Accepted Manuscript

Market segmentation: A multiple criteria approach combining preference analysis and segmentation decision

Jiapeng Liu, Xiuwu Liao, Wei Huang, Xianzhao Liao

PII: S0305-0483(17)30650-3
DOI: 10.1016/j.omega.2018.01.008
Reference: OME 1870

To appear in: *Omega*

Received date: 7 July 2017
Revised date: 25 January 2018
Accepted date: 25 January 2018

Please cite this article as: Jiapeng Liu, Xiuwu Liao, Wei Huang, Xianzhao Liao, Market segmentation: A multiple criteria approach combining preference analysis and segmentation decision, *Omega* (2018), doi: 10.1016/j.omega.2018.01.008

Highlights

- A new multiple criteria decision aiding approach is proposed for market segmentation.
- The preferences of each consumer are analyzed in a robust manner, even in the case where her/his preference information is inconsistent.
- The approach works out a representative preference model and the corresponding ranking of products for each consumer.

Market segmentation: A multiple criteria approach combining preference analysis and segmentation decision

Jiapeng Liu[a,*], Xiuwu Liao[a], Wei Huang[a], Xianzhao Liao[b]

[a]*School of Management, The Key Lab of the Ministry of Education for Process Control and Efficiency Engineering, Xi'an Jiaotong University, Xi'an, 710049, Shaanxi, PR China*
[b]*School of Software Engineering, Xi'an Jiaotong University, Xi'an, 710049, Shaanxi, PR China*

Abstract

We propose a new multiple criteria decision aiding approach for market segmentation that integrates preference analysis and segmentation decision within a unified framework. The approach employs an additive value function as the preference model and requires consumers to provide pairwise comparisons of some products as the preference information. To analyze each consumer's preferences, the approach applies the disaggregation paradigm and the stochastic multicriteria acceptability analysis to derive a set of value functions according to the preference information provided by each consumer. Then, each consumer's preferences can be represented by the distribution of possible rankings of products and associated support degrees by applying the derived value functions. On the basis of preference analysis, a new metric is proposed to measure the similarity between preferences of different consumers, and a hierarchical clustering algorithm is developed to perform market segmentation. To help firms serve consumers from different segments with targeted marketing policies and appropriate products, the approach proposes to work out a representative value function and the univocal ranking of products for each consumer so that products that rank in the front of the list can be presented to her/him. Finally, an illustrative example of a market segmentation problem details the application of the proposed approach.

Keywords: Multiple criteria decision aiding, Market segmentation, Preference modeling, Clustering analysis

1. Introduction

Market segmentation is a strategy that involves the division of a large market into segments of consumers with different needs, characteristics, or behavior which might require separate marketing policies [29]. It can help firms know more about preferences and needs of consumers and tailor different policies for targeted segments in order to improve consumer satisfaction and increase revenue. The development of market segmentation theory has been contingent on the availability of marketing data, the advances in analytical techniques, and the progress of segmentation methodology [46]. Comprehensive reviews on various market segmentation approaches and solution methods for different scenarios can be found in Ref. [20, 46].

*Corresponding author
Email address: jiapengliu@mail.xjtu.edu.cn (Jiapeng Liu)

In general, existing market segmentation methods can be classified into two main streams [15, 46, 47]: the priori approaches and the post-hoc approaches. In the priori approaches, such as Ref. [9, 15], firms need to determine the type and number of segments in advance according to prior knowledge or speculated factors that are associated with consumers, services or products (such as demographic characteristics, purchase amounts and geographic areas) [18]. Differently from the priori approaches, the post-hoc approaches perform the segmentation decision by analyzing market data. A wide range of segmentation techniques have been applied in the post-hoc approaches, including clustering [3, 6, 43], category management [18], classification and regression trees (CART) [7], self-organizing map (SOM) [28], and multi-objective evolutionary algorithms (MOEA) [33].

The key point of market segmentation problem consists in accounting for consumer preferences, because the exploitation of consumer preferences provides a basis for the following segmentation decision. The preferences of each consumer are usually composed of her/his multiple points of view on evaluation of products. For example, when buying an automobile, consumers would evaluate potential models with respect to price, acceleration, max speed and fuel consumption [5]. These points of view are formally represented by criteria [29, 46]. Thus, consumers' evaluation process is based on the combination of all criteria describing the characteristics of products. Modeling of consumer preferences on multiple criteria has been one of the major activities in marketing research [16]. To represent consumer preferences, the additive value function is widely used as the preference model, which aggregates marginal values of a product into a total value. Such a preference model is proved to be consistent with the descriptive models used to represent consumer behavior [19] and appropriate for practical decision support due to the high interpretability of numerical scores that can be decomposed into per-criterion marginal values [11, 23].

The additive value function is constructed from the preference information provided by consumers, which is usually collected with the use of questionnaires during the conduction of a market survey. For example, in order to analyze the preferences of consumers and develop a new olive oil product, Ref. [43] requires consumers to taste six different olive oils, evaluate them against six criteria (including image, color, odor, etc.) and provide the ranking of olive oils according to their purchase probability as their preference information. Then, it applies the UTASTAR method [21] for each consumer to infer her/his value function and perform a cluster-based market segmentation based on consumers' criteria importance weights. For the same data set, Ref. [35] proposes a descriptive post hoc segmentation method which considers the significance of criteria. A criterion is considered significant when its weight is larger than a significance threshold. In this method, an overlapping clustering scheme is adopted to perform market segmentation where a consumer may belong to multiple segments. Ref. [34] designs an intelligent decision support system that integrates various models including UTASTAR for the storage, analysis and retrieval of consumers' preference information.

In this paper, we propose a new multiple criteria decision aiding (MCDA) approach for market segmentation that integrates the analysis of consumer preferences and the implementation of segmentation decision within a unified framework. It requires consumers who participate in the market survey to provide pairwise comparisons of some products based on their evaluation on multiple criteria. Asking consumers to provide preference information by means of pairwise comparisons is natural and consistent with intuitive reasoning of consumers [10]. To analyze the preferences of each consumer in a robust manner, the approach uses the additive value function composed of general monotone marginal value functions [12, 13] to act as consumer's preference model, instead of piecewise-linear marginal value functions in UTASTAR. Moreover, we consider all instances of the preference model compatible with the provided preference information, rather than a specific one, thus avoiding arbitrarily selecting one to represent the preferences of each consumer. For this purpose, the approach applies the disaggregation paradigm and the stochastic multicriteria acceptability analysis (SMAA) [30, 31, 32, 40] to derive a set of additive value functions compatible with the preference information supplied by each consumer. In case of inconsistent judgments in the provided preference information, the approach proposes to define the priority for each piece of preference information by taking into account all maximal subsets of consistent pairwise comparisons and then adjust the original preference information into a consistent one based on the relative importance of each piece of preference information.

Since any instance of the additive value function generates a complete ranking of products, the preferences of each consumer could be further represented by the distribution of possible rankings of products, each of which is associated with a support degree defined as the share of corresponding additive value functions. Then, on the basis of the distribution of possible rankings of products and associated support degrees, a new metric is proposed to measure the similarity between the preferences of different consumers. This metric can reflect the degree of coincidence of consumers' attitudes and preferences about product features. With the measurement of similarity between the preferences of different consumers, we can form groups of consumers (market segments) and each segment of consumers can be addressed with targeted marketing policies and appropriate products [44]. Accordingly, we develop a hierarchical clustering algorithm to group consumers into clusters, and generated clusters can be deemed as segments of consumers. Moreover, we define two metrics for measuring the certainty of each consumer's preferences. For those consumers whose preferences are certain, their preference structures on the set of products would not vary substantially and thus the clustering outcomes for them are meaningful. While for those consumers whose preferences are uncertain, we cannot arbitrarily assign them to any cluster and need to collect more preference information from them for the following analysis.

In order to help firms serve consumers from different segments with targeted marketing policies and appropriate products, our approach proposes to average all the instances of additive value functions to derive the representative one so that a univocal ranking of products for each consumer can be obtained. Then, products that rank in the front of the list can be presented to the corresponding consumer. Moreover, through the representative value function, firms can learn more about each consumer's preferences on multiple criteria, which is reflected by the trade-offs between criteria, the impact of each criterion for each product, and the difference between marginal values of different criteria evaluation as well as the strengths and weaknesses of each product.

In comparison with Ref. [34, 35, 43], the proposed approach employs the additive value function composed of general monotone marginal value functions as the preference model, thus avoiding arbitrarily specifying the number of breakpoints and also increasing the expressiveness of the preference model. Unlike previous

UTASTAR-based market segmentation studies, the proposed approach does not use criteria weights from a specific value function to group consumers into clusters. Instead, the present work proposes a new metric to measure similarity between consumer preferences based on the derived distribution of possible rankings of products and associated support degrees. In this robust manner, the proposed approach avoids arbitrarily selecting one specific value function to generate the clustering outcomes.

The proposed approach has several new features that are found to be useful to support market segmentation. First, the approach enriches the market segmentation literature by considering consumer preference information in form of pairwise comparisons. The disaggregation analysis of consumer preferences based on preference information of the simplest form helps firms to gain a better understanding of the needs of consumers in the market which, in turn, results in delivering better products and services. Second, the preferences of each consumer are analyzed in a robust manner, even in the case where the provided preference information is inconsistent. This is accomplished by employing the general monotone value function as the consumer's preference model, considering all compatible instances of the preference model, representing each consumer's preferences with the distribution of possible rankings of products and corresponding support degrees, and measuring the certainty of each consumer's preferences. Third, our approach works out a representative preference model and the corresponding ranking of products for each consumer. In this way, firms can better understand the preferences and needs of consumers from different segments and make targeted marketing policies and offers in order to improve consumer satisfaction and increase revenue.

The rest of this paper is organized as follows. In Section 2, we present the proposed approach to address the multiple criteria market segmentation problem. Section 3 demonstrates the proposed approach through an illustrative example. Finally, Section 4 ends with conclusions and discussion regarding future research.

2. The proposed approach

2.1. Problem description

We consider the following multiple criteria market segmentation problem: $A = \{a_1, a_2, ..., a_i, ..., a_n\}$ is a set of n products. $G = \{g_1, g_2, ..., g_j, ..., g_m\}$ is a family of m evaluation criteria; $g_j : A \to \mathbb{R}$ for all $j = 1, ..., m$. Let $D = \{d_1, d_2, ..., d_r, ..., d_p\}$ be a group of consumers participating in the market survey, who share the same set of products A and the same family of criteria G. $g_j^{d_r}(a_i)$ is the evaluation of product a_i on criterion g_j for consumer d_r, $a_i \in A$, $j = 1, ..., m$, $d_r \in D$. Without loss of generality, we assume that for any consumer $d_r \in D$, all criteria have an increasing direction of preference, i.e., the greater $g_j^{d_r}(a_i)$, the more preferred product a_i on criterion g_j, for all $a_i \in A$ and $j = 1, ..., m$. Note that for some criteria $g_j \in G$ (e.g., price), all the consumers have the same evaluations of products on g_j, i.e., $g_j^{d_1}(a_i) = g_j^{d_2}(a_i) = ... = g_j^{d_p}(a_i)$ for all $a_i \in A$; for some criteria $g_{j'} \in G$ (e.g., comfort), consumers may have different evaluations on $g_{j'}$ for some products, i.e., $\exists d_r, d_{r'} \in D$ and $a_i \in A$ such that $g_j^{d_r}(a_i) \neq g_j^{d_{r'}}(a_i)$. Let $X_j^{d_r} = \{g_j^{d_r}(a_i) \mid a_i \in A\}$ be the set of all different evaluations on criterion g_j provided by consumer d_r, $j = 1, ..., m$, $d_r \in D$. We sort the values in $X_j^{d_r}$

in an ascending order and then derive the ordered values $x_j^{d_r,1}, x_j^{d_r,2}, ..., x_j^{d_r,n_j^{d_r}(A)}$ such that $x_j^{d_r,k} < x_j^{d_r,k+1}$, $k = 1, ..., n_j^{d_r}(A) - 1$, where $n_j^{d_r}(A) = \left|X_j^{d_r}\right|$ and $n_j^{d_r}(A) \leqslant n$.

We assume that each consumer provides preference information in form of pairwise comparisons of some reference products. Let PC^{d_r} be the set of pairwise comparisons provided by consumer d_r, and $PC_k^{d_r}$ the kth pairwise comparison, $k = 1, ..., \left|PC^{d_r}\right|$, where $\left|PC^{d_r}\right|$ is the number of pairwise comparisons in PC^{d_r}. For any pair of products (a^*, b^*) in PC^{d_r}, the consumer d_r can state that a^* is at least as good as (weakly preferred to) b^* ($a^* \succsim^{d_r} b^*$), or a^* is indifferent to b^* ($a^* \sim^{d_r} b^*$), or a^* is strictly preferred to b^* ($a^* \succ^{d_r} b^*$) [11]. Note that all the preference relations $\succsim^{d_r}$, $\sim^{d_r}$ and $\succ^{d_r}$ are transitive [37].

To represent each consumer d_r's preferences, $d_r \in D$, we shall use an additive value function as the preference model [22]:

$$U^{d_r}(a) = \sum\nolimits_{j=1}^{m} u_j^{d_r}(g_j^{d_r}(a)), \quad a \in A,$$

where $U^{d_r}(a)$ is the comprehensive value of product a, and $u_j^{d_r}(\cdot) : X_j^{d_r} \to \mathbb{R}$ is the marginal value of product a on criterion g_j, $j = 1, ..., m$. The additive value function assigns a numerical score to the overall evaluation of any product. Here, we assume mutual preferential independence of a consumer's preferences [27] and therefore $U^{d_r}(a)$ has an additive form. The marginal value function $u_j^{d_r}(\cdot)$ is monotone, non-decreasing and normalized so that the comprehensive value is bounded within the interval [0,1]:

$$\left.\begin{array}{l} u_j^{d_r}(x_j^{d_r,k+1}) - u_j^{d_r}(x_j^{d_r,k}) \geqslant 0, \quad k = 1, ..., n_j^{d_r}(A) - 1, \; j = 1, ..., m, \\ u_j^{d_r}(x_j^{d_r,1}) = 0, \quad j = 1, ..., m, \\ \sum_{j=1}^{m} u_j^{d_r}(x_j^{d_r,n_j^{d_r}(A)}) = 1. \end{array}\right\} E_{BASE}^{d_r}$$

2.2. Outline of the proposed approach

The general framework of the proposed approach is depicted in Figure 1, and its phases are described as follows.

Phase 1. Conduct a market survey and require consumers to provide preference information in form of pairwise comparisons of some products.

Phase 2. Analyze the preferences of each consumer individually. For each consumer, check the consistency of her/his preference information (see Section 2.3.1), derive the distribution of possible rankings and associated support degrees (see Section 2.3.2) and measure the certainty of her/his preferences (see Section 2.3.3).

Phase 3. Perform market segmentation. For any pair of consumers, calculate the similarity degree between their preferences (see Section 2.4.1). Then, use the agglomerative hierarchical clustering algorithm to group consumers whose preferences are certain into clusters (see Section 2.4.2). For each consumer, work out a representative preference model and the corresponding ranking of products (see Section 2.4.3).

The approach utilizes the disaggregation paradigm and the SMAA to analyze the preference information provided by consumers in a robust manner. Firstly, a Linear Programming (LP) model is used to check the consistency of the preference information provided by each consumer. In case of inconsistent judgments in the provided preference information, the approach proposes to adjust the original preference information

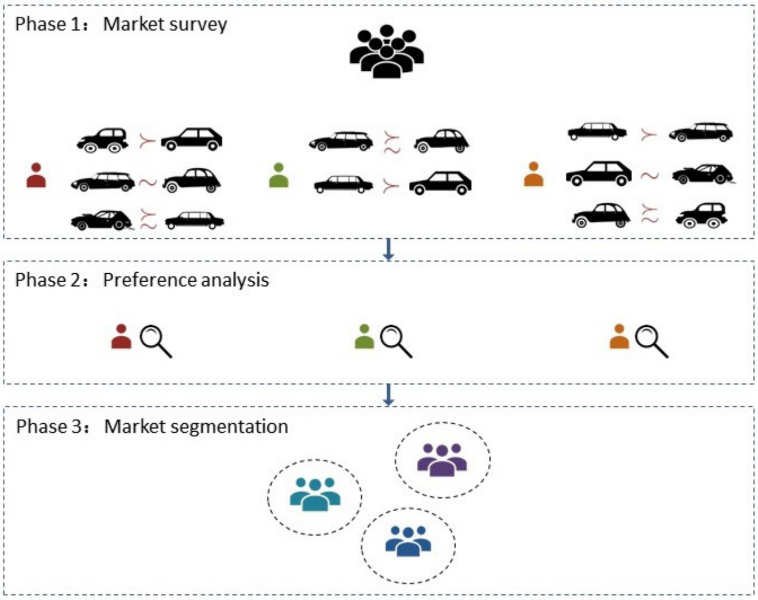


Figure 1: General framework of the proposed approach.

into a consistent one by taking into account all maximal subsets of consistent pairwise comparisons. Then, a set of additive value functions are derived to represent each consumer's preferences by sampling from the feasible space generated by the set of preference information. All derived additive value functions constitute the distribution of possible rankings of products and each possible ranking is associated with a support degree defined as the share of corresponding additive value functions. Moreover, the approach proposes two metrics for measuring the certainty of each consumer's preferences. In order to perform market segmentation, a similarity degree is defined to measure the similarity between the preferences of different consumers on the basis of the distribution of possible rankings of products and associated support degrees. Then, an agglomerative hierarchical clustering algorithm is used to group consumers into clusters, and each cluster of consumers can be deemed as a market segment. Finally, in order to learn more about each consumer's preferences and provide appropriate products for her/him, the approach proposes to average all compatible additive value functions to derive the representative one so that a univocal ranking of products can be obtained.

2.3. Analyzing preferences of each consumer

2.3.1. Checking the consistency of each consumer's preference information

The preference information provided by each consumer $d_r \in D$ can be translated into the following set of linear constraints [22]

$$\left.\left.\begin{array}{ll} U^{d_r}(a^*) \geqslant U^{d_r}(b^*), & \text{if } a^* \succsim^{d_r} b^*, \\ U^{d_r}(a^*) = U^{d_r}(b^*), & \text{if } a^* \sim^{d_r} b^*, \\ U^{d_r}(a^*) \geqslant U^{d_r}(b^*) + \varepsilon, & \text{if } a^* \succ^{d_r} b^*, \end{array}\right\} PC_k^{d_r} = (a^*, b^*), \quad k = 1, ..., |PC^{d_r}| \right\} E_{PC^{d_r}}$$

where ε is an arbitrarily small positive value. One can observe that each pairwise comparison induces a linear constraint on the parameters of the preference model. The preference information supplied by each consumer

d_r can be represented by a set of value functions U^{d_r}, denoted by $\mathcal{U}_{PC^{d_r}}$, that are able to restore the provided pairwise comparisons. $\mathcal{U}_{PC^{d_r}}$ is defined with the following set of constraints:

$$\left.\begin{array}{l} E_{PC^{d_r}}, \\ E^{d_r}_{BASE}. \end{array}\right\} E^{d_r}$$

If E^{d_r} is feasible and $\varepsilon^* = \max \varepsilon$, s.t. E^{d_r}, is greater than 0, $\mathcal{U}_{PC^{d_r}}$ is not empty and we say the preference information provided by the consumer d_r is consistent; otherwise, $\mathcal{U}_{PC^{d_r}}$ is empty and some pairwise comparisons are inconsistent with the preference model [22].

Traditionally, in case of incompatibility (i.e., $\mathcal{U}_{PC^{d_r}}$ is empty), we can identify the minimal subset of inconsistent pairwise comparisons which need to be removed from the whole set of preference information or presented to the consumer, who is expected to revise her/his previous judgments, to restore consistency. The method to resolve inconsistency has been proposed in [12, 24, 38]. However, the removal of inconsistent pairwise comparisons from the whole set of preference information is not a good method as it may lose important information to capture the preferences of a consumer especially when the amount of preference information is relatively small. Moreover, for a large group of consumers, it would be unrealistic to require all the consumers to learn about their preferences and understand where and why their judgments do not comply with the consistency principle, because some consumers are inactive to participate in the interactive process due to their different locations and time limitation.

In this paper, we propose a method to deal with the incompatibility in the preference information of d_r so as to obtain a non-empty set of value functions U^{d_r}. Such a method considers the priority of each pairwise comparison $PC_k^{d_r}$ and does not need consumer d_r to change her/his preference information. The definition of the priority of each pairwise comparison $PC_k^{d_r}$ is based on the analysis of all maximal subsets of consistent pairwise comparisons provided by d_r.

Definition 1. A maximal subset of consistent pairwise comparisons provided by the consumer d_r is a subset of pairwise comparisons compatible with the preference model, which is maximal in the sense that any of its proper supersets is inconsistent. Let us denote it by CPC^{d_r}.

According to the above definition, for any maximal subset of consistent pairwise comparisons CPC^{d_r}, the rest of pairwise comparisons in $PC^{d_r} \backslash CPC^{d_r}$ lead to the inconsistency in the preference information. Thus, in order to obtain a non-empty set of value functions U^{d_r}, we can relax the linear constraints underlying the pairwise comparisons $PC_k^{d_r} \in PC^{d_r} \backslash CPC^{d_r}$. Note that there may exist more than one maximal subset of consistent pairwise comparisons CPC^{d_r} for d_r, and thus there could be several solutions to relax the linear constraints underlying the pairwise comparisons that contribute to the inconsistency. To avoid arbitrarily selecting one to restore consistency, we propose to identify all maximal subsets of consistent pairwise comparisons and then consider them jointly. Algorithm 1 is used to find all maximal subsets of consistent pairwise comparisons for d_r, which starts with solving the following Mixed-Integer Linear Programming (MILP) [22]

$$\text{MILP1 : Maximize } f = \sum_{k=1}^{\left|PC^{d_r}\right|} v_k, \ \text{ s.t. } \left(E^{d_r}\right)',$$

and $\left(E^{d_r}\right)'$ is defined as follows:

$$\left.\begin{array}{l}\left.\begin{array}{ll}U^{d_r}(a^*) \geqslant U^{d_r}(b^*) - (1 - v_k), & \text{if } a^* \succsim^{d_r} b^*, \\ \left.\begin{array}{l}U^{d_r}(a^*) \geqslant U^{d_r}(b^*) - (1 - v_k), \\ U^{d_r}(b^*) \geqslant U^{d_r}(a^*) - (1 - v_k),\end{array}\right\} & \text{if } a^* \sim^{d_r} b^*, \\ U^{d_r}(a^*) \geqslant U^{d_r}(b^*) + \varepsilon - (1 - v_k), & \text{if } a^* \succ^{d_r} b^*, \\ v_k \in \{0, 1\},\end{array}\right\} PC_k^{d_r} = (a^*, b^*), \;\; k = 1, ..., \left|PC^{d_r}\right| \\ E_{BASE}^{d_r},\end{array}\right\} \left(E^{d_r}\right)'$$

where ε is a small positive value representing the smallest discernible difference. v_k is a binary variable such that $v_k = 1$ if $PC_k^{d_r}$ is selected into the maximal subset of consistent pairwise comparisons; otherwise, $v_k = 0$ and the corresponding constraint is always satisfied, which is equivalent to elimination of this constraint. Let f^* and v_k^* denote the values of the objective function f and the variable v_k at the optimum, respectively. MILP1 indicates the first maximal subset of consistent pairwise comparisons by the consumer d_r, i.e.

$$CPC_1^{d_r} = \left\{PC_k^{d_r} | v_k^* = 1, \; k \in \left\{1, ..., \left|PC^{d_r}\right|\right\}\right\}.$$

Algorithm 1 iteratively solves MILP1 to find all maximal subsets of consistent pairwise comparisons by the consumer d_r. Let w denote the index of iteration, Λ_w the subset of pairwise comparisons $PC_k^{d_r}$ whose $v_k^* = 1$ in the iteration w, and $CPC_s^{d_r}$ the sth maximal subset of pairwise comparisons. We iteratively add the constraint $\sum\limits_{PC_k^{d_r} \in \Lambda_w} v_k \leqslant f^* - 1$ to $(E^{d_r})'$ to forbid finding again the same solution as obtained in previous iterations $1, ..., w-1$. Note that Λ_w cannot be directly identified as a maximal subset of pairwise comparisons, because in some iteration w, Λ_w might not be maximal for one of its supersets being a maximal subset of consistent pairwise comparisons. Therefore, in step 6 of Algorithm 1, we need to check whether Λ_w is a subset of a maximal subset of consistent pairwise comparisons that have been found in previous iterations. If so, we proceed to the next iteration (step 11); otherwise, Λ_w is identified as a new maximal subset of pairwise comparisons, and then we proceed to the next iteration. Appendix A provides an illustrative example to identify all maximal subsets of consistent pairwise comparisons.

In order to obtain a non-empty set of value functions for d_r, we consider jointly all maximal subsets of consistent pairwise comparisons $CPC_s^{d_r}$, $s = 1, ..., K^{d_r}$, and relax the linear constraints underlying pairwise comparisons $PC_k^{d_r} \in PC^{d_r} \backslash CPC_s^{d_r}$ that lead to the inconsistency. For this purpose, we define the priority for each pairwise comparison $PC_k^{d_r}$ in the following.

Definition 2. The priority $\lambda(PC_k^{d_r})$ of pairwise comparison $PC_k^{d_r} \in PC^{d_r}$ for the consumer d_r is defined as the number of maximal subsets of consistent pairwise comparisons $CPC_s^{d_r}$, $s = 1, ..., K^{d_r}$, such that $PC_k^{d_r} \in CPC_s^{d_r}$, i.e.,

$$\lambda(PC_k^{d_r}) = \sum\nolimits_{s=1}^{K^{d_r}} \chi_{\{PC_k^{d_r} \in CPC_s^{d_r}\}},$$

where $\chi_{\{PC_k^{d_r} \in CPC_s^{d_r}\}} = 1$ if the condition $PC_k^{d_r} \in CPC_s^{d_r}$ is true and $\chi_{\{PC_k^{d_r} \in CPC_s^{d_r}\}} = 0$ otherwise.

Algorithm 1 Identify all maximal subsets of consistent pairwise comparisons for the consumer d_r.

Input:

Pairwise comparisons provided by the consumer d_r: $PC_k^{d_r}$, $k = 1, ..., \left|PC^{d_r}\right|$;

1: Index of iteration $w \leftarrow 1$;

2: Index of maximal subset of consistent pairwise comparisons $s \leftarrow 1$;

3: Solve MILP1;

4: **while** $f^* > 0$ **do**

5: $\quad \Lambda_w \leftarrow \{PC_k^{d_r} \mid v_k^* = 1,\ k \in \{1, ..., \left|PC^{d_r}\right|\}\}$;

6: $\quad$ **if** $w > 1$ and $\exists z \in \{1, ..., s\} : \Lambda_w \subseteq CPC_z^{d_r}$ **then**

7: $\quad\quad$ Go to step 11;

8: $\quad$ **end if**

9: $\quad CPC_s^{d_r} \leftarrow \Lambda_w$;

10: $\quad s \leftarrow s + 1$;

11: $\quad w \leftarrow w + 1$;

12: $\quad (E^{d_r})' \leftarrow (E^{d_r})' \cup \{\sum_{PC_k^{d_r} \in \Lambda_w} v_k \leqslant f^* - 1\}$;

13: $\quad$ Solve MILP1;

14: **end while**

Output:

Number of maximal subsets of consistent pairwise comparisons for the consumer d_r: $K^{d_r} \leftarrow s - 1$;

All maximal subsets of consistent pairwise comparisons for the consumer d_r: $CPC_1^{d_r}, ..., CPC_{K^{d_r}}^{d_r}$.

According to the above definition, a pairwise comparison $PC_k^{d_r}$ with a higher priority occurs in more maximal subsets of consistent pairwise comparisons than another pairwise comparison $PC_{k'}^{d_r}$ with a lower priority. Thus, the priority of a pairwise comparison reflects the relative importance of this piece of preference information, and we want to infer a set of value functions that are as consistent with those pairwise comparisons with higher priorities as possible. In this way, we consider jointly all maximal subsets of consistent pairwise comparisons and avoid arbitrarily selecting a solution to restore consistency. To reflect the compliance between the inferred value functions U^{d_r} and the pairwise comparisons $PC_k^{d_r}$, $k = 1, ..., |PC^{d_r}|$, we introduce a set of error variables $\sigma(PC_k^{d_r})$ to measure the violation with the corresponding linear constraints as follows:

$$\left.\begin{array}{l} \left.\begin{array}{ll} U^{d_r}(a^*) + \sigma(PC_k^{d_r}) \geqslant U^{d_r}(b^*), & \text{if } a^* \succsim^{d_r} b^*, \\ \left.\begin{array}{l} U^{d_r}(a^*) + \sigma(PC_k^{d_r}) \geqslant U^{d_r}(b^*), \\ U^{d_r}(b^*) + \sigma(PC_k^{d_r}) \geqslant U^{d_r}(a^*), \end{array}\right\} & \text{if } a^* \sim^{d_r} b^*, \\ U^{d_r}(a^*) + \sigma(PC_k^{d_r}) \geqslant U^{d_r}(b^*) + \varepsilon, & \text{if } a^* \succ^{d_r} b^*, \\ \sigma(PC_k^{d_r}) \geqslant 0 & \end{array}\right\} PC_k^{d_r} = (a^*, b^*), \ \ k = 1, ..., |PC^{d_r}| \\ E_{BASE}^{d_r}. \end{array}\right\} (E^{d_r})''$$

Then, we solve the following LP model that aims to minimize the sum of weighted errors:

$$\text{LP2}: \ \ \text{Minimize } \delta = \sum\nolimits_{k=1}^{|PC^{d_r}|} \lambda(PC_k^{d_r})\sigma(PC_k^{d_r}), \ \ \text{s.t.}, \ (E^{d_r})''.$$

Let δ^* be the value of the objective function $\delta = \sum_{k=1}^{|PC^{d_r}|} \lambda(PC_k^{d_r})\sigma(PC_k^{d_r})$ at the optimum. Then, the following set of linear constraints defines a non-empty set of value functions U^{d_r}, denoted by $\mathcal{U}'_{PC^{d_r}}$

$$\left.\begin{array}{l} \sum\nolimits_{k=1}^{|PC^{d_r}|} \lambda(PC_k^{d_r})\sigma(PC_k^{d_r}) = \delta^*, \\ (E^{d_r})''. \end{array}\right\} (E^{d_r})'''$$

2.3.2. Deriving distribution of possible rankings and associated support degrees

In the above section, we translate the preference information supplied by each consumer into a set of linear constraints and then obtain a non-empty set of value functions to represent each consumer's preferences, even in the case of inconsistency existing in the preference information. However, there are usually many compatible value functions in $\mathcal{U}_{PC^{d_r}}$ or $\mathcal{U}'_{PC^{d_r}}$, whose number is, in general, infinite and which are neither explicitly known nor calculated [8]. To efficiently approximate the distribution of compatible value functions in $\mathcal{U}_{PC^{d_r}}$ or $\mathcal{U}'_{PC^{d_r}}$, we use the SMAA to uniformly sample a set of general monotone value functions from $\mathcal{U}_{PC^{d_r}}$ or $\mathcal{U}'_{PC^{d_r}}$, which is denoted by $\mathcal{U}_{PC^{d_r}}^{SMAA}$. The SMAA applies Monte Carlo simulation to sample from a convex space restricted by a set of linear constraints, and only 10,000 functions are needed to approximate the distribution of compatible value functions with a sufficient accuracy. The following procedure presents the way of sampling from $\mathcal{U}_{PC^{d_r}}$ or $\mathcal{U}'_{PC^{d_r}}$ [41]:

Step 1 : For each criterion $g_j \in G$, the marginal value function $u_j^{d_r}(\cdot)$ with $n_j^{d_r}(A)$ levels are obtained by sampling uniformly $n_j^{d_r}(A) - 2$ random numbers from [0,1]. Then, sorting the $n_j^{d_r}(A) - 2$ numbers in an ascending order, and adding 0 and 1 to the beginning and the end of the sequence, respectively.

Step 2 : Sampling uniformly $m-1$ random numbers from [0,1] and sort the $m-1$ numbers along with 0 and 1 in an ascending order to get $0 = \theta_0 \leqslant \theta_1 \leqslant ... \leqslant \theta_m = 1$. Then, obtain the criteria weights as $w_1 = \theta_1 - \theta_0$, $w_2 = \theta_2 - \theta_1$, ..., $w_m = \theta_m - \theta_{m-1}$.

Step 3 : Use the criteria weights $w_1, ..., w_m$ to scale the ordered numbers from *Step* 1 for each criterion $g_j \in G$ to obtain a general monotone value function $U^{d_r}(\cdot)$.

Step 4 : Test the compatibility of $U^{d_r}(\cdot)$ with the set of constraints E^{d_r} or $(E^{d_r})'''$. If compatible, add $U^{d_r}(\cdot)$ to $\mathcal{U}^{SMAA}_{PC^{d_r}}$; otherwise, discard $U^{d_r}(\cdot)$ and go to *Step* 1 to sample a new general monotone value function until $|\mathcal{U}^{SMAA}_{PC^{d_r}}| = 10000$. $\square$

Any value function $U^{d_r} \in \mathcal{U}^{SMAA}_{PC^{d_r}}$ generates a specific ranking of all products, denoted by $\Gamma(U^{d_r})$, and the rank of product $a \in A$ is determined by the following ranking function.

Definition 3. [26] The rank of any product a relative to all products in A is defined with the ranking function

$$rank(U^{d_r}, a) = 1 + \sum_{b \in A \setminus \{a\}} h(U^{d_r}, a, b),$$

where

$$h(U^{d_r}, a, b) = \begin{cases} 1, & \text{if } U^{d_r}(b) > U^{d_r}(a), \\ 0, & \text{otherwise.} \end{cases}$$

Note that if $U^{d_r}(a) = U^{d_r}(b)$, then a and b have the same position in the ranking.

Definition 4. Let $\Gamma(U_1^{d_r})$ and $\Gamma(U_2^{d_r})$ be two rankings of all products determined by value functions $U_1^{d_r}$ and $U_2^{d_r}$, respectively. $\Gamma(U_1^{d_r})$ is equal to $\Gamma(U_2^{d_r})$, denoted by $\Gamma(U_1^{d_r}) = \Gamma(U_2^{d_r})$, if and only if

$$rank(U_1^{d_r}, a) = rank(U_2^{d_r}, a), \quad \text{for all } a \in A.$$

Definition 5. The set of possible rankings of all products with regard to the consumer d_r's preferences can be represented by

$$\Psi^{d_r} = \left\{ \Gamma \mid \Gamma(U^{d_r}) = \Gamma, \;\; U^{d_r} \in \mathcal{U}^{SMAA}_{PC^{d_r}} \right\}$$

and for each $\Gamma \in \Psi^{d_r}$, its support degree is defined as

$$\eta^{d_r}(\Gamma) = \frac{1}{|\mathcal{U}^{SMAA}_{PC^{d_r}}|} \left| \left\{ U^{d_r} \mid \Gamma(U^{d_r}) = \Gamma, \;\; U^{d_r} \in \mathcal{U}^{SMAA}_{PC^{d_r}} \right\} \right|.$$

The set Ψ^{d_r} consists of all possible rankings of $a \in A$, each of which is generated by sampled value functions in $\mathcal{U}^{SMAA}_{PC^{d_r}}$. The support degree $\eta^{d_r}(\Gamma)$ is defined as the share of sampled value functions which generate the same ranking Γ. Note that $\eta^{d_r}(\Gamma)$ is bounded within the interval (0,1] for any $\Gamma \in \Psi^{d_r}$, and $\sum_{\Gamma \in \Psi^{d_r}} \eta^{d_r}(\Gamma) = 1$.

2.3.3. Measuring certainty of consumer preferences

Consumers provide pairwise comparisons of some reference products as their preference information, and we sample a set of value functions to represent their preferences. When the preference information supplied by a particular consumer d_r is insufficient, the ranking outcomes by applying sampled value functions are not univocal in the sense that for some value functions product a is ranked better than product b while for other value functions product b is ranked better than product a. In this case, d_r's preference structure on the set A is uncertain and thus we cannot assign d_r to a market segment arbitrarily or provide products for her/him. For such consumers, we need to collect more preference information from them and then analyze their preferences for the following segmentation decision.

In the following, we propose two metrics for measuring the certainty of a particular consumer d_r's preference. The first metric is an Average Stability Index (ASI) which investigates the variety of value functions that generate different rankings of products. Such a metric is adapted from Ref. [17, 39] which use ASI to measure the robustness of the inferred value function in the post-optimality analysis of UTASTAR method. In this paper, ASI for consumer d_r is assessed as the mean of the normalized standard deviation of each $u_j^{d_r}(x_j^{d_r,k}),\ k = 1, ..., n_j^{d_r}(A),\ j = 1, ..., m$:

$$ASI^{d_r} = \frac{1}{m}\sum\nolimits_{j=1}^{m} ASI^{d_r}(j),$$

where $ASI^{d_r}(j)$ is the average stability index for criterion g_j which is calculated as

$$ASI^{d_r}(j) = 1 - \frac{1}{n_j^{d_r}(A)} \sum_{k=1}^{n_j^{d_r}(A)} \sqrt{\frac{1}{|\mathcal{U}_{PC^{d_r}}^{SMAA}| - 1}\left(\sum_{U^{d_r} \in \mathcal{U}_{PC^{d_r}}^{SMAA}} \left(u_j^{d_r}(x_j^{d_r,k})\right)^2 - \frac{1}{|\mathcal{U}_{PC^{d_r}}^{SMAA}|}\left(\sum_{U^{d_r} \in \mathcal{U}_{PC^{d_r}}^{SMAA}} u_j^{d_r}(x_j^{d_r,k})\right)^2\right)}.$$

Note that the larger ASI^{d_r}, the more certain consumer d_r's preferences.

In addition to ASI, we propose an Ranking Variation Index (RVI) to quantify the certainty of each consumer's preferences. RVI is defined based on the ranking outcomes of each product by applying all sampled value functions. RVI for consumer d_r is defined as follows:

$$RVI^{d_r} = \frac{1}{n}\sum\nolimits_{a \in A}\left(\sqrt{\sum\nolimits_{p=1}^{n} \phi(a,p)^2}\right),$$

where

$$\phi(a,p) = \frac{1}{|\mathcal{U}_{PC^{d_r}}^{SMAA}|}\left|\left\{U^{d_r} \in \mathcal{U}_{PC^{d_r}}^{SMAA} \mid rank(U^{d_r}, a) = p\right\}\right|$$

is the share of sampled value functions $U^{d_r} \in \mathcal{U}_{PC^{d_r}}^{SMAA}$ that rank product a to the p-th position.

Proposition 1. *For any $a \in A$, let $\Phi(a) = \sum_{p=1}^{n} \phi(a,p)^2$. Then, we have $\frac{1}{n} \leqslant \Phi(a) \leqslant 1$.*

Proof. The proof of Proposition 1 is provided in Appendix C.

Proposition 2. *Let $I = \{1, ..., n\}$ and $\Phi(a) = \sum_{p=1}^{n} \phi(a,p)^2$, $a \in A$. For any $a, b \in A$, if there exists two subsets $I_1, I_2 \subseteq I$, such that $\phi(a,p) = \frac{1}{|I_1|}$, $p \in I_1$, and $\phi(a,p) = \frac{1}{|I_2|}$, $p \in I_2$, and $|I_1| < |I_2|$, then we have $\Phi(a) > \Phi(b)$.*

Proof. The proof is obvious since in this setting $\Phi(a) = \frac{1}{|I_1|} > \Phi(b) = \frac{1}{|I_2|}$.

Proposition 3. *Let $I = \{1, ..., n\}$ and $\Phi(a) = \sum_{p=1}^{n} \phi(a,p)^2$, $a \in A$. For any $a \in A$, if there exists a subset $I' \subseteq I$, such that $\phi(a,p) > 0$, $p \in I'$, and $\phi(a,p) = 0$, $p \in I \backslash I'$, then the minimum value of $\Phi(a)$ is $\frac{1}{|I'|}$ if $\phi(a,p) = \frac{1}{|I'|}$, $p \in I'$.*

Proof. The proof is similar to that of Proposition 1.

The above three propositions indicate that RVI^{d_r} is large when the preference relations between products are certain according to d_r's preferences and the main ranking range of each product by applying all sampled value functions is narrow and concentrated. Thus, RVI can be used to quantify the certainty of each consumer's preferences. In practice, the manager may eliminate those consumers whose preferences are uncertain and then make segmentation decision. According to the above propositions, Table 1 lists some reference values of RVI for different length of ranking range so that the manager can eliminate those consumers whose preferences are uncertain. For example, the manager specifies that the preferences of any consumer, according to which each product ranks in less than two positions (i.e., the length of main ranking range is two), are certain. Then, any consumer d_r with $RVI^{d_r} \geqslant 0.7071$ ($\sqrt{1/2} = 0.7071$) should be considered as the ones whose preferences are certain.

Table 1: Reference values of RVI

Ranking Range	RVI
1	1.0000
2	0.7071
3	0.5573
4	0.5000
5	0.4472
6	0.4082

2.4. Performing market segmentation

2.4.1. Measuring similarity between consumer preferences

We propose to perform market segmentation based on consumer preferences and each segment of consumers can be addressed with targeted marketing offers and appropriate products. For this purpose, we first give a definition of similarity degree between consumer preferences and then use an agglomerative hierarchical clustering algorithm to group consumers into clusters. Generated clusters can be deemed as segments of consumers.

Definition 6. For any rankings Γ and Γ', the consistency degree $\tau(\Gamma, \Gamma')$ is defined as the proportion of concordant pairwise preference relations of products, i.e.,

$$\tau(\Gamma, \Gamma') = \frac{2}{n(n-1)} \cdot \sum\nolimits_{a,b \in A} q(\Gamma, \Gamma', a, b),$$

where

$$q(\Gamma, \Gamma', a, b) = \begin{cases} 1, & \text{if } (a \succ^{\Gamma} b \text{ and } a \succ^{\Gamma'} b) \text{ or } (a \sim^{\Gamma} b \text{ and } a \sim^{\Gamma'} b) \text{ or } (b \succ^{\Gamma} a \text{ and } b \succ^{\Gamma'} a), \\ 0, & \text{otherwise.} \end{cases}$$

$a \succ^{\Gamma} b$ and $a \sim^{\Gamma} b$ mean that in the ranking Γ, a is ranked higher than b and a is ranked the same with b, respectively.

Definition 7. The similarity degree between the preferences of the consumers d_r and $d_{r'}$ is defined as

$$sim(d_r, d_{r'}) = \frac{\Omega_1(d_r, d_{r'})}{\Omega_1(d_r, d_{r'}) + \Omega_2(d_r, d_{r'})},$$

where

$$\Omega_1(d_r, d_{r'}) = \sum\nolimits_{\Gamma\in\Psi^{d_r},\Gamma'\in\Psi^{d_{r'}}} \eta^{d_r}(\Gamma)\cdot\eta^{d_{r'}}(\Gamma')\cdot\tau(\Gamma,\Gamma'),$$

$$\Omega_2(d_r, d_{r'}) = \sum\nolimits_{\Gamma\in\Psi^{d_r}-\Psi^{d_{r'}}} \eta^{d_r}(\Gamma)^2 + \sum\nolimits_{\Gamma'\in\Psi^{d_{r'}}-\Psi^{d_r}} \eta^{d_{r'}}(\Gamma')^2 + \sum\nolimits_{\Gamma\in\Psi^{d_r}\cap\Psi^{d_{r'}}} \left(\eta^{d_r}(\Gamma) - \eta^{d_{r'}}(\Gamma)\right)^2.$$

In the above definition, $\Omega_1(d_r, d_{r'})$ is built by aggregating agreement measures of all pairs of rankings $(\Gamma, \Gamma') \in \Psi^{d_r} \times \Psi^{d_{r'}}$. It not only includes the support degrees for the same ranking (i.e., $\Gamma = \Gamma'$), but also takes into account those for different rankings (i.e., $\Gamma \neq \Gamma'$) with the "discount" $\tau(\Gamma, \Gamma')$. From our point of view, the similarity measure of two different rankings cannot be neglected as they share concordant pairwise preference relations between some products. Meanwhile, $\tau(\Gamma, \Gamma')$, which counts concordant pairwise preference relations in Γ and Γ', is incorporated into the similarity degree, because, for example, the agreement between "$a_1 \succ a_2 \succ a_3 \succ a_4$" and "$a_1 \succ a_2 \succ a_4 \succ a_3$" is larger than that between "$a_1 \succ a_2 \succ a_3 \succ a_4$" and "$a_4 \succ a_3 \succ a_2 \succ a_1$". On the other hand, $\Omega_2(d_r, d_{r'})$ can be regarded as the absolute difference between the support degrees of the two consumers' possible rankings. As a result, the similarity degree constitutes a synthetic representation of these two aspects. $sim(d_r, d_{r'})$ is increasing with respect to $\Omega_1(d_r, d_{r'})$ and decreasing with respect to $\Omega_2(d_r, d_{r'})$. An illustrative example of measuring similarity between consumer preferences is given in Appendix B.

Proposition 4. *For any $d_r, d_{r'} \in D$,*

- $sim(d_r, d_{r'}) = sim(d_{r'}, d_r)$,
- $0 \leqslant sim(d_r, d_{r'}) \leqslant 1$,
- $sim(d_r, d_{r'}) = 0$ *if and only if* Ψ^{d_r} *contains only one possible ranking* Γ, $\Psi^{d_{r'}}$ *contains only one possible ranking* Γ', *and* $q(\Gamma, \Gamma', a, b) = 0$, $\forall a, b \in A$.
- $sim(d_r, d_{r'}) = 1$ *if and only if* $\Psi^{d_r} = \Psi^{d_{r'}}$ *and* $\eta^{d_r}(\Gamma) = \eta^{d_{r'}}(\Gamma),\ \Gamma \in \Psi^{d_r}$.

Proof. The proof of Proposition 4 is provided in Appendix D.

Several measures for quantifying the agreement between ranking results delivered by different instances of a preference model have been proposed in literature, such as Kendall's τ, rank difference measure, and rank agreement measure [25]. These measures can only be applied to the case of two rankings, each of which is generated by a specific instance of the preference model. In contrast, the similarity degree proposed in this paper takes into account all possible rankings generated by sampled value functions. Note that we do not associate the above definition of similarity degree directly with the support degrees for possible preference relations between products generated by sampled value functions, but rather with the support degrees for obtained possible rankings. To explain this proposal, let us consider two consumers d_r and $d_{r'}$, whose possible rankings are $\Psi^{d_r} = \{\Gamma_1, \Gamma_2\}$ and $\Psi^{d_{r'}} = \{\Gamma_3, \Gamma_4\}$, respectively. Each possible ranking and its support degrees of each consumer are $\Gamma_1 : a_1 \succ a_2 \succ a_3$, $\eta^{d_r}(\Gamma_1) = 0.5$; $\Gamma_2 : a_3 \succ a_2 \succ a_1$, $\eta^{d_r}(\Gamma_2) = 0.5$; $\Gamma_3 : a_1 \succ a_3 \succ a_2$, $\eta^{d_{r'}}(\Gamma_3) = 0.5$; $\Gamma_4 : a_2 \succ a_3 \succ a_1$, $\eta^{d_r}(\Gamma_4) = 0.5$. In this example, we can observe that for each consumer and any pair of products $a, b \in A$, one half of the set of sampled value functions support that $a \succ b$ and the other half claim that $b \succ a$. In this sense, the two consumers have the same preferences. However, the sets of sampled value functions for different possible rankings are not the same, and thus the preferences of the two consumers are different. This example proves why it is not justified to define the similarity degree between the preferences of the consumers d_r and $d_{r'}$ at the level of support degrees for possible preference relations between products.

2.4.2. Grouping consumers into clusters

With the definition of similarity degree between consumer preferences, we apply the agglomerative hierarchical clustering methodology [4] to perform market segmentation. The agglomerative hierarchical clustering algorithm is given in Algorithm 2, in which the similarity degree between two clusters of consumers is defined as follows:

Definition 8. The similarity between two clusters of consumers, denoted by ξ and ξ', is obtained by aggregating the similarity measures between the preferences of all consumers in ξ and ξ', i.e.,

$$sim(\xi, \xi') = \frac{1}{|\xi| \cdot |\xi'|} \sum\nolimits_{d_r \in \xi, d_{r'} \in \xi'} sim(d_r, d_{r'}),$$

where $|\xi|$ and $|\xi'|$ are the numbers of consumers in ξ and ξ', respectively.

The number of clusters can be specified a priori, or determined by observing the variation of the following measure with different number of clusters. Such a measure is defined as the average similarity degree between consumer preferences in the same cluster, which quantifies the quality of the clustering result. During the clustering process, this measure decreases as more clusters are merged and the number of clusters decreases. We can find a certain number of clusters at which this measure decreases significantly and consider it as the optimal number of clusters.

Definition 9. The average similarity degree between consumer preferences in the same cluster is defined as

$$\overline{sim} = \frac{1}{n_{cluster}} \sum\nolimits_{\xi} \left(\frac{1}{|\xi|} \sum\nolimits_{d_r, d_{r'} \in \xi} sim\left(d_r, d_{r'}\right) \right),$$

where $n_{cluster}$ is the number of clusters in a certain iteration of Algorithm 2.

2.4.3. Estimating representative value functions

In order to supply consumers with appropriate products and improve products to meet consumer expectations, we need to generate a univocal ranking of products for each consumer according to the analysis of her/his preferences. Then, products that rank in the front of the list can be presented to the corresponding consumer. For this purpose, a simple idea of generating the ranking list of products for consumer d_r consists in

Algorithm 2 The clustering algorithm to group consumers into clusters.

Input:

Initial clusters: $\xi_r = \{d_r | d_r\text{'s preferences are certain}, r = 1, ..., p\}$;

Number of clusters: N (if the number of clusters is not specified a priori, set $N = 1$);

1: $n_{cluster} \leftarrow$number of initial clusters;

2: **while** $n_{cluster} > N$ **do**

3: Find the two clusters $\xi_r, \xi_{r'}$ which have the maximal similarity degree $sim\,(\xi_r, \xi_{r'})$;

4: Merge $\xi_r, \xi_{r'}$ to generate a new cluster $\xi_{r''}$;

5: $n_{cluster} \leftarrow n_{cluster} - 1$;

6: **end while**

Output:

Generated clusters of consumers.

averaging the sampled value functions in $\mathcal{U}_{PC^{d_r}}^{SMAA}$ to generate the representative value function $U_{Rep}^{d_r}$, in which the marginal value $u_{j,Rep}^{d_r}(x_j^{d_r,k})$ of the performance value $x_j^{d_r,k}$ is calculated as follows:

$$u_{j,Rep}^{d_r}(x_j^{d_r,k}) = \frac{1}{|\mathcal{U}_{PC^{d_r}}^{SMAA}|} \sum\nolimits_{U^{d_r} \in \mathcal{U}_{PC^{d_r}}^{SMAA}} u_j^{d_r}(x_j^{d_r,k}), \;\; k = 1, ..., n_j^{d_r}(A), \; j = 1, ..., m.$$

Then, according to Definition 3, we apply the representative value function $U_{Rep}^{d_r}$ to determine the ranking of products, i.e., $rank(U_{Rep}^{d_r}, a)$ for $a \in A$. The products that rank in the front of the list (i.e., $\left\{a \in A \mid rank(U_{Rep}^{d_r}, a) \leqslant n^*\right\}$ where n^* is the number of products that can be presented to the consumer and usually specified by the manager) can be presented to consumer d_r.

Moreover, for each consumer d_r, firms can learn more about her/his preferences on multiple criteria from the representative value function $U_{Rep}^{d_r}$, which is reflected by the corresponding comprehensive and marginal value functions. Specifically, firms can be aware of the trade-offs between criteria of d_r, the impact of each criterion for each product, and the difference between marginal values of different criteria evaluation as well as the strengths and weaknesses of each product.

3. Illustrative example

In this section, we illustrate the application of the proposed approach to a hypothetical market segmentation problem. An automobile seller sells 24 models of economy car and the manager wants to learn about consumer preferences so that he can make different marketing policies. The manager designs a questionnaire that contains the evaluation of these 24 cars with respect to four criteria Price (g_1), Acceleration (g_2), Max speed (g_3), Fuel consumption (g_4), Comfort (g_5), and Appearance (g_6). The evaluations of these cars on criteria g_1, g_2, g_3 and g_4 are the same for all consumers, while for the last two criteria g_5 and g_6, consumers could give their own evaluations (the two criteria are evaluated using a 5-point ordinal scale). The evaluations of cars on criteria g_1, g_2, g_3 and g_4 are presented in Table 2, which are originally considered in Ref. [5], while those on criteria g_5 and g_6 from each consumer are given in Appendix E. Note that Price (g_1), Acceleration (g_2), and Fuel consumption

(g_4) are cost criteria while Max speed (g_3), Comfort (g_5), and Appearance (g_6) are gain criteria. The manager delivers this questionnaire to potential consumers and ask them to compare these cars. The manager gets back 538 responses and each one consists of pairwise comparisons provided by a particular consumer. The preference information provided by all consumers are provided in Appendix F.

Table 2: Set of considered cars and their evaluations with respect to criteria g_1, g_2, g_3 and g_4.

Cars	Price [Euro]	Acceleration [seconds from 0 to 100 km/h]	Max speed [km/h]	Fuel consumption [l/km]
(a_1) Audi A3 (3-doors)	22,140	10.3	193	4.9
(a_2) BMW 1 Series (3-doors)	23,089	11.2	195	5.5
(a_3) Hyunday ix20	14,000	12.9	167	6
(a_4) Ford C-Max	19,500	12.6	174	5.1
(a_5) Toyota Aygo	10,350	13.7	157	4.4
(a_6) Seat Ibiza (5-doors) Style	13,000	13.9	163	5.4
(a_7) VolksWagen Polo highline 1.4 (3-doors)	16,550	12.1	177	5.9
(a_8) BMW Serie 1 (3-doors)	23,069	11.2	195	5.5
(a_9) Chevrolet Spark	9,952	15.3	152	5
(a_{10}) FIAT Punto (3-doors)	13,711	11.2	182	4.2
(a_{11}) Ford Fiesta (3-doors)	12,750	14.9	165	4.6
(a_{12}) Honda Civic	18,900	13.4	187	5.4
(a_{13}) Kia Rio	11,650	13.1	172	5.1
(a_{14}) Lancia Ypsilon	14,568	11.9	176	4.2
(a_{15}) Mazda2 3-door Sporty	14,900	13.6	172	5
(a_{16}) Mercedes A-Class	23,630	9.2	202	5.5
(a_{17}) Mini Cooper	20,700	7.9	210	4.5
(a_{18}) Mitsubishi Space Star	11,490	13.6	172	4
(a_{19}) Nissan Micra	11,250	13.7	170	5
(a_{20}) Opel Corsa	11,330	18.2	155	5.1
(a_{21}) Peugeot 208	12,100	14	163	4.3
(a_{22}) Renault Clio	16,200	12.2	182	4.5
(a_{23}) Skoda Citygo	9,260	14.4	160	4.5
(a_{24}) Suzuki Swift	12,100	11.5	165	5

To analyze the preferences of each consumer, we firstly check whether the preference information of each consumer is consistent. If the preference information of consumer $d_r \in D$ is consistent, we adopt the SMAA to sample 10000 compatible value functions from $\mathcal{U}_{PC^{d_r}}$; otherwise, we use Algorithm 1 to identify all maximal subsets of consistent pairwise comparisons for d_r, and define the priority $\lambda(PC_k^{d_r})$ for each pairwise comparison $PC_k^{d_r}$, $k = 1, ..., |PC^{d_r}|$, and use the model LP2 to relax linear constraints in order to obtain a set of compatible value functions $\mathcal{U}'_{PC^{d_r}}$, and then sample 10000 compatible value functions from $\mathcal{U}'_{PC^{d_r}}$. Let us emphasize that each $CPC_s^{d_r}$, $s = 1, ..., K^{d_r}$, is maximal in the sense that any of its proper super sets is inconsistent. For example, the consumer d_{75} provides nine initial pairwise comparisons, and we identify three maximal subsets of consistent pairwise comparisons for d_{75}, which are provided in Table 3.

Table 3: Maximal subsets of consistent pairwise comparisons for consumer d_{75}.

CPC	Pairwise comparisons
$CPC_1^{d_{75}}$	$a_1 \succ a_{20}$, $a_2 \succ a_{14}$, $a_4 \succ a_{23}$, $a_6 \succ a_9$, $a_6 \succ a_{21}$, $a_7 \succ a_{17}$, $a_{11} \succ a_2$, $a_{24} \succ a_6$
$CPC_2^{d_{75}}$	$a_1 \succ a_{20}$, $a_2 \succ a_{14}$, $a_4 \succ a_{23}$, $a_6 \succ a_9$, $a_6 \succ a_{21}$, $a_7 \succ a_{17}$, $a_{11} \succ a_2$, $a_{21} \succ a_{24}$
$CPC_3^{d_{75}}$	$a_1 \succ a_{20}$, $a_2 \succ a_{14}$, $a_4 \succ a_{23}$, $a_6 \succ a_9$, $a_7 \succ a_{17}$, $a_{11} \succ a_2$, $a_{21} \succ a_{24}$, $a_{24} \succ a_6$

The maximal subsets of consistent pairwise comparisons for each consumer is provided in Appendix G. Table 4 summarizes the number of maximal subsets of consistent pairwise comparisons for consumers. One can observe that preference information of most consumers (i.e., 479) is consistent; for those consumers whose preference information is inconsistent, most of them (i.e. 47) have two maximal subsets of consistent pairwise comparisons, and only a small proportion of them (less than 3%) have three.

Table 4: Summary of the number of maximal subsets of consistent pairwise comparisons for consumers.

K^{d_r}	Number of consumers
1 (i.e., consistent)	479
2	47
3	12

Each value function sampled from $\mathcal{U}_{PC^{d_r}}$ or $\mathcal{U}'_{PC^{d_r}}$, generates a specific ranking of cars, and there could be more than one value function which generates the same ranking. For each consumer $d_r \in D$, we obtain the set of possible rankings Ψ^{d_r} to represent d_r's preferences. For each $\Gamma \in \Psi^{d_r}$, its support degree $\eta^{d_r}(\Gamma)$ can be regarded as the possibility of the ranking Γ occurring according to d_r's preferences. For example, the set of possible rankings for the consumer d_{75} is composed of 329 possible rankings of cars, and the rankings with ten largest support degrees (ranging from 0.1382 to 0.0476) are presented in Table 5. The support degree of any other possible ranking is less than 0.04. In fact, the sum of the support degrees of the possible rankings given in Table 5 accounts for a large proportion (i.e. 74.75%) of that of all possible rankings.

Table 6 and Figure 2 summarizes the distribution of the number of possible rankings for consumers. It is obvious that most consumers have less than 500 possible rankings. This inspires us to calculate the similarity degree between consumer preferences, which can be finished in a reasonable amount of computational time.

On the other hand, we also calculate the ASI and RVI for each individual consumer to measure the certainty of her/his preferences. The ASI and RVI of each consumer is reported in Appendix H. The Pearson correlation

Table 5: Possible rankings with ten largest support degrees for consumer d_{75}.

No.	Γ	$\eta^{d_{75}}(\Gamma)$
1	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{21} \succ a_{24} \succ a_{15} \succ a_{17} \succ a_{12} \succ a_{11}$ $\succ a_{6} \succ a_{4} \succ a_{2} \succ a_{14} \succ a_{23} \succ a_{1} \succ a_{8} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.1382
2	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{15} \succ a_{6} \succ a_{4} \succ a_{17} \succ a_{12} \succ a_{24}$ $\succ a_{21} \succ a_{11} \succ a_{2} \succ a_{23} \succ a_{1} \succ a_{8} \succ a_{14} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0943
3	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{12} \succ a_{24} \succ a_{21} \succ a_{15} \succ a_{17} \succ a_{4}$ $\succ a_{6} \succ a_{11} \succ a_{1} \succ a_{2} \succ a_{14} \succ a_{8} \succ a_{23} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0855
4	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{15} \succ a_{7} \succ a_{6} \succ a_{12} \succ a_{21} \succ a_{24} \succ a_{17}$ $\succ a_{11} \succ a_{4} \succ a_{1} \succ a_{2} \succ a_{23} \succ a_{8} \succ a_{14} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0812
5	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{7} \succ a_{19} \succ a_{15} \succ a_{6} \succ a_{17} \succ a_{12} \succ a_{21} \succ a_{4}$ $\succ a_{11} \succ a_{24} \succ a_{2} \succ a_{1} \succ a_{23} \succ a_{8} \succ a_{14} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0734
6	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{12} \succ a_{15} \succ a_{4} \succ a_{6} \succ a_{17} \succ a_{24}$ $\succ a_{21} \succ a_{11} \succ a_{2} \succ a_{23} \succ a_{1} \succ a_{8} \succ a_{14} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0687
7	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{12} \succ a_{4} \succ a_{24} \succ a_{21} \succ a_{15} \succ a_{6}$ $\succ a_{17} \succ a_{11} \succ a_{1} \succ a_{2} \succ a_{23} \succ a_{14} \succ a_{8} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0553
8	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{6} \succ a_{11} \succ a_{12} \succ a_{15} \succ a_{21} \succ a_{17}$ $\succ a_{24} \succ a_{4} \succ a_{2} \succ a_{23} \succ a_{1} \succ a_{14} \succ a_{8} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0532
9	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{24} \succ a_{21} \succ a_{6} \succ a_{7} \succ a_{15} \succ a_{17} \succ a_{4}$ $\succ a_{12} \succ a_{11} \succ a_{2} \succ a_{14} \succ a_{1} \succ a_{23} \succ a_{8} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0501
10	$a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_{7} \succ a_{15} \succ a_{6} \succ a_{21} \succ a_{4} \succ a_{12} \succ a_{11}$ $\succ a_{24} \succ a_{17} \succ a_{2} \succ a_{1} \succ a_{23} \succ a_{14} \succ a_{8} \succ a_{5} \succ a_{9} \succ a_{20} \succ a_{3} \succ a_{16}$	0.0476

Table 6: Summary of the number of possible rankings for the group of consumers.

Number of possible rankings	Number of consumers
< 500	427
$500 - 1000$	77
$1000 - 1500$	25
> 1500	9
Max.	1784
Min.	11
Avg.	379.25

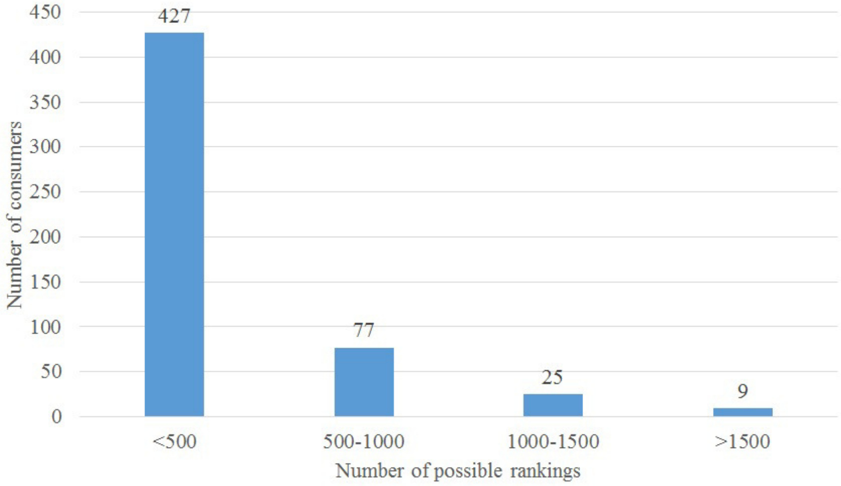


Figure 2: Distribution of number of possible rankings for the group of consumers.

coefficient between ASI and RVI is 0.4253, which means the two measures have a high correlation. The manager does not plan to provide products for those consumers whose preferences are uncertain and decides to eliminate those consumers according to whose preferences the average length of each product's ranking range is larger than four. Thus, we find those consumers whose RVI is less than 0.5 ($\sqrt{1/4} = 0.5$). In this step, 44 consumers are eliminated.

When it comes to measuring the similarity between consumer preferences, we adopt the new definition of similarity degree proposed in Section 2.4. Let us emphasize that the definition of similarity degree takes into account support degrees for different rankings ($\Gamma \neq \Gamma'$) with the "discount" $\tau(\Gamma, \Gamma')$, as we cannot neglect concordant pairwise preference relations between some cars in the two rankings. To save space, we only report ten pairs of consumers whose preferences are the most similar and dissimilar, respectively, in Table 7.

With the obtained similarity degrees between consumer preferences, we apply Algorithm 2 to group consumers into clusters so that the manager can learn about the preferences of each cluster and make different marketing policies. The number of clusters is not specified in advance, and thus we determine the optimal clustering result by observing the variation of the average similarity degree $\overline{sim}$ according to Definition 9. Table 8 and Figure 3 report average similarity degrees $\overline{sim}$ for different number of clusters. Obviously, when the number of clusters becomes two from three, $\overline{sim}$ decreases significantly. Thus, the optimal number of clusters is three. In the optimal clustering result which is given in Appendix I, the obtained three clusters of consumers are composed of 223, 99, and 172 consumers, respectively.

Then, we aggregate the sampled value functions in $\mathcal{U}^{SMAA}_{PC^{d_r}}$ for each consumer d_r into the representative value function $U^{d_r}_{Rep}$, which is used to obtain the ranking of products. The representative value function and the ranking of products of each consumer is presented in Appendix J. For each consumer, those products that rank in the front of the ranking list are the most preferred ones that can be presented to her/him. For example, the representative value function of consumer d_{75} is reported in Table 9 and depicted in Figure 4. One can observe that the weights of the six criteria are 0.4109, 0.1198, 0.1600, 0.2064, 0.0861, and 0.0164,

Table 7: Pairs of consumers whose preferences are the most similar and dissimilar.

Most similar pairs of consumers			Most dissimilar pairs of consumers		
Consumer d_i	Consumer d_j	Similarity degree	Consumer d_i	Consumer d_j	Similarity degree
d_{342}	d_{493}	0.9276	d_{148}	d_{481}	0.759
d_{118}	d_{292}	0.9272	d_{369}	d_{537}	0.2160
d_{181}	d_{519}	0.9267	d_{31}	d_{133}	0.2162
d_{64}	d_{69}	0.9262	d_{99}	d_{201}	0.2162
d_{365}	d_{417}	0.9259	d_{107}	d_{197}	0.2163
d_{48}	d_{299}	0.9256	d_{161}	d_{297}	0.2163
d_{134}	d_{326}	0.9248	d_{5}	d_{182}	0.2165
d_{201}	d_{336}	0.9248	d_{62}	d_{109}	0.2167
d_{191}	d_{260}	0.9245	d_{274}	d_{425}	0.2168
d_{436}	d_{535}	0.9240	d_{212}	d_{222}	0.2168

Table 8: Average similarity degree $\overline{sim}$ for different number of clusters.

Number of clusters	Average similarity degree $\overline{sim}$
1	0.2983
2	0.3287
3	0.5487
4	0.5692
5	0.5703
6	0.5729
7	0.5872
8	0.5938
9	0.6209
10	0.6381

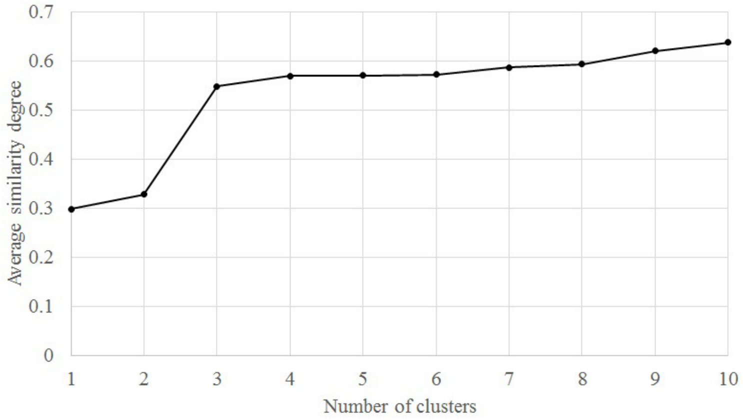


Figure 3: Average similarity degree $\overline{sim}$ for different number of clusters.

respectively, which are retrieved from the marginal values corresponding to the most preferred performance on each criterion and indicates that consumer d_{75} cares more about the criteria Price (g_1) and Fuel consumption (g_4). According to the representative value function, we work out the ranking list of products for consumer d_{75} as $a_{18} \succ a_{10} \succ a_{13} \succ a_{22} \succ a_{19} \succ a_7 \succ a_{12} \succ a_{15} \succ a_6 \succ a_{21} \succ a_{17} \succ a_{24} \succ a_4 \succ a_{11} \succ a_2 \succ a_1 \succ a_{23} \succ a_{14} \succ a_8 \succ a_5 \succ a_9 \succ a_{20} \succ a_3 \succ a_{16}$. Thus, the manager can provide the cars that rank in the front of the list for consumer d_{75} and emphasize the advantages of these cars on price and fuel consumption.

On the other hand, we find that for most consumers in the first cluster, the weight of the coalition of Price (g_1) and Fuel consumption (g_4) are larger than that of the coalition of Acceleration (g_2) and Max speed (g_3) and that of the coalition of Comfort (g_5) and Appearance (g_6), which indicates that these consumers care more about Price (g_1) and Fuel consumption (g_4). Then, for most consumers in the second cluster, the weight of the coalition of Acceleration (g_2) and Max speed (g_3) are larger than that of the coalition of Price (g_1) and Fuel consumption (g_4) and that of the coalition of Comfort (g_5) and Appearance (g_6), which shows that Acceleration (g_2) and Max speed (g_3) are more concerned by these consumers. Finally, for most consumers in the second cluster, the weight of the coalition of Comfort (g_5) and Appearance (g_6) are larger than that of the coalition of Price (g_1) and Fuel consumption (g_4) and that of the coalition of Acceleration (g_2) and Max speed (g_3), which illustrates that these consumers pay more attention to Comfort (g_5) and Appearance (g_6). The analysis suggests that the manager should consider the first and the third clusters (segments), since they are consisted of most consumers. The second cluster (segment) is probably the worst choice, as it contains the smallest number of consumers.

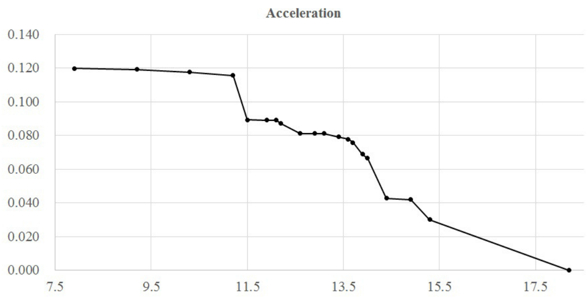
4. Conclusions

In this paper, we propose a multiple criteria market segmentation that integrates consumer preference analysis and segmentation decision within a unified framework. It requires consumers who participate in the market survey to evaluate a set of products based on multiple criteria and specify some pairwise comparisons on products that they are familiar with or have experienced. To analyze each consumer's preferences in

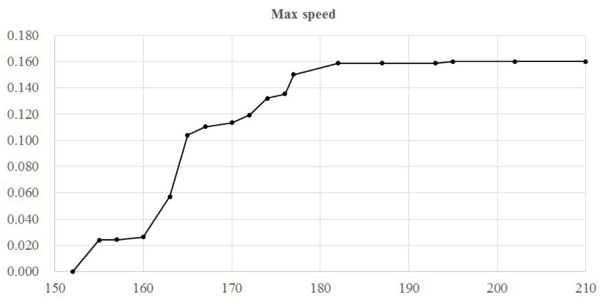
Table 9: The representative value function for consumer d_{75}.

Criteria	Performance and corresponding marginal value								
g_1 - Price	Performance	23630	23089	23069	22140	20700	19500	18900	16550
	Value	0.0000	0.2016	0.2066	0.2078	0.2275	0.2886	0.2962	0.2962
	Performance	16200	14900	14568	14000	13711	13000	12750	12100
	Value	0.3110	0.3130	0.3130	0.3698	0.3718	0.3748	0.3812	0.3837
	Performance	11650	11490	11330	11250	10350	9952	9260	
	Value	0.3955	0.3997	0.4056	0.4085	0.4097	0.4109	0.4109	
g_2 - Acceleration	Performance	18.2	15.3	14.9	14.4	14	13.9	13.7	13.6
	Value	0.0000	0.0301	0.0420	0.0427	0.0664	0.0689	0.0758	0.0778
	Performance	13.4	13.1	12.9	12.6	12.2	12.1	11.9	11.5
	Value	0.0791	0.0811	0.0811	0.0812	0.0871	0.0890	0.0890	0.0892
	Performance	11.2	10.3	9.2	7.9				
	Value	0.1156	0.1176	0.1191	0.1198				
g_3 - Max speed	Performance	152	155	157	160	163	165	167	170
	Value	0.0000	0.0242	0.0243	0.0263	0.0571	0.1039	0.1105	0.1134
	Performance	172	174	176	177	182	187	193	195
	Value	0.1193	0.1319	0.1353	0.1500	0.1588	0.1588	0.1588	0.1600
	Performance	202	210						
	Value	0.1600	0.1600						
g_4 - Fuel consumption	Performance	6	5.9	5.5	5.4	5.1	5	4.9	4.6
	Value	0.0000	0.1156	0.1171	0.1179	0.1180	0.1180	0.1180	0.1180
	Performance	4.5	4.4	4.3	4.2	4			
	Value	0.1186	0.1191	0.1191	0.1244	0.2064			
g_5 - Comfort	Performance	1	2	3	4	5			
	Value	0.0000	0.0624	0.0725	0.0764	0.0861			
g_6 - Appearance	Performance	1	2	3	4	5			
	Value	0.0000	0.0089	0.0089	0.0155	0.0164			

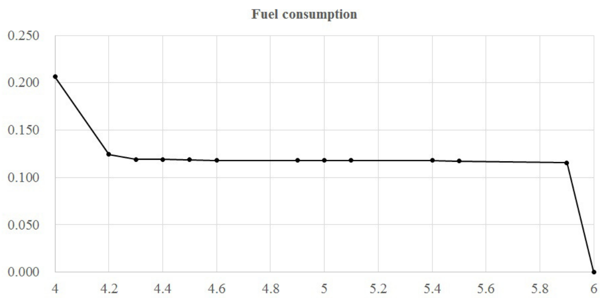
(a) g_1 - Price



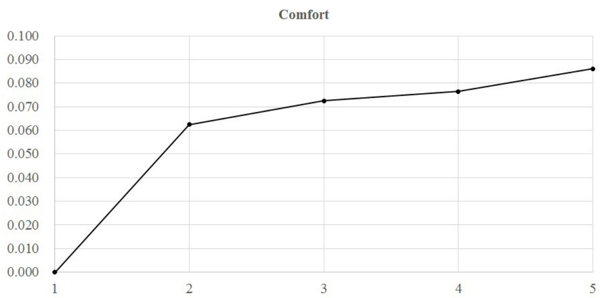
(b) g_2 - Acceleration



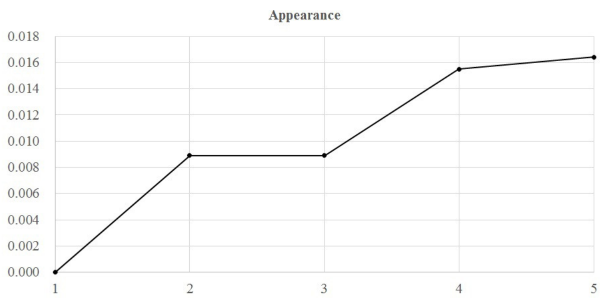
(c) g_3 - Max speed



(d) g_4 - Fuel consumption



(e) g_5 - Comfort



(f) g_6 - Appearance

Figure 4: The representative value function for consumer d_{75}.

a robust manner, the approach employs the general monotone value function as the consumer's preference model and utilizes the disaggregation paradigm and the SMAA to derive a set of compatible value functions according to the preference information provided by each consumer. Then, the approach proposes to represent each consumer's preferences through the distribution of possible rankings and associated support degrees. On the basis of such representation of consumer preferences, the approach defines the similarity degree between consumer preferences and uses the hierarchical clustering algorithm to perform market segmentation. Finally, the approach works out a representative value function and the univocal ranking of products for each consumer so that products that rank in the front of the list can be presented to her/him. In this way, firms can get thorough insight into the strengths and weaknesses of each product and serve consumers from different segments with targeted marketing policies and appropriate products.

We perceive the proposed approach as a complementary methodology to the market segmentation literature which has paid little attention to the situation where consumers provide preference information in form of pairwise comparisons, although such statements have been widely used in marketing management practice. On the other hand, differently from existing MCDA methods, the approach allows a large group of consumers to participate in the decision process, and reveals several new characteristics including the way of analyzing consumer preferences, the procedure of grouping consumers into segments, and the recommendation of a univocal result.

We envisage further research towards several interesting directions. First of all, the approach can be extended to consider various types of preference information, such as product ratings. Allowing consumers to provide various types of preference information would increase the flexibility of the market survey procedure and reduce the cognitive burden on consumers. Secondly, natural language processing techniques (e.g., word2vector [36]) can be utilized to extract relevant criteria from online product reviews and then the proposed approach can be applied to real word problems. Thirdly, the rejection rate in our sampling method presented in Section 2.3.2 grows significantly when there are more criteria or pairwise comparisons. Future research should investigate application of more efficient sampling techniques (e.g., Hit-And-Run [42, 45]). Finally, an experimental analysis can be performed to investigate the expressiveness of the underlying preference model and the robustness of the recommendation suggested by the preference model [23]. In particular, other preference models such as the extended value functions [1, 14] and the Choquet integral [2] can be employed to take into account the interaction effects between criteria when consumers express preferences.

Acknowledgement

The research is supported by the National Natural Science Foundation of China (#71701160, #91546119) and the China Postdoctoral Science Foundation (#2017M623201). We are grateful to the anonymous reviewers for their constructive and detailed comments, which helped us improve the previous version of the paper.

References

[1] Angilella, S., Corrente, S., Greco, S., and Słowiński, R. (2014). MUSA-INT: Multicriteria customer satisfaction analysis with interacting criteria. *Omega*, 42(1):189–200.

[2] Angilella, S., Greco, S., and Matarazzo, B. (2010). Non-additive robust ordinal regression: A multiple criteria decision model based on the Choquet integral. *European Journal of Operational Research*, 201(1):277–288.

[3] Balakrishnan, P., Kumar, S., and Han, P. (2011). Dual objective segmentation to improve targetability: An evolutionary algorithm approach. *Decision Sciences*, 42(4):831–857.

[4] Berkhin, P. (2006). A survey of clustering data mining techniques. In *Grouping multidimensional data*, pages 25–71. Springer.

[5] Corrente, S., Greco, S., and Ishizaka, A. (2015). Combining analytical hierarchy process and Choquet integral within non-addititive robust ordinal regression. *Omega*, 61:2–18.

[6] Dowling, G. R. and Midgley, D. F. (1988). Identifying the coarse and fine structures of market segments. *Decision Sciences*, 19(4):830–847.

[7] Fan, B. and Zhang, P. (2009). Spatially enabled customer segmentation using a data classification method with uncertain predicates. *Decision Support Systems*, 47(4):343–353.

[8] Figueira, J. R., Greco, S., and Słowiński, R. (2009). Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. *European Journal of Operational Research*, 195(2):460–486.

[9] Frank, R. E. and Strain, C. E. (1972). A segmentation research design using consumer panel data. *Journal of Marketing Research*, pages 385–390.

[10] Greco, S., Kadziński, M., Mousseau, V., and Słowiński, R. (2011). ELECTRE$^{\mathrm{GKMS}}$: Robust ordinal regression for outranking methods. *European Journal of Operational Research*, 214(1):118–135.

[11] Greco, S., Kadziński, M., Mousseau, V., and Słowiński, R. (2012). Robust ordinal regression for multiple criteria group decision: UTA$^{\mathrm{GMS}}$-GROUP and UTADIS$^{\mathrm{GMS}}$-GROUP. *Decision Support Systems*, 52(3):549–561.

[12] Greco, S., Mousseau, V., and Słowiński, R. (2008). Ordinal regression revisited: Multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):416–436.

[13] Greco, S., Mousseau, V., and Słowiński, R. (2010). Multiple criteria sorting with a set of additive value functions. *European Journal of Operational Research*, 207(3):1455–1470.

[14] Greco, S., Mousseau, V., and Słowiński, R. (2014). Robust ordinal regression for value functions handling interacting criteria. *European Journal of Operational Research*, 239(3):711–730.

[15] Green, P. E. (1977). A new approach to market segmentation. *Business Horizons*, 20(1):61–73.

[16] Green, P. E. and Srinivasan, V. (1978). Conjoint analysis in consumer research: issues and outlook. *Journal of consumer research*, 5(2):103–123.

[17] Grigoroudis, E. and Siskos, Y. (2002). Preference disaggregation for measuring and analysing customer satisfaction: The MUSA method. *European Journal of Operational Research*, 143(1):148–170.

[18] Han, S., Ye, Y., Fu, X., and Chen, Z. (2014). Category role aided market segmentation approach to convenience store chain category management. *Decision Support Systems*, 57:296–308.

[19] Hauser, J. R. (1978). Consumer preference axioms: Behavioral postulates for describing and predicting stochastic choice. *Management Science*, 24(13):1331–1341.

[20] Huerta-Muñoz, D. L., Ríos-Mercado, R. Z., and Ruiz, R. (2017). An iterated greedy heuristic for a market segmentation problem with multiple attributes. *European Journal of Operational Research*, 261(1):75–87.

[21] Jacquet-Lagrèze, E. and Siskos, Y. (1982). Assessing a set of additive utility functions for multicriteria decision-making, the UTA method. *European Journal of Operational Research*, 10(2):151–164.

[22] Kadziński, M., Corrente, S., Greco, S., and Słowiński, R. (2014). Preferential reducts and constructs in robust multiple criteria ranking and sorting. *OR Spectrum*, 36(4):1021–1053.

[23] Kadziński, M., Ghaderi, M., Wasikowski, J., and Agell, N. (2017). Expressiveness and robustness measures for the evaluation of an additive value function in multiple criteria preference disaggregation methods: an experimental analysis. *Computers & Operations Research*, 87:146–164.

[24] Kadziński, M., Greco, S., and Słowiński, R. (2013). Selection of a representative value function for robust ordinal regression in group decision making. *Group Decision and Negotiation*, 22(3):1–34.

[25] Kadziński, M. and Michalski, M. (2016). Scoring procedures for multiple criteria decision aiding with robust and stochastic ordinal regression. *Computers & Operations Research*, 71:54–70.

[26] Kadziński, M. and Tervonen, T. (2013). Robust multi-criteria ranking with additive value models and holistic pair-wise preference statements. *European Journal of Operational Research*, 228(1):169–180.

[27] Keeney, R. and Raiffa, H. (1976). *Decisions with multiple objectives: preferences and value trade-offs*. Cambridge University Press.

[28] Kiang, M. Y., Hu, M. Y., and Fisher, D. M. (2006). An extended self-organizing map network for market segmentation - a telecommunication example. *Decision Support Systems*, 42(1):36–47.

[29] Kotler, P. and Armstrong, G. (2010). *Principles of marketing.* Pearson Education.

[30] Lahdelma, R., Hokkanen, J., and Salminen, P. (1998). SMAA-stochastic multiobjective acceptability analysis. *European Journal of Operational Research*, 106(1):137–143.

[31] Lahdelma, R., Miettinen, K., and Salminen, P. (2003). Ordinal criteria in stochastic multicriteria acceptability analysis (SMAA). *European Journal of Operational Research*, 147(1):117–127.

[32] Lahdelma, R. and Salminen, P. (2001). SMAA-2: Stochastic multicriteria acceptability analysis for group decision making. *Operations Research*, 49(3):444–454.

[33] Liu, Y., Ram, S., Lusch, R. F., and Brusco, M. (2010). Multicriterion market segmentation: a new model, implementation, and evaluation. *Marketing Science*, 29(5):880–894.

[34] Matsatsinis, N. and Siskos, Y. (1999). Markex: An intelligent decision support system for product development decisions. *European Journal of Operational Research*, pages 336–354.

[35] Matsatsinis, N. and Siskos, Y. (2003). *Intelligent support systems for marketing decisions.* Kluwer, Heidelberg.

[36] Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., and Dean, J. (2013). Distributed representations of words and phrases and their compositionality. In *Advances in neural information processing systems*, pages 3111–3119.

[37] Moretti, S., Öztürk, M., and Tsoukiàs, A. (2016). Preference modelling. In *Multiple Criteria Decision Analysis: State of the Art Surveys*, pages 43–95.

[38] Mousseau, V., Figueira, J., Dias, L., da Silva, C. G., and Clmaco, J. (2003). Resolving inconsistencies among constraints on the parameters of an MCDA model. *European Journal of Operational Research*, 147(1):72 – 93.

[39] Siskos, Y. and Grigoroudis, E. (2010). New trends in aggregation-disaggregation approaches. *Handbook of Multicriteria Analysis*, pages 189–214.

[40] Tervonen, T. and Figueira, J. R. (2008). A survey on stochastic multicriteria acceptability analysis methods. *Journal of Multi-Criteria Decision Analysis*, 15(1-2):1–14.

[41] Tervonen, T. and Lahdelma, R. (2007). Implementing stochastic multicriteria acceptability analysis. *European Journal of Operational Research*, 178(2):500–513.

[42] Tervonen, T., van Valkenhoef, G., Baştürk, N., and Postmus, D. (2013). Hit-and-run enables efficient weight generation for simulation-based multiple criteria decision analysis. *European Journal of Operational Research*, 224(3):552–559.

[43] Tsafarakis, S., Grigoroudis, E., and Matsatsinis, N. (2008). Targeting the undecided customer. In *Proceedings of the 37th EMAC Conference*.

[44] Tsafarakis, S., Lakiotaki, K., and Matsatsinis, N. (2010). Applications of MCDA in marketing and e-commerce. In *Handbook of Multicriteria Analysis*, pages 425–448. Springer.

[45] van Valkenhoef, G., Tervonen, T., and Postmus, D. (2014). Notes on hit-and-run enables efficient weight generation for simulation-based multiple criteria decision analysis. *European Journal of Operational Research*, 239(3):865–867.

[46] Wedel, M. and Kamakura, W. A. (2012). *Market segmentation: Conceptual and methodological foundations*, volume 8. Springer Science & Business Media.

[47] Wind, Y. (1978). Issues and advances in segmentation research. *Journal of marketing research*, pages 317–337.