COMPOSITION OF INTERNAL THEORIES

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ABSTRACT. XXX

1. INTRODUCTION

XXX state main lemma (to be proved in Section 3)

Definition 1.1. A geometric morphism $\mathcal{F} \to \mathcal{E}$ is *of presheaf type* if and only if it is equivalent (XXX be more precise) to the canonical functor $\mathcal{E}^{\mathcal{C}^{\mathrm{op}}} \to \mathcal{E}$ for an internal category $\mathcal{C}$ in $\mathcal{E}$.

XXX This notion is the relativization of the notion of a presheaf category; a geometric theory $\mathbb{T}$ is of presheaf type if and only if the canonical geometric morphism $\mathrm{Set}[\mathbb{T}] \to \mathrm{Set}$ is of presheaf type.

Proposition 1.2. *The composition of geometric morphisms of presheaf type is of presheaf type.*

Proof. (This is what Sina confirmed for us.) Spell out construction. (Maybe in Section 4.) $\square$

2. BACKGROUND ON INTERNAL GEOMETRIC THEORIES

XXX: State definition of geometric theory, being careful to only use geometric constructions

XXX: State definition of model of a geometric theory, being careful to only use geometric constructions

XXX: Explain internal geometric theories

XXX: Remark subtletly with respect to infinitary disjunctions and infinitary proofs

3. PROOF OF THE MAIN RESULT

XXX: State and prove main lemma about $\mathrm{Set}[\mathbb{T}][\mathbb{T}']$

4. CRITERIA FOR THEORIES TO BE OF PRESHEAF TYPE

XXX: Give semantic criterion for being of presheaf type: Let $\mathbb{T}$ be of presheaf type and let $\mathbb{T}''$ be an extension of $\mathbb{T}$ such that there is a theory $\mathbb{T}'$ internal to $\mathrm{Set}[\mathbb{T}]$ such that $\mathrm{Set}[\mathbb{T}][\mathbb{T}'] = \mathrm{Set}[\mathbb{T}'']$. If $\mathbb{T}'$ is of presheaf type from the point of view of $\mathrm{Set}[\mathbb{T}]$, then $\mathbb{T}''$ is of presheaf type.

XXX: Explain that in the previous paragraph, the existence of $\mathbb{T}'$ is not actually an assumption, but is in fact always satisfied. (?)

XXX: Draw conclusions, for instance that adding a further constant preserves being of presheaf type (find lots of further such syntactical examples).

5. Applications

XXX: Results from the Master thesis of Matthias

XXX: Show that the naive definition of the big Zariski topos does not work. (From the point of view of Sh(S), the big Zariski topos of S is *not* the topos of local $\mathcal{O}_S$-algebras, but the topos of local $\mathcal{O}_S$-algebras which are local over $\mathcal{O}_S$.)

6. XXX general todo

XXX cite Martha Bunge's characterization of functors of presheaf type

XXX cite Shawn Henry's PhD thesis

XXX cite that other guy who mentioned internal geometric theories (forgot whom right now, might be Freyd or Andreas Blass, either in the result that the object of Kuratowski-finite subsets is preserved under pullback or in the paper about existential fixed-point logic)

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