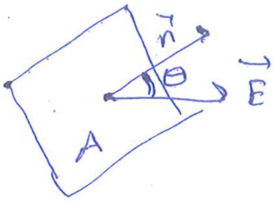
Gauss's Law

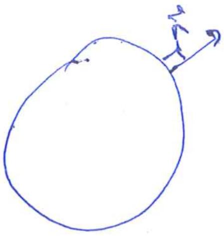
Electric flux



$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\phi = E_\perp A$$

not flat surface $\Phi_E = \int E\cos\phi \, dA = \int E_\perp dA = \int \vec{E}\cdot d\vec{A}$

Gauss's law

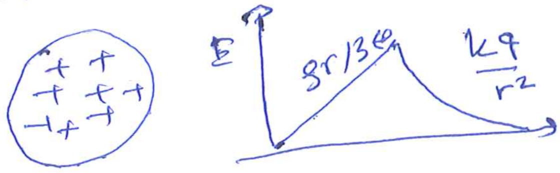


$$\Phi_E = \frac{\sum q_{inside}}{\epsilon_0}$$

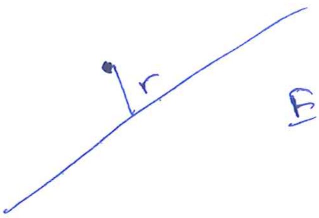
Electric Field Magnitude by Gauss's law

Conducting sphere

Uniform charged sphere

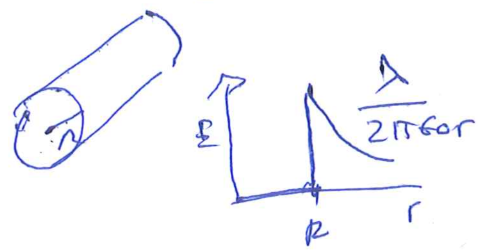


Linear charge distribution

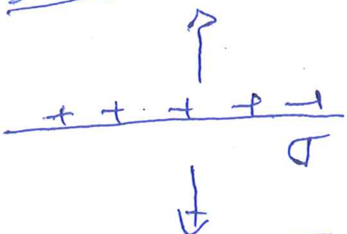


$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

Conducting cylinder

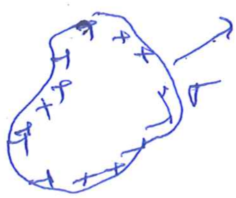


Surface



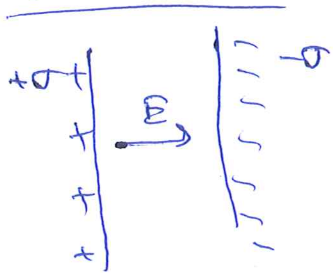
$$E = \frac{\sigma}{2\epsilon_0}$$

Outside conductor



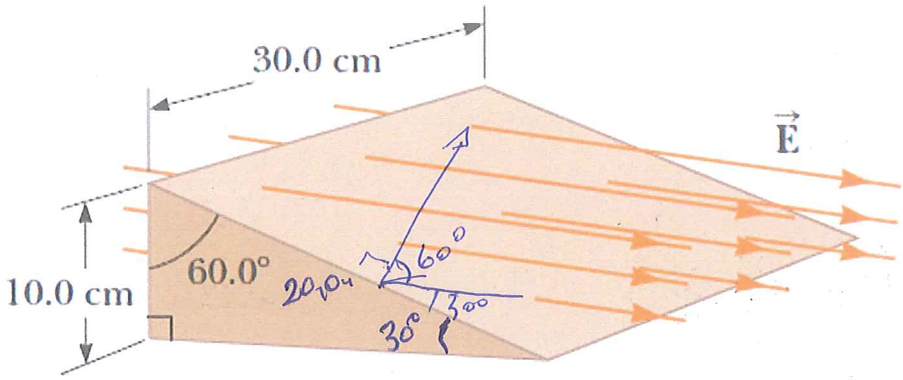
$$E = \frac{\sigma}{\epsilon_0}$$

Parallel plates



$$E = \frac{\sigma}{\epsilon_0}$$

4. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4$ N/C as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.



(a) Surface area = 0,1 m × 0,3 m = 0,03 m²

$\vec{E}$ is perpendicular to the surface

$$|\Phi_E| = EA = 7{,}80 \times 10^4 \frac{N}{C} \cdot 0{,}03 = 2{,}34 \times 10^3 \frac{N m^2}{C}$$

$\vec{E}$ in to a closed surface, flux is negative

$$\Phi_E = -2{,}34 \times 10^3 \frac{N}{C} m^2$$

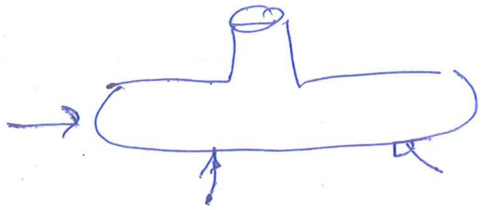
(b) $\Phi_E = EA \cos(60°)$

$$\Phi = 7{,}8 \times 10^4 \cdot 0{,}2 \cdot 0{,}3 \cdot \frac{1}{2} = 2{,}34 \times 10^3 \frac{N}{C} m^2$$

(c) The flux through other surfaces are zero.

$$\Phi_{tot} = -2{,}34 \times 10^3 + 2{,}34 \times 10^3 = 0$$

9. M The following charges are located inside a submarine: 5.00 μC, −9.00 μC, 27.0 μC, and −84.0 μC. (a) Calculate the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?



$\sum q_{in} = 5 - 9 + 27 - 84 \ \mu C$

$\sum q_{in} = -61{,}0 \ \mu C$

(a) $\Phi_E = \dfrac{\sum q_{in}}{\varepsilon_0} = -\dfrac{61\times10^{-6}}{8{,}85\times10^{-12}} = -6{,}89\times10^{6} \ \dfrac{N m^2}{C}$

the inner electric field lines are towords volume.

(b) Negative means more lines enter than leave the surface

19. A particle with charge $Q = 5.00\ \mu C$ is located at the center of a cube of edge $L = 0.100$ m. In addition, six other identical charged particles having $q = -1.00\ \mu C$ are positioned symmetrically around Q as shown in Figure P24.19. Determine the electric flux through one face of the cube.

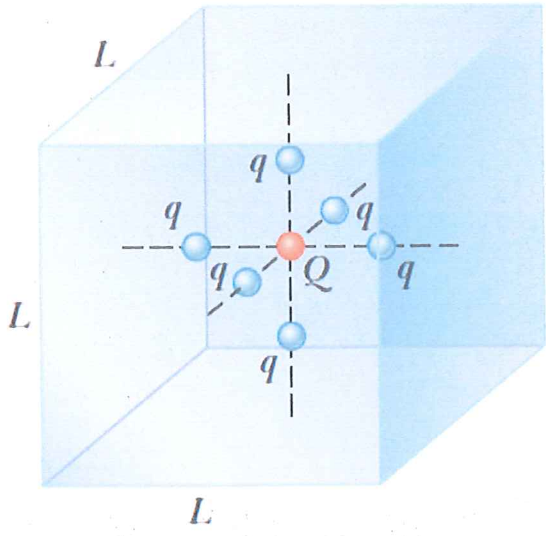


Figure P24.19
Problems 19 and 20.

$$\sum q_{in} = Q + 6q = 5\mu C + 6(-1\mu C) = -1{,}0\mu C$$

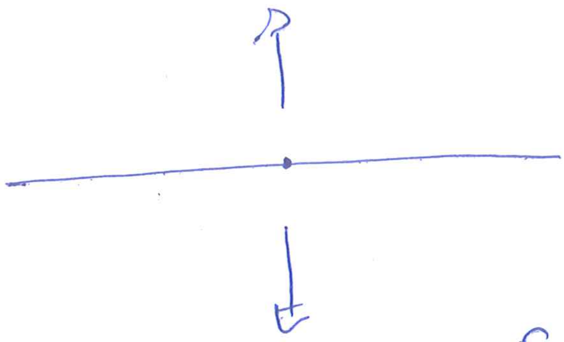
Due to symmetry

$$\Phi_{face} = \frac{\Phi_{total}}{6} = -\frac{1{,}0\times10^{-6}/\varepsilon_0}{6}$$

$$\Phi_{face} = -\frac{10^{-6}}{6\times8{,}85\times10^{-12}}$$

$$\Phi_{face} = -1{,}88\times10^{4}\ \frac{Nm^2}{C}$$

27. **M** A large, flat, horizontal sheet of charge has a charge per unit area of 9.00 $\mu C/m^2$. Find the electric field just above the middle of the sheet.



$E = \frac{\sigma}{2\varepsilon_0} = \pi 2 k \sigma$

$E = \pi \cdot 2 \cdot 8{,}99 \times 10^9 \cdot 9 \times 10^{-6} \frac{C}{m^2}$

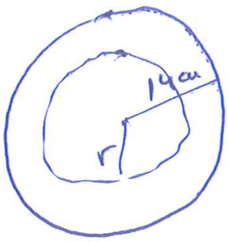
$E = 5{,}08 \times 10^5 \frac{N}{C}$

up

$\vec{E} = \left(5{,}08 \times 10^5 \, \hat{j}\right) \frac{N}{C}$

29. **M** Consider a thin, spherical shell of radius 14.0 cm with a total charge of 32.0 μC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

(a)



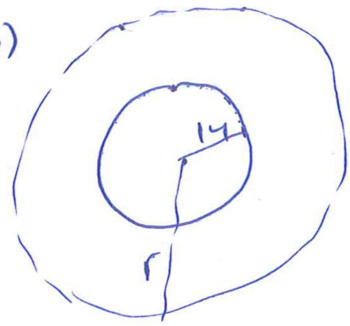
$r = 10.0 \text{ cm}.$

$$\Phi_E = E \cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$E\, 4\pi r^2 = 0$$

$$E = 0$$

(b)



$r = 20.0 \text{ cm}$

$$E\, 4\pi r^2 = \frac{q_{in}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

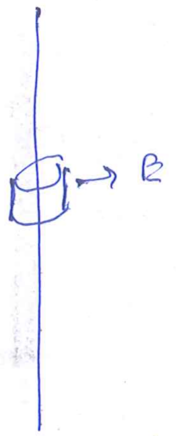
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$E = 8.99 \times 10^9 \cdot \frac{32 \times 10^{-6}}{(0.2)^2}$$

$$E = 7.19 \times 10^6 \text{ N/C}$$

31. **M** A uniformly charged, straight filament 7.00 m in length has a total positive charge of 2.00 μC. An uncharged cardboard cylinder 2.00 cm in length and 10.0 cm in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.

filament is long, assume that $\vec{E}$ near the center is radial.



(a) $\Phi_E = E\, 2\pi r L = \dfrac{\lambda L}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi \epsilon_0 r} =$

$E = \dfrac{2k\lambda}{r} = \dfrac{2 \cdot 8.99 \times 10^{9}}{(0.1)} \cdot \dfrac{2\times10^{-6}}{7}$

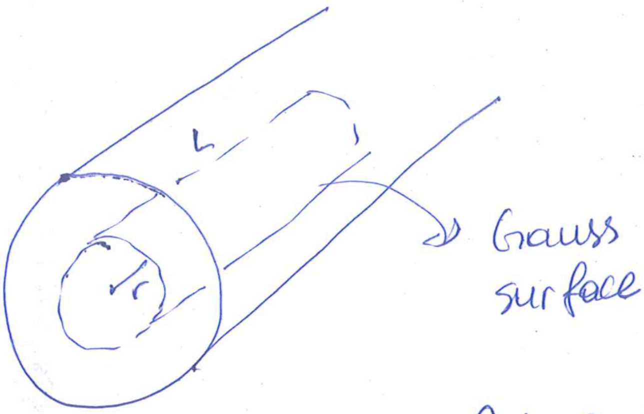
$\lambda = \dfrac{2.00\times10^{-6}\, C}{7.0\ m}$

$E = 5.14 \times 10^{4}\ \dfrac{N}{C}$

b) $\Phi_E = \dfrac{\lambda L}{\epsilon_0} = \dfrac{2\times10^{-6}}{7} \cdot 2\times10^{-2} \cdot \dfrac{1}{8.85\times10^{-12}}$

$= 6.46 \times 10^{2}\ \dfrac{N m^2}{C}$

33. Consider a long, cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis, where $r < R$.



$$\Phi_E = \frac{q_{in}}{\epsilon_0}$$

$$\boxed{E\, 2\pi r L = \frac{q_{in}}{\epsilon_0}} \qquad r < R$$

$$q_{in} = \rho \times \text{volume}$$

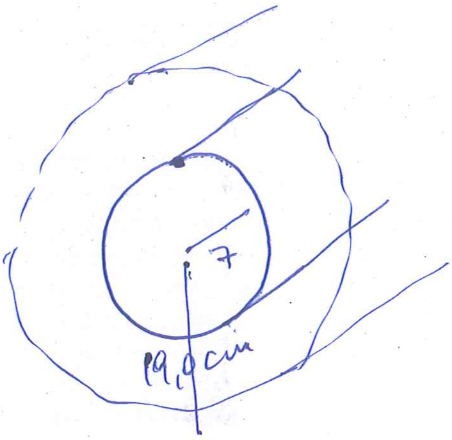
$$q_{in} = \rho \cdot \pi r^2 L$$

$$E\, 2\cancel{\pi} r \cancel{L} = \rho \cdot \cancel{\pi} r^2 \cancel{L} / \epsilon_0$$

$$\boxed{E = \frac{\rho r}{2\epsilon_0}} \qquad \text{radial}$$

$$\vec{E} = \frac{\rho r}{2\epsilon_0}\,\hat{r}$$

34. A cylindrical shell of radius 7.00 cm and length 2.40 m **W** has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.



(a) $\Phi_E = \frac{q}{\epsilon_0}$

$q = \epsilon_0 \Phi_E$

$q = \epsilon_0 \, 2\pi r L \, E$

$q = 8{,}85 \times 10^{-12} \cdot 2 \cdot \pi \cdot 19 \times 10^{-2} \cdot 2{,}40 \cdot 36 \times 10^{3} \frac{N}{C}$

$= 9{,}13 \times 10^{-7}\, C = 913\, nC$

b) 

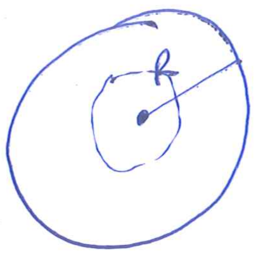
$\Phi = E \cdot 2\pi r L = \frac{q_{in}}{\epsilon_0}$

$E \cdot 2\pi r L = 0$

no charge inside $r = 4$ cm

$\boxed{E = 0}$

35. A solid sphere of radius 40.0 cm has a total positive **W** charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.



$r < R$

$$E \cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_0}$$

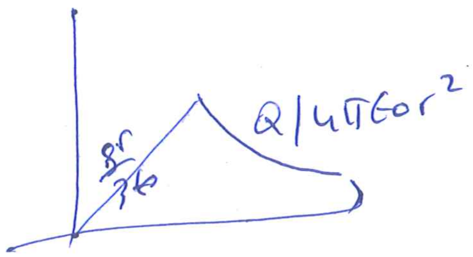
$r \geq R$

$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$E = \frac{\rho r}{3\epsilon_0}$$

$$= \frac{Q\, r}{\frac{4\pi}{3} R^3\, 3\, \epsilon_0}$$



$$= \frac{3Qr}{4\pi R^3 \epsilon_0}$$

(a) $r = 0$, $E = 0$

(b) $$E = \frac{3Qr}{4\pi\epsilon_0 R^3} = \frac{3 \cdot 26\times10^{-6} \cdot 0{,}1\,m}{(0{,}4\,m)^3} \cdot 8{,}99\times10^9 \frac{Nm^2}{C^2}$$

$$E = 3{,}65\times10^5\,N/C$$

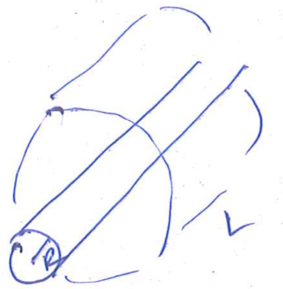
(c) $r = R = 0{,}4m$

$$E = \frac{kQ}{R^2} = \frac{8{,}99\times10^9 \times 26\times10^{-6}}{(0{,}4)^2} = 1{,}46\times10^6 \frac{N}{C}$$

(d) $r > R$, $r = 0{,}6m$

$$E = \frac{kQ}{r^2} = \frac{8{,}99\times10^9 \times 26\times10^{-6}}{(0{,}6)^2} = 6{,}49\times10^5 \frac{N}{C}$$

37. M A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the axis of the rod, where distances are measured perpendicular to the rod's axis.



$r > R$

$$E \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

$r < R$, conductor

$E = 0$

(a) $r < 5{,}00$ cm, $E = 0$

(b)

$$E = \frac{2 \cdot 8{,}99 \times 10^9 \cdot 30{,}0 \frac{10^{-9}\,c}{m}}{(0{,}1\,m)}$$

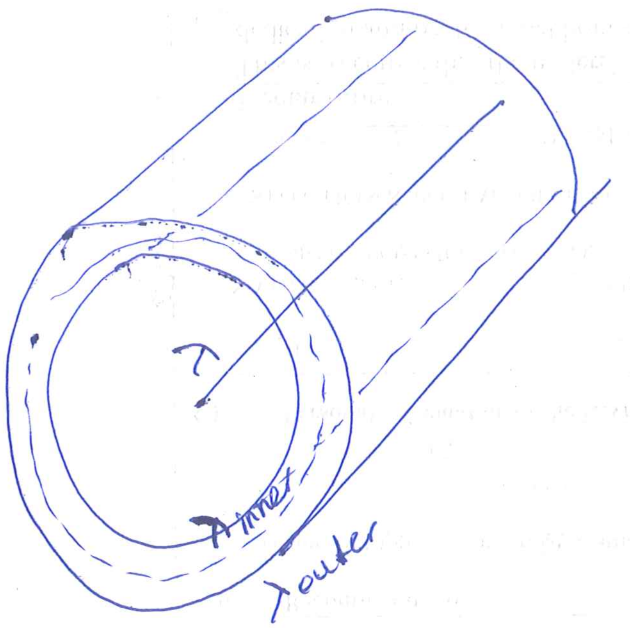
$$= 5400 \frac{N}{c}$$ radially outward

(c)

$$E = \frac{2 \times 8{,}99 \cdot 10^9 \times 30 \times 10^{-9}\ c/m}{1\,m}$$

$$= 540\ N/c$$

45. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner surface of the cylinder, (b) the charge per unit length on the outer surface of the cylinder, and (c) the electric field outside the cylinder a distance r from the axis.



$\lambda_{inner} + \lambda_{outer} = 2\lambda$

a) Select the Gauss surface inside the conducting shell

$E=0$

$$\oint_{E=0} = \frac{q_{in}}{\epsilon_0}$$

$$0 = \frac{\lambda L + \lambda_{inner} L}{\epsilon_0}$$

$$\boxed{\lambda_{inner} = -\lambda}$$

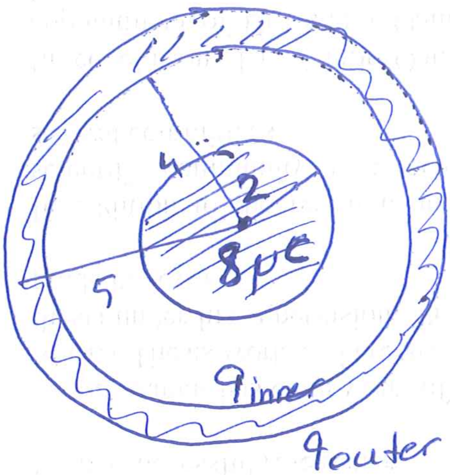
(b) $\lambda_{outer} = 2\lambda - \lambda_{inner} = 2\lambda - (-\lambda) = 3\lambda$

(c) Takin Gauss surface outsid

$$E\, 2\pi r L = \frac{q_{in}}{\epsilon_0} = \frac{2\lambda L + \lambda L}{\epsilon_0}$$

$$E = \frac{3\lambda}{2\pi\epsilon_0 r} = \frac{6k\lambda}{r}$$ radially outward

47. A solid conducting sphere of radius 2.00 cm has a **M** charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of $-4.00\ \mu$C. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.



$q_{inner} + q_{outer} = -4\mu C$

(a) $r = 1{,}00$ cm, inside the Conducting sphere

$E = 0$

(b) $r = 3$ cm, in the cavity

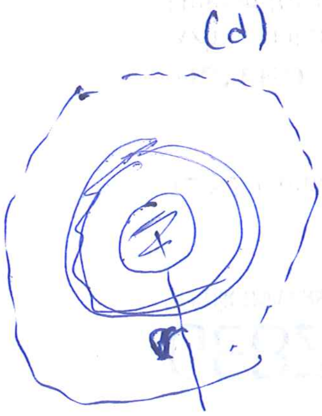
$$E \cdot 4\pi r^2 = \frac{8\times10^{-6}\,C}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{8\times10^{-6}}{(3\times10^{-2})^2} = \frac{8{,}99\times10^{9}\cdot 8\cdot10^{-6}}{9\times10^{-4}}$$

$$E = 7{,}99\times10^{7}\ N/C$$

(c) $r = 4{,}50$ m, inside the conductor. $E = 0$

(d) $r = 7{,}00$ cm, outside



$$E \cdot 4\pi r^2 = \frac{(8{,}00 - 4{,}00)\times10^{-6}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{4\times10^{-6}}{(7\times10^{-2})^2} = \frac{8{,}99\times10^{9}\cdot 4\cdot10^{-6}}{(7\times10^{-2})^2}$$

$$= 7{,}34\times10^{6}\ \frac{N}{C}$$ radially outward

64. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. Assume the material does not affect the electric field. A spherical cavity of radius a is now removed from the sphere as shown in Figure P24.64. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a/3\epsilon_0$.

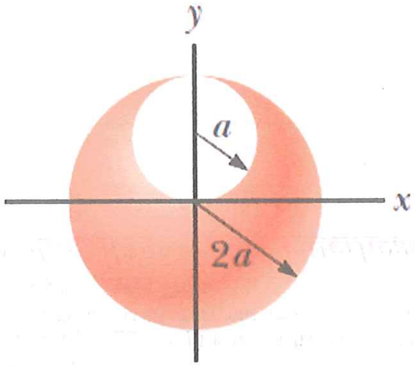
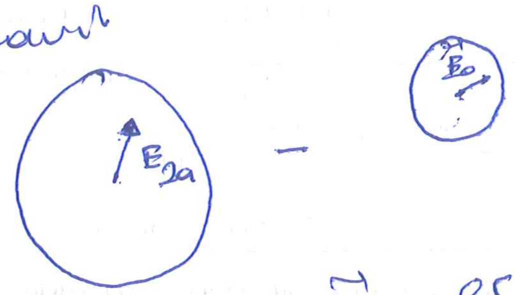


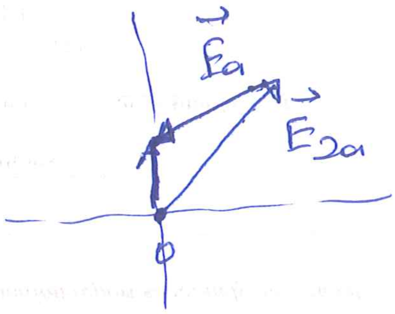
Figure P24.64

inside the cavit

$\vec{E} =$ 

$$\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}$$

$$= \frac{\rho \vec{r}}{3\epsilon_0}$$



$$\vec{E}_{total} = \vec{E}_{2a} - \vec{E}_a$$

$$\vec{E}_{total} = \frac{\rho}{3\epsilon_0} a\hat{j}$$

$E_x = 0, \quad E_y = \dfrac{\rho a}{3\epsilon_0}$ inside the cavity

64. A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ. Assume the material does not affect the electric field. A spherical cavity of radius a is now removed from the sphere as shown in Figure P24.64. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a/3\epsilon_0$.

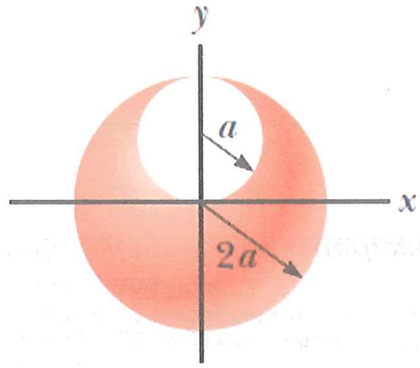
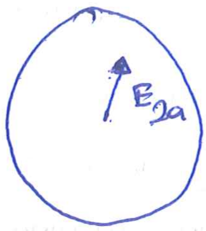


Figure P24.64

inside the cavit

$\vec{E} =$ 

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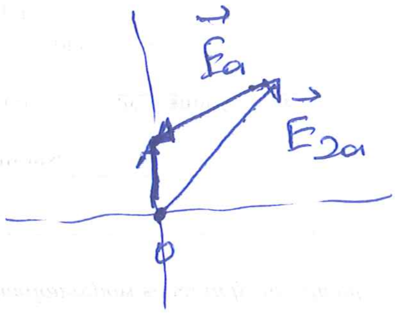

$$\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}$$

$$= \frac{\rho \vec{r}}{3\epsilon_0}$$

$$\vec{E}_{total} = \vec{E}_{2a} - \vec{E}_a$$

$$\vec{E}_{total} = \frac{\rho}{3\epsilon_0} a\hat{j}$$

$E_x = 0, \quad E_y = \dfrac{\rho a}{3\epsilon_0}$ inside the cavity