IberoSing International Workshop 2024

Low-dimensional Topology & Singularity Theory

25 - 29 NOVEMBER

MADRID (SPAIN)

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POLITÉCNICA

UNIVERSIDAD POLITÉCNICA DE MADRID

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MINISTERIO DE CIENCIA E INNOVACIÓN

AGENCIA ESTATAL DE INVESTIGACIÓN

Red de Geometría Algebraica y Singularidades

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About

IBEROSING

This is the fourth edition of regular conferences organized by the main staff of the Iberoamerican Webminar of Young Researchers in Singularity Theory and related topics. The event will be held at Ed. Retiro, ETSI Caminos, Canales y Puertos (Universidad Politécnica de Madrid) from 25 to 29 of November 2024.

It aims to be an international meeting place for both young and senior researchers in Singularity theory, where some of the recent topics in the theory are addressed in detail through courses and various specialized talks in a highly stimulating environment. This is a face-to-face event.

This year event will be focusing on the interactions between **Low-dimensional topology** and **Singularity Theory** such as the theory of contact loci or lattice cohomology.

The event consists of two courses, a series of invited talks and a selection of 5mins lightning-talks followed by poster sessions. The idea of the lightning-talks is to give the opportunity to some participants to briefly present their posters to the entire audience before the poster session.

Organizing committee

Pablo Portilla-Cuadrado	Univ. Politécnica de Madrid
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Laura Starkston	UC Davis

Timetable

	Monday 25 Nov	Tuesday 26 Nov	Wednesday 27 Nov	Thursday 28 Nov	Friday 29 Nov
9-10	Registration & Opening 9:30-10:30 **Paula Truöl**	15:30-17:00 **LC** course	9:30-10:30 **Eduardo Fernández**	9:30-11:00 **SMH** course	9:30-10:30 **Viktória Földvári**
10-11	**5mins ϟ-talks**		**5mins ϟ-talks**		**5mins ϟ-talks**
11-12	Coffee break / Posters I	Coffee break / Posters III	Coffee break / Posters II	Coffee break	Coffee break / Posters III
12-13	11:30-13:00 **SMH** course	11:30-13:00 **SMH** course	11:30-13:00 **LC** course	11:30-12:30 **Alexander Kubasch** 12:30-13:30 **Gergő Schefler**	11:30-13:00 **SMH** course
13-14	**LUNCH TIME**	**LUNCH TIME**	**LUNCH TIME**		Closing
14-15	**LUNCH TIME**	**LUNCH TIME**	**LUNCH TIME**	**LUNCH TIME**	**LUNCH TIME**
15-16	15:30-17:00 **LC** course	15:30-16:30 **Javier de la Bodega I**		15:30-17:30 **Discussion time**	
16-17		Break			
17-18		17:00-18:00 **Javier de la Bodega II**			

21:00 **Conference dinner**

Courses

The event offers two specialization mini-courses:

1. **Lattice cohomology: then and now** by Tamás László (Babeş-Bolyai University).

2. **Singularities of maps and regular homotopy** by Roberto Giménez Conejero (Mid Sweden University).

Lattice cohomology: then and now

Tamás László C

Babeş-Bolyai University (Cluj-Napoca, Romania)

The topic of this lecture series is the lattice cohomology. During these three lectures we will discuss the concepts from two different points of view.

First of all, we start from the beginnings by motivating the development of the theory as part of the so-called Artin-Laufer-Némethi program. We define the first version which nowadays is called the topological lattice cohomology of normal surface singularities.

Then we look at it from a new perspective by defining the combinatorial lattice cohomology, and we also specify it to the analytic theory of surface singularities.

Returning to the topological theory, we present the reduction theorem and a couple of its applications which, on one hand, made the theory available at the computational level, on the other hand, placed it in the context of the ALN program. At the end of the course, we formulate a couple of problems and an idea for an “embedded" version that directs the course towards to the talks of Kubasch and Schefler.

Prerequisites: some basics on the resolution and topology of normal surface singularities, sheaf theory, algebraic topology.

References

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Singularities of maps and regular homotopy

Roberto Giménez Conejero C

Mid Sweden University

This course will be self contained, but a recommended reference for singularities of maps is [MNB20, Chapter 3]. I also invite attendees to read about regular homotopy. I will prioritise a working knowledge rather than a detailed explanation, but I will give precise references. The tentative schedule is as follows.

Day 1: Origin, algebra of maps

Understanding key concepts of map germs and the state of the art of the subject. There will be examples.

Day 2: Topology of maps

I will cover some recent results regarding how to work with the topology of the singularities of map germs, including a work in progress, with examples!

Day 3: Regular homotopy

I will introduce regular homotopy, Smale-Hirsch theorem, applications and (possibly) a work in progress that is purely topological. If there is time I may show several approaches to this topic.

Day 4: Everything at once

I will explain how everything comes together to study certain Milnor fibers of non-isolated surface singularities, and more things that relate all subjects.

References

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List of Abstracts

Main talk	⟶	T
Lightning talk	⟶	LT
Poster	⟶	P

Monday 25th

3-braid knots with maximal 4-genus

Paula Truöl T

Max Planck Institute for Mathematics

This talk deals with the problem of determining the topological 4-genus for the special case of 3-braid knots. The 4-genus of a knot is the minimal genus of a "nicely" embedded surface in the 4-dimensional ball with boundary the given knot. Asking whether a knot has 4-genus zero, i.e. whether it bounds a disk in the 4-ball, is a natural generalization in dimension 4 of the question whether it is isotopic to the trivial knot. It is one of the curiosities of low-dimensional topology that constructions such as finding these disks can sometimes be done in the topological category, but fail to work smoothly. The first examples of this phenomenon followed Freedman's famous work on 4-manifolds.

Four decades later, the topological 4-genus of knots, even torus knots, remains difficult to determine. In a joint work with S. Baader, L. Lewark and F. Misev, we classify 3-braid knots whose topological 4-genus is maximal (i.e. equal to their 3-dimensional Seifert genus). In the talk, we will define the relevant terms and provide some context for our results.

Tuesday 26th

Contact loci in singularity theory

Javier de la Bodega

T

BCAM-KU Leuven

Since Nash introduced them in 1968, *arc spaces* have proven to be rich objects when studying singularities of algebraic varieties. At that time, the *Nash problem* kept the attention of many mathematicians until it was finally solved by Fernández de Bobadilla and Pe Pereira. Arc spaces returned to the frontline of singularity theory in the 90's with the foundation of *motivic integration* by Denef and Loeser, for they served as the building blocks of the theory.

Nonetheless, the possibilities of arcs do not end there, and they can also be used to study singularities of pairs (X, D). In this new setting, the objects we have to look at are *contact loci*, i.e. subsets of arcs of X with a prescribed intersection multiplicity with D. These subsets are essential to introduce the motivic zeta function, hence their importance in the monodromy conjecture. Moreover, classical singularity invariants such as the log canonical threshold or the minimal log discrepancy can also be expressed in terms of contact loci. Strikingly, in the last few years, a potential connection between contact loci and the symplectic properties of the Milnor fibration has been found.

In this talk, I will review what is known about contact loci and their connection to singularity theory. In particular, I will address the first difficulty that arises when studying the geometry of contact loci, i.e. that they are highly non-irreducible. Determining its irreducible components geometrically is known as the *embedded Nash problem*, for its analogy to the classical Nash problem that pushed the development of arc spaces more than fifty years ago.

Wednesday 27th

Strongly overtwisted contact 3-manifolds

Eduardo Fernández T

Univ. of Georgia

Overtwisted contact structures in 3 dimensions were introduced by Y. Eliashberg in his seminal 1989 paper. One of their key properties is that two overtwisted contact structures are homotopic among contact structures if and only if they are homotopic among plane fields. However, the same is not true for the classification problem of families of overtwisted contact structures, up to homotopy: T. Vogel (2018) exhibited a non-contractible loop of overtwisted contact structures on the 3-sphere that is contractible as a loop of plane fields. In this talk, I will introduce a new subclass of overtwisted contact structures in dimension 3, called strongly overtwisted, for which the classification problem for families can indeed be reduced to the classification problem for families of plane fields.

Thursday 28th

Introduction to the lattice cohomology of curve singularities

Alexander Kubasch T

Rényi Institute of Mathematics

The lattice cohomology of isolated curve singularities was introduced by T. Ágoston and A. Némethi in 2023. It is an embedded topological invariant of plane curves and analytic in higher codimensions. Similarly to the topological lattice cohomology of surfaces which can be thought of as an analytic version of Heegard Floer homology, the lattice cohomology of plane curves is closely related to Heegard Floer Link homology.

In this introductory talk, I will define the lattice cohomology of a curve singularity, show numerous examples, and compare it to various other classical invariants, such as the delta invariant, the Seifert form, or the multivariate Poincaré series. I will also outilne how it detects both the Gorenstein property and the multiplicity via the notion of local minima. Joint work with A. Némethi and G. Schefler.

Lattice cohomology of curve singularities and beyond

Gergő Scheffer T

Rényi Institute of Mathematics

The analytic lattice cohomology of curve singularities has strong connection to other lattice cohomology theories and even Heegaard Floer Link theory. We will present how the weight function corresponding to irreducible plane curves can be computed, through a generalized Laufer sequence of universal cycles, from the weight function of the topological lattice cohomology corresponding to its minimal embedded resolution graph. This observation connects the embedded topology with the abstract analytic setup and also allows us to provide a new characterization of the Apéry set of the abstract semigroup of values in more geometric terms. These results are joint with A. Kubasch and A. Némethi.

The lattice cohomology of plane curve singularities is defined via valuations of the normalization. However, if the singularity is Newton nondegenerate, it is natural to use another set of valuations determined from the combinatorics of the Newton boundary. This provides a lattice cohomology with the same Euler characteristic, but with (usually) different weight functions. Our result with A. Némethi shows however, that the two lattice cohomologies agree. The methods allow us to extend the definition of the lattice cohomology to a more general algebraic setup, to certain ideals cut out by valuations having some special properties.

Friday 29th

Convex surface theory for Legendrian classification

Viktória Földvári

T

Eötvös Loránd Univ.

Legendrian knots in contact 3-manifolds form a richer family than classical knots, due to the fact that there exist several distinct Legendrian realizations of the same topological knot type. Despite significant progress over the last few decades, the complete classification of Legendrian knots remains a distant goal.

A central question is how many distinct Legendrian realizations exist for a given knot type with prescribed classical invariants. If there is only one, we call the knot type Legendrian simple. Since the early 2000s, it has been known that Legendrian non-simple knot types exist. In 2019, using knot Floer homology and the contact invariant, we established lower bounds on the number of distinct realizations, identifying new examples of Legendrian non-simple knot types.

In this talk, I will present an approach for establishing upper bounds as well. I will introduce the necessary concepts from convex surface theory, a powerful tool in contact topology, and explain how we can use these techniques to classify Legendrian knots with respect to Legendrian simplicity. As an application, I will present a joint result with Vera Vértesi, which provides an upper bound on the number of distinct Legendrian realizations of certain double twist knots with maximal Thurston-Bennequin invariant.