AMATH101 — Real Analysis
Quiz: Series & Continuity & Differentiability

M6 & M11

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Series (13 Questions)

(1) Let $u_n = 1 + \frac{1}{n^2}$. Then $\sum u_n$ converges.

☐ True ☐ False

(2) Let $u_n = \sin(n+1) - \sin(n)$. Then $\sum u_n$ diverges.

☐ True ☐ False

(3) Let $u_n = \frac{(-1)^n}{\sqrt{n}}$. Then $\sum u_n$ converges conditionally.

☐ True ☐ False

(4) Let $u_n = \frac{2^n}{n!}$. Then $\sum u_n$ diverges.

☐ True ☐ False

(5) Let $u_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$. Then $\sum u_n$ diverges.

☐ True ☐ False

(6) Let $u_n = \sqrt[n]{\frac{n+1}{3n+10}}$. Then $\sum u_n$ diverges.

☐ True ☐ False

(7) Let (u_n) be a sequence such that $\sum u_n$ converges. Then $\sum u_n^2$ converges.

☐ True ☐ False

(8) Let (u_n) and (v_n) be sequences with $u_n < v_n$ for all n. If $\sum v_n$ converges, then $\sum u_n$ converges.

☐ True ☐ False

(9) Let $u_n = \frac{1}{n^2 \ln(n)}$. Then $\sum u_n$ diverges.

☐ True ☐ False

(10) If $\sum u_n$ converges, then $\sum |u_n|$ converges.

☐ True ☐ False

(11) Let (u_n) and (v_n) be sequences such that $\sum u_n$ diverges and $\sum v_n$ diverges. Then $\sum(u_n + v_n)$ diverges.

☐ True ☐ False

(12) If $\sum_{n=1}^{\infty} a_n$ converges only conditionally, then $\sum_{n=1}^{\infty} a_n^2$ diverges.

☐ True ☐ False

(13) Let (u_n) be a sequence such that $\sum u_n$ converges. Then $\lim_{n\to\infty} \cos(n) \cdot u_n = 0$.

☐ True ☐ False

Continuity (12 Questions)

(1) Let $f(x) = \frac{\sin x}{x}$ Then f is continuous at $x = 0$.

☐ True ☐ False

(2) Let $X = \left\{ \frac{n^3+1}{n+1} \mid n \in \mathbb{N} \right\}$. Then X is bounded.

☐ True ☐ False

(3) $\sqrt{2} \in \overline{\mathbb{Q}}$ (the closure of $\mathbb{Q}$).

☐ True ☐ False

(4) Let $X = \mathbb{Z}$. Then X is compact (closed and bounded).

☐ True ☐ False

(5) $\lim\limits_{x\to+\infty} x^3 \cos(x) = +\infty$.

☐ True ☐ False

(6) Let $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Then f is discontinuous at $x = 0$.

☐ True ☐ False

(7) Let $a, b \in \mathbb{R}$ with $a < b$, and let f be continuous on $[a, b]$. Then f is bounded on $[a, b]$.

☐ True ☐ False

(8) Let $f : [a, b] \to \mathbb{R}$ be continuous with $f(a) \cdot f(b) < 0$. Then there exists $c \in (a, b)$ such that $f(c) = 0$.

☐ True ☐ False

(9) Let $f : [0, 1] \to [0, 1]$ be continuous. Then the equation $f(x) = x$ has at least two solutions in $[0, 1]$.

☐ True ☐ False

(10) Let f be continuous on (a, b). Then f is bounded and attains its bounds on (a, b).

☐ True ☐ False

(11) Let f be uniformly continuous on $(0, 1]$. Then f is continuous on $(0, 1]$.

☐ True ☐ False

(12) The function $f(x) = x^4 + 1$ is uniformly continuous on $\mathbb{R}$.

☐ True ☐ False

Differentiability (Questions 1-8)

(1) The function $f(x) = x|x|$ is not differentiable at $x = 0$.

☐ True ☐ False

(2) The function $f(x) = |x^6|$ is differentiable at $x = 0$.

☐ True ☐ False

(3) If a function f is continuous at $x = 0$, then f is differentiable at $x = 0$.

☐ True ☐ False

(4) Let I be an interval of $\mathbb{R}$, let $f : I \to \mathbb{R}$ be differentiable, and let $x_0 \in I$ such that $f'(x_0) = 0$. Then f admits a local extremum at x_0.

☐ True ☐ False

(5) Let I be an interval and $f : I \to \mathbb{R}$ be differentiable. If f has a local extremum at $x_0 \in I$, then $f'(x_0) = 0$.

☐ True ☐ False

(6) Let I be an interval of $\mathbb{R}$ and $f : I \to \mathbb{R}$. If f is strictly increasing, then f' is strictly positive.

☐ True ☐ False

(7) Let $f : I \to \mathbb{R}$ be differentiable, where I is an interval. Let $x, y \in I$ with $x \neq y$ such that $f(x) - f(y) = y - x$. Then there exists $c \in I$ such that $f'(c) = -1$.

☐ True ☐ False

(8) Let f be differentiable on $[a, b]$. Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

☐ True ☐ False

Differentiability (Questions 9-15)

(8) Let f be differentiable on I with bounded derivative. Then f is uniformly continuous on I.

☐ True ☐ False

(9) Let $I \subset \mathbb{R}$ and f be differentiable with $f'(x) = 0$ for all $x \in I$. Then f is constant on I.

☐ True ☐ False

(10) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. If f' is even, then f is odd.

☐ True ☐ False

(11) Let $f, g : \mathbb{R}^+ \to \mathbb{R}$ be differentiable on $\mathbb{R}^+$, such that $f(x) \leq g(x)$ for all $x \in \mathbb{R}^+$ and $f(0) = g(0)$. Then $f'(0) \leq g'(0)$.

☐ True ☐ False

(12) Let $f(x) = \begin{cases} (x(x-1))^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Then f is differentiable at $x = 0$ and $x = 1$.

☐ True ☐ False

(13) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. If f is odd, then f' is even.

☐ True ☐ False

(14) The floor function $f(x) = \lfloor x \rfloor$ is differentiable at $x = 1$.

☐ True ☐ False

Series Score

___ / 13

Continuity Score

___ / 12

Differentiability Score

___ / 15

Total Score & Time

Total: ___ / 40

Time: 1h 20min (80 minutes)