Current, Resistance, and Electromotive Force

Current: $I = \frac{\Delta Q}{\Delta t}$, instantaneous $I = \frac{dQ}{dt}$

$I = n|q|v_d A$ current

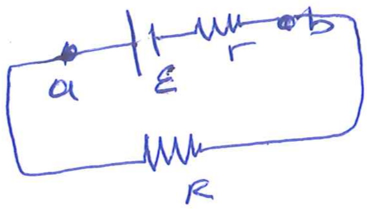
$\vec{J} = nq\vec{v}_d$ current density

Resistivity: $\vec{E} = \rho\vec{J}$ Ohm's law

$\rho(T) = \rho_0(1+\alpha(T-T_0))$

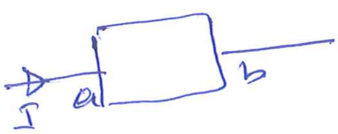
Resistors: $R = \frac{\rho L}{A}$, $V = IR$

Circuits and emf:



$V_{ab} = \mathcal{E} - Ir$
terminal voltage

Power



power drawn from the circuit

$P = IV_{ab} = I(V_a - V_b)$

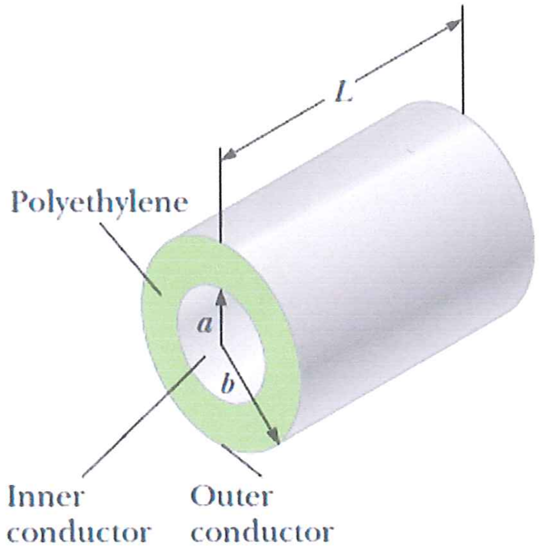
resistor: $P = I^2R = \frac{V^2}{R} = IV$

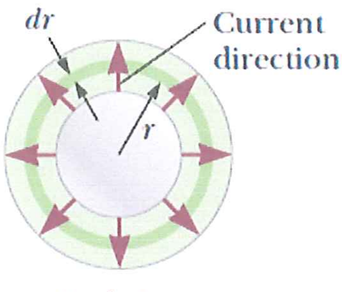
Conduction in metals:

τ: mean free time

$\rho = \frac{m}{ne^2\tau}$

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in Figure 27.8a. Current leakage through the plastic, in the *radial* direction, is unwanted. (The cable is designed to conduct current along its length, but that is *not* the current being considered here.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length is $L = 15.0$ cm. The resistivity of the plastic is $1.0 \times 10^{13}\ \Omega \cdot \text{m}$. Calculate the resistance of the plastic between the two conductors.





End view

$$R = \rho \frac{\ell}{A}$$

For the radial current

$$\ell = \Delta r$$

$$A = 2\pi r L$$

resistance between r and r+dr

$$dR = \rho \frac{dr}{2\pi r L}$$

resistance between a and b

$$R = \int_a^b dR = \int_a^b \rho \frac{dr}{2\pi r L}$$

$$= \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r}$$

$$= \frac{\rho}{2\pi L} \left[\ln r\right]_a^b = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$$

$$R = 1\times10^{13}\ \Omega\cdot\text{m} \cdot \frac{1}{2\pi\ 15\times10^{-2}\ \text{m}} \cdot \ln\left(\frac{1,75}{0,5}\right)$$

$$= 1,33\times10^{13}\ \Omega$$

1. A 200-km-long high-voltage transmission line 2.00 cm in diameter carries a steady current of 1 000 A. If the conductor is copper with a free charge density of 8.50×10^{28} electrons per cubic meter, how many years does it take one electron to travel the full length of the cable?

AMT

M

$$J = \frac{I}{\pi r^2} = \frac{I}{\pi \left(\frac{d}{2}\right)^2} = n|e|v_d$$

$$v_d = \frac{I}{n|e|\pi\left(\frac{d}{2}\right)^2}$$

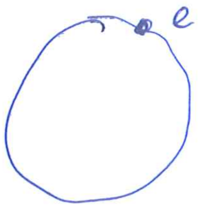
$$v_d = \frac{4I}{\pi d^2 n|e|}$$

$$t = \frac{L}{v_d} = \frac{200\times10^3\ m}{4I / \pi d^2 n|e|}$$

$$t = \frac{2.10^5 m . \pi . (2\times10^{-2})^2\ 8.50\times10^{28} . 1.6.10^{-19}}{4.\ 1000\ A}$$

$$= (8.55\times10^8 s)\ \frac{1\ year}{(3.156\times10^7 s)} = 27.1\ year$$

4. In the Bohr model of the hydrogen atom (which will AMT be covered in detail in Chapter 42), an electron in the lowest energy state moves at a speed of 2.19×10^6 m/s in a circular path of radius 5.29×10^{-11} m. What is the effective current associated with this orbiting electron?



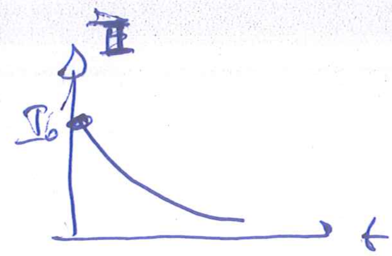
$$I = \frac{|e|}{T} \quad , \quad T = \frac{2\pi r}{v}$$

$$I = \frac{|e| v}{2\pi r}$$

$$I = \frac{1.6 \times 10^{-19} \cdot 2.19 \times 10^{6}}{2 \cdot \pi \cdot 5.29 \times 10^{-11}}$$

$$I = 1.05 \times 10^{-3} \ \frac{C}{s} = 1.05 \ mA$$

7. Suppose the current in a conductor decreases exponentially with time according to the equation $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$) and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) What If? How much charge passes this point between $t = 0$ and $t = \infty$?



$$I = \frac{dq}{dt}$$

$$dq = I\,dt = I_0 e^{-\frac{t}{\tau}} dt$$

$$\int_{t_i}^{t_f} dq = \int_{t_i}^{t_f} I_0 e^{-\frac{t}{\tau}} dt \quad \text{between } t_i \text{ and } t_f$$

$$Q = \left[-\tau I_0 e^{-\frac{t}{\tau}}\right]_{t_i}^{t_f}$$

(a) $t_i = 0$, $t_f = \tau$

$$Q = \left[-\tau I_0 e^{-t/\tau}\right]_0^{\tau} = \left[-\tau I_0 e^{-\frac{\tau}{\tau}} + \tau I_0\right]$$

$$Q = \tau I_0 (1 - e^{-1}) = 0.632\, \tau I_0$$

(b) $t_f = 10\tau$

$$Q = \left[-\tau I_0 e^{-\frac{10\tau}{\tau}} + \tau I_0\right]$$

$$Q = \tau I_0 \left[1 - e^{-10}\right] = 0.99995\, I_0 \tau$$

(c) $t_f = \infty$

$$Q = \left[-\tau I_0 \underset{0}{\underbrace{e^{-\infty}}} + \tau I_0\right] = \tau I_0$$

9. The quantity of charge q (in coulombs) that has passed
W through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1.00$ s? (b) What is the value of the current density?

$$A = 2 \times 10^{-4}\ m^2$$

$$(a)\quad I = \frac{dq}{dt} = \frac{d}{dt}(4t^3 + 5t + 6)$$

$$I = 12t^2 + 5$$

$$t = 1.0\,s \Rightarrow I = 12 + 5 = 17A$$

$$b)\quad J = \frac{I}{A} = \frac{17\ A}{2 \times 10^{-4} m^2} = 85 \times 10^3 \frac{A}{m^2}$$

$$= 85 \frac{kA}{m^2}$$

12. An electric current in a conductor varies with time according to the expression $I(t) = 100 \sin (120\pi t)$, where I is in amperes and t is in seconds. What is the total charge passing a given point in the conductor from $t = 0$ to $t = \frac{1}{240}$ s?

$$Q = \int_{t_i}^{t_f} I\,dt$$

$$Q = \int_0^{\frac{1}{240}} 100 \sin(120\pi t)\,dt$$

$$= \frac{100}{120\pi} \Big[-\cos(120\pi t) \Big]_0^{1/240}$$

$$= \frac{100}{120\pi} \left[-\cos\left(\frac{120\pi}{240}\right) + \cos(0) \right]$$

$$= \frac{100}{120\pi} \Big[\underbrace{-\cos\left(\frac{\pi}{2}\right)}_{0} + 1 \Big]$$

$$Q = \frac{100}{120\pi} \text{ coulomb}$$

$$Q = 0{,}265 \text{ coulomb}$$

19. **M** Suppose you wish to fabricate a uniform wire from 1.00 g of copper. If the wire is to have a resistance of $R = 0.500\ \Omega$ and all the copper is to be used, what must be (a) the length and (b) the diameter of this wire?

d: gravimetric density

$$m = dV = dAL \quad , \quad A = \frac{m}{dL}$$

$$R = \rho \frac{L}{A} = \rho \frac{L}{\frac{m}{dL}} = \rho \frac{dL^2}{m}$$

$$\sqrt{L^2} = \sqrt{\frac{mR}{\rho d}} \Rightarrow L = \sqrt{\frac{mR}{\rho d}}$$

$$L = \sqrt{\frac{1\times10^{-3}\,kg \cdot 0,5}{(1,7\times10^{-8}\,\Omega.m) \cdot 8,92\times10^{3}\,\frac{kg}{m^3}}}$$

$$L = 1,82\,m$$

b) $$A = \frac{m}{dL}$$

$$\pi r^2 = \frac{m}{dL}$$

$$r = \sqrt{\frac{m}{\pi d L}}$$

$$\text{diameter} = 2r = 2\sqrt{\frac{m}{\pi d L}}$$

$$= 2\sqrt{\frac{1\times10^{-3}}{\pi \cdot 8,92\cdot10^{3} \cdot 1,82}} = 2,8\times10^{-4}\,m$$

$$= 280\,\mu m$$

25. If the magnitude of the drift velocity of free electrons
M in a copper wire is 7.84×10^{-4} m/s, what is the electric field in the conductor?

in Drude model   E and v_d proportional

$E = \rho J$

$E = \rho n e v_d$

$\rho = 1.7 \times 10^{-8}\ \Omega \cdot m$

$e = 1.6 \times 10^{-19} C$

$n = \frac{mass}{M} \cdot N_A / Volume = \frac{\# \text{ of atoms}}{Volume}$

$n = \frac{mass}{Volume} \cdot \frac{N_A}{M} = d \frac{N_A}{M}$

$= 8920\ kg/m^3 \cdot \frac{6.02 \times 10^{23}}{63.5 \times 10^{-3}\ kg}$

$\boxed{n = 8.46 \times 10^{28}\ m^{-3}}$

$E = 1.7 \times 10^{-8} \cdot 8.46 \times 10^{28} \cdot 1.6 \times 10^{-19} \cdot 7.84 \times 10^{-4}$

$\boxed{E = 0.18\ \frac{V}{m}}$

45. Batteries are rated in terms of ampere-hours (A · h). **W** For example, a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A · h. (a) What is the total energy, in kilowatt-hours, stored in a 12.0-V battery rated at 55.0 A · h? (b) At $0.110 per kilowatt-hour, what is the value of the electricity produced by this battery?

(a) $6\ Ah = 6 \cdot \frac{C}{s} \cdot 3600s = 6 \cdot 3600$ coulomb

amount of charge

$55\ Ah = 55\ \frac{C}{s} \cdot 3600s = 55\ \frac{C}{s} \cdot h$

$\Delta U = 12$ volt · charge

$\Delta U = 12\ V\ 55\ \frac{C}{s} \cdot h$

$= 660\ \frac{VC}{s} \cdot h$, $VC = $ joule , $\frac{joule}{s} = $ watt

$= 660$ wh

$\Delta U = 0.66$ kWh

b) price $= 0.66\ \text{kwh} \cdot \frac{0{,}11\ \$}{1\ \text{kwh}}$

$= 0{,}0726$ USD

57. A particular wire has a resistivity of $3.0 \times 10^{-8}\ \Omega \cdot m$ and a cross-sectional area of $4.0 \times 10^{-6}\ m^2$. A length of this wire is to be used as a resistor that will receive 48 W of power when connected across a 20-V battery. What length of wire is required?

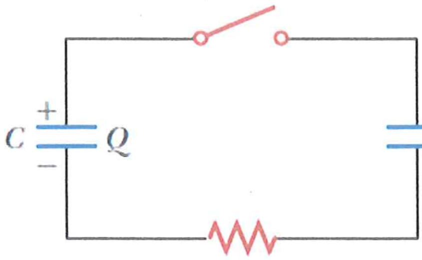
M

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(20)^2}{48} = \frac{\cancel{400}}{\cancel{48}} = \frac{25}{3}\ \Omega$$

$$R = \rho \frac{L}{A} \Rightarrow L = \frac{AR}{\rho} = \frac{4 \cdot 10^{-6} \cdot 25/3}{3 \cdot 10^{-8}\ \Omega \cdot m}$$

$$L = 1{,}1 \times 10^3\ m = 1{,}1\ km$$

63. A charge Q is placed on a capacitor of capacitance C. The capacitor is connected into the circuit shown in Figure P27.63, with an open switch, a resistor, and an initially uncharged capacitor of capacitance $3C$. The



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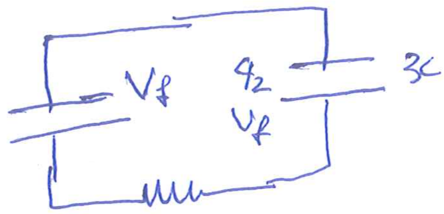

switch is then closed, and the circuit comes to equilibrium. In terms of Q and C, find (a) the final potential difference between the plates of each capacitor, (b) the charge on each capacitor, and (c) the final energy stored in each capacitor. (d) Find the internal energy appearing in the resistor.

final voltage on the capacitors are the same

$$\frac{q_1}{C} = \frac{q_2}{3C} \Rightarrow q_2 = 3q_1$$

$q_1 + q_2 = Q$ conservation of charge

$4q_1 = Q \Rightarrow q_1 = \frac{Q}{4}, \quad q_2 = \frac{3Q}{4}$

(a) $V_f = \frac{q_1}{C} = \frac{Q}{4C}$

(b) $q_1 = Q/4$, on C $\quad q_2 = 3Q/4$ on 3C

(c) on C $\quad \frac{1}{2}qV = \frac{q^2}{2C} = \frac{Q^2}{32C}$

on 3C $\quad \frac{(3Q/4)^2}{2 \cdot 3C} = \frac{3\cancel{9}Q^2}{{}_2\cancel{6} \cdot 16} = \frac{3Q^2}{32C}$

(d) Final potential energy $\frac{3Q^2 + Q^2}{32C} = \frac{Q^2}{8C}$

initial energy $\frac{Q^2}{2C}$

lost energy $= \frac{Q^2}{2C} - \frac{Q^2}{8C} = \frac{3Q^2}{8C}$

66. AMT M An all-electric car (not a hybrid) is designed to run from a bank of 12.0-V batteries with total energy storage of 2.00×10^7 J. If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, (a) what is the current delivered to the motor? (b) How far can the car travel before it is "out of juice"?

(a) $P = 8 \times 10^3 \text{ watt} = I \cdot \Delta V$

$$I = \frac{8 \cdot 10^3 \text{ watt}}{12} = 667 A$$

b) $$\Delta t = \frac{U}{P} = \frac{2 \times 10^7 \text{ J}}{8 \times 10^3 \text{ J/s}}$$

$$\Delta t = 2{,}50 \times 10^3 \text{ s}$$

$$\Delta x = v \Delta t$$

$$= 20 \frac{m}{s} \cdot 2{,}5 \times 10^3 \text{ s}$$

$$= 50 \times 10^3 \text{ m}$$

$$\Delta x = 50 \text{ km}$$

67. A straight, cylindrical wire lying along the x axis has a length of 0.500 m and a diameter of 0.200 mm. It is made of a material described by Ohm's law with a resistivity of $\rho = 4.00 \times 10^{-8}\ \Omega \cdot m$. Assume a potential of 4.00 V is maintained at the left end of the wire at $x = 0$. Also assume $V = 0$ at $x = 0.500$ m. Find (a) the magnitude and direction of the electric field in the wire, (b) the resistance of the wire, (c) the magnitude and direction of the electric current in the wire, and (d) the current density in the wire. (e) Show that $E = \rho J$.

4V   V=0

x=0   x=0,5m

(a) $\vec{E}$ from high pot. to lower potential

$\vec{E} \rightarrow$ $E = \frac{\Delta V}{d} = \frac{4V}{0,5m} = 8\ \frac{V}{m}$

(b) $R = \rho \frac{L}{A} = 4 \times 10^{-8}\ \Omega.m. \frac{0,5m}{\pi (0,1 \times 10^{-3})^2} = 0,637\ \Omega$

(c) I is parallel to $\vec{E}$

$I \rightarrow$ $I = \frac{V}{R} = \frac{4}{0,637} = 6,28A$

(d) $J = \frac{I}{A} = \frac{6,25}{\pi (0,1 \times 10^{-3})^2} = 2 \times 10^8\ \frac{A}{m^2}$

in +x-direction

(e) $8\ \frac{V}{m} \stackrel{?}{=} 4 \times 10^{-8}\ \Omega.m.\ 2 \times 10^8\ \frac{A}{m^2}$

$8\ \frac{V}{m} \stackrel{?}{=} 8\ \frac{V}{m}$

indeed $\boxed{E = \rho J}$

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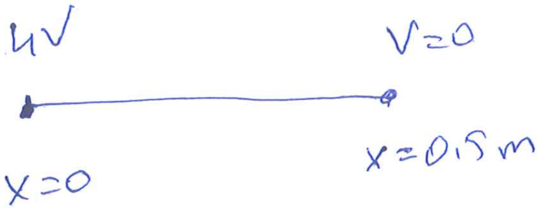

(a) $\vec{E}$ from high pot. to lower potential

$\vec{E}$ $\longrightarrow$ $\quad E = \dfrac{\Delta V}{d} = \dfrac{4\,V}{0{,}5\,m} = 8\,\dfrac{V}{m}$

(b) $R = \rho \dfrac{L}{A} = 4\times10^{-8}\ \Omega.m.\dfrac{0{,}5\,m}{\pi(0{,}1\times10^{-3})^2} = 0{,}637\ \Omega$

(c) I is paralel to $\vec{E}$

I $\longrightarrow$ $\quad I = \dfrac{V}{R} = \dfrac{4}{0{,}637} = 6{,}28\,A$

(d) $J = \dfrac{I}{A} = \dfrac{6{,}25}{\pi(0{,}1\times10^{-3})^2} = 2\times10^{8}\ \dfrac{A}{m^2}$

in +x-direction

(e) $8\,\dfrac{V}{m} \stackrel{?}{=} 4\times10^{-8}\ \Omega.m.\ 2\times10^{8}\ \dfrac{A}{m^2}$

$8\,\dfrac{V}{m} \stackrel{?}{=} 8\,\dfrac{V}{m}$ $\qquad$ indeed $\boxed{E = \rho J}$