IberoSing International Workshop 2024

Low-dimensional Topology & Singularity Theory

25 - 29 NOVEMBER

MADRID (SPAIN)

https://iberosing.github.io/IW24/

POLITÉCNICA

UNIVERSIDAD POLITÉCNICA DE MADRID

Instituto de Matemáticas UNAM

MINISTERIO DE CIENCIA E INNOVACIÓN

AGENCIA ESTATAL DE INVESTIGACIÓN

Red de Geometría Algebraica y Singularidades

Contents

About

IBEROSING

This is the fourth edition of regular conferences organized by the main staff of the Iberoamerican Webminar of Young Researchers in Singularity Theory and related topics. The event will be held at Ed. Retiro, ETSI Caminos, Canales y Puertos (Universidad Politécnica de Madrid) from 25 to 29 of November 2024.

It aims to be an international meeting place for both young and senior researchers in Singularity theory, where some of the recent topics in the theory are addressed in detail through courses and various specialized talks in a highly stimulating environment. This is a face-to-face event.

This year event will be focusing on the interactions between **Low-dimensional topology** and **Singularity Theory** such as the theory of contact loci or lattice cohomology.

The event consists of two courses, a series of invited talks and a selection of 5mins lightning-talks followed by poster sessions. The idea of the lightning-talks is to give the opportunity to some participants to briefly present their posters to the entire audience before the poster session.

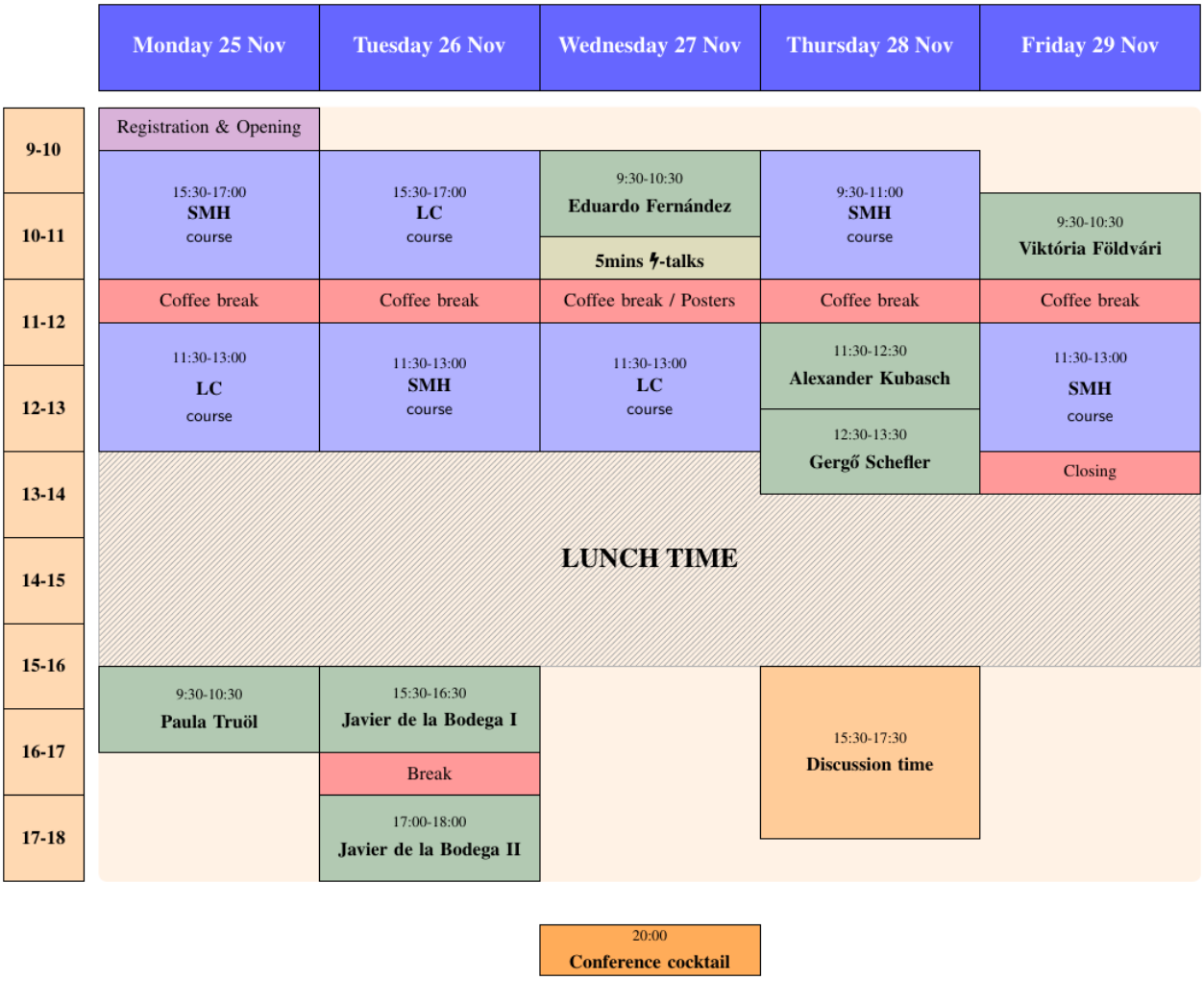
Organizing committee

Pablo Portilla-Cuadrado	Univ. Politécnica de Madrid
Juan Viu-Sos	Univ. Politécnica de Madrid
Agustín Romano-Velázquez	Univ. Nacional Autónoma de México

Scientific committee

Patricio Almirón-Cuadros	Univ. de Granada
Roi do Campo	Univ. of Oklahoma
Eva Elduque	Univ. Autónoma de Madrid
José Seade	Univ. Nacional Autónoma de México
Laura Starkston	UC Davis

Timetable


Monday 25 Nov
Tuesday 26 Nov
Wednesday 27 Nov
Thursday 28 Nov
Friday 29 Nov
9-10
10-11
11-12
12-13
13-14
14-15
15-16
16-17
17-18
Registration & Opening
15:30-17:00 SMH course
15:30-17:00 LC course
9:30-10:30 Eduardo Fernández
5mins ϟ-talks
9:30-11:00 SMH course
9:30-10:30 Viktória Földvári
Coffee break
Coffee break
Coffee break / Posters
Coffee break
Coffee break
11:30-13:00 LC course
11:30-13:00 SMH course
11:30-13:00 LC course
11:30-12:30 Alexander Kubasch
12:30-13:30 Gergő Schefler
11:30-13:00 SMH course
Closing
LUNCH TIME
9:30-10:30 Paula Truöl
15:30-16:30 Javier de la Bodega I
Break
17:00-18:00 Javier de la Bodega II
15:30-17:30 Discussion time
20:00 Conference cocktail

Courses

The event offers two specialization mini-courses:

1. **Lattice cohomology: then and now** by Tamás László (Babeş-Bolyai University).

2. **Singularities of maps and regular homotopy** by Roberto Giménez Conejero (Mid Sweden University).

Course 1

Lattice cohomology: then and now

Tamás László

Babeş-Bolyai University (Cluj-Napoca, Romania)

The topic of this lecture series is the lattice cohomology. During these three lectures we will discuss the concepts from two different points of view.

First of all, we start from the beginnings by motivating the development of the theory as part of the so-called Artin-Laufer-Némethi program. We define the first version which nowadays is called the topological lattice cohomology of normal surface singularities.

Then we look at it from a new perspective by defining the combinatorial lattice cohomology, and we also specify it to the analytic theory of surface singularities.

Returning to the topological theory, we present the reduction theorem and a couple of its applications which, on one hand, made the theory available at the computational level, on the other hand, placed it in the context of the ALN program. At the end of the course, we formulate a couple of problems and an idea for an “embedded" version that directs the course towards to the talks of Kubasch and Schefler.

Prerequisites: some basics on the resolution and topology of normal surface singularities, sheaf theory, algebraic topology.

References

[1] Artin, M.: On isolated rational singularities of surfaces, *Amer. J. of Math.* **88** (1966), 129–136.

[2] Ágoston, T. and Némethi, A.: The analytic lattice cohomology of surface singularities, arXiv 2108.12294, 2021.

[3] Ágoston, T. and Némethi, A.: Analytic lattice cohomology of surface singularities, II (the equivariant case), arXiv 2108.12429, 2021.

[4] Ágoston, T. and Némethi, A.: Lattice Cohomology. In: de Bobadilla, J.F., Marengon, M., Némethi, A., Stipsicz, A. (eds) Singularities and Low Dimensional Topology. Bolyai Society Mathematical Studies, vol 30. Springer, Cham.

[5] Barth, W., Peters, C. and Van de Ven, A.: Compact complex surfaces, *Springer–Verlag*, Berlin & Heidelberg, 1984.

[6] László, T. and Némethi, A.: Reduction theorem for lattice cohomology, *Int. Math. Research Notices* 2015 (1) (2015), 2938–2985.

[7] László, T.: Lattice cohomology and Seiberg-Witten invariants of normal surface singularities, PhD thesis, CEU Edt collection: https://www.etd.ceu.edu/2014/laszlo_tamas.pdf (2013)

[8] Laufer, H.B.: Normal two–dimensional singularities, *Ann. of Math. Studies* **71**, Princeton University Press, 1971.

[9] Laufer, H.B.: On rational singularities, *Amer. J. of Math.* **94** (1972), 597-608.

[10] Laufer, H.B.: On minimally elliptic singularities, *Amer. J. of Math.* **99** (1977), 1257–1295.

[11] Luengo-Velasco, I., Melle-Hernández, A. and Némethi, A.: Links and analytic invariants of superisolated singularities, *Journal of Alg. Geom.* **14** (2005), 543–565.

[12] Némethi, A.: Five lectures on normal surface singularities, lectures at the Summer School in *Low dimensional topology* Budapest, Hungary, 1998; Bolyai Society Math. Studies **8** (1999), 269–351.

[13] Némethi, A.: "Weakly" elliptic Gorenstein singularities of surfaces, *Invent. Math.* **137** (1999), 145–167.

[14] Némethi, A.: On the Ozsváth-Szabó invariant of negative definite plumbed 3-manifolds, *Geometry and Topology*, **9** (2005), 991–1042.

[15] Némethi, A.: Graded roots and singularities, in Singularities in Geometry and Topology, World Scientific 394–463 (2007).

[16] Némethi, A.: Lattice cohomology of normal surface singularities, *Publ. RIMS. Kyoto Univ.*, **44** (2008), 507–543.

[17] Némethi, A.: Normal surface singularities, *Ergebnisse der Math. und ihrer Grenzgebiete 74*, Springer 2022.

[18] Némethi, A.: Filtered lattice homology of surface singularities, *arXiv:2307.16581.* (2023)

[19] Némethi, A. and Sigurdsson, B.: The geometric genus of hypersurface singularities, *J. Eur. Math. Soc.* 18 (2016), 825–851.

[20] Ozsváth, P., Szabó, Z. and Stipsicz, A.: Knot lattice homology in L-spaces, *Journal of Knot Theory and Its Ramifications*, Vol. 25, No. 01, 1650003 (2016).

[21] Zemke, I.: The equivalence of lattice and Heegaard Floer homology, arXiv:2111.14962. Springer, Cham, 2020.

Course 2

Singularities of maps and regular homotopy

Roberto Giménez Conejero C

Mid Sweden University

This course will be self contained, but a recommended reference for singularities of maps is [MNB20, Chapter 3]. I also invite attendees to read about regular homotopy. I will prioritise a working knowledge rather than a detailed explanation, but I will give precise references. The tentative schedule is as follows.

Day 1: Origin, algebra of maps

Understanding key concepts of map germs and the state of the art of the subject. There will be examples.

Day 2: Topology of maps

I will cover some recent results regarding how to work with the topology of the singularities of map germs, including a work in progress, with examples!

Day 3: Regular homotopy

I will introduce regular homotopy, Smale-Hirsch theorem, applications and (possibly) a work in progress that is purely topological. If there is time I may show several approaches to this topic.

Day 4: Everything at once

I will explain how everything comes together to study certain Milnor fibers of non-isolated surface singularities, and more things that relate all subjects.

References

[MNB20] D. Mond and J. J. Nuño-Ballesteros. *Singularities of mappings*, volume 357 of *Grundlehren der mathematischen Wissenschaften*. Springer, Cham, 2020.

Talks

Main talk ⟶ T
Lightning talk ⟶ LT
Poster ⟶ P

Monday 25th

3-braid knots with maximal 4-genus

Paula Truöl T

Max Planck Institute for Mathematics

This talk deals with the problem of determining the topological 4-genus for the special case of 3-braid knots. The 4-genus of a knot is the minimal genus of a "nicely" embedded surface in the 4-dimensional ball with boundary the given knot. Asking whether a knot has 4-genus zero, i.e. whether it bounds a disk in the 4-ball, is a natural generalization in dimension 4 of the question whether it is isotopic to the trivial knot. It is one of the curiosities of low-dimensional topology that constructions such as finding these disks can sometimes be done in the topological category, but fail to work smoothly. The first examples of this phenomenon followed Freedman's famous work on 4-manifolds.

Four decades later, the topological 4-genus of knots, even torus knots, remains difficult to determine. In a joint work with S. Baader, L. Lewark and F. Misev, we classify 3-braid knots whose topological 4-genus is maximal (i.e. equal to their 3-dimensional Seifert genus). In the talk, we will define the relevant terms and provide some context for our results.

Tuesday 26th

Contact loci in singularity theory

Javier de la Bodega T

BCAM-KU Leuven

Since Nash introduced them in 1968, *arc spaces* have proven to be rich objects when studying singularities of algebraic varieties. At that time, the *Nash problem* kept the attention of many mathematicians until it was finally solved by Fernández de Bobadilla and Pe Pereira. Arc spaces returned to the frontline of singularity theory in the 90's with the foundation of *motivic integration* by Denef and Loeser, for they served as the building blocks of the theory.

Nonetheless, the possibilities of arcs do not end there, and they can also be used to study singularities of pairs (X, D). In this new setting, the objects we have to look at are *contact loci*, i.e. subsets of arcs of X with a prescribed intersection multiplicity with D. These subsets are essential to introduce the motivic zeta function, hence their importance in the monodromy conjecture. Moreover, classical singularity invariants such as the log canonical threshold or the minimal log discrepancy can also be expressed in terms of contact loci. Strikingly, in the last few years, a potential connection between contact loci and the symplectic properties of the Milnor fibration has been found.

In this talk, I will review what is known about contact loci and their connection to singularity theory. In particular, I will address the first difficulty that arises when studying the geometry of contact loci, i.e. that they are highly non-irreducible. Determining its irreducible components geometrically is known as the *embedded Nash problem*, for its analogy to the classical Nash problem that pushed the development of arc spaces more than fifty years ago.

Wednesday 27th

Strongly overtwisted contact 3-manifolds

Eduardo Fernández T

Univ. of Georgia

Overtwisted contact structures in 3 dimensions were introduced by Y. Eliashberg in his seminal 1989 paper. One of their key properties is that two overtwisted contact structures are homotopic among contact structures if and only if they are homotopic among plane fields. However, the same is not true for the classification problem of families of overtwisted contact structures, up to homotopy: T. Vogel (2018) exhibited a non-contractible loop of overtwisted contact structures on the 3-sphere that is contractible as a loop of plane fields. In this talk, I will introduce a new subclass of overtwisted contact structures in dimension 3, called strongly overtwisted, for which the classification problem for families can indeed be reduced to the classification problem for families of plane fields.

Thursday 28th

Introduction to the lattice cohomology of curve singularities

Alexander Kubasch T

Rényi Institute of Mathematics

The lattice cohomology of isolated curve singularities was introduced by T. Ágoston and A. Némethi in 2023. It is an embedded topological invariant of plane curves and analytic in higher codimensions. Similarly to the topological lattice cohomology of surfaces which can be thought of as an analytic version of Heegard Floer homology, the lattice cohomology of plane curves is closely related to Heegard Floer Link homology.

In this introductory talk, I will define the lattice cohomology of a curve singularity, show numerous examples, and compare it to various other classical invariants, such as the delta invariant, the Seifert form, or the multivariate Poincaré series. I will also outilne how it detects both the Gorenstein property and the multiplicity via the notion of local minima. Joint work with A. Némethi and G. Schefler.

Lattice cohomology of curve singularities and beyond

Gergő Scheffer T

Rényi Institute of Mathematics

The analytic lattice cohomology of curve singularities has strong connection to other lattice cohomology theories and even Heegaard Floer Link theory. We will present how the weight function corresponding to irreducible plane curves can be computed, through a generalized Laufer sequence of universal cycles, from the weight function of the topological lattice cohomology corresponding to its minimal embedded resolution graph. This observation connects the embedded topology with the abstract analytic setup and also allows us to provide a new characterization of the Apéry set of the abstract semigroup of values in more geometric terms. These results are joint with A. Kubasch and A. Némethi.

The lattice cohomology of plane curve singularities is defined via valuations of the normalization. However, if the singularity is Newton nondegenerate, it is natural to use another set of valuations determined from the combinatorics of the Newton boundary. This provides a lattice cohomology with the same Euler characteristic, but with (usually) different weight functions. Our result with A. Némethi shows however, that the two lattice cohomologies agree. The methods allow us to extend the definition of the lattice cohomology to a more general algebraic setup, to certain ideals cut out by valuations having some special properties.

Friday 29th

Convex surface theory for Legendrian classification

Viktória Földvári T

Rényi Institute of Mathematics/Eötvös Loránd Univ.

Legendrian knots in contact 3-manifolds form a richer family than classical knots, due to the fact that there exist several distinct Legendrian realizations of the same topological knot type. Despite significant progress over the last few decades, the complete classification of Legendrian knots remains a distant goal.

A central question is how many distinct Legendrian realizations exist for a given knot type with prescribed classical invariants. If there is only one, we call the knot type Legendrian simple. Since the early 2000s, it has been known that Legendrian non-simple knot types exist. In 2019, using knot Floer homology and the contact invariant, we established lower bounds on the number of distinct realizations, identifying new examples of Legendrian non-simple knot types.

In this talk, I will present an approach for establishing upper bounds as well. I will introduce the necessary concepts from convex surface theory, a powerful tool in contact topology, and explain how we can use these techniques to classify Legendrian knots with respect to Legendrian simplicity. As an application, I will present a joint result with Vera Vértesi, which provides an upper bound on the number of distinct Legendrian realizations of certain double twist knots with maximal Thurston-Bennequin invariant.

Lightning talks & Posters

A τ/μ-type inequality for frontal map germs

Christian Muñoz-Cabello

Universitat de València

LT + P

A holomorphic map germ $f\colon (\mathbb{C}^n, S) \to (\mathbb{C}^{n+1}, 0)$ is frontal if it admits a smoothly-varying field of tangent hyperplanes to the image of f.

The image of a stable deformation of such a map germ is homotopically equivalent to a bouquet of spheres, and we can define an analogue to Mond's image Milnor number, as well as stating a Mond-type conjecture for frontal map germs.

In this joint work with J.J. Nuño-Ballesteros and R. Oset Sinha, we present formulas to count the number of spheres and show conditions under which the Mond conjecture holds for frontals.

From Differential Values to Roots of the Bernstein-Sato Polynomial

David Senovilla-Sanz

University of Cantabria

LT + P

Let C be a plane branch in $(\mathbb{C}^2, \mathbf{0})$ with one Puiseux pair. The goal of this presentation is to show a relationship between two analytic invariants of C: the semimodule of differential values of C and its Bernstein-Sato polynomial. In particular, we show how we can detect roots of the Bernstein-Sato polynomial by means of the semimodule of differential values.

Good real pictures

Ignacio Breva Ribes

Universitat de València

LT + P

It is always difficult to construct graphic representations of complex objects. In fact, the two dimensions of the paper often restrict us to just drawing their real representatives. When is the homology of the complex image of a mapping completely represented by the homology of its real representative? We study this problem for stabilizations of map-germs, giving a necessary condition in terms of the multiple-point spaces. This is ongoing work in collaboration with Roberto Giménez Conejero.

List of Participants

PARTICIPANT	INSTITUTION
Pablo Portilla	Universidad Politecnica de Madrid
Juan Viu Sos	Universidad Politécnica de Madrid
Eduardo de Lorenzo Poza	KU Leuven-BCAM
Benjamin Bode	Universidad Politécnica de Madrid
Eduardo Fernández	University of Georgia
Javier de la Bodega	Alfréd Rényi Institute of Mathematics
Alexander Kubasch	Eötvös Loránd University / Alfréd Rényi Institute of Mathematics
Jorge Martin-Morales	University of Zaragoza
Tamás László	Babes-Bolyai University, Cluj
Baldur Sigurðsson	UPM
Gergő Schefler	ELTE, Rényi Institute
Paula Truöl	Max Planck Institute for Mathematics in Bonn
Hanine AWADA	BCAM
György Tőtős	Babeș-Bolyai University
Wim Veys	KU Leuven
Ignacio Breva Ribes	Universitat de València
Maria Pe Pereira	Universidad Complutense de Madrid
Marko Čmrlec	KU Leuven, BCAM
David Senovilla Sanz	Universidad de Cantabria
Pedro Daniel González Pérez	Universidad Complutense de Madrid
Elvira Pérez Callejo	Universitat Jaume I & Universidad de Valladolid
Christian Muñoz Cabello	Universitat de València
Mark Spivakovsky	Intistut des Mathematiques de Toulouse and CNRS
Maria del Rosario Gonzalez Dorrego	Universidad Autónoma de Madrid
Eva Elduque	Universidad Autónoma de Madrid
Zsolt Baja	BBU
Santiago Pareja Pérez	Unizar+UCM
Juan Carlos Castro Rivera	Universidad de Zaragoza
Ananyo Dan	CUNEF Universidad, Spain
Patricio Almirón	Universidad de Zaragoza
Edwin León Cardenal	Universidad de la Rioja
Bruno VALVERDE MORALES	Universidad Complutense de Madrid
Roger Gómez López	Universitat Politècnica de Catalunya
Alberto López Moral	Universidad Complutense de Madrid
Wei Zhou	UCM - ICMAT
Roberto Giménez Conejero	Mid Sweden University

Viktória Földvári	Rényi Institute of Mathematics - Eötvös Loránd Univ.

Useful Information

Wi-Fi & Conference room

Talks will take place at **Aula 19 (1st floor)** of the Retiro Building of the ETSI Caminos, Canales y Puertos (Univ. Politécnica de Madrid).

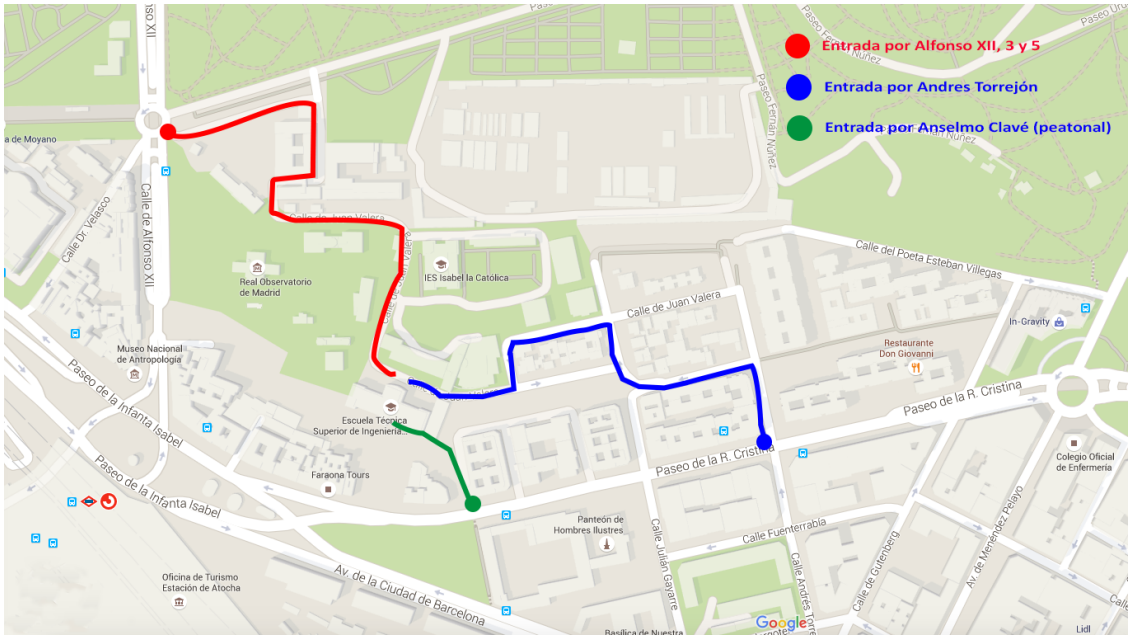
Wi-Fi will be available during the conference:

- **Network name:** InvitadosUPM
- **Username:** WIFI-Iberosing24
- **Password:** MilnorFiber68

Additional workspace

Participants have access to an additional workspace (**Aula 17-Bolonia**) next to the conference room. There are several blackboards, tables & multiple power outlets available for charging electric devices. You may use it during the Discussion Time session.

How to get to the conference room



The Retiro building of the ETSI Caminos, Canales y Puertos has two entrances:

Main entrance

Back entrance

Main entrance (green path)

The easiest way to enter the building is to exit Atocha metro station or Atocha train station and then to take the stairs from Paseo de la Coruña which brings you to the main entrance of the building.

Secondary entrance (red & blue paths)

An alternative way to enter the building is using a back door placed nearby Retiro's Park entrance. You can use that one if you plan to go to the City Center.

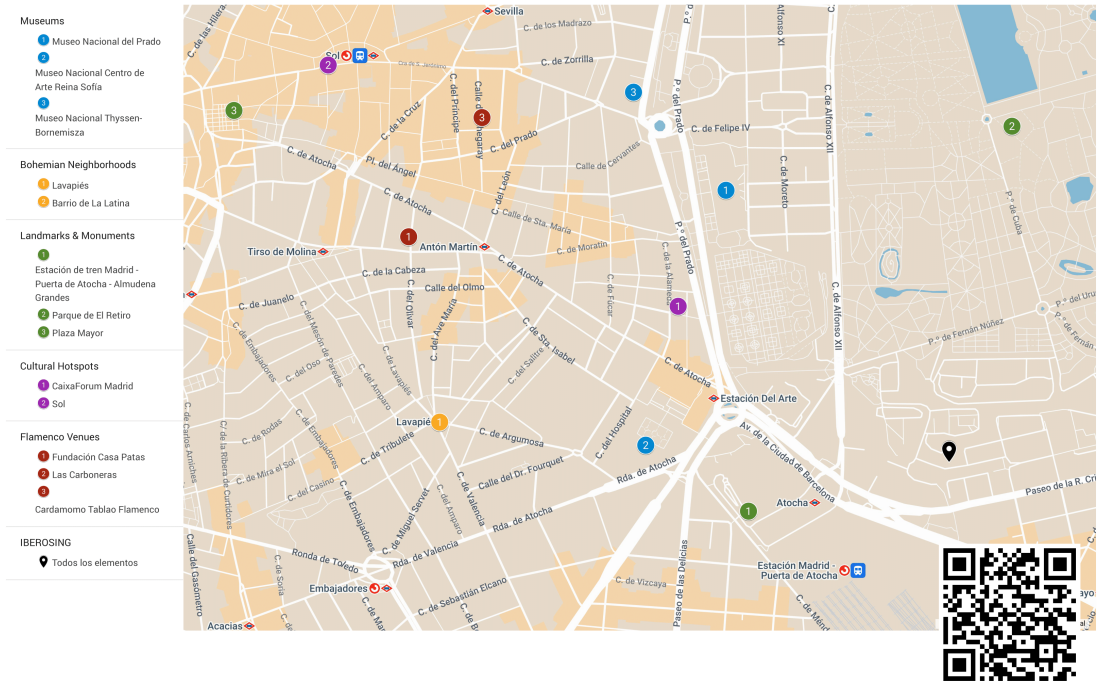
Conference cocktail

A **conference cocktail** with vegetarian options will be held at the main hall of the conference building at 20:00. **Price is 20€ by person. Participants should pay at the moment of their registration.**

What's around?

Atocha, located in the heart of Madrid, is a vibrant area rich in culture and history. From its iconic train station to world-famous museums, lively neighborhoods, and historic landmarks, the area offers something for every visitor. Below is an overview of some must-visit spots near Atocha.

IberoSing24_Maps



Museums

1. **Museo Nacional del Prado:** The *Museo del Prado* is one of the most renowned art museums in the world, housing masterpieces by artists such as Velázquez, Goya, and El Greco. Located just a short walk from Atocha, it's a must-see for art lovers.

2. **Museo Reina Sofía:** The *Reina Sofía Museum* is another cultural gem near Atocha, focusing on contemporary and modern art. It features iconic works, including Picasso's *Guernica*.

3. **Thyssen-Bornemisza Museum:** Part of Madrid's "Golden Triangle of Art," the Thyssen Museum offers a diverse collection ranging from Renaissance to modern art. It is also within

easy reach from Atocha.

Bohemian neighborhoods

1. **Lavapiés:** The *Lavapiés* neighborhood, located near Atocha, is one of the most multicultural and eclectic areas in Madrid. Known for its vibrant street art, diverse cuisine, and lively atmosphere, Lavapiés is a great place to explore local culture and discover unique shops, bars, and theaters.

2. **La Latina:** Just a bit farther from Atocha, *La Latina* is famous for its charming narrow streets, historic buildings, and a lively tapas scene. It's also home to the famous *El Rastro* flea market, which takes place every Sunday, attracting locals and visitors alike.

Landmarks and Monuments

1. **Atocha Train Station:** The *Estación de Atocha* is not just a transportation hub but also an architectural landmark. Inside, you can find a beautiful tropical garden and a memorial dedicated to the victims of the 2004 Madrid train bombings.

2. **El Retiro Park:** Just a short stroll from Atocha lies the *Parque del Buen Retiro*, a large and historic park where visitors can relax, rent boats on the lake, or explore monuments such as the Crystal Palace.

3. **Plaza Mayor:** A bit farther but still walkable, the *Plaza Mayor* is a historic square surrounded by stunning architecture and bustling with restaurants, shops, and street performers.

Cultural Hotspots

1. **CaixaForum Madrid:** This cultural center near Atocha offers temporary art exhibitions, a vertical garden on its facade, and a variety of events and activities.

2. **Puerta del Sol:** One of Madrid's most iconic locations, *Puerta del Sol*, is a vibrant plaza that marks the city's geographic and symbolic center. It's a great spot for shopping and people-watching.

Flamenco Venues

1. **Casa Patas:** Located in the nearby Lavapiés neighborhood, *Casa Patas* is one of Madrid's most iconic flamenco venues. It offers an intimate setting with performances by some of the

best flamenco artists in the world, along with traditional Spanish cuisine.

2. **Tablao Flamenco Las Carboneras:** Situated near Plaza Mayor, this venue blends traditional flamenco with a modern twist. It's known for its passionate performances and cozy ambiance, making it a favorite for visitors and locals alike.

3. **Cardamomo Tablao Flamenco:** A short distance from Atocha, *Cardamomo* is renowned for its vibrant and energetic shows. It's a great spot to experience authentic flamenco in a lively atmosphere.


IberoSing