

$$U \models s = t : \mathcal{F} \quad :\Longleftrightarrow \quad s|_U = t|_U \in \Gamma(U, \mathcal{F})$$

$$U \models \top \quad :\Longleftrightarrow \quad U = U \text{ (always fulfilled)}$$

$$U \models \perp \quad :\Longleftrightarrow \quad U = \emptyset$$

$$U \models \varphi \wedge \psi \quad :\Longleftrightarrow \quad U \models \varphi \text{ and } U \models \psi$$

$$U \models \varphi \vee \psi \quad :\Longleftrightarrow \quad \cancel{U \models \varphi \text{ or } U \models \psi}$$

there exists a covering $U = \bigcup_i U_i$ such that for all i :

$$U_i \models \varphi \text{ or } U_i \models \psi$$

$$U \models \varphi \Rightarrow \psi \quad :\Longleftrightarrow \quad \text{for all open } V \subseteq U: V \models \varphi \text{ implies } V \models \psi$$

$$U \models \forall s : \mathcal{F}. \varphi(s) \quad :\Longleftrightarrow \quad \text{for all sections } s \in \Gamma(V, \mathcal{F}), \text{ open } V \subseteq U: V \models \varphi(s)$$

$$U \models \exists s : \mathcal{F}. \varphi(s) \quad :\Longleftrightarrow \quad \cancel{\text{there exists a section } s \in \Gamma(U, \mathcal{F}) \text{ such that } U \models \varphi(s)}$$

there exists an open covering $U = \bigcup_i U_i$ such that for all i :

$$\text{there exists } s_i \in \Gamma(U_i, \mathcal{F}) \text{ such that } U_i \models \varphi(s_i)$$

Table 1: The Kripke–Joyal semantics of a sheaf topos.