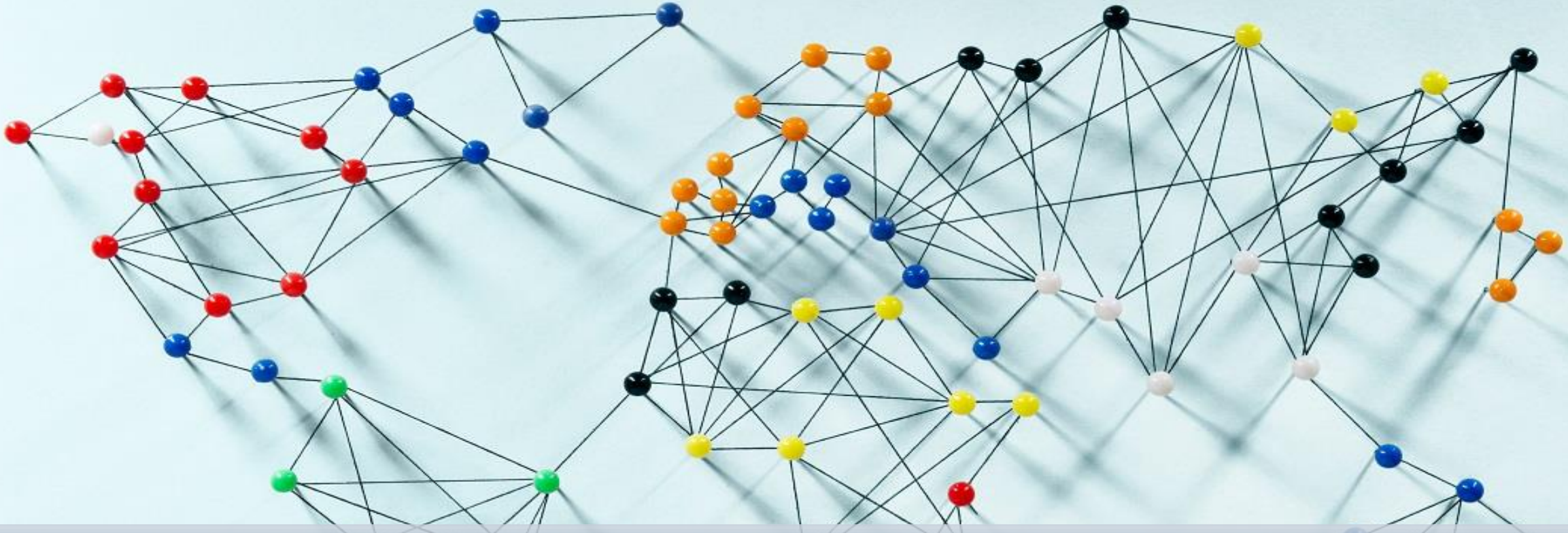

Graph theory, Recursion and its implementation in board games

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Muhammad Tahir Ahmed 24151
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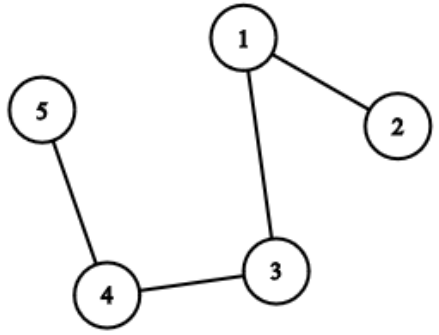
GRAPH

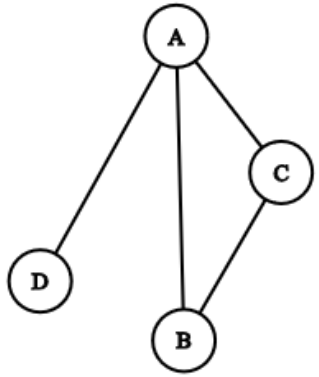
A list of pairs of “things” called vertex/vertices, and lines between those points, called edges

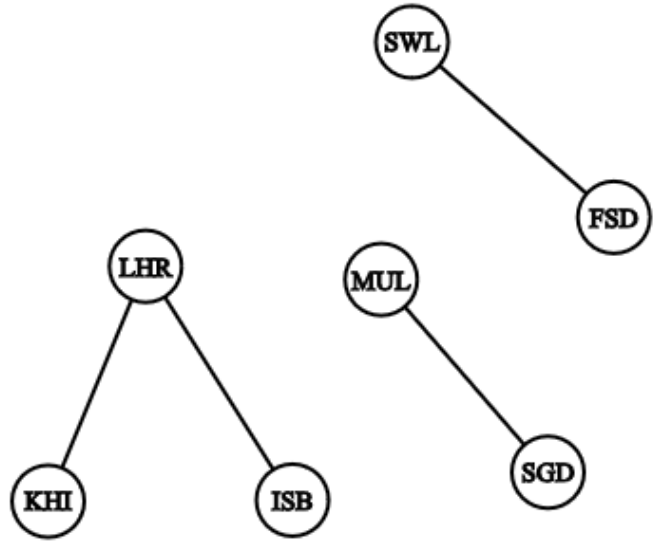
A = {(1, 2), (1, 3), (3, 4), (4, 5)}

B = {(A, B), (B, C), (A, C), (A, D)}

C = {(LHR,KRA), (LHR,ISB), (FSD,SWL), (STD,MUL)}







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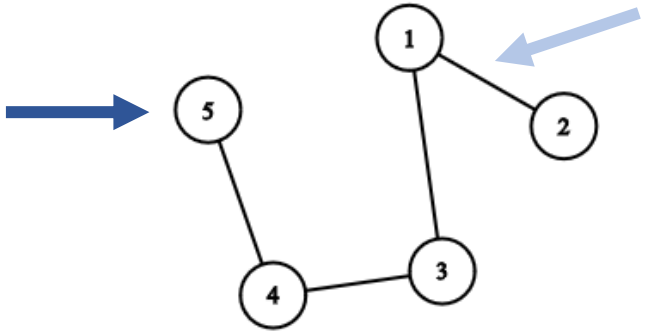

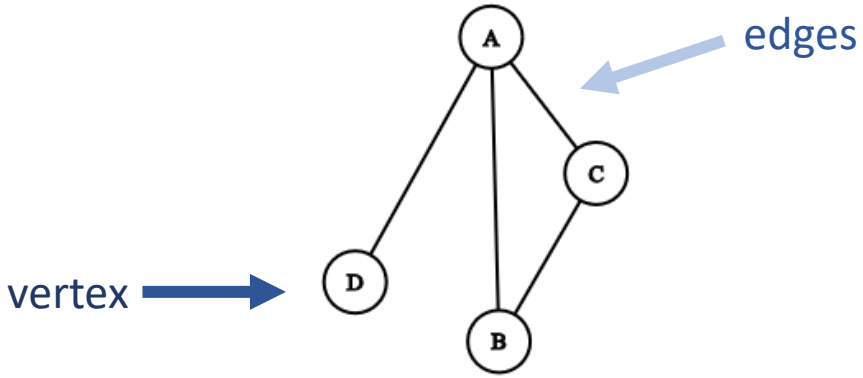

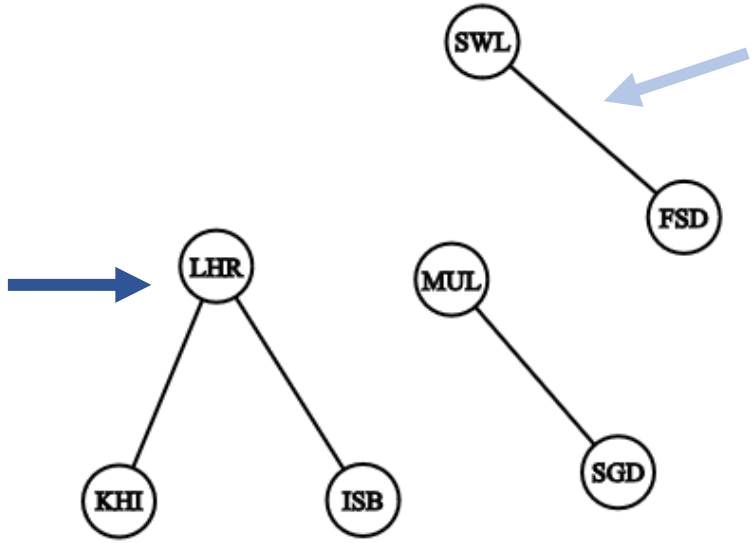

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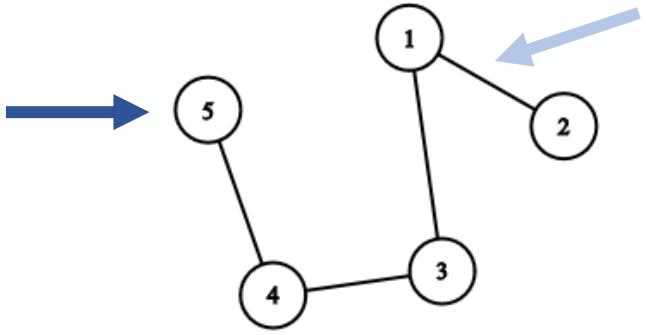
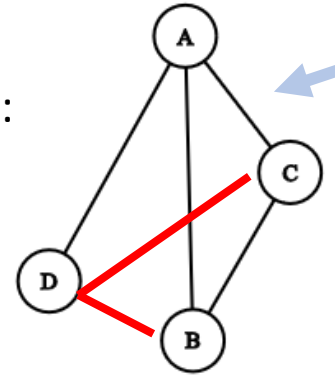
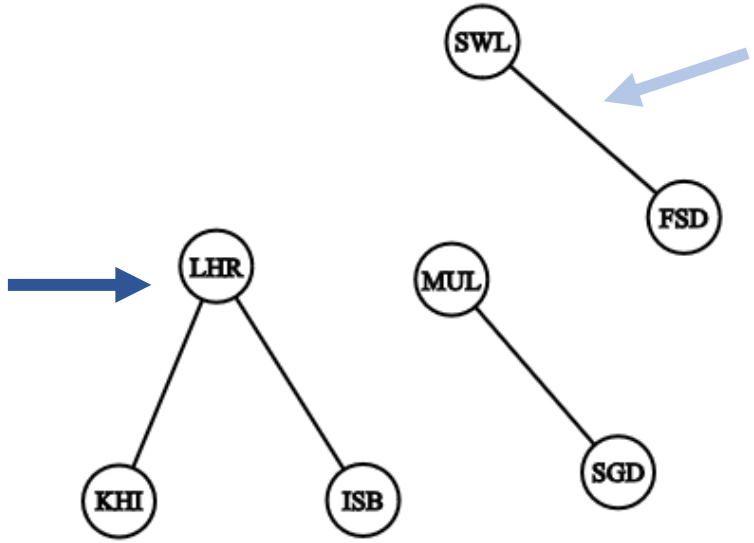


Total edges:
(n(n-1))/2

vertex

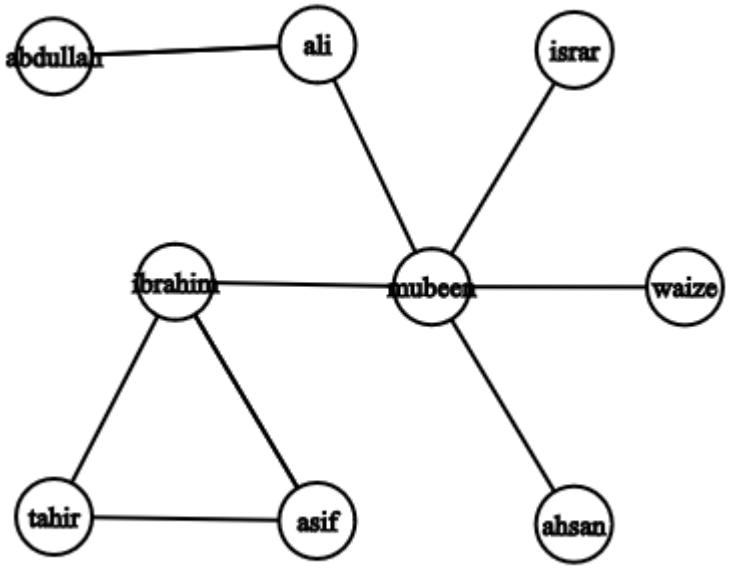


edges

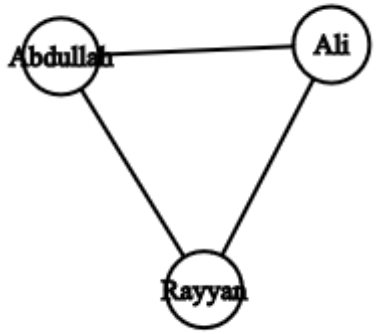


Difference between Directed and undirected graphs?

Instagram followers



Siblings Relation



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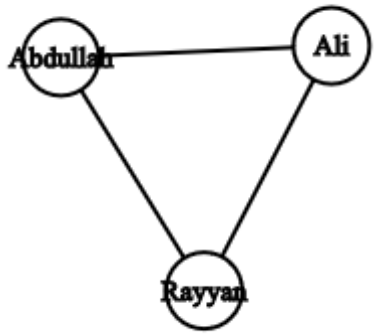
Siblings Relation



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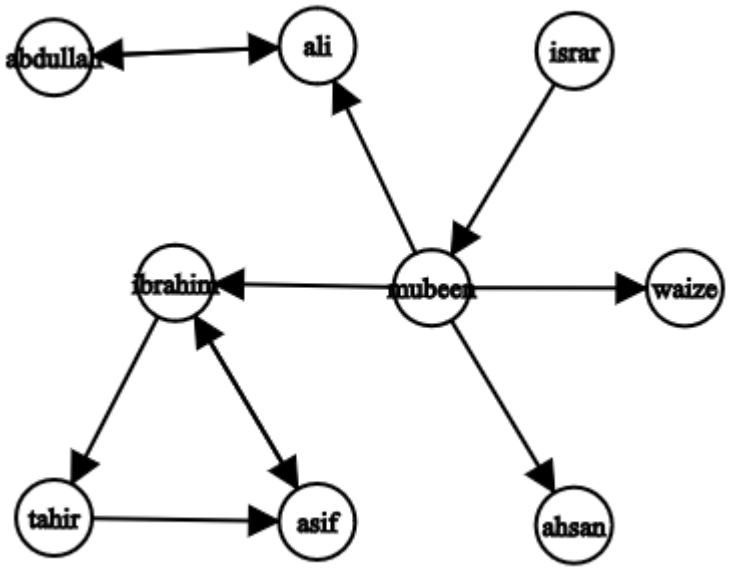
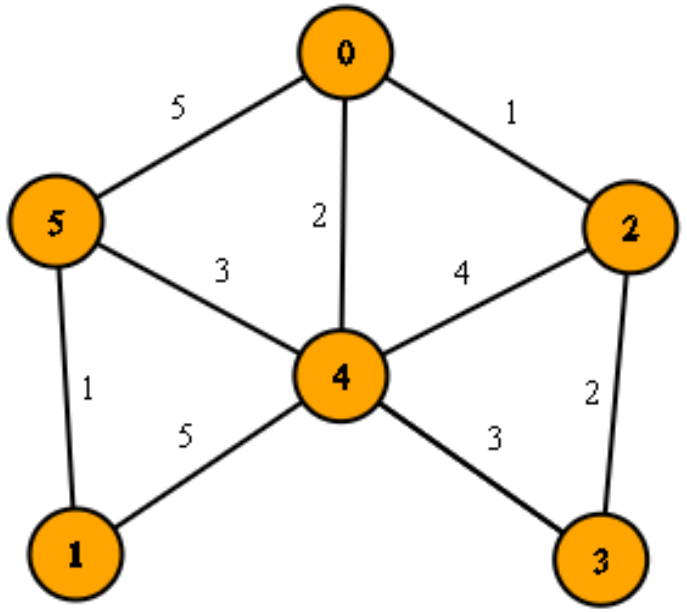


Weighted Graphs

Many graphs can have edges that contain a certain weight to represent an arbitrary value such as cost, distance, quantity, etc.
This suggests that all edges cannot be treated equally in certain cases.

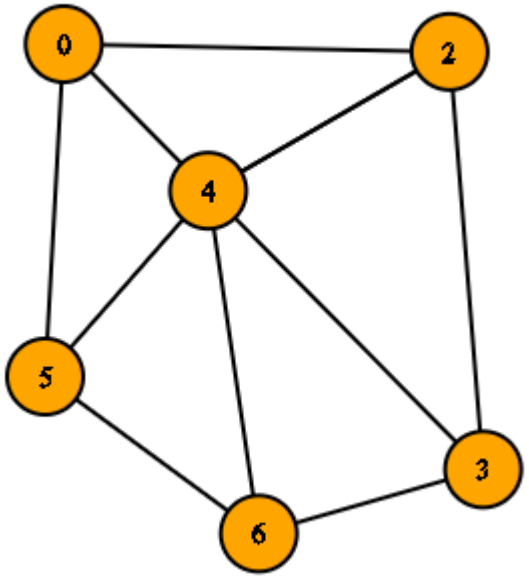


USE IN GAME:

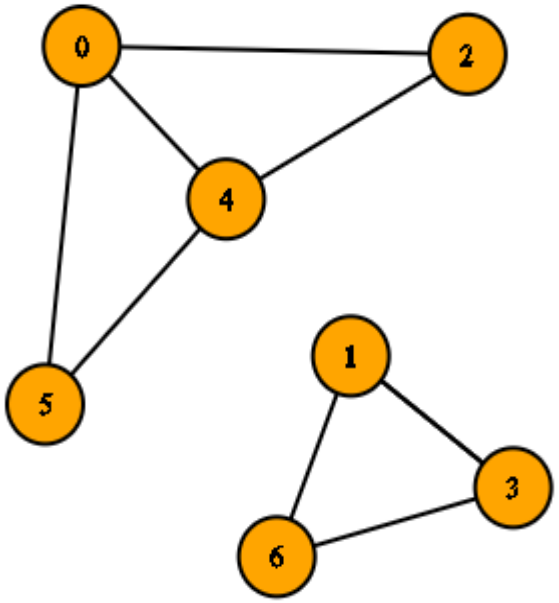
- to represent the potential impact or strategic value of certain moves.
- Weights could represent how close a sequence is to completion
- In more advanced AI implementations, weighted graphs can help find paths that maximize player's advantage while minimizing the opponent’s.

Connected vs Disconnected Graphs

A graph is called connected when a path exist between all of its vertices.



A graph is called disconnected when a path doesn’t exist between all of its vertices.



GRAPH REPRESENTATION

1. Edge List
    1. Individual track of all edges
    2. Need to check all edges

2. Adjacency List
    1. Track of all adjacent to a particular edge
    2. Need to check relevant list of adjacent

3. Adjacency Matrix
    1. Table in form of rows and columns that keep track of all possible edges in terms of YES/NO
    2. Only need to check one cell.

Things we need for our area of research:

1. How to traverse a graph
2. Count neighbors of a vertex
3. Count degree of a vertex(number of edges meeting at that vertex)
4. Deleting and inserting edges
5. Path and cycle of vertices

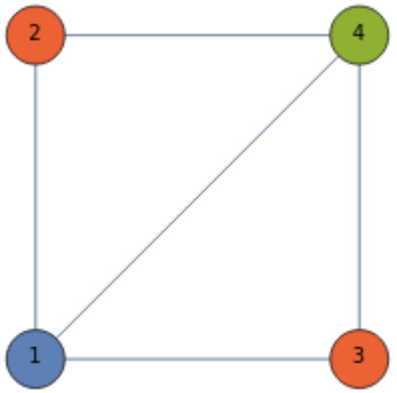
Graph in Board Games

Most board games are played two-dimensional grid.

- Nodes represent the cells where game pieces are placed.
- Winning conditions involve finding paths (edges) with consecutive stones.

Graph Coloring in Board Games

- involves assigning different colors to the vertices of a graph such that no two adjacent vertices share the same color.
- Simplifies game design by managing player interactions and organizing regions.



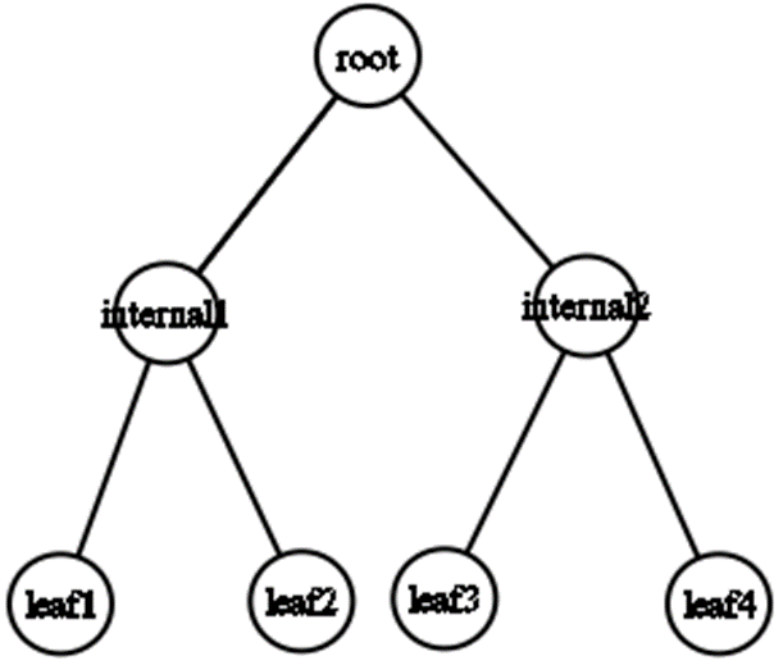
1. Territory Colouring: Assigning different colours to neighbouring territories.
2. Player moves: Useful in multiplayer games to minimize conflicts.
3. Puzzle Solving: No two adjacent players have the same colours.

TREES AND SPANNING TREES

Tree is a simple, connected ,undirected ,acyclic(having a unique path between any two vertices) graph.

A **spanning tree** of a graph is a subgraph that includes all vertices of the graph and is a tree.
- Spanning trees connect all vertices with the minimum possible edges.

In games, spanning trees can help create unique paths with no loops.



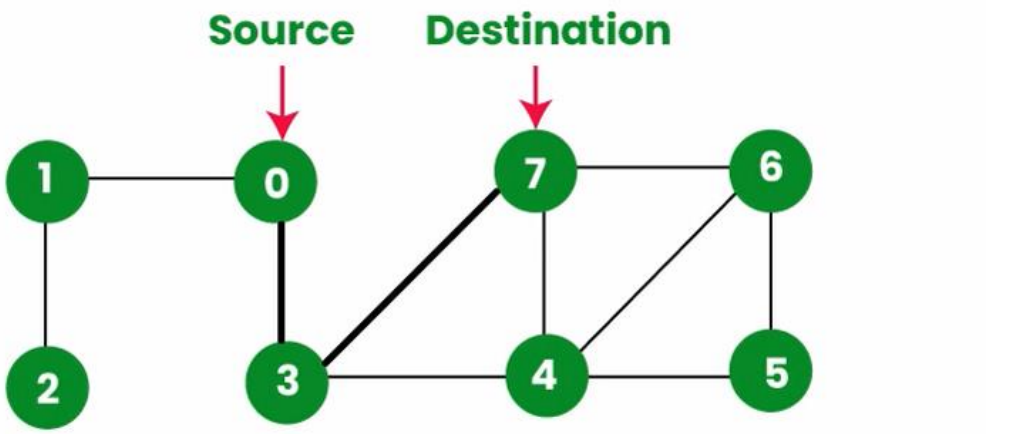
Path finding

Definition:

“Pathfinding refers to the process of finding a path from one point (vertex) to another in a graph.”

A **path** in a graph is a sequence of edges that connects a series of vertices.

It can be represented as a list of vertices starting from a source node and ending at a destination node, with each consecutive vertex connected by an edge.



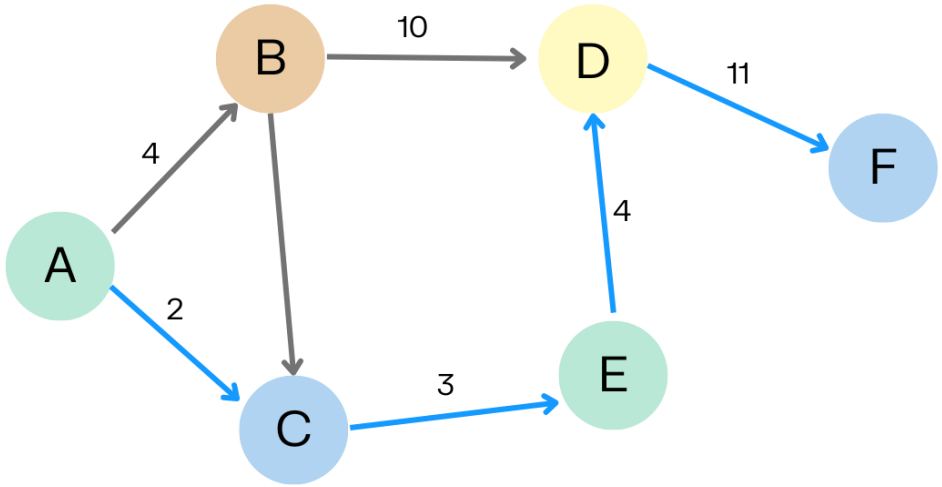
Shortest Path

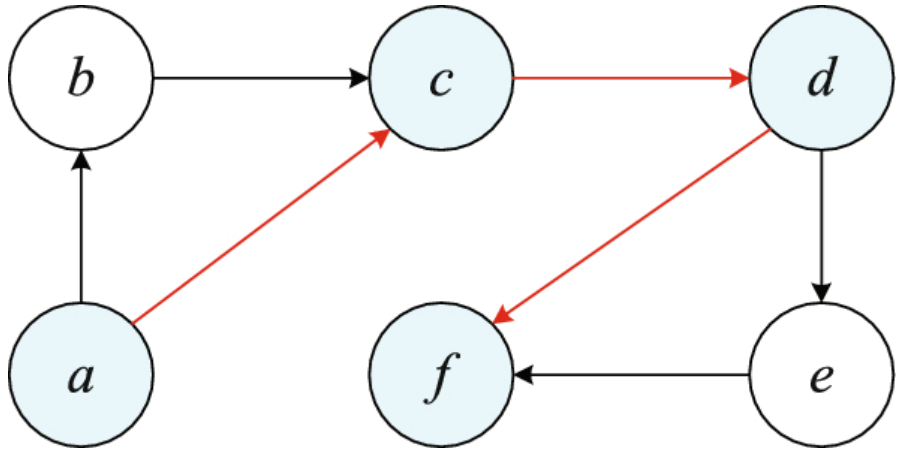
Definition:

"The **shortest path** is a path between two vertices such that the sum of the weights of the edges in the path is minimized."

In unweighted graphs, the shortest path is simply the path with the least number of edges.

In weighted graphs, it's the path with the minimum total weight.





Some Algorithms Used for Path Finding

- Dijkstra's Algorithm
- A* Algorithm
- Bellman-Ford Algorithm
- Floyd-Warshall Algorithm
- Breadth-First Search (BFS)
- Depth-First Search (DFS)
- Johnson's Algorithm

Some Algorithms Used for Path Finding

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We have used Breadth-First Search (BFS) in our code..

Breadth-First Search Algorithm

BFS is the best algorithm used for path finding in **unweighted graphs**.

This BFS implementation explores close moves first and extends outward, efficiently finding potential moves for the AI to play based on nearby pieces.

RECURSION:

- **BASE CASE:**
  a condition that allows an algorithm stops further recursive calls and return a result.
- **Recursive case:**

  It is the part of a recursive function that breaks down a problem into smaller subproblems, which moves the problem closer to the base case.

RECURSIVE FUNCTION:

Recursive function is a function that values at any point can be calculated from the values of the function at some previous points

HOW RECURSION WORKS :

- Step By Step :

Step 2: Step 1 + lowest step.

Step 3: Step 2 + Step 1 + lowest step.

Step 4: Step 3 + step 2 + step 1+ lowest step, and so on.

The smallest argument is donated by f(0) or f(1) .

The nth argument is donated by f(n).

- **Example:**

  fibonacci sequence.

Fibonacci Series Formula

$F_0 = 0, F_1 = 1$

$F_n = F_{n-1} + F_{n-2}$

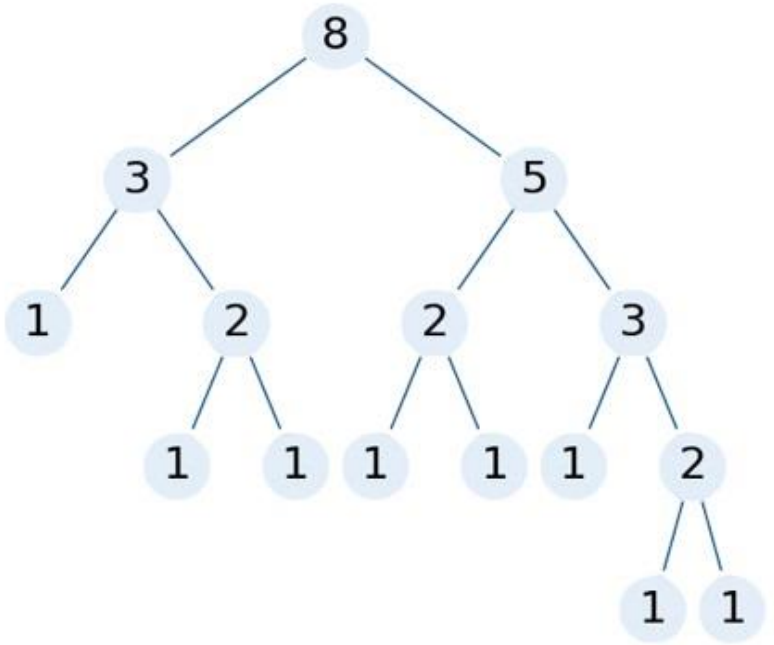
For n > 1

⇒ Fibonacci Series:

0, 1, 1, 2, 3, 5, 8, 13, 21,

Recursive function:

$$F(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{if } n > 1 \end{cases}$$



1. Fibonacci(5) calls → Fibonacci(4) + Fibonacci(3)
2. Fibonacci(4) calls → Fibonacci(3) + Fibonacci(2)
3. Fibonacci(3) calls → Fibonacci(2) + Fibonacci(1)
4. Fibonacci(2) calls → Fibonacci(1) + Fibonacci(0)
5. Base case:
   Fibonacci(1) returns 1
   Fibonacci(0) returns 0

```python
def fibonacci(n):
    # Base cases
    if n == 0:
        return 0
    elif n == 1:
        return 1
    # Recursive case
    return fibonacci(n - 1) + fibonacci(n - 2)
```

Implementation

Topics like graph theory, and recursion play crucial roles in modeling and solving the game.

Introduction to Gomoku

Gomoku rules and win condition

Graph Model: Gomoku's grid can be represented as a graph where:

- **Vertices (Nodes)** represent each board position.
- **Edges (Connections)** represent possible moves between adjacent positions (up, down, left, right, or diagonally).

- **Significance**: This representation enables the game to leverage graph algorithms to analyze the board for optimal moves and winning patterns.
- **Recursion in Gomoku:**
- **Exploring Moves Recursively**: AI can use recursion to evaluate multiple future moves, creating a **game tree** structure. Each recursive call simulates a potential move, building layers of possible future game states.
- **AI Strategy**:
  - **Block Opponent Moves**: The AI recursively checks potential moves that could lead to an opponent's victory, blocking these to defend.
  - **Find Winning Paths**: It evaluates sequences of moves to identify paths where it can form five in a row.

- **Graph Theory**: By modeling the board as a graph, the AI can:
  - Track connections between moves.
  - Evaluate clusters of pieces that could lead to potential wins.
- **Recursion**: Recursively explores all potential moves, simulating different sequences and choosing the path that maximizes the chance of winning.
- **Python**: Recommended for ease of use and development speed, especially with libraries that simplify board representation and recursive functions.
- **Basic Algorithm for AI Move Selection**
- **Recursive Evaluation**:
  - Start with an empty board state, checking all valid moves.
  - For each move, recursively simulate possible opponent responses.
  - Limit recursive checks