

# Which One Looks Better?

| signal processing concept  | algebraic concept<br>(coordinate free)  | in coordinates   |
|--|---|--|
| filter<br>signal<br>filtering<br>impulse<br>impulse response of $h \in \mathcal{A}$  | $h \in \mathcal{A}$ (algebra)<br>$s = \sum s_i b_i \in \mathcal{M}$ ( $\mathcal{A}$ -module)<br>$h \cdot s$<br>base vector $b_i \in \mathcal{M}$<br>$h \cdot b_i \in \mathcal{M}$ | $\phi(h) \in \mathbb{C}^{I \times I}$<br>$\mathbf{s} = (s_i)_{i \in I} \in \mathbb{C}^I$<br>$\phi(h) \cdot \mathbf{s}$<br>$\mathbf{b}_i = (\dots, 0, 1, 0, \dots)^T \in \mathbb{C}^I$<br>$\phi(h) \cdot \mathbf{b}_i = (\dots, h_{-1}, h_0, h_1, \dots)^T \in \mathbb{C}^I$  |
| Fourier transform<br>spectrum of signal<br>frequency response of $h \in \mathcal{A}$ | $\Delta : \mathcal{M} \rightarrow \bigoplus_{\omega \in W} \mathcal{M}_\omega$<br>$\Delta(s) = (s_\omega)_{\omega \in W} = \omega \mapsto s_\omega$                               | $\mathcal{F} : \mathbb{C}^I \rightarrow \bigoplus_{\omega \in W} \mathbb{C}^{d_\omega}$<br>$\Leftrightarrow \phi \rightarrow \bigoplus_{\omega \in W} \phi_\omega$<br>$\mathcal{F}(\mathbf{s}) = (\mathbf{s}_\omega)_{\omega \in W} = \omega \mapsto \mathbf{s}_\omega$<br>$(\phi_\omega(h))_{\omega \in W} = \omega \mapsto \phi_\omega(h)$ |

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|---|--|---|
| filter                                    | $h \in \mathcal{A}$ (algebra)  | $\phi(h) \in \mathbb{C}^{I \times I}$   |
| signal                                    | $s = \sum s_i b_i \in \mathcal{M}$ ( $\mathcal{A}$ -module)                    | $\mathbf{s} = (s_i)_{i \in I} \in \mathbb{C}^I$   |
| filtering                                 | $h \cdot s$  | $\phi(h) \cdot \mathbf{s}$  |
| impulse                                   | base vector $b_i \in \mathcal{M}$  | $\mathbf{b}_i = (\dots, 0, 1, 0, \dots)^T \in \mathbb{C}^I$   |
| impulse response of $h \in \mathcal{A}$   | $h \cdot b_i \in \mathcal{M}$  | $\phi(h) \cdot \mathbf{b}_i = (\dots, h_{-1}, h_0, h_1, \dots)^T \in \mathbb{C}^I$  |
| Fourier transform                         | $\Delta : \mathcal{M} \rightarrow \bigoplus_{\omega \in W} \mathcal{M}_\omega$ | $\mathcal{F} : \mathbb{C}^I \rightarrow \bigoplus_{\omega \in W} \mathbb{C}^{d_\omega} \Leftrightarrow \phi \rightarrow \bigoplus_{\omega \in W} \phi_\omega$ |
| spectrum of signal                        | $\Delta(s) = (s_\omega)_{\omega \in W} = \omega \mapsto s_\omega$              | $\mathcal{F}(\mathbf{s}) = (\mathbf{s}_\omega)_{\omega \in W} = \omega \mapsto \mathbf{s}_\omega$   |
| frequency response of $h \in \mathcal{A}$ | n.a.   | $(\phi_\omega(h))_{\omega \in W} = \omega \mapsto \phi_\omega(h)$   |

Easy decision, isn't it?