Real Analysis — Quiz Sequences

Mohammed VI Polytechnic University
School of Applied Science and Engineering
Prof IBNJAA Said

Part 1: Theory & Exercises

Part 1

Question 1: Write definitions using ϵ

Write the ϵ-definition for the following assertions:

(1) (a_n) is a Cauchy sequence

(2) (a_n) converges to L

(3) (a_n) diverges

Question 2: Using the limit definition

Using the definition of limit, prove that:

(1) If a sequence admits a limit, then this limit is unique

(2) Every convergent sequence is Cauchy

(3) Every convergent sequence is bounded

Question 3: Examples and counterexamples

(1) Give an example of a bounded sequence that is not convergent, with justification

(2) Give an example of a divergent sequence that has two convergent subsequences (specify them)

(3) Give an example of a sequence (a_n) such that $\liminf_{n\to\infty} a_n = 0$ and $\limsup_{n\to\infty} a_n = 3$

Question 4: Theorems and applications

(1) State the Bolzano-Weierstrass theorem and give an example of its application

(2) Give an example of a sequence that is not Cauchy

(3) State and prove the Squeeze theorem

(4) Give two sequences, one strictly increasing and the other strictly decreasing, such that the limit of their difference is 0

Part 2: Multiple Choice Questions

Q1

Let $H_n = \sum_{k=1}^{n} \frac{1}{k}$ **and** $V_n = H_{3n} - H_n$. **Then:**

(A) $V_n \geq \frac{1}{3}$
(B) $V_n \geq \frac{2}{3}$
(C) $V_n \geq \frac{1}{2}$
(D) $V_n \geq \frac{1}{4}$

Q2

Let $H_n = \sum_{k=1}^{n} \frac{1}{k}$. **Then:**

(A) H_n is decreasing
(B) H_n converges
(C) H_n is Cauchy
(D) H_n is not Cauchy

Q3

Let (u_n) **be a sequence converging to** L **with** $L < 1$, **and let** $S_n = \frac{1}{n} \sum_{k=1}^{n} u_k$. **Then** $\lim_{n\to\infty} S_n$ **is:**

(A) 0
(B) 1
(C) $+\infty$
(D) L

Q4

Let (u_n) **be a sequence converging to** L **with** $L < 1$ **and** $\frac{1}{n} \sum_{k=1}^{n} u_k = L^2$. **Then** $L =$**:**

(A) 1
(B) 0
(C) $\frac{1}{2}$
(D) 4

Q5

Let $x_n = x^n \cdot n$ **with** $0 < x < 1$. **Then** $\lim_{n\to\infty} x_n =$**:**

(A) 1
(B) 0
(C) $+\infty$
(D) 2

Q6

A sequence with values in $\mathbb{Z}$ **that converges is necessarily:**

(A) Eventually constant
(B) Increasing
(C) Decreasing
(D) Has limit 0

Q7

Let $a_n = (-1)^n$. **Then:**

(A) (a_n) converges
(B) $\limsup a_n = 1$
(C) $\liminf a_n = 0$
(D) (a_n) is increasing

Q8

Let $a_n = n$. **Then:**

(A) (a_n) converges
(B) (a_n) is decreasing
(C) $\limsup a_n = +\infty$
(D) $\limsup a_n = 1$

Q9

A monotone increasing sequence bounded above is:

(A) Eventually constant
(B) Convergent
(C) Divergent
(D) Bounded below

Q10

A bounded sequence on $\mathbb{R}$**:**

(A) Is necessarily convergent
(B) Has a limit
(C) Has a divergent subsequence
(D) Has a convergent subsequence