Thermal and Economic Efficiency of Progressive Retrofit Strategies for School Buildings by a Statistical Analysis based Tool

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Abstract

Design alternatives in air conditioned buildings may be easily compared just by summing the hourly consumption of primary energy, while quantitative approachs for bioclimatic design strategies are difficult to be assessed and compared. A actively heated and passively cooled school building is considered as an application field of a novel methodology to promote an informed choice about the retrofit strategies to be adopted for buildings, defined as the Gained Comfort Cost (GCC). A functional and significant unit (i.e. a classroom), is used to test different energy retrofit solutions and their performances were compared with a baseline, in terms of the capacity to reduce the indoor air temperature variation. The novel methodology is a visual tool allowing to understand the "distance" of indoor conditions from comfort; the retrofit strategies are promoted to reduce this distance considering however the associated costs (LCC) to deal with actual feasibility.

Introduction

Dynamic measurements and simulations are crucial to define quantitative advantages of bioclimatic design strategies; nevertheless, they are complex and time consuming due to the amount of hourly data that are managed and finally the passive behaviour of a building is not effortlessly synthetized. A comparison of hourly consumption can be used for air-conditioned buildings, meanwhile buildings with no active thermal control in summer need more sophisticated statistical analyses to account for the thermal inertia effect (Di Perna et al. 2011). The existing school buildings' stock is the main field of application of the study inasmuch the National plan of renovation includes a 24 hours a day occupancy and consequently comfort conditions shall be maintained and energy consumption calculated throughout all the day and during the whole year.

Energy efficiency is a main driver of the Government actions (ENEA 2012), more than 62,000 building units compose the National School Buildings stock from which 35% are in need of maintenance and refurbishment to achieve the required levels of environmental well-being, health, attractiveness and cost-effectiveness, through the accurate design and renovation of schools' spaces (MIUR 2013). The strong correlation between users and built environment states comfort levels and can affect proficiency of students (Chatzidiakou, 2014). Moreover

health and safety of the indoor spaces is a main topic. Improving indoor conditions and space quality could upgrade the learning performance of students from an average 16% (BB90 2006, BB93 2014, BB101 2014) to a maximum of 50% if adequate Indoor Air Quality related to ventilation and Daylighting are considered (BB90 2006, BB93 2014, BB101 2014). Additionally to evaluate the energy use profile depending on the real building use is a main issue when extended time of use are promoted. The occupancy profiles and users' habits can help to predict the variability of the energy performance of the building (Tagliabue 2016). The occupants' awareness about energy use combined with low cost strategies has an estimated 20% effectiveness on energy reduction (ENEA 2012).

The evaluation of the users' behaviour is as well crucial to define the payback time of investment of the retrofit strategies regarding the building envelope and systems refurbishment, especially in case of total replacement or integration of thermal plants and without smart control devices.

Italian School buildings are mainly equipped with heating systems for winter, avoiding cooling systems for summer, however climate changes and the extended use of the buildings entail the need of mitigation measures for overheating in the middle and summer seasons. In this paper are mainly proposed refurbishment strategies referred to an adaptive comfort approach, considering the building envelope as a passive control system of the indoor conditions. Moreover, since 2009 (DPR 59/09) national regulations introduced dynamic thermal properties to be assessed for building envelope in order to reduce and effectively control the heat gains (Decreto Interministeriale 2015).

In any case, the bioclimatic approach encompasses evident advantages such as a lean and cost effective implementation in addition to its affordability.

The Italian school building stock scenario

The National school building stock counts over 62,000 schools which are for the 70% public buildings. The total annual energy consumption is estimated in about 1 million TOE (70% for heating and 30% for electricity). The specific heating and hot water consumption for public schools is about 180 kWh/m^2year whereas the requirement for new construction is less than 40 kWh/m^2year, according to current standards (Basarir 2012) and EU Directives (Directive 2010/31/EU 2010).

Only the 25% of Italian school buildings have been realized following the energy laws defined after the 70s, such as L. 373/76 and L.10/91. Thus the average stock is old and 31% of buildings dates from before 1900 up to 1960. This distribution does not change in the territory (Fig 1).

In order to fastly and cheaply reduce the energy consumption of the building stock a minimal intervention could consist in the lighting and thermal systems controls upgrade; this would be cost competitive in comparison with envelope improvements such as vertical and horizontal opaque surfaces insulation or enhancement of transparent surfaces performance (Citterio 2009). On the other hand the 40% of the school buildings need maintenance and energy saving retrofit measures focused on envelope and thermal plants should be beneficial. The average cost distribution depending on the retrofit strategy is presented in Fig. 2.
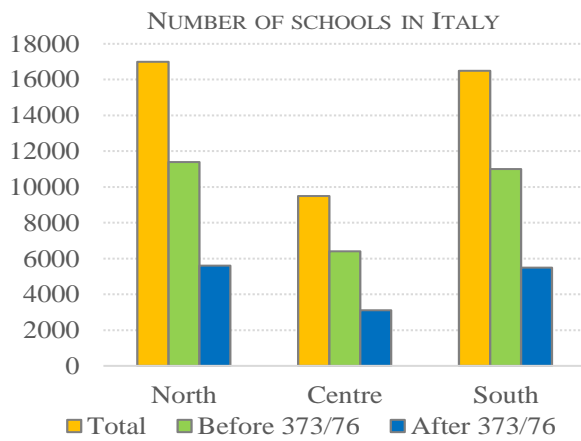

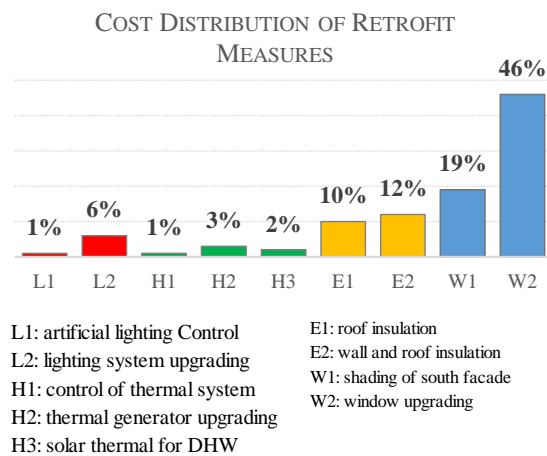*Figure 1: Distribution of schools and their age in the Italian territory (Energy Law 373/76).*

L1: artificial lighting Control
L2: lighting system upgrading
H1: control of thermal system
H2: thermal generator upgrading
H3: solar thermal for DHW

E1: roof insulation
E2: wall and roof insulation
W1: shading of south facade
W2: window upgrading

Figure 2: Cost distribution of energy retrofit measures for schools (Citterio 2009).

Average building schools construction conditions and related performances

The 70% of the national school buildings are realized with reinforced concrete frame structure, brick infill walls and they are equipped with gas boiler systems for heating (average efficiency $\eta \leq 0.9$). In any case, for buildings realized after the L. 373/76 was established, a thin insulation layer in the opaque envelope can be expected (Aste 2009).

Focusing only on the schools built from 1976 to 1990 the average and most frequently adopted envelope typologies in Italian School Buildings are presented in Table 1 to define the framework in which the envelope technologies and thermal properties of the simulation baseline scenario are limited. The main reported parameter are: U-factor is the Thermal Transmittance, Y_{ie} represents the Periodic Thermal Transmittance Value and SHGC the Solar Heat Gain Coefficient.

Table 1: Frequently adopted envelope typologies for the Italian school building stock.

Opaque Envelope component	U_{factor} [W/m^{2}K]	Y_{ie} [W/m^{2}K]
ROOF: Flat with reinforced brick-concrete slab, low insulation	1.01	0.19
WALL: Hollow brick masonry, low insulation (25 cm)	0.80	0.19
WALL: Hollow brick masonry, low insulation (40 cm)	0.76	0.06
FLOOR: with reinforced brick-concrete slab, low insulation	0.98	0.19
FLOOR: Concrete floor on soil, low insulation	1.24	0.11
Transparent Envelope component	U_{factor} [W/m^{2}K]	SHGC [W/m^{2}K]
Double glass, air filled, wood frame	2.8	0.75
Double glass, air filled, metal frame without thermal break	3.7	0.75

It is worthy to note that, in addition to thermal transmittance for both transparent and opaque envelope, and solar heat gains control strategies, a suitable level of thermal inertia is crucial to improve comfort conditions and energy savings in particular when adaptive thermal comfort models are assumed. Depending on the calculation methodology, the building type and use (Aste 2009, Karlsson 2013), the influence of the inertia in the thermal behaviour of a building can vary from 30 to 80%.

In old school buildings where the transparent/opaque envelope surface ratio is low, the effect of thermal inertia decreases while air change rate and permeable coverings interact more efficiently with time constant and energy saving (Di Perna 2011). Nevertheless, thresholds of suitable internal areal heat capacity related to periodic thermal transmittance (Y_{ie}) have also been defined for school buildings envelopes ranging between 50 kJ/m^{2}K for $Y_{ie} \leq 0.04$ to 70 kJ/m^{2}K for $0.04 \leq Y_{ie} \leq 0.08$ and 90 kJ/m^{2}K for $0.08 \leq Y_{ie} \leq 0.12$.

Methods

The methodology adopted in the present study focuses on the assessment of the thermal indoor conditions into a representative unit or classroom of a school building in

northern Italy, equipped with traditional envelope (Table 1) and compared to improved scenarios including refurbishment strategies.

Comfort analysis

The comparison parameter is the thermal zone temperature referred to comfort condition.

The indoor air temperature inside the thermal zone (T_{zone}) is calculated through dynamic simulation with the hourly climate data simulating different strategies (e.g. windows replacement, opaque envelope insulation, enhanced ventilation, addition of shading devices, etc.) with a progressive retrofit upgrading.

The evaluation is visualized in a cloud diagram in which the following quantities are co-related:

$$y = T_{zone} - T_{24} \tag{1}$$

where T_{24} is the moving average of the indoor air temperature inside the thermal zone, and:

$$x = T_{24} - T_{comfort} \tag{2}$$

where the comfort temperature $T_{comfort}$ in winter is fixed to the set-point room temperature equal to 20°C and considering the heating season going from 04/15 to 10/15 (Climate zone E) (DPR 412/93).

During the summer season the comfort temperature is evaluated in accordance with the adaptive comfort model (Ashrae 2013) and with T_{air} as the outdoor air temperature.

$$T_{comfort,summer} = 17.6 + 0.31 * T_{air} \tag{3}$$

The origin of the diagrams (point x=0 and y=0) has been considered, both for summer and winter, as an optimal comfort condition reference state in accordance with (1), (2) because of the coincidence between the zone's air temperature and comfort temperature, and for the small variance of the temperature during the day.

The data are then plotted in Mathematica (Wolfram 2016) for every x,y couple using a colour scale obtained from a hue colour function referring to the following condition:

$$hue\left(\frac{d}{dmax}, 1, 1\right) \tag{4}$$

where d is a measure of the hourly comfort conditions defined as:

$$d = |x| + |y| \tag{5}$$

Hue corresponds to a cylindrical transformation of RGB colour scale and is defined as:

$$hue(h, s, b) \tag{6}$$

Where h represent a specific colour in a hue colour palette defined by a real number in a domain [0,1], s is saturation and b brightness.

The novel visual tool is thus the diagram showing on the Y axis the number of hours when d is out of a range of adaptive comfort (d<2) divided by the total number of hours of each simulation, hereby defined as *discomfort frequency*. This value is related to the mean of the differences between T_{24} (1) and $T_{comfort}$ (2) for each cloud point and represented on the X axis.

The radius value represents the average measured d of the point cloud and its average dispersion.

In this way it was possible to compare strategies, defining the most effective strategy as the one with the smaller radius and the nearest centre coordinate to the graph origin. The origin is the comfort condition and ±2°C is considered the bounding adaptive comfort level.

LCC based techno-economic assessment

Although the potential of energy saving is high, insulating the opaque envelope and replacing transparent surfaces are the most expensive options, but are considered with the aim of simulating possible investment strategies and to consider the associated cost during the service life through the Life Cycle Costing (LCC).

The alternative scenarios derived by the retrofit strategies have been calculated according to the ISO 15686-5:2008, considering all the related costs: (A) construction; (B) operation; and (C) maintenance.

The three categories of costs contributing to the definition of each alternative LCC have been calculated as follows and without including VAT.

A) The construction cost is given by the sum of the costs of installation of each layer accounting only the costs related to the new layer installed for each retrofit solution. The bearing layer (hollow clay bricks) has not been considered, as it is equal in all the cases and it is not interested by maintenance operations. Costs have been gathered from the local price list 2016 (Comune di Milano 2016).

B) Operation costs have been calculated referred to the energy consumption value. The annual energy demand for heating has been multiplied by the cost of energy with the actualized value (discount rate 3%) to get the value over the analyzed period (75 years).

C) Maintenance costs have been estimated according to two different approaches: corrective and preventive. The former has no maintenance until replacement, while the latter includes two maintenance operations (i.e. light and heavy) and replacement. The preventive approach allows to increase the useful service life of a component, keeping a defined performance level (agreed with the client). Maintenance costs, occurring at year 5 (light operations), year 15 (heavy operations) and at the end of the service life (different for each component analyzed) are modified (discount rate 5%) to obtain the actualized maintenance costs for each alternative in the defined period of time (75 years). For each alternative, maintenance operations of all the finishing and insulation have been calculated; no maintenance is performed on the bearing layer (i.e. the brick wall).

The sum of the three categories of costs provided the LCC of each alternative. Costs related to end-of-life have not been considered as they are strongly affected by

uncertainty, with a low influence on the total LCC and with almost no variance among the options.

Gained Comfort Cost (GCC) variation

A Gained Comfort Cost (GCC) is finally calculated to include the main parameters of the presented analysis.
The Gained Comfort Cost (GCC) variation that is defined as follows:

$$GCC_i = \frac{LCC_1 - LCC_i}{d_1 - d_i} \qquad (7)$$

Where LCC_1 is the Life Cycle Cost and d_1 is the comfort measure of the base case.

In the discussion and conclusion section, a diagram is provided referred to the base case and the retrofit options in which different lines describe the whole comfort and techno economic performance of the solution. The slope of the lines derives from equation (7) that represent the GCC variation.

Case study

Definition

The case study building is a school building located in Milan (45° 28' N, 09°10' E) with three floors, where a standard classroom with 8.7 m length, 7.6 m depth and 3 m height, is located (Fig. 3). The baseline model is an existing primary school in northern Italy organized with a main corridor and two sides of classrooms (Fig. 3a).

The simulation test cell is a single classroom, south oriented with three identical windows (1.25 x 2.5 m) on the only wall facing outdoors (Fig. 3b).
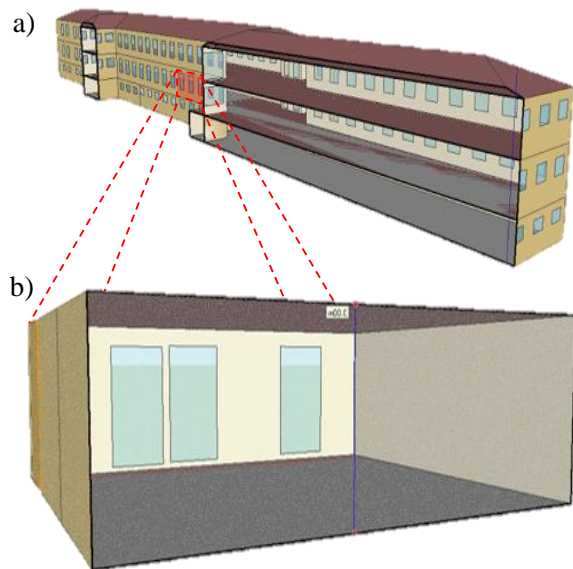

Figure 3: a) Model of an existing school; (b) single classroom space adopted as test cell for the simulation.

The main objective of the analysis is to assess the energy performance of a representative classroom by the use of new windows in accordance with the current performance requirement (Decreto Interministeriale 2015) and with different SHGC values. Alternatives with both transparent

and opaque envelope improvements are also presented to define the energy saving ratios, considering that renovation strategies are able to improve energy performance in winter period, but might worse it during summer.

An existing base case was defined from the values presented in Table 1. The envelope performance is described through an average thermal transmittance U_{av} value which refers to the area weighted average thermal transmittance of the whole building envelope, considering floors, roofs, walls and windows. Specifically the window thermal transmittance U_w refers to the wood framed window reported in Table 1.

Simulation alternatives

The base case has been compared with five improved alternatives with different energy retrofit strategies for enhancing energy performances and improving indoor thermal comfort (Table 2).

Table 2: Tested combinations of energy saving retrofit strategies.

n.	Evaluated test case	U_{av} W/m² K	U_w W/m² K	SHGC [-]
1	Base case	0.96	2.8	0.75
2	Improved U_w	0.96	1.0	0.50
3	Improved U_w and SHGC	0.96	1.0	0.35
4	U_{av} reduced	0.29	2.8	0.75
5	U_{av} and U_w reduced	0.29	1.0	0.50
6	Best case	0.29	1.0	0.35

The thermal properties of the opaque envelope used in the base case and in test case 5 and 6, after renovation, are summarized in Table 3, including the parameter M_s which refers to surface mass value.

Table 3: Opaque envelope components used as existing and renovated case.

Case	Thickness [m]	$U_{-factor}$ [W/m² K]	M_s [kg/m²]	Y_{ie} [W/m²K]
1	0.40	0.64	366.2	0.065
5; 6	0.49	0.26	368.0	0.012

Simulation set up

All the alternatives were modelled using Energy plus 8.2 (Crawley 2000).

Other parameters used for the dynamic simulations are listed in the Table 4 and mainly refer to the thermal zone settings. The classroom occupancy is derived from the MIUR guidelines (ENEA 2016) which assess the occupancy rate and the minimum required ventilation per student.

Table 4: Zone thermal settings.

Parameter	Value	Unit

Occupancy density index	0.5	person/m^2
Occupancy schedule weekdays	7:00-19:00	h
Internal gains	4	W/m^2
Ventilation rate	2.8	ac/h

Results

The proposed methodological approach eases the identification of the thermal and economic efficiency of the retrofit strategies to improve indoor comfort condition, energy savings and affordability in the long term which are key factors for a real application of retrofitting in the public sector.

Discomfort Frequency and Thermal comfort assessment

The performance of the six different cases (i.e. 1 base case and 5 retrofit strategies used to improve the indoor conditions) were evaluated considering a progressive retrofit, which means the progressive application of all the listed strategies.
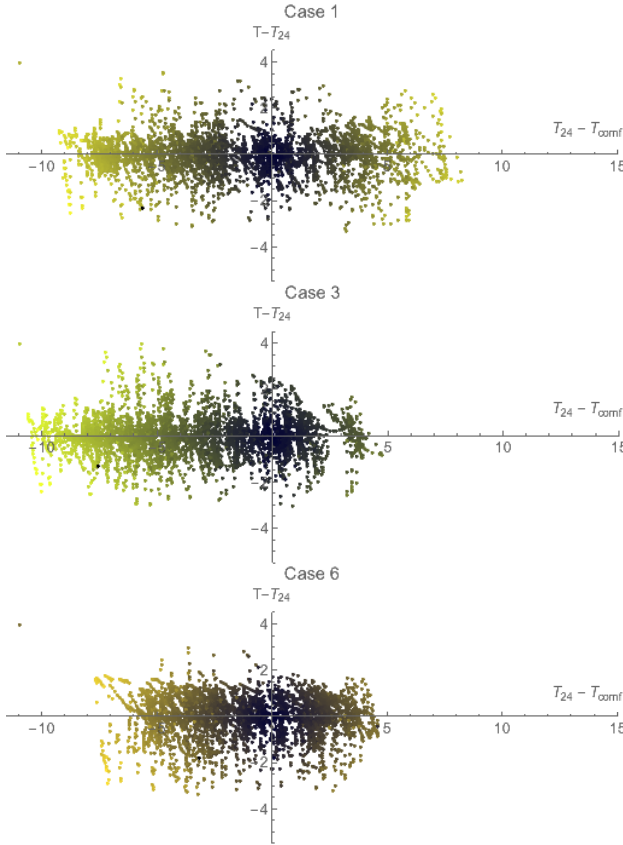

Figure 4: Winter Conditions: Temperature difference plot [(T_{zone}-T_{24})=(T_{24}-$T_{comfort}$)], energy saving retrofit strategies 1, 3, and 6.

The assessment is based on the hourly indoor air temperature as a comfort parameter under free-floating conditions. The diagrams representing the effectiveness of the most significant refurbishment alternatives, are plotted for both winter (Fig. 4) and summer conditions (Fig. 5). They show the results for the base case (Case 1), and for

the most significant cases such as for the replacement of the window with improved SHGC and U_w (Case 3), and for the best case (Case 6) in which both wall insulation and window replacement take place.

The colour hues of plot is in accordance with the measure of the hourly comfort conditions by the use of equations (4) and (5). Because of the nature of the colouring function, dark points represent the alternatives with a better comfort conditions.
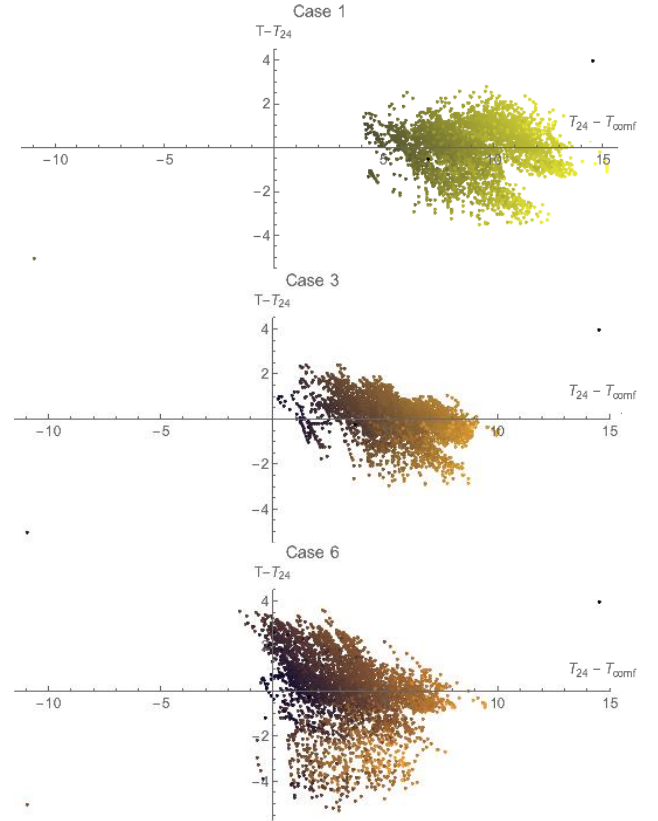

Figure 5: Summer Conditions: Temperature difference plot [(T_{zone}-T_{24})=(T_{24}-$T_{comfort}$)], energy saving retrofit strategies 1, 3, and 6.

The effects of the energy saving retrofit strategies are evaluated through the average dimension of the cloud points, i.e. comparing their statistical moments (Table 4).

The more the point distribution is centred and compact around the graph origin (i.e. with mean and skewness close to zero and a low variance), the more the proposed strategy avoid discomfort hours during seasons. The span between the cloud extremes along the Y axis, defines the thermal inertia efficacy and thus the responsiveness of the envelope in mitigating temperature fluctuation through the year.

Table 4: Statistical description of the clouds (summer cases)

Case	Mean	Variance	Skewness
1	9.17	3.73	0.05
2	5.55	2.94	-0.15
3	5.52	2.78	-0.15
4	7.66	2.69	-0.02

| 5 | 7.29 | 2.72 | -0.02 |
| 6 | 3.36 | 3.67 | 0.21 |

Taking as an example the use of a window component with a low SHGC value (Case 3), it shows the reduction of the overheated period during summer, as stated by the cloud points moving toward the graph origin (Fig. 5), but slightly increases the amount of under heated hours, as stated in Figure 4, with an increase of the dimension of the cloud along the negative part of the x axis. Improving window SHGC, especially using films, and increasing window to wall ratio, could be a lean alternative to improve the whole energy use of a building (Mainini 2015) in the Italian context even though the winter solar gains are reduced.

Comparing the results with the base case (Case 1) that is far from adaptive comfort optimal temperatures and presents over-heated condition, a general improvement of the comfort conditions is always granted with any of the proposed refurbishment strategies.

The effectiveness of every scenario can be synthetically introduced in an alternative way as presented in Figure 6 as an example only for summer conditions. Here a Synthetic plot of comfort conditions is provided: the diagram reports the discomfort frequency as a function of the mean (T_{24}-$T_{comfort}$) and the mean of measure d (radius).
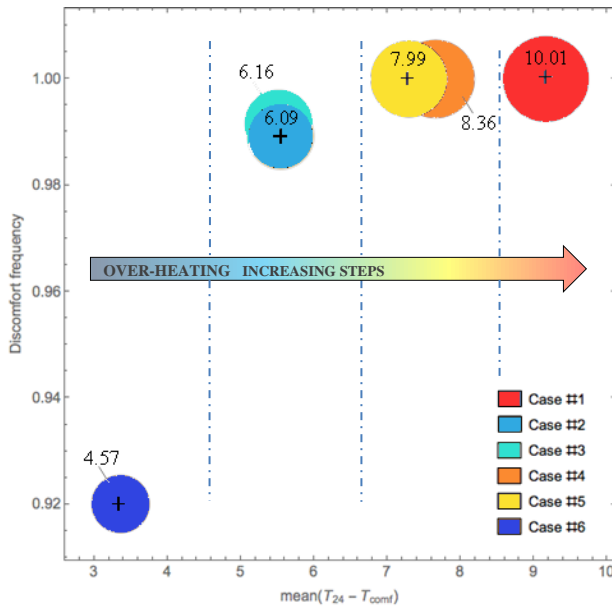

Figure 6: Summer Condition: discomfort frequency vs mean (T_{24}-$T_{comfort}$).

Cost Diagram

The LCC has been calculated for the 6 cases to show how the cost categories (i.e. construction, operation, maintenance) influence the total cost.

As it is shown in Fig. 7, most of the costs are associated to energy demand (96% to 93%), while construction/installation is around 1% and maintenance ranges from 3% to 5%.

Installation costs have a variance of 27%; preventive maintenance costs vary 9%; energy costs vary 41%. The total costs have a variance of 39%.

The installation cost is proportional to the maintenance cost, while operational energy costs are related to the performance of the component. The energy cost has a predominant role because installation and maintenance costs are strictly related to the envelope (opaque and transparent); in case of installation and maintenance costs of different components (e.g. finishing, floor, partitions, systems) were included, the ratio would change.
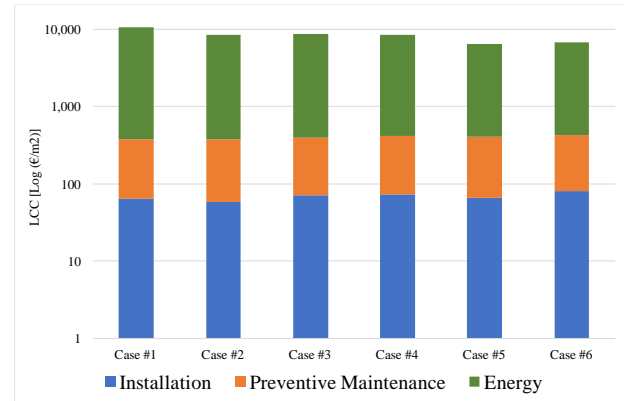

Figure 7: LCC of the retrofit options (values are in logarithmic scale).

The preventive maintenance strategy has been chosen, as more convenient than the corrective strategy.

Discussion and conclusions

The energy saving retrofit strategies and the variation in associated costs GCC are represented by the segments in the graph in Fig. 8 for winter (blue lines) and summer (yellow lines).
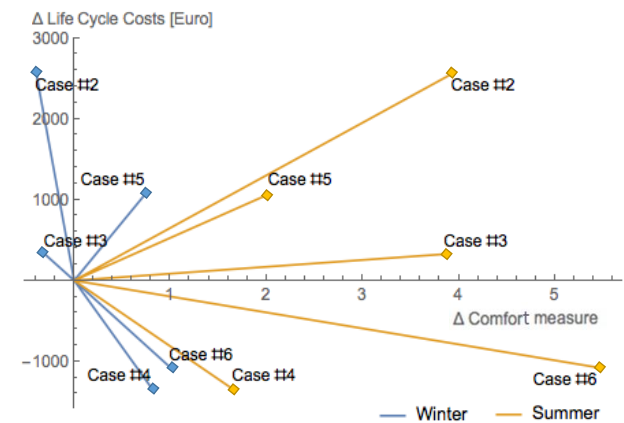

Figure 8: Gained Comfort Cost variation graph.

Fig. 8 shows basically the cost for achieving one unit of comfort. This is done comparing the LCC and the comfort (in winter and in summer) for each alternative with the base case. According to the LCC (Y axis), the best solutions are the positive ones, with a LCC lower than the base case. According to the comfort (X axis), the best solutions are the most distant from the origin; a negative value means that the alternative has a lower comfort than the base case. According to what was mentioned above,

case 6 (best case) is the most suitable in terms of comfort, while case 2 is the most suitable in terms of LCC.

Acknowledgements

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