Question 2:

a)

That is, every string derivable in G_1 is derivable in G_2 and vice-versa.

iff for every position $n \geq 1$, the n-th value produced by G_1 equals the n-th value produced by G_2.

Formally, if we write

$v_1(n) = G_1$.next().value,

$v_2(n) = G_2$.next().value,

then

$G_1 \equiv G_2 \Leftrightarrow \forall n \in \mathbb{N}^+, v_1(n) = v_2(n)$.

In other words, they yield the same infinite sequence of numbers.

c)

נוכיח באינדוקציה על איבר n

בסיס: Fib1 הגדרנו ש Fib1(1)=1

Fib2 —> Fib2(1) = $(((1+\sqrt{5})/2) - ((1-\sqrt{5})/2))/\sqrt{5}$ => $\frac{2\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}} = 1$

אז Fib1(1) =Fib2(1)

טענה: נניח כי עד מספר n מתקיים ש Fib1(n)=Fib2(n)

צעד:נוכיח עבור n+1

$Fib1(n+1) = Fib1(n) + Fib1(n-1) = Fib2(n) + Fib2(n-1)$:הנ"ל

$Fib2(n+1) = \frac{\phi^{n+1} - \psi^{n+1}}{\sqrt{5}} = \frac{\phi^n \phi - \psi^n \psi}{\sqrt{5}}$

$= Fib_2(n) \cdot \phi + Fib_2(n-1) \cdot \phi^0 = Fib1(n+1)$

Question 3.b:

Proposition: For any lists lst1 and lst2 and a continuation procedure cont, (append$ lst1 lst2 cont) = (cont (append lst1 lst2)).

Proof: By induction on the length of lst1

Base: For the case of a lst1 of length 0 [the empty list], the value of (append lst1 lst2) is lst2, and the value of (append$ lst1 lst2 cont) is (cont lst2), which implies (append$ lst1 lst2 cont) = (cont (append lst1 lst2)).

Induction step: We assume the proposition holds for lst1 of length n, and show the proposition holds for lst1 of a length n+1.

(a) According to the code, the value of (append lst1 lst2) is (cons (car lst1) (append (cdr lst1) lst2)).

(b) According to the code ,the value of (append$ lst1 lst2 cont) is (append$ (cdr lst1) lst2 cont2), where cont2 is the continuation procedure defined in lines 6-7.

Since the first operand of (append$ (cdr lst1) lst2 cont2) is a list of length n, according to the induction assumption: (cont2 (append (cdr lst1) lst2)) = (append$ (cdr lst1) lst2 cont2).

—> (cont (cons (car lst1) (append (cdr lst1) lst2))) = (append$ (cdr lst1) lst2 cont2) ;;; code of cont 2

—> (cont (append lst1 lst2)) = (append$ (cdr lst1) lst2 cont2) ;;; (a)

—> (cont (append lst1 lst2)) = (append$ lst1 lst2 cont) ;;; (b)

5.1)

a)

אוסף משוואות: $\{t(s(s), G, s(U), P, t(K), s),\ t(s(G), G, K, P, t(K), U)\}$

הצבה: $\{\}$

בחירת משוואה מהאוסף:

$t(s(s), G, s(U), P, t(K), s) = t(s(G), G, K, P, t(K), U)$

אוסף המשוואות: $[s(s)=s(G),\ G=G,\ s(U)=K,\ P=P,\ t(K)=t(K),\ s=U]$

הצבה: $\{\}$

נפתור: $s(s)=s(G)$, נוסיף להצבה: $\{s=G\}$

$G=G$ טריוויאלי

נפתור $s(U)=K$, נוסיף להצבה: $\{s=G,\ s(U)=K\}$

$P=P$, $t(K)=t(K)$ טריוויאליים

נפתור $s=U$, נוסיף להצבה: $\{s=G,\ s(U)=K,\ s=U\}$

נציב $K=s(U)=s(s)$

נקבל: $\{G=s,\ U=s,\ K=s(s)\}$

b) $[[W|U] \mid [V|K]] = [[V|U] \mid W]$

הצבה: $\{\}$

נפתור: $[W|U]=[V|U]$: $\{W=V\}$

נפתור: $[V|K]=V$, היוניפיקציה כושלת כי V אטומי ו $[V|K]$ הוא רשימה

5.3) plus(s(s(zero)), X, s(s(zero)))

%3 {X1 = s(s(zero)), X = zero}

plus(s(s(zero)), zero, s(s(zero)))

%2 {X1 = s(s(zero))}

natural-number(s(s(zero)))

%2 {X1 = s(zero)}

natural-number(s(zero))

%1 {X1 = zero}

natural-number(zero)

לכן X = zero, זהו עץ הצלחה סופי.