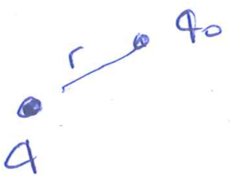
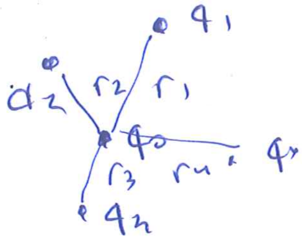
Electric Potential

Work done by electrical forces $W_{a\to b} = U_a - U_b$



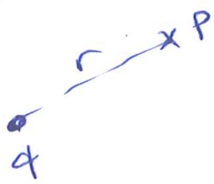
potential energy of two change $U = \frac{kqq_0}{r}$



Potential energy of q_0 at point P $U = kq_0 \sum \frac{q_i}{r_i}$

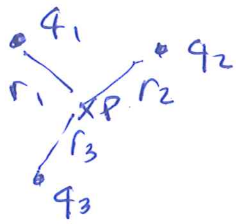
<u>Potential energy</u>: Work that must be done by external forces to bring the change from infinity to that position

- Potential by point change

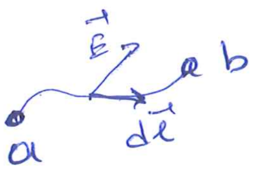


V at $P = \frac{kq}{r}$

- Superposition principle



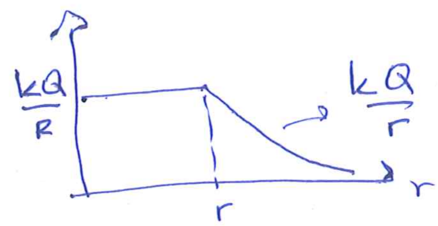
$V = \sum \frac{kq_i}{r_i}$



$$V_a - V_b = \int_a^b \vec{E} \cdot \vec{dl} \qquad \left(\frac{N}{C} \equiv \frac{V}{m}\right)$$

$$\boxed{\hat{r} \cdot \vec{dl} = dr}$$

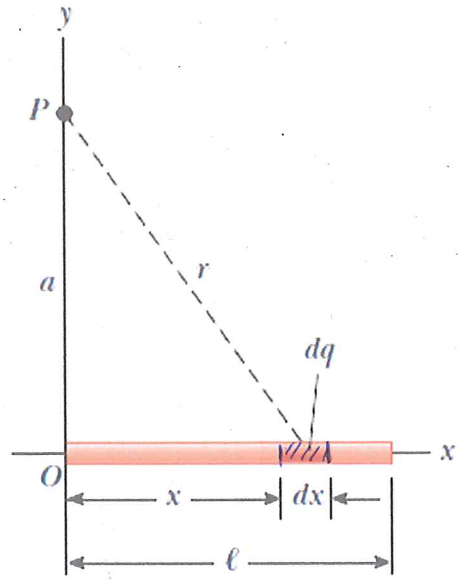
- Potential of a changed conductor



- $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$\vec{E} =$

A rod of length ℓ located along the x axis has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance a from the origin (Fig. 25.16).



dV potential by dq

$$dV = k\frac{dq}{r} = \frac{k\,dq}{\sqrt{a^2+x^2}}$$

and $dq = \lambda\,dx$

$$dV = \frac{k\lambda\,dx}{\sqrt{a^2+x^2}}$$

$$V = \underset{\text{over the rod}}{\int dV} = \int_{x=0}^{x=\ell} \frac{k\lambda\,dx}{\sqrt{a^2+x^2}}$$

$$V = k\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2+x^2}} = k\lambda \ln\left(x+\sqrt{x^2+a^2}\right)\Big|_0^{\ell}$$

$$V = k\lambda\left[\ln\left(\ell+\sqrt{\ell^2+a^2}\right) - \ln(a)\right]$$

$$V = k\lambda \ln\left(\frac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$$

and $\lambda = \dfrac{\text{charge}}{\text{length}} = \dfrac{Q}{\ell}$

$$V = \frac{kQ}{\ell}\ln\left(\frac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$$

1. Oppositely charged parallel plates are separated **M** by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned 2.90 mm from the positive plate?

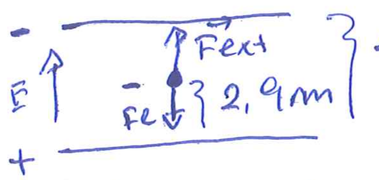
$\Delta V = 600\,V$ $5{,}33\,mm$ $\Delta V = \int \vec{E} \cdot d\ell = Ed$

a) $$E = \frac{\Delta V}{d}$$

$$E = \frac{600\,V}{5{,}33 \times 10^{-3}\,m} = 1{,}13 \times 10^{5}\,\frac{N}{C} = 1{,}13 \times 10^{5}\,\frac{V}{m}$$

b) $$F = |q|\,E = 1{,}6 \times 10^{-19}\,C \cdot 1{,}13 \times 10^{5}\,\frac{N}{C}$$
$$= 1{,}80 \times 10^{-14}\,N$$

c)



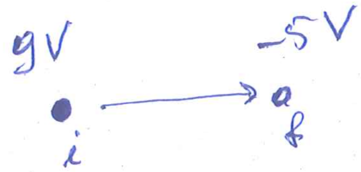
$W_{ext} = F_{ext}\,\Delta x$

$\vec{F}_{ext} = -\vec{F}_e$, $\vec{F}_{ext}$ is up , Δx is up

$$W_{ext} = 1{,}80 \times 10^{-14}\,N \cdot (5{,}33 - 2{,}9) \times 10^{-3}\,m$$
$$= 4{,}37 \times 10^{-17}\,Nm$$
$$= 4{,}37 \times 10^{-17}\,J$$

This is the work done by the external force

4. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the electric potential is −5.00 V? (The potential in each case is measured relative to a common reference point.)



$$W_{ext} = -W_{electrical} = -Q(V_i - V_f)$$

$$W_{ext} = Q(V_f - V_i) = U_f - U_i$$

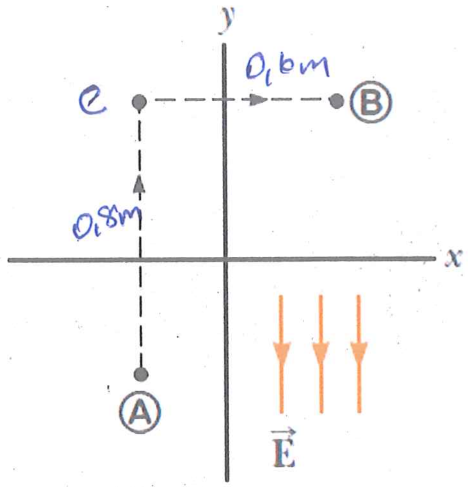
$$Q = N_A \cdot q = -N_A e = -6.02\times10^{23} \cdot 1.6\times10^{-19}\,C$$

$$Q = -9.63\times10^{4}\,C$$

$$W_{ext} = -9.63\times10^{4}\,C\,(-5-9)\,V$$

$$W_{ext} = 1.35\times10^{6}\,J$$

This is the force that must be done by external forces.

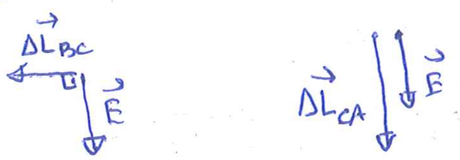


5. A uniform electric field **W** of magnitude 325 V/m is directed in the negative y direction in Figure P25.5. The coordinates of point Ⓐ are (−0.200, −0.300) m, and those of point Ⓑ are (0.400, 0.500) m. Calculate the electric potential difference $V_Ⓑ - V_Ⓐ$ using the dashed-line path.

$\vec{E}$ is uniform

$$V_B - V_A = \int_B^C \vec{E}\cdot d\vec{\ell} + \int_C^A \vec{E}\cdot d\vec{\ell}$$

$$= \vec{E}\cdot \Delta\vec{L}_{BC} + \vec{E}\cdot\Delta\vec{L}_{CA}$$



$$= 325\,\frac{V}{m}\cdot 0.6\cdot \underset{0}{\underset{\shortparallel}{\cos 90^\circ}} + 325\,\frac{V}{m}\times 0{,}8\,m\cdot \underset{1}{\underset{\shortparallel}{\cos(0^\circ)}}$$

$$V_B - V_A = 325\,\frac{V}{m}\cdot 0{,}8\,m$$

$$V_B - V_A = 260\,V$$

7. AMT M An electron moving parallel to the x axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point $x = 2.00$ cm. (a) Calculate the electric potential difference between the origin and that point. (b) Which point is at the higher potential?

$v_i = 3{,}70 \times 10^6 \frac{m}{s}$ $\qquad$ $v_f = 1{,}40 \times 10^5$ m/s

$x_i = 0$ $\qquad$ $x_f = 2{,}00$ cm

(a) Totale energy Conserved

$$qV_i + \frac{1}{2} m v_i^2 = qV_f + \frac{1}{2} m v_f^2$$

$$V_i - V_f = \frac{\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2}{q}$$

$$V_i - V_f = \frac{m}{2q}\left(v_f^2 - v_i^2\right)$$

$$V_f - V_i = -\frac{9{,}11 \times 10^{-31}}{-2 \cdot 1{,}6 \times 10^{-19}}\left((1{,}4 \times 10^5)^2 - (3{,}7 \times 10^6)^2\right)$$

$$V_f - V_i = -38{,}9\ V$$

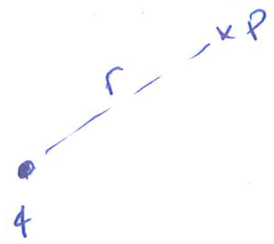
b) $V_f < V_i$

origin is at a higher potential

$+$ $\qquad \vec{E} \qquad$ $-$

0 $\qquad\qquad$ $x = 2$ cm

20. **M** At a certain distance from a charged particle, the magnitude of the electric field is 500 V/m and the electric potential is −3.00 kV. (a) What is the distance to the particle? (b) What is the magnitude of the charge?



$$E = \frac{k|q|}{r^2}\ , \quad |V| = \frac{k|q|}{r}$$

(a) $$\frac{|V|}{E} = \frac{k|q|/r}{k|q|/r^2} = r$$

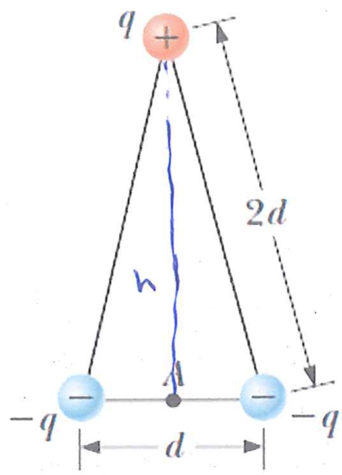
$$r = \frac{3.0\times10^{3}\ V}{500\ \frac{V}{m}} = 6.00\ m$$

(b) $$V = -3000 = \frac{kq}{6}$$

$$q = \frac{-18000}{k} = \frac{-18\cancel{000}}{9\times10^{\cancel{9}6}} = -2\times10^{-6}\,C$$

$$q = -2\ \mu C$$

22. **M** The three charged particles in Figure P25.22 are at the vertices of an isosceles triangle (where $d = 2.00$ cm). Taking $q = 7.00\ \mu C$, calculate the electric potential at point A, the midpoint of the base.



$$V = \sum \frac{kq_i}{r_i}$$

$$V = -\frac{kq}{d/2} - \frac{kq}{d/2} + \frac{kq}{h}$$

$$h = \sqrt{4d^2 - \frac{d^2}{4}}$$

$$h = \frac{\sqrt{15}\,d}{2}$$

$$V = -\frac{2kq}{d} - \frac{2kq}{d} + \frac{2kq}{\sqrt{15}\,d}$$

$$V = \frac{2kq}{d}\left(\frac{1}{\sqrt{15}} - 2\right)$$

$$V = \frac{2 \cdot 9 \cdot 10^{9} \cdot 7 \cdot 10^{-6}}{2 \cdot 10^{-2}}\left(\frac{1}{\sqrt{15}} - 2\right)$$

$$V = -1{,}10 \times 10^{7} \text{ volt}$$

27. Four identical charged particles ($q = +10.0\ \mu C$) are **W** located on the corners of a rectangle as shown in Figure P25.27. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the change in electric potential energy of the system as the particle at the lower left corner in Figure P25.27 is brought to this position from infinitely far away. Assume the other three particles in Figure P25.27 remain fixed in position.

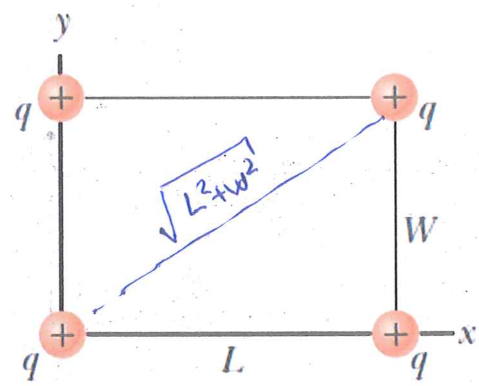


Figure P25.27

The question is the potential energy of the charge at lower left corner

$$U_q = \sum \frac{k q_i q}{r_i}$$

$$U_q = \frac{kqq}{L} + \frac{kqq}{W} + \frac{kqq}{\sqrt{L^2+W^2}}$$

$$= kq^2\left(\frac{1}{L} + \frac{1}{W} + \frac{1}{\sqrt{L^2+W^2}}\right)$$

$$= 9\times10^9 \times (10^{-5}C)^2 \left(\frac{1}{0.6} + \frac{1}{0.15} + \frac{1}{\sqrt{0.6^2+0.15^2}}\right)$$

$$U_q = 8.95\,J$$

39. **W** Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. (a) Find the expressions for the x, y, and z components of the electric field over this region. (b) What is the magnitude of the field at the point P that has coordinates (1.00, 0, −2.00) m?

(a) $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(5x - 3x^2y + 2yz^2\right)$

$E_x = -\left(5 - 6xy\right) = 6xy - 5$

$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(5x - 3x^2y + 2yz^2\right)$

$= -\left(-3x^2 + 2z^2\right) = 3x^2 - 2z^2$

$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left(5x - 3x^2y + 2yz^2\right)$

$= -\left(4yz\right) = -4yz$

$\vec{E} = \left(6xy - 5\right)\hat{i} + \left(3x^2 - 2z^2\right)\hat{j} - 4yz\hat{k}$

(b) $x = 1$, $y = 0$, $z = -2$

$\vec{E} = \left(6 \cdot 1 \cdot 0 - 5\right)\hat{i} + \left(3 \cdot 1 - 2 \cdot 4\right)\hat{j} - 4 \cdot 0 \cdot (-2)\hat{k}$

$\vec{E} = \left(-5\hat{i} - 5\hat{j}\right)\frac{V}{m} = \left(-5\hat{i} - 5\hat{j}\right)\frac{N}{C}$

$E = \sqrt{(-5)^2 + (-5)^2}\ \frac{V}{m} = 5\sqrt{2}\ \frac{V}{m}$

44. A uniformly charged insulating rod of **W** length 14.0 cm is bent into the shape of a semicircle as shown in Figure P25.44. The rod has a total charge of $-7.50\ \mu C$. Find the electric potential at O, the center of the semicircle.

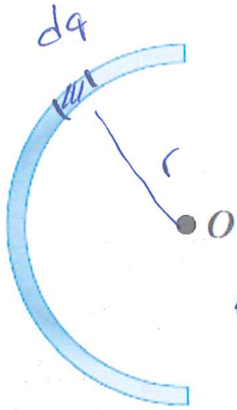


Figure P25.44

$L = \pi r$

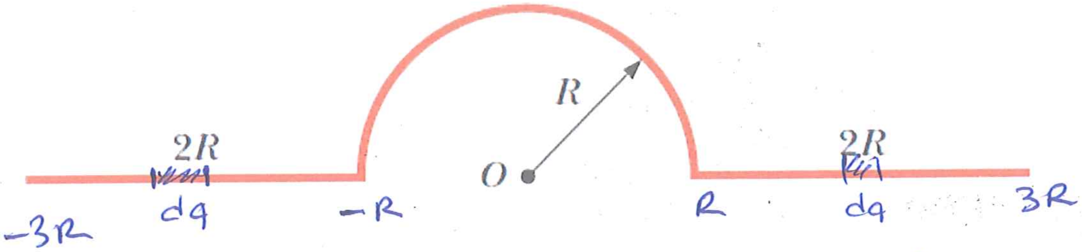
$r = \frac{L}{\pi}$

$$V = \int \frac{k\,dq}{r} = \frac{k}{r}\int dq$$

$$V = \frac{kQ}{r} = \frac{9\times10^{9} \cdot (-7{,}50\times10^{-6})}{(14.0/\pi \times 10^{-2})}$$

$$V = -1{,}51\times10^{6}\ \text{Volt}$$

47. A wire having a uniform linear charge density λ is bent **W** into the shape shown in Figure P25.47. Find the electric potential at point O.



$dq = \lambda\, dx$

$dq = \lambda\, dx$

$$V = V_{left} + V_{circle} + V_{righ}$$

$$= \int_{-3R}^{-R} \frac{k\, dq}{|x|} + \int \frac{k\, dq}{R} + \int_{R}^{3R} \frac{k\, dq}{x}$$

$$= \int_{-3R}^{-R} \frac{k \lambda\, dx}{|x|} + \frac{k}{R}\int dq + \int_{R}^{3R} \frac{k\lambda\, dx}{x}$$

$$= \int_{-3R}^{-R} \frac{k\lambda\, dx}{-x} + \frac{k}{R}\int dq + \int_{R}^{3R} \frac{k\lambda\, dx}{x}$$

$x \to -x$

$$= -\int_{3R}^{R} \frac{k\lambda\, dx}{x} + \frac{k}{R}\cdot \lambda \cdot \pi R + \int_{R}^{3R} \frac{k\lambda\, dx}{x}$$

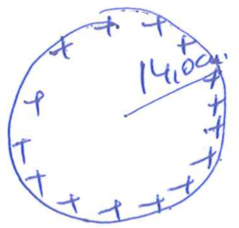
$$= \int_{R}^{3R} \frac{k\lambda\, dx}{x} + k\lambda\pi + \int_{R}^{3R} \frac{k\lambda\, dx}{x}$$

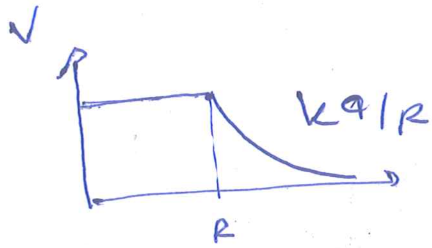
$$= 2\int_{R}^{3R} \frac{k\lambda\, dx}{x} + k\lambda\pi = 2k\lambda\big[\underbrace{\ln 3R - \ln R}_{\ln(3R/R)}\big] + k$$

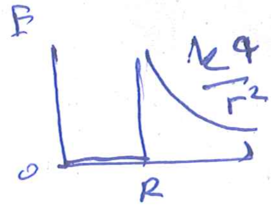
$$= k\lambda\left[2\ln 3 + \pi\right]$$

50. **M** A spherical conductor has a radius of 14.0 cm and a charge of 26.0 μC. Calculate the electric field and the electric potential at (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

(a)







(a) $r < R$, $\quad V = V_R = \frac{kq}{R} = \frac{9\times10^9 \cdot 26\times10^{-6}}{(14\times10^{-2})}$

$V = 1{,}67\times10^6$ volt

$E = 0$

(b) $r = 20$ cm $= 0{,}2$ m

$r > R$

$V = \frac{kq}{r} = \frac{9\times10^9 \cdot 26\times10^{-6}}{0{,}2\,m} = 1{,}17\times10^6$ volt

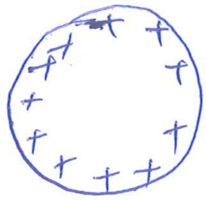
$E = \frac{kq}{r^2} = \frac{9\times10^9 \cdot 26\times10^{-6}}{(0{,}2\,m)^2} = 5{,}84\times10^6\ \frac{N}{C}$

(c) $r = 14$ cm $= 0{,}14$ m $= R$

$V = V_R = \frac{kq}{R} = 1{,}67\times10^6$ volt

$E = \frac{kq}{R^2} = \frac{9\times10^9 \cdot 26 \cdot 10^{-6}}{(0{,}140\,m)^2} = 11{,}9\times10^6\ \frac{N}{C}$

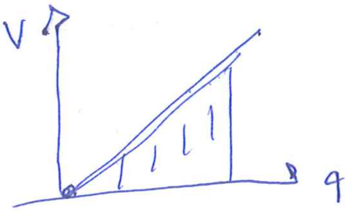
61. Calculate the work that must be done on charges brought from infinity to charge a spherical shell of radius $R = 0.100$ m to a total charge $Q = 125\ \mu C$.



$$dW = V\,dq$$

$$dW = \frac{kq}{R}\,dq$$

$$W = \frac{k}{R}\int_0^Q q\,dq$$



$$W = \frac{k}{r}\left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{kQ^2}{2R} = \frac{9.10^9\,(125.10^{-6})^2}{2\cdot 0.1}$$

$$= 702\ \mathrm{J}$$

65. From Gauss's law, the electric field set up by a uniform line of charge is

$$\vec{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{r}$$

where $\hat{r}$ is a unit vector pointing radially away from the line and λ is the linear charge density along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

$$V_2 - V_1 = \int_2^1 \vec{E} \cdot d\vec{\ell}$$

$$V_2 - V_1 = \int_2^1 \frac{\lambda}{2\pi\epsilon_0 r} \underbrace{\hat{r} \cdot d\hat{\ell}}_{dr}$$

$$V_2 - V_1 = \int_2^1 \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0} \left[\ln r\right]_{r_2}^{r_1}$$

$$V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \left[\ln r_1 - \ln r_2\right]$$

$$V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

65. From Gauss's law, the electric field set up by a uniform line of charge is

$$\vec{\mathbf{E}} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the line and λ is the linear charge density along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

$$V_2 - V_1 = \int_2^1 \vec{E} \cdot d\vec{l}$$

$$V_2 - V_1 = \int_2^1 \frac{\lambda}{2\pi\epsilon_0 r} \underbrace{\hat{r} \cdot d\hat{l}}_{dr}$$

$$V_2 - V_1 = \int_2^1 \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0} \left[\ln r\right]_{r_2}^{r_1}$$

$$V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \left[\ln r_1 - \ln r_2\right]$$

$$V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$