Bifurcation in parameter dependent systems

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Jonas Ibn-Salem

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Overview

Introduction: Fixed Point Analysis

Given the system of differential equations:

$$y' = f(y)$$

Definition

A *fixed point* y^* is defined by $f(y^*) = 0$.

- Solve the equation $f(y) = 0$
- Analyse eigenvalues of the Jacobian at fixed points.

Now: System with *controle parameter* μ.

$$y' = f(y, \mu)$$

$\Rightarrow$ How does μ influence the number, location and stability of fixed points?

Bifurcation

Definition

Bifurcation is the changing of the character of an equalibrium point and/or the creation of extra ones by alteration of a control parameter.

Example: Logistig growth with harvesting

Growth of a population:

$$y' = \frac{1}{10} y(10 - y) - \mu$$

Solving $f(y, \mu) = 0$ for any parameter μ.

Bifurcation Diagram: