

Die wundersame Welt der vierdimensionalen Geometrie

Weihnachtsvorlesung am 18. Dezember 2017

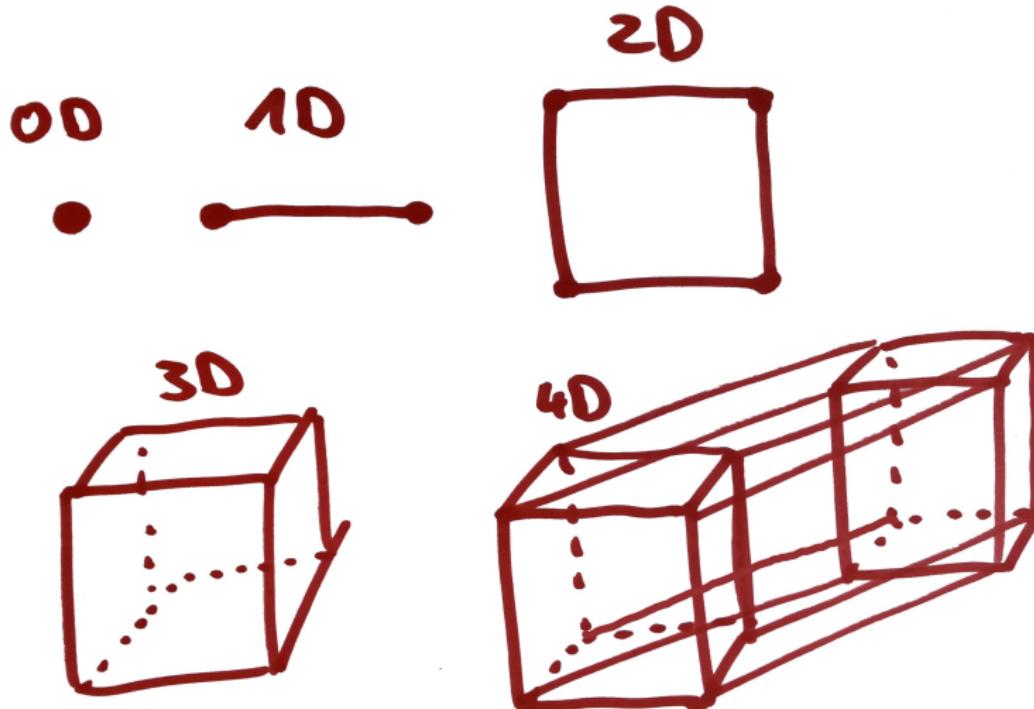
Fragen sind jederzeit willkommen! Bitte nicht bis zum Ende aufsparen.

Ingo Blechschmidt und Matthias Hutzler
Lehrstuhl für Algebra und Zahlentheorie

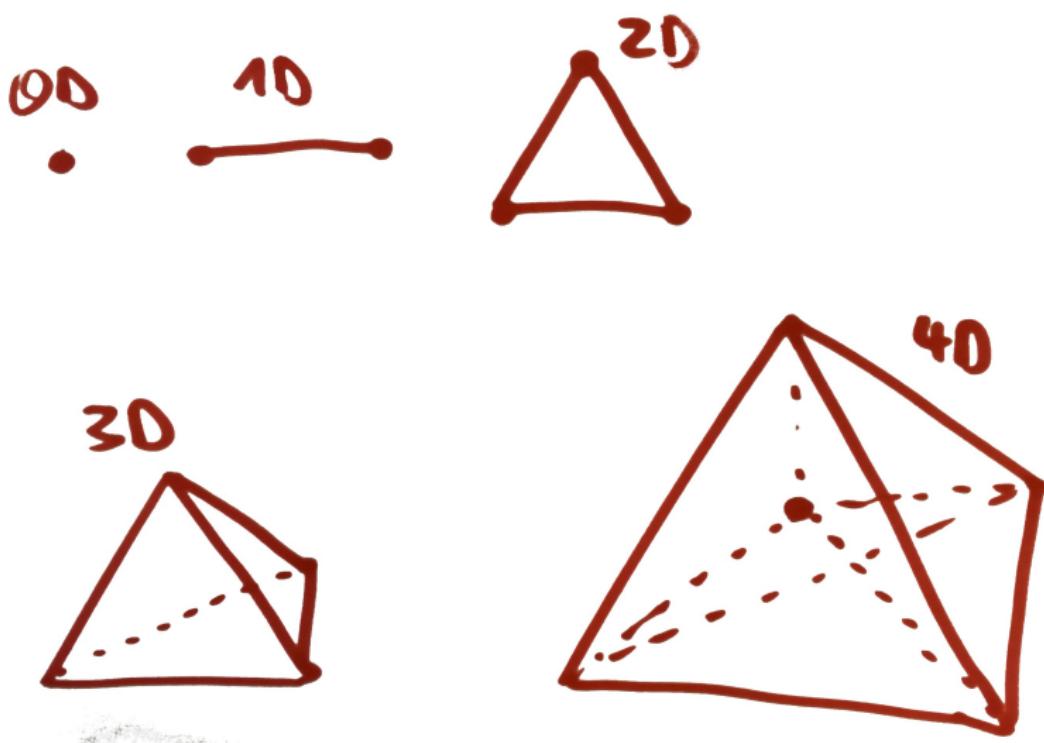
Wie geht's weiter?

1, ∞ , 5, 6, ??

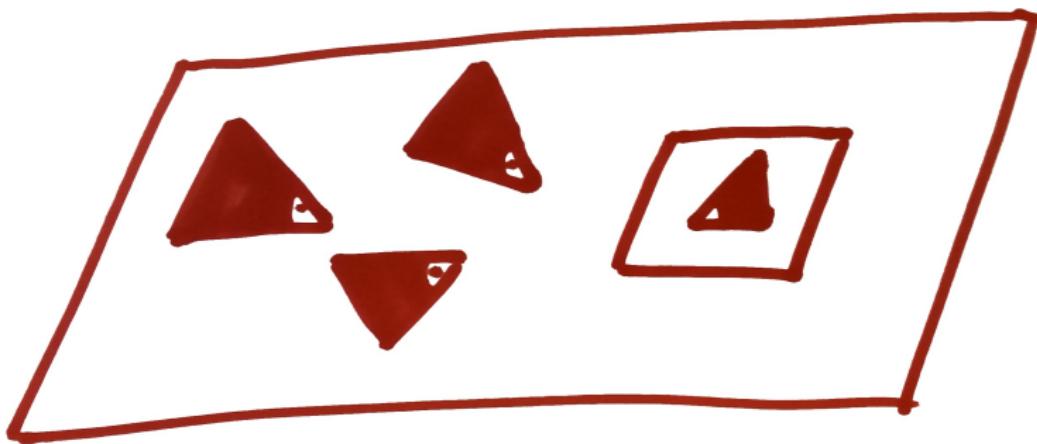
Vier Dimensionen?



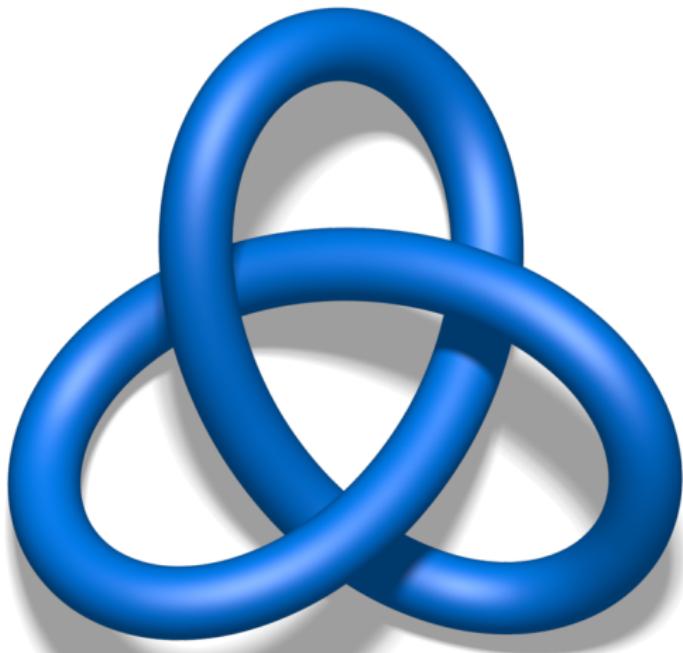
Vier Dimensionen?



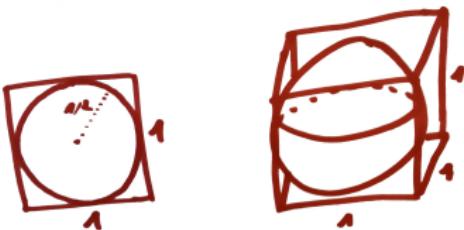
Vier Dimensionen?



Schnürsenkel binden



Hypervolumen von Hyperkugeln

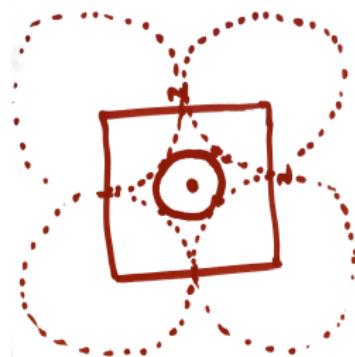


Dimension	Hypervolumen	
$n = 2$	$\pi/4$	$\approx 0,785 \text{ m}^2$
$n = 3$	$\pi/6$	$\approx 0,524 \text{ m}^3$
$n = 4$	$\pi^2/32$	$\approx 0,308 \text{ m}^4$
$n = 5$	$\pi^2/60$	$\approx 0,164 \text{ m}^5$
$n = 6$	$\pi^3/384$	$\approx 0,081 \text{ m}^6$
$n = 7$	$\pi^3/840$	$\approx 0,037 \text{ m}^7$
$n \rightarrow \infty$	$\rightarrow 0$	

Liebe ist
wichtig.



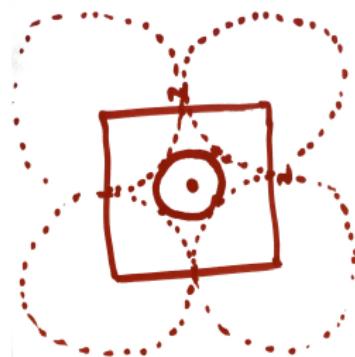
Küssende Hyperkugeln



Dimension Radius der inneren Hyperkugel

$n = 2$

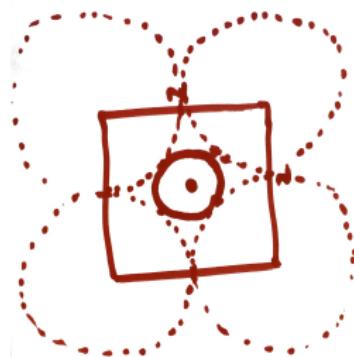
Küssende Hyperkugeln



Dimension	Radius der inneren Hyperkugel
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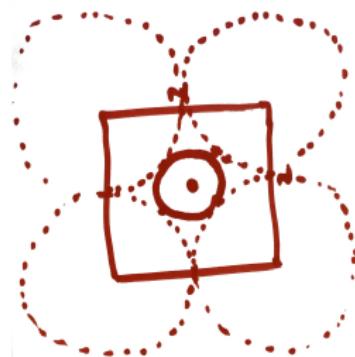
$$n = 2 \quad \sqrt{2} - 1$$

Küssende Hyperkugeln



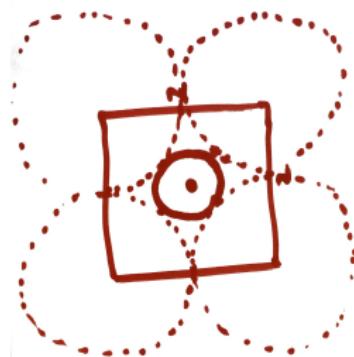
Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	

Küssende Hyperkugeln



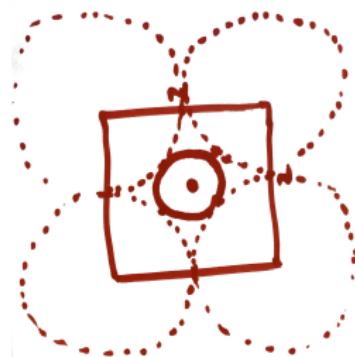
Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$

Küssende Hyperkugeln



Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$

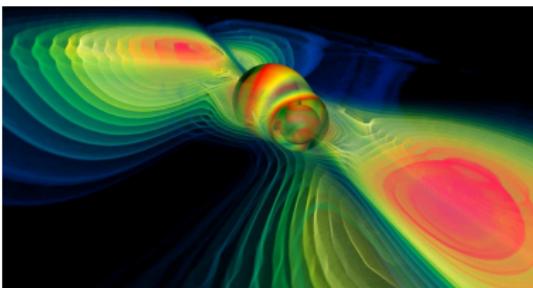
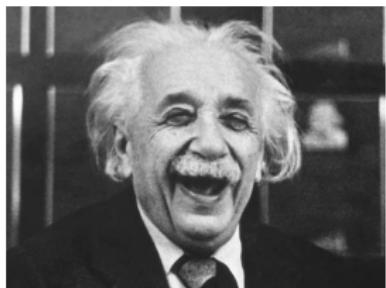
Küssende Hyperkugeln



Dimension	Radius der inneren Hyperkugel
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$
n	$\sqrt{n} - 1$

Die Entfernung zu den Ecken wird immer größer.

Allgemeine Relativitätstheorie



Einstiens gefeierte **Feldgleichung** besagt

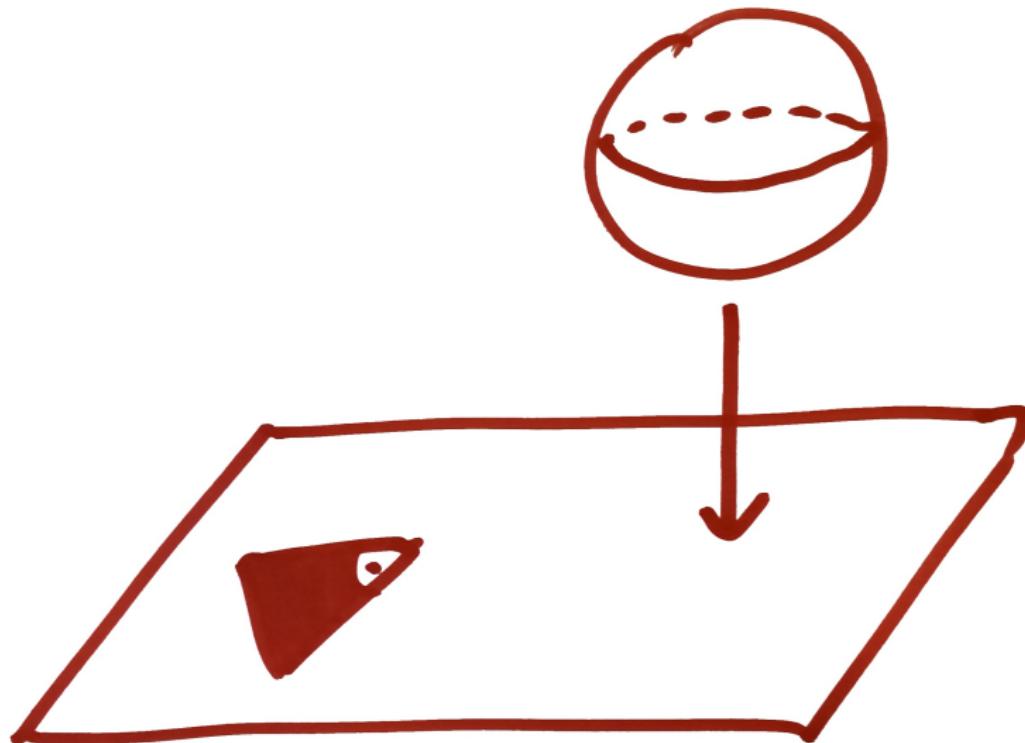
$$G = \kappa \cdot T,$$

wobei

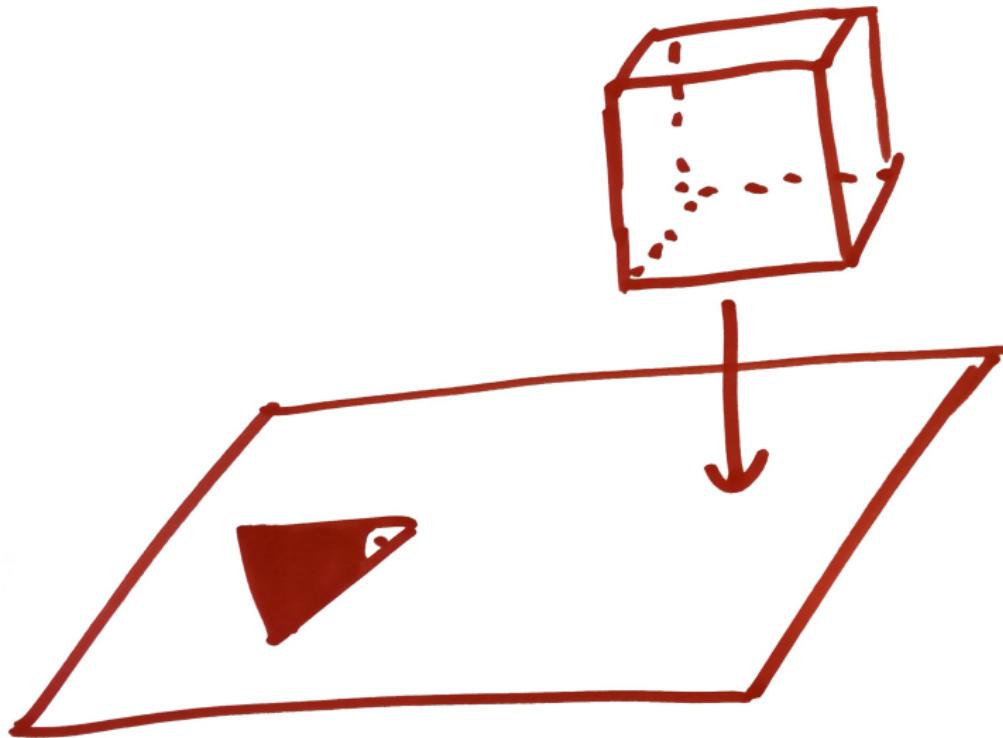
- G die **Raumkrümmung** angibt,
- T die **Massenverteilung** misst und
- κ eine Konstante ist.

In $2 + 1$ Dimensionen impliziert die Gleichung $T = 0$. Nur in vier und mehr Dimensionen ist die Theorie nichttrivial.

Ankunft eines Hyperballs

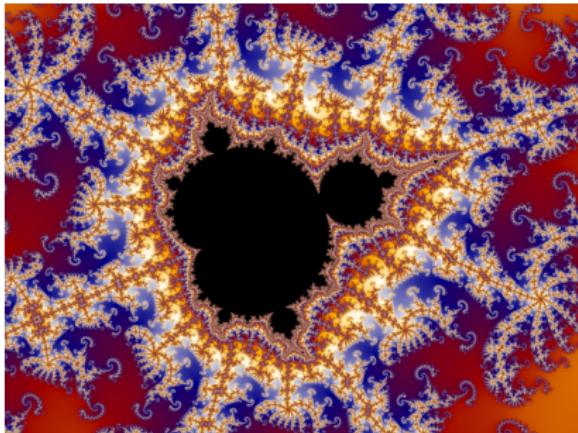


Ankunft eines Tesserakts



Ein vierdimensionales Fraktal

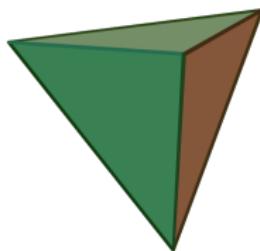
Ihr kennt das Mandelbrotfraktal. Vielleicht kennt ihr auch die Juliafraktale, von denen es je eins für jeden Punkt der Ebene gibt. Aber wusstet ihr, dass diese unendlich vielen Fraktale nur zweidimensionale Schnitte eines vereinheitlichten vierdimensionalen Fraktals ist? Wir laden euch ein, **damit zu spielen**.



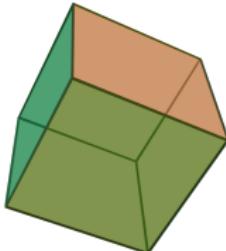
Platonische Körper in 3d

Tetraeder

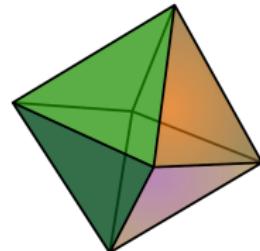
4 E, 6 K, 4 F

**Hexaeder**

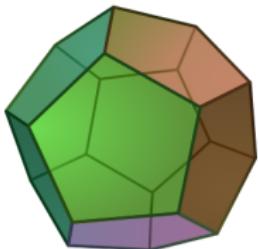
8 E, 12 K, 6 F

**Oktaeder**

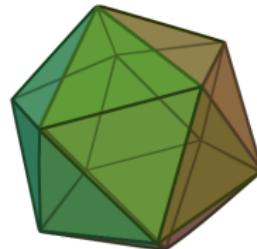
6 E, 12 K, 8 F

**Dodekaeder**

20 E, 30 K, 12 F

**Ikosaeder**

12 E, 30 K, 20 F



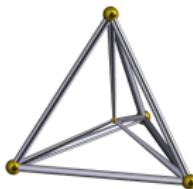
Platonische Körper in 4d

Tetraeder

4E, 6K, 4F

**Pentachoron**

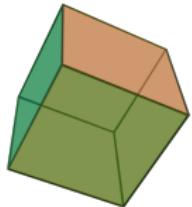
5E, 10K, 10F, 5Z



Platonische Körper in 4d

Hexaeder

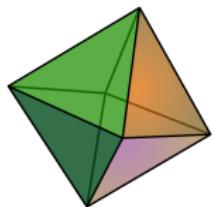
8E, 12K, 6F

**Octachoron**

16E, 32K, 24F, 8Z

**Oktaeder**

6K, 12K, 8F

**Hexadecachoron**

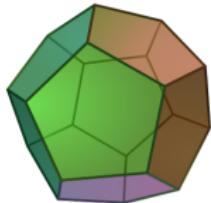
8E, 24K, 32F, 16Z



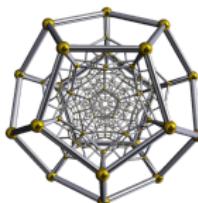
Platonische Körper in 4d

Dodekaeder

20E, 30K, 12F

**Hecatonicosachoron**

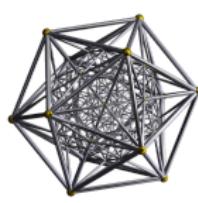
600E, 1200K, 720F, 120Z

**Ikosaeder**

12E, 30K, 20F

**Hexacosichoron**

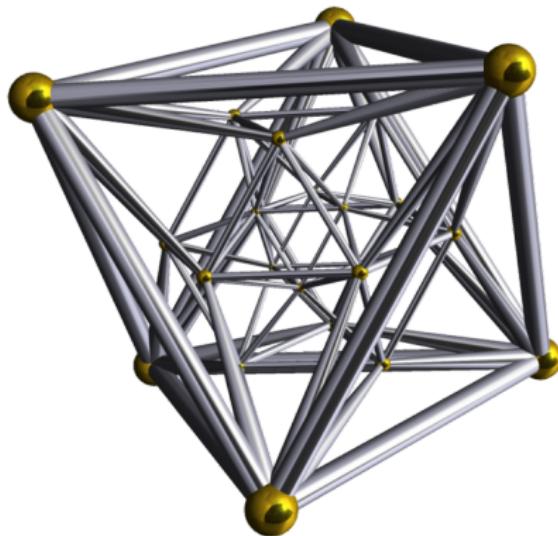
120E, 720K, 1200F, 600Z



Platonische Körper in 4d

Icositetrachoron

24E, 96K, 96F, 24Z

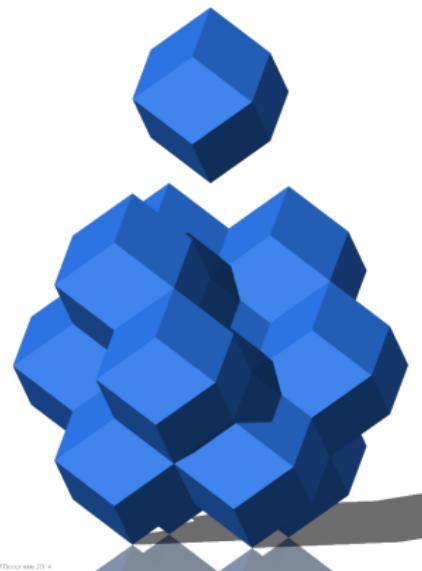
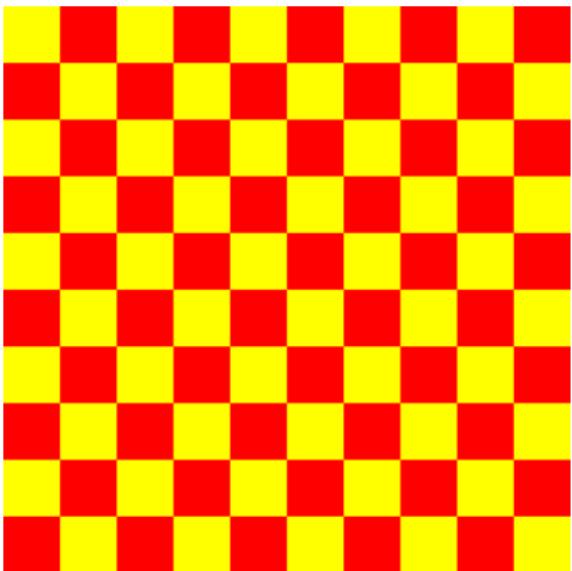


Platonische Körper in 4d

Icositetrachoron

24E, 96K, 96F, 24Z

Pflasterungen



Der 24-Zeller pflastert den vierdimensionalen Raum.

Übersicht

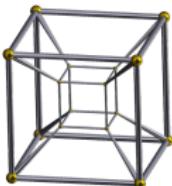
Pentachoron

5E, 10K, 10F, 5Z



Octachoron

16E, 32K, 24F, 8Z



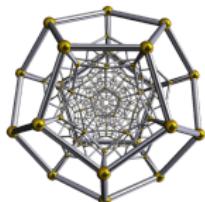
Hexadecachoron

8E, 24K, 32F, 16Z



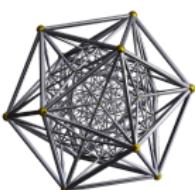
Hecatonicosachoron

600E, 1200K, 720F, 120Z



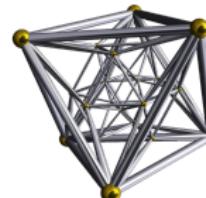
Hexacosichoron

120E, 720K, 1200F, 600Z



Icositetrachoron

24E, 96K, 96F, 24Z

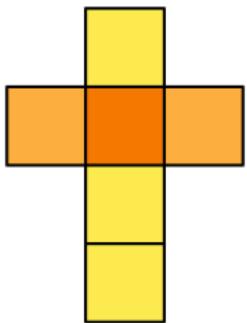


In beliebigen Dimensionen

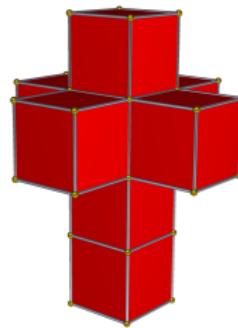
Dimension	Anzahl platonischer Körper
$n = 1$	1 (nur die Linie)
$n = 2$	∞ (Dreieck, Quadrat, Fünfeck, Sechseck, ...)
$n = 3$	5
$n = 4$	6
$n = 5$	3 (nur Simplex, Hyperwürfel und sein Duales)
$n = 6$	3 (nur Simplex, Hyperwürfel und sein Duales)
$n = 7$	3 (nur Simplex, Hyperwürfel und sein Duales)
$n = 8$	3 (nur Simplex, Hyperwürfel und sein Duales)
<i>und so weiter</i>	

Kleben vierdimensionaler Formen

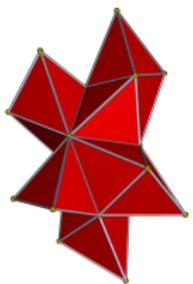
Würfel



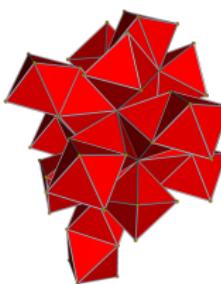
Tesseract



16-Zeller



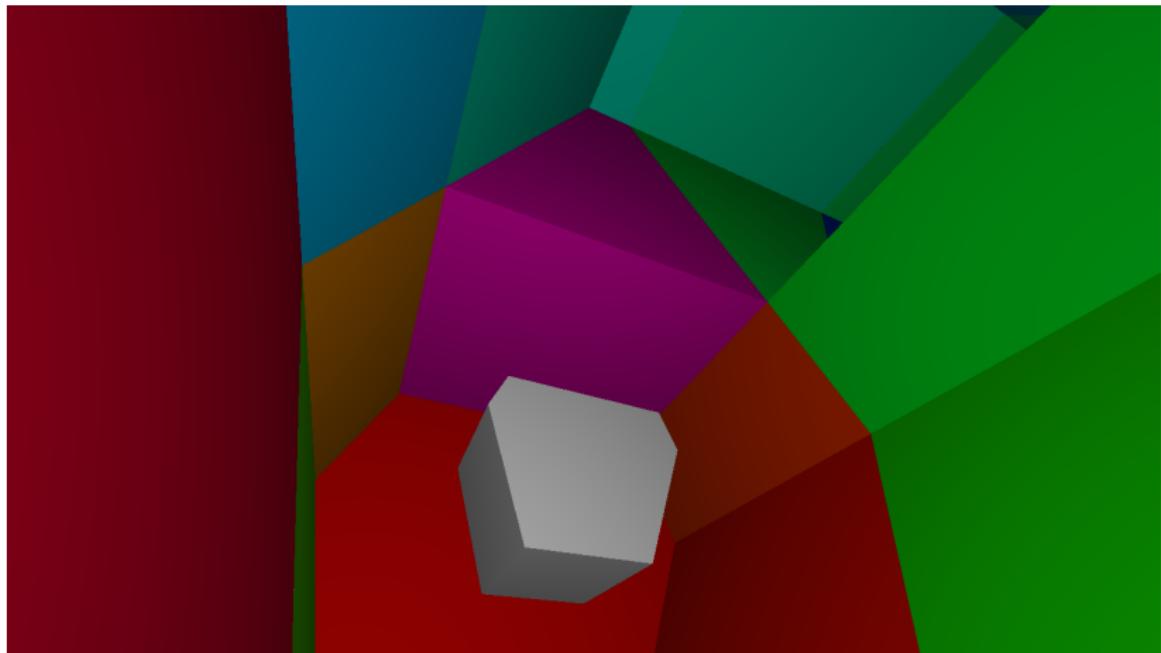
24-Zeller





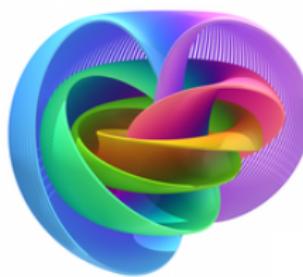
Salvador Dalí: **Corpus Hypercubus** (1954)

Ein vierdimensionales Labyrinth



Die vierte Dimension ...

- 1 ist faszinierend schön,
- 2 hilft beim Verständnis der dritten Dimension,



- 3 ist unabdingbar für die moderne Physik und
- 4 ist als einzige Dimension noch größtenteils unverstanden.

Dimension	1	2	3	4	5	6	7	8	9	...
Anzahl Sphären	1	1	1	??	1	1	28	2	8	...

Catharina Stroppel
Knotentheoretikerin

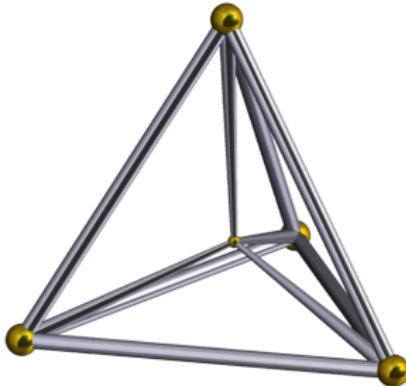


Julia Grigsby
niedrigdimensionale Topologin



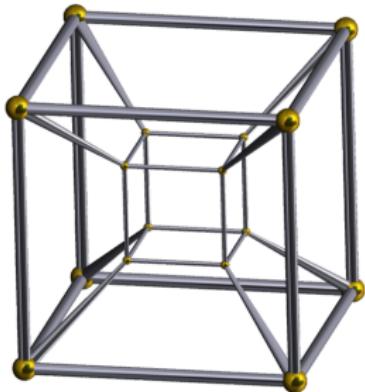
Applaus für unsere Helden!

Pentachoron

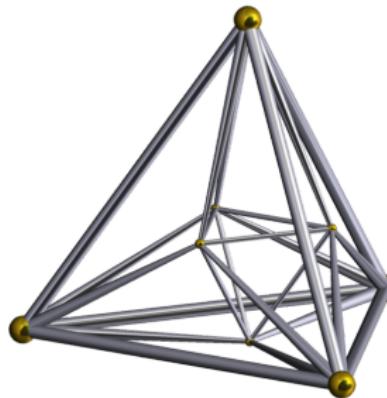


Applaus für unsere Helden!

Tesserakt

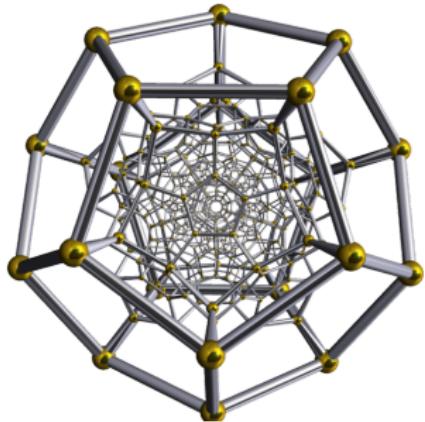


Hexadecachoron

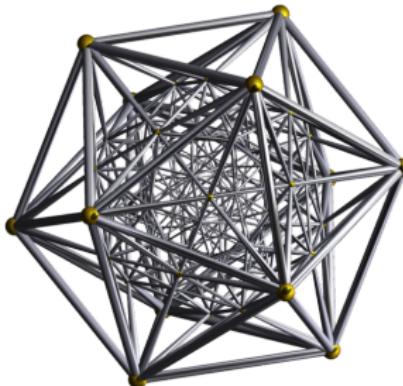


Applaus für unsere Helden!

Hecatonicosachoron

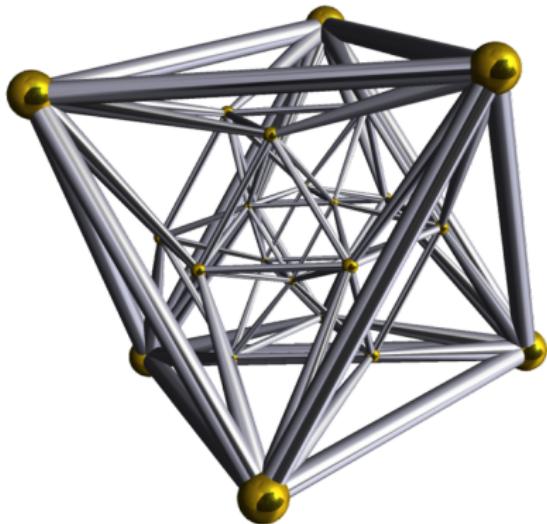


Hexacosichoron



Applaus für unsere Helden!

Icositetrachoron



Unterlagen zum Vortrag:
<http://4d.speicherleck.de/>

Image sources

Rendered images of four-dimensional bodies created by Robert Webb with his Stella software:

https://en.wikipedia.org/wiki/File:Ortho_solid_011-uniform_polychoron_53p-t0.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_5-cell.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_8-cell.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_16-cell.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_24-cell.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_120-cell.png

https://en.wikipedia.org/wiki/File:Schlegel_wireframe_600-cell_vertex-centered.png

Image sources

Miscellaneous pictures:

http://4.bp.blogspot.com/_TbkIC-eqFNM/S-dK9dd1cUI/AAAAAAAFAjA/d8qdTHhKy1U/s320/tesseract+unfolded.png
<http://gwydir.demon.co.uk/jo/tess/optical6.gif>
https://commons.wikimedia.org/wiki/File:Blue_Trefoil_Knot.png
<https://en.wikipedia.org/wiki/File:Dodecahedron.svg>
<https://en.wikipedia.org/wiki/File:Hexahedron.svg>
<https://en.wikipedia.org/wiki/File:Icosahedron.svg>
<https://en.wikipedia.org/wiki/File:Octahedron.svg>
<https://en.wikipedia.org/wiki/File:Tetrahedron.svg>
<https://mathlesstraveled.files.wordpress.com/2017/01/villarceau-torus-small.jpg>
<https://upload.wikimedia.org/wikipedia/commons/1/1e/600-cell.gif>
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https://upload.wikimedia.org/wikipedia/commons/thumb/b/b9/Hopf_Fibration.png/250px-Hopf_Fibration.png
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http://www.gnuplotting.org/figs/klein_bottle.png
http://www.math.uni-bonn.de/ag/stroppel/Picture_cs2.jpg