of the previously presented literature.

SCFG is a strong syntactic pattern recognition formalism that has been commonly used for string patterns. However, it is possible to lightly modify this formalism so that grammars can model 2D problems. The authors (Chou, 1989) articulated the fundamentals of stochastic context-free grammars, including parsing and parameter re-estimation algorithms. They extended these algorithms to two dimensions and demonstrated their use in a system that recognizes images of noisy equations by translating them into commands and learning the noise probabilities.

The authors (Yamamoto et al., 2006) formulated and devised the problem of ME identification as a search problem for the most likely ME

Table 7 (*continued*).

<table>
<tr><td>(Kacem et al., 2001)</td><td>No details of writers involved</td><td>Training MSs=1,182 Implicit Operators = 200 Testing Symbols = 460, Implicit Operators = 100 Formulas = 300</td><td>The formulae extraction rate has a variable complexity to close to 93%.</td></tr>
<tr><td>(Ahmed et al., 2004)</td><td>No writers involved</td><td>Online Encyclopaedia of integers. OEIS dataset contains 90000 sequences of integers, arranged lexicographically</td><td>Not specified</td></tr>
<tr><td>(Toyozumi et al., 2004)</td><td>Different writers</td><td>200 handwritten mathematical formulas including 4803 strokes and 3579 symbols</td><td>Table a: Performance of CCLM
<table>
<tr><th></th><th>Correct seg.</th><th>Over seg.</th><th>Under seg.</th></tr>
<tr><td>(a) $\alpha = 0$</td><td>90.2%</td><td>220</td><td>26</td></tr>
<tr><td>(b) $\alpha = 14$</td><td>93.3%</td><td>88</td><td>51</td></tr>
<tr><td>(c) $\alpha = 28$</td><td>90.8%</td><td>35</td><td>116</td></tr>
</table>
Table b: Results of Symbol Segmentation
<table>
<tr><th></th><th>C_β</th><th>C_γ</th><th>Correct seg</th><th>Over seg.</th><th>Under seg.</th></tr>
<tr><td rowspan="3">S1</td><td>0</td><td>0</td><td>90.2%</td><td>220</td><td>26</td></tr>
<tr><td>31</td><td>0</td><td>96.2%</td><td>36</td><td>36</td></tr>
<tr><td>31</td><td>51</td><td>97.1%</td><td>36</td><td>23</td></tr>
<tr><td rowspan="3">S2</td><td>0</td><td>0</td><td>88.7%</td><td>341</td><td>55</td></tr>
<tr><td>31</td><td>0</td><td>94.3%</td><td>80</td><td>69</td></tr>
<tr><td>31</td><td>51</td><td>95.3%</td><td>86</td><td>47</td></tr>
</table>
The correct recognition rate is 97.1%</td></tr>
<tr><td>(Thammano and Rugkunchon, 2006)</td><td>61 writers</td><td>This database consists of 61 examples, which were written by 61 people. Each example contains 89 symbols. For each experiment, thirty examples were randomly chosen from the database as the training data, while the remaining 31 examples were used as the testing data.</td><td>Experimental Results
<table>
<tr><th></th><th>Exp. 1</th><th>Exp. 2</th><th>Exp. 3</th><th>Exp. 4</th><th>Exp. 5</th><th>AVG</th></tr>
<tr><td>% Correct</td><td>87.17</td><td>87.02</td><td>86.95</td><td>87.86</td><td>88. 84</td><td>87.57</td></tr>
</table>
Recognition rate: 87.57%</td></tr>
<tr><td>(Ramteke and Mehrotra, 2006)</td><td>No details of writers involved</td><td>The authors used 2000 numerals images obtained from different individuals of different professions. The dataset contains a variety of writing styles.</td><td>Recognition rate 92% success rate</td></tr>
<tr><td>(Genoe et al., 2006)</td><td>Two users.</td><td>The system was tested on a database of 60 expressions entered by two different users. Thirty of the expressions were simple expressions containing less than eight symbols. The remaining 30 expressions were more elaborate, including 10–25 symbols.</td><td>86.67%, 76.67% of the simple expressions and complex expressions were recognized, respectively, and the overall recognition rate was found to be 81.67%.</td></tr>
<tr><td>(LaViola and Zeleznik, 2007)</td><td>11 writers (seven males and four females)</td><td>A total of 6,336 symbols were tested in each of the experiments, which were taken from 11 subjects</td><td>Recognition rate:91.37%</td></tr>
<tr><td>(Shi et al., 2007)</td><td>NA</td><td>Written expressions=2574 with 59,166 strokes Symbols = 43,300.</td><td>Symbol Accuracy
<table>
<tr><th>System</th><th>Symbol Accuracy</th><th>Error Reduction</th></tr>
<tr><td>Baseline System</td><td>85.8%</td><td>–</td></tr>
<tr><td>Bigram Decoding</td><td>92.0%</td><td>43.7%</td></tr>
<tr><td>Trigram Rescoring</td><td>96.6%</td><td>76.1%</td></tr>
</table>
The formula recognition rate was improved from 74.8%to 80.9%</td></tr>
<tr><td>(Fitzgerald et al., 2007)</td><td>Eight writers</td><td>Eight different users wrote the dataset of 245 MEs. Each ME contained an average of 15–20 symbols.</td><td>ME RecognitionRate: 77.6%
<table>
<tr><td>Correct Structure of ME Determined</td><td>87.8%</td></tr>
<tr><td>Every Symbol in ME recognized</td><td>84.9%</td></tr>
<tr><td>ME Recognition Rate</td><td>77.6%</td></tr>
<tr><td>ME Rec. Rate without Multiple Parses</td><td>65.7%</td></tr>
<tr><td>Avg Number of Parses per ME</td><td>4.93</td></tr>
<tr><td>Avg Processing Time per ME</td><td>836ms</td></tr>
<tr><td>Optimum Limit on Number of Parses</td><td>10</td></tr>
</table></td></tr>
<tr><td>(Tian et al., 2007)</td><td>No writers involved</td><td>The authors have scanned 100 Chinese mathematical pieces of literature in the experiment, including about 3000 mathematical formulas.</td><td>(Recognition rate) High quality images: 97.8% 99.5% Low quality images: 97.1% 96.7%</td></tr>
</table>

(*continued on next page*)

candidate in the sense of SCFG. In this method, stroke order was used to minimize the search space, and the CYK algorithm was deployed to parse the series of input strokes. Experimentally, it was proven that the outcome of recognition is strengthened in terms of various grammatical restriction levels. In work (Muñoz, 2010), the authors were stimulated to focus on modeling MEs using SCFG. The authors (Álvaro et al., 2014a; Alvaro et al., 2014b) proposed an improved form of SCFG for the adequate formalization of the recognition model that deals appropriately with the two fundamental problems associated with this kind of model, that is, the learning problem and the interpretation problem.

Table 7 (*continued*).

<table>
<tr><td>(Keshari and Watt, 2007)</td><td>Four writers</td><td>Symbol Set= 137 unique MSs (Latin characters, digits, Greek characters, relational operators, arrows, basic operators, logical operators, delimiters, special characters, and miscellaneous characters.) The tangent distance method was used by the author to increase the dataset by five times.</td><td>Maximum increment in the accuracy Online recognizer = 5% Offline recognizer = 22%.</td></tr>
<tr><td>(Průša and Hlaváč, 2007)</td><td>No details of writers involved</td><td>The dataset included images. There were 330 handwritten images and 210 printed images.</td><td>Recognition accuracy for handwritten images 96.2% Recognition accuracy on printed images 98.0%</td></tr>
<tr><td>(Vuong et al., 2008)</td><td>Ten writers</td><td>A total of 53 expressions were selected from different domains (10 expressions each from elementary algebra, integration, differential calculus, geometry, trigonometric equations, and three from number theory)</td><td>Recognition Rates Trigonometric functions and differential calculus = 97% Number theory Recognition rate = 90%. Average Recognition rates (all domains) = above 90%</td></tr>
<tr><td>(Rhee and Kim, 2009)</td><td>Thirteen graduate students and 31 students from different universities.</td><td>13 Graduate student wrote 30 ME patterns and each expression contain min 5 and max 24 symbols
Total symbols=388 Classes = 50 Total collections are 5044 handwritten symbols and 390 MEs. Thirty-one students of different universities wrote 100 MEs patterns, which contain min 6 and max 35 symbols. Total symbols = 1401 Classes = 79 Total collections are 43431 handwritten symbols and 3100 MEs.</td><td>Symbol Level Accuracy
<table>
<tr><td colspan="3">Symbol level accuracy</td></tr>
<tr><td>Database</td><td>KMW-1(390 MEs)</td><td>KME-II(3100 MEs)</td></tr>
<tr><td>MEs recognized within one minute</td><td>381 (97.7%)</td><td>2955 (95.3%)</td></tr>
<tr><td>Total number of recognized symbols</td><td>4919</td><td>41302</td></tr>
<tr><td>Correctly segmented symbols</td><td>4703(95.6%)</td><td>38576 (93.4%)</td></tr>
<tr><td>Correctly placed symbols in the constructed structures</td><td>4548(92.5%)</td><td>36356 (88.0%)</td></tr>
<tr><td>Correctly labeled symbols</td><td>4316(87.7%)</td><td>34276 (83.0%)</td></tr>
<tr><td colspan="3">ME level accuracy</td></tr>
<tr><td>Database</td><td>KMW-1(390 MEs)</td><td>KME-II(3100 MEs)</td></tr>
<tr><td>Correctly recognized MEs</td><td>151(38.7%)</td><td>677 (21.8%)</td></tr>
<tr><td>MEs whose symbols were all correctly identified</td><td>155(39.7%)</td><td>754 (24.3%)</td></tr>
<tr><td>MEs whose symbols were all correctly placed</td><td>269(69.0%)</td><td>1707 (55.1%)</td></tr>
<tr><td>MEs whose symbols were all correctly segmented</td><td>289(74.1%)</td><td>2001 (64.5%)</td></tr>
</table></td></tr>
<tr><td>(Kim et al., 2009)</td><td>15 writers</td><td>1500 ME data consists of 15 sets of 100 high school level ME examples.
Each ME contains 5 to 25 symbols.</td><td>The proposed system reduced 22.0% of errors in the ME level evaluation Comparison of system performance
<table>
<tr><td></td><td>Incorrect symbols</td><td>Incorrect spatial relationships</td><td>Incorrect MEs</td></tr>
<tr><td>Base system</td><td>1375</td><td>878</td><td>815</td></tr>
<tr><td>Proposed system</td><td>941 (31.6% reduction)</td><td>674 (23.2% reduction)</td><td>635 (22.0% reduction)</td></tr>
</table></td></tr>
<tr><td>(Qi et al., 2009)</td><td>11 undergraduate writers</td><td>Expression from books = 412
Table structure expression = 60 Character samples = 26617</td><td>Test Results
<table>
<tr><td>Test no.</td><td>Input Character</td><td>Error</td><td>Error rate (%)</td><td>Input Expression</td><td>Error</td><td>Error rate (%)</td></tr>
<tr><td>1</td><td>729</td><td>6</td><td>0.82</td><td>50</td><td>3</td><td>6</td></tr>
<tr><td>2</td><td>729</td><td>7</td><td>0.96</td><td>50</td><td>3</td><td>6</td></tr>
<tr><td>3</td><td>729</td><td>19</td><td>2.60</td><td>50</td><td>6</td><td>12</td></tr>
<tr><td>4</td><td>729</td><td>16</td><td>2.19</td><td>50</td><td>5</td><td>10</td></tr>
<tr><td>5</td><td>729</td><td>3</td><td>0.41</td><td>50</td><td>1</td><td>2</td></tr>
<tr><td>6</td><td>729</td><td>28</td><td>3.84</td><td>50</td><td>8</td><td>16</td></tr>
<tr><td>7</td><td>729</td><td>14</td><td>1.92</td><td>50</td><td>3</td><td>6</td></tr>
<tr><td>8</td><td>729</td><td>2</td><td>0.27</td><td>50</td><td>2</td><td>4</td></tr>
<tr><td>9</td><td>729</td><td>5</td><td>0.69</td><td>50</td><td>2</td><td>4</td></tr>
<tr><td>10</td><td>729</td><td>5</td><td>0.69</td><td>50</td><td>3</td><td>6</td></tr>
<tr><td>11</td><td>729</td><td>1</td><td>0.14</td><td>50</td><td>1</td><td>2</td></tr>
</table></td></tr>
</table>

(*continued on next page*)

In *Definite clause Grammar*, a hierarchical decomposition parsing is implemented to apply the replacement rules for parsing MEs. With DCG, the replacement rules were concisely defined and, their definitions were also in a readily executable form. The main idea of using DCG was to detect the errors at the early stage of processing, and error-correcting parsing was implemented to extend the grammar to include all the expected errors into its productions (i.e., grammar rules). The resulting system developed using DCG parser could handle lexical, syntactic, and some semantic errors.

Other *parsing methods* comprises the recognition systems, which included the methods based majorly on parsers such as DRACULAE parsers (Zanibbi et al., 2001, 2002), fuzzy logic-oriented recognition concepts (Kacem et al., 2001; Fitzgerald et al., 2006; MacLean and

Table 7 (*continued*).

(Aly et al., 2009))	No details of writers involved	Total symbols and characters of MEs=1,58,308 Character–Character type = 37,263 Symbols–character type = 54,057 Character–Symbols = 54,924 Symbols–Symbols = 14,064	Accuracy rate relative size = 99.57% Accuracy rate relative position = 99.57%
(MacLean and Labahn, 2010)	20 writers	Expression = 3610 from 20 writers Common expression = 53 Rest all expressions were unique to all writers	Recognition accuracy 80%
(Hu and Zanibbi, 2011)	20 writers	5,119 transcriptions were collected from the 20 participants. Of these, 109 were blank, and 355 were discarded, resulting in 4,655 usable hand-drawn expressions. Comprising these expressions were 25,963 symbols drawn and 21,264 relationships between subexpressions. Training Set Samples = 20281 Testing Set Samples = 2202	The top-5 recognition rate is 98.9%; the top-5 recognition rate is 99.1%.
(Celik and Yanikoglu, 2011)	15 different writers.	Fifty-seven equations each from 15 different writers. For testing, 20 equations are used, which are written by five different writers.	Correctly Recognized Expressions = 17% Correct Structural Analysis = 50% Correct Character Recognition = 79% 1100/1410
(Clark et al., 2013)	Two writers, both of whom have engaged in mathematics at the university level, were selected to partake in the tests	The database consists of 62 expressions, a total of 839 symbols in 48 different classes. Aster mathematical database.	Symbol Classification in SVM = 94.83% accuracy. The expression recovery rate for DRACULAE = 80% The expression recovery rate for custom adjacency matrix = 43.3%
(Gharde et al., 2013)	Three writers from Engineering Field	Mathematical Equation=12 MSs = 124	The handwritten mathematical equation recognition rate is 98.26%
(Hu et al., 2014)	41 different writers	A dataset consisting of 30 model expressions with a total of about 15000 symbols	Segmentation: 82.1% Segmentation and Classification rate 77.9% Tree Relations rate 42.2%
(Bharambe, 2015)	Twenty writers write one symbol ten times.	The dataset contains Logical Symbols = 2000 Alphabets = 1560. Handwritten logical expressions were 50 consisting of 273 char.	Recognition rate 93.8%.
(Phan et al., 2015)	62 elementary school children, 27 junior high school students, and 26 members of the laboratory.	Dataset of 10864 MEs	Recognition rate is 62.74%
(Le and Nakagawa, 2015)	62 primary school children, 27 junior high school students, and 26 students	Hands-Math is a database of MEs which included 11,069 MEs. The number of symbol classes is 94, including math symbols in Japanese elementary and secondary schools. training and testing sets consist of 8,266 and 2,803 MEs, respectively	Symbol segmentation rate: 92.46% Classification rate: 88.24% Tree relations rate: 89.79% Expression Recognition rate: 68.07%
(Chajri and Bouikhalene, 2016)	Different writers	Mathematical logic = 50 Expression Mathematical analysis = 50 Expression Mathematical algebra = 50 Expression Mathematical probability = 50 Expression	Success Rate in Expression segmentation Mathematical logic = 100% Mathematical analysis = 96% Mathematical algebra = 100% Mathematical probability = 98% Recognition rate: Using SVM 98% Using ANN 93%
(He et al., 2016)	No details of users mentioned	To test the proposed method, a database containing 416 ME images taken by mobile phones in various conditions. The authors consider 97 classes of symbols that cover all the digits and letters, as well as widely used MSs	Clear images accuracy is 87% Cluttered accuracy is 45%
(Ung et al., 2018)	21 students	Each student wrote three times 50 HMEs on three kinds of writing surfaces. The total number of online HMEs is 3150, and they belong to 50 classes.	The clustering result is 0.9185 for purity
(Shinde et al., 2018)	Different writers	500 handwritten mathematical equations. Each class is having a minimum of 50 equations.	The segmentation and recognition rate is 97.11%.
(Dai et al., 2019)	20 writers	Handwritten MEs	70% accuracy
(Fontenele Marques Junior et al., 2019)		264780 images containing symbols and numbers. The first dataset contains 70,000 28 × 28 binary images, with 10 classes 2nd dataset: 360,000 45 × 45 images with both numerical and non-numerical symbols. 82 classes	99% accuracy

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Labahn, 2010; Farulla et al., 2016) and other methods based on relational grammars (MacLean et al., 2013)). DRACULAE applies a tree-transformation approach to the recognition of MEs syntax and semantics. Both Fuzzy logic-based and relational grammars have been used to model geometric features to identify mathematical notation spatial characteristics.

- **Artificial Intelligence-based approach**

Neural network-based methods are emerging and are widely used in the past decades as the collection of review studies depicts the implementation of CNN (Mohan and Lu, 2015; Drsouza and Mascarenhas, 2018; Shan et al., 2019; Choudhary et al., 2021), RNN (Zhang et al., 2018d; Ren et al., 2019; Wu et al., 2020), BiRNN (Hong et al., 2019), and other neural network-based systems.

Table 7 (*continued*).

(Abirami and Jaganathan, 2019)	Different writers	There are a total of 92 symbolsEach writer writes each symbol ten times. Dataset — Number of Symbols Latin — 62 Greek — 10 Special Symbols — 20 Equation Dataset — 70	98% symbol classification
(Wan et al., 2019)	No writers involved	The dataset contains 134 categories and a total of 6,300 formula symbol images	94.25% recognition rate
(Khuong et al., 2019)	A questionnaire was used for the experiment. A total of 30 people was used from both sexes and different age groups	Totally 500 synthetic HME patterns were generated by the proposed method. Even the CROHME 2016 dataset was increased to an extent using pattern generation strategies.	The result shows that there is no significant difference in the naturalness between them $t = -1.87$ and p-value $0.07 > 0.05$
(Kumar et al., 2019)	No details of users mentioned	Testing was performed on 50 equations includes symbols such as integral, differential, square, square root.	Recognition rate of 92%
(Zhang et al., 2020)	No details of users mentioned	The dataset containing 81214 equations was obtained to generate ME images. Besides, 6000 background images were collected and split into two parts: one part contains 4000 images, used as the background of the training set, validation set, and the first test set; the other part 2000 images were used as the background of the second test set.	Expression Recognition Rate of the proposed model is 74.3%

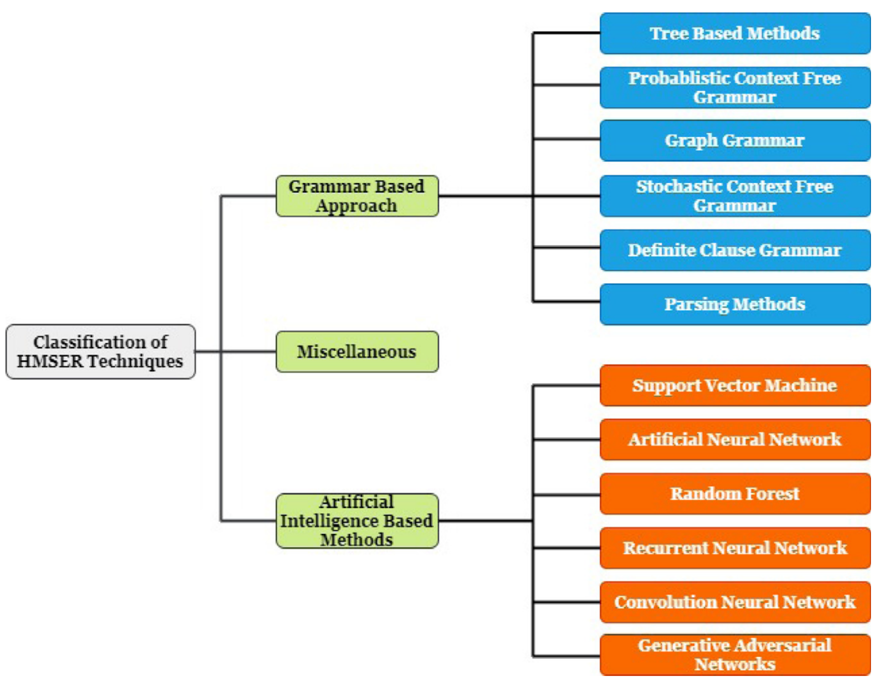


Fig. 9. Classification of HMSER techniques.

The neural networks-based approach has distinguished phases compared to the conventional steps involved in the recognition process. The structural analysis phase is fragmented, and the system constitutes the training and testing of the model. The present research works based on neurons and layered architectures subsume steps of preprocessing, segmentation, feature extraction, and classification. The step towards the enhancement in this approach is the advent of introducing an encoder–decoder-based framework (Shan et al., 2021) along with the neural method technologies. Also, the generality of the *encoder–decoder* framework is suggestive in many ways. The researchers (Zhang et al., 2017a) proposed a model that comprised of an FCN encoder and a GRU decoder equipped with a coverage-based attention model, while (Deng et al., 2017) employed a CRNN as the encoder and the decoder is equipped with a coarse-to-fine attention model. However, both (Zhang et al., 2017a) and (Deng et al., 2017) treated the HMEs input as static images that ignore the handwriting dynamics (namely the temporal order and trajectory).

Recently most of the neural networks (Altan et al., 2021), as well as the encoder–decoder-based approaches are merged and used in alignment with the *attention-based mechanism*. In the works of Zhang et al. (2017c), the authors introduced an encoder–decoder with a coverage-based attention model for recognizing online HMEs. The proposed approach makes the use of GRU-RNN based encoder for complete utilization of online trajectory information. The coverage model is entirely sufficient for attention by making the use of alignment history information. Similarly, the literature witnesses the trend of an attention-based network approach embedded with an encoder–decoder system that dominates the current trend era of the recognition approaches for HMSE.

Table 8
Other datasets Analysis.

<table>
<tr><th>Author</th><th>Dataset name</th><th>Type of database</th><th>Accuracy rate</th></tr>
<tr><td>(Garain et al., 2004)</td><td>Corpus by Garain and Chaudhuri (2005)</td><td>Calligraphic letters, Roman letters and digits, Greek symbols, and some mathematical operators-2,459. The number of symbols found in these expressions- 59,288. Distinct symbols classes-274</td><td>Recognition Rate 98.30%</td></tr>
<tr><td>(Shi and Soong, 2008)</td><td>Corpus by Shi et al. (2007), Hill (1991)</td><td>Test set –2,574 written expressions, 59,166 strokes, 43,300 symbols. It contains English and Greek letters, digits, mathematical operators, integration operators, trigonometric functions, and some special symbols, altogether about 150ME.</td><td>Expression Recognition rate 75.2% (threshold 500)</td></tr>
<tr><td>(Malon et al., 2008)</td><td>Infty CDB-3A</td><td>Infty CDB-3A=1629 mathematical characters</td><td>97.70% of the accuracy of symbol recognition rate</td></tr>
<tr><td>(Aly et al., 2008)</td><td>InftyCDB-1 and InftyCDB-2</td><td>41,581 pairs adjacent alphanumeric characters in math expressions. It consists of 65 English articles (published between 1949 ~2000)4 French articles (available between 1974 and 1988)
7 German articles (available between 1956 and 1987) on pure mathematics.
Total pages = 908.</td><td>Discrimination can be carried out almost entirely ~99.89%</td></tr>
<tr><td>(Awal et al., 2009)</td><td>Aster MEFMmathen database</td><td>The dataset contains 62 different expressions, and each expression length us 13 symbols in each ME. Total symbol = 839 Distinct classes = 48 Dataset created from digits, greek and roman letters, functions, etc.</td><td>Segmentation rate = 91.2% Symbol recognition rate = 75% Global ME recognition rate = 37.1%</td></tr>
<tr><td>(Garain, 2009)</td><td>INFTY</td><td>Scanned pages were taken by the author to conduct the experiment, and a total of 200 scanned pages were considered. Fifty pages of the INFTY were also included in the selected dataset.</td><td>Accuracy rate for Extracting embedded = 88.3% Displayed expressions = 97.2%.</td></tr>
<tr><td>(Awal et al., 2010c)</td><td>Aster Database</td><td>Expression = 36 Each expression have 11 symbols Classes = 34 An expression is collected from 280 writers Total MEs' = 10,080 Each expression was written two times by ten writers, so a total of 72 expressions are there.</td><td>Segmentation rate:91.4% Recognition rate:88.7% Expression rate:64.9%</td></tr>
<tr><td>(Awal et al., 2010a)</td><td>Calculate dataset and ASTER database</td><td>Calculate database contribution
<table>
<tr><th></th><th># writers</th><th>#isolated symbols</th><th>#Expression</th><th>#symbols</th></tr>
<tr><td>Train</td><td>180</td><td>180 x 15 = 2700</td><td>180 x 5 = 900</td><td>5448</td></tr>
<tr><td>Test</td><td>100</td><td>100 x 15 = 1500</td><td>100 x 5 = 500</td><td>3051</td></tr>
</table>
Aster database contribution
<table>
<tr><th></th><th># writers</th><th>#isolated symbols</th><th>#Expression</th><th>#symbols</th></tr>
<tr><td>Train</td><td>180</td><td>180 x 34 = 6120</td><td>180 x 36 = 6480</td><td>180 x 412 = 74160</td></tr>
<tr><td>Test</td><td>100</td><td>100 x 34 = 3400</td><td>100 x 36 = 3600</td><td>100 x 412 = 41200</td></tr>
</table></td><td>SVM Hybrid 73.6 72.9 50.6 TDNN Hybrid 89.8 87.4 51.8 (ASTER DATABASE)</td></tr>
<tr><td>(Medjkoune et al., 2011)</td><td>CIEL database,</td><td>Isolated Math Symbols-74, Latin Alphabet Digits-10, Greek alphabet-6 remaining classes taken from various MEs symbols</td><td>Recognition Rate 81.55</td></tr>
<tr><td>(Álvaro et al., 2011)</td><td>(InftyCDB-1) dataset</td><td>InftyCDB-1=21K ME Includes MSs = 157K Classes = 212</td><td>Symbol error rate of 4.68%.</td></tr>
<tr><td>(Álvaro et al., 2012)</td><td>MathBrush corpus.</td><td>The authors randomly split the database for training and testing purposes. The training set was self-possessed of 70% of the samples (3227 expressions), and the remaining 30% were selected as the test set (1383 expressions).</td><td>Segmentation recognition rate 85%, Symbol recognition rate 80%., and Expression-recognition rate was: 25.31%</td></tr>
<tr><td>(Zhu et al., 2013)</td><td>InftyCDB-3 dataset</td><td>Alphanumeric characters and math symbols, 188,752 symbols</td><td>Top-1, recognition rate = 96.90% and Area Under Curve (AUC) of precision/recall = 47.27%</td></tr>
<tr><td>(Pillay, 2014b)</td><td>INFTY dataset</td><td>INFTY dataset = 21,056 MEs</td><td>Recognition Rate 74%</td></tr>
<tr><td>(Farulla et al., 2016)</td><td>MNIST</td><td>Handwritten digits=60,000 MNIST validation = 10000 handwritten digits.</td><td>Segmentation rate 76.5%</td></tr>
<tr><td>(Iwatsuki et al., 2017)</td><td>ACL Anthology</td><td>The authors selected 74 papers from ACL Anthology. Each paper contained a minimum of one ME, a maximum of 699 MEs, and an average of 156 MEs.</td><td>88.95% F-measure value</td></tr>
</table>

(*continued on next page*)

After briefly analyzing the trend patterns of ML and DL aspects, the ML approaches such as SVM (Malon et al., 2008; Simistira et al., 2014), K-means (Hu and Zanibbi, 2011; Nazemi et al., 2019), Random Forests (Hu and Zanibbi, 2016a; Lods et al., 2019), Decision Trees (Davila et al., 2014; Lavanya et al., 2017) and GANs (Wu et al., 2020; Kundu et al., 2020) are few notable names of the algorithms identified in the selected studies for review.

- **Miscellaneous**

The gathered literature of the review also showcased the occurrence of few methods and hybrid techniques that do not wholly lie under one specific heading. These techniques were different and held no scope for generalization. These techniques were segregated and thus analyzed separately. The literature witnessed the recognition and implementation methods that were based on computer vision concepts such as PSA (Vuong et al., 2008), HMM (Winkler, 1996; Reddy et al., 2012), and other methods like x–y cut stroke reordering method (Le et al., 2016), statistical approach (Jan and Daniel Průša, 2014), pattern recognition techniques like elastic matching distance (Dimitriadis and López Coronado, 1995; Chan and Yeung, 1998; Simistira et al., 2012) and so on.

5.2.2. What is the intensity of implementation of each category of technique classified for recognition of HMSE?

It is crucial to analyze the intensity to refine and align well with the trend line of recognition techniques. As mentioned above, the reviewers have categorized the recognition techniques into three categories: an AI approach and a Grammar-Driven Approach and the miscellaneous approaches comprising some techniques based on computer vision, statistical and hybrid approaches. The frequency of each category of classification has been synthesized with an objective to extract out the popularity of individual techniques. It has been found that almost 68% of the total selected studies implemented the technologies and methods associated with the AI approach.

Table 8 (*continued*).

(Phan et al., 2018)	Hands-Math dataset	Hands-Math dataset=10,864 OHMEs Written by School students=62 Junior High School student=27 Own employee of Lab=26 For training=8266 OHMEs For Testing=2598 OHMEsSymbol Classes=94	Segmentation rate: 92.11% Relations Tree:89.49% Classification+ Segmentation: 87.54% Expression rate:66.96%
(Hossain et al., 2018)	The modified version of the NIST dataset	The authors used 2000 data items for the training of the network and 1000 data items for testing.	98.5% training accuracy
(Zhang et al., 2019)	IM2LATEX-100K dataset	Prebuilt dataset for OpenAI's task for the image-2-latex system. The system included 100k formulas and images, further split into training, validation, and test purposes.	Symbol accuracy rate= 97.5%, BLEU=88.42%
(Nazemi et al., 2019)	MNIST	MNIST = 6000 Images from inkML, taken from the CROHME dataset, and all images were converted to PNG through the application, which the author developed. Handwritten Digit Images were 2000, which were published by Computer Vision Group.	SqueezeNet achieved 90% Accuracy.
(Fu et al., 2020)	ME-20K and ME-98K	ME-20K: Dataset Formula collects printed math expression images and corresponding LaTeX representations from high school math exercises in Zhixue.com, an online education system. Due to many duplicates in the dataset, authors removed the duplicates ME-98K: Dataset IM2LATEX collects the printed formula and corresponding LaTeX representations from 60,000 research papers. As there are 4881 instances in the IM2LATEX dataset, which are tables or graphs, rather than math expressions, authors removed these LaTeX strings and corresponding images from IM2LATEX	EDSL has achieved 92.7% and 89.0% in evaluation metric Match, which is 3.47% and 4.04% higher than the state-of-the-art method

In contrast, approximately 30% of the studies executed grammar-driven strategies for recognition tasks. The miscellaneous category has an implementation intensity rate of 22.5%. In the process of synthesizing AI-based methods, it was discovered that the neural-based approach was the most popular, as seen in Fig. 10. Also, other ML methods SVM was flourishing and the most used method in the recognition process. Among the grammar-driven methods, Stochastic Context-Free Grammar was a ubiquitous approach, as almost 28% of the grammar-driven methods deployed this form of grammar for the task of recognition. The parsing methods included fuzzy logic-based grammars, and the parsers like DRACULAE parsers showed the evidential count of 11 studies in this review.

Summary of findings (RQ2) The prominent recognition techniques were based on the AI approach. Among this category, neural network-based methods like CNN, RNN, and other ML approaches like SVM were eminently employed.

5.3. Dataset used for the recognition process

The choice of the appropriate dataset is one of the crucial decisions made before beginning a quality implementation process and deciding the domain of the recognition technique that is to be deliberately employed. Thus, for a profound understanding of the present literature on HMSER, the reviewers have attempted to extract, identify and contemplate the datasets involved in the recognition process and also made a laborious and extensive search for corresponding accuracy rates with each dataset to anatomize and determine the dependency of accuracy results on the size of the dataset involved in the recognition procedures.

5.3.1. What are the datasets used and available for HMSE recognition?

The research trend in terms of datasets witnessed the use of around ten significant sources of dataset collection and also the central inclination of interest towards two kinds of corpus associated—CROHME and self-created dataset. CROHME series of competitions have genuinely influenced the pace of research and completely altered the researchers' priorities. With the aim to perform the standardization of the dataset for comparison in the competition series, the researchers' interest has become far more inclined towards the dataset been available by CROHME periodically (CROHME 2011, CROHME 2012, CROHME 2014, CROHME 2016, CROHME 2019). Though before the availability of the CROHME dataset, the researchers used to develop and construct their dataset by including different writers, users, and volunteers. This trivial and conducive practice of crafting and generating own datasets continued till 2011. After that, with the eventuality of the CROHME series, the custom of inventing and creating individual datasets seems to abolish, and the rising interest in the CROHME series provoked the usage of the CROHME dataset and thus, lead to the standardization of the regular dataset for HMSER. Apart from the mentioned two mentioned datasets, significant instances of other datasets were also observed in the review process. These datasets entailed few datasets developed by the researchers and made available and accessible for others like the corpus developed by Raman (Raman, 1994) known as the Aster database and the one created by Utpal Garain (Garain and Chaudhuri, 2005). Some other names in the datasets list are INFTY, MNIST, MathBrush, and Hands-math.

As per the intensity analysis, the recognition tasks on the CROHME dataset are consummated to a portentous extend. Almost 45% of the chosen studies put in practice the CROHME dataset, which had a periodical launch of versions since 2011. Thus, the analysis report also ascertained the contribution of each version of CROHME to be varying amounts. The CROHME 2014 was observed to most prominently and interminably used, while other versions of CROHME were comparatively less incessant. The CROHME 2014 had the higher intensity rate of 37% approximately (among the CROHME dataset), followed by the other CROHME datasets of the years 2016, 2013, 2011, 2012, and 2019 with intensity rate of 22%,18%,11%,9%, and 1% respectively as depicted in Fig. 11.

The Self-Created dataset, which had a sufficient trend before the onset of CROHME, was brought into use for research about 33% of the total chosen studies of this review. The remaining research papers showed the dependency on distinguished datasets and the researcher-oriented corpus, as 27.5% of the total studies implemented this third group of identified datasets.

5.3.2. Is the size of the dataset affect the accuracy of the recognition system?

The gathered literature witnesses several instances of different metrics used for HMSER. These metrics were chosen as per the recognition implementation models. If the last decade's literature was considered, various accuracy measures trend was a bit regulated and standardized. The reason for settling a limit to a decade and standardization of accuracy measures was due to CROHME, as it was started ten years back in 2011. It also introduced a set of metrics for the recognition of handwritten text. Still, the lack of standard accuracy metrics before 2011 led to the uneven metric trend in the research history of HMSER

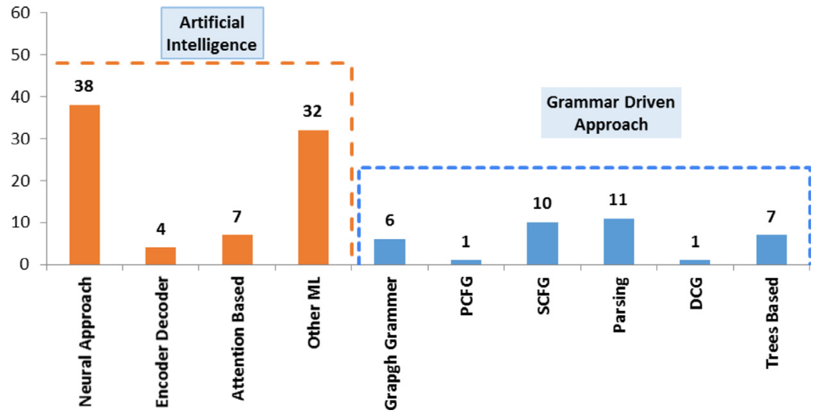


Fig. 10. Intensity of employed HMSER technique.

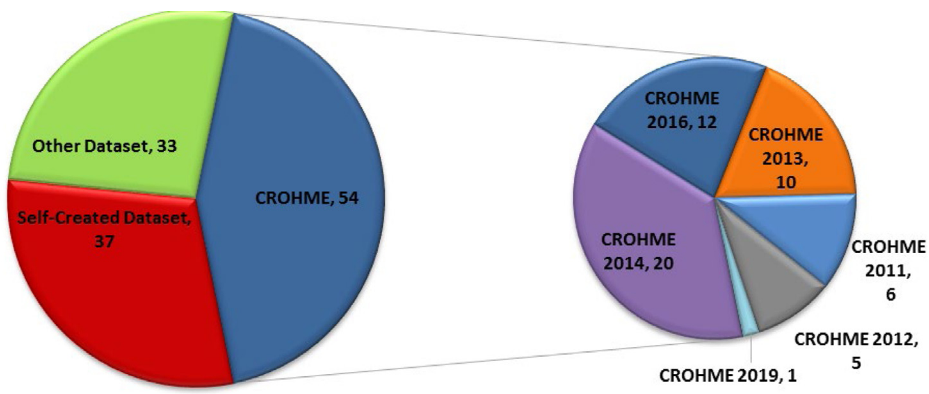


Fig. 11. Datasets and their frequencies.

as the collected studies for evaluation of the performance produced clear evidence to support our conclusion. Thus, the non-availability of standard accuracy measures made it obsolete to perform direct comparisons between the accuracy values derived using different accuracy metrics.

Apart from using various accuracy metrics, another factor that affected the comparison process is the non-availability of the public dataset. The non-availability of an open public dataset of HMSE before the year 2011 forced the researchers to collect and develop their own datasets. So, the use of different accuracy measures and the lack of standard datasets on HMSE has made the comparisons thoroughly challenging and complicated. The datasets were traced in the literature summary. The reviewers categorized the datasets into three categories: CROHME datasets, self-created datasets, and other datasets (corpus compiled by different research groups). The details of each dataset and their corresponding accuracy metric and accuracy details are presented in Tables 6–8. On investigating each of the selected studies carefully, it was observed that the most prominently used accuracy metric is the recognition rate. Recognition rate has defined the percentage of correctly recognized expressions to total expressions considered for recognition. If the datasets are scrutinized, the tabulated data depicts well that CROHME has been widely used as a dataset in the selected studies. Most predominantly, CROHME 2014 has been of wide choice. Yet, there is no fair procedure that formulates the exact solution to the framed research question of whether the size plays a significant role in determining the accuracy or not. The two hindering factors (different datasets and different accuracy metrics), as already discussed, make the discriminating and arduous task to be accomplished.

Summary of Findings (RQ3): The substantially used dataset is CROHME, and among the several distinct versions of CROHME, the CROHME 2014 was found to be voluminously utilized, and there has been a lack of standard accuracy measures and datasets which made it discriminant to generalize whether the size of the dataset is a factor affecting the accuracy or not.

5.4. Stages of the recognition process

The objective behind the formulation of this research question is to anthologize the works to construct a study pattern for the researchers to identify the stages and problems encountered during the recognition process. Several researchers attempted different recognition procedures in the entire time range of four decades of varying research monomania in this domain. They endeavored to achieve proficient accuracy rates using emerging techniques pertinent to parsing systems, computer vision, and AI. It becomes essential to study and identify these standard stages involved in recognition systems to align the varying thought frequencies of new researchers and existing practitioners and make them resonate according to few headings of steps identified in this phenomenon. The authors are hopeful that the classification and identification of sub-stages in the recognition process would purvey a broad outline for the stepwise procedure to be followed whenever involving the concepts around recognition tasks.

5.4.1. What are the sub-processes involved in the recognition processes?

The standard sub-processes identified during the recognition process of studies are Preprocessing, Segmentation, Symbol Recognition, Structural Analysis, Feature Extraction, and Classification. It was observed that the recognition process that applied AI techniques (ML and DL-oriented methods) usually has phases like preprocessing, segmentation,

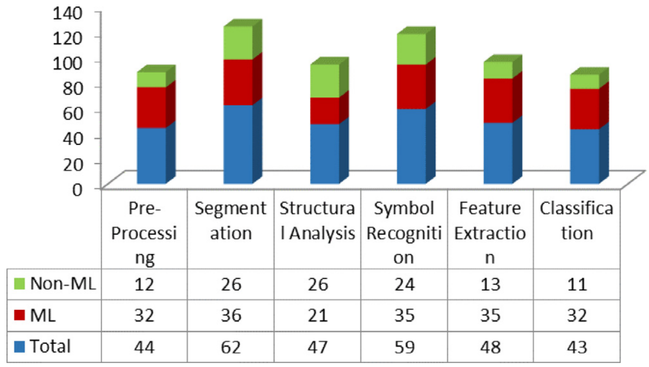


Fig. 12. Sub-processes analysis.

feature extraction, and classification (as presented in Table 9). In contrast, the research studies involving grammar-driven or conventional approaches undergo the stages of segmentation, and symbol recognition, and structural analysis. Thus, it has been concluded by the reviewers that segmentation is a common sub-stage in both approaches. The following Table 9 represents the summary of the sub-processes used by the considered studies for review.

5.4.2. How frequently is work done using the sub-processes identified?

Table 9 shows the frequency of sub-processes. Earlier, we discussed that segmentation was the only common sub-stage identified among the AI approach and the grammar-driven approach. But the frequency analysis of this research question reveals that apart from being a common sub-stage, segmentation is the most implemented sub-process as almost 52% of the total studies included this sub-stage.

On analyzing the recognition methods, it is further found that 30% of the studies implementing segmentation are using ML methods, i.e., AI approach, and 22% (approx.) of the non-ML origin. The next popular sub-process after the segmentation is the phase of symbol recognition. Both ML and non-ML methods steadily involve the symbol recognition phase with a frequency percentage of 29% (approx.) and 20%, respectively. The third highest trend of the sub-process is phase feature extraction. It has been mostly found concentrating ML studies (29% approx.) than non-ML studies (11% approx.), followed on the trend-line by structural analysis (39% approx.), preprocessing (37% approx.), and classification phase (36%) of the recognition process. The calculated percentages have been derived using the frequency values presented in Fig. 12.

Summary of Findings (RQ4) Among the synthesized sub-processes of HMSER, segmentation is found to be the comprehensively used sub-stage in both ML and non-ML-based techniques of recognition.

5.5. Representation models, algorithms, and tools

One of the objectives of this review is to perform a statistical analysis of various aspects of HMSER. This dimension of investigating the representation models, algorithms, and tools will be taking us a step ahead to understand the recognition process along this less noticed dimension. It has been perpetually seen that these dimensions of research review have been considerably ignored, and taking the review on this aspect will not only guide the current researchers working on this research domain but will also help to serve as an in-depth prerequisite for understanding what all data-structures, algorithms, and tools are engaged while researching HMSER.

5.5.1. What are the representation models used in the recognition process?

The reviewers have identified numerous representation models that are used in the recognition tasks while synthesizing the selected set of studies. It has been concluded that the most prominent representation type is of Box and Tree, which further had so many variations and versions as per the research requirement of each research proposal mentioned in the chosen research studies. A simple representation type of Box had variations in terms of the bounding box, gray box, solid box, hidden writing area, bounding rectangle, etc. The primary of all these versions are more or less the same, but they have been optimally used by the researchers as per their research objectives. Table 10 elucidates several identified representation models of majorly of type box and tree with their associated versions. The description column is accrued to enhance the understanding of each of the representation types. Also, the mentions of references would help the readers to get direct instances of the study where these representation models and structures have been deployed. On performing statistical analysis on the data collected under this head, another fact that has been discovered is that tree structures (associated variations) are more frequently used in the recognition than the box representation type.

Apart from the above-mentioned representative models and structures used to recognize HMSE, a few other structures and models can be categorized thoroughly. It includes the concepts like Symbol Class (Zhang et al., 2005), Linked List (Dai et al., 2019), and MST (Hu and Zanibbi, 2016b; Tapia and Rojas, 2004; Hu and Zanibbi, 2016b; Ahmed et al., 2004; Qi et al., 2009).

5.5.2. What are the algorithms used in the recognition process?

There have been varying trends of algorithms that are used in the recognition process of HMSE. Our review studies have investigated to extract out various algorithms used in the recognition phases apart from the primary ML algorithms identified like SVM algorithm, K-means algorithms, and other associated algorithms. The commonly used algorithms used in the recognition process are listed below and mentioned in Table 11.

Dynamic Programming algorithm, Gradient Descant Learning algorithm Ostu algorithm Projection-based algorithm CYK algorithm Viterbi algorithm Forward–backward algorithm, MST algorithm Parsing algorithm, Best First Search algorithm, Prim algorithm, Skin and Bone algorithm, Adaboost algorithm, Edmonds algorithm, LOS algorithm

5.6. What are the tools available for HMSER?

The gathered literature depicts a scarce presence and usage of tools used in the recognition process. But the available studies did not show many significant stances for the usage of tools in the collected research papers. It would be completely unfair to generalize that there are no tools used in the recognition process, but yes, there are several evidential stances of tools used. The identified tools are MathBrush (Labahn et al., 2008; Julca-Aguilar et al., 2015; Álvaro et al., 2014a; Alvaro et al., 2014b), Mathpad (Fitzgerald et al., 2007), WebMath (Vuong et al., 2010), MathFor (Tapia and Rojas, 2007) Grid search tools, latex2ink (Awal et al., 2010b), Tensor flow (Fontenele Marques Junior et al., 2019; Fang and Zhang, 2020) libraries like LibSVM (Malon et al., 2008), RNLib (Dai Nguyen et al., 2016), data-mining tools such as weka (Tapia and Rojas, 2007; Celar et al., 2015; Veres et al., 2019) and other CROHME based tools (Zhang et al., 2018b) example, LgEval (Zhang et al., 2018a,b,c).

Summary Of findings (RQ5): The predominantly used representation models are tree and box, whereas there are no significant stances of any algorithm (except the CYK algorithm used in grammar-driven approaches) or tool that has been largely used for HMSER.

Table 9
Sub-processes Analysis.

Title of paper	Pre-processing	Segmentation	Structural analysis	Symbol recognition	Feature extraction	Classification
(Lee and Lee, 1994)	Yes	No	Yes	Yes	No	No
(Dimitriadis and López Coronado, 1995)	Yes	Yes	No	Yes	No	No
(Xuejun et al., 1997)	No	No	No	No	No	No
(Matsakis, 1999)	Yes	Yes	Yes	Yes	Yes	Yes
(Chan and Yeung, 2000a,b)	Yes	Yes	Yes	Yes	No	No
(Kanahori et al., 2000)	No	Yes	Yes	Yes	No	No
(Suzuki, 2000)	No	Yes	Yes	Yes	No	No
(Eto and Suzuki, 2001)	No	No	No	No	No	No
(Kacem et al., 2001)	No	Yes	No	No	Yes	No
(Zanibbi et al., 2001)	No	No	No	No	No	No
(Zanibbi et al., 2002)	No	No	No	No	No	No
(Tapia and Rojas, 2003)	Yes	Yes	Yes	Yes	Yes	No
(Ahmed et al., 2004)	No	Yes	Yes	Yes	No	No
(Garain et al., 2004)	No	No	No	Yes	No	No
(Tapia and Rojas, 2004)	No	Yes	Yes	Yes	No	No
(Toyozumi et al., 2004)	No	Yes	No	No	No	No
(Zhang et al., 2005)	No	No	No	No	No	No
(Genoe et al., 2006)	No	No	No	No	Yes	Yes
(Ramteke and Mehrotra, 2006)	No	No	No	No	Yes	Yes
(Thammano and Rugkunchon, 2006)	Yes	No	No	Yes	No	No
(Yamamoto et al., 2006)	No	Yes	Yes	Yes	No	No
(Fitzgerald et al., 2007)	No	No	Yes	Yes	No	No
(Huang et al., 2007)	Yes	No	No	No	Yes	Yes
(Keshari and Watt, 2007)	Yes	No	No	Yes	Yes	No
(LaViola and Zeleznik, 2007)	No	No	No	Yes	Yes	No
(Průša and Hlaváč, 2007)	No	Yes	Yes	No	No	No
(Shi et al., 2007)	Yes	No	Yes	No	No	No
(Tian et al., 2007)	Yes	No	No	Yes	No	No
(Aly et al., 2008)	No	No	No	No	Yes	No
(Malon et al., 2008)	No	No	No	No	Yes	Yes
(Shi and Soong, 2008)	No	Yes	No	No	No	No
(Vuong et al., 2008)	No	No	Yes	Yes	No	No
(Awal et al., 2009)	No	Yes	Yes	Yes	No	No
(Aly et al., 2009)	Yes	No	No	No	Yes	Yes
(Garain, 2009)	No	No	Yes	Yes	No	No
(Kim et al., 2009)	No	No	No	No	Yes	No
(Qi et al., 2009)	No	No	No	No	No	No
(Rhee and Kim, 2009)	No	Yes	Yes	Yes	No	No
(Awal et al., 2010a)	No	Yes	Yes	Yes	No	No
(Awal et al., 2010c)	No	No	Yes	Yes	No	No
(Baker et al., 2010)	No	No	No	No	No	No
(MacLean and Labahn, 2010)	No	No	No	No	No	No
(Álvaro et al., 2011)	Yes	Yes	Yes	Yes	No	No
(Celik and Yanikoglu, 2011)	Yes	Yes	Yes	Yes	No	No
(Hu and Zanibbi, 2011)	Yes	No	No	No	Yes	Yes
(Medjkoune et al., 2011)	Yes	No	No	Yes	No	No
(Álvaro et al., 2012)	No	Yes	Yes	Yes	No	No
(Hu et al., 2012)	No	Yes	No	No	No	Yes
(Lin et al., 2012)	Yes	Yes	No	No	Yes	Yes
(Simistira et al., 2012)	No	No	Yes	Yes	No	No
(Bage et al., 2010)	Yes	Yes	No	No	Yes	Yes
(Clark et al., 2013)	Yes	No	No	Yes	Yes	Yes
(Gharde et al., 2013)	Yes	Yes	No	No	Yes	Yes
(MacLean et al., 2013)	Yes	Yes	Yes	Yes	No	No
(Mohan and Lu, 2013)	No	No	No	No	No	No
(Zhu et al., 2013)	Yes	No	No	No	No	No
(Awal et al., 2014))	No	Yes	Yes	Yes	Yes	Yes
(Álvaro et al., 2014a)	No	No	No	No	Yes	Yes
(Alvaro et al., 2014b)	No	Yes	Yes	Yes	No	No
(Le et al., 2014)	No	Yes	Yes	Yes	No	No
(Davila et al., 2014)	Yes	No	No	Yes	No	No
(Davila et al., 2014)	Yes	No	No	No	Yes	Yes
(Jan and Daniel Průša, 2014)	No	Yes	Yes	Yes	No	No
(Julca-Aguilar et al., 2014)	Yes	No	No	Yes	No	No
(Pillay, 2014b)	Yes	Yes	Yes	Yes	No	No
(Simistira et al., 2014)	No	No	No	No	Yes	Yes
(Hu et al., 2014)	No	Yes	Yes	Yes	No	No
(Le and Nakagawa, 2015)	No	Yes	Yes	Yes	No	No
(Bharambe, 2015)	Yes	Yes	Yes	No	Yes	Yes
(Julca-Aguilar et al., 2015)	No	Yes	Yes	No	Yes	No
(Phan et al., 2015)	No	Yes	Yes	Yes	Yes	No
(Mohan and Lu, 2015)	No	Yes	No	Yes	Yes	Yes

(*continued on next page*)

Table 9 (*continued*).

Title of paper	Pre-processing	Segmentation	Structural analysis	Symbol recognition	Feature extraction	Classification
(MacLean and Labahn, 2015)	No	Yes	Yes	No	No	Yes
(Simistira et al., 2015)	No	Yes	Yes	No	Yes	Yes
(Álvaro et al., 2016)	No	Yes	Yes	Yes	Yes	Yes
(Le and Nakagawa, 2016a)	No	Yes	Yes	Yes	No	No
(Le et al., 2016)	No	No	Yes	Yes	No	No
(Chajri and Bouikhalene, 2016)	Yes	Yes	No	Yes	Yes	Yes
(Dai Nguyen et al., 2016)	Yes	No	No	No	Yes	Yes
(Farulla et al., 2016)	No	No	No	No	Yes	Yes
(He et al., 2016)	Yes	No	No	Yes	Yes	Yes
(Hu and Zanibbi, 2016a)	Yes	Yes	Yes	No	No	No
(Hu and Zanibbi, 2016b)	No	Yes	Yes	Yes	Yes	Yes
(Ramadhan et al., 2016)	Yes	No	No	Yes	No	No
(Zhang et al., 2016)	No	Yes	Yes	Yes	Yes	Yes
(Le and Nakagawa, 2017a,b)	No	No	No	No	Yes	Yes
(Iwatsuki et al., 2017)	No	No	No	No	No	No
(Jain and Zanibbi, 2017)	No	No	No	No	Yes	No
(Zhang et al., 2017c)	Yes	Yes	Yes	Yes	Yes	Yes
(Zhang et al., 2017a)	Yes	Yes	Yes	Yes	No	Yes
(Zhang et al., 2017b)	Yes	Yes	No	No	Yes	No
(Drsouza and Mascarenhas, 2018)	Yes	Yes	No	No	No	Yes
(Hossain et al., 2018)	Yes	Yes	No	No	No	Yes
(Zhang et al., 2018b)	No	No	No	No	No	No
(Zhang et al., 2018a)	Yes	Yes	No	No	No	No
(Phan et al., 2018)	No	Yes	Yes	Yes	Yes	No
(Shinde et al., 2018)	Yes	Yes	No	Yes	Yes	Yes
(Zhang et al., 2018a)	No	Yes	No	No	Yes	Yes
(Zhang et al., 2018b)	No	Yes	Yes	No	No	No
(Ung et al., 2018)	No	Yes	No	No	Yes	Yes
(Abirami and Jaganathan, 2019)	Yes	Yes	No	Yes	No	Yes
(Le et al., 2019a)	No	No	No	No	No	No
(Le et al., 2019b)	No	No	No	No	No	No
(Dai et al., 2019)	Yes	Yes	Yes	Yes	No	No
(Fang et al., 2019)	No	No	No	No	Yes	Yes
(Fontenele Marques Junior et al., 2019)	No	Yes	No	Yes	No	No
(Khuong et al., 2019)	No	No	No	Yes	No	Yes
(Kumar et al., 2019)	Yes	Yes	No	Yes	No	No
(Nazemi et al., 2019)	No	Yes	No	Yes	No	Yes
(Shan et al., 2019)	No	No	No	No	No	No
(Ohyama et al., 2019)	No	No	No	No	No	No
(Zhang et al., 2019)	No	No	No	No	Yes	No
(Wan et al., 2019)	Yes	No	No	Yes	No	No
(Le, 2020)	Yes	No	No	No	No	No
(Fang and Zhang, 2020)	Yes	No	No	No	Yes	No
(Fu et al., 2020)	No	Yes	No	No	Yes	Yes
(Zhang et al., 2020)	No	Yes	Yes	No	No	Yes
(Julca-Aguilar et al., 2020)	No	Yes	No	No	No	Yes
(Nguyen et al., 2020)	No	No	No	No	Yes	No
(Wang and Shan, 2020)	No	No	No	No	Yes	Yes
(Wu et al., 2020)	No	No	No	No	Yes	No

6. Concluding remarks

Several facts and conclusive analyses have been identified at the end of this research. All the research questions are potentially well enough to surface the main extractions in the context of targeted objectives. Some inferences are drawn after careful interpretations and intellections. These are presented below:

- The first evidence of study on HMSER was thesis and research reports of Anderson (Anderson, 1967), and the reported year was 1967.
- The year in which the research on this domain got accelerated and paced was 2011. The reported cause of this hype and acceleration of research on HMSER was the advent of CROHME (Mouchère, 2011) series of competitions.
- The identified continent which is found to be a proactive hub of research on HMSER is Asia, as most of the researchers (Wang et al., 2019; Phan et al., 2018; Wang et al., 2020) are from Asian countries (refer Fig. 5) and consistently engaged in this research direction.
- Among these Asian countries, China and Japan are the most occupied countries with the highest evidential research in this direction (refer to Fig. 4).
- The most proactive researcher of this domain is found to be Masaki Nakagawa (Le et al., 2019a; Nguyen et al., 2020). He is working as Professor at Media Interaction at Tokyo University of Agriculture and Technology. There are many other researchers active in this domain, namely Viard Gaudin (Julca-Aguilar et al., 2020), Richard Zanibbi (Mahdavi et al., 2019), Harold Mouchere, and Anch Duc Le (Le, 2020).
- The widely used recognition approach is found to be an AI-based approach (refer to Fig. 9). Among all AI-based approaches, the neural network-based recognition systems and models are pre-eminently and mostly deployed.
- The widely used machine learning algorithm for recognition is found to be SVM (Davila et al., 2014; Neves et al., 2011).
- The eminent tool and databases are used for experimentation in HMSER are found to be CROHME (Mouchère, 2011) and its associated datasets.
- The enormously frequent algorithm used in the grammar-driven recognition model is found to be the CYK algorithm (Le et al., 2014), and the representation model used in the process of HMSER is box and tree structure (refer to Table 10) (Zhang et al., 2018a,b,c).

Table 10
Representation Models Analysis.

Representation type	Description	Reference
BOX		
BOUNDING BOX	Bounding boxes are the enclosed annotations or boxes that outline and mark the two-dimensional math symbols. Data annotators draw the rectangles in the form of these bounding boxes over images, highlighting the object of interest within each image by defining its X and Y coordinates. Bounding boxes of symbols and sub-expressions are used to extract geometric features. They have often been employed to determine structural relationships between the symbols in isolated or embedded forms within a mathematical equation or an expression. Though the main objective of adding a bounding box is for object detection, yet in terms of HMSER, it is used for determining the spatial relation.	(Zanibbi et al., 2001; Huang et al., 2007; Shi et al., 2007; Zanibbi et al., 2002; Zhang et al., 2005; Celik and Yanikoglu, 2011; Le et al., 2016; Yamamoto et al., 2006; Simistira et al., 2012; Álvaro et al., 2011; Vuong et al., 2008; Dai et al., 2019; Simistira et al., 2015; Awal et al., 2010a; Clark et al., 2013; Le et al., 2016; Zhang et al., 2020; LaViola and Zeleznik, 2007; Garain, 2009; Průša and Hlaváč, 2007; Simistira et al., 2014; He et al., 2016; Suzuki, 2000; Bharambe, 2015; Gharde et al., 2013; Kacem et al., 2001; Phan et al., 2018; Aly et al., 2008; Shinde et al., 2018)
SOLID BOX	Solid boxes are a kind of variation of bounding boxes with a specific purpose. The rectangular hulls of the root symbol and its argument are known as solid boxes and are the (union of) geometric bounding boxes of the symbols.	(Kim et al., 2009; MacLean et al., 2013)
BODY BOX	HMSEs are complicated, and their structural relations are ambiguous. To overcome this problem, we propose a body box for a symbol or a sub-expression. The body box is more stable than the bounding box.	(Le et al., 2014; Le and Nakagawa, 2015; Phan et al., 2015)
Hidden Writing Area	Structure probability is calculated by a concept called Hidden Writing Area. Yamamoto et al. (2006) proposed this new 2D structure model for MEs by the Hidden Writing Area's original idea, which a structure model trained statistically by data. The HWA was designed for a more precise estimation of structural and logical relations as compared to the bounding box.	(Le and Nakagawa, 2015; Le et al., 2016; Yamamoto et al., 2006)
Bounding Rectangle	Its another name variation, but it serves more and less like the bounding box. The rectangles are used to determine spatial relations while identifying symbols with their subscripts, superscripts, etc.	(Jan and Daniel Průša, 2014)
Dotted Box	They are used to identify minute symbolic entities and resolve the spatial ambiguity in the process of recognizing spatial relations.	(Rhee and Kim, 2009)
Gray boxes	As per the description in the works by (Thimbleby, 2004) gray boxes are data annotators and placeholders for different templates or simple expressions that could be added.	(Toyozumi et al. 2004), (Thimbleby, 2004)
TREE		
Baseline Structure Tree (BST)	The hierarchical structure of baselines in any expression is represented by BST (Zanibbi et al., 2001). It captures the crucial features of symbols without any syntactic or semantic interpretation.	(Zanibbi et al., 2001, 2002; Pillay, 2014b)

(*continued on next page*)

7. Inferences and conclusion

The main objective of the research is to characterize the body of knowledge pertaining to HMSER. To achieve that, a systematic literature study has been conducted to identify primary studies relevant to the scope. One of the other objectives of this research is to compile the works related to the recognition process that outlines and summarizes the entire research literature in a way that it must become depictive and suggestive for leading future research in this domain. From an initial set of articles retrieved from multiple digital sources, we have identified 120 primary studies relevant to our research scope. These primary studies are precisely investigated, classified, and analyzed concerning defined dimensions in terms of research questions, and a summary is presented in Appendix C. The investigation and review performed on the research history of HMSER sublimed several inferences and conclusions, as mentioned below:

- There has been significant work done on HMSER since the initial launch of the concept, but a catalyzing event that paced up the research process in this domain was CROHME. Thus, the year of the significance of the research history of HMSER was 2011, which denotes the year when the CROHME series of competitions commenced.
- The recognition techniques are synthesized to assign them categories based on the originating domain of a particular technique. Thus, most of the recent works operate on AI-based techniques. Though there have been significant stances of the grammar-driven

Table 10 (*continued*).

Representation type	Description	Reference
Parse Tree	For representing the stochastic context-free grammars and other derivatives of expression, it is convenient to use the parse tree. Many syntactic and semantic structures, while recognition is represented using parse trees or parses forests.	(Zanibbi et al., 2001; MacLean et al., 2013; MacLean and Labahn, 2010; Julca-Aguilar et al., 2020; MacLean et al., 2013)
Expression Tree	In simple words, it is a syntactic structure used to represent the ME for their recognition and evaluation in the form of a tree-like structure.	(Shi et al., 2007; Zanibbi et al., 2002; Celik and Yanikoglu, 2011; Genoe et al., 2006; Vuong et al., 2008; Fitzgerald et al., 2007)
BLSTM based TREE	While implementing the BLSTM based architecture, this BLSTM based tree representation is used that labeling the nodes or symbols and their edges or relationships from a graph.	(Zhang et al., 2018a,b,c)
Layered Search Tree	For resolving the ambiguities at symbol identities and spatial relationships, a layered search tree has been implemented, which is represented as the number of branches in the expansion.	(Rhee and Kim, 2009)
Merging Subtrees	Merging subtrees are used in the events where there is no relationship between the symbols used in the expression and the symbols, which will add new to the expression.; a new subtree is created for the new symbol by merging it with the prior created subtree.	(Genoe et al., 2006)
Label Graph	Label Graph (LG) represents the strokes of symbols and labels on the edges, translating the segmentation information or layout information. It can also be derived from the symbol relation tree.	(Zhang et al., 2016; Julca-Aguilar et al., 2015; Zanibbi et al., 2013)
Left Blocking Tree	In expressions, it is the primary need that we have to identify the leftmost symbols and leftmost strokes of that symbol. It can be done with the help of data structure and a Left-Blocking Tree (LBT).	(Hu et al., 2012)
Tree	For recognition of HMSE, there have good stances of the implementation of a tree structure. MSs and especially expressions are represented through trees. It will define their operational semantics or visual layout.	(Zhang et al., 2017b; Kim et al., 2009; Drsouza and Mascarenhas, 2018; Hu et al., 2014; Chan and Yeung, 2000a,b; Dai Nguyen et al., 2016; Zhang et al., 2018a; Fitzgerald et al., 2007)
Symbol Relation Tree	The recognition-based features are obtained from a symbol relation tree produced by the recognition system, representing symbols and relations in an HMSE.	(Khuong et al., 2019; Ung et al., 2018; Zhang et al., 2016)
Fuzzy Parse Tree	In fuzzy parse trees, nodes contain fuzzy attribute values, and branches have fuzzy values returned by constraints.	(Fitzgerald et al., 2007)
Substitution tree	A substitution tree is used to represent the layout-based indexing and retrieval of MEs. They were used for indexing MEs in the operator tree representation.	(Schellenberg et al., 2012)

techniques and parsing methods, the current trend of recognition techniques kept oscillating around the ML and DL-based models.

- Most of the research articles have implemented the tools and datasets launched by CROHME. Still, the standardization of datasets is perceived as a major limitation during our comparative analysis, investigations, and research.
- Moreover, the corpus used for implementation lacked large expressions. Thus, there is an urging need to standardize the dataset. It has also been found that the unavailability of the open public datasets has made the researchers to develop other small corpus or develop their own datasets and customize the recognition process as well as the recognition metrics
- Along with the unavailability of the standard dataset, there have been no standard metrics adopted by the domain researchers, which has made the comparative analysis more challenging.
- To summarize the two significant lacks in the research history of HMSER, we discovered that there is a need to adopt the standard datasets and standard metrics so as the comparative analysis could be performed efficiently.
- The highlighting part of this review is a distinct analysis performed on the recognition models that have been used in the recognition process. To the best of our knowledge, it is eventually the first review analysis that covers this dimension of research as the different aspects of all representative models have been scrutinized heedfully. Also, there is a need to standardize and regulate the accuracy norms and datasets so that a more judicious comparative analysis can be carried in the future.

After we sum up our conclusions, the other extracted future recommendations are documented. To broaden the scope of research questions and delve into extracting details of the recognition techniques and datasets used, there is a need to perform a distinct review analysis covering this perspective of the domain. There is also a significant scope for research to be performed in a way that addresses the recognition of mathematical text that is used in several distinct languages. While completing the selection process of primary studies for this review, the studies concerning the math symbols and expressions belonging to other languages have been ignored, constraining the defined scope of

Table 11
Algorithms' Analysis.

Algorithm	Description	References
Dynamic Programming algorithm	A better way of segmentation and identification of the input is discovered by a dynamic programming algorithm. Thus, the traditional dynamic programming algorithm is frequently implemented at sub-stages of the recognition process. It is also used for the computation of a probabilistic parse table.	(MacLean and Labahn, 2015; Awal et al., 2014; Hu et al., 2014; Clark et al., 2013; Shi et al., 2007; Awal et al., 2014; Le et al., 2014; MacLean and Labahn, 2010)
Cocke Younger Kasami algorithm	CYK algorithm is used to analyze and find the significant recognition among the several MEs taken as candidates in the processing step. While implementing the CYK algorithm, a CYK table is constructed in ascending order vertically and is updated from both upper right and bottom left. This CYK algorithm is majorly used while we are dealing with the parsing techniques or stochastic context-free grammars.	(Phan et al., 2018, 2016; Hu and Zanibbi, 2016b; Le et al., 2014; Simistira et al., 2015; Álvaro et al., 2014a; Alvaro et al., 2014b; Awal et al., 2014; Le and Nakagawa, 2015; Hu et al., 2014; Davila et al., 2014; MacLean et al., 2013; Álvaro et al., 2012, 2011; MacLean and Labahn, 2010; Shi et al., 2007; Yamamoto et al., 2006)
Depth First Search algorithm	When using the derivatives of the tree for representing the mathematical structures and their spatial relations, the DFS algorithm is used for traversal for this tree in the recognition process.	(Khuong et al., 2019; Zhang et al., 2018a,b,c)
Best First Search algorithm	In the process of HMSER, by introducing symbol hypotheses, representing the various identities of symbols, expression structures can be generalized. A new branch can be added from each structural ambiguity, creating a search tree. The most accurately calculated search is carried out by the "best first search" algorithm in this case.	(Awal et al., 2014)
Viterbi algorithm	Viterbi algorithm is a dynamic programming algorithm for verdict the most probable sequence of hidden patterns in a mathematical text.	(Álvaro et al., 2014a; Alvaro et al., 2014b; Dimitriadis and López Coronado, 1995)
Edmonds algorithm	Edmonds' algorithm is used to obtain a directed MST representing symbol layout, with only visual and geometric features used to segment, classify, and determine relationships between symbols.	(Hu and Zanibbi, 2016b)
Skin and Bone algorithm	The skin and bone algorithm is a basic but successful symbol recognition algorithm. In this algorithm, the images of each symbol in the database are used to develop a skin image and a bone image. Binary-AND operations are performed on the database images to gain the skin. The bone is obtained by performing a binary-OR operation on the symbol images. There will be two images (Skin Image and Bone Image) for each mark.	(Kumar et al., 2019)
Prims algorithm	It is used on MST for the calculation of weight during the process of recognition of HMSE.	(Tapia and Rojas, 2004)

this study. Yet, there is a need to compile the works of recognition that involve HMSEs from languages like Arabic, Chinese, and others. Moreover, the applications of this domain is another work that calls attention, and features involved during classification or recognition of HMSE needed to be considered and abstracted distinctly.

CRediT authorship contribution statement

Sakshi: Data curator, Writing - original draft, Investigation. **Vinay Kukreja:** Conceptualization, Methodology, Supervision, Reviewing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Acronyms

AI	Artificial Intelligence
ANN	Artificial Neural Network
BLSTM	Bidirectional Long Short-Term Memory
CNN	Convolutional Neural Network
CROHME	Competition on Recognition of Online Handwritten Mathematical Expression
CYK	Cock-Younger Kasami (algorithm)
DL	Deep Learning
DCG	Definite clause Grammar
DRACULAE	Diagram Recognition Application for Computer Understanding of Large Algebraic Expressions
HMM	Hidden Markov Model
HMR	Handwritten Mathematics Recognition
HMSER	Handwritten Mathematical Symbols and Expressions Recognition
HMSEs	Handwritten Mathematical Symbols and Expressions

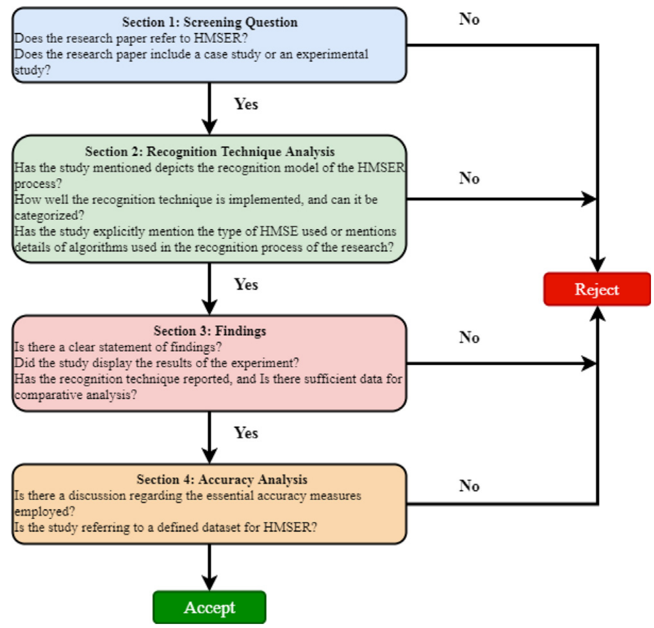
ICDAR	International Conference on Document Analysis and Recognition
IJDAR	International Journal of Document Analysis and Recognition
IR	Information Retrieval
ME	Mathematical Expression
ML	Machine Learning
MS	Mathematical Symbol
MSE	Mathematical Symbols and expressions
MST	Minimum Spanning Tree
OCR	Optical Character Recognition
PSA	Progressive Structure Analysis
QA	Quality Assessment
RNN	Recurrent Neural Network
SCFG	Stochastic Context-Free Grammar
SLM	Systematic Literature Mapping
SLR	Systematic Literature Review
SVM	Support Vector Machine

Appendix B

B.1. Quality assessment framework

Section 1 Screening question Does the research paper refer to HMSER? Does the research paper include a case study or an experimental study? Section1 — is evaluated first. If a positive reply is received, then proceed to Section 2, else reject the study.	Yes
Section 2 Recognition Technique Analysis Has the study mentioned depicts the recognition model of the HMSER process? How well the recognition technique is implemented, and can it be categorized? Has the study explicitly mention the type of HMSE used or mentions details of algorithms used in the recognition process of the research? If the above details are vividly mentioned, then move to section 3.	Yes
Section 3 Findings Is there a clear statement of findings? Did the study display the results of the experiment? Has the recognition technique reported, and Is there sufficient data for comparative analysis? If the above findings are appropriately mentioned, then accept the study, else reject it.	Yes
Section 4 Accuracy Analysis Is there a discussion regarding the essential accuracy measures employed? Is the study referring to a defined dataset for HMSER? Are the questions mentioned above satisfactorily answered, then accept it else reject it.	Yes

B.2. Quality assessment flow



Appendix C. Key extracts and summary

Extracted facts	Associated conclusions
The year in which the research idea on HMSER was initiated	1968 by Andersons
The year in which the research in this domain was accelerated	2011
The cause of this research acceleration and catalyzation	CROHME series of handwriting recognition competition
The leading continent actively involved in the research on HMSER	Asia
The leading country actively involved in the research on HMSER	China and Japan
The researcher with a significant contribution to the works on HMSER	Masaki Nakagawa
The university proactively engaged in research activities on HMSER	Tokyo University of Agriculture and Technology, Japan
The prominent research publication channel and platform to represent research on HMSER	Conference (ICDAR)
The widely used recognition approach	Artificial intelligence-based approach

Extracted facts	Associated conclusions
The prominently implemented category of artificial intelligence-based methods	Neural Network-based approach
The vastly used ML algorithm for HMSER	SVM
The eminent tool and database are used for experimentation in HMSER.	CROHME tools and datasets.
The enormously frequent algorithm used in the grammar-driven recognition model	CYK algorithm
The representation model used in the process of HMSER	Box and Tree structure

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