Goal-oriented Updating Technique Applied to Building Thermal model

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Abstract

The present paper introduces a goal-oriented approach for parameter calibration of thermal building models. In a context of reducing the global energy consumption and greenhouse gas emissions, the goal-oriented method may be used for a robust prediction of a quantity of interest. Contrary to standard inverse methods, we do not aim at identifying all the model parameters in view of simulating the global thermal behavior of the building. Only the model parameters involved in the computation of the quantity of interest are updated. The proposed inverse strategy can lead to low computational time and reduced instrumentation. To validate the method, a first application on a steady state heat transfer problem is presented.

Introduction

In a context of reducing the global energy consumption, the building sector is the first lever to act on. In this sector a special attention has to be put on the existing buildings that constitute a significant potential in energy savings. Hence, appropriate renovation works have to be undertaken. The principal difficulty with the existing buildings is to properly know their constitutive materials and thermal properties in order to set up the energy performance diagnosis. To achieve this purpose, in France conventional methods are widely used. They are based on statistical data on the weather conditions and the building properties. Nevertheless, their results may not be representative of the actual building energy consumption. A suitable solution can be the inverse modelling. Combining a physical model with in-situ sensor outputs, inverse modelling techniques enable the identification of the model parameters that are essential to compute the real building energy consumption. The major drawback is that inverse problems are generally ill-posed in the sense of Hadamard (HADAMARD (1923)). In fact, inverse problems can have more than one solution and are highly sensitive to the measurement error. To overcome the ill-posed features of inverse models, regularization methods are classicaly used such as Tikhonov method (TIKHONOV and ARSENIN (1977)) and the Constitutive Relation Error method (LADEVEZE and CHOUAKI (1999)).

Two categories of inverse methods can be distinguished: global and local approaches. In thermal building applications, global inverse methods such as (NASSIOPOULOS et al. (2014)) are mainly used. All the model parameters are sought to minimize the gap between the measurement data and the simulation. It leads to the prediction of the global thermal behavior of the building. One of the most commonly encountered issues using these global methods can be the lack of data. Indeed, the deployed instrumentation may not be sufficient to identify a large number of model parameters. Moreover, the resolution of the global inverse problem can be very costly. In case of low instrumentation, the calibration result may be inaccurate. When the simulation results are only exploited in part, local inverse methods should be preferred. It consists in selectively calibrating the model parameters in view of a robust prediction of a quantity of interest as illustrated in Figure 1.

In the present article, we deal with a local inverse method firstly applied to mechanical models in (CHAMOIN et al. (2014)). This goal-oriented inverse method aims to identify only the parameters involved in the computation of the selected quantity of interest. This restriction enables a more accurate prediction of the quantity of interest in a reduced computation time and using an optimized number of sensors.

The paper is organized as follows: first, we describe the case study and we introduce the goal-oriented technique for model parameter calibration. In the next section, we present the results of the calibration process. The last section is dedicated to conclusions and prospects.

Goal-oriented technique for model parameter calibration

Most of the time it is not interesting to compute the entire physical behavior of the studied building by solving an inverse problem. Hence, a goal-oriented method for model parameter identification was firstly introduced in (CHAMOIN et al. (2014)). Depending on the objective of the user, one or more physical quantities corresponding to a partial reponse of the structure are sought. These quantities are called "quantities of interest". An example of a quantity of interest is shown in Figure 1. In this case, while traditional inverse techniques such as the regularization

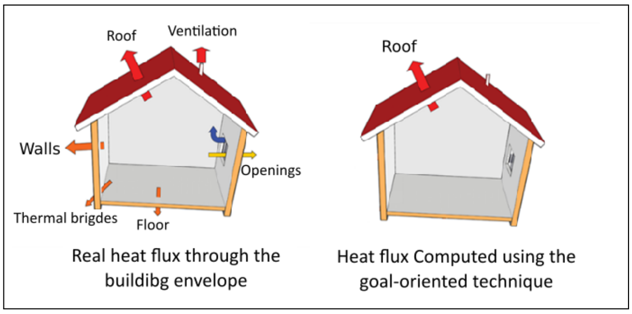

Figure 1: Prediction of all heat fluxes by standard inverse methods (left), focus on the prediction of heat flux through the roof by goal-oriented inverse method (right)

methods (TIKHONOV and ARSENIN (1977)) and constitutive relation error strategies (LADEVEZE and CHOUAKI (1999)) attempt to predict the entire building thermal behavior (*i.e.*, all the heat fluxes) by calibrating all the model parameters, the quantity of interest based method reproduces only the heat flux through the building roof and updates only the parameters involved in its computation.

One-dimensional steady state heat transfer problem

The studied problem (Figure 2) streams from a monozone model that describes the thermal behavior of a chalet, part of the "Sense-City" equipment (DERKX et al., 2012) (Figure 3). For the sake of simplicity, the problem is restricted to the study of the envelope subjected to a constant heat flux q_0 on its internal surface and a convective heat exchange on its external surface. As a first step, only the steady state problem is investigated. The one-dimensional heat transfer equations through the building envelope are given by:

$$\begin{cases} \frac{\partial q(x)}{\partial x} = 0, \ x \in]0, L_e[\\ q(x) = -k\frac{\partial \mathbf{T}(x)}{\partial x}, \ x \in]0, L_e[\\ q(x=0) = q_0 \\ q(x=L_e) = h(T(x=L_e) - T_{out}) \end{cases} \tag{1}$$

where q_0 is the prescribed heat flux on the inside envelope surface, k denotes the global thermal conductivity of the wall and h is the convective heat transfer coefficient between the external wall surface and the outside air.

The parameters to be updated are k and h. After setting a spatial discretization, the problem (1) can be recast as a linear system in the following matrix form:

$$\mathbf{KT} = \mathbf{F} \tag{2}$$

where $\mathbf{K}$ is the matrix of the envelope thermal transfer coefficients (including k and h), $\mathbf{T}$ the sought temperature vector and $\mathbf{F}$ the thermal loading vector.

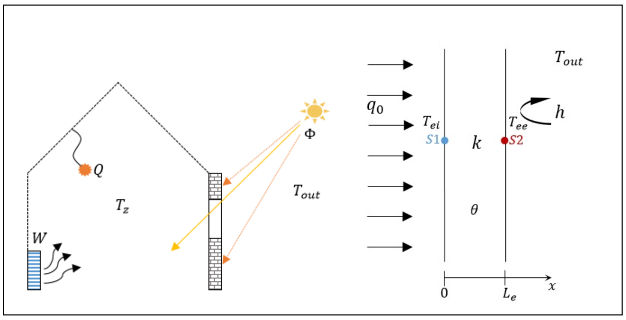

Figure 2: Monozone model for the studied chalet (left), simplified heat transfer through the building wall (right)

Figure 3: Chalets of the "Sense-City" equipment located at IFSTTAR Marne-la-Valle, France

Analytical solution

The advantage of studying a simple problem such as the steady state one-dimensional heat transfer problem is to enable the expression of its analytical solution that is useful for the validation of the numerical results.

One can show that the analytical solution of the problem (1) is

$$\begin{cases} q(x) = q_0, x \in [0, L_e] \\ T_{ee} = \frac{q_0}{h} + T_{out} \\ T_{ei} = \frac{q_0 L_e}{k} + \frac{q_0}{h} + T_{out} \end{cases} \tag{3}$$

where the temperature on the inner surface (resp. on the outer surface) of the wall is T_{ei} (resp. T_{ee}). Note that the temperature inside the envelope varies linearly from T_{ei} to T_{ee}.

Formulation of the goal-oriented technique

In this paragraph the goal-oriented technique for model parameter identification is formulated for the steady state one-dimensional heat transfer problem described above.

Unlike the traditional methods used for the resolution of inverse problems where the cost functional is defined to minimize the gap between the measured data and the simulated solution of the model, in the proposed method the functional minimizes the error on the quantity of interest (CHAMOIN et al. (2014)). In other words, it minimizes the gap between the quantity of interest Q_1 derived from the simulated solution of the model and the quantity of interest Q_2 derived from an extrapolation of measurement. This method acts as a coupling between sensitivity analysis and model updating.

Herein we choose as quantity of interest Q the gap between the temperature $T_{ee} = T(x = L_e)$ of the external surface of the envelope and the outside air temperature T_{out} (4).

$$Q = T_{ee} - T_{out} \tag{4}$$

The goal-oriented cost functional is then given by

$$\mathcal{J} = \frac{1}{2}\frac{\alpha}{\alpha+1}(Q_1 - Q_2)^2 \tag{5}$$

where α is a weighting coefficient determined by the discrepancy principal introduced in (MOROZOV (1966)). The quantity of interest Q_1 derived from the model and the quantity of interest Q_2 obtained from an extrapolation of the measurements are computed from:

$$\begin{cases} Q_1 = Q(\mathbf{T_1}) \\ \\ \mathbf{K}\mathbf{T_1} = \mathbf{F} \end{cases} \tag{6}$$

and

$$\begin{cases} Q_2 = Q(\mathbf{T_2}) \\ \\ (\mathbf{K} + \alpha s \boldsymbol{\beta}^T\boldsymbol{\beta})\mathbf{T_2} = \mathbf{F} + \alpha s \boldsymbol{\beta}^T \mathbf{T_{mes}} \end{cases} \tag{7}$$

where s is a scaling parameter chosen to ensure the physical homogeneity of the problem. In the present case, we take $s = q_0/T_{comf}$ where $T_{comf} = 20°C$ is the comfort temperature inside the building. The matrix $\boldsymbol{\beta}$ is defined to extract the measured temperatures. All the components of $\boldsymbol{\beta}$ vanish except those corresponding to the positions of the sensors. $\mathbf{T_{mes}}$ is the measured temperature vector. Equation (7) derives from the minimization of the modified constitutive relation error (LADEVEZE and CHOUAKI (1999)). The inverse problem can be expressed in (8) and solved by an iterative gradient based method that consists of seeking the parameter vector $\mathbf{P} = \{k, h\}$ that minimizes the cost functional $\mathcal{J}$ at each iteration.

$$\begin{cases} \underset{\mathbf{P}=\{k,h\}}{\mathcal{M}in}\ \mathcal{J}(\mathbf{T},\mathbf{P}) \\ \\ \mathcal{J}(\mathbf{T},\mathbf{P}) = \frac{1}{2}\frac{\alpha}{\alpha+1}(Q_1 - Q_2)^2 \end{cases} \tag{8}$$

To solve this constrained minimization problem, the most common approach is to rewrite it as an unconstrained problem by introducing the Lagrangian

$$\mathcal{L} = \mathcal{J} - \boldsymbol{\lambda}_1^T(\mathbf{K}\mathbf{T_1} - \mathbf{F})$$

$$- \boldsymbol{\lambda}_2^T[(\mathbf{K} + \alpha s \boldsymbol{\beta}^T\boldsymbol{\beta})\mathbf{T_2} - (\mathbf{F} + \alpha s \boldsymbol{\beta}^T \mathbf{T_{mes}})] \tag{9}$$

where $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are Lagrange multipliers.

On the one hand, expressing the stationarity of (9) according to the Lagrange multipliers leads to the equation of the direct heat transfer problem (second equation in (6)) and that of the heat transfer problem combining the numerical model and the measurements(second equation in (7)). These two equations respectively allow the computation of the temperature vectors $\mathbf{T_1}$ and $\mathbf{T_2}$. On the other hand, the stationarity of the Lagrangian according to $\mathbf{T_1}$ and $\mathbf{T_2}$ leads to the adjoint heat transfer problem (10) and one additional heat transfer problem (11), their resolution enables the computation of the Lagrange multipliers $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$.

$$\mathbf{K}\boldsymbol{\lambda}_1 = \frac{\alpha}{1+\alpha}(Q_1 - Q_2)\frac{\partial Q_1}{\partial \mathbf{T_1}} \tag{10}$$

$$(\mathbf{K} + \alpha s \boldsymbol{\beta}^T\boldsymbol{\beta})\boldsymbol{\lambda}_2 = -\frac{\alpha}{1+\alpha}(Q_1 - Q_2)\frac{\partial Q_2}{\partial \mathbf{T_2}} \tag{11}$$

The system of equations to be solved at each iteration of the calibration process can then be summarized in (12)

$$\begin{cases} \mathbf{K}\mathbf{T_1} = \mathbf{F} \\ \\ (\mathbf{K} + \alpha s \boldsymbol{\beta}^T\boldsymbol{\beta})\mathbf{T_2} = \mathbf{F} + \alpha s \boldsymbol{\beta}^T \mathbf{T_{mes}} \\ \\ \mathbf{K}\boldsymbol{\lambda}_1 = \frac{\alpha}{1+\alpha}(Q_1 - Q_2)\frac{\partial Q_1}{\partial \mathbf{T_1}} \\ \\ (\mathbf{K} + \alpha s \boldsymbol{\beta}^T\boldsymbol{\beta})\boldsymbol{\lambda}_2 - \frac{\alpha}{1+\alpha}(Q_1 - Q_2)\frac{\partial Q_2}{\partial \mathbf{T_2}} \end{cases} \tag{12}$$

Selection of the model parameters to be updated

In the widely used Tikhonov regularization method, all the model parameters are updated at each iteration. In the proposed method, only one parameter is updated at each iteration. This parameter is automatically selected based on the gradient of the goal-oriented functional. Only the parameter that leads to the highest value of the gradient is calibrated. The gradient of the functional with respect to the parameters is obtained by the derivation of the Lagrangian $\mathcal{L}$.

$$\frac{\partial \mathcal{J}}{\partial \mathbf{P}} = \frac{\partial \mathcal{L}}{\partial \mathbf{P}} = \frac{\alpha}{1+\alpha}(Q_1 - Q_2)\left(\frac{\partial Q_1}{\partial \mathbf{P}} - \frac{\partial Q_2}{\partial \mathbf{P}}\right)$$

$$-\boldsymbol{\lambda}_1^T \frac{\partial \mathbf{K}}{\partial \mathbf{P}}\mathbf{T_1} - \boldsymbol{\lambda}_2^T \frac{\partial \mathbf{K}}{\partial \mathbf{P}}\mathbf{T_2} + (\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2)^T \frac{\partial \mathbf{F}}{\partial \mathbf{P}} \quad (13)$$

Summary of the calibration process based on the goal-oriented technique

In practice, after the definition of a quantity of interest, the cost functional is written to minimize the error on this quantity of interest (5) and at each iteration, the calibration process is achieved according to the following steps:

1. Solve the standard thermal problem (first equation in (12)), this allows the computation of the temperature vector $\mathbf{T_1}$ that is used for the calculation of the quantity of interest $Q(\mathbf{T_1})$;
2. Solve the thermal problem (second equation in (12)) coupling measurements and numerical model. Its solution gives the temperature vector $\mathbf{T_2}$ used for the computation of the quantity of interest $Q(\mathbf{T_2})$;
3. Solve the adjoint thermal problem (third equation in (12)) to obtain the first Lagrange multiplier $\boldsymbol{\lambda}_1$;
4. Solve the additional thermal problem (fourth equation in (12)) for the calculation of the second Lagrange multiplier $\boldsymbol{\lambda}_2$;
5. Compute the gradient (13) of the cost functional with respect to each parameter of the model. This step allows the selection of the parameter P_i to be updated at the current iteration (14). It also gives the descent direction $\nabla \mathcal{J}_i = \partial \mathcal{J} / \partial P_i$ for the updating process.

$$P_{i_{new}} = P_{i_{old}} - \gamma \nabla \mathcal{J}_i \quad (14)$$

where $P_{i_{new}}$ is the updated value of the normalized parameter P_i, $P_{i_{old}}$ its old value and γ the descent step.

Numerical results

The proposed method is applied to the stationary one-dimensional heat transfer problem presented before. The method is tested on three instrumentation scenarios as mentioned hereafter an illustrated on Figure 2. The calibration results are compared with standard global inverse methods (TIKHONOV and ARSENIN (1977)) and (LADEVEZE and CHOUAKI (1999)).

- Scenario 1: only the sensor S_1 on the internal envelope surface is taken into account,
- Scenario 2: only the sensor S_2 on the external envelope surface is taken into account,
- Scenario 3: both S_1 and S_2 are considered.

Our objective is to identify the model parameters that enable the accurate computation of the sought quantity of interest Q defined in (4). We recall that the parameters to be calibrated are the global thermal conductivity k of the envelope and the convective heat transfer coefficient h between the external wall surface and the outside air temperature.

A set of exact parameters $\mathbf{P_{ex}} = \{k_{ex}, h_{ex}\}$ is used for the simulation of the measurements and another one $\mathbf{P_0} = \{k_0, h_0\}$ as the initial parameter vector. Herein, we take $k_0/k_{ex} = 0.7$ and $h_0/h_{ex} = 0.5$. The precision on the prediction of the quantity of interest Q and on calibration of the model parameters k and h is evaluated by the ratios Q/Q_{ex}, k/k_{ex} and h/h_{ex}. The closer the ratio is to 1, the higher the precision.

Case of idealized measurement data

As a first step, the inverse methods are studied on idealized sensor outputs, where no measurement noise is considered. The results for Scenario 1, Scenario 2 and Scenario 3 are respectively presented in Tables 1, 2 and 3.

Table 1: Results of the calibration process for Tikhonov, Constitutive Relation Error and Quantity of interest based methods - Scenario 1: only the sensor S_1 on the internal envelope surface is considered.

Method	k/k_{ex}	h/h_{ex}	Q/Q_{ex}
Tikhonov	1.11	0.52	1.92
CRE	1.12	0.5	2.00
QI	1.12	0.5	2.00

Table 2: Results of the calibration process for Tikhonov, Constitutive Relation Error and Quantity of interest based methods - Scenario 2: only the sensor S_2 on the external envelope surface is considered.

Method	k/k_{ex}	h/h_{ex}	Q/Q_{ex}
Tikhonov	0.7	1.00	1.00
CRE	0.7	1.00	1.00
QI	0.7	1.00	1.00

Table 3: Results of the calibration process for Tikhonov, Constitutive Relation Error and Quantity of interest based methods - Scenario 3: both sensors S_1 and S_2 are considered.

Method	k/k_{ex}	h/h_{ex}	Q/Q_{ex}
Tikhonov	1.01	0.88	1.14
CRE	1.00	1.00	1.00
QI	1.00	1.00	1.00

The results presented in Table 1 show that the measurement of the temperature at the internal surface

of the wall (Scenario 1) enables, with all the three methods, to adjust only the global thermal conductivity k of the envelope which is the most sensitive parameter to the available data in this case. It can be noticed that the Tikhonov regularization method slightly attempts to calibrate the parameter h, but the final result is still erroneous. This is because this method calibrates all the parameters at once. Lastly, we note that the quantity of interest Q is poorly computed by the three methods.
From the results given in Table 2, it appears that when the only available data is the measured temperature at the external surface of the wall (Scenario 2), the three methods identify properly the convective heat transfer parameter h. This result can be justified by the fact that the temperature T_{ee} on the external surface of the building envelope is governed only by the parameter h (see analytical solution (3)). Contrary to the previous Scenario, here the quantity of interest Q is well reproduced.
Finally, in Table 3, the results show that when the two temperatures on both the internal and the external wall surfaces are available, both the global thermal conductivity k and the heat exchange coefficient h are correctly identified by the Constitutive Relation Error based method and the quantity of interest based method, while an error of 12% is found for the identification of the parameter h by the Tikhonov method. Hence the quantity of interest Q is well reproduced by the Constitutive Relation Error based approach and the quantity of interest based approach, while an error of 14% is recorded in the case of the Tikhonov method.

Case of noisy measurements

To assess the performance of the quantity of interest based method with respect to the measurement noise, the method is studied on Scenarios 2 and 3 for three different measurement noise levels $\delta T = 0.1°C$, $\delta T = 0.5°C$ and $\delta T = 1°C$ that are added to the idealized data. The results of the calibration process obtained by the goal-oriented method are compared to those obtained by the Tikhonov regularization method and the Constitutive Relation Error based method.
When considering only the sensor on the external envelope surface (Scenario 2), Figure 4 shows that with all the three methods, only the convective heat exchange parameter h and the quantity of interest Q are updated and well identified. For each inverse method, we notice in this case a low sensitivity to measurement noise.
The calibration results of Scenario 3, where both sensors on the internal and the external envelope surfaces are taken into account, are given in Figure 5. It shows that the goal-oriented method enables to achieve a better accuracy on the identification of the parameters, particularly of the parameter h. This can be explained by the fact that this method is formulated

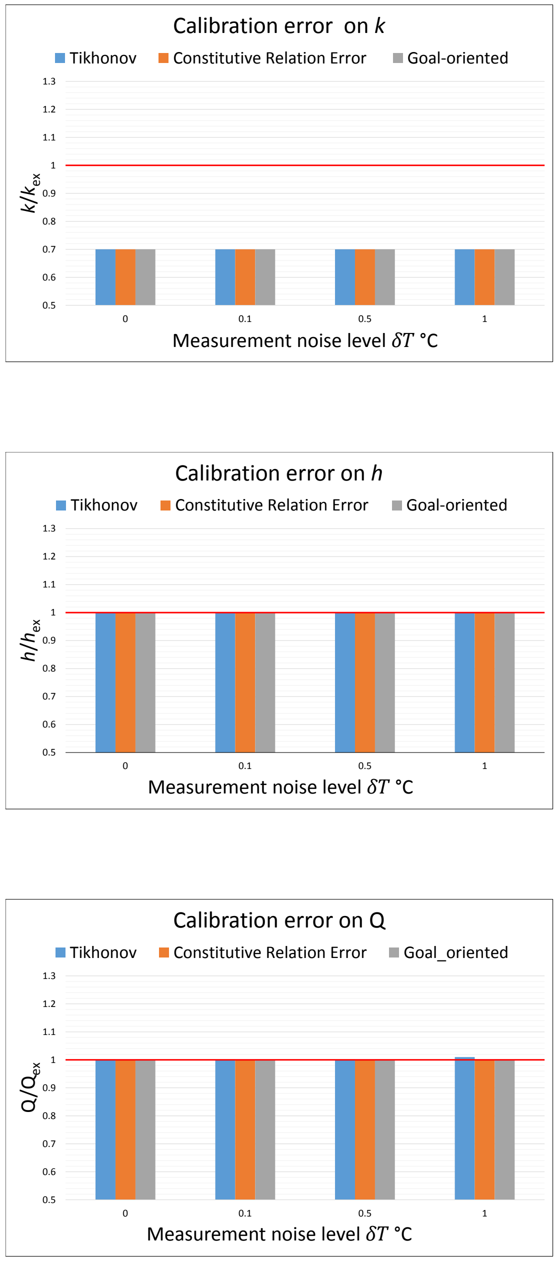

Figure 4: Calibration results for k (top), h (middle) and Q (bottom) for each measurement noise level and with the three methods - Scenario 2: only one sensor on the external surface of the wall is considered.

for the prediction of a quantity of interest and the sought quantity of interest herein is a function of the external wall temperature which depends only on the parameter h. The quantity of interest Q is more accurately computed with the goal-oriented method. It also appears from these results that the Tikhonov and the Constitutive Relation Error based regularization methods present a certain sensitivity to the measurement noise while the goal-oriented method seams to be less sensitive.

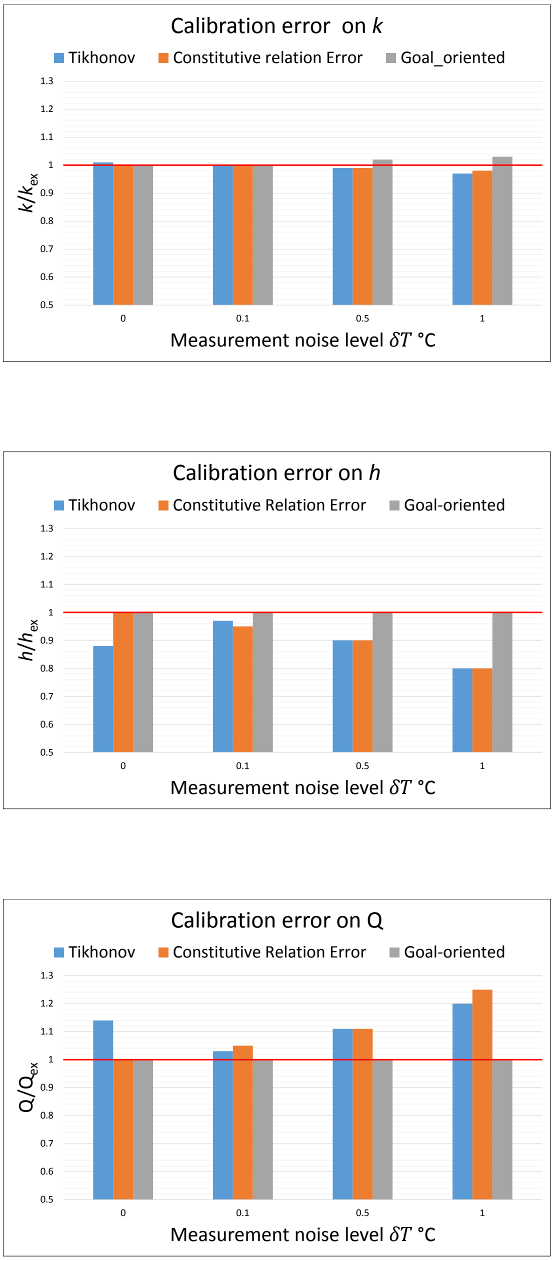

Figure 5: Calibration results for k (top), h (middle) and Q (bottom) for each measurement noise level and with the three methods - Scenario 3: two sensors on the internal and external surfaces of the wall are considered.

Concerning the computation time, a clear reduction is observed for the Constitutive Relation Error based and the goal-oriented methods comparing to the Tikhonov regularization method. For instance, in Scenario 3, while the Tikhonov regularization method required around one hundred resolutions of the direct problem to update the parameters and compute the quantity of interest, this number is divided by four with the Constitutive Relation Error and the goal-oriented methods. These results show a faster convergence of the Constitutive Relation Error and the goal-oriented inverse methods. Nevertheless, when considering the updating process results, we observe that the best accuracy is achieved by the goal-oriented inverse method.

Conclusion and prospects

We introduced a new inversion method which selects the thermal model parameters to be updated in view of accurately predict a quantity of interest. To assess the performance of the goal-oriented technique, the first step consisted on the study of a simple one-dimensional heat transfer problem in steady state. The results of the calibration process have shown that with the same amount of data, the goal-oriented method predicts the sought quantity of interest and the convective heat transfer coefficient more accurately than the traditional inverse methods. In the considered application, it also appeared from the results that the goal-oriented method shows a low sensitivity to the measurement noise compared to the usual techniques.
In future works, the goal-oriented technique will be extended to transient heat transfer problem and applied to real buildings in the “Sense-City” equipment. It will help to understand the method behavior on larger inverse problems and assess its ability to reduce the computation time and the number of sensors necessary for the estimation of the quantity of interest.

Acknowledgments

The authors acknowledge support from the project Sense-City financed by ANR (France) within the Investment for the Future Program under reference number ANR-10-EQPX-48.

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