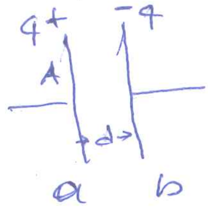
# Capacitance and Dielectrics

## Capacitance

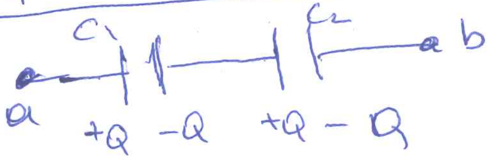
$$C = \frac{q}{V_{ab}} \quad ; V_{ab} = \int_{+}^{-} \vec{E} \cdot d\vec{l}$$




## Paralel Plate Capacitor:

$$C = \frac{\epsilon_0 A}{d}$$

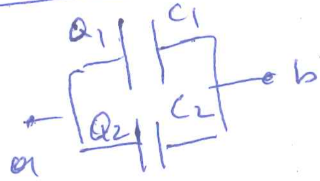
## Capacitance in series




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

$$Q = C_1 \Delta V_1 = C_2 \Delta V_2 = C_{eq} \cdot V_{ab}$$

## Capacition in parallel




$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V_{ab}$$

$$\frac{Q_1 + Q_2}{C_1 + C_2} = V_{ab}$$

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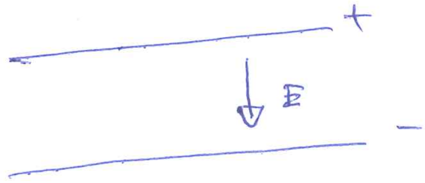
## Energy in a capacitor




$$U = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V$$

$$\text{energy density} = \frac{U}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2$$

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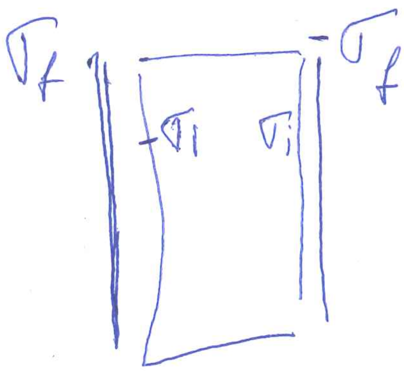
## Dielectrics

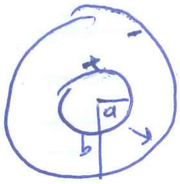



$$C = K C_0 = K \epsilon_0 \frac{A}{d}$$

$$E = \frac{E_0}{K}$$

$$\sigma_i = \sigma_{free} \left( \frac{K - 1}{K} \right)$$

**4.** M An air-filled spherical capacitor is constructed with inner- and outer-shell radii of 7.00 cm and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a 4.00-$\mu$C charge on the capacitor?




$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$|\Delta V| = \int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\, dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{1}{\frac{1}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0\, ab}{(b-a)} = \frac{ab}{k(b-a)}$$

(a) $$C = \frac{(0,07)(0,14)}{8,99\times10^{9} \times (0,14 - 0,07)}$$

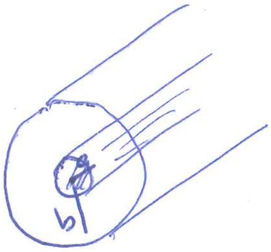
$$C = 15,6 \times 10^{-12}\ F$$

(b) $$C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C}$$

$$\Delta V = \frac{4\times10^{-6}\ C}{15.6\times10^{-12} F} = 2,57\times10^{5}\ V$$

$$\Delta V = 257\ kV$$

**5.** **M** A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 $\mu$C. The surrounding conductor has an inner diameter of 7.27 mm and a charge of $-8.10$ $\mu$C. Assume the region between the conductors is air. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors?




$$E \cdot 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi\epsilon_0 r L} = \frac{Q/L}{2\pi\epsilon_0 r}$$

$$E = \frac{2kQ}{Lr}$$

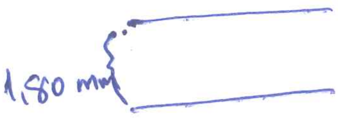
$$\Delta V = \int_a^b \frac{2kQ}{Lr}\,dr = \frac{2kQ}{L}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{L}{2k\ln(b/a)}$$

(a) $$C = \frac{8{,}10\times10^{-6}C}{\Delta V} = \frac{50\,m}{2\times8{,}99\times10^{9}\times\ln\left(\frac{7{,}27}{2{,}58}\right)}$$

$$C = 2.68\times10^{-9}F$$

**9.** An air-filled capacitor consists of two parallel plates, each with an area of 7.60 $cm^2$, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.




$A = 7{,}60\ cm^2 = 7{,}60\times10^{-4}\ m^2$

$\Delta V = 20\ V$

(a) $E = \dfrac{\Delta V}{d} = \dfrac{20\ V}{1{,}80\times10^{-3}\ m} = 1{,}11\times10^{4}\ \dfrac{V}{m}$

(b) $\sigma = \dfrac{\text{charge}}{\text{area}} = \varepsilon_0 E$ , as $E = \dfrac{\sigma}{\varepsilon_0}$ between plates

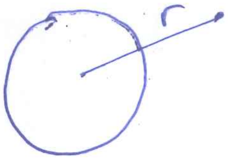
$\sigma = (8{,}854\times10^{-12})\cdot(1{,}11\times10^{4})$

$\sigma = 9{,}83\times10^{-8}\ \dfrac{C}{m^2}$

$= 98{,}3\ \dfrac{nC}{m^2}$

**11.** An isolated, charged conducting sphere of radius 12.0 cm creates an electric field of $4.90 \times 10^4$ N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

$r > R$




$$E = \frac{kQ}{r^2}$$

(a) $$Q = \frac{Er^2}{k}$$

$$Q = \frac{4{,}90 \times 10^4 \cdot (0{,}21)^2}{(8{,}99 \times 10^9)}$$

$$Q = 0{,}240 \times 10^{-6}\ C$$

$$\sigma = \frac{Q}{area} = \frac{0{,}240 \times 10^{-6} C}{4 \cdot \pi\,(0{,}12)^2} = 1{,}33 \times 10^{-6}\ \frac{C}{m^2}$$

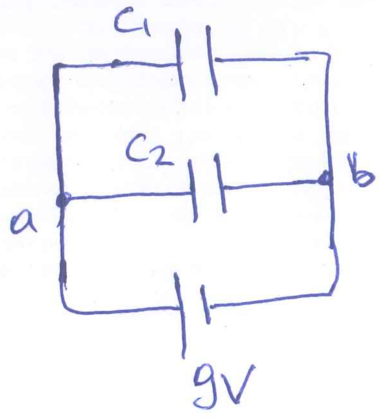
(b) $$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0 r}} = 4\pi\epsilon_0 r$$

$$C = 4\pi\,(8{,}854 \times 10^{-12}) \cdot (0{,}120\ m)$$

$$C = 13{,}3 \times 10^{-12}\ F$$

$$C = 13{,}3\ pF$$

**13.** Two capacitors, $C_1 = 5.00\ \mu F$ and $C_2 = 12.0\ \mu F$, are **W** connected in parallel, and the resulting combination is connected to a 9.00-V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge stored on each capacitor.




(a) $C_{eq} = C_1 + C_2$

$= 5\mu F + 12 \mu F$

$C_{eq} = 17 \mu F$

b) They are connected to the same potential

$\Delta V = 9.00$ volt

c) $C_1 = \dfrac{Q_1}{\Delta V} \Rightarrow Q_1 = C_1 \Delta V$

$= 5\mu F \cdot 9$ volt

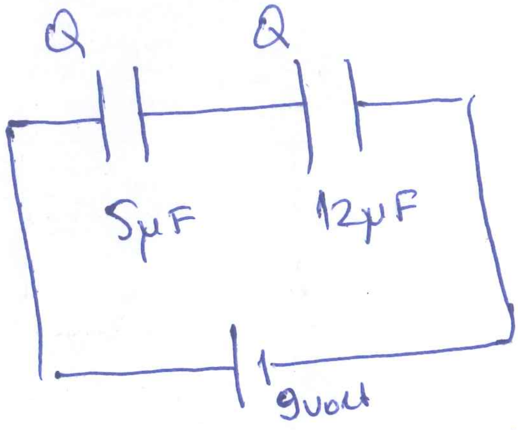
$\boxed{Q_1 = 45\ \mu C}$

$Q_2 = C_2 \cdot \Delta V$

$= 12 \mu F \cdot 9$

$Q_2 = 108 \mu C$

**14. What If?** The two capacitors of Problem 13 ($C_1 = 5.00\ \mu F$ **W** and $C_2 = 12.0\ \mu F$) are now connected in series and to a 9.00-V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.




(a) $\frac{1}{C} = \frac{1}{5} + \frac{1}{12} = \frac{17}{60}$

$C_{eq} = \frac{60}{17}\ \mu F$

(b) $Q = C_{eq} \cdot \Delta V = \frac{60}{17} \cdot 9\ \mu C$

$\Delta V_1 = \frac{Q}{C_1} = \frac{\frac{60 \cdot 9}{17}\ \mu C}{5\ \mu F} = \frac{12 \cdot 9}{17}$ volt

$V_1 = 6{,}35$ volt

$\Delta V_2 = \frac{Q}{C_2} = \frac{\frac{60 \cdot 9}{17}}{12} = \frac{5 \cdot 9}{17}$ volt

$\Delta V_2 = 2.65$ volt

or $\Delta V_2 = 9 - \Delta V_1$

(c) $Q = \frac{60 \cdot 9}{17}\ \mu C = 31.8\ \mu C$

19. For the system of four capacitors shown in Figure P26.19, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.

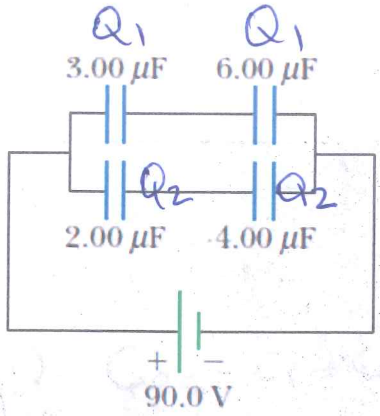



Figure P26.19

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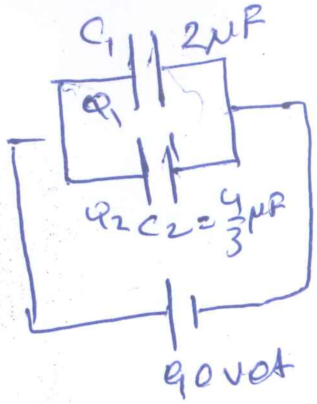



|||


$Q = Q_1 + Q_2$

(a) $\frac{1}{C_1} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$ (2)

$C_1 = 2\mu F$

$\frac{1}{C_2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ (2)

$C_2 = \frac{4}{3}\mu F$

$C_{eq} = C_1 + C_2 = 2 + \frac{4}{3} = \frac{10}{3}\mu F$

(b) $Q_1 = C_1 \cdot 90 = 180\mu C$

$Q_2 = C_2 \cdot 90 = \frac{4}{3} \cdot 90\ \mu C = 120\mu C$

charge on 3 and 6 μF are 180 μC

Charge on 2 and 4 μF are 120 μC

(c) $\Delta V_{3\mu F} = \frac{Q_1}{C} = \frac{180\mu C}{3\mu F} = 60$ volt

$\Delta V_{6\mu F} = \frac{Q_1}{C} = \frac{180\mu C}{6\mu F} = 30$ volt

$\Delta V_{2\mu F} = \frac{Q_2}{2\mu F} = \frac{120\mu C}{2\mu F} = 60$ volt , $\Delta V_{4\mu F} = \frac{120\mu C}{4\mu F} = 30$ volt

22. (a) Find the equivalent capacitance **W** between points $a$ and $b$ for the group of capacitors connected as shown in Figure P26.22. Take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$. (b) What charge is stored on $C_3$ if the potential difference between points $a$ and $b$ is 60.0 V?

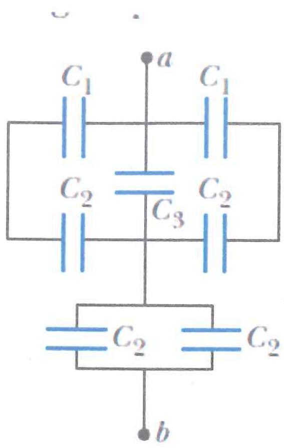
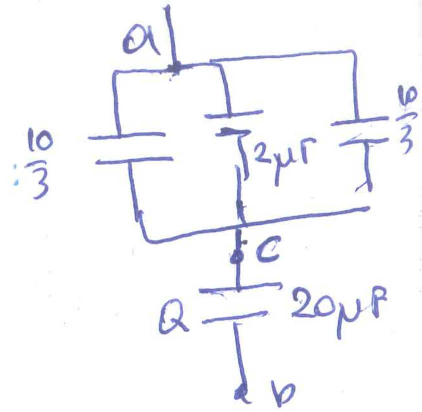
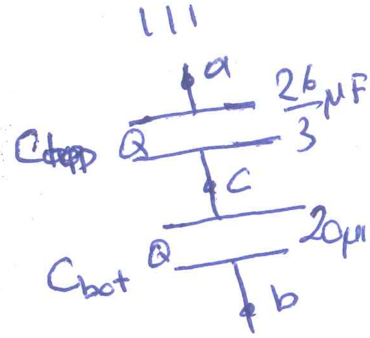
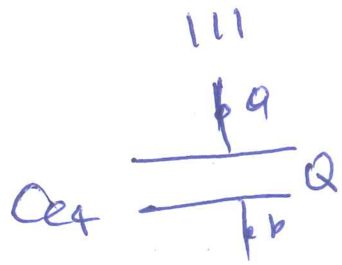



Figure P26.22




$C_1 + C_2$ in series

$$\frac{1}{C_{eq1}} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

$$C_{eq1} = \frac{10}{3}\ \mu F$$

2 $C_2$'s are parallel

$$C_{eq2} = 2C_2 = 20\ \mu F$$

$$C_{top} = \frac{10}{3} + 2 + \frac{10}{3} = \frac{26}{3}$$

$$C_{bot} = 20\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{3}{26} = \frac{26 + 3 \cdot 20}{20 \cdot 26}$$

$$C_{eq} = \frac{20 \cdot 26}{26 + 3 \cdot 20} = 6.05\ \mu F$$

$$Q = C_{eq} \cdot \Delta V = 6.05 \times 60 = 363\ \mu C$$

$$|\Delta V_{ac}| = \frac{363\ \mu C}{\frac{26}{3}\ \mu F} = 41.9$$

$$Q_3 = |\Delta V_{ac}| \cdot 2\ \mu F = 41.9 \times 2.00\ \mu F$$

$$= 83.7\ \mu C$$

**23.** Four capacitors are connected as shown in Figure P26.23. (a) Find the equivalent capacitance between points $a$ and $b$. (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0$ V.

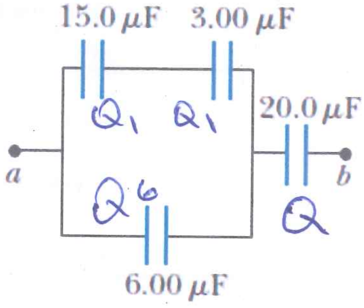
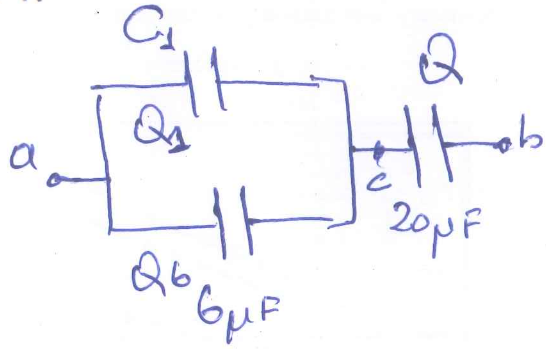


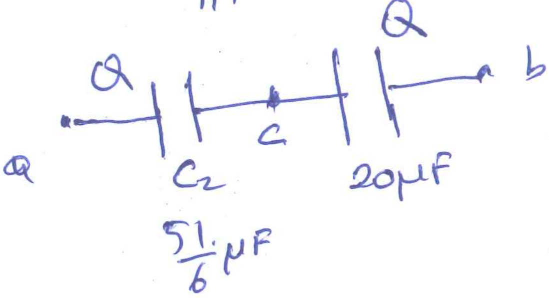

Figure P26.23

(a)



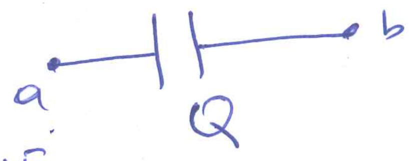

$$\frac{1}{C_1} = \frac{1}{15} + \frac{1}{3} = \frac{6}{15}$$

(5)




$$C_1 = \frac{15}{6}\ \mu F$$

$$C_2 = C_1 + 6\mu F = 6 + \frac{15}{6} = \frac{51}{6}\ \mu F$$




$$\frac{1}{C_{eq}} = \frac{6}{51} + \frac{1}{20}$$

$$C_{eq} = \frac{120+51}{51 \cdot 20} \Rightarrow C_{eq} = \frac{51 \cdot 20}{171} = 5.96\ \mu F$$

(b) $Q = C_{eq} \cdot 150 = 89.5\ \mu C$ charge on 20.0 μF

$$V_{ac} = \frac{Q}{C_2} = \frac{89.5}{51/6} = 10.5 \text{ volt}$$

$$Q_1 = C_1 \cdot V_{ac} = \frac{15}{6} \cdot 10.5 = 26.3\ \mu C$$ charge on 15 μF and 3 μF

$$Q_6 = 6\mu C \cdot 10.5 \text{ volt} = 63.2\ \mu C$$ charge on 6.00 μF

**24.** **M** Consider the circuit shown in Figure P26.24, where $C_1 = 6.00\ \mu F$, $C_2 = 3.00\ \mu F$, and $\Delta V = 20.0$ V. Capacitor $C_1$ is first charged by closing switch $S_1$. Switch $S_1$ is then opened, and the charged capacitor is connected to the uncharged capacitor by closing $S_2$. Calculate (a) the initial charge acquired by $C_1$ and (b) the final charge on each capacitor.

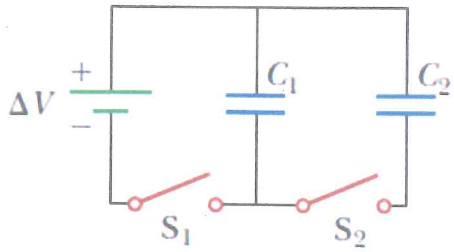
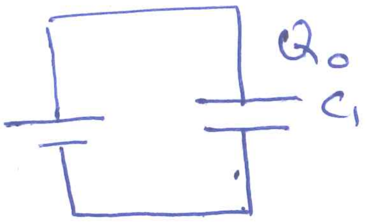


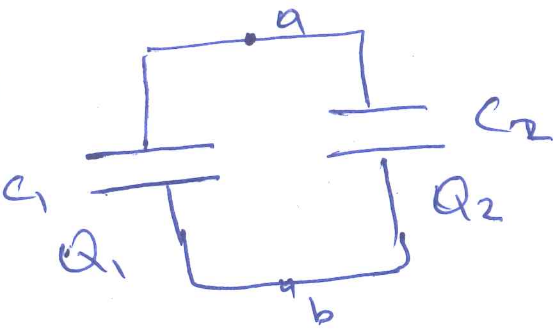

Figure P26.24

(a) initially




$$Q_0 = C_1 V = 20 \cdot 6\ \mu F = 120\ \mu C$$

(b)




$$Q_1 + Q_2 = Q_0$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \Delta V$$

$$Q_2 + Q_1 = C_1 \Delta V + C_2 \Delta V$$

$$Q_0 = (C_1 + C_2)\Delta V$$

$$\Delta V = \frac{Q_0}{C_1 + C_2}$$

$$Q_1 = C_1 \cdot \frac{Q_0}{C_1 + C_2} = \frac{2\,\cancel{6}}{\cancel{9}\,3} \cdot 120\ \mu C = 80\ \mu C$$

$$Q_2 = C_2 \frac{Q_0}{C_1 + C_2} = \cancel{3} \cdot \frac{120\ \mu C}{\cancel{9}\,3} = 40\ \mu C$$

32. (a) A 3.00-$\mu$F capacitor is connected to a 12.0-V battery. **W** How much energy is stored in the capacitor? (b) Had the capacitor been connected to a 6.00-V battery, how much energy would have been stored?

(a) $U = \frac{1}{2} C V^2 = \frac{1}{2} (3 \times 10^{-6}) \cdot (12)^2$

$= 216 \times 10^{-6}\ J$

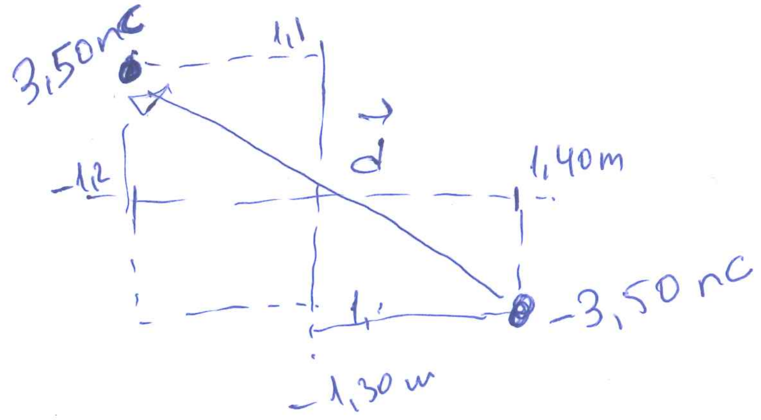
$= 216\ \mu J$

(b) $U = \frac{1}{2} C \cdot V^2 = \frac{1}{2} (3 \times 10^{-6}) \cdot (6)^2$

$= 54 \times 10^{-6}\ J$

$= 54\ \mu J$

**50.** **M** A small, rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates (−1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, −1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\vec{\mathbf{E}} = (7.80 \times 10^3\,\hat{\mathbf{i}} - 4.90 \times 10^3\,\hat{\mathbf{j}})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object–field system when the object is in this orientation. (d) Assuming the orientation of the object can change, find the difference between the maximum and minimum potential energies of the system.




(a) Displacemen vector from (−) to (+)

$$\vec{d} = \left[-1{,}20\,\hat{i} + 1{,}10\,\hat{j} - (1{,}40\,\hat{i} - 1{,}30\,\hat{j})\right] \text{mm}$$

$$\vec{d} = \left[-2{,}60\,\hat{i} + 2{,}40\,\hat{j}\right] \text{mm}$$

$$= \left[-2{,}60\,\hat{i} + 2{,}40\,\hat{j}\right] \times 10^{-3}\,\text{m}$$

$$\vec{p} = q\vec{d} = 3{,}50 \times 10^{-9}\,C \left[-2{,}60\,\hat{i} + 2{,}40\,\hat{j}\right]$$

$$\vec{p} = (-9{,}10\,\hat{i} + 8{,}40\,\hat{j})\,10^{-12}\,\text{Cm}$$

b)

$$\vec{\tau} = \vec{p} \times \vec{E} = 10^{-12}(-9{,}10\,\hat{i} + 8{,}40\,\hat{j}) \times (7{,}80 \times 10^3\,\hat{i} - 4{,}90 \times 10^3\,\hat{j})\,10^3$$

$$\vec{\tau} = (9{,}10 \cdot 4{,}90\,\hat{k} - 8{,}40 \cdot 7{,}80\,\hat{k})\,10^{-12}$$

$$\vec{\tau} = (-2{,}09 \cdot 10^{-8}\,\hat{k})\,\text{Nm}$$

c) $U = -\vec{p} \cdot \vec{E}$

$= -\left(-9{,}10\,\hat{i} + 8{,}40\,\hat{j}\right) \cdot \left(7{,}80\,\hat{i} - 4{,}90\,\hat{j}\right) 10^{-9}\ Nm$

$U = \left(9{,}10 \cdot 7{,}80 + 8{,}40 \cdot 4{,}90\right) 10^{-9}\ J$

$U = 112 \times 10^{-9}\ J = 112\ nJ$

d) $|\vec{p}| = \sqrt{9{,}10^2 + 8{,}40^2} \times 10^{-12}\ Cm = 12{,}4 \times 10^{-12}\ C.m$

$|\vec{E}| = \sqrt{(7{,}80)^2 + (4{,}90)^2} \times 10^3\ \frac{N}{C} = 9{,}21 \times 10^3\ N/C$

$U_{min} = -|\vec{p}||\vec{E}| = -114 \times 10^{-9}\ Nm = -114\ nJ$

$U_{max} = |\vec{p}||\vec{E}| = 114\ nJ$

$\Delta U = U_{max} - U_{min} = 228\ nJ$

**59.** A parallel-plate capacitor is constructed using a

**M** dielectric material whose dielectric constant is 3.00 and whose dielectric strength is $2.00 \times 10^8$ V/m. The desired capacitance is 0.250 $\mu$F, and the capacitor must withstand a maximum potential difference of 4.00 kV. Find the minimum area of the capacitor plates.

$$E_{max} = 2\times10^8 \frac{V}{m} = \frac{\Delta V}{d_{min}}$$

$$d_{min} = \frac{\Delta V}{2\times10^8 \frac{V}{m}} = \frac{\cancel{4}^{2}000\ V}{\cancel{2}\times10^8 \frac{V}{m}} = 2\times10^{-5}\ m$$

$$C = K \frac{\epsilon_0 A}{d}$$

$$A = \frac{C d}{K \epsilon_0} = \frac{0{,}250\times10^{-6} \cdot 2\times10^{-5}}{3 \cdot 8{,}854\times10^{-12}} = 0{,}188\ m^2$$

63. A 10.0-$\mu$F capacitor is charged to 15.0 V. It is next connected in series with an uncharged 5.00-$\mu$F capacitor. The series combination is finally connected across a 50.0-V battery as diagrammed in Figure P26.63. Find the new potential differences across the 5.00-$\mu$F and 10.0-$\mu$F capacitors after the switch is thrown closed.

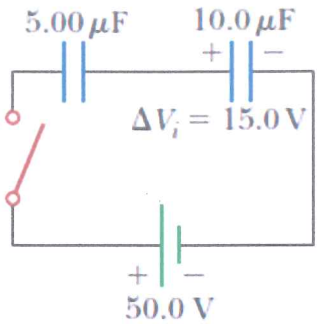
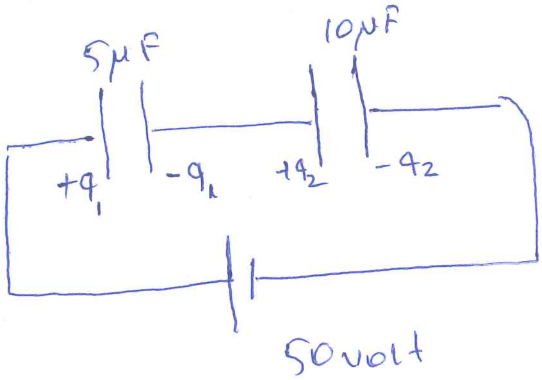



Figure P26.63

$q_i = C \cdot \Delta V_i = 10\,\mu F \cdot 15\,V = 150\,\mu C$




$q_2 - q_1 = +150\,\mu C$

$\Delta V_1 = q_1 / 5\,\mu F$

$\Delta V_2 = q_2 / 10\,\mu F$

$$\frac{q_1}{5\,\mu F} + \frac{q_2}{10\,\mu F} = 50$$

$$2q_1 + q_2 = 500\,\mu C$$

$$-\quad q_2 - q_1 = 150\,\mu C$$

$$3q_1 = 350\,\mu C$$

$$q_1 = \frac{350}{3}\,\mu C$$

$$q_2 = q_1 + 150\,\mu C$$

$$q_2 = \frac{800}{3}\,\mu C$$

$$\Delta V_1 = \frac{q_1}{5\,\mu F} = \frac{350}{3 \cdot 5} = \frac{70}{3}\text{ volt}$$

$$\Delta V_2 = \frac{q_2}{10\,\mu F} = \frac{800}{3 \cdot 10} = \frac{80}{3}\text{ volt}$$