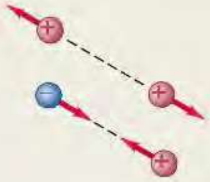
SOLUTIONS TO ALL EXAMPLES

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

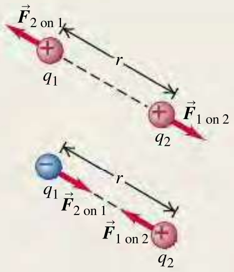


Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product q_1q_2 and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \tag{21.2}$$

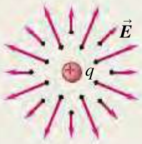
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$



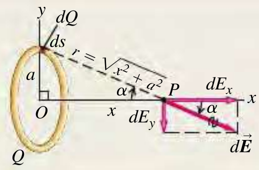
Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \tag{21.3}$$

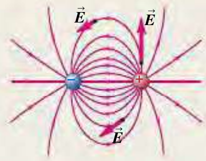
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{21.7}$$



Superposition of electric fields: The electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ, surface charge density σ, and volume charge density ρ. (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.

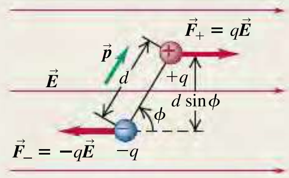


Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ has magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.13 and 21.14.)

$$\tau = pE\sin\phi \tag{21.15}$$

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{21.16}$$

$$U = -\vec{p}\cdot\vec{E} \tag{21.18}$$

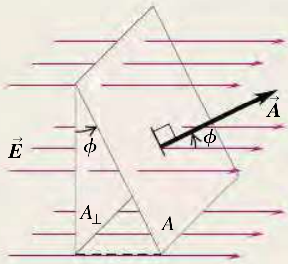


CHAPTER 22 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Electric flux: Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of an area element and the perpendicular component of $\vec{E}$, integrated over a surface. (See Examples 22.1–22.3.)

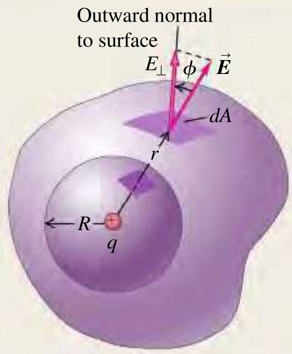
$$\Phi_E = \int E \cos\phi \, dA$$
$$= \int E_\perp \, dA = \int \vec{E} \cdot d\vec{A} \qquad (22.5)$$



Gauss’s law: Gauss’s law states that the total electric flux through a closed surface, which can be written as the surface integral of the component of $\vec{E}$ normal to the surface, equals a constant times the total charge Q_{encl} enclosed by the surface. Gauss’s law is logically equivalent to Coulomb’s law, but its use greatly simplifies problems with a high degree of symmetry. (See Examples 22.4–22.10.)

When excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, and $\vec{E} = \mathbf{0}$ everywhere in the material of the conductor. (See Examples 22.11–22.13.)

$$\Phi_E = \oint E \cos\phi \, dA$$
$$= \oint E_\perp \, dA = \oint \vec{E} \cdot d\vec{A}$$
$$= \frac{Q_{encl}}{\epsilon_0} \qquad (22.8), (22.9)$$



Electric field of various symmetric charge distributions: The following table lists electric fields caused by several symmetric charge distributions. In the table, q, Q, λ, and σ refer to the *magnitudes* of the quantities.

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Single point charge q	Distance r from q	$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$
Charge q on surface of conducting sphere with radius R	Outside sphere, $r > R$	$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$
	Inside sphere, $r < R$	$E = 0$
Infinite wire, charge per unit length λ	Distance r from wire	$E = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda}{r}$
Infinite conducting cylinder with radius R, charge per unit length λ	Outside cylinder, $r > R$	$E = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda}{r}$
	Inside cylinder, $r < R$	$E = 0$
Solid insulating sphere with radius R, charge Q distributed uniformly throughout volume	Outside sphere, $r > R$	$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}$
	Inside sphere, $r < R$	$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3}$
Infinite sheet of charge with uniform charge per unit area σ	Any point	$E = \dfrac{\sigma}{2\epsilon_0}$
Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$	Any point between plates	$E = \dfrac{\sigma}{\epsilon_0}$
Charged conductor	Just outside the conductor	$E = \dfrac{\sigma}{\epsilon_0}$

CHAPTER 23 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Electric potential energy: The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by the change in a potential-energy function U.

The electric potential energy for two point charges q and q_0 depends on their separation r. The electric potential energy for a charge q_0 in the presence of a collection of charges q_1, q_2, q_3 depends on the distance from q_0 to each of these other charges. (See Examples 23.1 and 23.2.)

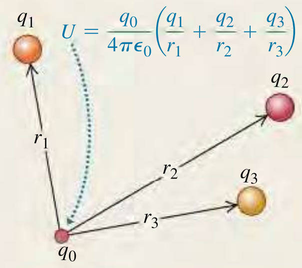
$$W_{a\to b} = U_a - U_b \qquad (23.2)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \qquad (23.9)$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.10)$$

(q_0 in presence of other point charges)



Electric potential: Potential, denoted by V, is potential energy per unit charge. The potential difference between two points equals the amount of work per charge that would be required to move a positive test charge between those points. The potential V due to a quantity of charge can be calculated by summing (if the charge is a collection of point charges) or by integrating (if the charge is a distribution). (See Examples 23.3, 23.4, 23.5, 23.7, 23.11, and 23.12.)

The potential difference between two points a and b, also called the potential of a with respect to b, is given by the line integral of $\vec{E}$. The potential at a given point can be found by first finding $\vec{E}$ and then carrying out this integral. (See Examples 23.6, 23.8, 23.9, and 23.10.)

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad (23.14)$$

(due to a point charge)

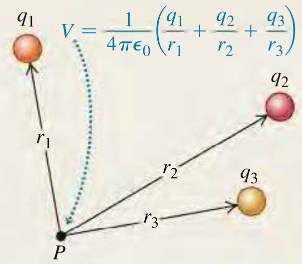
$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.15)$$

(due to a collection of point charges)

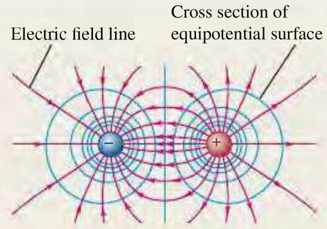
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad (23.16)$$

(due to a charge distribution)

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl \qquad (23.17)$$



Equipotential surfaces: An equipotential surface is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface and all points in the interior of a conductor are at the same potential. When a cavity within a conductor contains no charge, the entire cavity is an equipotential region and there is no surface charge anywhere on the surface of the cavity.



Finding electric field from electric potential: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field $\vec{E}$ at any point are given by partial derivatives of V. (See Examples 23.13 and 23.14.)

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \qquad (23.19)$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \qquad (23.20)$$

(vector form)

EXAMPLE 24.12 A SPHERICAL CAPACITOR WITH DIELECTRIC

SOLUTION

Use Gauss's law to find the capacitance of the spherical capacitor of Example 24.3 (Section 24.1) if the volume between the shells is filled with an insulating oil with dielectric constant K.

SOLUTION

IDENTIFY and SET UP: The spherical symmetry of the problem is not changed by the presence of the dielectric, so as in Example 24.3, we use a concentric spherical Gaussian surface of radius r between the shells. Since a dielectric is present, we use Gauss's law in the form of Eq. (24.23).

EXECUTE: From Eq. (24.23),

$$\oint K\vec{E} \cdot d\vec{A} = \oint KE\,dA = KE \oint dA = (KE)(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi K\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon r^2}$$

where $\epsilon = K\epsilon_0$. Compared to the case in which there is vacuum between the shells, the electric field is reduced by a factor of $1/K$. The potential difference V_{ab} between the shells is reduced by the same factor, and so the capacitance $C = Q/V_{ab}$ is *increased* by a factor of K, just as for a parallel-plate capacitor when a dielectric is inserted. Using the result of Example 24.3, we find that the capacitance with the dielectric is

$$C = \frac{4\pi K\epsilon_0 r_a r_b}{r_b - r_a} = \frac{4\pi\epsilon r_a r_b}{r_b - r_a}$$

EVALUATE: If the dielectric fills the volume between the two conductors, the capacitance is just K times the value with no dielectric. The result is more complicated if the dielectric only partially fills this volume.

TEST YOUR UNDERSTANDING OF SECTION 24.6 A single point charge q is embedded in a very large block of dielectric of dielectric constant K. At a point inside the dielectric a distance r from the point charge, what is the magnitude of the electric field? (i) $q/4\pi\epsilon_0 r^2$; (ii) $Kq/4\pi\epsilon_0 r^2$; (iii) $q/4\pi K\epsilon_0 r^2$; (iv) none of these. ▌

CHAPTER 24 SUMMARY

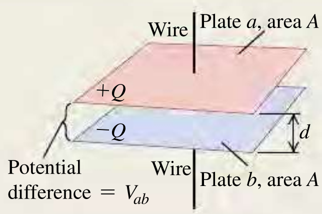
SOLUTIONS TO ALL EXAMPLES

Capacitors and capacitance: A capacitor is any pair of conductors separated by an insulating material. When the capacitor is charged, there are charges of equal magnitude Q and opposite sign on the two conductors, and the potential V_{ab} of the positively charged conductor with respect to the negatively charged conductor is proportional to Q. The capacitance C is defined as the ratio of Q to V_{ab}. The SI unit of capacitance is the farad (F): 1 F = 1 C/V.

A parallel-plate capacitor consists of two parallel conducting plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance depends on only A and d. For other geometries, the capacitance can be found by using the definition $C = Q/V_{ab}$. (See Examples 24.1–24.4.)

$$C = \frac{Q}{V_{ab}} \tag{24.1}$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \tag{24.2}$$



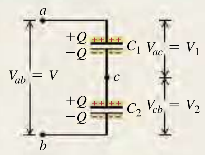
Capacitors in series and parallel: When capacitors with capacitances $C_1, C_2, C_3, \ldots$ are connected in series, the reciprocal of the equivalent capacitance C_{eq} equals the sum of the reciprocals of the individual capacitances. When capacitors are connected in parallel, the equivalent capacitance C_{eq} equals the sum of the individual capacitances. (See Examples 24.5 and 24.6.)

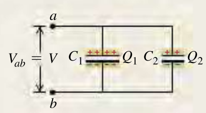
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{24.5}$$

(capacitors in series)

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{24.7}$$

(capacitors in parallel)

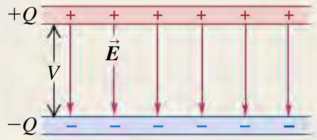




Energy in a capacitor: The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor. This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is proportional to the square of the electric-field magnitude. (See Examples 24.7–24.9.)

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \quad (24.9)$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 \quad (24.11)$$



Dielectrics: When the space between the conductors is filled with a dielectric material, the capacitance increases by a factor K, the dielectric constant of the material. The quantity $\epsilon = K\epsilon_0$ is the permittivity of the dielectric. For a fixed amount of charge on the capacitor plates, induced charges on the surface of the dielectric decrease the electric field and potential difference between the plates by the same factor K. The surface charge results from polarization, a microscopic rearrangement of charge in the dielectric. (See Example 24.10.)

Under sufficiently strong electric fields, dielectrics become conductors, a situation called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.

In a dielectric, the expression for the energy density is the same as in vacuum but with ϵ_0 replaced by $\epsilon = K\epsilon_0$. (See Example 24.11.)

Gauss's law in a dielectric has almost the same form as in vacuum, with two key differences: $\vec{E}$ is replaced by $K\vec{E}$ and Q_{encl} is replaced by $Q_{\text{encl-free}}$, which includes only the free charge (not bound charge) enclosed by the Gaussian surface. (See Example 24.12.)

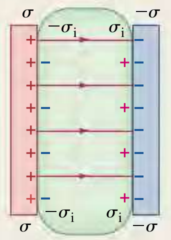
$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \quad (24.19)$$

(parallel-plate capacitor filled with dielectric)

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \quad (24.20)$$

$$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \quad (24.23)$$

Dielectric between plates



BRIDGING PROBLEM ELECTRIC-FIELD ENERGY AND CAPACITANCE OF A CONDUCTING SPHERE

SOLUTION

A solid conducting sphere of radius R carries a charge Q. Calculate the electric-field energy density at a point a distance r from the center of the sphere for (a) $r < R$ and (b) $r > R$. (c) Calculate the total electric-field energy associated with the charged sphere. (d) How much work is required to assemble the charge Q on the sphere? (e) Use the result of part (c) to find the capacitance of the sphere. (You can think of the second conductor as a hollow conducting shell of infinite radius.)

SOLUTION GUIDE

IDENTIFY and SET UP

1. You know the electric field for this situation at all values of r from Example 22.5 (Section 22.4). You'll use this to find the electric-field energy density u and *total* electric-field energy U. You can then find the capacitance from the relationship $U = Q^2/2C$.
2. To find U, consider a spherical shell of radius r and thickness dr that has volume $dV = 4\pi r^2 dr$. (It will help to make a drawing of such a shell concentric with the conducting sphere.) The energy stored in this volume is $u\,dV$, and the total energy is the integral of $u\,dV$ from $r = 0$ to $r \rightarrow \infty$. Set up this integral.

EXECUTE

3. Find u for $r < R$ and for $r > R$. (*Hint:* What is the field inside a solid conductor?)
4. Substitute your results from step 3 into the expression from step 2. Then calculate the integral to find the total electric-field energy U.
5. Use your understanding of the energy stored in a charge distribution to find the work required to assemble the charge Q.
6. Find the capacitance of the sphere.

EVALUATE

7. Where is the electric-field energy density greatest? Where is it least?
8. How would the results be affected if the solid sphere were replaced by a hollow conducting sphere of the same radius R?
9. You can find the potential difference between the sphere and infinity from $C = Q/V$. Does this agree with the result of Example 23.8 (Section 23.3)?

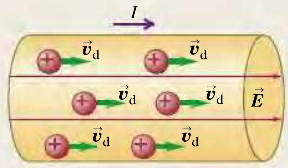
CHAPTER 25 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Current and current density: Current is the amount of charge flowing through a specified area, per unit time. The SI unit of current is the ampere $(1\text{ A} = 1\text{ C/s})$. The current I through an area A depends on the concentration n and charge q of the charge carriers, as well as on the magnitude of their drift velocity $\vec{v}_d$. The current density is current per unit cross-sectional area. Current is usually described in terms of a flow of positive charge, even when the charges are actually negative or of both signs. (See Example 25.1.)

$$I = \frac{dQ}{dt} = n|q|v_d A \tag{25.2}$$

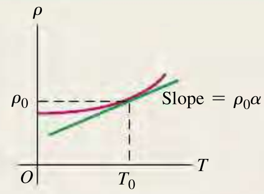
$$\vec{J} = nq\vec{v}_d \tag{25.4}$$



Resistivity: The resistivity ρ of a material is the ratio of the magnitudes of electric field and current density. Good conductors have small resistivity; good insulators have large resistivity. Ohm's law, obeyed approximately by many materials, states that ρ is a constant independent of the value of E. Resistivity usually increases with temperature; for small temperature changes this variation is represented approximately by Eq. (25.6), where α is the temperature coefficient of resistivity.

$$\rho = \frac{E}{J} \tag{25.5}$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \tag{25.6}$$

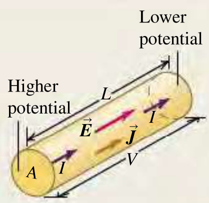


Metal: ρ increases with increasing T.

Resistors: The potential difference V across a sample of material that obeys Ohm's law is proportional to the current I through the sample. The ratio $V/I = R$ is the resistance of the sample. The SI unit of resistance is the ohm $(1\ \Omega = 1\text{ V/A})$. The resistance of a cylindrical conductor is related to its resistivity ρ, length L, and cross-sectional area A. (See Examples 25.2 and 25.3.)

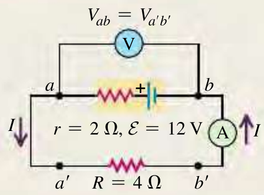
$$V = IR \tag{25.11}$$

$$R = \frac{\rho L}{A} \tag{25.10}$$



Circuits and emf: A complete circuit has a continuous current-carrying path. A complete circuit carrying a steady current must contain a source of electromotive force (emf) $\mathcal{E}$. The SI unit of electromotive force is the volt (V). Every real source of emf has some internal resistance r, so its terminal potential difference V_{ab} depends on current. (See Examples 25.4–25.7.)

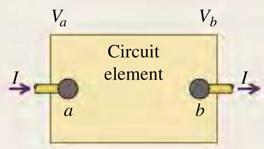
$$V_{ab} = \mathcal{E} - Ir \tag{25.15}$$
(source with internal resistance)



Energy and power in circuits: A circuit element puts energy into a circuit if the current direction is from lower to higher potential in the device, and it takes energy out of the circuit if the current is opposite. The power P equals the product of the potential difference $V_a - V_b = V_{ab}$ and the current I. A resistor always takes electrical energy out of a circuit. (See Examples 25.8–25.10.)

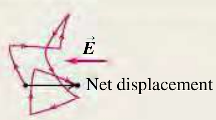
$$P = V_{ab}I \tag{25.17}$$
(general circuit element)

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \tag{25.18}$$
(power delivered to a resistor)



Conduction in metals: In a metal, current is due to the motion of electrons. They move freely through the metallic crystal but collide with positive ions. In a crude classical model of this motion, the resistivity of the material can be related to the electron mass, charge, speed of random motion, density, and mean free time between collisions. (See Example 25.11.)

$$\rho = \frac{m}{ne^2\tau} \tag{25.24}$$

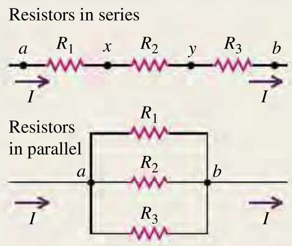


SOLUTIONS TO ALL EXAMPLES

Resistors in series and parallel: When several resistors $R_1, R_2, R_3, \ldots$ are connected in series, the equivalent resistance R_{eq} is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of equivalent resistance R_{eq} is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2.)

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad (26.1)$$
(resistors in series)

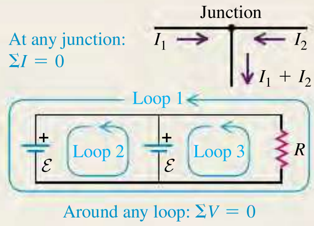
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad (26.2)$$
(resistors in parallel)



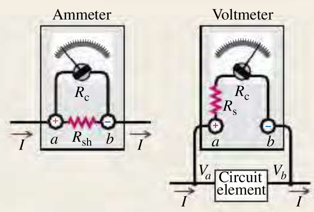
Kirchhoff's rules: Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules. (See Examples 26.3–26.7.)

$$\sum I = 0 \quad \text{(junction rule)} \quad (26.5)$$

$$\sum V = 0 \quad \text{(loop rule)} \quad (26.6)$$



Electrical measuring instruments: In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil. Such an instrument is called an ammeter. If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage. The instrument is then called a voltmeter. A good ammeter has very low resistance; a good voltmeter has very high resistance. (See Examples 26.8–26.11.)



R-C circuits: When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13.)

Capacitor charging:

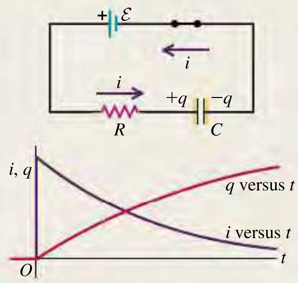
$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \quad (26.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.13)$$

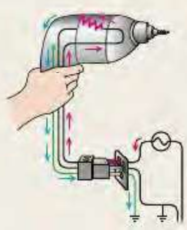
Capacitor discharging:

$$q = Q_0 e^{-t/RC} \quad (26.16)$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.17)$$



Household wiring: In household wiring systems, the various electrical devices are connected in parallel across the power line, which consists of a pair of conductors, one "hot" and the other "neutral." An additional "ground" wire is included for safety. The maximum permissible current in a circuit is determined by the size of the wires and the maximum temperature they can tolerate. Protection against excessive current and the resulting fire hazard is provided by fuses or circuit breakers. (See Example 26.14.)



EXAMPLE 27.12 A HALL-EFFECT MEASUREMENT

SOLUTION

You place a strip of copper, 2.0 mm thick and 1.50 cm wide, in a uniform 0.40-T magnetic field as shown in Fig. 27.41a. When you run a 75-A current in the $+x$-direction, you find that the potential at the bottom of the slab is 0.81 μV higher than at the top. From this measurement, determine the concentration of mobile electrons in copper.

SOLUTION

IDENTIFY and SET UP: This problem describes a Hall-effect experiment. We use Eq. (27.30) to determine the mobile electron concentration n.

EXECUTE: First we find the current density J_x and the electric field E_z:

$$J_x = \frac{I}{A} = \frac{75\text{ A}}{(2.0 \times 10^{-3}\text{ m})(1.50 \times 10^{-2}\text{ m})} = 2.5 \times 10^6\text{ A/m}^2$$

$$E_z = \frac{V}{d} = \frac{0.81 \times 10^{-6}\text{ V}}{1.5 \times 10^{-2}\text{ m}} = 5.4 \times 10^{-5}\text{ V/m}$$

Then, from Eq. (27.30),

$$n = \frac{-J_x B_y}{qE_z} = \frac{-(2.5 \times 10^6\text{ A/m}^2)(0.40\text{ T})}{(-1.60 \times 10^{-19}\text{ C})(5.4 \times 10^{-5}\text{ V/m})}$$
$$= 11.6 \times 10^{28}\text{ m}^{-3}$$

EVALUATE: The actual value of n for copper is $8.5 \times 10^{28}\text{ m}^{-3}$. The difference shows that our simple model of the Hall effect, which ignores quantum effects and electron interactions with the ions, must be used with caution. This example also shows that with good conductors, the Hall emf is very small even with large current densities. In practice, Hall-effect devices for magnetic-field measurements use semiconductor materials, for which moderate current densities give much larger Hall emfs.

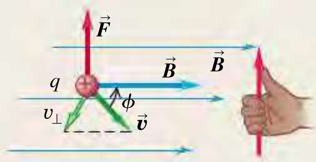
TEST YOUR UNDERSTANDING OF SECTION 27.9 A copper wire of square cross section is oriented vertically. The four sides of the wire face north, south, east, and west. There is a uniform magnetic field directed from east to west, and the wire carries current downward. Which side of the wire is at the highest electric potential? (i) North side; (ii) south side; (iii) east side; (iv) west side.

CHAPTER 27 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Magnetic forces: Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is the tesla ($1\text{ T} = 1\text{ N/A}\cdot\text{m}$). (See Example 27.1.)

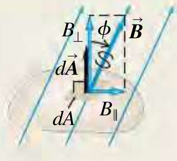
$$\vec{F} = q\vec{v} \times \vec{B} \qquad (27.2)$$



Magnetic field lines and flux: A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together, the field magnitude is large, and vice versa. Magnetic flux Φ_B through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is the weber ($1\text{ Wb} = 1\text{ T}\cdot\text{m}^2$). The net magnetic flux through any closed surface is zero (Gauss's law for magnetism). As a result, magnetic field lines always close on themselves. (See Example 27.2.)

$$\Phi_B = \int B\cos\phi\, dA = \int B_\perp\, dA = \int \vec{B}\cdot d\vec{A} \qquad (27.6)$$

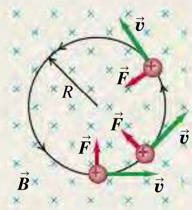
$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \text{(closed surface)} \qquad (27.8)$$



Motion in a magnetic field: The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B and the particle mass m, speed v, and charge q. (See Examples 27.3 and 27.4.)

Crossed electric and magnetic fields can be used as a velocity selector. The electric and magnetic forces exactly cancel when $v = E/B$. (See Examples 27.5 and 27.6.)

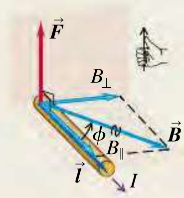
$$R = \frac{mv}{|q|B} \tag{27.11}$$



Magnetic force on a conductor: A straight segment of a conductor carrying current I in a uniform magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{B}$ and the vector $\vec{l}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current-carrying segment $d\vec{l}$. (See Examples 27.7 and 27.8.)

$$\vec{F} = I\vec{l} \times \vec{B} \tag{27.19}$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B} \tag{27.20}$$

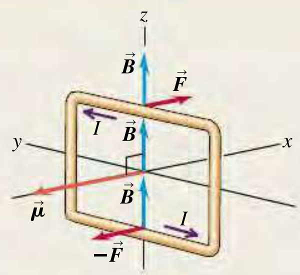


Magnetic torque: A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can the potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop. (See Examples 27.9 and 27.10.)

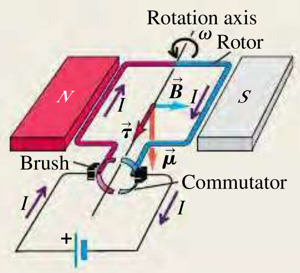
$$\tau = IBA\sin\phi \tag{27.23}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{27.26}$$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B\cos\phi \tag{27.27}$$

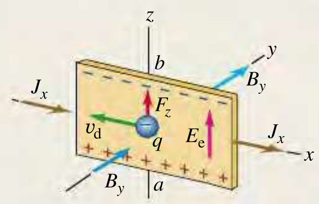


Electric motors: In a dc motor a magnetic field exerts a torque on a current in the rotor. Motion of the rotor through the magnetic field causes an induced emf called a back emf. For a series motor, in which the rotor coil is in series with coils that produce the magnetic field, the terminal voltage is the sum of the back emf and the drop Ir across the internal resistance. (See Example 27.11.)



The Hall effect: The Hall effect is a potential difference perpendicular to the direction of current in a conductor, when the conductor is placed in a magnetic field. The Hall potential is determined by the requirement that the associated electric field must just balance the magnetic force on a moving charge. Hall-effect measurements can be used to determine the sign of charge carriers and their concentration n. (See Example 27.12.)

$$nq = \frac{-J_x B_y}{E_z} \tag{27.30}$$

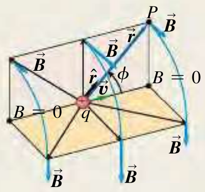


CHAPTER 28 SUMMARY

SOLUTIONS TO ALL EXAMPLES

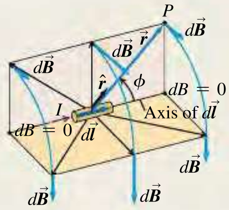
Magnetic field of a moving charge: The magnetic field $\vec{B}$ created by a charge q moving with velocity $\vec{v}$ depends on the distance r from the source point (the location of q) to the field point (where $\vec{B}$ is measured). The $\vec{B}$ field is perpendicular to $\vec{v}$ and to $\hat{r}$, the unit vector directed from the source point to the field point. The principle of superposition of magnetic fields states that the total $\vec{B}$ field produced by several moving charges is the vector sum of the fields produced by the individual charges. (See Example 28.1.)

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \tag{28.2}$$



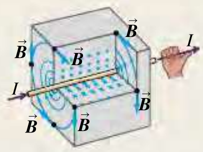
Magnetic field of a current-carrying conductor: The law of Biot and Savart gives the magnetic field $d\vec{B}$ created by an element $d\vec{l}$ of a conductor carrying current I. The field $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$, the unit vector from the element to the field point. The $\vec{B}$ field created by a finite current-carrying conductor is the integral of $d\vec{B}$ over the length of the conductor. (See Example 28.2.)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \tag{28.6}$$



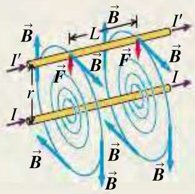
Magnetic field of a long, straight, current-carrying conductor: The magnetic field $\vec{B}$ at a distance r from a long, straight conductor carrying a current I has a magnitude that is inversely proportional to r. The magnetic field lines are circles coaxial with the wire, with directions given by the right-hand rule. (See Examples 28.3 and 28.4.)

$$B = \frac{\mu_0 I}{2\pi r} \tag{28.9}$$



Magnetic force between current-carrying conductors: Two long, parallel, current-carrying conductors attract if the currents are in the same direction and repel if the currents are in opposite directions. The magnetic force per unit length between the conductors depends on their currents I and I' and separation r. The definition of the ampere is based on this relationship. (See Example 28.5.)

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \tag{28.11}$$



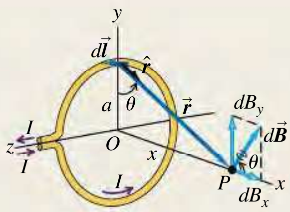
Magnetic field of a current loop: The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a circular conducting loop of radius a carrying current I. The field depends on the distance x along the axis from the center of the loop to the field point. If there are N loops, the field is multiplied by N. At the center of the loop, $x = 0$. (See Example 28.6.)

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \tag{28.15}$$

(circular loop)

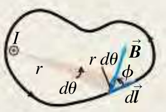
$$B_x = \frac{\mu_0 N I}{2a} \tag{28.17}$$

(center of N circular loops)



Ampere's law: Ampere's law states that the line integral of $\vec{B}$ around any closed path equals μ_0 times the net current through the area enclosed by the path. The positive sense of current is determined by a right-hand rule. (See Examples 28.7–28.10.)

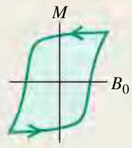
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \tag{28.20}$$



Magnetic fields due to current distributions: The table lists magnetic fields caused by several current distributions. In each case the conductor is carrying current I.

Current Distribution	Point in Magnetic Field	Magnetic-Field Magnitude
Long, straight conductor	Distance r from conductor	$B = \dfrac{\mu_0 I}{2\pi r}$
Circular loop of radius a	On axis of loop	$B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$
	At center of loop	$B = \dfrac{\mu_0 I}{2a}$ (for N loops, multiply these expressions by N)
Long cylindrical conductor of radius R	Inside conductor, $r < R$	$B = \dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2}$
	Outside conductor, $r > R$	$B = \dfrac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near center	$B = \mu_0 n I$
	Outside solenoid	$B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis	$B = \dfrac{\mu_0 N I}{2\pi r}$
	Outside the space enclosed by the windings	$B \approx 0$

Magnetic materials: When magnetic materials are present, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic-field expressions by $\mu = K_m \mu_0$, where μ is the permeability of the material and K_m is its relative permeability. The magnetic susceptibility χ_m is defined as $\chi_m = K_m - 1$. Magnetic susceptibilities for paramagnetic materials are small positive quantities; those for diamagnetic materials are small negative quantities. For ferromagnetic materials, K_m is much larger than unity and is not constant. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed. (See Examples 28.11 and 28.12.)



BRIDGING PROBLEM MAGNETIC FIELD OF A CHARGED, ROTATING DIELECTRIC DISK

SOLUTION

A thin dielectric disk with radius a has a total charge $+Q$ distributed uniformly over its surface (**Fig. 28.30**). It rotates n times per second about an axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

SOLUTION GUIDE

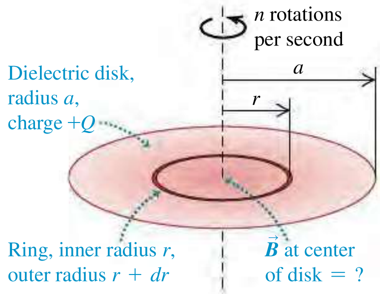
IDENTIFY and SET UP

1. Think of the rotating disk as a series of concentric rotating rings. Each ring acts as a circular current loop that produces a magnetic field at the center of the disk.
2. Use the results of Section 28.5 to find the magnetic field due to a single ring. Then integrate over all rings to find the total field.

EXECUTE

3. Find the charge on a ring with inner radius r and outer radius $r + dr$ (Fig. 28.30).
4. How long does it take the charge found in step 3 to make a complete trip around the rotating ring? Use this to find the current of the rotating ring.
5. Use a result from Section 28.5 to determine the magnetic field that this ring produces at the center of the disk.

28.30 Finding the $\vec{B}$ field at the center of a uniformly charged, rotating disk.



6. Integrate your result from step 5 to find the total magnetic field from all rings with radii from $r = 0$ to $r = a$.

EVALUATE

7. Does your answer have the correct units?
8. Suppose all of the charge were concentrated at the rim of the disk (at $r = a$). Would this increase or decrease the field at the center of the disk?

CHAPTER 29 SUMMARY

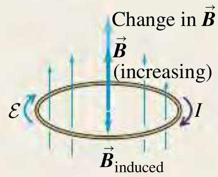
SOLUTIONS TO ALL EXAMPLES

Faraday's law: Faraday's law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop. This relationship is valid whether the flux change is caused by a changing magnetic field, motion of the loop, or both. (See Examples 29.1–29.6.)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad (29.3)$$

The magnet's motion causes a *changing* magnetic field through the coil, inducing a current in the coil.

Lenz's law: Lenz's law states that an induced current or emf always tends to oppose or cancel out the change that caused it. Lenz's law can be derived from Faraday's law and is often easier to use. (See Examples 29.7 and 29.8.)



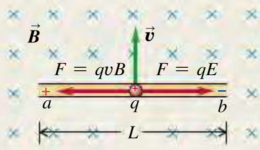
Motional emf: If a conductor moves in a magnetic field, a motional emf is induced. (See Examples 29.9 and 29.10.)

$$\mathcal{E} = vBL \qquad (29.6)$$

(conductor with length L moves in uniform $\vec{B}$ field, $\vec{L}$ and $\vec{v}$ both perpendicular to $\vec{B}$ and to each other)

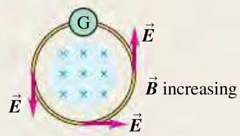
$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (29.7)$$

(all or part of a closed loop moves in a $\vec{B}$ field)



Induced electric fields: When an emf is induced by a changing magnetic flux through a stationary conductor, there is an induced electric field $\vec{E}$ of nonelectrostatic origin. This field is nonconservative and cannot be associated with a potential. (See Example 29.11.)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.10)$$



Displacement current and Maxwell's equations: A time-varying electric field generates displacement current i_D, which acts as a source of magnetic field in exactly the same way as conduction current. The relationships between electric and magnetic fields and their sources can be stated compactly in four equations, called Maxwell's equations. Together they form a complete basis for the relationship of $\vec{E}$ and $\vec{B}$ fields to their sources.

$$i_D = \epsilon \frac{d\Phi_E}{dt} \qquad (29.14)$$

(displacement current)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad (29.18)$$

(Gauss's law for $\vec{E}$ fields)

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad (29.19)$$

(Gauss's law for $\vec{B}$ fields)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.20)$$

(Faraday's law)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad (29.21)$$

(Ampere's law including displacement current)

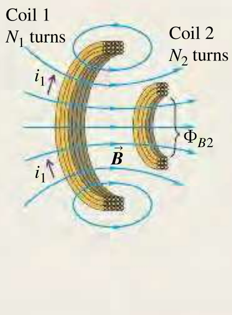
CHAPTER 30 SUMMARY

SOLUTIONS TO ALL EXAMPLES

Mutual inductance: When a changing current i_1 in one circuit causes a changing magnetic flux in a second circuit, an emf $\mathcal{E}_2$ is induced in the second circuit. Likewise, a changing current i_2 in the second circuit induces an emf $\mathcal{E}_1$ in the first circuit. If the circuits are coils of wire with N_1 and N_2 turns, the mutual inductance M can be expressed in terms of the average flux Φ_{B2} through each turn of coil 2 caused by the current i_1 in coil 1, or in terms of the average flux Φ_{B1} through each turn of coil 1 caused by the current i_2 in coil 2. The SI unit of mutual inductance is the henry, abbreviated H. (See Examples 30.1 and 30.2.)

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt} \tag{30.4}$$

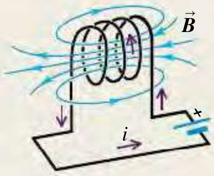
$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2} \tag{30.5}$$



Self-inductance: A changing current i in any circuit causes a self-induced emf $\mathcal{E}$. The inductance (or self-inductance) L depends on the geometry of the circuit and the material surrounding it. The inductance of a coil of N turns is related to the average flux Φ_B through each turn caused by the current i in the coil. An inductor is a circuit device, usually including a coil of wire, intended to have a substantial inductance. (See Examples 30.3 and 30.4.)

$$\mathcal{E} = -L\frac{di}{dt} \tag{30.7}$$

$$L = \frac{N\Phi_B}{i} \tag{30.6}$$

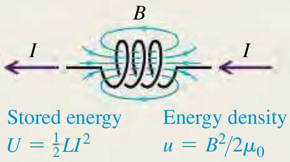


Magnetic-field energy: An inductor with inductance L carrying current I has energy U associated with the inductor's magnetic field. The magnetic energy density u (energy per unit volume) is proportional to the square of the magnetic-field magnitude. (See Example 30.5.)

$$U = \tfrac{1}{2}LI^2 \tag{30.9}$$

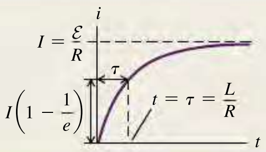
$$u = \frac{B^2}{2\mu_0} \quad \text{(in vacuum)} \tag{30.10}$$

$$u = \frac{B^2}{2\mu} \quad \text{(in a material with magnetic permeability } \mu\text{)} \tag{30.11}$$



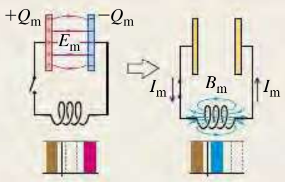
***R-L* circuits:** In a circuit containing a resistor R, an inductor L, and a source of emf, the growth and decay of current are exponential. The time constant τ is the time required for the current to approach within a fraction $1/e$ of its final value. (See Examples 30.6 and 30.7.)

$$\tau = \frac{L}{R} \tag{30.16}$$



***L-C* circuits:** A circuit that contains inductance L and capacitance C undergoes electrical oscillations with an angular frequency ω that depends on L and C. This is analogous to a mechanical harmonic oscillator, with inductance L analogous to mass m, the reciprocal of capacitance $1/C$ to force constant k, charge q to displacement x, and current i to velocity v_x. (See Examples 30.8 and 30.9.)

$$\omega = \sqrt{\frac{1}{LC}} \tag{30.22}$$



***L-R-C* series circuits:** A circuit that contains inductance, resistance, and capacitance undergoes damped oscillations for sufficiently small resistance. The frequency ω' of damped oscillations depends on the values of L, R, and C. As R increases, the damping increases; if R is greater than a certain value, the behavior becomes overdamped and no longer oscillates. (See Example 30.10.)

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{30.29}$$