جامعة بيروت العربية
BEIRUT ARAB UNIVERSITY

Date: 13-11-2022

Duration: 1hr 15min

Mark: 30

No. of pages including cover:

Mid-Term Exam

Semester: Fall 2022/2023

Faculty : Faculty of Science

Department : Math & Computer Science

Division/ program: Computer Science

Course Name : Discrete Structures II

Course Code: CMPS 345

Student's Name: Model Answer v1 **ID:**

Section/ Group: **Seat Number:**

INSTRUCTIONS:

1- Any kind of cheating will subject the student to the penalties specified by the University rules
2- Use of cell phone is strictly prohibited

Question	Mark	Out of
One		6
Two		6
Three		6
Four		6
Five		6
Six		
Seven		
Eight		
Nine		
Ten		
Total		

Total marks in letters

Examiner's Name: Dr May Itani

Signature:

Question 1.

Prove or disprove

$\forall n \in N$(n *maybe even or odd*), $n^2 + 3n + 2$ is even

Assume n even

n = 2K $\quad (2k)^2 + 3(2k) + 2$

$= 4k^2 + 6k + 2 = 2(\text{int}) \Rightarrow$ even. $\quad \overset{\text{integer}}{(2k^2+3k+1)}$

Assume n odd $\quad (2k+1)^2 + 3(2k+1) + 2$

$= 4k^2 + 4k + 1 + 6k + 3 + 2$ $\quad (3+3=6)$

$= 2(2k^2 + 2k + 3k + 3) = 2(\text{int}) \Rightarrow$ even.

$\therefore$ even.

$a_n = 3.2^{n-1} + 2.(-1)^n$ is a solution for the recurrence relation $a_n = a_{n-1} + 2.a_{n-2}$

$3 \times 2^{n-1} + 2(-1)^n \overset{?}{=} 3 \times 2^{n-2} + 2(-1)^{n-1} + 2\left[3 \times 2^{n-3} + 2(-1)^{n-2}\right]$

$3 \times 2^{n-1} + 2(-1)^n \overset{?}{=} 3 \times 2^{n-2} + 2 \times (-1)^{n-1} + 2 \times 3 \times 2^{n-3} + 2 \times 2 \times (-1)^{n-2}$

$\overset{?}{=} 2^{n-1}\left(3 \times 2^{-1} + 2 \times 3 \times 2^{-2}\right] + (-1)^n\left[2 \times (-1)^{-1} + 2 \times 2 \times (-1)^{-2}\right]$

$\overset{?}{=} 2^{n-1}\left[\frac{3}{2} + \frac{\cancel{2} \times 3}{\cancel{2} \times 2}\right] + (-1)^n\left[\frac{2}{-1} + \frac{4}{(-1)^2}\right]$

$3 \times 2^{n-1} + (-1)^n[-2 + 4]$

Proved ✓ $\quad = 3 \times 2^{n-1} \quad (-1)^n \times 2$

correct. $\therefore$ sol'n.

Question 2.

Use mathematical induction to prove that:

$$\sum_{i=1}^{n} i\,2^{i} = (n-1)2^{n+1} + 2$$

Basecase 2 pts

Base Case $P(1)$ $n=1$

$\sum_{i=1}^{1} i\,2^{i} \stackrel{?}{=} \cancel{(1-1)2^{1+1}} + 2$

$2^{1} = 2$ ✓

Induc 3pts

Inductive Case $P(k)$ true $\sum_{i=1}^{k} i\,2^{i} = (k-1)2^{k+1} + 2$

$P(k+1)$: $\sum_{i=1}^{k+1} i\,2^{i} \stackrel{?}{=} (k+1-1)2^{k+1+1} + 2 = \boxed{k\,2^{k+2} + 2}$

$P(k+1)$: $\sum_{i=1}^{k} i\,2^{i} + (k+1)2^{k+1} = (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$

$= 2^{k+1}[(k-1) + (k+1)] + 2 = 2^{k+1} 2k + 2 = \boxed{2^{k+2} \cdot k + 2}$

$\{2^{k+1} \times 2k\}$

1pt Base case + inductive case ∴ $P(n)$ true ∀n.

∴ $P(k+1)$ Proved LHS=RHS

Question 3 (You can use back of page for answer)

Let x be an integer. **Prove or disprove** whether the following statements are equivalent:

$3x - 2$ is odd (2)
$x + 8$ is odd (1)
$x^2 + 2$ is odd (3)

$x+8$ odd

$x + 8 = 2k+1 \Rightarrow x = 2k+1-8 = 2k-7$
$2(k-4)+1$
odd

x odd $\rightarrow$ $3x-2$ odd

$3(2k+1) - 2 = 6k+3-2 = 6k+1$ odd

$3x-2$ odd $\rightarrow$ x^2+2 odd

$3x-2 = 2k+1$

$3x-2$ odd $\rightarrow$ x^2+2 odd

$\neg$ $\neg$

x^2+2 even $\Rightarrow$ x^2 even x even

$3x-2$ even $3x-2 = 3(2k)-2$

∴ $2(\text{int})$ even.

$x^2+2 \rightarrow x+8$

x^2 ~~odd~~ odd x odd $x+8 = 2k+1+8 = 2(k+4)+1$

∴ odd

Proved

Question 4

Part A.
Give a recursive definition with initial condition for the sequence
$a_1 = 24, a_2 = 20, a_3 = 16, a_4 = 12, \ldots$

Your answer

Basis Step: $a_1 = 24$

Recursive Step:

$a_n = a_{n-1} - 4$

Part B.

Give a **recursive algorithm** for finding the **sum of the first n odd positive integers**

if n = 1 return 1

if = odd return

return (n-1 , sum + n.)

sum(n-1) + N

Question 5.

A 5-word is a word composed of five letters from the English alphabet. (Ignore case sensitivity of letters)

1. How many 5-words are there?

$$26^5$$

2. How many 5-words that contain at least one T are there?

$$26^5 - 25^5$$

3. How many 5-words that begin **or** end with an A are there?

$$26^4 + 26^4 - 26^3$$

جامعة بيروت العربية
BEIRUT ARAB UNIVERSITY

Date: 13-11-2022

Duration: 1hr 15min

Mark: 30

No. of pages including cover:

Mid-Term Exam

Semester: Fall 2022/2023

Faculty : Faculty of Science

Department : Math & Computer Science

Division/ program: Computer Science

Course Name : Discrete Structures II

Course Code: CMPS 345

Student's Name: Model Answer V2 **ID:** ......

Section/ Group: ...... **Seat Number:** ......

INSTRUCTIONS:

1- Any kind of cheating will subject the student to the penalties specified by the University rules

2- Use of cell phone is strictly prohibited

Question	Mark	Out of
One		**6**
Two		**6**
Three		**6**
Four		**6**
Five		**6**
Six		
Seven		
Eight		
Nine		
Ten		
Total		

Total marks in letters

Examiner's Name: Dr May Itani

Signature: ......

Question 1.

Prove or disprove

$\forall n \in N$ (n *maybe even or odd*), $n^2 + 3n + 1$ is odd

$a_n = 5.(-1)^n - n + 2$ is a solution for the recurrence relation $a_n = a_{n-1} + 2.a_{n-2} + 2n - 9$

Question 2.

Use mathematical induction to prove that:

$$\sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n}$$

2

Base Case $P(0)$ $n=0$

$\sum_{i=0}^{0} \frac{1}{2^i} = \frac{1}{2^0} = 1$ ✓ true.

$2 - \frac{1}{2^0} = 2-1 = 1$

Inductive Step.

3

Assume $P(k)$ $\sum_{i=1}^{k} \frac{1}{2^i} = 2 - \frac{1}{2^k}$ true.

$P(k+1)$: $\sum_{i=1}^{k+1} \frac{1}{2^i} \stackrel{?}{=} 2 - \frac{1}{2^{k+1}}$

$\sum_{i=1}^{k} \frac{1}{2^i} + \frac{1}{2^{k+1}} = 2 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 2 + \frac{-2+1}{2^{k+1}} = 2 - \frac{1}{2^{k+1}}$ true 1

Base Case + Ind. Case ∴ P(n) true

Question 3 (You can use back of page for answer)

Let x be an integer. **Prove or disprove** whether the following statements are equivalent:

$3x-1$ *is odd*
$x+7$ *is odd*
x^2+2 *is even*

2 each proof.

$3x-1$ odd $\rightarrow$ $x+7$ odd

contrap. $x+7$ even $3x-1$ even

$x+7=2k$ $x=2k-7$

$3x-1 = 3(2k-7)-1$

$= 6k-21-1$

$= 6k-22 \equiv$ even.

so proved.

$x+7$ odd $\rightarrow$ x^2+2 even

$x+7 = 2k-1 \Rightarrow x = 2k-1-7 = 2k-8 \equiv$ even

x^2 even x^2+2 even.

$x^2+2 \xrightarrow{\text{even}} 3x-1$ odd x^2+2 even $\Rightarrow x^2$ even, x even

$3(2k)-1$ odd

Question 4

Part A.

Give a recursive definition with initial condition for the sequence
$a_1 = 16, a_2 = 13, a_3 = 10, a_4 = 7, \ldots$

Your answer

Basis Step: $a_1 = 16$

Recursive Step: $a_n = a_{n-1} - 3$

Part B.

Give a **recursive algorithm** for finding the **sum of the first n even positive integers**

2N + sum(N-1)

if n=1 return 2

if n even return

sum(N-1) + 2N

Question 5.

A 6-digit number is a number composed of six digits from 0 to 9 each

1. How many 6-digit numbers are there?

10^6

2. How many 6-digit numbers that contain at least one 7 are there?

$10^6 - 9^6$

3. How many 6-digit numbers that begin **or** end with a 5 are there?

$10^5 + 10^5 - 10^4$