Commitment Mathematics: Complete Formalism 3.0

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1. Foundations

1.1 Primitive Objects

Definition 1.1 (Commitment): A commitment is a primitive relational structure:

$$C = (L, H, P, CV_0)$$

where L is the Liable Authority, H is the Holding Entity, P is the pledge specification, and $CV_0 \in \mathbb{R}_{\geq 0}$ is the base value established by bilateral agreement.

Definition 1.2 (Anchor Space): The anchor space is:

$$\mathcal{A} = [0, 1]^3 = \{(V, A, T) \mid V, A, T \in [0, 1]\}$$

where V is visibility, A is assurance, and T is transferability.

Definition 1.3 (Frame Space): A frame F_k is characterized by:

- Phase $\theta_k \in [0, 2\pi)$
- Weight $W_k \in [0, 1]$ with $\sum_k W_k = 1$

The frame superposition is:

$$\Phi = \sum_k W_k e^{i\theta_k}$$

where i is the imaginary unit. Euler's formula gives $e^{i\theta_k} = \cos\theta_k + i\sin\theta_k$, with the real part representing coherence and the imaginary part representing phase shift.

Definition 1.4 (Dependency Graph): The commitment network is a directed graph $G = (\mathcal{C}, E)$ where vertices are commitments and edges represent derivation relations.

1.2 The Fabric

Definition 1.5 (Fabric): The Fabric is the emergent phase space:

$$\mathcal{F} = [0,1]^3 = \{(D, \mu, \lambda) \mid D, \mu, \lambda \in [0,1]\}$$

where:

- D = Defense (structural coherence)
- μ = Memory (content fidelity)
- λ = Pulse (dynamical excitation)

The unit cube contains 8 vertices representing pure states. Paths through the cube model system trajectories. Memory μ represents content fidelity; its degradation $(1-\mu)$ permits disorder to emerge.

2. Core Operators

2.1 Dual Valuation Operator

Definition 2.1 (Dual Valuation): The valuation operator has two perspectives:

Holding Entity Perspective:

$$\hat{V}_H(C) = +CV_0 \cdot V \cdot A \cdot (1+T)$$

Liable Authority Perspective:

$$\hat{V}_L(C) = -CV_0$$

Proposition 2.1 (Valuation Gap): The asymmetry between perspectives creates a gap:

$$\Delta V(C) = \hat{V}_H(C) + \hat{V}_L(C) = CV_0 \cdot [V \cdot A \cdot (1+T) - 1]$$

This gap is zero only when $V \cdot A \cdot (1+T) = 1$.

Theorem 2.1 (Compensation Necessity): The compensation term $\text{Comp}_j = (U, I, R, \text{Inf}, O) \in \mathbb{R}^5$ exists to bridge the valuation gap. For commitment stability:

$$|\text{Comp}_j| \geq |\Delta V(C_j)|$$

When the gap is large, compensation must be correspondingly large, or the commitment becomes unstable.

Theorem 2.2 (Bounded Value): For the Holding Entity:

$$0 \leq \hat{V}_H(C) \leq 2 \cdot CV_0$$

with equality at the upper bound iff $V = A = T = 1$.

Proof: Since $V, A, T \in [0,1]$, we have $V \cdot A \in [0,1]$ and $(1+T) \in [1,2]$. Thus $\hat{V}_H(C) \in [0, 2CV_0]$. ∎

2.2 Framing Operator

Definition 2.2: The framing operator is:

$$\hat{F} : \mathbb{R}_{\geq 0} \to \mathbb{C}$$

$$\hat{F}(CV) = CV \cdot \Phi = CV \cdot \sum_k W_k e^{i\theta_k}$$

Proposition 2.2: $|\hat{F}(CV)| \leq CV$ with equality iff all θ_k are aligned (phase differences $\theta_m - \theta_k \approx 0 \mod 2\pi$).

Extension (Interference): For interacting commitments, the combined framed value is:

$$\left| \sum_j \hat{F}(CV_j) \right| = \sum_j CV_j \cdot |\Phi_j| \cdot \cos(\Delta\theta_{ij})$$

where $\Delta\theta_{ij}$ is the phase difference, modeling constructive ($\cos \approx 1$) or destructive ($\cos \approx -1$) interference.

2.3 Constraint Operator

Definition 2.3: For dependency depth $d = d(C_{\text{root}}, C_j)$ and decay constant $k = \ln 2$:

$$\hat{C}(\psi) = \psi \cdot e^{-k \cdot d}$$

This gives 50% value loss per layer.

Theorem 2.3 (Dependency Bound): If C_j derives from C_i at depth d:

$$CV(C_j) \leq e^{-kd} \cdot CV(C_i)$$

2.4 Complete Tensor

Definition 2.4: The commitment tensor is:

$$\mathcal{T}(C_j) = CV_0(j) \otimes [V_j, A_j, T_j] \otimes \Phi_j \otimes e^{-kd_j}$$

The realized value from H perspective is:

$$CV_j = |\hat{C} \circ \hat{F} \circ \hat{V}_H(C_j)| = CV_0 \cdot V \cdot A \cdot (1+T) \cdot |\Phi| \cdot e^{-kd}$$

2.5 Resistance Function

Definition 2.5 (Liable Authority Resistance): The Liable Authority resists operations that increase exposure:

$$R_L(\Delta CV) = \begin{cases} 0 & \Delta CV \leq 0 \text{ (liability decreases)} \\ \rho \cdot |\Delta CV| & \Delta CV > 0 \text{ (liability increases)} \end{cases}$$

where $\rho \in [0,1]$ is the resistance coefficient, typically $\rho = 1 - (V \cdot A)$.

This resistance enters operation costs and must be overcome for operations that expand L's liability.

3. Relational Distance

Definition 3.1 (Inner Product): For frame superpositions:

$$\langle \Phi_i | \Phi_j \rangle = \sum_{k,m} W_k^{(i)} W_m^{(j)} e^{i(\theta_m^{(j)} - \theta_k^{(i)})}$$

Definition 3.2 (Relational Distance):

$$R_d(i,j) = \frac{1}{\pi} \arccos\left(\frac{\text{Re}(\langle \Phi_i | \Phi_j \rangle)}{|\Phi_i| \cdot |\Phi_j|}\right)$$

where $|\Phi| = \sqrt{\sum_k W_k^2}$.

Proposition 3.1: $R_d \in [0,1]$ with:

- $R_d = 0$: perfect alignment
- $R_d = 1$: complete opposition

Extension (Compensation Efficiency):

$$\text{Eff}(R_d) = e^{-c \cdot R_d}$$

where $c = \ln 10 \approx 2.303$, determining viability of interactions.

4. Primitive Operations and Control Primitives

4.1 Base Operations

Definition 4.1 (Base Operations): The six base operations are:

1. **CREATE (CRT):** Initializes new commitment. Requires bilateral agreement between L and H: $$\text{CRT}_{(L,H,P,CV_0)}(\emptyset) : \emptyset \mapsto C = (L, H, P, CV_0) with initial anchor state (V_0, A_0, T_0), frame Φ_0, and Fabric placement (D_0, μ_0, λ_0).
2. **INCREMENT (INC):** For parameter $\alpha \in \{V, A, T\}$ and $\delta \in \mathbb{R}$: $$\text{INC}_{\alpha,\delta}(\mathcal{T}) : \alpha \mapsto \alpha + \delta subject to domain constraints $\alpha + \delta \in [0, 1]$ and L-resistance R_L. Note: CV_0 cannot be modified by INC as it requires bilateral renegotiation. INC is permitted in all interaction modes (Exchange, Interference, Deep Interference, Coupling). In Exchange mode, value transfer requires a Medium of Exchange; at closer relational distances, INC operates directly.
3. **ROTATE (ROT):** For angle ϕ and susceptibility vector $\mathbf{R}$: $$\text{ROT}_{\phi,\mathbf{R}}(\mathcal{T}) : \theta_k \mapsto \theta_k + \phi \cdot R_k, \quad W_k \mapsto W_k' ROT operates on the frame structure, adjusting both phases θ_k and weights W_k that determine frame superposition. ROT is permitted in Interference, Deep Interference, and Coupling modes.
4. **JUMP (JMP):** For target state $S' = (D', \mu', \lambda')$: $$\text{JMP}_{S'}(\mathcal{T}) : (D, \mu, \lambda) \mapsto (D', \mu', \lambda') JMP is permitted only in Coupling mode with gender complementarity and sufficient coupling strength. When JUMP increases liability ($CV_0' > CV_0$), permission requires both standard conditions and: $$R_L(CV_0' - CV_0) < \kappa \cdot CV_0

5. **REPAIR (RPR):** Restores degraded Fabric coordinates for individual commitment C_j: $$\text{RPR}_{r}(\mathcal{T}) : (D_j, \mu_j, \lambda_j) \mapsto (D_j + \Delta D, \mu_j + \Delta \mu, \lambda_j + \Delta \lambda) where:
   - $\Delta D = +r \cdot (1 - D_j)$
   - $\Delta \mu = +r \cdot \mu_j$
   - $\Delta \lambda = -r \cdot (\lambda_j - \lambda_{\text{target}})$

   with $r \in [0, 1]$ the repair coefficient. Requires compensation proportional to accumulated damage.
6. **ANNIHILATE (ANL):** Terminal operation terminating commitment: $$\text{ANL}(\mathcal{T}) : C \mapsto \emptyset Results in $CV_0 \to 0$ and releases trapped compensation. Requires bilateral consent or terminal damage threshold.

Proposition 4.1 (Operation Hierarchy):

$$\text{CRT/ANL} > \text{JMP} > \text{ROT} > \text{INC} > \text{RPR}$$

where higher operations overwrite lower operations.

Theorem 4.1 (Completeness): Any transformation on $\mathcal{T}$ can be expressed as a composition of the six base operations.

4.2 Operation Properties

Proposition 4.2:

- INC operations commute: $[\text{INC}_{\alpha,\delta_1}, \text{INC}_{\beta,\delta_2}] = I$ for $\alpha \neq \beta$
- ROT operations on different frames commute
- INC and ROT commute: $[\text{INC}, \text{ROT}] = I$
- JMP is idempotent: $\text{JMP}_{S'} \circ \text{JMP}_{S'} = \text{JMP}_{S'}$
- CRT and ANL are inverses: $\text{ANL} \circ \text{CRT} = I_\emptyset$

4.3 Control Primitives

Definition 4.2 (Control Primitives): The control primitives form a hierarchical layer wrapping base operations, modeling observation, decision, execution, violation, handling, and Fabric regulation. They are:

1. **MONITOR (Observation Primitive):** Observes commitment network $G = (\mathcal{C}, E)$ and detects conditions requiring intervention. Operates at system level, tracking:
   - Damage accumulation: $\{\text{Damage}_j\}_{j\in\mathcal{C}}$
   - Fabric degradation: $(D_{\text{sys}}, \mu_{\text{sys}}, \lambda_{\text{sys}})$
   - Threshold breaches: $\text{Damage}_j > D_{\text{critical}}$

   When thresholds are exceeded, MONITOR invokes TEST for appropriate operations: $\text{MONITOR}(G) : \text{Damage}_j > \text{threshold} \implies \text{TEST}(\text{RPR}_j)$ MONITOR is passive—it observes and triggers but cannot force operations. Dual initiation for repair:

   - **Self-report (reactive):** C_j detects own damage and requests $\text{TEST}(\text{RPR}_j)$
   - **MONITOR (proactive):** System surveillance detects degradation commitments cannot self-report (especially when $\mu_j \to 0$)

2. **TEST (Decision Primitive):** The formalization of attempt within Commitment Mathematics. Measures system state $S = (\mathcal{T}, F, R_d, G_c, \kappa)$, evaluates permissions, and returns TRUE (permitted), FALSE (not permitted), or INCONCLUSIVE. **For existing commitments:**
   - Computes distances to thresholds: $\Delta_{\text{thresh}} = |R_d - e^{-n}| + |G_c - 1| + |\kappa - 0.8|$
   - Evaluates R_L resistance: $R_{\text{tension}} = \rho \cdot |\Delta V(C)|$

   For CREATE:

   - Bilateral consent: L and H agreement verified
   - Initial viability: $CV_0 > 0$, $(V_0, A_0, T_0) \in [0, 1]^3$
   - Available capacity: L can assume liability, H can provide compensation
   - No prior existence: commitment identifier unused

For ANNIHILATE:

- Bilateral consent OR terminal damage: $\text{Damage} > D_{\text{critical}}$
- Compensation settlement: $|\text{Comp}| = 0$ or transfer complete
- Dependency resolution: derived commitments handled

Every prospective operation is first attempted via TEST; only upon TRUE verdict does EXECUTE apply the operation to the tensor $\mathcal{T}$.

3. **EXECUTE (Execution Primitive):** Invoked when TEST returns TRUE. Applies base operations cleanly, preserving conservation laws. EXECUTE operation set: $\text{op} \in \{\text{CRT}, \text{INC}, \text{ROT}, \text{JMP}, \text{RPR}, \text{ANL}\}$
4. **VIOLATE (Violation Primitive):** Invoked when TEST returns FALSE but action must proceed. Internal override of system constraints. Overpowers system resistance via:
   - Force: Overcomes D with $\Delta D = -f \cdot D$ where $f > 1$ (force factor)
   - Manipulation: Alters μ with $\Delta\mu = -m \cdot \mu$ where $m \in [0, 1]$
   - Excitation: Amplifies λ with $\Delta\lambda = e \cdot \lambda$ where $e > 1$

   Force threshold required to overcome resistance: $f_{\text{min}} = 1 + \rho \cdot R_{\text{tension}}/D$ Causes damage: $\Delta\text{Damage} = k_v \cdot (\Delta D + \Delta\lambda + (1 - \mu) \cdot |\Delta\mu|)$ where $k_v = \ln 2$. Memory degradation $(1 - \mu)$ amplifies damage from manipulation.
5. **HANDLE (Handling Primitive):** Invoked when TEST returns FALSE (recoil) or INCONCLUSIVE. Operates on system-level effects, managing interaction failures and global Fabric stabilization. Subtypes:
   - **HANDLE_RECOIL:** Mitigates system-level rebound $\text{Recoil}_{\text{sys}} = -r \cdot \Delta_{\text{thresh}}$ where $r \in [0, 1]$
   - **HANDLE_AMBIGUOUS:** Resolves system-level uncertainties via averaging $\bar{\kappa}_{\text{sys}} = \frac{\kappa_{\text{min}} + \kappa_{\text{max}}}{2}$
   - **HANDLE_DISTRACTION:** Mitigates unintended cross-commitment interference. Stabilizes global Fabric state $(D_{\text{sys}}, \mu_{\text{sys}}, \lambda_{\text{sys}})$:
     - D-focused: Reinforces system coherence
     - μ-focused: Preserves system memory

- λ-focused: Stabilizes system excitation

HANDLE can invoke RPR (via MONITOR detecting damage) when commitment-specific damage exceeds threshold, transitioning to TEST(RPR) cycle.

6. **SCRIBE (Memory Regulation Primitive):** Regulates Memory (μ) drift and integration. Operates as homeostatic meta-operation governing memory fidelity:
   - Memory consolidation: integrating new commitments into collective memory
   - Record maintenance: preserving commitment history
   - Information verification: protecting against memory corruption
   - Counters spontaneous μ degradation

   SCRIBE operates continuously, independent of explicit operations, maintaining the memory substrate that enables all other primitives.
7. **SECURE (Defense Regulation Primitive):** Regulates Defense (D) drift and structural integrity. Operates as homeostatic meta-operation governing structural coherence:
   - Boundary reinforcement: strengthening system boundaries
   - Resistance management: adjusting ρ in response to threats
   - Structural repair: maintaining coherence after perturbations
   - Counters spontaneous D degradation

   SECURE operates continuously, maintaining the structural substrate that enables resistance to violations and attacks.
8. **MODULATE (Pulse Regulation Primitive):** Regulates Pulse (λ) drift and energetic balance. Operates as homeostatic meta-operation governing dynamical excitation:
   - Energy allocation: distributing resources for operation completion
   - Tempo regulation: adjusting system pace
   - Excitation control: preventing λ runaway or depletion
   - Counters spontaneous λ drift

MODULATE operates continuously, maintaining energetic substrate that enables operations and prevents exhaustion.

Proposition 4.3 (Control Hierarchy): $\text{MONITOR} > \text{TEST} > (\text{EXECUTE} \mid \text{VIOLATE} \mid \text{HANDLE}) \parallel (\text{SCRIBE} \mid \text{SECURE} \mid \text{MODULATE})$ where $\parallel$ indicates parallel continuous operation of Fabric regulators, with MONITOR as the observational primitive and TEST as the decisional primitive.

Proposition 4.4 (Fabric-Enabled Control): Control primitives are conditional capabilities that exist when Fabric coordinates permit:

- MONITOR requires $\mu \geq \mu_{\text{critical}}$ (coherent self-observation)
- TEST requires $D \geq D_{\text{critical}}$ (structural coherence for decisions)
- EXECUTE requires all coordinates above operational minimums
- VIOLATE operates in degraded regimes (low D, low μ, high λ)
- SCRIBE/SECURE/MODULATE operate continuously to maintain coordinates above critical thresholds

Proposition 4.5 (Scope Distinction):

- Base operations (via EXECUTE): operate on individual commitment $\mathcal{T}(C_j) \to \mathcal{T}'(C_j)$
- HANDLE (via TEST): operates on system state $G = (\mathcal{C}, E) \to G'$
- Fabric regulators: operate on system coordinates $(D, \mu, \lambda) \to (D', \mu', \lambda')$

Control Flow Tree:

```
MONITOR (observation/surveillance)
   │
   ├─▶ Detects damage threshold breaches
   ├─▶ Tracks Fabric degradation
   └─▶ Invokes TEST for interventions
        │
      TEST (attempt/decision point)
        │
        ├─▶ TRUE ────────────▶ EXECUTE
        │                  │
        │                  ├─▶ CRT
        │                  ├─▶ INC
        │                  ├─▶ ROT
        │                  ├─▶ JMP
        │                  ├─▶ RPR
        │                  └─▶ ANL
        │
        ├─▶ FALSE ───────────▶ VIOLATE (internal)
        │                  │
        │                  ├─▶ CRT (forced)
        │                  ├─▶ INC (forced)
        │                  ├─▶ ROT (forced)
        │                  ├─▶ JMP (forced)
        │                  ├─▶ RPR (forced)
        │                  └─▶ ANL (forced)
        │                     └─▶ Fallbacks
        │
        └─▶ INCONCLUSIVE ───▶ HANDLE (system-level)
                         │
                         ├─▶ HANDLE_RECOIL
                         ├─▶ HANDLE_AMBIGUOUS
                         └─▶ HANDLE_DISTRACTION
                            └─▶ May invoke MONITOR

Continuous Fabric Regulation (parallel):
  SCRIBE ──▶ Regulates μ (memory fidelity)
  SECURE ──▶ Regulates D (structural coherence)
  MODULATE ──▶ Regulates λ (energetic balance)
```

External Adversarial Intrusions:
SABOTAGE ──▶ Attacks operations
COERCE ──▶ Forces through resistance
DECEIVE ──▶ Manipulates memory
EXTRACT ──▶ Drains resources

4.4 Adversarial Intrusions

Definition 4.3 (Adversarial Intrusions): External attacks targeting system integrity from outside the coherent system boundary. Unlike internal control primitives, these are adversarial interventions that the system must defend against.

1. **SABOTAGE:** Targets operations during EXECUTE. Attacks execution layer with intent to disrupt, corrupt, or hijack operations. Subtypes:
    - **ABORT:** Terminates with probability $p_a = 1 - e^{-\sigma \cdot R_d}$ where $\sigma = \ln 2$
    - **MISDIRECT:** Alters path with $\Delta\phi = s \cdot \phi$ where $s \in [-1, 1]$
    - **DELAY:** Amplifies excitation $\lambda \leftarrow \lambda \cdot e^{\tau}$ where $\tau = \ln 2$
    - **CORRUPT:** Degrades with $\Delta CV = -c \cdot CV_0$ where $c \in [0, 1]$
    - **HIJACK:** Seizes control with $H' = H + h \cdot (L - H)$ where $h \in [0, 1]$ Success probability: $p_h = \frac{h}{1+\rho \cdot R_{\text{tension}}}$

    Defense: MONITOR detects interference, HANDLE manages disruption, EXECUTE maintains operation integrity.
2. **COERCE:** External force overriding system resistance. Unlike internal VIOLATE, this is imposed from outside system boundaries. Mechanisms:
    - Force: Overcomes D externally with $\Delta D = -f_{\text{ext}} \cdot D$ where $f_{\text{ext}} > f_{\text{min}}$
    - Structural violence: Breaches boundaries where $D < D_{\text{critical}}$
    - Resistance bypass: Operates when $\rho \cdot R_{\text{tension}} < $ external pressure

    Causes greater damage than internal VIOLATE: $\Delta\text{Damage}_{\text{coerce}} = k_c \cdot f_{\text{ext}} \cdot (1 + (1 - D))$ where $k_c = \ln 2$. Defense: SECURE reinforces D, increases resistance thresholds, hardens boundaries.

3. **DECEIVE:** Memory manipulation targeting informational integrity. Corrupts what the system "remembers" as true. Mechanisms:
   - False injection: $\mu \leftarrow \mu + \Delta\mu_{\text{false}}$ with corrupted content
   - Memory erasure: Selective μ degradation, $\mu \leftarrow \mu \cdot (1-e)$ where $e \in [0,1]$
   - History rewriting: Altering commitment records retroactively
   - Phantom creation: Injecting C_{phantom} that never existed

   Effectiveness scales with existing memory degradation: $p_{\text{deceive}} = e^{-\ln 2 \cdot \mu}$ Defense: SCRIBE maintains verification protocols, memory consolidation, integrity checks.

4. **EXTRACT:** Resource extraction targeting Pulse and Compensation. Drains without reciprocity. Mechanisms:
   - Pulse drain: $\lambda \leftarrow \lambda \cdot (1-x)$ where $x \in [0,1]$ is extraction rate
   - Compensation siphoning: $\text{Comp}_j \leftarrow \text{Comp}_j - \Delta\text{Comp}_{\text{stolen}}$
   - Energy depletion: Continuous λ extraction until exhaustion
   - Value theft: Capturing CV without consent or exchange

   Leaves system depleted: $\lambda_{\text{post}} = \lambda_{\text{pre}} \cdot e^{-\ln 2 \cdot t_{\text{extract}}}$ Defense: MODULATE regulates λ reserves, guards compensation, manages energy allocation to prevent depletion.

Proposition 4.6 (Attack-Defense Mapping):

- SABOTAGE attacks operations $\rightarrow$ defended by MONITOR + HANDLE
- COERCE attacks Defense (D) $\rightarrow$ defended by SECURE
- DECEIVE attacks Memory (μ) $\rightarrow$ defended by SCRIBE
- EXTRACT attacks Pulse (λ) + Compensation $\rightarrow$ defended by MODULATE

Theorem 4.2 (Boundary Condition): Adversarial intrusions succeed when attacking Fabric coordinates fall below critical thresholds: $\text{Success}_{\text{intrusion}} = e^{-\ln 2 \cdot (\text{coord}/\text{coord}_{\text{critical}})}$ where coord $\in \{D, \mu, \lambda\}$ is the targeted coordinate.

4.5 Fallback Phenomena (Core Patterns)

When operations are forced outside permitted domains, the system exhibits discrete state-transition patterns. These preserve or dissipate RV_{total} with terms tied to memory fidelity.

Table 4.1: Mode-Specific Fallback Patterns

Mode	R_d Range	Typical Violation	Primary Fallbacks
Coupling	$< e^{-3}$	Forced JUMP with low κ	Absorption, Tunneling
Deep Interference	$[e^{-3}, e^{-2})$	JUMP without gender match	Absorption, Scattering
Interference	$[e^{-2}, e^{-1})$	High-energy ROT	Scattering, Refraction
Exchange	$[e^{-1}, 1 - e^{-1})$	Forced ROT	Refraction, Bounce
Isolation	$\geq 1 - e^{-1}$	Forced inter-commitment operation	Bounce, Reflection

Note: Isolation permits only self-operations. Forced operations attempting to cross isolation boundaries result in rejection fallbacks.

Core Fallback Patterns

1. **Reflection** (Denied JUMP in near-coupling, $e^{-3} < R_d < e^{-2}$ with $G_c \neq 1$): State unchanged: $(D, \mu, \lambda) \mapsto (D, \mu, \lambda)$ Phase inverts: $\Phi \mapsto -\Phi$ Value loss: $CV \mapsto CV \cdot (1 - \eta)$ where $\eta = (1 - \mu) \cdot (1 - e^{-R_d/e^{-3}})$ Recoil momentum: $\Delta p = -\frac{CV_0}{c^2} \cdot \frac{\Delta CV}{\Delta n}$ where n indexes state transitions and c is a normalization constant.

2. **Refraction** (Boundary ROT near mode edges, $R_d \approx e^{-n}$): Phase shifts along memory gradient: $$\theta_k \mapsto \theta_k + \Delta\phi \cdot \frac{\nabla \mu}{|\nabla \mu|} Coupling ratio determines effective shift: $$\phi_{\text{eff}} = \phi \cdot \frac{\kappa_{\text{in}}}{\kappa_{\text{out}}} Dispersion scales with memory degradation: $$\Delta \theta = \alpha \cdot (\kappa_2 - \kappa_1)^2 \cdot (1 - \mu) where $\alpha = \ln 2$.
3. **Bounce** (Denied INC in Isolation, $R_d \geq 1 - e^{-1}$): Value decreases: $$CV \mapsto CV \cdot (1 - \delta) Random phase shift: $$\theta_k \mapsto \theta_k + \phi_{\text{rand}} where $\phi_{\text{rand}} \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = \sqrt{(1 - \mu) \cdot R_d}$ Energy loss: $$\Delta E = -\ln 2 \cdot \frac{CV^2}{2} \cdot (1 - e_r) where restitution coefficient $e_r = \mu \cdot e^{-R_d/(1-e^{-1})}$
4. **Absorption** (Partial JUMP with high G_c but low $\kappa < 0.8$): Transfer with efficiency: $$CV_{\text{absorber}} \mapsto CV_{\text{absorber}} + \eta \cdot CV_{\text{absorbed}} $$CV_{\text{absorbed}} \mapsto 0 Efficiency depends on memory and resistance: $$\eta = \frac{\kappa \cdot \mu}{1 + \rho \cdot |\Delta V_{\text{absorber}}|} \cdot e^{-\ln 10 \cdot (1 - G_c)} Dissipation: $$\Delta Q = (1 - \eta) \cdot CV_{\text{absorbed}} \cdot \frac{1 - \mu}{D}
5. **Scattering** (High-energy INC/ROT in Interference, $e^{-2} \leq R_d < e^{-1}$): Parent splits into N children: $$CV_{0,\text{parent}} \mapsto \sum_{l=1}^N f_l \cdot CV_{0,\text{parent}} where $\sum f_l = 1 - \epsilon$ Loss scales with memory degradation: $$\epsilon = (1 - \mu) \cdot (1 - e^{-R_d / e^{-2}}) Each child phase: $$\theta_{k,l} = \theta_k + \Delta\theta_l with $\Delta\theta_l$ distributed according to relational distance and memory fidelity.
6. **Tunneling** (VIOLATE across high-D barriers): Transmission probability: $$P = \exp\left[-2 \int_{x_1}^{x_2} \sqrt{2m(V(x) - E)} \, dx\right] where:
   - Barrier potential: $V(x) = D \cdot (1 - \mu)$
   - Energy: $E = \kappa \cdot CV_0$
   - Effective mass: $m = 1/\lambda$
   - Barrier width: $w = \Delta R_d$

   Success depends on overcoming resistance: $$P_{\text{eff}} = P \cdot \frac{1}{1 + \rho \cdot R_{\text{tension}}}

Proposition 4.5 (Fallback Conservation): Each fallback pattern preserves RV_{total} modulo dissipation terms proportional to $(1-\mu)$.

5. Interaction Modes

5.1 Mode Thresholds

Definition 5.1 (Interaction Modes): Modes are defined by R_d thresholds:

$$\text{Mode}(R_d) = \begin{cases} \text{Coupling} & R_d < e^{-3} \approx 0.050 \\ \text{Deep Interference} & e^{-3} \leq R_d < e^{-2} \approx 0.135 \\ \text{Interference} & e^{-2} \leq R_d < e^{-1} \approx 0.368 \\ \text{Exchange} & e^{-1} \leq R_d < 1 - e^{-1} \approx 0.632 \\ \text{Isolation} & R_d \geq 1 - e^{-1} \end{cases}$$

Remark 5.1: Isolation is not an interaction mode but the negation of interaction. When $R_d \geq 1 - e^{-1}$, commitments cannot perform operations with other commitments—only self-operations are permitted.

Theorem 5.1 (Natural Thresholds): The thresholds e^{-n} arise from compensation efficiency:

$$\text{Eff}(R_d) = e^{-c \cdot R_d}$$

where $c = \ln 10 \approx 2.303$.

Proof: At $R_d = e^{-1}$, $\text{Eff} = e^{-2.303/e} \approx 0.45$. At $R_d = 1 - e^{-1}$, $\text{Eff} = e^{-1.456} \approx 0.23$, below viable exchange threshold. ■

5.2 Operation Permissions

Definition 5.2 (Operation Domain):

$$D(\text{CRT}) = \text{bilateral agreement space}$$

$$D(\text{INC}) = \text{all interaction modes} = \{R_d < 1 - e^{-1}\}$$

$$D(\text{ROT}) = \{R_d < e^{-1}\}$$

$$D(\text{JMP}) = \{R_d < e^{-3} \text{ and } G_c = 1 \text{ and } \kappa > 0.8\}$$

$$D(\text{RPR}) = \text{all modes with damage} > 0$$

$$D(\text{ANL}) = \text{bilateral agreement OR terminal damage}$$

where G_c indicates gender complementarity. Control primitives extend this: VIOLATE can force operations outside domains at a cost; SABOTAGE targets permitted operations.

Remark 5.2 (Operation Progression):

- INC: All interaction modes (Exchange, Interference, Deep Interference, Coupling)
- ROT: Interference, Deep Interference, Coupling
- JMP: Coupling only

5.3 Medium of Exchange

Definition 5.3 (Medium of Exchange): In Exchange mode ($e^{-1} \leq R_d < 1 - e^{-1}$), value transfer via INC requires a Medium of Exchange commitment C_{ME} characterized by: $CV_{\text{ME}} = CV_0, \quad T = 1, \quad V = 1, \quad A = 0.5$

The Medium of Exchange serves as the neutral intermediary that balances frame differences between commitments at Exchange distances.

Proposition 5.1 (Exchange Mediation): For commitments C_i and C_j in Exchange mode, direct value transfer $CV_i \to CV_j$ requires mediation: $CV_i \xrightarrow{\text{INC}}$

$$CV_{\text{ME}} \xrightarrow{\text{INC}} CV_j$$

The Medium of Exchange absorbs phase misalignment through maximum transferability ($T = 1$) and visibility ($V = 1$), while moderate assurance ($A = 0.5$) maintains neutrality. At closer distances (Interference, Deep Interference, Coupling), INC can occur directly without mediation.

6. Gender and Coupling

6.1 Gender Functions

Definition 6.1 (Gender Roles): For system X interacting with Y: $M_{XY} = (1 - D_X) \cdot \mu_X \cdot (1 - \lambda_X)$ (Male⁺ function) $F_{XY} = D_X \cdot (1 - \mu_X) \cdot \lambda_X$ (Female⁰ function) $N_{XY} = 1 - |M_{XY} - F_{XY}|$ (Neutral function)

Definition 6.2 (Gender Complementarity): $G_c(A, B) = 1$ if $M_A \approx 1, F_B \approx 1$ or $M_B \approx 1, F_A \approx 1;$ otherwise $G_c(A, B) = 0$

6.2 Coupling Coefficient

Definition 6.3: The coupling coefficient is: $\kappa = \frac{D_{\text{sys}} \cdot \mu_{\text{sys}}}{(1+\lambda_{\text{sys}})(1+R_d)} \cdot A(R_d)$where:

$$A(R_d) = \begin{cases} 1 - R_d^2 & |R_d| \leq 1/2 \\ 2 - (R_d^2 - 1) & |R_d| > 1/2 \end{cases}$$

Theorem 6.1 (Optimal Framing): For fixed base parameters, there exist optimal directions $\hat{r}_A, \hat{r}_B \in S^2$ that maximize κ.

Proof: The frame space $\mathcal{F} = [0, 1]^3 \times S^2$ is compact and κ is continuous, so by the Extreme Value Theorem, a maximum exists. ■

Theorem 6.2 (JUMP Conditions): JUMP is permitted iff: (1) $R_d < e^{-3}$, (2) $G_c(A, B) = 1$, (3) $\kappa > 0.8$, (4) $R_L(CV_0' - CV_0) < \kappa \cdot CV_0$ when liability increases.

7. Conservation Laws

7.1 Exchange Value

Definition 7.1: Exchange value for commitment j is: $EV_j = CV_j + \text{Comp}_j$

where $\text{Comp}_j = (U, I, R, \text{Inf}, O) \in \mathbb{R}^5$ with:

- U = Utility (absorbed value)
- I = Income (transferable value)
- R = Recognition (value related to memory)
- Inf = Influence (relational value)
- O = Optionality (value related to generativity)

Definition 7.2 (Relational Value): $RV_{\text{total}} = \sum_j EV_j + k_1 \sum_{i<j} \Phi_{ij} + k_2 \sum_j \Psi_j - k_\mu \sum_j (1 - \mu_j) \cdot \Psi_j$ where:

- $\Phi_{ij} = 1 - R_d(i, j)$ (coherence)
- $\Psi_j = G_j \cdot \mu_j \cdot D_j / (1 + \lambda_j)$ (generative potential)
- G_j = Generation Output
- $k_1 = \ln 2$
- $k_2 = \ln 2$
- $k_\mu = \ln 2$

The term $-k_\mu (1 - \mu_j) \Psi_j$ penalizes generative potential when memory fidelity is low.

Theorem 7.1 (Conservation): In any closed interaction, RV_{total} is path-invariant.

Proof sketch: Each operation preserves value modulo compensation flows. INC redistributes value, ROT changes phase but not magnitude, JMP converts Ψ to EV, RPR requires compensation input, CRT/ANL maintain system closure. The sum along any path is preserved. ∎

Corollary 7.1 (Dual Perspective Conservation): For the system as a whole: $\sum_j [\hat{V}_H(C_j) + \hat{V}_L(C_j)] = \sum_j \Delta V(C_j) = \sum_j |\text{Comp}_j|$

The total compensation in the system equals the sum of all valuation gaps.

7.2 Mode-Specific Conservation

Proposition 7.1: In different modes:

- Exchange ($R_d > e^{-1}$): $\sum EV$ conserved, Φ unchanged
- Interference ($e^{-2} < R_d < e^{-1}$): $\sum(EV + k_1\Phi)$ conserved
- Coupling ($R_d < e^{-3}$): $\sum(EV + k_1\Phi + k_2\Psi)$ conserved
- Isolation ($R_d \geq 1 - e^{-1}$): No inter-commitment conservation; each commitment conserves independently

Extension: For Violations: $RV'_{\text{total}} = RV_{\text{total}} + I_{\text{TEST}} - D_{\text{HANDLE}}$where:

- $I_{\text{TEST}} = -\sum p_i \log p_i$ is information gain from TEST
- $D_{\text{HANDLE}} = \ln 2 \cdot \Delta^2_{\text{thresh}} \cdot (1 - \mu)$ is dissipation from handling

Violations degrade memory fidelity, permitting disorder to increase when μ is low.

8. Fabric Dynamics

8.1 Fabric Coordinates

Definition 8.1: From commitment network $\{C_j\}$: $D = \frac{1}{|\mathcal{C}|} \sum_j |\nabla_G CV_0(j)|$ $\mu = \frac{|\{(C_i,C_j):\text{phase coherent}\}|}{|\mathcal{C}|^2}$$\lambda = \text{implicit from } T, A \text{ interactions}$

Definition 8.2 (Generativity):$\text{Gen}(S) = G_S \cdot \mu_S \cdot D_S/(1 + \lambda_S)$where G_S is the Generation Output of system S.

Theorem 8.1: $\text{Gen} > 1$ is necessary for sustainable growth.

Extension: SABOTAGE targets low D (force), low μ (manipulation), or high λ (excitation) vulnerabilities, with attack efficiency: $\text{Eff}_{\text{sab}} = e^{-\ln 2\cdot(D+\mu-\lambda)}$

9. Constraint Algebra

9.1 Constraint Operations

Definition 9.1: Let $\mathcal{C}$ be the set of constraints.

Define:

- Conjunction: $C_1 \wedge C_2$
- Disjunction: $C_1 \vee C_2$

- Sequence: $C_1 \triangleright C_2$
- Compatibility: $\simeq$

Theorem 9.1 (Constraint Completeness): Any interaction condition is expressible as a Boolean combination of: R_d comparisons against e^{-n}, gender checks, anchor bounds, and resistance thresholds R_L.

9.2 Threshold Lattice

Definition 9.2: The mode lattice is: $(\mathcal{T}, \leq) = (\{\text{Coupling}, \text{Interference}, \text{Exchange}\}, <)$with meet $\wedge$ and join $\vee$ operations.

Extension: Constraints integrate with TEST. Violations override constraints at a cost to memory fidelity: $\Delta\mu = -\ln 2 \cdot \log(1 + \Delta_{\text{thresh}})$

10. Key Theorems

Theorem 10.1 (Weakest Link): For dependency chain $C_0 \to C_1 \to \cdots \to C_n$: $CV(C_n) \leq e^{-kn} \cdot \min_i CV(C_i)$

Theorem 10.2 (Phase Coherence):$\left|\sum_j \hat{F}(CV_j)\right| \leq \sum_j |CV_j|$with equality iff all phases align.

Theorem 10.3 (Triadic Threshold): For generative coupling producing third entity: $R_d < e^{-3}$ is necessary and sufficient given gender complementarity.

Theorem 10.4 (Efficiency Bound): For $R_d \geq 1 - e^{-1}$: $\text{Eff}(R_d) < 0.25$making interaction economically infeasible.

Theorem 10.5 (Violation Consequences): Forced operations (VIOLATE) cause fragmentation and memory degradation. Let $n = 1, 2, \ldots, N$ index the sequence of violation operations. The total memory degradation is: $\Delta\mu = -\ln 2 \cdot \sum_{n=1}^{N} \left[(\Delta D_n)^2 + (\Delta\lambda_n)^2 + (1 - \mu_n)^2\right]$where ΔD_n is the change in Defense during violation n, $\Delta\lambda_n$ is the change in Pulse during violation n, and μ_n is the Memory value before violation n. The summation is over ordered violation events.

Theorem 10.6 (Control Universality): The TEST → (EXECUTE | VIOLATE | HANDLE) pattern is structurally invariant across discrete state-transition systems.

Theorem 10.7 (Dual Perspective Tension): For any commitment C, the inherent tension between L and H creates resistance proportional to the valuation gap: $R_{\text{tension}} = \rho \cdot |\Delta V(C)| = \rho \cdot CV_0 \cdot |V \cdot A \cdot (1+T) - 1|$

This tension must be overcome for operations that increase liability, or absorbed through compensation mechanisms.

Theorem 10.8 (Lifecycle Completeness): The six base operations form a complete lifecycle algebra:

- CRT: initialization from void
- INC, ROT: continuous modification
- JMP: discrete state transition
- RPR: degradation reversal
- ANL: termination to void

satisfying $\text{ANL} \circ \text{CRT} = I_\emptyset$ and $\text{RPR} \circ \text{VIOLATE}^n \approx I$ for finite n.

11. Path Dependence

Definition 11.1 (State Trajectory): A path through the state space is an ordered sequence: $\Pi = (S_0, \text{OP}_1, S_1, \text{OP}_2, S_2, \ldots, \text{OP}_N, S_N)$where S_i are commitment states and $\text{OP}_i \in \{\text{CRT}, \text{INC}, \text{ROT}, \text{JMP}, \text{RPR}, \text{ANL}\}$ are operations.

Definition 11.2 (Path Space): The complete state graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ contains:

- $\mathcal{S}$: all possible commitment configurations
- $\mathcal{E}$: all valid state transitions under permitted operations

Proposition 11.1 (Path Conservation): Conservation laws apply per-path: $RV_{\text{total}}(\Pi)$ is invariant along trajectory Π.

Proposition 11.2 (History Dependence): The set of available transitions from state S_n depends on the path history $(S_0, \ldots, S_{n-1})$ through accumulated memory degradation and phase relationships.

12. Summary

Commitment Mathematics is a complete formal system with:

Primitives: Commitments $C = (L, H, P, CV_0)$ in anchor space $\mathcal{A} = [0, 1]^3$, where CV_0 is established by bilateral agreement

Dual Valuation: $\hat{V}_H(C) = +CV_0 \cdot V \cdot A \cdot (1 + T)$ and $\hat{V}_L(C) = -CV_0$, creating valuation gap $\Delta V(C)$ bridged by compensation

Operators: $\hat{V}, \hat{F}, \hat{C}$ forming tensor $\mathcal{T}$

Geometry: Fabric $\mathcal{F} = [0, 1]^3$ with coordinates (D, μ, λ), where μ is memory fidelity whose degradation permits disorder

Operations: Six base operations (CRT, INC, ROT, JMP, RPR, ANL) with hierarchical algebra and mode-dependent domains, subject to L-resistance R_L

Control: Eight internal primitives (MONITOR, TEST, EXECUTE, VIOLATE, HANDLE, SCRIBE, SECURE, MODULATE) for observation, decision, execution, violation, handling, and Fabric regulation. Four external adversarial intrusions (SABOTAGE, COERCE, DECEIVE, EXTRACT) targeting operations, Defense, Memory, and Pulse+Compensation respectively.

Interactions: Governed by R_d with natural thresholds at e^{-n}. Isolation ($R_d \geq 1 - e^{-1}$) is the negation of interaction, permitting only self-operations.

Medium of Exchange: In Exchange mode, value transfer requires Medium of Exchange commitment C_{ME} with $T = 1, V = 1, A = 0.5$ to balance frame differences

Gender: Relational roles M, F, N with gender complementarity G_c and coupling coefficient κ

Conservation: Relational Value $RV_{\text{total}} = \sum EV + k_1 \sum \Phi + k_2 \sum \Psi - k_\mu \sum (1 - \mu)\Psi$ where $\Psi_j = G_j \cdot \mu_j \cdot D_j/(1 + \lambda_j)$ with canonical constants $k_1 = k_2 = k_\mu = \ln 2$, extended for violations, with dual perspective constraint $\sum \Delta V = \sum |\text{Comp}|$. Compensation components: U (Utility), I (Income), R (Recognition), Inf (Influence), O (Optionality)

Fallbacks: Six core state-transition patterns (reflection, refraction, bounce, absorption, scattering, tunneling) for boundary and adversarial conditions, with mode-specific mappings

Path Structure: State graph $\mathcal{G}$ exists atemporally; paths Π are ordered traversals through this space; conservation and memory degradation are path-dependent

Canonical Constants:

- Decay: $k = \ln 2 \approx 0.693$
- Efficiency: $c = \ln 10 \approx 2.303$
- Relational Value: $k_1 = k_2 = k_\mu = \ln 2$
- Violation: $k_v = k_s = k_d = \ln 2$
- Sabotage: $\sigma = \tau = \ln 2$
- Refraction: $\alpha = \ln 2$

Theorems: Completeness, Conservation, Optimal Framing, Natural Thresholds, Violation Consequences, Control Universality, Dual Perspective Tension, Lifecycle Completeness

This system is atemporal, discrete, substrate-independent, and foundational—a geometric structure through which all observable relational phenomena trace paths.