

# Die wundersame Welt der vierdimensionalen Geometrie

**Schuljahresausklangsmachvortrag am 20. Juli 2022**

*Fragen sind jederzeit willkommen! Bitte nicht bis zum Ende aufsparen.*

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Universität Augsburg

Matthias Hutzler  
University of Gothenburg

**Wie geht es weiter?**

1,

**Wie geht es weiter?**

1,  $\infty$ ,

**Wie geht es weiter?**

1,  $\infty$ , 5,

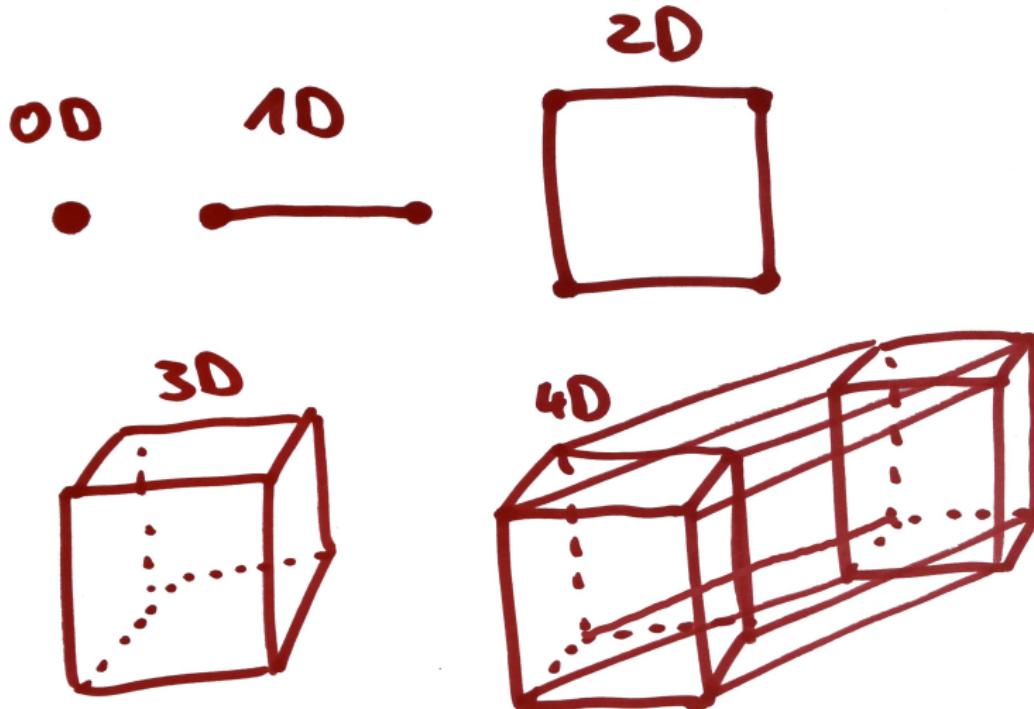
**Wie geht es weiter?**

1,  $\infty$ , 5, 6,

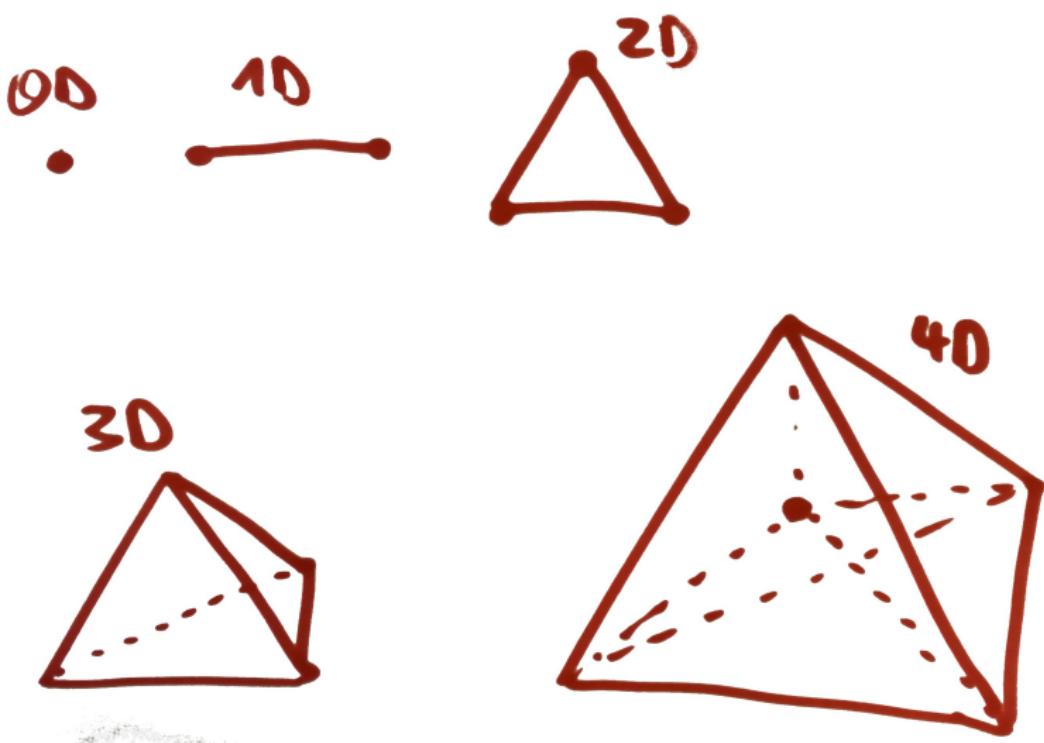
**Wie geht es weiter?**

1,  $\infty$ , 5, 6, ??

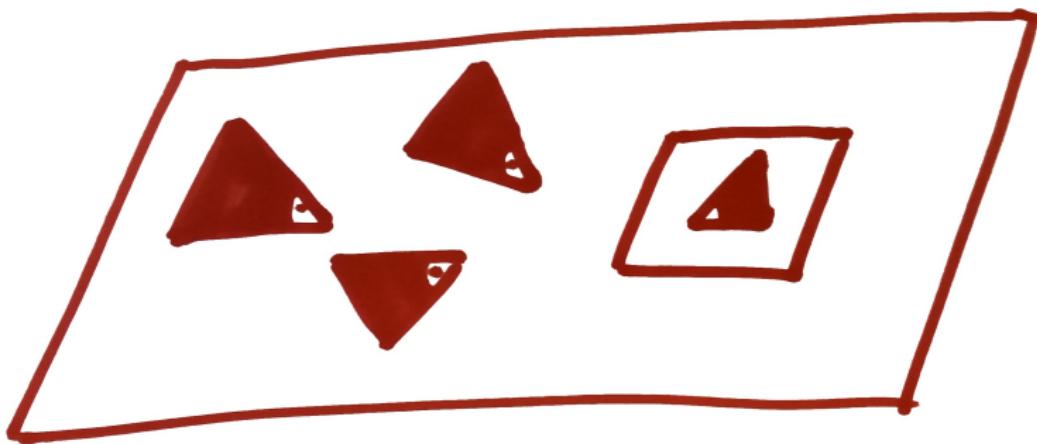
# Vier Dimensionen?



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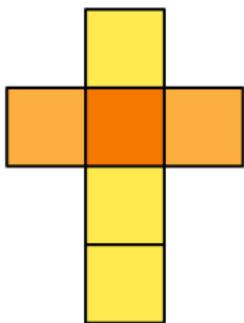


# Vier Dimensionen?

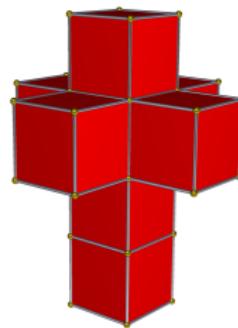


# Kleben vierdimensionaler Formen

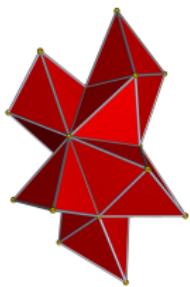
Würfel



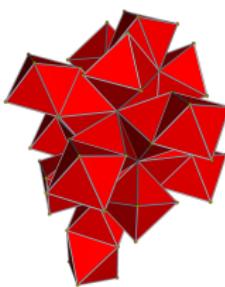
Tesserakt



16-Zeller



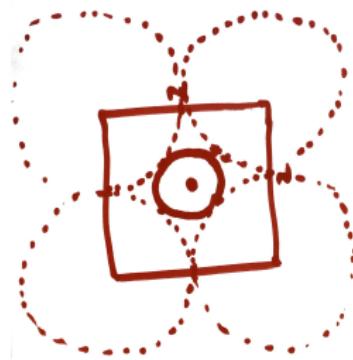
24-Zeller



Liebe ist  
wichtig.



# Küssende Hyperkugeln

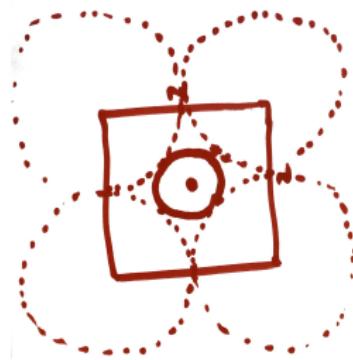


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Dimension	Radius der inneren Hyperkugel
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$n = 2$	
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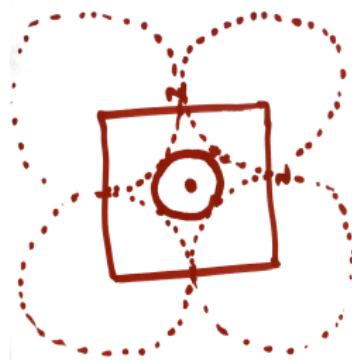
# Küssende Hyperkugeln



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Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$

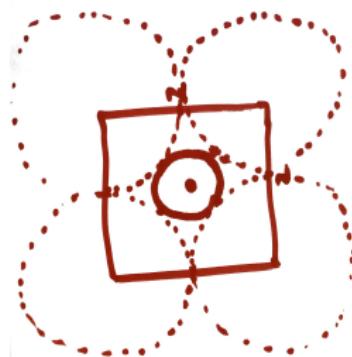
# Küssende Hyperkugeln



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Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	

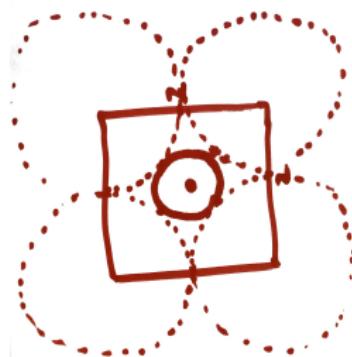
# Küssende Hyperkugeln



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Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$

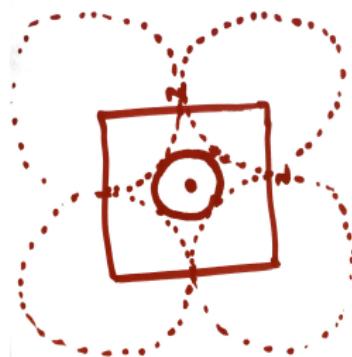
# Küssende Hyperkugeln



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Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$

# Küssende Hyperkugeln



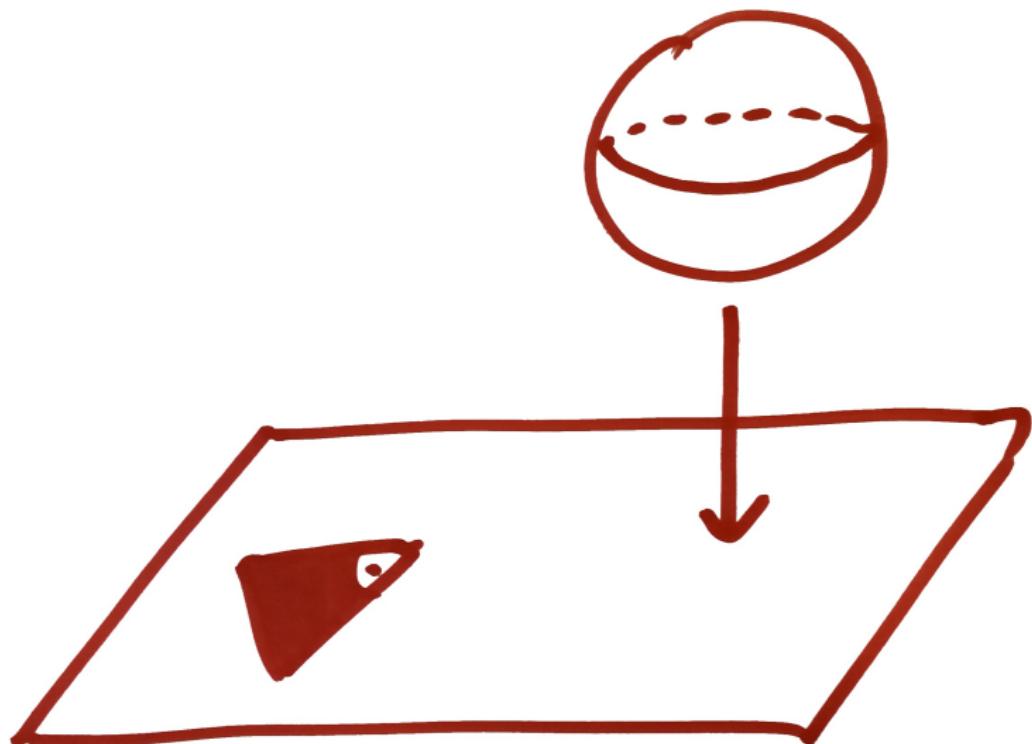
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Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$
$n$	$\sqrt{n} - 1$

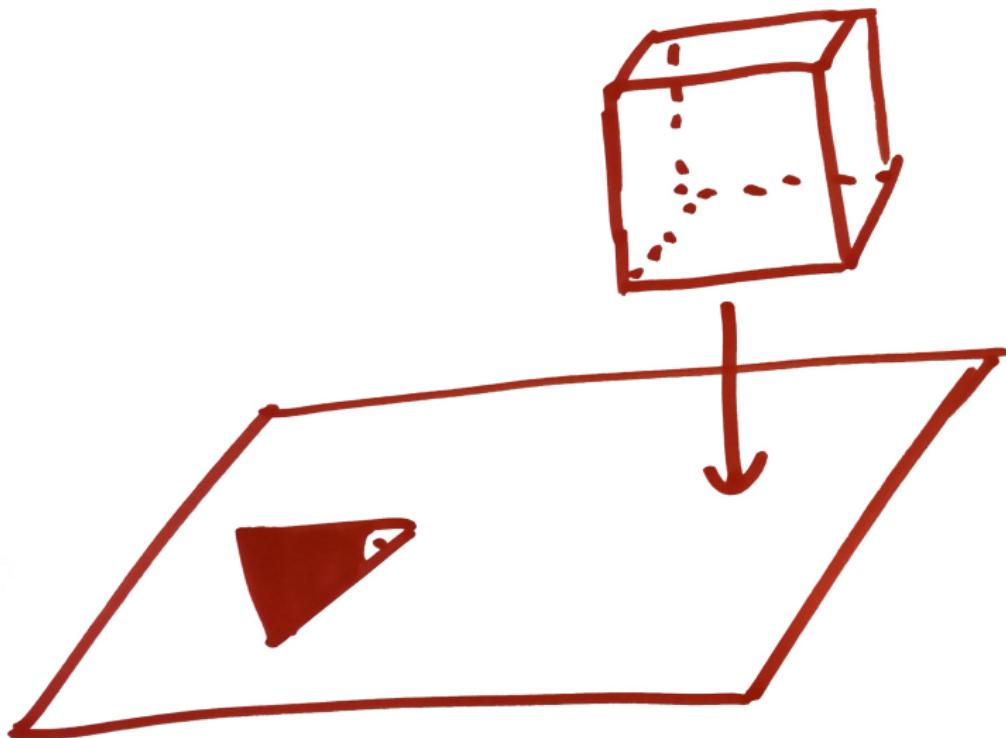
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Die Entfernung vom Mittelpunkt zu den Ecken wird  
immer größer.

# Ankunft eines Hyperballs



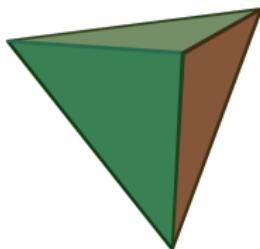
# Ankunft eines Tesserakts



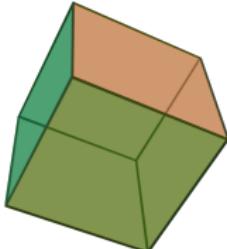
# Platonische Körper in 3d

**Tetraeder**

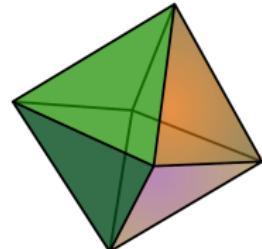
4E, 6K, 4F

**Hexaeder**

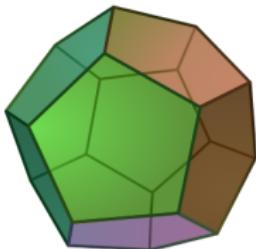
8E, 12K, 6F

**Oktaeder**

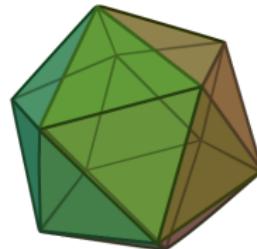
6E, 12K, 8F

**Dodekaeder**

20E, 30K, 12F

**Ikosaeder**

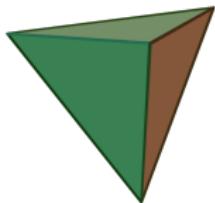
12E, 30K, 20F



# Platonische Körper in 4d

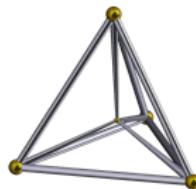
**Tetraeder**

4E, 6K, 4F



**Pentachoron**

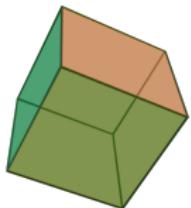
5E, 10K, 10F, 5Z



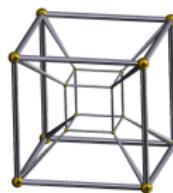
# Platonische Körper in 4d

**Hexaeder**

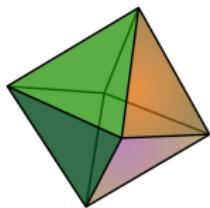
8E, 12K, 6F

**Octachoron**

16E, 32K, 24F, 8Z

**Oktaeder**

6E, 12K, 8F

**Hexadecachoron**

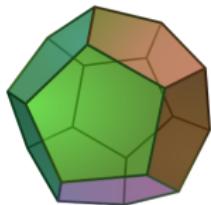
8E, 24K, 32F, 16Z



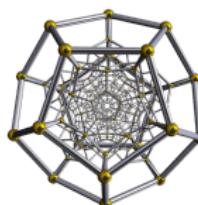
# Platonische Körper in 4d

**Dodekaeder**

20E, 30K, 12F

**Hecatonicosachoron**

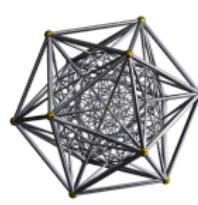
600E, 1200K, 720F, 120Z

**Ikosaeder**

12E, 30K, 20F

**Hexacosichoron**

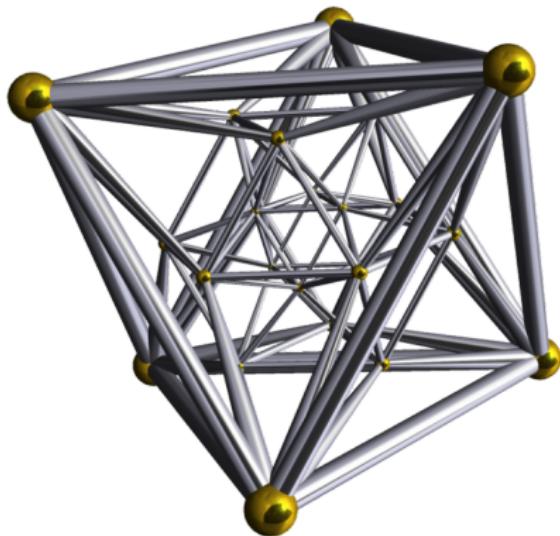
120E, 720K, 1200F, 600Z



# Platonische Körper in 4d

Icositetrachoron

24E, 96K, 96F, 24Z

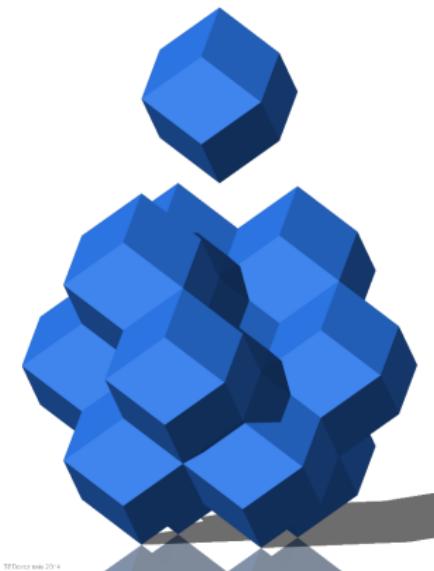
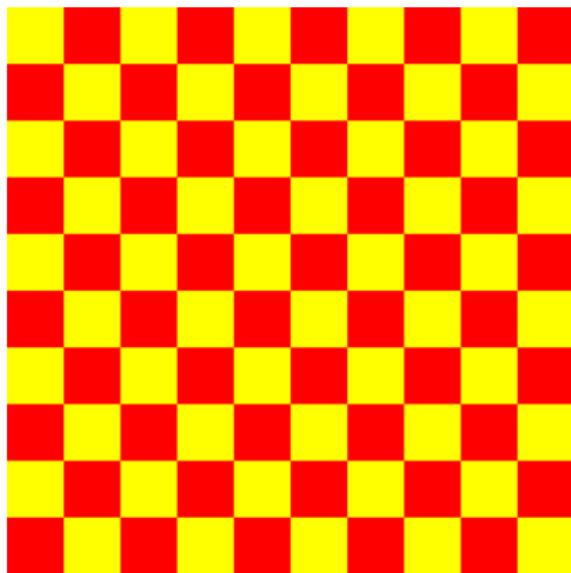


# Platonische Körper in 4d

## Icositetrachoron

24E, 96K, 96F, 24Z

# Pflasterungen



Der 24-Zeller pflastert den vierdimensionalen Raum.

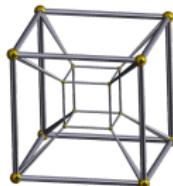
# Übersicht

**Pentachoron**

5E, 10K, 10F, 5Z

**Octachoron**

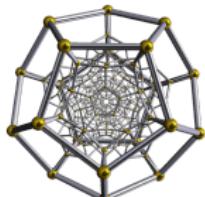
16E, 32K, 24F, 8Z

**Hexadecachoron**

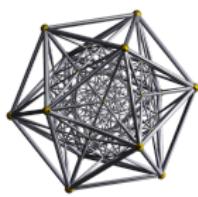
8E, 24K, 32F, 16Z

**Hecatonicosachoron**

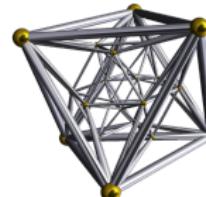
600E, 1200K, 720F, 120Z

**Hexacosichoron**

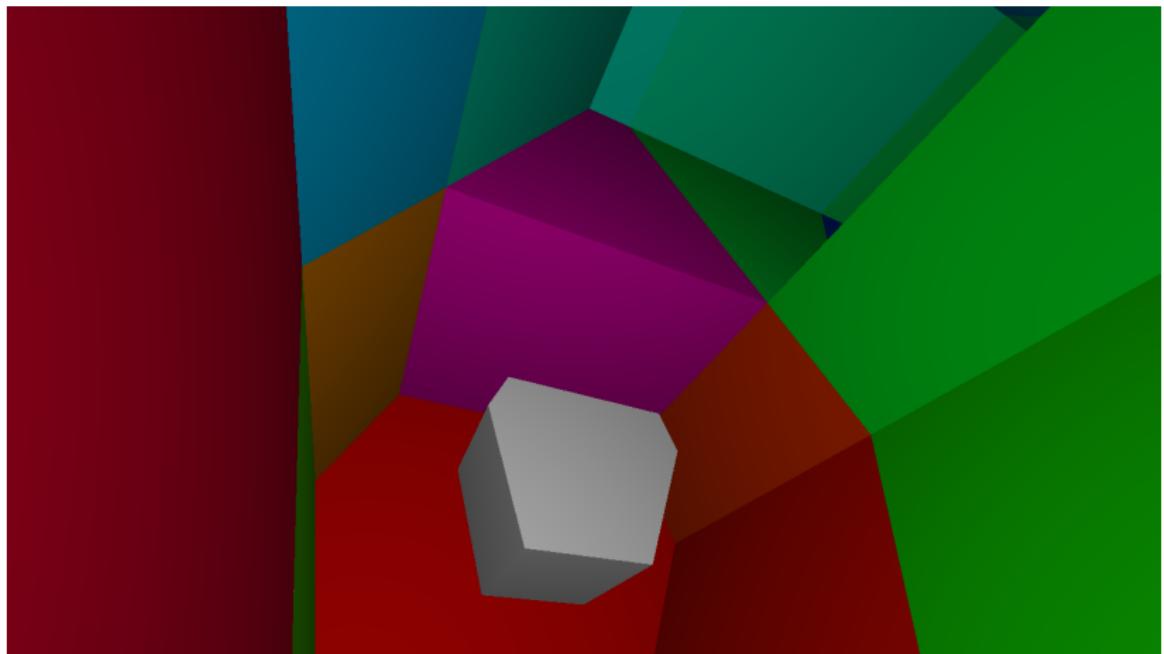
120E, 720K, 1200F, 600Z

**Icositetrachoron**

24E, 96K, 96F, 24Z

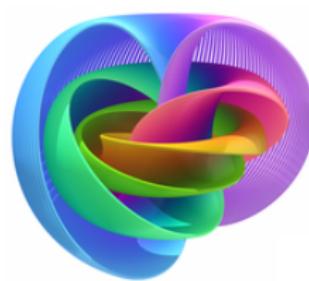


# Ein vierdimensionales Labyrinth



# Die vierte Dimension ...

- 1 ist faszinierend schön,
- 2 hilft beim Verständnis der dritten Dimension,



- 3 ist unabdingbar für die moderne Physik und
- 4 ist als einzige Dimension noch größtenteils unverstanden.

Dimension	1	2	3	4	5	6	7	8	9	...
Anzahl Sphären	1	1	1	??	1	1	28	2	8	...

**Catharina Stroppel**  
Knotentheoretikerin



**Julia Grigsby**  
niedrigdimensionale Topologin





Salvador Dalí: **Corpus Hypercubus** (1954)

# Image sources

Rendered images of four-dimensional bodies created by Robert Webb with his Stella software:

[https://en.wikipedia.org/wiki/File:Ortho\\_solid\\_011-uniform\\_polychoron\\_53p-t0.png](https://en.wikipedia.org/wiki/File:Ortho_solid_011-uniform_polychoron_53p-t0.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_5-cell.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_5-cell.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_8-cell.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_8-cell.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_16-cell.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_16-cell.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_24-cell.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_24-cell.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_120-cell.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_120-cell.png)

[https://en.wikipedia.org/wiki/File:Schlegel\\_wireframe\\_600-cell\\_vertex-centered.png](https://en.wikipedia.org/wiki/File:Schlegel_wireframe_600-cell_vertex-centered.png)

# Image sources

Miscellaneous pictures:

[http://4.bp.blogspot.com/\\_TbkIC-eqFNM/S-dK9dd1cUI/AAAAAAAFAjA/d8qdTHhKy1U/s320/tesseract+unfolded.png](http://4.bp.blogspot.com/_TbkIC-eqFNM/S-dK9dd1cUI/AAAAAAAFAjA/d8qdTHhKy1U/s320/tesseract+unfolded.png)  
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