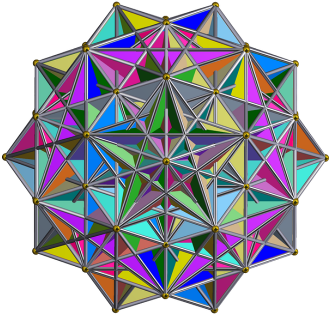
$$V_n = \int_0^1 \int_0^{2\pi} V_{n-2}(\sqrt{1 - r^2})^{n-2} \, r \, d\theta \, dr$$

The curious world of four-dimensional geometry

Ingo Blechschmidt and Matthias Hutzler
with thanks to Sven Prüfer
Universität Augsburg
December 29th, 2016



The curious world of four-dimensional geometry

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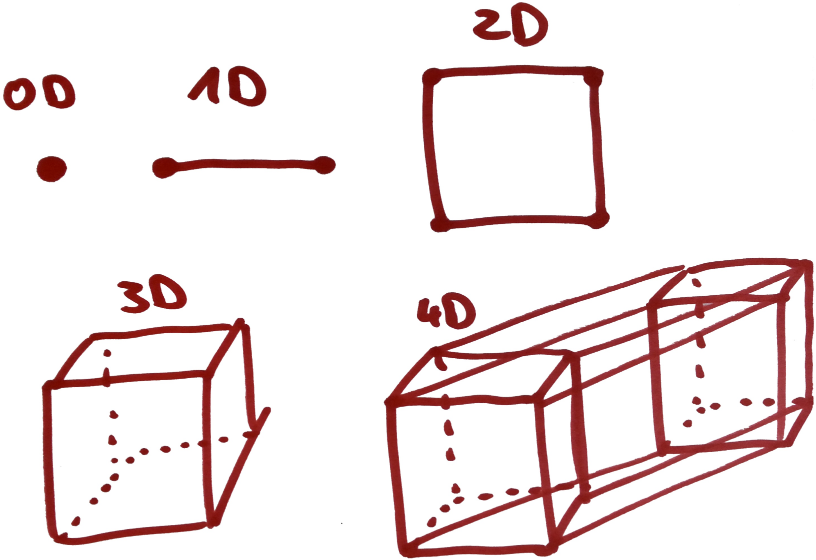
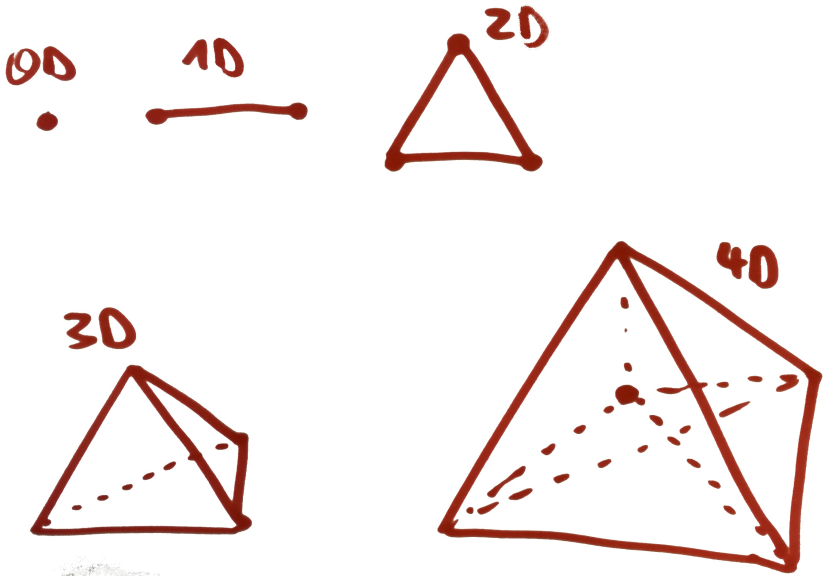
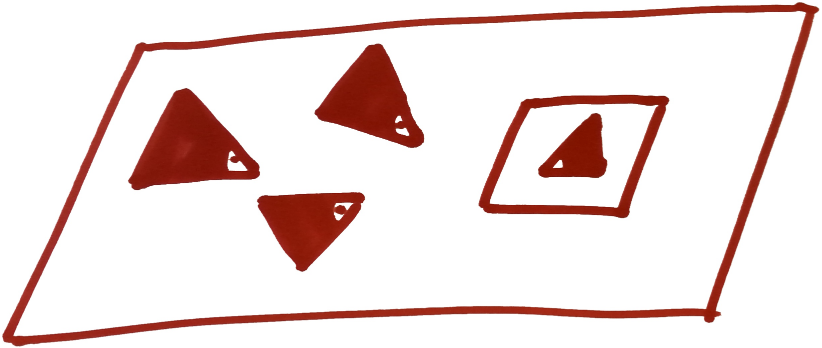
Four dimensions?



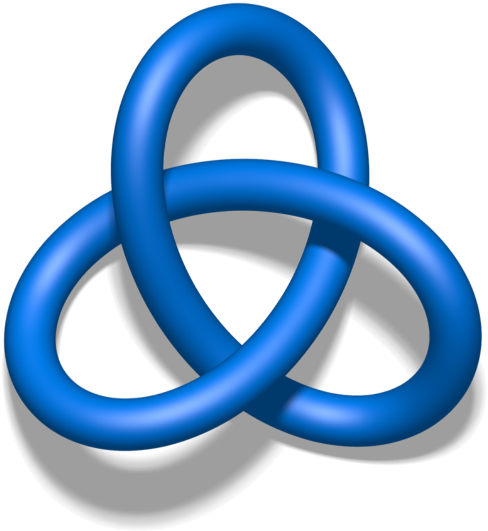
Four dimensions?



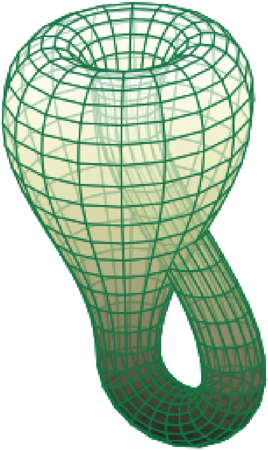
Four dimensions?



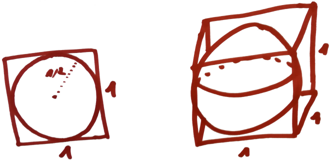
Tieing your shoelaces



The Klein bottle

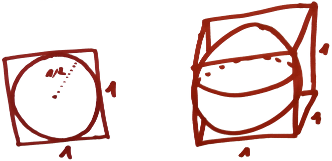


Hypervolume of hyperballs



dimension	hypervolume	
$n = 2$	$\pi/4$	≈ 0.785
$n = 3$	$\pi/6$	≈ 0.524
$n = 4$	$\pi^2/32$	≈ 0.308
$n = 5$	$\pi^2/60$	≈ 0.164
$n = 6$	$\pi^3/384$	≈ 0.081
$n = 7$	$\pi^3/840$	≈ 0.037
$n \to \infty$	$\to 0$	

Hypervolume of hyperballs



dimension	hypervolume	
$n = 0$	1	≈ 1.000
$n = 1$	1	≈ 1.000
$n = 2$	$\pi/4$	≈ 0.785
$n = 3$	$\pi/6$	≈ 0.524
$n = 4$	$\pi^2/32$	≈ 0.308
$n = 5$	$\pi^2/60$	≈ 0.164
$n = 6$	$\pi^3/384$	≈ 0.081
$n = 7$	$\pi^3/840$	≈ 0.037
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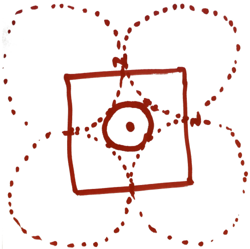
Love is important.

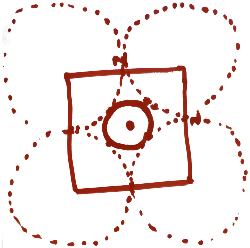
♡

Kissing hyperspheres



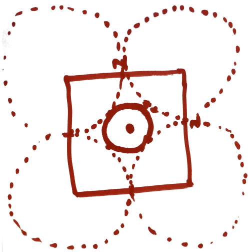
dimension	radius of the inner hypersphere
$n = 2$	$\sqrt{2} - 1$

Kissing hyperspheres



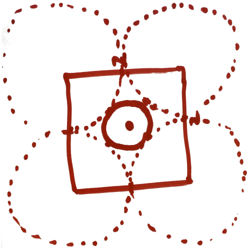
dimension	radius of the inner hypersphere
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$

Kissing hyperspheres



dimension	radius of the inner hypersphere
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$

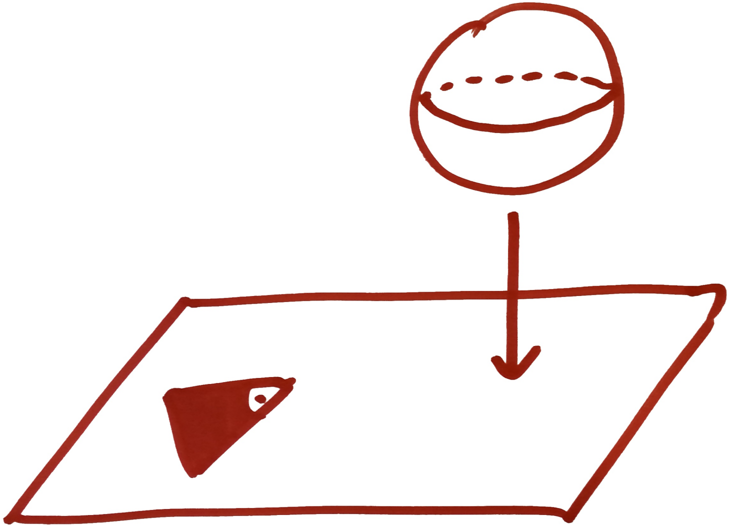
Kissing hyperspheres



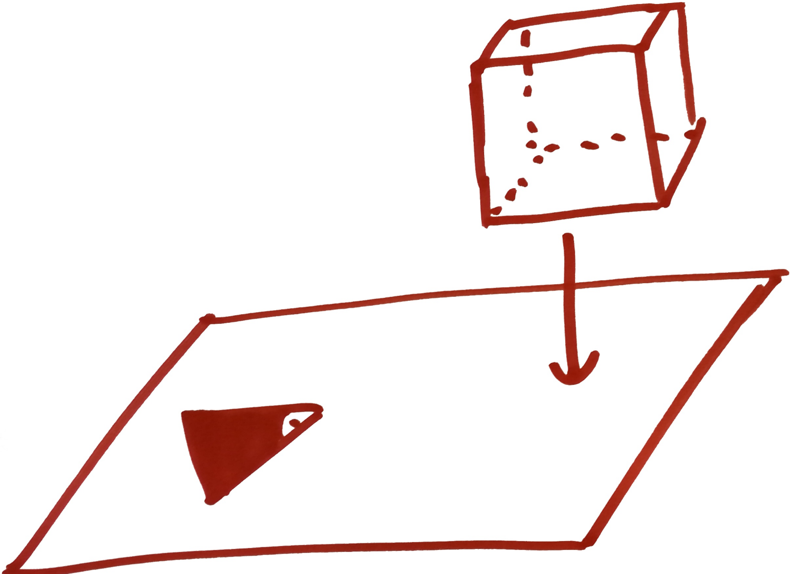
dimension	radius of the inner hypersphere
$n = 2$	$\sqrt{2} - 1$
$n = 3$	$\sqrt{3} - 1$
$n = 4$	$\sqrt{4} - 1$
n	$\sqrt{n} - 1$

The distance to the corners gets bigger and bigger.

A hyperball arrives



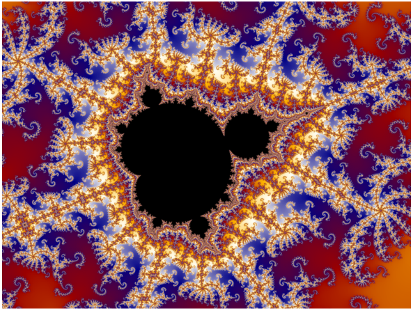
A tesseract arrives



A four-dimensional fractal

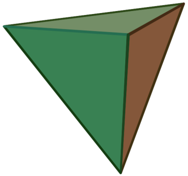
You know the Mandelbrot set. Maybe you also know the Julia sets associated to any point of the plane.

But did you know that these infinitely many fractals are just two-dimensional cuts of an unifying four-dimensional fractal? We invite you to play with it.

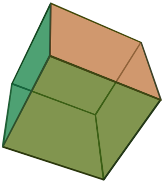


Platonic solids in 3D

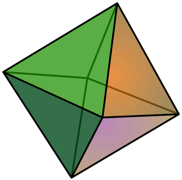
Tetrahedron
4 faces, 4 vertices



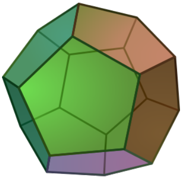
Hexahedron
6 faces, 8 vertices



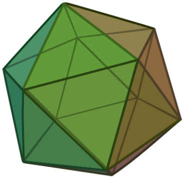
Octahedron
8 faces, 6 vertices



Dodecahedron
12 faces, 20 vertices

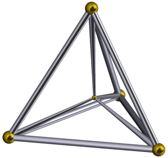


Icosahedron
20 faces, 12 vertices

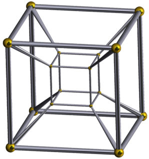


Platonic solids in 4D

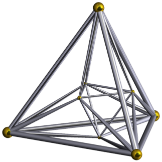
Pentachoron
5v, 10e, 10f, 5c



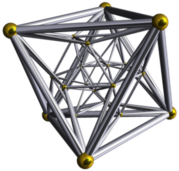
Octachoron
16v, 32e, 24f, 8c



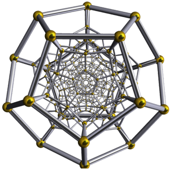
Hexadecahedron
8v, 24e, 32f, 16c



Icositetrachoron
24v, 96e, 96f, 24c

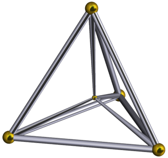


Hecatonicosachoron
600v, 1200e, 720f, 120c

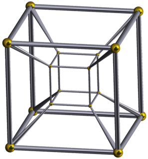


Platonic solids in 4D

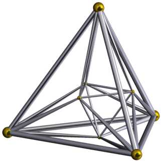
Pentachoron
5v, 10e, 10f, 5c



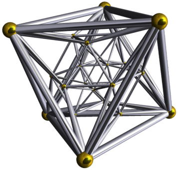
Octachoron
16v, 32e, 24f, 8c



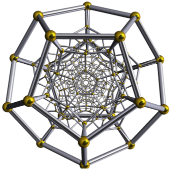
Hexadecahedron
8v, 24e, 32f, 16c



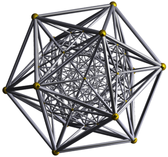
Icositetrachoron
24v, 96e, 96f, 24c



Hecatonicosachoron
600v, 1200e, 720f, 120c



Hexacosichoron
120v, 720e, 1200f, 600c



Glueing four-dimensional shapes