

Homework 11

①

$$P(s_1|s, a_1) = 0.6 \quad P(s_2|s, a_1) = 0.4$$

$$P(s_1|s, a_2) = 0.5 \quad P(s_2|s, a_2) = 0.5$$

$$R(s, a_1, s_1) = 1$$

$$R(s, a_1, s_2) = 0$$

$$R(s, a_2, s_1) = 2$$

$$R(s, a_2, s_2) = 2$$

$$a) \quad E[R_{t+1} | s_t = s, A_t = a_1] = ?$$

$$= P(s_1|s, a_1) \cdot R(s, a_1, s_1) + P(s_2|s, a_1) \cdot R(s, a_1, s_2)$$

$$= 0.6 \times 1 + 0.4 \times 0$$

$$= 0.6$$

$$E[R_{t+1} | s_t = s, A_t = a_2] = ?$$

$$= P(s_1|s, a_2) R(s, a_2, s_1) + P(s_2|s, a_2) R(s, a_2, s_2)$$

$$= 0.5 \times 2 + 0.5 \times 2$$

$$= 1 + 1$$

$$= 2$$

$$b) \quad R(s, a_1, s) = 0.6, \quad R(s, a_2, s) = 2$$

$$\begin{aligned} v(s; a_i) &= \sum_{t=0}^{\infty} \gamma^t \cdot E[R_{t+1} | s, a_i] \\ &= \frac{E[R_{t+1} | s, a_i]}{1 - \gamma} \end{aligned}$$

$$v(s; a_1) = \frac{0.6}{1 - 0.9} = \frac{0.6}{0.1} = 6$$

$$v(s; a_2) = \frac{2}{1 - 0.9} = \frac{2}{0.1} = 20$$

②

$$a) \quad S = \{s_1, s_2\} \quad A = \{a\}$$

$$P(s_2 | s_1, a) = 1, \quad R(s_1, a, s_2) = 3$$

$$P(s_1 | s_2, a) = 1, \quad R(s_2, a, s_1) = 2$$

$$\pi(a | s_1) = \pi(a | s_2) = 1$$

$$\gamma = 0.5$$

$$v_{\pi}(s_1) = R(s_1, a, s_2) + \gamma v_{\pi}(s_2)$$

$$v_{\pi}(s_2) = R(s_2, a, s_1) + \gamma v_{\pi}(s_1)$$

$$V_{\pi}(s_1) = 3 + 0.5 V_{\pi}(s_2) \quad \text{--- (1)}$$

$$V_{\pi}(s_2) = 2 + 0.5 V_{\pi}(s_1) \quad \text{--- (2)}$$

b) Inserting the value of (1) in (2)

$$V_{\pi}(s_2) = 2 + 0.5 (3 + 0.5 V_{\pi}(s_2))$$

$$V_{\pi}(s_2) = 2 + \frac{3}{2} + \frac{1}{4} V_{\pi}(s_2)$$

$$\frac{3}{4} V_{\pi}(s_2) = \frac{7}{2}$$

$$\boxed{V_{\pi}(s_2) = \frac{14}{3}}$$

$$V_{\pi}(s_1) = 3 + \frac{1}{2} \times \frac{14}{3}$$

$$\boxed{V_{\pi}(s_1) = \frac{16}{3}}$$