INDEX

1. If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

   (a) 4-cell reuse. (b) 7-cell reuse. (c) 12-cell reuse.

   If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

2. If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

   (a) $n = 4$. (b) $n = 3$.

   Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

3. How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

   (a) 1, (b) 5, (c) 10, (d) 20, (e) 100.

   Assume each user generates 0.1 Erlangs of traffic.

4. An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

5. A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

   a) The number of cells in the service area,
   b) The number of channels per cell,
   c) Traffic intensity of each cell,

d) The maximum carried traffic,
e) The total number of users that can be served for 2% GOS,
f) The number of mobiles per channel, and
g) The theoretical maximum number of users that could be served at one time by the system.

6. If a transmitter produces 50 watts of power, express the transmit power in units of

a) dBm and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,
a) Find the received power in dBm at a free space distance of 100 m from the antenna,
b) What is P (10 km)?
Assume unity gain for the receiver antenna.

7. Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4Km.

8. Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.

9. A mobile is located 5 km away from a base station and uses a vertical λ /4 monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.

a) Find the length and the gain of the receiving antenna.
b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.

10. A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ= call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-

a) How many users per square kilometer will this system support?
b) What is the probability that a delayed call will have to wait for more than 10s?
c) What is the probability that a call will be delayed for more than 10 seconds?

Problem-1: If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

(a) 4-cell reuse. (b) 7-cell reuse. (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

Solution:

Have given,

Total bandwidth = 33 MHz,
= 33,000 kHz

Channel bandwidth = 25 kHz x 2 simplex channels
= 50 kHz / duplex channel

Total available channels = Total Bandwidth / Channel Bandwidth
= 33,000 / 50
= 660 channels

Control channel bandwidth = 1000 kHz

The number of available control channel = Control channel bandwidth / Channel bandwidth

= 1000 / 50

= 20 channels

(a)

Have given, Cluster size, N = 4

Total number of channels available per cell = Total available channels / N
= 660/4
≈165 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N

= (660 – 20) / 4

≈ 160 channels

Control Channel = Total number of channels available per cell – Voice Channel

= 165 – 160

= 5 channels

(b) Have given, Cluster size, N = 7

Total number of channels available per cell = Total available channels / N
= 660/7
≈94 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N

= (660 – 20) / 7

≈ 91 channels

Control Channel = Total number of channels available per cell – Voice Channel

= 94 – 91

= 3 channels

(c) Have given, Cluster size, N = 12

Total number of channels available per cell = Total available channels / N
= 660/12 ≈
55 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N

= (660 – 20) / 12

≈ 53 channels

Control Channel = Total number of channels available per cell – Voice Channel

= 55 – 53

= 2 channels

Source Code:

```
bw = 30000  # Bandwidth in Hz
schannel_bw = 25  # Single channel bandwidth in Hz
print("Channel Bandwidth...")
dup_ch_bw = 2 * schannel_bw  # Duplex channel bandwidth (both directions)
t_ch = bw / dup_ch_bw  # Total number of available channels
print(dup_ch_bw)
print("Total available channels:", t_ch)
# Control channel bandwidth
cc_bw = 1000  # Control channel bandwidth in Hz
t_cc = cc_bw / dup_ch_bw  # Number of control channels
print("Total control channels:", t_cc)
# N represents different cluster sizes
N = [4, 7, 12]  # Cluster sizes
# Loop through each cluster size in N
for i in range(3):
```

```
    # Calculate channels per cell for each cluster size
    ch = t_ch / N[i]
    ch_per_cell = round(ch)  # Rounded channels per cell
    print("Cluster size:", N[i])
    print("Channels per cell:", ch_per_cell)
    # Calculate control channels per cell
    c = t_cc / N[i]
    cc = round(c)
    # Calculate voice channels per cell
    v = (t_ch - t_cc) / N[i]
    vc = round(v)
    # Display the results for control and voice channels
    print("Control channels:", cc)
    print("Voice channels:", vc)
```

Output:

Cluster size: 4

Channels per cell: 165

Control channels: 5

Voice channels: 160

Cluster size: 7

Channels per cell: 94

Control channels: 3

Voice channels: 91

Cluster size: 12

Channels per cell: 55

Control channels: 2

Voice channels: 53

Total control channels: 20.0

Channel Bandwidth...

50

Total available channels: 660.0

Problem-2: If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a) $n = 4$. (b) $n = 3$.

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

Solution: Have given,

Minimum Required Signal-to-Noise interference ratio, $S/I = 15$ dB,
The number of Co-channel interfering cells, $i_o = 6$

We Know,

Number of cell reuse, $N = i^2 + i*j + j^2$ ………………………………………….. (i)

First, let us consider a 7-cell reuse pattern, $N = 7$ For $i=1, j=2$

Also,

The Frequency Reuse Factor, $Q = D/R$

$= \sqrt{(3N)}$ …………….……………………….. (ii)

$= 4.583$.

Where,

D = Distance between centers of the nearest Co-channel cells.

R = Radius of the cell.

(a) Have given, Path Loss exponent; n = 4, Frequency Reuse Factor, Q = 4.583.

We know,

Signal-to-Noise interference ratio, $S/R = 10 \log (Q^n / i_0)$

$= 10 \log ((4.583)^4 / 6)$

$= 18.66$ dB.

Since this is greater than the minimum required S/I (18.66 > 15), N = 7 can be used.

(b) Have given, Path Loss exponent, n = 3

We know,

Signal-to-Noise interference ratio, $S/R = 10 \log(Q^n / i_0)$ …………………..…. (iii)

$= 10 \log ((4.583)^3 / 6) = 12.05$ dB.

Since this is less than the minimum required S/I (12.05 < 15), we need to use a larger N.

Using equation (i), the next possible value of N = 12; For i = j = 2.

The corresponding co-channel ratio is given by equation (ii)

as- **Frequency Reuse Factor**, Q = 6.

Using equation (iii) the signal-to-interference ratio, S/I = 15.56 dB.

Since, this is greater than the minimum required S/I (15.56 > 15), N = 12 can be used.

Source Code:

```
import math
# Initialize variables
R_SI = 15  # Reference Signal to Interference ratio
io = 6  # Interference
n = [4, 3]  # Power values

# Loop through each element of n
for a in range(2):
    N = 7  # Initial reuse pattern size
    Q = math.sqrt(3 * N)  # Frequency reuse factor
    print('n:', n[a])
    print('Frequency reuse factor:', Q)

    # Calculate Signal to Interference Ratio (SI)
    SI = 10 * math.log10((1 / io) * (Q ** n[a]))
    print('Signal to interference ratio:', SI)

    # Check if SI is less than the reference value R_SI
    if SI < R_SI:
        i, j = 2, 2  # Parameters for new N
        N = (i ** 2) + (i * j) + (j ** 2)  # New reuse pattern size
        Q = math.sqrt(3 * N)  # Recalculate frequency reuse factor
        print('n:', n[a])
        print('New frequency reuse factor:', Q)

        # Calculate new Signal to Interference Ratio (SI1)
        SI1 = 10 * math.log10((1 / io) * (Q ** n[a]))
        print('New Signal to interference ratio:', SI1)
```

Output:

n: 4
Frequency reuse factor: 4.58257569495584
Signal to interference ratio: 18.66287339084195
n: 3
Frequency reuse factor: 4.58257569495584
Signal to interference ratio: 12.051776917172353
n: 3
New frequency reuse factor: 6.0
New Signal to interference ratio: 15.563025007672874

Problem-3: How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

(b) 1, (b) 5, (c) 10, (d) 20, (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

Solution:

Have given,

Blocking Probability, P_B = 0.5%,

Traffic Intensity, A_u = 0.1 Erlangs

We Know,

For Erlangs B, Grade of Service, GOS = P_B

= 0.005

And, Total number of user, U = A / A_u ……………………………….. (i)

Where,

A = Offered Traffic Intensity.

Also,

Table 3.1: Capacity of an Erlang B System.

Number of Channels C	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

From Table 3.1, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels.

(a) Have given, Trunked channels, C = 1

From table 3.1, For C = 1 we obtain, A = 0005

From equation (i), we have- Total number of user, U = A / A_u = 0.05 users.

But, actually one user could be supported on one channel. So, U = 1.

(b) Have given, Trunked channels, C = 5

From table 3.1, For C = 5 we obtain, A = 1.13 Erlang.

From equation (i), we have- Total number of user, U = A / $A_u \approx$ 11 users.

(c) Have given, Trunked channels, C = 10

From table 3.1, For C = 10 we obtain, A = 3.96 Erlang

From equation (i), we have-
Total number of users, U = A / $A_u \approx$ 39 users.
(d) Have given, Trunked channels, C = 20

From table 3.1, For C = 20 we obtain, A = 11.10 Erlang

From equation (i), we have- Total number of users, U = A / $A_u \approx$ 110 users.
(e) Have given, Trunked channels, C = 100

From table 3.1, For C = 100 we obtain, A = 80.9 Erlang.

From equation (i), we have- Total number of users, U = A / $A_u \approx$ 809 users.

Source Code (MATLAB):
```
import numpy as np

# Initialize variables
Gos = 0.5 / 100  # Grade of service (blocking probability)
Au = 0.1  # Traffic intensity per user
A = np.array([0.005, 1.13, 3.96, 11.1, 80.9])  # Traffic intensity
c = np.array([1, 5, 10, 20, 100])  # Number of channels

# Displaying the given values
print('Blocking probability:', Gos)
print('Traffic intensity per user:', Au)
print('Traffic intensity:', A)
print('Number of channels:', c)

# Calculate the number of users for each value of A
U = A / Au  # Total traffic divided by traffic per user gives number of users
u = np.round(U)  # Round to nearest whole number
print('Number of users:', u)
```

Output:
Blocking probability: 0.005
Traffic intensity per user: 0.1
Traffic intensity: [5.00e-03 1.13e+00 3.96e+00 1.11e+01 8.09e+01]
Number of channels: [1 5 10 20 100]
Number of users: [0. 11. 40. 111. 809.]

Problem-4: An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution:

Have given,

Blocking Probability, P_B = 2%, The
average number of call requests per unit time λ = 2.
The average duration of a call, H = 3/60 seconds
There are 2 million residents in the given urban area = 2000000

We Know, For Erlangs B, Grade of Service, GOS = P_B = 0.02. And, Traffic Intensity,
$A_u = \lambda H$= 0.1 Erlangs
Also, Total number of user, $U = A / A_u$ ……………………………....... (i)
Where, A = Offered Traffic Intensity.
Also,

Table 4.1: Capacity of an Erlang B System.

Erlang B Traffic Table

Maximum Offered Load Versus B and N
B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
94	64.25	68.07	69.98	75.41	78.43	82.17	89.10	97.53	105.3	113.2	131.2	154.3
95	65.08	68.93	70.85	76.33	79.37	83.13	90.12	98.63	106.4	114.4	132.6	155.9
100	69.27	7~.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

N is the number of servers. The numerical column headings indicate blocking probability B in %. Table generated by Dan Dexter

From Table 4.1, we can find the total capacity in Erlangs for the 2% GOS for different numbers of channels.

For System-A

Have given, Number of channels per cell used in the system, C = 19

From table 4.1, For C = 19 and GOS = 0.02 we obtain, A = 12 Erlangs

From equation (i), we have- Total number of user, U = A / A_u = 120 users.
*Since there are 394 cells, the total number of' subscribers that can be supported by **System A** is equal to 120 x 394 = 47280.*

Since, the percentage market penetration = 47280/2000000 = 2.36%

For System-B

Have given, Number of channels per cell used in the system, C = 57
From table 4.1, For C = 57 and GOS = 0.02 we obtain, A = 45 Erlangs

From equation (i), we have- Total number of user, U = A / A_u = 450 users.
*Since there are 98 cells, the total number of' subscribers that can be supported by **System B** is equal to 450 x 98 = 44,100.*

Since, the percentage market penetration = 44100/2000000 = 2.205%

For System-C

Have given, Number of channels per cell used in the system, C = 100
From table 4.1, For C = 100 and GOS = 0.02 we obtain, A = 88 Erlangs
From equation (i), we have- Total number of user, U = A / A_u = 880 users.
*Since there are 49 cells, the total number of' subscribers that can be supported by **System C***

is equal to 880 x 49 = 43,120.

Since, the percentage market penetration = 43,120/2000000 = 2.156%

Therefore, total number of cellular subscribers that can be supported by these three systems are (47280 + 44100 + 43120) = 134500 users.

The market penetration of the three systems combined is equal to 134500/2000000 = 6.725

Source Code:

```
blocking_p = 2 / 100  # Blocking probability
lamda = 2  # Arrival rate (calls per hour)
H = 3 / 60  # Average call duration in hours (3 minutes)
Au = lamda * H  # Traffic intensity per user (Erlangs)
print('For system A')
channel_a = 19
cell_A = 394
A = 12  # Total traffic intensity in system A
Ua = A / Au
print('Number of users in System A:', Ua)
subscriber_A = Ua * cell_A
print('Total number of subscribers in system A:', subscriber_A)
percentage_market_penetration_for_A = (subscriber_A / 2000000) * 100
print('Percentage market penetration for A:', percentage_market_penetration_for_A)
print('For system B')
channel_b = 57
cell_B = 98
```

```
Ab = 45  # Total traffic intensity in system B
Ub = Ab / Au
print('Number of users in System B:', Ub)
subscriber_B = Ub * cell_B
print('Total number of subscribers in system B:', subscriber_B)
percentage_market_penetration_for_B = (subscriber_B / 2000000) * 100
print('Percentage market penetration for B:', percentage_market_penetration_for_B)
print('For system C')
channel_c = 100
cell_C = 49
Ac = 88  # Total traffic intensity in system C
Uc = Ac / Au
print('Number of users in System C:', Uc)
subscriber_C = Uc * cell_C
print('Total number of subscribers in system C:', subscriber_C)
percentage_market_penetration_for_C = (subscriber_C / 2000000) * 100
print('Percentage market penetration for C:', percentage_market_penetration_for_C)
Total_number_of_subscriber = subscriber_A + subscriber_B + subscriber_C
print('Total number of subscribers across all systems:', Total_number_of_subscriber)
Market_penetration_for_three_systems = (Total_number_of_subscriber / 2000000) * 100
print('Market       penetration       for       all       three       systems:',
Market_penetration_for_three_systems)
```

Output:

For system A

Number of users in System A: 120.0

Total number of subscribers in system A: 47280.0

Percentage market penetration for A: 2.3640000000000003

For system B

Number of users in System B: 450.0

Total number of subscribers in system B: 44100.0

Percentage market penetration for B: 2.205

For system C

Number of users in System C: 880.0

Total number of subscribers in system C: 43120.0

Percentage market penetration for C: 2.156

Total number of subscribers across all systems: 134500.0

Market penetration for all three systems: 6.7250000000000005

Problem-5: A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

a) The number of cells in the service area,
b) The number of channels per cell,
c) Traffic intensity of each cell,
d) The maximum carried traffic,
e) The total number of users that can be served for 2% GOS,
f) The number of mobiles per channel, and
g) The theoretical maximum number of users that could be served at one time by the system.

Solution: (a) Have given, Total coverage area = 1300 miles, Cell radius = 4 miles

We know,

The area of a cell (hexagon) can be shown to be $2.5981R^2$

Thus each cell covers $2.5981 \times (4)^2 = 41.57$ sq km.

Hence, the total number of cells, N_c = 1300/41.57 = 31 cells

(b) Have given,

Allocated spectrum = 40, 000,000 Hz
Channel width = 60,000 Hz
Frequency reuse factor, N = 7 cells

We know,
The total number of channels per cell, C= Allocated spectrum / (Channel width × N)
= 40, 000,000 / (60,000 × 7) = 95 channels/cell

(c) Have given,From (b) No, C = 95 And, GOS = 0.02

From the table 4.1 (Erlang B chart) For C = 95 and GOS = 0.02, we have- Traffic intensity per cell, A = 84 Erlangs/cell

(d)

Have given,
From (a), Number of cells = 31 cells
From (c), Traffic intensity per cell = 84 Erlangs/cell

We Know,

Maximum carried traffic = Number of cells × Traffic intensity per cell
= 31 × 84 = 2604 Erlangs.

(e) Have given, Traffic per user, A_u = 0.03 Erlangs

From (d), Total traffic, A = 2604 Erlangs.

We Know,

Total number of users, $U = A / A_u$

$= 2604 / 0.03$

$= 86,800$ users.

(f)

Have given, Allocated spectrum = 40, 000,000 Hz, Channel width = 60,000 Hz
From (e), Number of users, U = 86,800 users.

We Know,

Number of channels = Allocated Spectrum / Channel Width = 40, 000,000/60,000 ≈ 666
Number of mobiles per channel = Number of users/Number of channels = 86,800 / 666
≈ 130 mobiles/channel

(g)

Have given,

From (b) No, C = 95 channels/cell
From (a), the total number of cells, N_c= 31 cells.
From (e) Total number of users, U = 86,800 users.

We Know,

The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied) $= C \times N_c$

$= 95 \times 31 = 2945$ users,

Which is (2945/86,800) × 100 = 3.4% of the customer base.

Source Code:

```
import math
area = 1300
radius = 4
each_cell_covers = math.floor(2.5981 * radius ** 2)
print('(a)')
number_of_cells = math.floor(area / each_cell_covers)
print(f'Number of cells: {number_of_cells}')
allocated_spectrum = 40000
channel_width = 60
frequency_reuse_factor = 7
print('(b)')
number_of_channel_per_cell = math.floor(allocated_spectrum / (channel_width *
frequency_reuse_factor))
print(f'Number of channels per cell: {number_of_channel_per_cell}')
print('(c)')
traffic_intensity_per_cell = 84
print(f'Traffic intensity per cell: {traffic_intensity_per_cell}')
print('(d)')
maximum_carried_traffic = number_of_cells * traffic_intensity_per_cell
print(f'Maximum carried traffic: {maximum_carried_traffic}')
traffic_per_user = 0.03
print('(e)')
total_number_of_users = maximum_carried_traffic / traffic_per_user
```

```
print(f'Total number of users: {total_number_of_users}')
number_of_channels = number_of_channel_per_cell * frequency_reuse_factor
print('(f)')
number_of_mobile_per_channel = math.floor(total_number_of_users /
number_of_channels)
print(f'Number of mobiles per channel: {number_of_mobile_per_channel}')
print('(g)')
theoretical_maximum_users = number_of_cells * number_of_channel_per_cell
print(f'Theoretical maximum number of users: {theoretical_maximum_users}')
```

Output:

(a)

Number of cells: 31

(b)

Number of channels per cell: 95

(c)

Traffic intensity per cell: 84

(d)

Maximum carried traffic: 2604

(e)

Total number of users: 86800.0

(f)

Number of mobiles per channel: 130

(g)

Theoretical maximum number of users: 2945

Problem-6: If a transmitter produces 50 watts of power, express the transmit power in units
of

a) dBm, and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

c) Find the received power in dBm at a free space distance of 100 m from the antenna,
d) What is P (10 km)?

Assume unity gain for the receiver antenna.

<u>Solution:</u> Have given, Transmitter power, $P_t = 50$ W, Carrier frequency, $f_c = 900$ MHz

(a) We know,

Transmitter power, $P_t(\text{dBm}) = 10 \log[P_t(\text{mW})/(1\text{mW})]$
$= 10 \log [50 \times 10^3]$
$= 47.0$ dBm

(b) We know,

Transmitter power, $P_t(\text{dBW}) = 10 \log[P_t(\text{W})/(1\text{W})]$
$= 10 \log [50]$
$= 17.0$ dBW

(c) If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

Have given,

Transmitter Gain, G_t	$= 1$
Receiver Gain, G_r	$= 1$
Wave length λ	$= c / f = 1 / 3$ m
The T-R separation distance, d	$= 100$m
The system loss factor, L	$= 1$

We know,

The received power, $P_r = (P_t \times G_t \times G_r \times \lambda^2) / (4\pi^2 \times d^2 \times L)$
$= (50 \times 1 \times 1 \times (1/3)^2) / ((4\pi)^2 \times 100^2 \times 1)$
$= 3.5 \times 10^{-3}$ mW

Received power, $P_r(\text{dBm}) = 10 \log[P_r(\text{mW})]$
$= 10 \log[P_r(3.5 \times 10^{-3})]$
$= -24.5$ dBm

(d) Have given, d_o = 10 km = 10000 m

We Know,

The received power at 10 km can be expressed in terms of dBm, we have

$\therefore P_r(10\text{ km}) = P_r(100) + 20 \log[d / d_o]$
$= P_r(100) + 20 \log[100 / 10000]$
$= -24.5 - 40 =$
-64.5 dBm

Source Code:

```
import numpy as np
# Define constants
pt = 50          # Transmitted power in watts
fc = 900         # Carrier frequency in MHz
gt = 1           # Transmitter antenna gain
gr = 1           # Receiver antenna gain
d = 100          # Free space distance in meters
L = 1            # System loss factor
c = 3 * 10**8    # Speed of light in m/s

# Calculate wavelength (lambda)
lambda_ = c / (fc * 10**6)  # Converting fc from MHz to Hz
# Question (a): Transmitter power in dBm
tr_dBm = np.ceil(10 * np.log10(pt * 1000))
print(f"(a) Transmitter power, Pt in dBm: {tr_dBm:.0f} dBm")
# Question (b): Transmitter power in dBW
tr_dBW = np.ceil(10 * np.log10(pt))
print(f"(b) Transmitter power, Pt in dBW: {tr_dBW:.0f} dBW")
# Question (c): Received power at 100 meters in dBm
c_value = (pt * gt * gr * (lambda_ ** 2)) / ((4 * np.pi) ** 2 * d ** 2 * L) * 1000
Pr = 10 * np.log10(c_value)
print(f"(c) Received power, Pr in dBm: {Pr:.2f} dBm")
# Question (d): Received power at 10 km in dBm
d_km = 10000  # Distance of 10 km in meters
Pr_10km = Pr + (20 * np.log10(d / d_km))
print(f"(d) Received power, Pr at 10km in dBm: {Pr_10km:.2f} dBm")
```

Output:

(a) Transmitter power, Pt in dBm: 47 dBm
(b) Transmitter power, Pt in dBW: 17 dBW
(c) Received power, Pr in dBm: -24.54 dBm
(d) Received power, Pr at 10km in dBm: -64.54 dBm

Problem-7: Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4 km.

Solution:

Have given,

The frequency, f_c = 900 MHz (150 MHz to 1500MHz)

The effective transmitter (base station) antenna height, h_{te} = 100m

The effective transmitter (mobile) antenna height, h_{re} = 2m

T-R separation distance, d = 4 km

Now, The correction factor for effective mobile antenna height, $a(h_{re}) = 3.2 (\log 11.75\ h_{re})^2 - 4.97$ dB for fc ≥ 300 MHz

From Okumura-Hata Model we know,

The path loss in urban areas is given by

$$L_{50}(urban)(\text{dB}) = 69.55 + 26.16\log f_c - 13.82\log h_{te} - a(h_{re}) + (44.9 - 6.55\log h_{te})\log d$$

$$= 69.55 + 26.16 \times 2.954 - 13.82 \times 2 - 1.045 + (44.9 - 13.1) \times 0.6 = 137.3 \text{ dB}$$

Source Code (MATLAB):

```
import numpy as np
# Define constants
hte = 100  # Effective transmitter (base station) antenna height in meters
hre = 2    # Effective receiver (mobile) antenna height in meters
fc = 900   # Frequency in MHz
d = 4      # T-R separation distance in kilometers
# Calculate the correction factor using the Okumura-Hata model
a_hre = (3.2 * (np.log10(11.75 * hre)) ** 2) - 4.97

# Calculate Path Loss in urban areas
Lp = 69.55 + 26.16 * np.log10(fc) - 13.82 * np.log10(hte) - a_hre + \
     ((44.9 - 6.55 * np.log10(hte)) * np.log10(d))

print(f'The path loss in urban areas, Lp = {Lp:.2f} dB')
```

Output:

The path loss in urban areas, Lp = 137.29 dB

Problem-8: Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.

Solution:

Have given, The frequency, f_c = 1.8 GHz (0.9 to 2 GHz)
The effective transmitter (base station) antenna height, h_b = 20m
T-R separation distance, d = $\sqrt{(20^2 + 30^2)}$ = 0.036 km

From Okumura-Hata Model we know, The path loss in a high-rise urban areas with Perpendicular Street to the location of the Base Station is given by-

$$L_p = 135.41 + 12.49 \log f_c - 4.99 \log h_b + [46.84 - 2.34 \log h_b] \log d$$

$$= 135.41 + 12.49 \times \log (1.8) - 4.99 \times \log 20 + [46.84 - 2.34 \log 20] \times \log 0.036$$

$$= 68.91 \text{ dB}$$

Source Code (MATLAB):

```
import numpy as np

# Define constants
fc = 1.8  # Frequency in GHz
hb = 20   # Effective transmitter (base station) antenna height in meters

# Calculate T-R separation distance in kilometers
d = (np.sqrt(20**2 + 30**2)) / 1000  # Convert meters to kilometers

# Calculate Path Loss in high-rise urban areas
Lp = 135.41 + (12.49 * np.log10(fc)) - (4.99 * np.log10(hb)) + \
     ((46.84 - 2.34 * np.log10(hb)) * np.log10(d))

print(f'The path loss in high-rise urban areas, Lp = {Lp:.2f} dB')
```

Output:

The path loss in a high-rise urban areas, L_p = 68.91 dB

Problem-9: A mobile is located 5 km away from a base station and uses a vertical λ /4 monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.

a) Find the length and the gain of the receiving antenna.
b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.

Solution:

Have given,
Frequency of operation, f = 900 MHz
Gain of antenna, G= 1.8 = 2.55 dB

(a) We Know, Wave length,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.333 \text{ m.}$$

Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833 \text{ m} = 8.33 \text{ cm.}$

And, Gain of antenna, G = 2.55 dB.

(b) Have given,

T-R separation distance, d	= 5 km
E-field at a distance of 1 km, E_o	= 10^{-3} V/m
Transmitter distance d_o	= 1km
Transmitting antenna height, h_t	= 50m
Receiving antenna height, h_r	= 1.5m
Wave length, λ	= 0.333

We Know,

Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0}{d}\frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

$$= \frac{2\times10^{-3}\times1\times10^3}{5\times10^3}\left[\frac{2\pi(50)(1.5)}{0.333(5\times10^3)}\right]$$

$$= 113.1\times10^{-6} \text{ V/m.}$$

Here, Effective Aperture

$$A_e = \frac{\lambda^2}{4\pi}G$$

$= 0.016\ m^2$

Now, the received power at a distance d can be obtained using

$$P_r(d) = P_d A_e = \frac{|E|^2}{120\pi} A_e$$

$= ((113.1 \times 10^{-6})^2 \times 0.016)/337$
$= 5.4 \times 10^{-13}$ W
$= -122.68$ dbW
$= -92.68$ dBm

Source Code (MATLAB):

```
import numpy as np
f = 900         # Frequency in MHz
g = 2.55        # Gain of antenna in dB
# Question (a)
gain = 10 ** (g / 10)  # Convert gain from dB to linear scale
lemda = (3 * 10**8) / (f * 10**6)  # Wavelength in meters
L = lemda / 4  # Antenna length
print('For (a)')
print('---------')
print(f'Length of the antenna: {L:.3f} m')
print(f'Gain of the antenna: {gain:.1f} = {g:.2f} dB\n')
# Question (b)
print('For (b)')
print('---------')
d = 5000                # T-R separation distance in meters
E0 = 10**-3             # Electric field
d0 = 1000               # Transmitter distance in meters
ht = 50                 # Transmitting antenna height in meters
hr = 1.5                # Receiving antenna height in meters
# Electric Field
Er_d = (2 * E0 * d0 * 2 * np.pi * ht * hr) / (lemda * d**2)
# Effective Aperture
Ae = (gain * lemda**2) / (4 * np.pi)
# Received power at a distance d
Pr_d = (Er_d**2 / (120 * np.pi)) * Ae
Pr_dB = 10 * np.log10(Pr_d)
print(f'Electric Field, Er(d): {Er_d:.9f} V/m')
print(f'Effective Aperture, Ae: {Ae:.3f} m^2')
print(f'Received power at 5 km distance Er(5 km): {Pr_dB:.3f} dBW')
```

Output:

For (a)

Length of the antenna: 0.083 m

Gain of the antenna: 1.8 = 2.55 dB

For (b)

Electric Field, Er(d): 0.000113097 V/m

Effective Aperture, Ae: 0.016 m^2

Received power at 5 km distance Er(5 km): -122.679 dBW

Problem 10: A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ= call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-

a) How many users per square kilometer will this system support?
b) What is the probability that a delayed call will have to wait for more than 10s?
c) What is the probability that a call will be delayed for more than 10 seconds?

Solution:

Have given,

Cell radius, R = 1.387 km
Area covered per cell is $2.598 \times (1.387)^2$ ≈ 5 sq km
Number of cells per cluster,n = 4
Total number of channels, N = 60
Therefore, number of channels per cell = 60 / 4 = 15 channels.
From Erlang C chart, for 5% probability of delay with C = 15,
Traffic intensity, A = 9.0 Erlangs.

(a)

Have given, Traffic per user, A_u = 0.029 Erlangs.
We know, The number of users, $U = A / A_u$ = 9.0/0.029 = 310 users
The number of users per square km = 310 users /5 sq km = 62 users /sq km

(b)

Have given, Wave length, λ = 1 call/hour, Holding time, $H = A_u / \lambda$
= 0.029 hour
= 104.4 seconds.
Time, t = 10s

We know,
The conditional probability that a delayed call will have to wait for more than t seconds is $P_r[\text{delay} > t \mid \text{delay}] = \exp(-(C-A)t/H)$
$= \exp(-(15-9)10/104.4)$
= 56.29 %

(c) Have given, The probability of delayed call, $P_r[\text{delay} > 0]$ = 5 % = 0.05
We know, Probability that a call is delayed more than 10 seconds,
$P_r[\text{delay} > 10] = P_r[\text{delay} > 0] \times P_r[\text{delay} > t \mid \text{delay}]$
= 0.05 × 0.5629 =2.81 %

Source Code (MATLAB):

```
import numpy as np
# Define constants
R = 1.387               # Cell Radius
n = 4                   # Number of cells
N = 60                  # Total number of channels
```

```
area = round(2.5981 * R**2)  # Area covered per cell
C = N / n              # Number of channels per cell
A = 9                  # Traffic intensity at c=15, GOS=0.05, Au=0.029 from Erlang
C chart

# Question (a)
Au = 0.029             # Traffic per user
U = np.floor(A / Au)   # Number of users
U_per = round(U / area)  # Number of users per square km
print(f'(a) Number of users per square km: {U_per} users/sq km\n')

# Question (b)
lemda = 1              # lambda = 1 hour
H = (Au / lemda) * 3600  # Holding time in seconds
t = 10                 # Delay time in seconds
Prb = np.exp(-((C - A) * t) / H)  # Probability that a delayed call will have to
wait
print(f'(b) The probability that a delayed call will have to wait: {Prb *
100:.2f}%\n')

# Question (c)
Prc = 0.05 * Prb * 100  # 5% probability of delayed call
print(f'(c) The probability that a call will be delayed: {Prc:.2f}%\n')
```

Output:

(a) Number of users per square km: 62 users/sq km

(b) The probability that a delayed call will have to wait: 56.29%

(c) The probability that a call will be delayed: 2.81%