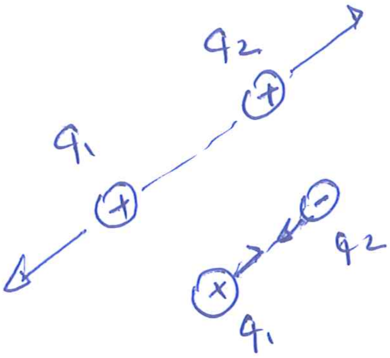
# Electric Charge and Electric field

## Coulomb's law

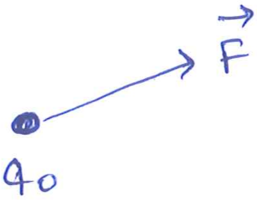



$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = k \frac{|q_1 q_2|}{r^2}$$

$$k \simeq 9 \times 10^9 \frac{Nm^2}{C^2}$$

## Electric field

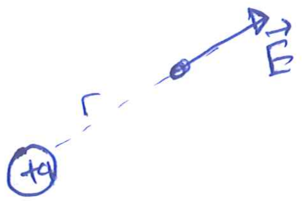
$$\vec{E} = \frac{\vec{F}}{q_0}$$ force per unit charge




$$\vec{F} = q_0 \vec{E}$$

## Electric field by a point charge:

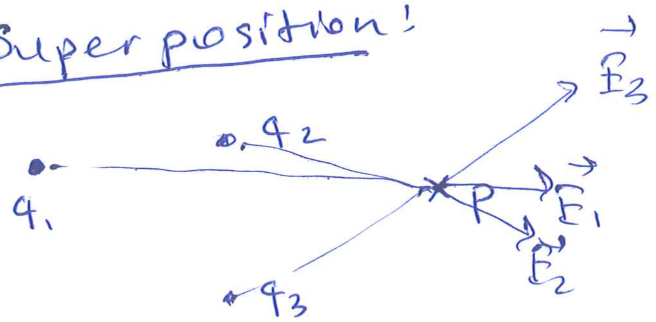
$\vec{E}$ in radial direction




$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$ radial unit vector

## Superposition:

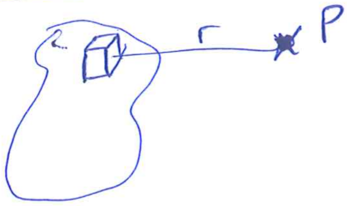



$$\vec{E} = \sum_i \vec{E}_i$$

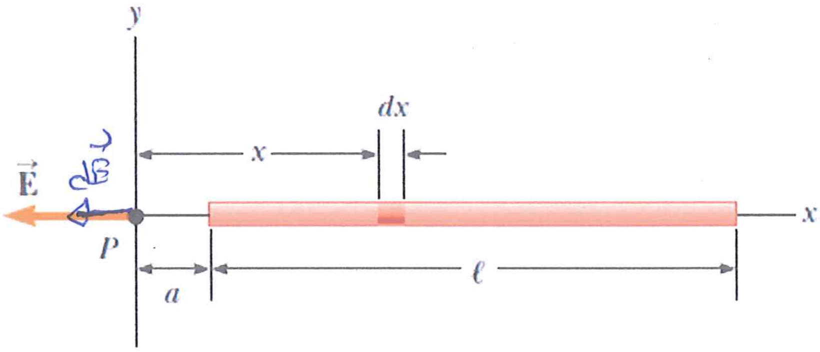
$$\vec{E} = \sum_i \frac{kq_i}{r_i^2} \hat{r}_i$$

## Continuous Charge Distribution:

$$\vec{E} = \int \frac{kdq}{r^2} \hat{r}$$



A rod of length $\ell$ has a uniform positive charge per unit length $\lambda$ and a total charge $Q$. Calculate the electric field at a point $P$ that is located along the long axis of the rod and a distance $a$ from one end (Fig. 23.15).




electric field by length dx

$$dE = \frac{k\,dq}{x^2} \quad , \quad dq = \lambda\, dx \quad , \quad \lambda = \frac{Q}{\ell}$$

in $-x$-direction

$$dE = \frac{k \lambda\, dx}{x^2} = \frac{kQ}{\ell x^2}\, dx$$

$$E = \int_{x=a}^{x=\ell+a} \frac{kQ}{\ell x^2}\, dx = \frac{kQ}{\ell}\left[-\frac{1}{x}\right]_{x=a}^{x=\ell+a}$$

$$E = \frac{kQ}{\ell}\left[-\frac{1}{\ell+a} + \frac{1}{a}\right] = \frac{kQ}{\ell}\left[\frac{-\cancel{a}+\ell+\cancel{a}}{a(\ell+a)}\right]$$

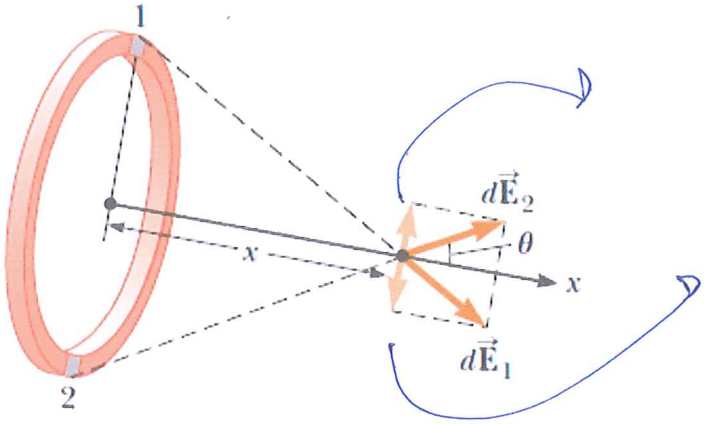
$(a)$ $(\ell+a)$

$$E = \frac{kQ}{\ell}\cdot\frac{\ell}{a(\ell+a)} = \frac{kQ}{a(\ell+a)}$$

in $-x$ direction

A ring of radius $a$ carries a uniformly distributed positive total charge $Q$. Calculate the electric field due to the ring at a point $P$ lying a distance $x$ from its center along the central axis perpendicular to the plane of the ring (Fig. 23.16a).

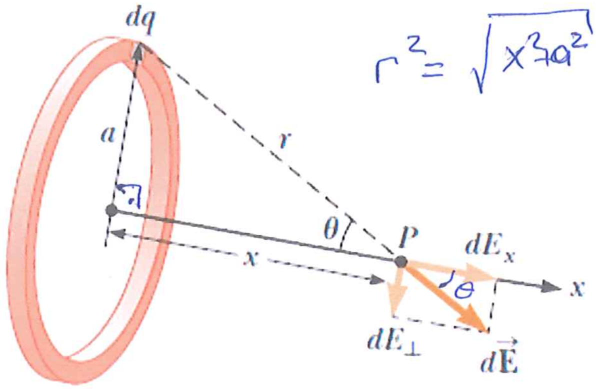
Electric field is the vector sum of charge elements $\int \frac{k\,dq}{r^2}\hat{r}$




y-components cancel out.

Only x-component is enough

$r^2 = \sqrt{x^2+a^2}$




$$dE = \frac{k\,dq}{r^2} = \frac{k\,dq}{(x^2+a^2)}$$

$$dE_x = dE \cdot \cos\theta$$

$$dE_x = \frac{k\,dq}{(x^2+a^2)} \cdot \frac{x}{r}$$

$$dE_x = \frac{k\,dq}{(x^2+a^2)} \cdot \frac{x}{\sqrt{x^2+a^2}}$$

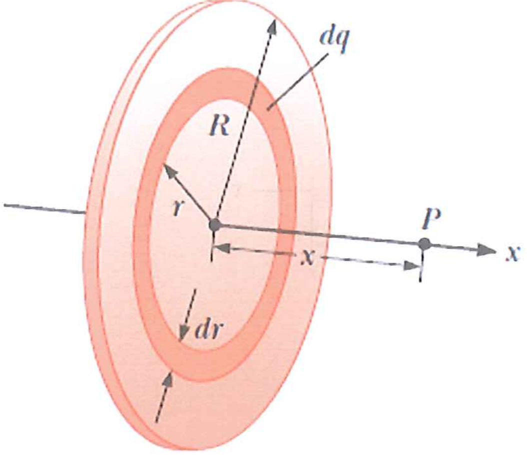
$$dE_x = \frac{k\,x}{(x^2+a^2)^{3/2}}\,dq$$

integrate over the whole ring

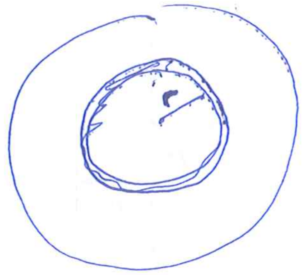
$$\int dE_x = \int \frac{kx}{(x^2+a^2)^{3/2}}\,dq = \frac{kx}{(x^2+a^2)^{3/2}} \underbrace{\int dq}_{\text{whole charge on the ring}}$$

$$E_x = \frac{kxQ}{(x^2+a^2)^{3/2}}, \quad E_y = 0 \quad \Rightarrow \quad \vec{E} = \frac{kxQ}{(x^2+a^2)^{3/2}}\,\hat{\imath}$$

A disk of radius $R$ has a uniform surface charge density $\sigma$. Calculate the electric field at a point $P$ that lies along the central perpendicular axis of the disk and a distance $x$ from the center of the disk (Fig. 23.17).




Disk consists of rings




$dq = \sigma \cdot \text{area}$

$\text{area} = d(\pi r^2)$

$= 2\pi r\,dr$

$dq = \sigma \cdot 2\pi r\,dr \quad , \quad \sigma = \dfrac{Q}{\pi R^2}$

$dq = \dfrac{Q}{\cancel{\pi} R^2}\, 2\cancel{\pi} r\,dr = \dfrac{2Q r\,dr}{R^2}$

electric field by a ring of radius r

$dE_x = \dfrac{k x}{(r^2+x^2)^{3/2}} \cdot dq = \dfrac{k x}{(r^2+x^2)^{3/2}} \dfrac{2Q r\,dr}{R^2}$

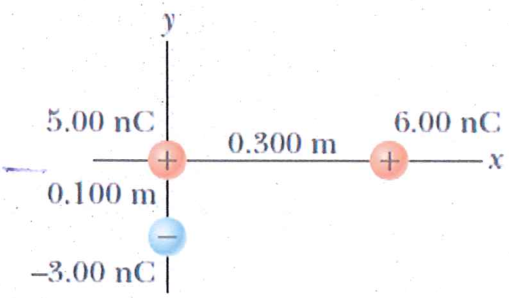
$\int dE_x = \dfrac{2kQx}{R^2} \int_0^R \dfrac{r\,dr}{(r^2+x^2)^{3/2}}$

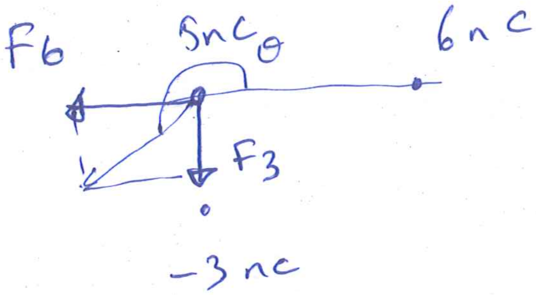
$E_x = \dfrac{2kQx}{R^2}\left[-(r^2+x^2)^{-1/2}\right]_0^R$

$= \dfrac{2kQx}{R^2}\left[-\dfrac{1}{\sqrt{R^2+x^2}} + \dfrac{1}{x}\right]$

$E_x = \dfrac{2kQ}{R^2}\left[1 - \dfrac{x}{\sqrt{R^2+x^2}}\right]$

**11.** Three point charges are arranged as shown in Figure **M** P23.11. Find (a) the magnitude and (b) the direction of the electric force on the particle at the origin.




$$\vec{F} = \vec{F}_6 + \vec{F}_3$$

$$F_x = -F_6 = -8{,}99 \times 10^9 \frac{Nm^2}{C^2} \; \frac{6\times10^{-9} \cdot 5\times10^{-9} \, C^2}{(0{,}3m)^2}$$

$$F_x = -3{,}00 \times 10^{-6} N$$

$$F_y = -F_3 = -8{,}99\times10^9 \frac{Nm^2}{C^2} \cdot \frac{5\times10^{-9} \cdot 3\cdot10^{-9} C^2}{(0{,}1m)^2}$$

$$= -1{,}35\times10^{-5} N$$

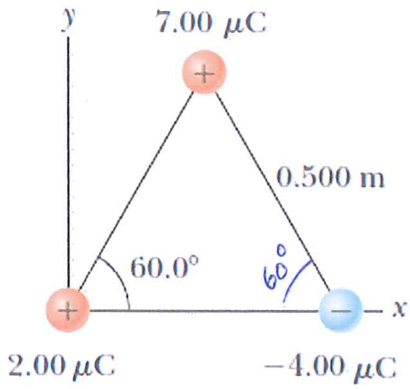
a) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1{,}35)^2 + 3^2} \; 10^{-5} N$

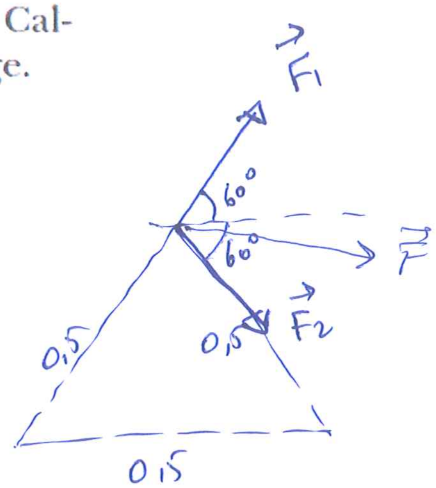
$= 1{,}38 \times 10^{-5} N$

b) $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1{,}35}{3}\right) = 77{,}5°$, or $77{,}5° + 180$

$= 257{,}5°$

**15.** **M** Three charged particles are located at the corners of an equilateral triangle as shown in Figure P23.15. Calculate the total electric force on the $7.00\text{-}\mu C$ charge.




- $\vec{F} = \vec{F}_1 + \vec{F}_2$
- $F_1 = k \dfrac{(2\times10^{-6}c)(7\times10^{-6}c)}{(0{,}5)^2} = \dfrac{(8{,}99\times10^9)\cdot(2\times10^{-6})(7\times10^{-6})}{(0{,}5)^2}$

$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j} = F_1 \cos 60^\circ \hat{i} + F_1 \sin 60 \hat{j}$

$\vec{F}_1 = (0{,}252\hat{i} + 0{,}436\hat{j})\ N$

- $F_2 = 8{,}99\times10^9 \dfrac{Nm^2}{c^2} \dfrac{(7\times10^{-6}c)\cdot|-4\times10^{-6}c|}{(0{,}5)^2}$

$\vec{F}_2 = F_{2x}\hat{i} + F_{2y}\hat{j} = F_2 \cos 60^\circ \hat{i} - F_2 \sin 60^\circ \hat{j}$

$\vec{F}_2 = (0{,}503\hat{i} - 0{,}872\hat{j})\ N$

$\vec{F} = 0{,}252\hat{i} + 0{,}436\hat{j} + 0{,}503\hat{i} - 0{,}872\hat{j}$

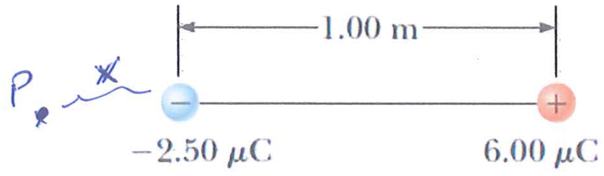
$\vec{F} = (0{,}755\hat{i} - 0{,}436\hat{j})\ N$

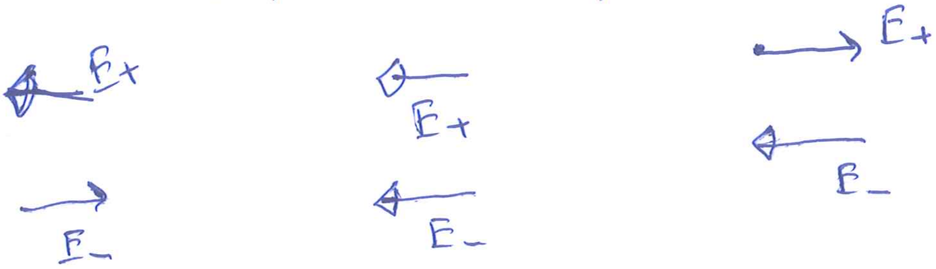
magnitude $F = \sqrt{(0{,}755)^2 + (0{,}436)^2} = 0{,}872\ N$

angle $\theta = \tan^{-1}\left(\dfrac{-0{,}436}{0{,}755}\right) = 330^\circ\ (-30^\circ)$

29. In Figure P23.29, determine the point (other than infinity) at which the electric field is zero.

M




impossible to have zero field

$E_+ = E_-$ for zero field. The point should be closer to the smaller charge.

$$\frac{\cancel{k}\,|2{,}50\,\cancel{\mu C}|}{x^2} = \frac{\cancel{k}\,|6{,}00\,\cancel{\mu C}|}{(1+x)^2}$$

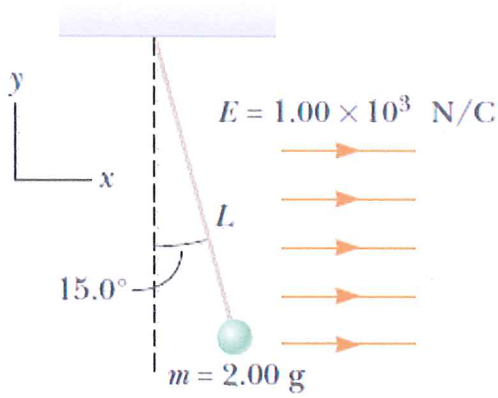
$$6x^2 = 2{,}50\,(1+x)^2$$

$$\sqrt{6}\,x = \sqrt{2{,}50}\,(1+x)$$

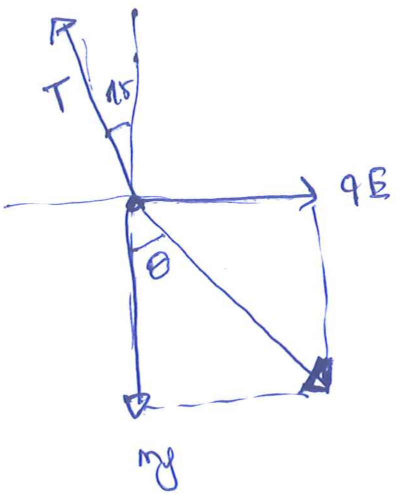
$$x = \frac{\sqrt{2{,}50}}{\sqrt{6} - \sqrt{2{,}5}} = 1.82\ \text{m}$$

33. A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in Figure P23.33. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

AMT




$F_E = qE$




$T \sin 15^\circ = qE$

$T \cos 15 = mg$

$\tan 15^\circ = \frac{qE}{mg}$

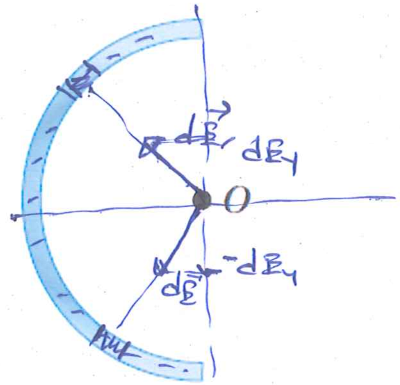
$q = \frac{mg \tan 15^\circ}{E}$

$q = \frac{2 \times 10^{-3} \cdot 9{,}80 \tan(15^\circ)}{1 \times 10^{3}}$

$q = 5{,}25 \times 10^{-6}\ C$

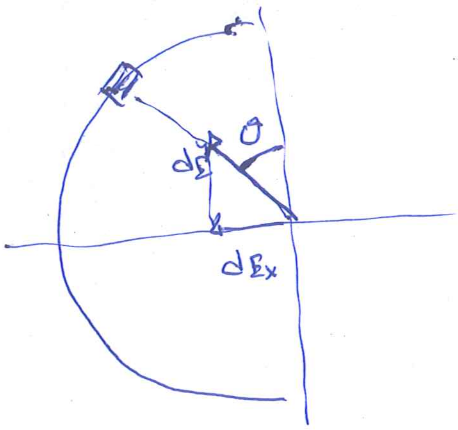
$q = 5{,}25\ \mu C$

**45.** **M** A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.45. The rod has a total charge of $-7.50\ \mu C$. Find (a) the magnitude and (b) the direction of the electric field at $O$, the center of the semicircle.




y-components cancel out

Due to symmetry, we need only the x-component




$$dq = \lambda\, ds = \lambda r\, d\theta$$

$$\lambda = \frac{-7{,}50\ \mu C}{14\times10^{-2}\,m}, \quad r = \frac{L}{\pi} = \frac{14\times10^{-2}}{\pi}\,m$$

$$dE_x = \frac{k\,dq}{r^2} = \frac{k\lambda r\,d\theta}{r^2}\sin\theta$$

$$E_x = \int_0^{\pi}\left(\frac{k\lambda}{r}\right)\sin\theta\,d\theta = \frac{k\lambda}{r}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{k\lambda}{r}\left[1+1\right] = \frac{2k\lambda}{r}$$

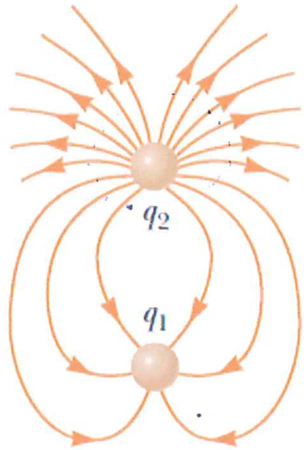
$$E_x = \frac{2\cdot 8{,}99\times10^{9}\cdot\dfrac{-7{,}50\times10^{-6}}{14\times10^{-2}}}{\dfrac{14\times10^{-2}}{\pi}} = -\frac{2\times8{,}99\times10^{9}\times7{,}50\times10^{-6}\,\pi}{(14\times10^{-2})^2}$$

$$E_x = -2{,}16\times10^{7}\ N/C$$

(a) $|\vec{E}| = |E_x| = 2{,}16\times10^{7}\ N/C$

(b) $\leftarrow$ to the left

**49.** Figure P23.49 shows the electric **W** field lines for two charged particles separated by a small distance. (a) Determine the ratio $q_1/q_2$. (b) What are the signs of $q_1$ and $q_2$?




Field lines emerge from a positive charge, but enter a negative charge.

$q_1$ is negative

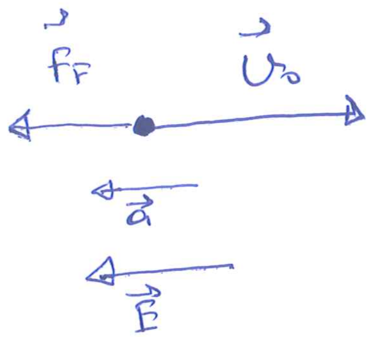
$q_2$ positive

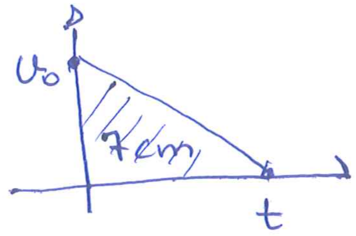
(a) $\frac{q_1}{q_2} = -\frac{\text{\# of lines}}{\text{\# of lines}}$

$= -\frac{6}{16} = -\frac{1}{3}$

(b) $q_1$ is negative, $q_2$ is positive

52. A proton is projected in the positive $x$ direction **W** into a region of a uniform electric field $\vec{\mathbf{E}} = (-6.00 \times 10^5)\,\hat{\mathbf{i}}$ N/C at $t = 0$. The proton travels 7.00 cm as it comes to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time interval over which the proton comes to rest.




(a) $|a| = \dfrac{qE}{m} = \dfrac{1{,}6\times10^{-19} \cdot 6\times10^{5}}{1{,}67\times10^{-27}\,kg} = 5{,}76\times10^{13}\ m/s^2$

$\vec{a} = -5{,}76\times10^{13}\,\hat{i}\ \left(\frac{m}{s^2}\right)$

(b) $v^2 = v_0^2 - 2|a|\Delta x$

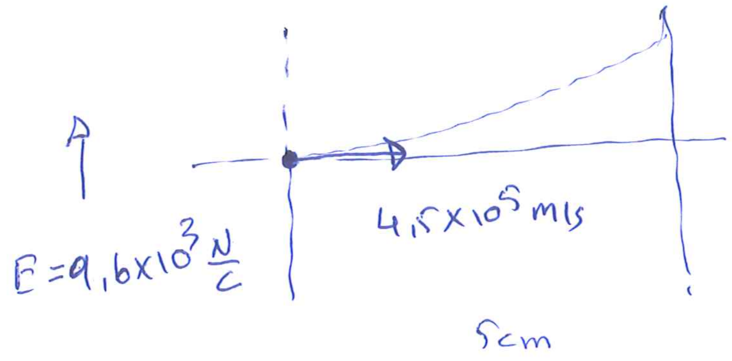
$v_0 = \sqrt{2|a|\Delta x} = \sqrt{2\times5{,}76\times10^{13} \cdot 7\times10^{-2}}$

$v_0 = 2{,}84\times10^{6}\ m/s$

$\vec{v}_0 = (+2{,}84\times10^{6}\,\hat{i})\ m/s$

(c) $t = \dfrac{|\Delta v|}{|a|} = \dfrac{2{,}84\times10^{6}\ m/s}{5{,}76\times10^{13}\ m/s^2} = 4{,}93\times10^{-8}\,s$

**57.** **M** A proton moves at $4.50 \times 10^5$ m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of $9.60 \times 10^3$ N/C. Ignoring any gravitational effects, find (a) the time interval required for the proton to travel 5.00 cm horizontally, (b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.




a) $\Delta x = v_x \cdot \Delta t \Rightarrow \Delta t = \dfrac{5 \times 10^{-2}\,m}{4{,}5 \times 10^5\,m/s} = 1{,}11 \times 10^{-7}\,s$

b) $\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{1}{2} \frac{qE}{m} (\Delta t)^2$

$\Delta y = \frac{1}{2} \dfrac{1.6 \times 10^{-19} \cdot 9.6 \times 10^3}{1.67 \times 10^{-27}\,kg} \cdot (1{,}11 \times 10^{-7})^2$

$\Delta y = 5.68 \times 10^{-3}\,m = 5{,}68\,mm$

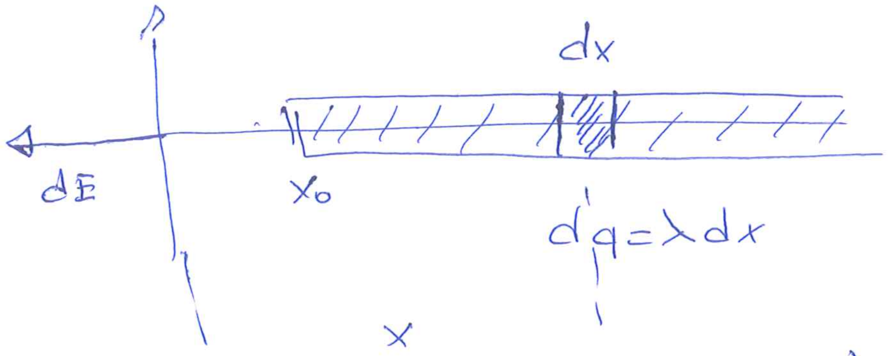
c) $v_x = v_0 = 4{,}5 \times 10^5\,m/s$

$v_y = a_y \cdot \Delta t = \frac{qE}{m} \cdot \Delta t = \dfrac{1.6 \times 10^{-19} \times 9{,}6 \times 10^3}{1.67 \times 10^{-27}} \cdot (1{,}11 \times 10^{-7})$

$v_y = 1{,}02 \times 10^5\,m/s$

$\vec{v} = (4{,}5 \times 10^5\,\hat{i} + 1.02 \times 10^5\,\hat{j})\,m/s$

63. A line of charge starts at $x = +x_0$ and extends to positive infinity. The linear charge density is $\lambda = \lambda_0 x_0/x$, where $\lambda_0$ is a constant. Determine the electric field at the origin.




$$dE = \frac{k\,dq}{x^2} = \frac{k\lambda\,dx}{x^2} = \frac{k \cdot \frac{\lambda_0 x_0}{x}\,dx}{x^2}$$

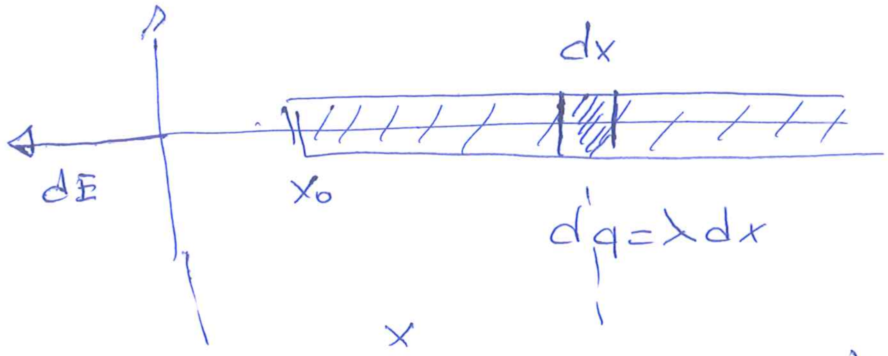
$$dE = \frac{k\lambda_0 x_0}{x^3}\,dx$$

$$\int dE = \int \frac{k\lambda_0 x_0}{x^3}\,dx = k\lambda_0 x_0 \left[-\frac{1}{2x^2}\right]_{x_0}^{\infty}$$

$$= k\lambda_0 x_0 \left[0 + \frac{1}{2x_0^2}\right] = \frac{k\lambda_0}{2x_0}$$

$$\vec{E} = -\frac{k\lambda_0}{2x_0}\,\hat{e}$$

63. A line of charge starts at $x = +x_0$ and extends to positive infinity. The linear charge density is $\lambda = \lambda_0 x_0/x$, where $\lambda_0$ is a constant. Determine the electric field at the origin.




$$dE = \frac{k\,dq}{x^2} = \frac{k\lambda\,dx}{x^2} = \frac{k \cdot \frac{\lambda_0 x_0}{x}\,dx}{x^2}$$

$$dE = \frac{k\lambda_0 x_0}{x^3}\,dx$$

$$\int dE = \int \frac{k\lambda_0 x_0}{x^3}\,dx = k\lambda_0 x_0 \left[-\frac{1}{2x^2}\right]_{x_0}^{\infty}$$

$$= k\lambda_0 x_0 \left[0 + \frac{1}{2x_0^2}\right] = \frac{k\lambda_0}{2x_0}$$

$$\vec{E} = -\frac{k\lambda_0}{2x_0}\,\hat{e}$$