Example 3.1

A spectrum of 30 MHz is allocated to a wireless FDD cellular system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses (a) four-cell reuse, (b) seven-cell reuse, and (c) 12-cell reuse. If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

Solution

Given:

Total bandwidth = 30 MHz

Channel bandwidth = 25 kHz × 2 simplex channels = 50 kHz/duplex channel

Total available channels = 30,000/50 = 600 channels

(a) For $N = 4$,
total number of channels available per cell = 600/4 ≈ 150 channels.

(b) For $N = 7$,
total number of channels available per cell = 600/7 ≈ 85 channels.

(c) For $N = 12$,
total number of channels available per cell = 600/12 ≈ 50 channels.

A 1 MHz spectrum for control channels implies that there are 1000/50 = 20 control channels out of the 600 channels available. To evenly distribute the control and voice channels, simply allocate the same number of voice channels in each cell wherever possible.

(a) For $N = 4$, we can have 5 control channels and 145 voice channels per cell. In practice, however, each cell only needs a single control channel (the control channels have a greater reuse distance than the voice channels). Thus, 1 control channel and 145 voice channels would be assigned to each cell.

(b) Total number of voice channels for $N = 7$, (600 – 20)/7 = 82 voice channels are to be assigned to each cell approximately, 4 cells with 3 control channels and 82 voice channels, and 3 cells with 2 control channels are to be assigned along with 83 voice channels.
Note: there is no fixed distribution of control channel as control channel has longer reuse distance than voice channel.

(c) For $N = 12$, we can have eight cells with two control channels and 48 voice channels, and four cells with one control channel and 49 voice channels each. In an actual system, each cell would have 1 control channel, 8 cells would have 48 voice channels, and 4 cells would have 49 voice channels.

Example 3.2

For given path loss exponent (a) $n = 4$ and (b) $n = 3$, find the frequency reuse factor and the cluster size that should be used for maximum capacity. The signal-to-interference ratio of 15 dB is minimum required for satisfactory forward channel performance of a cellular system. There are six co-channel cells in the first tier, and all of them are at the same distance from the mobile. Use suitable approximations.

Solution

(a) $n = 4$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.4), Frequency reuse factor, $Q = D/R = \sqrt{(3\,N)} = \sqrt{21} = 4.583$.

Using Equation (3.9), the signal-to-noise interference ratio is given by

$$S/I = (1/6) \times (4.583)^4 = 75.3 = 18.66 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 7$ can be used.

$S/I = \frac{(D/R)^n}{i_0} = \frac{(\sqrt{3N})^n}{i_0}$

(b) $n = 3$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.9), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (4.583)^3 = 16.04 = 12.05 \text{ dB}$$

Since this is less than the minimum required S/I, we need to use a larger N.

Using Equation (3.3), the next possible value of N is 12, $(i = j = 2)$.

The corresponding co-channel ratio is given by Equation (3.4) as

$$D/R = 6.0$$

Using Equation (3.3), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (6)^3 = 36 = 15.56 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 12$ is used.

abscissa of Figure 3.7. With two of the parameters specified, it is easy to find the third parameter

Example 3.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

Solution

From Table 3.4, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels. By using the relation $A = UA_u$, we can obtain the total number of users that can be supported in the system.

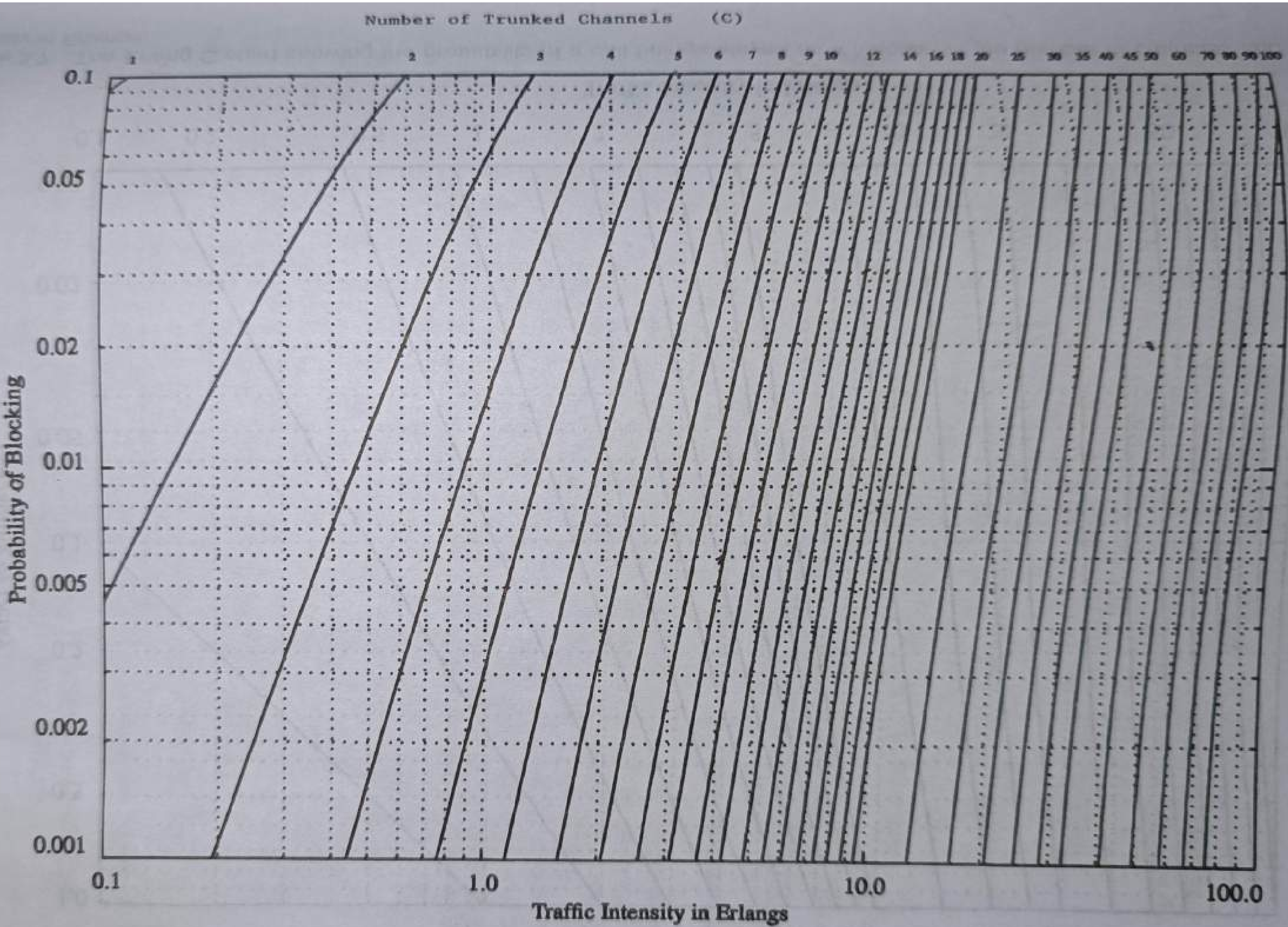

Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

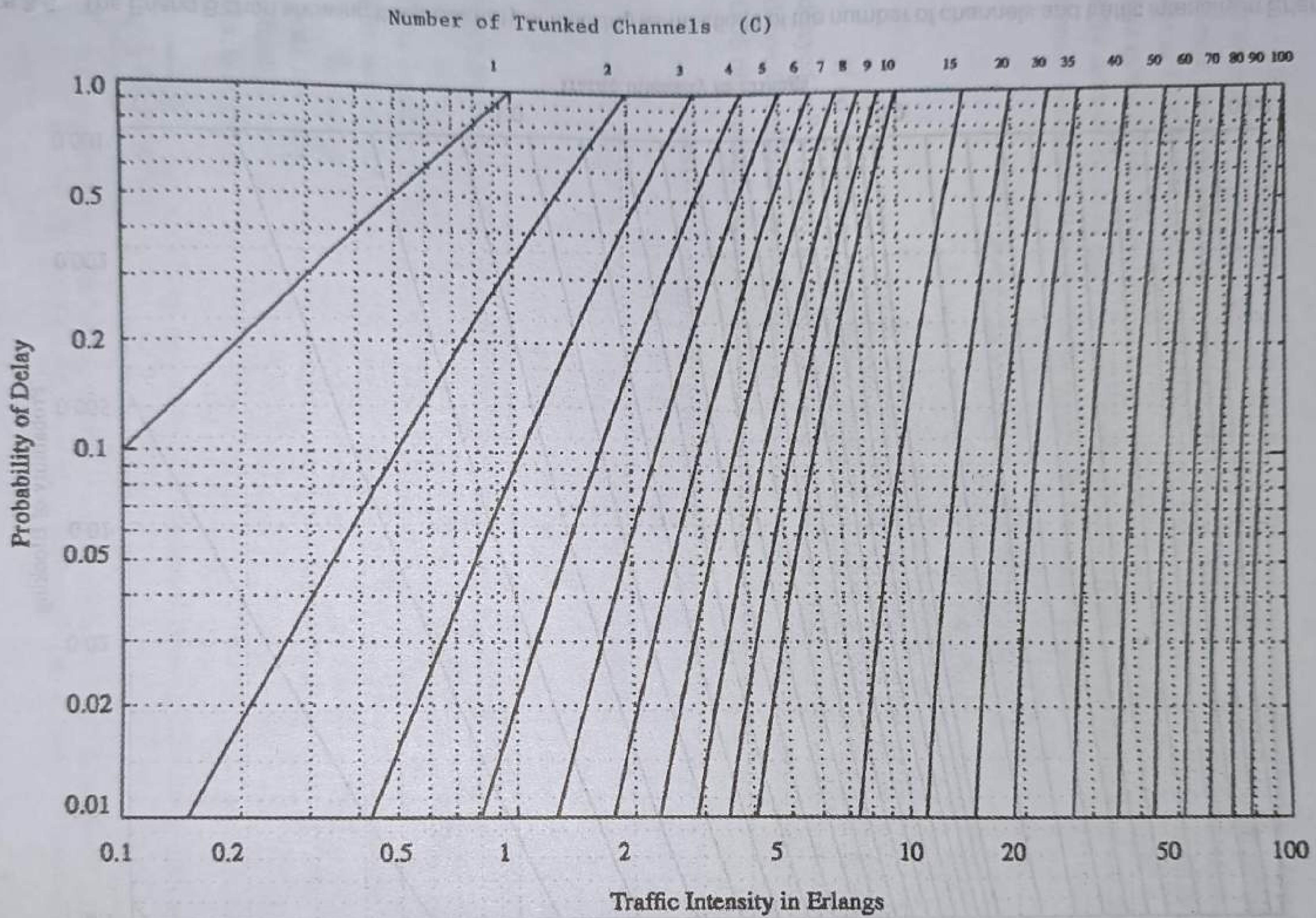

Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

(a) Given $C = 1$, $A_u = 0.1$, $GOS = 0.005$
From Figure 3.6, we obtain $A = 0.005$.
Therefore, total number of users, $U = A/A_u = 0.005/0.1 = 0.05$ users.
But, actually one user could be supported on one channel. So, $U = 1$.

(b) Given $C = 5$, $A_u = 0.1$, $GOS = 0.005$
From Figure 3.6, we obtain $A = 1.13$.
Therefore, total number of users, $U = A/A_u = 1.13/0.1 \approx 11$ users.

(c) Given $C = 10$, $A_u = 0.1$, $GOS = 0.005$
From Figure 3.6, we obtain $A = 3.96$.
Therefore, total number of users, $U = A/A_u = 3.96/0.1 \approx 39$ users.

(d) Given $C = 20$, $A_u = 0.1$, $GOS = 0.005$
From Figure 3.6, we obtain $A = 11.10$.
Therefore, total number of users, $U = A/A_u = 11.1/0.1 \approx 110$ users.

(e) Given $C = 100$, $A_u = 0.1$, $GOS = 0.005$,
From Figure 3.6, we obtain $A = 80.9$.
Therefore, total number of users, $U = A/A_u = 80.9/0.1 = 809$ users.

Example 3.5

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution

System A

Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 19$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.
Therefore, the number of users that can be supported per cell is
$U = A/A_u = 12/0.1 = 120$
Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$

System B

Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 57$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 45/0.1 = 450$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44,100$

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 88/0.1 = 880$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43,120$

Therefore, total number of cellular subscribers that can be supported by these three systems are $47,280 + 44,100 + 43,120 = 134,500$ users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

$47,280/2,000,000 = 2.36\%$

Similarly, market penetration of System B is equal to

$44,100/2,000,000 = 2.205\%$

and the market penetration of System C is equal to

$43,120/2,000,000 = 2.156\%$

The market penetration of the three systems combined is equal to

$134,500/2,000,000 = 6.725\%$

Example 3.6

A city with a coverage area of 1500 sq km is covered with a 12-cell system each with a radius of 1.387 km. If the total spectrum allocated is 28.5 MHz with a full duplex channel bandwidth of 25 MHz. Assume a GOS of 0.02 for an Erlang B system is specified and the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

Solution

(a) Given:

Total city coverage area = 500 sq km and cell radius, $R = 1.387$ miles

The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers $2.5981(1.387)^2 = 5$ sq km.

Hence, the total number of cells are $N_c = 500/5 = 100$ cells.

(b) The total number of channels per cell (C)

= allocated spectrum/(channel width × frequency reuse factor)

= 28500/(25 × 12) = 95 channels/cell

(c) Given:

$C = 95$ and $GOS = 0.02$

From the Erlang B chart, we have

traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell

= 100 × 84 = 8400 Erlangs.

(e) Given traffic per user = 0.03 Erlangs

Total number of users = Total traffic/traffic per user

= 8400/0.03 = 2,80,000 users.

(f) Number of mobiles per channel = number of users/number of channels

= 280000/1140 = 245 mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)

$= C \times N_C = 95 \times 100 = 9500$ users, which is 3.4% of the customer base.

Example 3.7

A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(b) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

Solution

Given:

Cell radius, $R = 1.387$ km

Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km

Number of cells per cluster = 4

Total number of channels = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.

In mobile radio systems, it is not uncommon to find that P_r may change by many orders of magnitude over a typical coverage area of several square kilometers. Because of the large dynamic range of received power levels, often dBm or dBW units are used to express received power levels. Equation (4.8) may be expressed in units of dBm or dBW by simply taking the logarithm of both sides and multiplying by 10. For example, if P_r is in units of dBm, the received power is given by

$$P_r(d)\ \text{dBm} = 10\log\left[\frac{P_r(d_0)}{0.001\ \text{W}}\right] + 20\log\left(\frac{d_0}{d}\right) \qquad d \geq d_0 \geq d_f \tag{4.9}$$

where $P_r(d_0)$ is in units of watts.

The reference distance d_0 for practical systems using low-gain antennas in the 1–2 GHz region is typically chosen to be 1 m in indoor environments and 100 m or 1 km in outdoor environments, so that the numerator in Equations (4.8) and (4.9) is a multiple of 10. This makes path loss computations easy in dB units.

10 b.t

Example 4.1

Find the Fraunhofer distance for an antenna with maximum dimension of 1 m and operating frequency of 900 MHz. If antennas have unity gain, calculate the path loss.

Solution

Operating frequency, $f = 900$ MHz

$\lambda = c/f = 3 \times 10^8$ m/s$/900 \times 10^6$ Hz $= 0.33$ m

Fraunhofer distance, $d_f = 2D^2 / \lambda = 2(1)^2 / 0.33 = 6$ m

Path loss $P_L(\text{dB}) = -10 \log [(\lambda^2)/(4\pi)^2 d^2] = -10 \log [(0.33)^2 / (4 \times 3.14)^2 \times 36] = 47\text{dB}$

9 b.t

Example 4.2

If a transmitter produces 50 W of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 W is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is P_r(10 km)? Assume unity gain for the receiver antenna.

Solution

Given:

Transmitter power, $P_t = 50$ W

Carrier frequency, $f_c = 900$ MHz

Using Equation (4.9),

(a) Transmitter power,

$$P_t(\text{dBm}) = 10\log[P_t(\text{mW})/(1\text{ mW})]$$
$$= 10\log[50 \times 10^3] = 47.0\text{ dBm}.$$

(b) Transmitter power,

$$P_t(\text{dBW}) = 10\log[P_t(\text{W})/(1\text{ W})]$$
$$= 10\log[50] = 17.0\text{ dBW}.$$

The received power can be determined using Equation (4.1)

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} = \frac{50(1)(1)(1/3)^2}{(4\pi)^2(100)^2(1)} = (3.5 \times 10^{-6})\text{ W} = 3.5 \times 10^{-3}\text{ mW}$$

$$P_r(\text{dBm}) = 10\log P_r(\text{mW}) = 10\log(3.5 \times 10^{-3}\text{ mW}) = -24.5\text{ dBm}.$$

The received power at 10 km can be expressed in terms of dBm using Equation (4.9), where $d_0 = 100$ m and $d = 10$ km

$$P_r(10\text{ km}) = P_r(100) + 20\log\left[\frac{100}{10000}\right] = -24.5\text{ dBm} - 40\text{ dB}$$
$$= -64.5\text{ dBm}.$$

4.3 Relating Power to Electric Field

The free space path loss model of Section 4.2 is readily derived from first principles. It can proven that any radiating structure produces electric and magnetic fields [Gri87], [Kra50]. Co sider a small linear radiator of length L, that is placed coincident with the z-axis and has its ce ter at the origin, as shown in Figure 4.2.

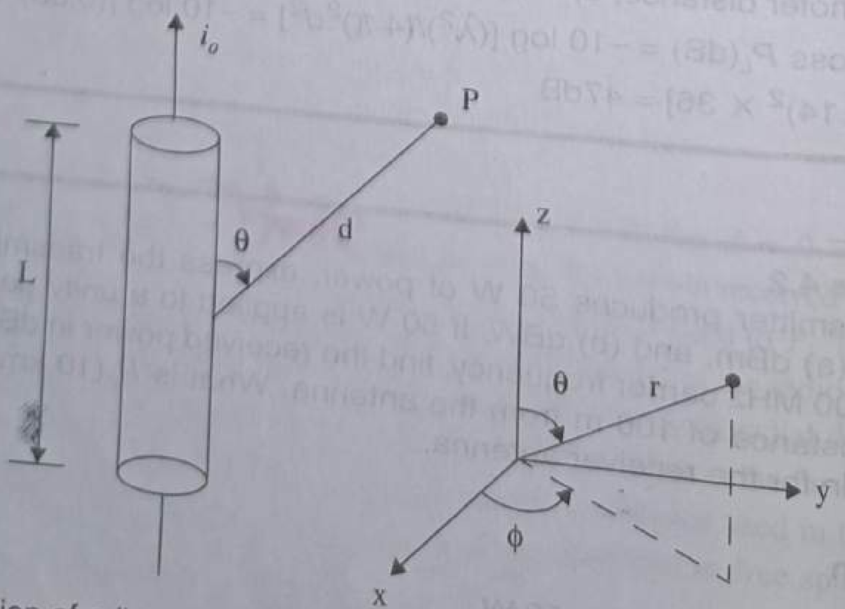

Figure 4.2 Illustration of a linear radiator of length $L(L \ll \lambda)$, carrying a current of amplitude i_o and making an angle θ with a point, at distance d.

Therefore, number of users = total traffic intensity/traffic per user
$= 9.0/0.029 = 310$ users
$= 310$ users/5 sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time
$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.
The probability that a delayed call will have to wait longer than 10 s is
$$Pr[delay > t|delay] = \exp(-(C-A)t/H)$$
$$= \exp(-(15-9.0)10/104.4) = 56.29\%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$
Probability that a call is delayed more than 10 seconds,
$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t|delay]$$
$$= 0.05 \times 0.5629 = 2.81\%$$

Trunking efficiency is a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels. The way in which channels are grouped can substantially alter the number of users handled by a trunked system. For example, from Table 3.4, 10 trunked channels at a GOS of 0.01 can support 4.46 Erlangs of traffic, whereas two groups of five trunked channels can support 2 × 1.36 Erlangs, or 2.72 Erlangs of traffic. Clearly, 10 channels trunked together support 60% more traffic at a specific GOS than do two five channel trunks! It should be clear that the allocation of channels in a trunked radio system has a major impact on overall system capacity.

3.7 Improving Coverage and Capacity in Cellular Systems

As the demand for wireless service increases, the number of channels assigned to a cell eventually becomes insufficient to support the required number of users. At this point, cellular design techniques are needed to provide more channels per unit coverage area. Techniques such as *cell splitting, sectoring*, and *coverage zone approaches* are used in practice to expand the capacity of cellular systems. Cell splitting allows an orderly growth of the cellular system. Sectoring uses directional antennas to further control the interference and frequency reuse of channels. The *zone microcell* concept distributes the coverage of a cell and extends the cell boundary to hard-to-reach places. While cell splitting increases the number of base stations in order to increase capacity, sectoring and zone microcells rely on base station antenna placements to improve capacity by reducing co-channel interference. Cell splitting and zone microcell techniques do not suffer the trunking inefficiencies experienced by sectored cells, and enable the base station to oversee all handoff chores related to the microcells, thus reducing the computational load at the MSC. These three popular capacity improvement techniques will be explained in detail.

3.7.1 Cell Splitting

Cell splitting is the process of subdividing a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitter power. Cell splitting increases the capacity of a cellular system since it increases the number of times the power and path loss become independent of frequency. The path loss for the two-ray model (with antenna gains) can be expressed in dB as

$$PL(\text{dB}) = 40\log d - (10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r) \tag{4.53}$$

At small T–R separation distances, Equation (4.39) must be used to compute the total E-field. When Equation (4.42) is evaluated for $\theta_\Delta = \pi$, then $d = (4h_t h_r)/\lambda$ is where the ground appears in the first *Fresnel zone* between the transmitter and receiver (Fresnel zones are treated in Section 4.7.1). The first Fresnel zone distance is a useful parameter in microcell path loss models [Feu94].

Example 4.6

A mobile is located 5 km away from a base station and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E-field at 1 km from the transmitter is measured to be 10^{-3} V/m. The carrier frequency used for this system is 900 MHz.

(a) Find the length and the effective aperture of the receiving antenna.

(b) Find the received power at the mobile using the two-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5 m above ground.

Solution

Given:

T–R separation distance = 5 km

E-field at a distance of 1 km = 10^{-3} V/m

Frequency of operation, $f = 900$ MHz

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{900\times 10^6} = 0.333\,\text{m}.$$

(a) Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833$ m = 8.33 cm.

Effective aperture of $\lambda/4$ monopole antenna can be obtained using Equation (4.2).

Effective aperture of antenna = 0.016 m^2.

(b) Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0}{d}\frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2}\ \text{V/m}$$

$$= \frac{2\times 10^{-3}\times 1\times 10^3}{5\times 10^3}\left[\frac{2\pi(50)(1.5)}{0.333(5\times 10^3)}\right]$$

$$= 113.1\times 10^{-6}\ \text{V/m}.$$

The received power at a distance d can be obtained using Equation (4.15)

$$P_r(d) = \frac{(113.1\times 10^{-6})^2}{377}\left[\frac{1.8(0.333)^2}{4\pi}\right]$$

$$P_r(d = 5\ \text{km}) = 5.4\times 10^{-13}\ \text{W} = -122.68\ \text{dBW or } -92.68\ \text{dBm}.$$
