HW6

Q1) a) Given $P_{X_i}(x_i, \lambda) = \dfrac{e^{-\lambda}\lambda^{x_i}}{x_i!}$

$$L(x_1, \dots, x_n; \lambda) = P_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n; \lambda)$$
$$= \prod_{i=1}^{n} P(x_i; \lambda)$$
$$= \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

b)
$$L(x_1, x_2, \dots, x_n; \lambda) = \sum_{i=1}^{n} (-\lambda + x_i \log\lambda - \log x_i!)$$
$$\log L(x_1, x_2, \dots, x_n; \lambda) = -n\lambda + \left(\sum_{i=1}^{n} x_i\right)\log\lambda \underbrace{- \sum_{i=1}^{n}\log x_i!}_{\text{constant}}$$

$$\ell(\lambda) = -n\lambda + \left(\sum_{i=1}^{n} x_i\right)\log\lambda$$

for MLE, differentiate wrt λ

$$\frac{dl}{d\lambda} = -n + \frac{\sum x_i}{\lambda}$$

$$-n + \frac{\sum x_i}{\lambda} = 0$$

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i \implies \boxed{\hat{\lambda}_{ML} = \frac{1}{n} \sum_{i=1}^{n} x_i}$$