

USING THE INTERNAL LANGUAGE OF TOPOSES IN ALGEBRAIC GEOMETRY

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ABSTRACT. We describe how the internal language of certain toposes, the associated petit and gros Zariski toposes of a scheme, can be used to give simpler definitions and more conceptual proofs of the basic notions and observations in algebraic geometry.

The starting point is that, from the internal point of view, sheaves of rings and sheaves of modules look just like plain rings and plain modules. In this way, some concepts and statements of scheme theory can be reduced to concepts and statements of intuitionistic linear algebra.

Furthermore, modal operators can be used to model phrases such as “on a dense open subset it holds that” or “on an open neighbourhood of a given point it holds that”. These operators define certain subtoposes; a generalization of the double-negation translation is useful in order to understand the internal universe of those subtoposes from the internal point of view of the ambient topos.

A particularly interesting task is to internalize the construction of the relative spectrum, which, given a quasicoherent sheaf of algebras on a scheme X , yields a scheme over X . From the internal point of view, this construction should simply reduce to an intuitionistically sensible variant of the ordinary construction of the spectrum of a ring, but it turns out that this expectation is too naive and that a refined approach is necessary.

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