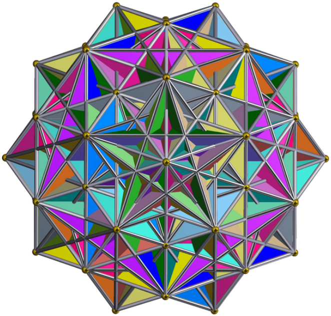


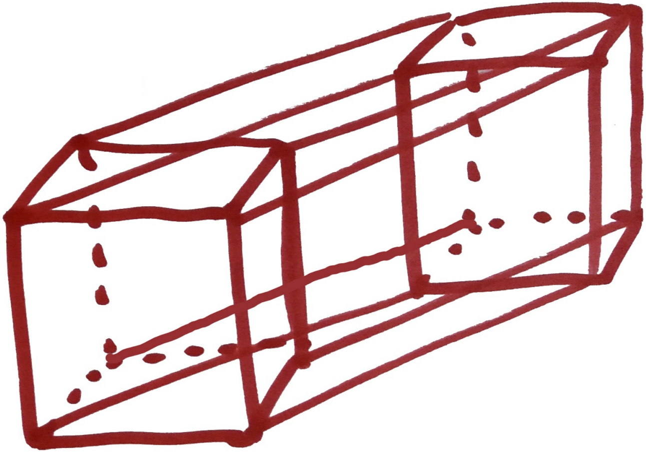
Die wundersame Welt der vierdimensionalen Geometrie

Weihnachtsvorlesung am 18. Dezember 2017

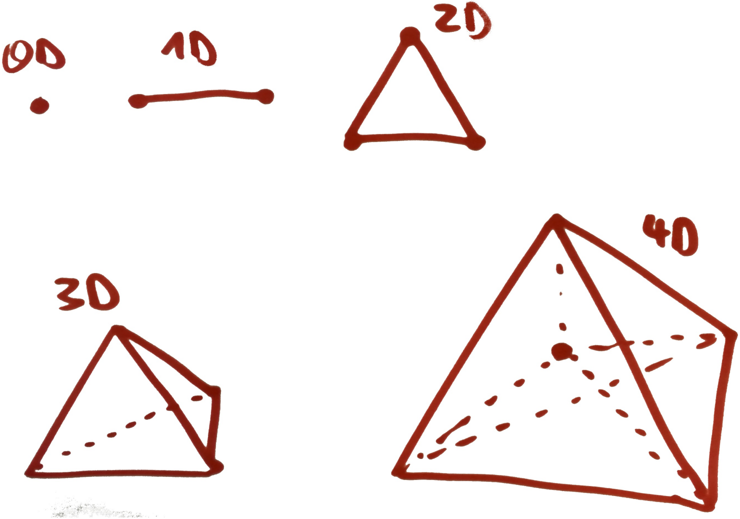
Fragen sind jederzeit willkommen! Bitte nicht bis zum Ende aufsparen.

Ingo Blechschmidt und Matthias Hutzler
Lehrstuhl für Algebra und Zahlentheorie

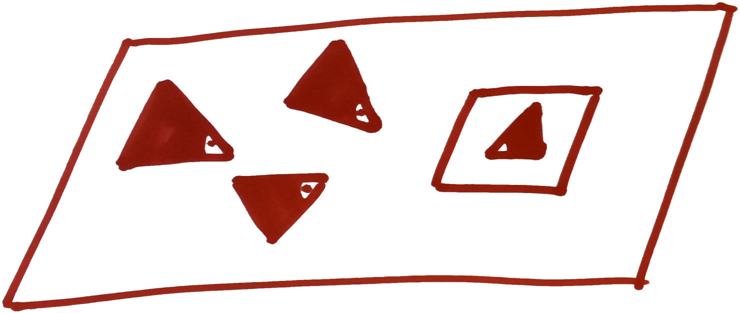
Vier Dimensionen?



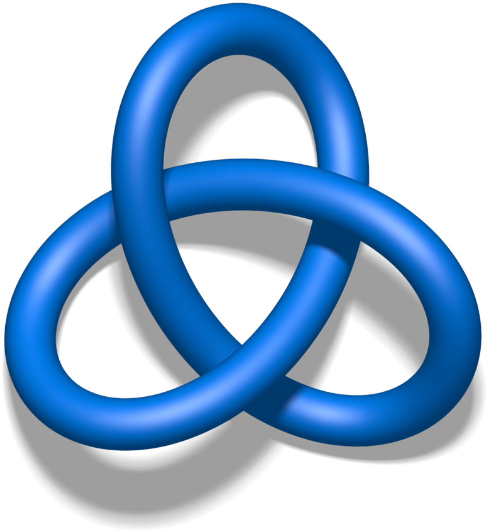
Vier Dimensionen?



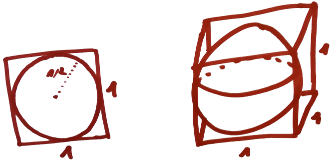
Vier Dimensionen?



Schnürsenkel binden



Hypervolumen von Hyperkugeln

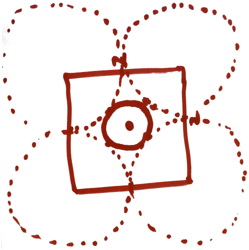


Dimension	Hypervolumen	
$n = 2$	$\pi/4$	$\approx 0{,}785\,\mathrm{m}^2$
$n = 3$	$\pi/6$	$\approx 0{,}524\,\mathrm{m}^3$
$n = 4$	$\pi^2/32$	$\approx 0{,}308\,\mathrm{m}^4$
$n = 5$	$\pi^2/60$	$\approx 0{,}164\,\mathrm{m}^5$
$n = 6$	$\pi^3/384$	$\approx 0{,}081\,\mathrm{m}^6$
$n = 7$	$\pi^3/840$	$\approx 0{,}037\,\mathrm{m}^7$
$n \to \infty$	$\to 0$	

Liebe ist wichtig.

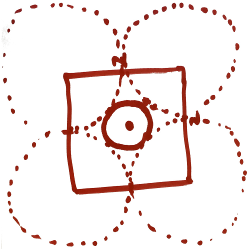
♡

Küssende Hyperkugeln



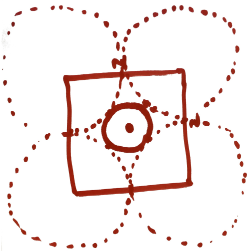
Dimension	Radius der inneren Hyperkugel
$n = 2$	

Küssende Hyperkugeln



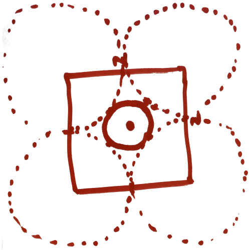
Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$

Küssende Hyperkugeln



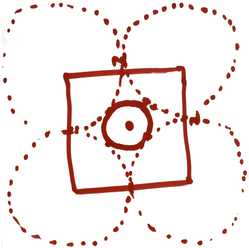
Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	

Küssende Hyperkugeln



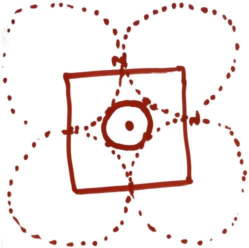
Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$

Küssende Hyperkugeln



Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$

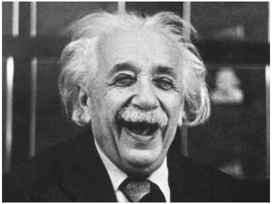
Küssende Hyperkugeln

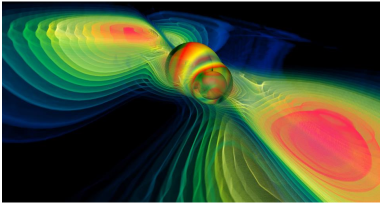


Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$
n	$\sqrt{n} - 1$

Die Entfernung zu den Ecken wird immer größer.

Allgemeine Relativitätstheorie





Einsteins gefeierte **Feldgleichung** besagt

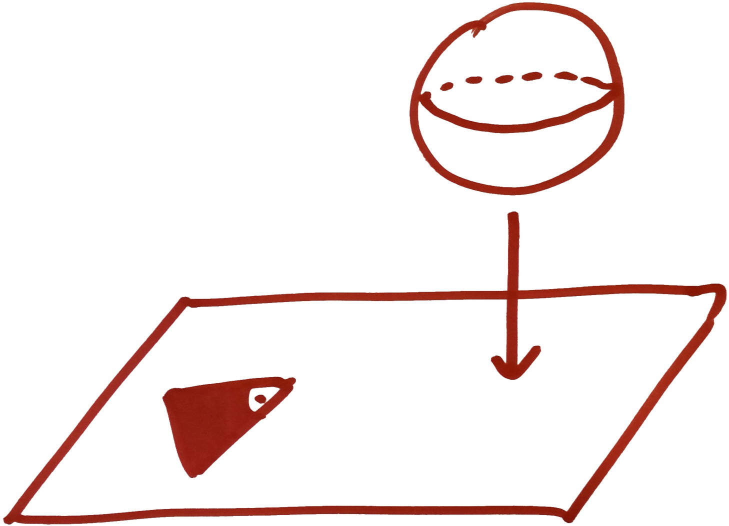
$$G = \kappa \cdot T,$$

wobei

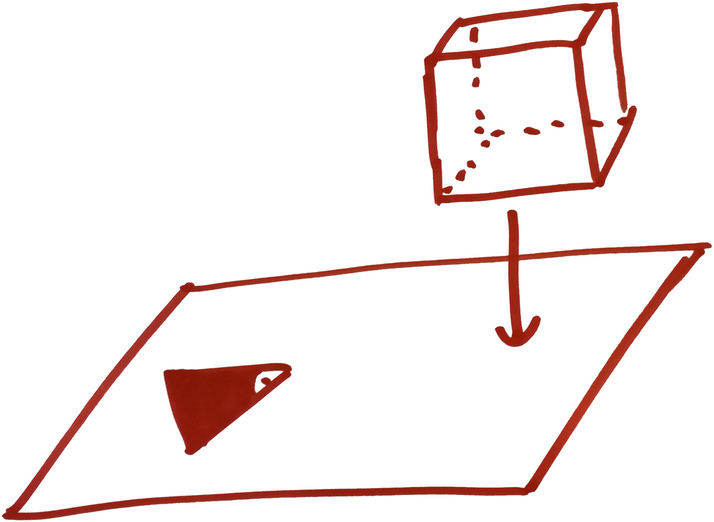
- G die **Raumkrümmung** angibt,
- T die **Massenverteilung** misst und
- κ eine Konstante ist.

In $2+1$ Dimensionen impliziert die Gleichung $T = 0$. Nur in vier und mehr Dimensionen ist die Theorie nichttrivial.

Ankunft eines Hyperballs

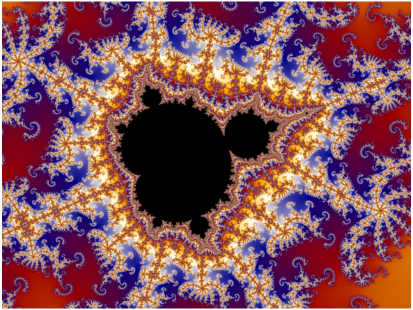


Ankunft eines Tesserakts



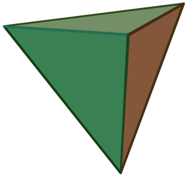
Ein vierdimensionales Fraktal

Ihr kennt das Mandelbrotfraktal. Vielleicht kennt ihr auch die Juliafraktale, von denen es je eins für jeden Punkt der Ebene gibt. Aber wusstet ihr, dass diese unendlich vielen Fraktale nur zweidimensionale Schnitte eines vereinheitlichten vierdimensionalen Fraktals ist? Wir laden euch ein, damit zu spielen.

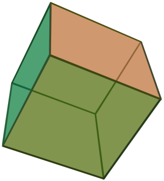


Platonische Körper in 3d

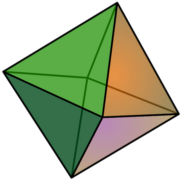
Tetraeder
4 E, 6 K, 4 F



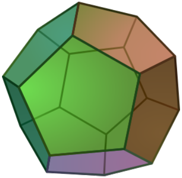
Hexaeder
8 E, 12 K, 6 F



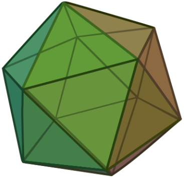
Oktaeder
6 E, 12 K, 8 F



Dodekaeder
20 E, 30 K, 12 F



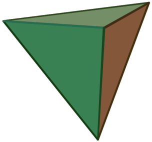
Ikosaeder
12 E, 30 K, 20 F



Platonische Körper in 4d

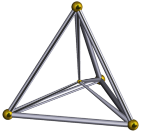
Tetraeder

4E, 6K, 4f



Pentachoron

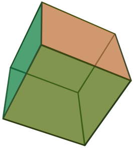
5E, 10K, 10F, 5Z



Platonische Körper in 4d

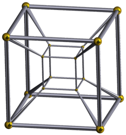
Hexaeder

8E, 12K, 6f



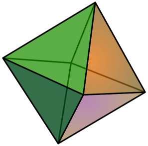
Octachoron

16E, 32K, 24F, 8Z



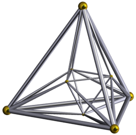
Oktaeder

6K, 12K, 8f



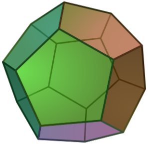
Hexadecachoron

8E, 24K, 32F, 16Z

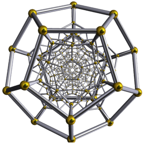


Platonische Körper in 4d

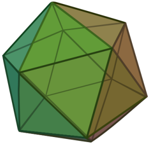
Dodekaeder
20E, 30K, 12f



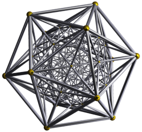
Hecatonicosachoron
600E, 1200K, 720F, 120Z



Ikosaeder
12E, 30K, 20f



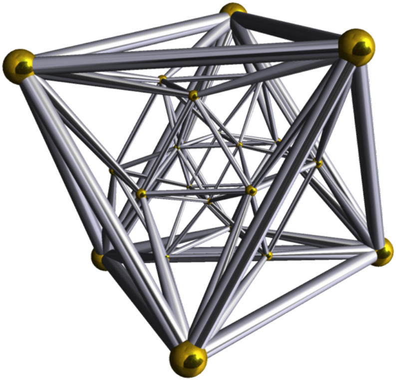
Hexacosichoron
120E, 720K, 1200F, 600Z



Platonische Körper in 4d

Icositetrachoron

24E, 96K, 96F, 24Z

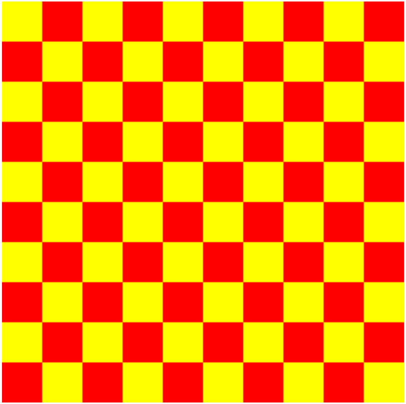


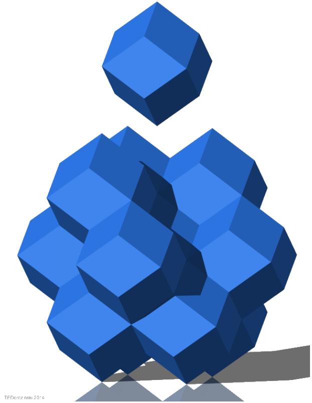
Platonische Körper in 4d

Icositetrachoron

24E, 96K, 96F, 24Z

Pflasterungen

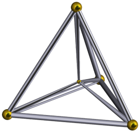




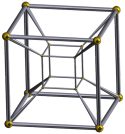
Der 24-Zeller pflastert den vierdimensionalen Raum.

Übersicht

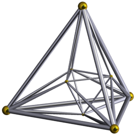
Pentachoron
5E, 10K, 10F, 5Z



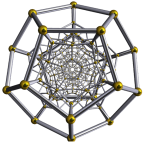
Octachoron
16E, 32K, 24F, 8Z



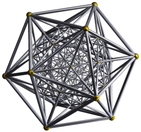
Hexadecachoron
8E, 24K, 32F, 16Z



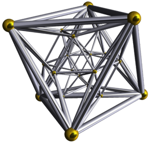
Hecatonicosachoron
600E, 1200K, 720F, 120Z



Hexacosichoron
120E, 720K, 1200F, 600Z

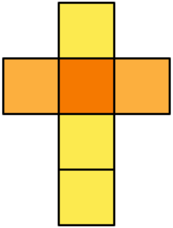


Icositetrachoron
24E, 96K, 96F, 24Z

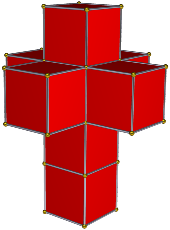


Kleben vierdimensionaler Formen

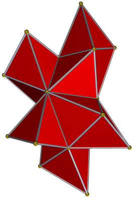
Würfel



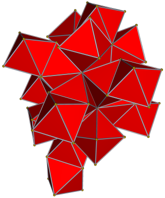
Tesserakt

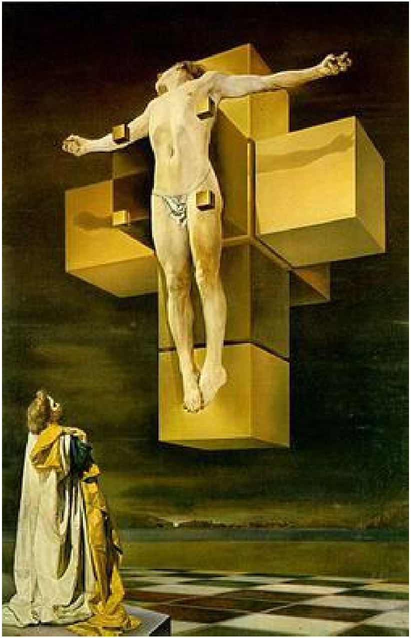


16-Zeller



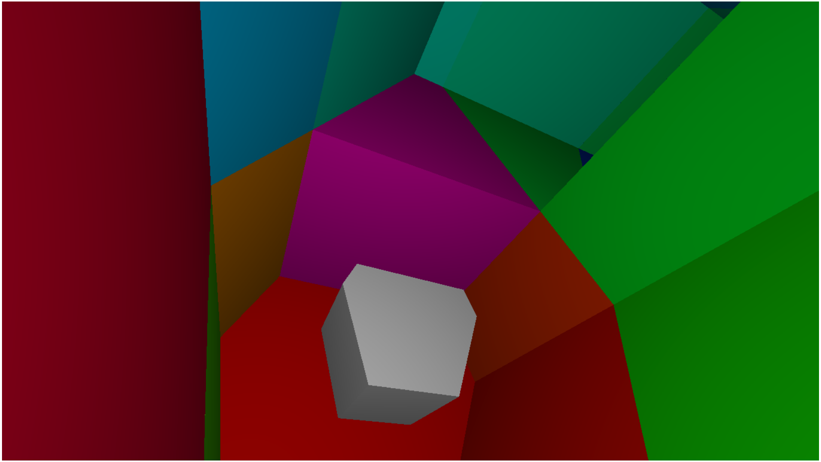
24-Zeller





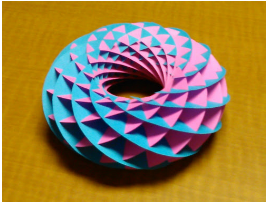
Salvador Dalí: **Corpus Hypercubus** (1954)

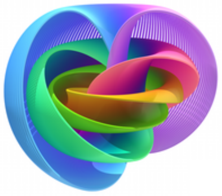
Ein vierdimensionales Labyrinth



Die vierte Dimension …

1. ist faszinierend schön,
2. hilft beim Verständnis der dritten Dimension,





3. ist unabdingbar für die moderne Physik und
4. ist als einzige Dimension noch größtenteils unverstanden.

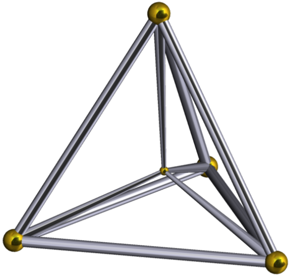
Dimension	1	2	3	4	5	6	7	8	9	…
Anzahl Sphären	1	1	1	??	1	1	28	2	8	…

Catharina Stroppel
Knotentheoretikerin

Julia Grigsby
niedrigdimensionale Topologin

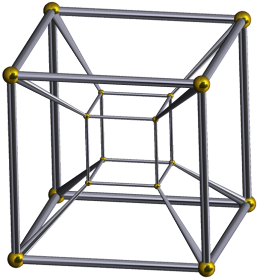
Applaus für unsere Helden!

Pentachoron

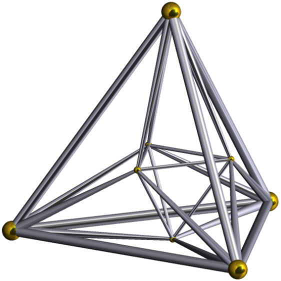


Applaus für unsere Helden!

Tesserakt

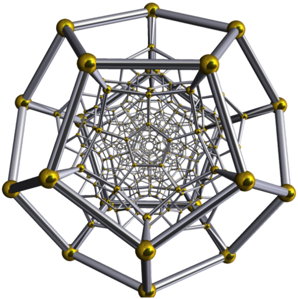


Hexadecachoron

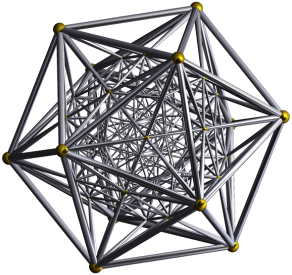


Applaus für unsere Helden!

Hecatonicosachoron

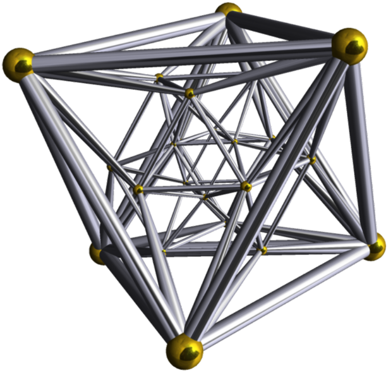


Hexacosichoron



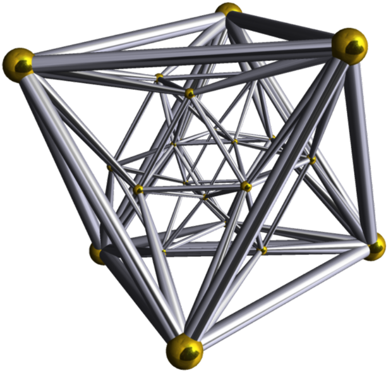
Applaus für unsere Helden!

Icositetrachoron



Applaus für unsere Helden!

Icositetrachoron



https://4d.speicherleck.de/

Image sources

Miscellaneous pictures:
https://commons.wikimedia.org/wiki/File:Blue_Trefoil_Knot.png
http://www.gnuplotting.org/figs/klein_bottle.png
http://4.bp.blogspot.com/_TbkIC-eqFNM/S-dK9dd1cUI/AAAAAAAAFjA/d8qdTHhKy1U/s320/tesseract+unfolded.png
https://en.wikipedia.org/wiki/File:Tetrahedron.svg
https://en.wikipedia.org/wiki/File:Hexahedron.svg
https://en.wikipedia.org/wiki/File:Octahedron.svg
https://en.wikipedia.org/wiki/File:Dodecahedron.svg
https://en.wikipedia.org/wiki/File:Icosahedron.svg

Rendered images of four-dimensional bodies created by Robert Webb with his Stella software:
https://en.wikipedia.org/wiki/File:Ortho_solid_011-uniform_polychoron_53p-t0.png
https://en.wikipedia.org/wiki/File:Schlegel_wireframe_5-cell.png
https://en.wikipedia.org/wiki/File:Schlegel_wireframe_8-cell.png
https://en.wikipedia.org/wiki/File:Schlegel_wireframe_16-cell.png
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https://en.wikipedia.org/wiki/File:Schlegel_wireframe_120-cell.png
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