Time Series Analysis & Forecasting

Objectives

- Be aware of alternatives for forecasting
- Understand components of a time series
- Be able to apply basic mathematical smoothing & forecasting techniques, using Excel

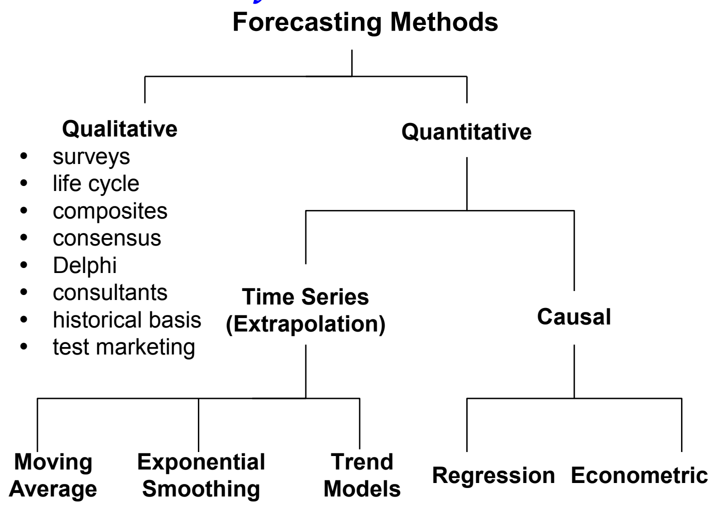
Who Needs to Forecast & Why?

- Businesses, Managers & Planners
  - marketing, production & operations
  - inventory, production, logistics, and purchasing
  - strategic and operational planning
- Financial & Economic analysts
  - in private and public sector
  - analysis and forecasting of financial and economic quantities such as interest rates, share prices, volatility, gross national product, and growth of production in economic sectors
- IT professionals
  - system demand planning

Judgment & Common Sense

- Much of what separates a good forecaster from a poor forecaster is their use of judgment and common sense
- Jan. 2000: "To justify this valuation, this company would have to grow at 80% compounded annually for 50 years
  - Commentary re no longer existing high tech
- Consider two examples:

Overview of Methods



Qualitative (judgmental) Methods

- Consumer market surveys
  - Questioning consumers on future plans
- Sales force composites
  - Aggregate joint estimates obtained from salespeople
- Executive Committee consensus
  - Finance, marketing and manufacturing managers join to prepare forecast
- Delphi technique
  - Iterative series of questionnaires answered anonymously by knowledgeable people; successive questionnaires are based on information obtained from previous surveys; seek "group consensus"
- Outside opinion
  - Consultants or other outside experts prepare forecast

Quantitative Methods

- Time series:
  - **Naïve: next value = previous, or past average**
  - **Moving avg**: Forecast based on avg of recent values
  - **Exponential smoothing**: More sophisticated form of averaging
  - **Classical decomposition**: Series is separated into trend, cyclical and seasonal components
- Causal (Associative) models:
  - Simple regression: Values of one variable are used to predict values of another variable
  - Multiple regression: Two or more variables are used to predict values of another variable
  - Econometrics: mult. equations, time series variables

Economic Indicators

- Leading Indicators: economic series that change in advance of the economy
  - Big Ticket Items, Employment, Financial
- Coincident Indicators: change with the economy (as a whole)
  - GDP, Index of Industrial Production, Personal Income, Retail Sales
- Lagging Indicators: confirm changes in economy
  - Business loans, Unemployment rate, Labour costs, Level of inventories, Inflation

Choosing Forecasting Method

- Availability of data
  - Historical
  - Other
- Resources
  - Time, $, Expertise
- Understanding of environment
  - Dependent vs independent variables
- Accuracy required

Forecasting Procedure

- Determine objectives re forecast (use)
- Select item(s) to be forecast
- Determine time horizon
  - Short, medium, long
- Select forecasting model(s)
- Gather data
- Validate forecasting model
- Make forecast
- Implement results; Monitor

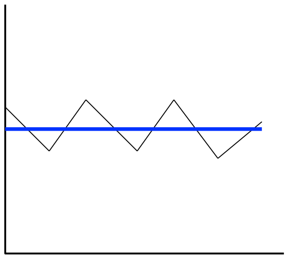
Time Series

Introduction

- Any variable measured over time in sequential order is called a **time series**
- Analyze time series to detect patterns, which help in forecasting future values
- A **time series** can consist of 5 components
  - Base (past average, starting point))
  - Long-term trend (T)
  - Cyclical effect (C)
  - Seasonal effect (S)
  - Random variation (R)

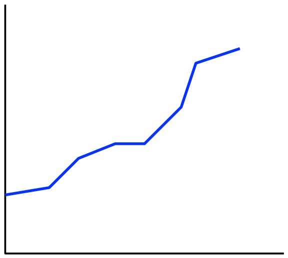
Base (Past Average)

- Also called “level component”
- Starting point for time series analysis



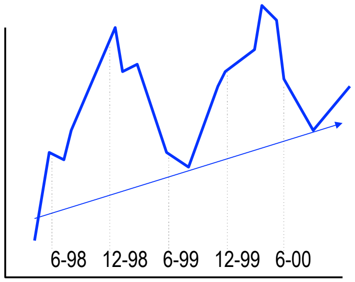
Long Term Trend (T)

- A trend is a long term relatively smooth pattern or direction, which usually persists for more than a year



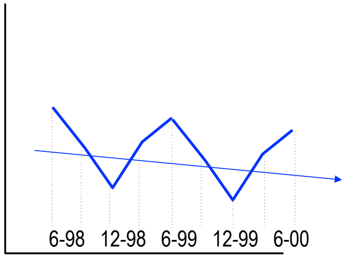
Cyclical Effects (C)

- A cycle is a wavelike pattern describing a long term behaviour (for more than one year)
- Cycles are seldom regular, and often appear in combination with other time series components



Seasonal Effects (S)

- Seasonal component of time series exhibits a short term (less than one year) calendar repetitive behaviour

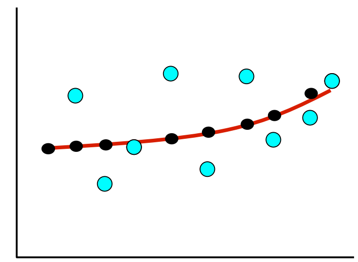


Seasonality Examples

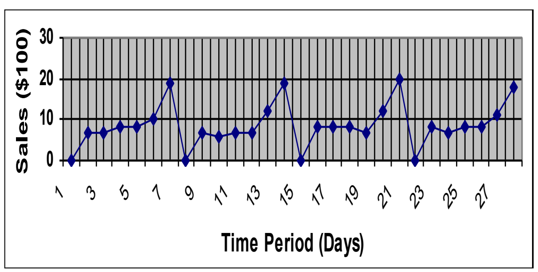
Length of Time Before Pattern is Repeated	Length of 'Season'	Number of 'Seasons' in Pattern
Year	Quarter	4
Year	Month	12
Year	Week	52
Month	Week	4
Month	Day	28-31
Week	Day	7 (5?)

Random Variation (R)

- Random variation comprises irregular, unpredictable changes in time series; tends to hide other (more predictable) components
- Can't forecast this
- Try to remove random variation & identify other components



Exercise: Identify Components

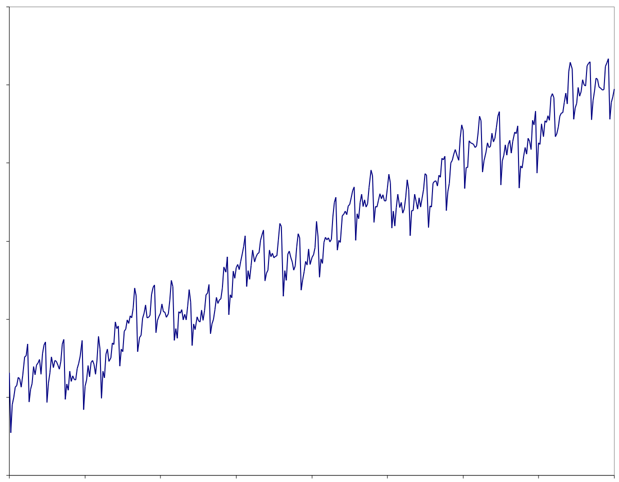


Component	Present	Not	Can't Tell
Base			
Trend			
Seasonalities			
Cycle			
Randomness			

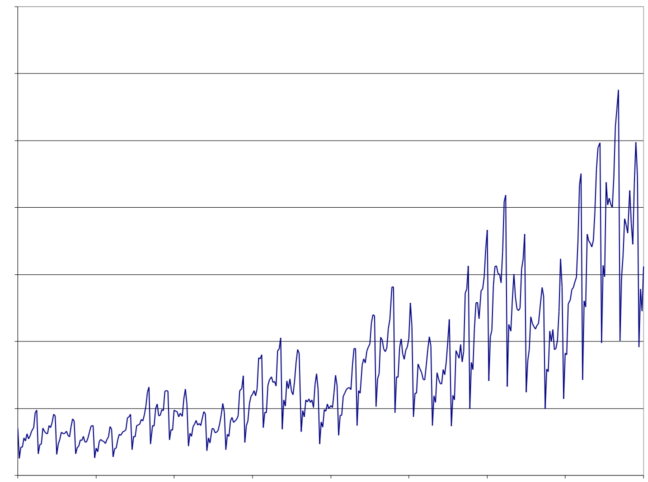
Time-series models

- Two commonly used time series models:
  - Additive model
    $$y_t = B + T_t + C_t + S_t + R_t$$
  - Multiplicative model
    $$y_t = B \times T_t \times C_t \times S_t \times R_t$$
    - Base (B)
    - Long-term trend (T)
    - Cyclical effect (C)
    - Seasonal effect (S)
    - Random variation (R)
- Only affects mathematical derivation & value of RHS terms

Example: Additive Model



Example: Multiplicative Model



Data Smoothing Techniques

Can we reduce **random** variation before further analysis?

Purpose

- To produce **better** forecast, need to determine which components are present in time series
- Always expect random variation (+/-)
- To identify components present, first need to reduce/remove random variation - easily done by smoothing techniques
- Possible to "over smooth"!!
- Also used as **forecasting method**

CHALLENGE: Reduce random variation, while leaving rest (trend, season, cycle) intact

Common Smoothing Techniques

- Moving average
- Exponential smoothing
- Combining data
  - aggregate over longer intervals (e.g., weekly, entire season, annual)
  - combine **like** intervals and work with this new data set; don't combine **unlike** intervals (or you will smooth more than random variation)

Moving Averages

- k-period moving average for time period t is arithmetic average of time series values starting at period t and counting k periods backward (**span** of k)
- Example: 3-period moving avg for period t is calculated by $(y_t + y_{t-1} + y_{t-2}) / 3$
- Note: lose k-1 values in doing this

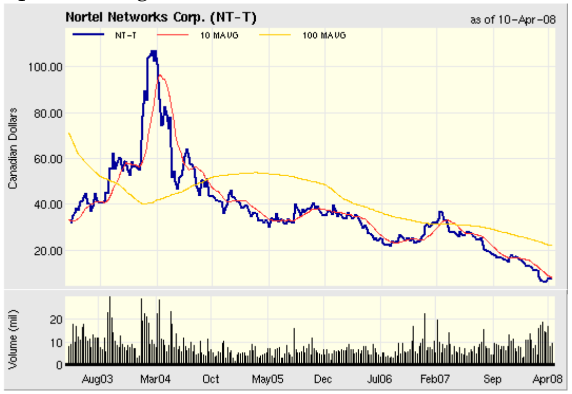
2-Minute Exercise

- You work for Waterloo Tire Inc. Smooth past sales data, using a 2, 3 and 5-period moving average.

YEAR	SALES
2001	20,000
2002	24,000
2003	22,000
2004	26,000
2005	25,000

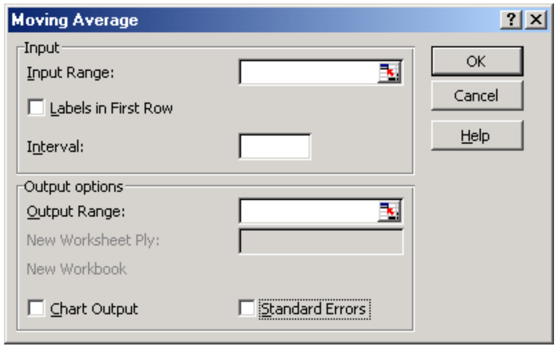
Example: TSX Charting

- GLOBEinvestor.com supports 10, 20, 50, 100 and 200 day moving averages
  - http://www.globeinvestor.com/static/hubs/charts.html



Excel

- Tools > Data Analysis ... > Moving Average



Caution re Averaging Period

- Weekly removes daily variation
- Monthly removes daily/weekly
- Yearly removes daily/weekly/seasonal
- Decade removes daily/weekly/seasonal & annual

Do you want to remove random variation + seasonalities & cycles?

Weighted Moving Average

- Simple Moving Average
  - equal weighting on each period in average
- Weighted Moving Average
  - uses unequal weights
  - allows more recent demand data to have greater/less effect on moving average
  - weights must add to 1.0; generally decrease with age of data

Exponential Smoothing

$$\boxed{S_t = \alpha y_t + (1-\alpha)S_{t-1}}$$

S_t = exponentially smoothed time series at time t
y_t = time series at time t
S_{t-1} = exp. smoothed time series at time t-1
α = smoothing constant, where $0 \le \alpha \le 1$
(complement of Excel damping factor)

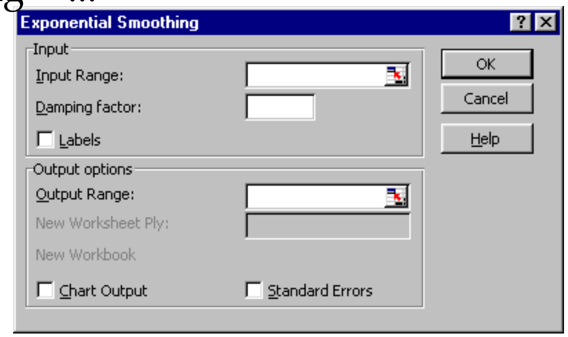
Use first period value (y_0) for S_{-1}

Smoothing Constant(α) Choice

- Values of 0.2 to 0.3 are common
  - 20-30% weight on latest observation; 70-80% weight on historical observations
  - Need to understand what we are smoothing. Is it just randomness, or seasonality/cycles or environment changes?
- Choice is both a "technical" & "managerial" decision
  - experience, skill, judgment
- Can use Excel to find "optimal" value

Excel

- Tools > Data Analysis > Exponential Smoothing > ...



Excel Damping Factor = 1 – α (Smoothing Constant)

5-Minute Exercise

- Reuse the Waterloo Tire data. Use exponential smoothing, with smoothing constants of 0.2 and 0.9

YEAR	SALES
2003	20,000
2004	24,000
2005	22,000
2006	26,000
2007	25,000

Issues re Smoothing

- Over smoothing possible
  - Large number of periods may smooth out real changes
- Don't directly identify trends
- Must keep sufficient past data
- For centered moving avg, must decide where to place averages in table or graph (which period)
  - Odd # - use middle
  - Even # - center by using average of adjacent moving average values

Smoothing Summary

- Exponential smoothing method provides smoothed values for **all time periods** observed
- When smoothing the time series at time t, exponential smoothing method **considers all data available** at t (y_t, y_{t-1},…)
- Moving average method does not provide smoothed values (moving average values) for initial and/or final periods
- Moving average method considers only observations included in calculation of average value

Forecasting (using Smoothing Techniques)

Assumption: Past can be used to predict the Future

Moving Average Forecasting

- Moving Average can be used to produce forecasts when time series
  - exhibits gradual (not a sharp) trend,
  - no cyclical effects,
  - no seasonal effects.
- Forecast for next (and any future period) is

$$F_{t+1} = (y_t + y_{t-1} + \ldots + y_{t-k+1})/k$$

k = # periods in moving avg

2-Minute Exercise

- Using the Waterloo Tire data, forecast 2004 sales. Use a 2, 3 and 5-period moving average.

YEAR	SALES
2003	20,000
2004	24,000
2005	22,000
2006	26,000
2007	25,000

Forecasting with Exp. Smoothing

- Exponential smoothing model can be used to produce forecasts when time series
  - exhibits gradual (not a sharp) trend,
  - no cyclical effects,
  - no seasonal effects.
- Forecast for future period t+k is

$$F_{t+k} = S_t$$

t is current period; $S_t = \alpha y_t + (1-\alpha)S_{t-1}$

- Usually forecast for next period only (k=1)

2-Minute Exercise

- As a planning analyst with Arctic Cat, you need to forecast next year's snowmobile sales. Use exponential smoothing, with α = 0.25 or 0.75.

YEAR	**SALES**	**S_t**
2003	30K	30.0
2004	40K	32.5
2005	30K	31.88
2006	10K	26.41
2007	20K	???

$$S_t = \alpha y_t + (1-\alpha)S_{t-1}$$

$$S_t = 0.25y_t + 0.75S_{t-1} \text{ or } S_t = 0.75y_t + 0.25S_{t-1}$$

Extensions

- Holt's Model for Trend
  - 2 smoothing constants (double smoothing):
    - » α - for noise
    - » β - for trend
- Winter's Model for Seasonality
  - 3 smoothing constants (triple smoothing):
    - » α - for noise
    - » β - for trend
    - » γ - for seasonality
- Available in all Forecasting s/w pkgs
  - Not built-in to Excel – but can be done

Choosing smoothing constant(s)

- Values of 0.2 to 0.3 are common
  - indicate that current forecast should be adjusted 20 to 30 percent for error in prior forecast
- Larger constants yield faster **impulse** response but can produce erratic projections
- Smaller constants can result in long lags in reacting to changes
- Choice is a "managerial decision"; involves possible trade-offs
- Need to monitor forecast vs actual, & adjust if necessary

Seasonality: Seasonal Relatives Method

Method

- Identify the seasons
- Compute the average per season over a number of years – use centered moving average if trend exists
- For each period, compute seasonal indices by dividing actuals by seasonal average
- Average the seasonal indices / normalize
- Apply these to the forecasts

Example: Seasonal Relatives

- **Identify seasons**
- This is trivial here as we are given quarterly data; usually plot will show

	Q1	Q2	Q3	Q4
1998	390	331	250	421
1999	372	324	280	457
2000	411	372	332	480
2001	430	384	342	486
2002	418	361	341	496
2003	445	384	365	484

Example (cont.)

- **Compute average per season for each year of data (smooth the data)**

	Q1	Q2	Q3	Q4	Average
1998	390	331	250	421	348.00
1999	372	324	280	457	358.25
2000	411	372	332	480	398.75
2001	430	384	342	486	410.50
2002	418	361	341	496	404.00
2003	445	384	365	484	419.50

Example (cont.)

- **Compute seasonal indices by dividing actual by seasonal average**

	Q1	Q2	Q3	Q4	Average	Q1	Q2	Q3	Q4
1998	390	331	250	421	348.00	1.12069	0.951149	0.718391	1.20977
1999	372	324	280	457	358.25	1.038381	0.904396	0.781577	1.275645
2000	411	372	332	480	398.75	1.030721	0.932915	0.832602	1.203762
2001	430	384	342	486	410.50	1.047503	0.935445	0.83313	1.183922
2002	418	361	341	496	404.00	1.034653	0.893564	0.844059	1.227723
2003	445	384	365	484	419.50	1.060787	0.915375	0.870083	1.153754

Example (cont.)

- **Average these seasonal indices over entire time period**

	Q1	Q2	Q3	Q4
	1.12069	0.951149	0.718391	1.20977
	1.038381	0.904396	0.781577	1.275645
	1.030721	0.932915	0.832602	1.203762
	1.047503	0.935445	0.83313	1.183922
	1.034653	0.893564	0.844059	1.227723
	1.060787	0.915375	0.870083	1.153754
Seasonal Factor	1.055456	0.922141	0.813307	1.209096

Example (cont.)

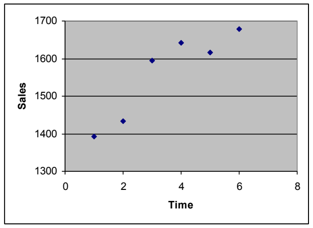
- **Apply these to forecast**

If forecast for next year is 1762, avg for quarter is 1762/4 = 440.5

	Q1	Q2	Q3	Q4
	1.12069	0.951149	0.718391	1.20977
	1.038381	0.904396	0.781577	1.275645
	1.030721	0.932915	0.832602	1.203762
	1.047503	0.935445	0.83313	1.183922
	1.034653	0.893564	0.844059	1.227723
	1.060787	0.915375	0.870083	1.153754
Seasonal Factor	1.055456	0.922141	0.813307	1.209096
Average Sales	440.5	440.5	440.5	440.5
Seasonalized	464.9283	406.2031	358.2618	532.6068

Example (cont.)

	Q1	Q2	Q3	Q4	Sum	t
1998	390	331	250	421	1392	1
1999	372	324	280	457	1433	2
2000	411	372	332	480	1595	3
2001	430	384	342	486	1642	4
2002	418	361	341	496	1616	5
2003	445	384	365	484	1678	6



b0	1356.733
b1	57.88571

Can forecast annual sales for t = 7; y = 1356.733 + 57.88571*t = 1761.993 (next topic is using regression)

Forecasting using Regression Techniques: Trend Component

Trend Analysis

- Trend component of a time series can be linear or non-linear
- It is easy to isolate trend component using linear regression
  - Start with scatterplot of data
  - For **linear trend**, use model $y = \beta_0 + \beta_1 t + \varepsilon$
  - For **non-linear trend** with one (major) change in slope use **quadratic model** $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon$

1-Minute Exercise

- As a marketing analyst for Northern Toys, you' ve gathered the following data on their MBA Relaxer Gadget
- What is the trend equation?

YEAR	SALES
2003	10,000
2004	15,000
2005	20,000
2006	25,000
2007	30,000

Example

- Annual sales for a pharmaceutical company are believed to change linearly over time
- Based on last 10 year sales records, measure trend component
- Data:

Year	Sales	Year	Sales
1993	18.0	1998	21.1
1994	19.4	1999	23.5
1995	18.0	2000	23.2
1996	19.9	2001	20.4
1997	19.3	2002	24.4

Excel Soln.

There is a relatively good fit. Be aware of possible cyclical effects and seasonality, that may reduce the fit of the model.

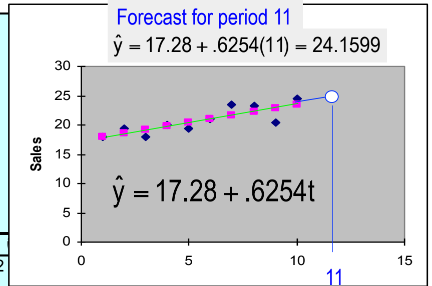
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.829573
R Square	0.688192
Adjusted F	0.649216
Standard E	1.351968
Observatio	10

ANOVA

	df	*SS*	*MS*	*F*
Regressior	1	32.27345	32.27345	17.65682
Residual	8	14.62255	1.827818	
Total	9	46.896		

	Coefficients	*Standard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	17.28	0.92357	18.71	6.88E-08	15.15024	19.40976
t	0.625455	0.148847	4.202002	0.002988	0.282213	0.968696



Forecasting using Regression Techniques: Seasonality

Linear regression forecast

- Linear regression can be used to forecast time series with trend and seasonality
- Multiplicative Model $\boxed{F_t = [b_0 + b_1 t] \cdot SI_t}$

Linear trend value for period t, obtained from linear regression

Seasonal index for period t

- Additive Model

$$\boxed{F_t = b_0 + b_1 t + \underbrace{b_2 S_1 + b_3 S_2 + b_4 S_3}_{\text{Indicator Variables}}}$$

Use of Indicator Variables

- 0-1 variables; indicate presence/absence of category
- If n categories, need n-1 variables
- Example: forecasting quarterly effects
  - 4 quarters, so need 3 indicator variables

	I1	I2	I3
Q1	1	0	0
Q2	0	1	0
Q3	0	0	1
Q4	0	0	0

Forecasting with Correlated Data

Autoregressive Models

Autoregressive models

- Autocorrelation among errors of regression model provides opportunity to produce accurate forecasts!
- Correlation between consecutive residuals leads to following autoregressive model:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$$

How would we identify autocorrelation among errors?

Identifying Autocorrelation

- Remember our assumptions when we studied regression analysis. Error terms are Normally distributed about mean zero, with constant standard deviation over all values of the independent variables.
- So look at a histogram of the error terms (or std residuals)
- Also, plot errors vs predicted y and see if there is a pattern
- Can also do correlation analysis, for 1, 2, … lags

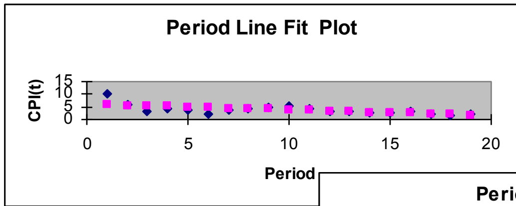
Example

- Forecast increase in Consumer Price Index (CPI) for year 1994, based on data collected for years 1980- 1999
- Data: CPI.xls

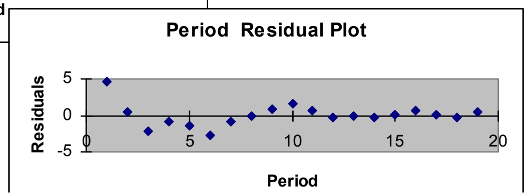
t	Y (t)
1	13.5
2	10.4
3	6.1
4	3.2
.	.
.	.

Excel Solution

- Regressing Percent increase (y_t) on the time variable (t), does not lead to great results: adj. $r^2 = 0.38$



Autocorrelation!



Soln (cont.)

	CPI(t)	Lag1	Lag2	Lag3
CPI(t)	1			
Lag1	0.500055	1		
Lag2	0.159033	0.667312	1	
Lag3	0.15991	0.480248	0.866425	1

Lose one data point for each lag calculated

Excel: just cut/paste data into new column (one cell over, one cell down; add approp. title)

Soln (cont.)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8748
R Square	0.7653
Adjusted R Square	0.7515
Standard Error	0.9898
Observations	19

ANOVA

	df	SS	MS	F	Significance F
Regression	1	54.31	54.31	55.44	0.0000
Residual	17	16.66	0.98		
Total	18	70.97			

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.20	0.42	2.90	0.0100
X Variable 1	0.59	0.08	7.45	0.0000

$$\hat{y}_t = 1.20 + .59 y_{t-1}$$

$$\textit{Forecast for } 2000\ (t = 21):$$

$$\hat{y}_{21} = 1.20 + .595 y_{20}$$

$$= 1.20 + .59(2.2) = 2.50$$

Choosing a Forecasting Method

Criteria for Selecting Method

- Cost
- Accuracy
- Data available
- Time Span
- Nature of product/service being forecast, and expected pattern
- Impulse response
- Noise dampening

Comparing Methods

- Moving Average
  - Best for randomness only; smoothes everything (so be careful); forecasts one period ahead
- Exponential Smoothing
  - Same as Moving Average
- Seasonal Relatives
  - Only for calculating seasonal indices
- Time Series Analysis
  - Can incorporate trend, seasonality, cycles; forecasts several periods ahead

Evaluating Forecast Accuracy

- To choose forecasting method, evaluate forecast accuracy using actual time series
- Most commonly used measures of forecast accuracy:
  - Mean Absolute Deviation $MAD = \sum_{t=1}^{n} | y_t - F_t | / n$
    - » MAE = mean absolute error
  - Mean Squared Error $MSE = \sum_{t=1}^{n} (y_t - F_t)^2 / n$
    - » RMSE = square root MSE
  - Mean Absolute Percent Error

$$MAPE = 100 \sum_{t=1}^{n} (| y_t - F_t | / F_t) / n$$

Model Selection

- Think about nature of problem
  - Qualitative?
  - Quantitative: time series or causal?
- Plot data to see nature of relationships
  - Identify time series components; if seasonality, decide if additive or multiplicative
- Use some observations to develop several competing forecasting models
- Run models on rest of observations (holdout)
- Calculate accuracy of each model
- Select model with best accuracy measure

Combining Forecasts

- Recent research has looked at combining forecasts
  - How many? 2-3 generally sufficient!
  - What weighting? Average them!
  - What models? Qualitative or Quantitative? Often does not matter!
- Suggests a "quick & dirty" approach can be just as good (or better) than a complex, costly quantitative approach

Monitoring Accuracy

- Must monitor forecast accuracy as well
- Continually assess confidence in model, and changes in environment
  - May require re-evaluation of approach
- MAD, MSE & MAPE commonly used
- Choose MSE if it is important to avoid (even a few) large errors. Otherwise, use MAD or MAPE.

Workshop – Forecasting Error

- Calculate error, error^2, sums, MSE, MAD

$$MSE = \sum_{t=1}^{n} (y_t - F_t)^2 / n$$

$$MAD = \sum_{t=1}^{n} | y_t - F_t | / n$$

	Actual (y_t)	**F_t**	**e**	**e^2**
1	15	15		
2	17	14.5		
3	12	16		
4	14	14.5		
5	18	13		
6	16	16		
Sum				
|e|				

Reasons for Ineffective Forecasting

- Not involving a broad cross section of people
- Nor recognizing that forecasting is integral to business planning
- Not recognizing that forecasts will always be wrong
- Not forecasting the right things
- Not selecting an appropriate forecasting method
- Not tracking accuracy of forecasting model

More Info

- Forecasting & Time Series on the Web:
  - http://www.buseco.monash.edu.au/units/forecasting/links.php
- Principles of Forecasting:
  - http://fourps.wharton.upenn.edu/forecast/
  - 139 principles; use to judge forecasting process quality; legal aspects!
- Forecasting software comparison
  - www.lionhrtpub.com/software-surveys.shtml
- Some bad forecasts: www.autobox.com/badfore.html