# Designing optimal cell factories: integer programming couples elementary mode analysis with regulation

Molecular Networks B SS13

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#### Paper and authors

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#### **METHODOLOGY ARTICLE**

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# Designing optimal cell factories: integer programming couples elementary mode analysis with regulation

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#### Overview

- 1 Background and Motivation
- 2 Theory
- 3 Illustrative example
- 4 Including regulation
- **5** (Optimizing metabolic functionality)
- **6** Results of realistic example
- Discussion
- 8 Summary and conclusions

# Background and Motivation

- microbes can be used for metabolic engineering e.g. ethanol production in *E. coli*
- Methabolic networks are available for some model organisms

#### Gaol

Gene deletion strategie to optimize the production efficiency of strains.

# Metabolic network and steady state assumption

#### Model of metabolic network with

- *m* internal metabolites
- *n* reactions
- $S \in \mathbb{R}^{m \times n}$  stoichiometric matrix
- $\hat{v} \in \mathbb{R}^n$  flux vector

#### Steady state assumption

Assume metabolite concentrations and reaction rates are constant.

$$S \cdot \hat{\mathbf{v}} = 0 \tag{1}$$

fig?

## Elementary mode analysis

#### Definition

An elementary mode (EM) is a minimal and indivisible set of reactions that operates under steady state conditions, while obeying all (ir-)reersibility constraints.

# Network of minimal functionality (NMF)

#### Theory: Elementary mode analysis

 $e = e(\hat{e})$  is the binary representation of a EM e.

$$e_i := e(\hat{e}_i)) = \begin{cases} 1 & \text{if } \hat{e}_i \neq 0 \\ 0 & \text{if } \hat{e}_i = 0 \end{cases}$$
 (2)

$$e^T v \le e^T e = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n e_i =: ||e||$$

#### Binary integer program

Group all q elementary modes:

$$G := (e_1, ..., e_r)^T$$
  
 $H := (e_{r+1}, ..., e_{r+s})^T$   
 $K := (e_{r+s+1}, ..., e_{r+s+t})^T$ 

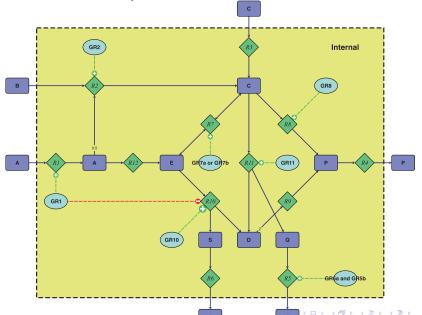
goal matrix with desirable EM
helper matrix tolerated EM
kill matrix unwanted EM

max||x|| s.t.

$$\begin{aligned} e_g^T x &= ||e_g|| \\ e_h^T x &\leq ||e_h|| \\ e_k^T x &\leq ||e_k|| - 1 \end{aligned}$$

$$g \in \{1, ..., r\}$$
$$h \in \{r + 1, ..., r + s\}$$
$$k \in \{r + s + 1, ..., r + s + t\}$$

# Illustrative example



#### Θ . 0 0 0 EM 1 EM 2 · EM 3 ---9 @ @ 0 **(C)** $\Diamond \odot \Box \Diamond \Box$ 0 EM 4 -----EM 5 ---@ **(** 0 · ----· Jonas Ibn-Salem () 25.04.13 11 / 13

Elementary Modes

## Introduction: Fixed Point Analysis

Given the system of differential equations:

$$y' = f(y)$$

#### **Definition**

A fixed point  $y^*$  is defined by  $f(y^*) = 0$ .

- Solve the equation f(y) = 0
- Analyse eigenvalues of the Jacobian at fixed points.

Now: System with controle parameter  $\mu$ .

$$y' = f(y, \mu)$$

How does  $\mu$  influence the number, location and stability of fixed points?

# Elementary flux mode

For  $v \in R$