Chapter 7

Revised exercises

7.1 Joint distributions

MATHEMATICAL TECHNIQUES

♠ For the following joint distributions describing the values of the random variables X and Y, find both marginal distributions and the conditional distribution requested. Are the two random variables independent?

• EXERCISE **7.1.1**

Find the marginal distributions and the distribution of Y conditional on $X = 0$.

	$X = 0$	$X = 1$
$Y = 0$	0.14	0.26
$Y = 1$	0.21	0.39

• EXERCISE **7.1.2**

Find the marginal distributions and the distribution of X conditional on $Y = 1$.

	$X = 0$	$X = 1$
$Y = 1$	0.45	0.25
$Y = 3$	0.05	0.25

• EXERCISE **7.1.3**

Find the marginal distributions and the distribution of Y conditional on $X = 1$.

	$X = 0$	$X = 1$	$X = 2$
$Y = 1$	0.1	0.2	0.3
$Y = 2$	0.05	0.15	0.2

• EXERCISE **7.1.4**

Find the marginal distributions and the distribution of Y conditional on $X = 2$.

	$X = 0$	$X = 1$	$X = 2$
$Y = 1$	0.05	0.04	0.01
$Y = 2$	0.1	0.08	0.02
$Y = 3$	0.35	0.28	0.07

♠ Find the expectations of the random variables from their marginal distributions.

• EXERCISE **7.1.5**

The random variables X and Y in exercise 7.1.1.

- EXERCISE **7.1.6**
  The random variables X and Y in exercise 7.1.2.
- EXERCISE **7.1.7**
  The random variables X and Y in exercise 7.1.3.
- EXERCISE **7.1.8**
  The random variables X and Y in exercise 7.1.4.

♠ Suppose the following random variables are independent. Find the joint distribution and the requested conditional distribution from the given marginal distributions.

- EXERCISE **7.1.9**
  The random variable X has probability distribution $\Pr(X=0)=0.7$ and $\Pr(X=1)=0.3$. The random variable Y has probability distribution $\Pr(Y=0)=0.3$ and $\Pr(Y=1)=0.7$. Find the distribution of X conditional on $Y=0$.
- EXERCISE **7.1.10**
  The random variable X has probability distribution $\Pr(X=0)=0.4$ and $\Pr(X=1)=0.6$. The random variable Y has probability distribution $\Pr(Y=1)=0.2$ and $\Pr(Y=3)=0.8$. Find the distribution of X conditional on $Y=3$.
- EXERCISE **7.1.11**
  The random variable X has probability distribution $\Pr(X=0)=0.8$ and $\Pr(X=1)=0.2$. The random variable Y has probability distribution $\Pr(Y=1)=0.3$, $\Pr(Y=2)=0.5$ and $\Pr(Y=3)=0.2$. Find the distribution of Y conditional on $X=0$.
- EXERCISE **7.1.12**
  The random variable X has probability distribution $\Pr(X=0)=0.3$, $\Pr(X=1)=0.4$ and $\Pr(X=2)=0.3$. The random variable Y has probability distribution $\Pr(Y=1)=0.6$, $\Pr(Y=2)=0.1$ and $\Pr(Y=3)=0.3$. Find the distribution of Y conditional on $X=2$.

♠ Use the given information to construct the entire joint distribution for the following pairs of random variables.

- EXERCISE **7.1.13**
  Suppose that the random variables X and Y are each Bernoulli random variables (and thus take on only the values 0 and 1). We know that $\Pr(X=0)=0.2$, $\Pr(Y=0)=0.4$ and $\Pr(X=0 \text{ and } Y=0)=0.1$.
- EXERCISE **7.1.14**
  Suppose that the random variables X and Y are each Bernoulli random variables and that $\Pr(X=0)=0.3$, $\Pr(Y=1)=0.5$ and $\Pr(X=1 \text{ and } Y=0)=0.4$.
- EXERCISE **7.1.15**
  Suppose that the random variables X and Y are each Bernoulli random variables, and that $\Pr(X=0)=0.3$, $\Pr(Y=0)=0.6$ and $\Pr(X=0|Y=0)=0.5$.
- EXERCISE **7.1.16**
  Suppose that the random variables X and Y are each Bernoulli random variables, and that $\Pr(X=1)=0.8$, $\Pr(Y=0)=0.4$ and $\Pr(X=0|Y=1)=0.1$.

♠ Consider the following joint distribution for the random variables T and N.

	$T=-1$	$T=0$	$T=1$	$T=2$
$N=0$	0.02	0.10	0.03	0.06
$N=1$	0.04	0.02	0.09	0.10
$N=2$	0.09	0.06	0.10	0.11
$N=3$	0.06	0.02	0.09	0.01

- EXERCISE **7.1.17**
  Suppose measurements can only distinguish two values of T, $T>0$ and $T\le 0$, and two values of N, $N=0$ and $N>0$. Find the joint distribution for these events.
- EXERCISE **7.1.18**
  Suppose measurements can only distinguish two values of T, $T>0$ and $T\le 0$, and two values of N, $N\le 1$ and $N>1$. Find the joint distribution for these events.

♠ When two baseball players bat in the same inning, the first gets a hit 25% of the time and the second gets a hit 35% of the time. In each of the following cases, find the joint distribution, the conditional distribution for the second player conditional on the first getting a hit and the conditional distribution for the first player conditional on the second getting a hit.

- EXERCISE **7.1.19**

  The case where the players have the highest possible probability of each getting a hit.
- EXERCISE **7.1.20**

  The case where the players have the lowest possible probability of each getting a hit.

♠ Write the joint distribution describing the states of the following Markov chains at times t and $t+1$. Assume that the marginal distributions at both time t and $t+1$ match the long-term probability.

- EXERCISE **7.1.21**

  The mutants described in exercises 6.2.19 and 6.6.15, where a gene has a 1.0% chance of mutating each time a cell divides, and a 1.0% chance of correcting the mutation.
- EXERCISE **7.1.22**

  The lemmings described in exercises 6.2.20 and 6.6.16, where a lemming has a probability 0.2 of jumping off the cliff each hour and a probability 0.1 of crawling back up.

APPLICATIONS

♠ Find the conditional distributions for the number of lice on birds with 0, 1 and 2 mites for the following birds from the text. Describe how the conditional distributions differ from each other.

- EXERCISE **7.1.23**

  Birds of type 2.
- EXERCISE **7.1.24**

  Birds of type 4.

♠ Draw the conditional distribution for the number of mites on birds with 0, 1 and 2 lice for the following birds from the text.

- EXERCISE **7.1.25**

  Birds of type 1.
- EXERCISE **7.1.26**

  Birds of type 2.
- EXERCISE **7.1.27**

  Birds of type 3.
- EXERCISE **7.1.28**

  Birds of type 4.

♠ Recall the ecologist observing eagles and rabbits in exercises 6.5.27 and 6.5.28. In each of the following cases, find the joint distribution, the marginal distributions, and the conditional distributions. Use the random variables E and J, where $E = 0$ represents seeing no eagle, $E = 1$ seeing an eagle where $J = 0$ seeing no jack rabbit and $J = 1$ seeing a jack rabbit. Use graphs of the conditional distributions to help interpret the results.

- EXERCISE **7.1.29**

  She sees an eagle with probability 0.2 during an hour of observation, a jack rabbit with probability 0.5, and both with probability 0.05 (as in exercise 6.5.27.
- EXERCISE **7.1.30**

  She sees an eagle with probability 0.2 during an hour of observation, a jack rabbit with probability 0.5, and both with probability 0.15 (as in exercise 6.5.28.

♠ Find the joint distribution of the two events in the rare disease model where a person either has the disease (event D) or not (event N), and either tests positive (event P) or not (event P^c) in the following cases. Use the joint distribution to find $\Pr(\mathrm{D}|\mathrm{P})$.

- EXERCISE **7.1.31**

  $\Pr(\mathrm{D}) = 0.2$, $\Pr(\mathrm{N}) = 0.8$, $\Pr(\mathrm{P}|\mathrm{D}) = 1.00$ and $\Pr(\mathrm{P}|\mathrm{N}) = 0.05$ (as in exercise 6.5.37).
- EXERCISE **7.1.32**

  $\Pr(\mathrm{D}) = 0.2$, $\Pr(\mathrm{N}) = 0.8$, $\Pr(\mathrm{P}|\mathrm{D}) = 0.95$ and $\Pr(\mathrm{P}|\mathrm{N}) = 0.05$ (as in exercise 6.5.38).

• EXERCISE **7.1.33**
Pr(D) = 0.8, Pr(P|D) = 1.0 and Pr(P|N) = 0.1 (as in exercise 6.5.39).
• EXERCISE **7.1.34**
Pr(D) = 0.8, Pr(P|D) = 0.95 and Pr(P|N) = 0.1 (as in exercise 6.5.40).

♠ Recall the cells in exercises 6.5.31 and 6.5.32. New cells stain properly with probability 0.95, one day old cells stain properly with probability 0.9, two day old cells stain properly with probability 0.8, three day old cells stain properly with probability 0.5. Suppose

$$\begin{aligned} \text{Pr(cell is 0 day old)} &= 0.4 \\ \text{Pr(cell is 1 day old)} &= 0.3 \\ \text{Pr(cell is 2 days old)} &= 0.2 \\ \text{Pr(cell is 3 days old)} &= 0.1. \end{aligned}$$

In each of the following cases, define two random variables, draw the joint distribution, find the marginal probability distributions, and compute and graph the conditional probability distributions for cell age. Compare the conditional distributions with the marginal distribution.
• EXERCISE **7.1.35**
With the probabilities as given.
• EXERCISE **7.1.36**
If the lab finds a way to eliminate the oldest cells (more than 3 days old) from its stock.

♠ Suppose immigration and emigration change the sizes of two populations with the following probabilities (as in exercises 6.8.29 and 6.8.30).

Population a		Population b	
Number	Probability	Number	Probability
-1	0.4	-1	0.1
0	0.2	0	0.3
1	0.3	1	0.2
2	0.1	2	0.4

Let I_a represent the change in population a and I_b the change in population b.
• EXERCISE **7.1.37**
Find the joint distribution if immigrants arrive into the two populations independently.
• EXERCISE **7.1.38**
Fill in the rest of the joint distribution.

	$I_a = -1$	$I_a = 0$	$I_a = 1$	$I_a = 2$
$I_b = -1$	0.05	0.03	0.01	?
$I_b = 0$	?	0.05	?	0.02
$I_b = 1$	0.12	?	0.05	?
$I_b = 2$	?	?	0.18	0.05

♠ We have seen that meiotic drive and lack of independence can create usual distributions of genotypes for offspring (exercises 6.6.25 and 6.6.26). In the following cases, find the joint distribution of genotypes in the offspring. Use your joint distribution to find the probability that an offspring is a heterozygote.
• EXERCISE **7.1.39**
Compare a case of meiotic drive where 60% of both pollen and ovules carry the **A** allele independently, with a case of non-independent assortment where an offspring gets an **A** allele from the pollen with probability 0.6 when the ovule provides an **A** and gets an **A** allele from the pollen with probability 0.4 when the ovule provides an **a**. The ovule provides **A** with probability 0.5 (from exercise 6.6.25).
• EXERCISE **7.1.40**
Compare a case of meiotic drive where 70% of the pollen and 40% of the ovules carry the **A** allele independently, with a case of non-independent assortment where an offspring gets an **A** allele from the pollen with probability 0.7

when the ovule provides an **A** and gets an **A** allele from the pollen with probability 0.3 when the ovule provides an **a**. The ovule provides **A** with probability 0.5 (from exercise 6.6.26).

♠ Many matings are observed in a species of bird. Both female and male birds come in three colors: red, blue and green. For each experiment, find the marginal distributions for both sexes and the conditional distributions of male color for red, blue and green females respectively. What might be going on with these birds?

- EXERCISE **7.1.41**

		Male		
		R	B	G
female	R	0.125	0.195	0.180
	B	0.225	0.027	0.048
	G	0.090	0.102	0.008

- EXERCISE **7.1.42**

		Male		
		R	B	G
female	R	0.19	0.06	0.00
	B	0.05	0.18	0.02
	G	0.13	0.08	0.29

Chapter 7

Answers

7.1.1. The marginal distributions are

	$X=0$	$X=1$	
$Y=0$	0.14	0.26	$\rightarrow$ $\Pr(Y=0)=0.4$
$Y=1$	0.21	0.39	$\rightarrow$ $\Pr(Y=1)=0.6$
	$\downarrow$ $\Pr(X=0)=0.35$	$\downarrow$ $\Pr(X=1)=0.65$	

The random variables are independent because the joint distribution is equal to the product of the marginal distributions. For example,

$$\Pr(X=0 \text{ and } Y=0)=0.14=\Pr(X=0)\cdot\Pr(Y=0)=0.35\cdot 0.4=0.14.$$

The distribution of Y conditional on $X=0$ is

$$\begin{aligned}
\Pr(Y=0|X=0) &= \frac{\Pr(Y=0 \text{ and } X=0)}{\Pr(X=0)}=\frac{0.14}{0.35}=0.4 \\
\Pr(Y=1|X=0) &= \frac{\Pr(Y=1 \text{ and } X=0)}{\Pr(X=0)}=\frac{0.21}{0.35}=0.6.
\end{aligned}$$

This matches the marginal distribution for these independent random variables.
7.1.3. The marginal distributions are

	$X=0$	$X=1$	$X=2$	
$Y=1$	0.1	0.2	0.3	$\rightarrow$ $\Pr(Y=1)=0.6$
$Y=2$	0.05	0.15	0.2	$\rightarrow$ $\Pr(Y=2)=0.4$
	$\downarrow$ $\Pr(X=0)=0.15$	$\downarrow$ $\Pr(X=1)=0.35$	$\downarrow$ $\Pr(X=2)=0.5$	

The random variables are not independent because

$$\Pr(X=0 \text{ and } Y=1)=0.1\neq\Pr(X=0)\cdot\Pr(Y=1)=0.15\cdot 0.6=0.09$$

The distribution of Y conditional on $X=1$ is

$$\begin{aligned}
\Pr(Y=1|X=1) &= \frac{\Pr(Y=1 \text{ and } X=1)}{\Pr(X=1)}=\frac{0.2}{0.35}=0.571 \\
\Pr(Y=2|X=1) &= \frac{\Pr(Y=2 \text{ and } X=1)}{\Pr(X=1)}=\frac{0.15}{0.35}=0.429.
\end{aligned}$$

7.1.5. $\mathrm{E}(X) = 0 \cdot 0.35 + 1 \cdot 0.65 = 0.65$. $\mathrm{E}(Y) = 0 \cdot 0.4 + 1 \cdot 0.6 = 0.6$.
7.1.7. $\mathrm{E}(X) = 0 \cdot 0.15 + 1 \cdot 0.35 + 2 \cdot 0.5 = 1.35$. $\mathrm{E}(Y) = 1 \cdot 0.6 + 2 \cdot 0.4 = 1.4$.
7.1.9. Because the random variables are independent, we can find the joint distribution by multiplying the marginal distributions, so

	$X = 0$	$X = 1$
$Y = 0$	0.21	0.09
$Y = 1$	0.49	0.21

Independence also ensures that the conditional distribution of X when $Y = 1$ matches the marginal distribution of X, so $\Pr(X = 0|Y = 1) = \Pr(X = 0) = 0.7$ and $\Pr(X = 1|Y = 1) = \Pr(X = 1) = 0.3$.
7.1.11. Because the random variables are independent, we can find the joint distribution by multiplying the marginal distributions, so

	$X = 0$	$X = 1$
$Y = 1$	0.24	0.06
$Y = 2$	0.4	0.1
$Y = 3$	0.16	0.04

Independence also ensures that the conditional distribution of Y when $X = 0$ matches the marginal distribution of Y, so $\Pr(Y = 1|X = 0) = \Pr(Y = 1) = 0.3$ $\Pr(Y = 2|X = 0) = \Pr(Y = 2) = 0.5$ $\Pr(Y = 3|X = 0) = \Pr(Y = 3) = 0.2$.
7.1.13. First, we use the fact that marginal probabilities for both X and Y must add to 1 to find that $\Pr(X = 1) = 0.8$ and $\Pr(Y = 1) = 0.6$. We can then include the following information on our joint distribution,

	$X = 0$	$X = 1$	
$Y = 0$	0.1	?	$\rightarrow$ $\Pr(Y = 0) = 0.4$
$Y = 1$	?	?	$\rightarrow$ $\Pr(Y = 1) = 0.6$
	$\downarrow$ $\Pr(X = 0) = 0.2$	$\downarrow$ $\Pr(X = 1) = 0.8$	

Because the probabilities in the first row must add to 0.4, we have that $\Pr(X = 1 \text{ and } Y = 0) = 0.3$. Because the probabilities in the first column must add to 0.2, we have that $\Pr(X = 0 \text{ and } Y = 1) = 0.1$. For all the probabilities to add to 1, the remaining probability is $\Pr(X = 1 \text{ and } Y = 1) = 0.5$. The final result is

	$X = 0$	$X = 1$	
$Y = 0$	0.1	0.3	$\rightarrow$ $\Pr(Y = 0) = 0.4$
$Y = 1$	0.1	0.5	$\rightarrow$ $\Pr(Y = 1) = 0.6$
	$\downarrow$ $\Pr(X = 0) = 0.2$	$\downarrow$ $\Pr(X = 1) = 0.8$	

7.1.15. First, we use the fact that marginal probabilities for both X and Y must add to 1 to find that $\Pr(X = 1) = 0.7$ and $\Pr(Y = 1) = 0.4$. We find

$$\Pr(X = 0 \text{ and } Y = 0) = \Pr(X = 0|Y = 0)\Pr(Y = 0) = 0.5 \cdot 0.6 = 0.3.$$

We can then include the following information on our joint distribution,

	$X = 0$	$X = 1$	
$Y = 0$	0.3	?	$\rightarrow$ $\Pr(Y = 0) = 0.6$
$Y = 1$	?	?	$\rightarrow$ $\Pr(Y = 1) = 0.4$
	$\downarrow$ $\Pr(X = 0) = 0.3$	$\downarrow$ $\Pr(X = 1) = 0.7$	

Using the row and column sums, we fill in the rest of the information as

	$X = 0$	$X = 1$	
$Y = 0$	0.3	0.3	$\rightarrow$ $\Pr(Y = 0) = 0.6$
$Y = 1$	0.0	0.4	$\rightarrow$ $\Pr(Y = 1) = 0.4$
	$\downarrow$ $\Pr(X = 0) = 0.3$	$\downarrow$ $\Pr(X = 1) = 0.7$	

7.1.17. The event $T > 0$ includes the simple events $T = 1$ and $T = 2$. Then

$$\Pr(T > 0 \text{ and } N = 0) = \Pr(T = 1 \text{ and } N = 0) + \Pr(T = 1 \text{ and } N = 0) = 0.03 + 0.06 = 0.09.$$

Similarly,

$$\Pr(T \leq 0 \text{ and } N = 0) = \Pr(T = -1 \text{ and } N = 0) + \Pr(T = 0 \text{ and } N = 0) = 0.02 + 0.10 = 0.12.$$

Finding $\Pr(T \leq 0 \text{ and } N > 0)$ requires adding up the six probabilities in the lower left hand corner of the matrix, giving 0.29. Finding $\Pr(T > 0 \text{ and } N > 0)$ requires adding up the six probabilities in the lower right hand corner of the matrix, giving 0.50. The joint distribution is then

	$T \leq 0$	$T > 0$
$N = 0$	0.12	0.09
$N > 0$	0.29	0.50

7.1.19. Using the notation H_1 means first player hit, M_1 that first missed and so forth, the maximum is

	H_1	M_1
H_2	0.25	0.1
M_2	0.0	0.65

The conditional distribution for the second if the first gets a hit is

$$\begin{aligned}
\Pr(\mathrm{H}_2|\mathrm{H}_1) &= \frac{\Pr(\mathrm{H}_2 \text{ and } \mathrm{H}_1)}{\Pr(\mathrm{H}_1)} = \frac{0.25}{0.25} = 1 \\
\Pr(\mathrm{M}_2|\mathrm{H}_1) &= \frac{\Pr(\mathrm{M}_2 \text{ and } \mathrm{H}_1)}{\Pr(\mathrm{H}_1)} = \frac{0.0}{0.25} = 0.
\end{aligned}$$

The second player is sure to hit if the first does. The conditional distribution for the first if the second gets a hit is

$$\begin{aligned}
\Pr(\mathrm{H}_1|\mathrm{H}_2) &= \frac{\Pr(\mathrm{H}_1 \text{ and } \mathrm{H}_2)}{\Pr(\mathrm{H}_2)} = \frac{0.25}{0.35} = 0.714 \\
\Pr(\mathrm{M}_1|\mathrm{H}_2) &= \frac{\Pr(\mathrm{M}_1 \text{ and } \mathrm{H}_2)}{\Pr(\mathrm{H}_2)} = \frac{0.1}{0.35} = 0.286.
\end{aligned}$$

The first player is very likely, but not certain, to hit if the first does.
7.1.21. Let M_t denote the event "mutant at generation t" and N_t denote the event "non-mutant at generation t". We know that

$$\begin{aligned}
\Pr(\mathrm{M}_{t+1}|\mathrm{M}_t) &= 0.99 \\
\Pr(\mathrm{M}_{t+1}|\mathrm{N}_t) &= 0.01 \\
\Pr(\mathrm{N}_{t+1}|\mathrm{M}_t) &= 0.01 \\
\Pr(\mathrm{N}_{t+1}|\mathrm{N}_t) &= 0.99.
\end{aligned}$$

Also, we found that $\Pr(M_t) = 0.5$ after a long time. Therefore,

$$\begin{aligned}
\Pr(\mathrm{M}_{t+1} \text{ and } \mathrm{M}_t) &= \Pr(\mathrm{M}_{t+1}|\mathrm{M}_t)\Pr(\mathrm{M}_t) = 0.99 \cdot 0.5 = 0.495 \\
\Pr(\mathrm{M}_{t+1} \text{ and } \mathrm{N}_t) &= \Pr(\mathrm{M}_{t+1}|\mathrm{N}_t)\Pr(\mathrm{N}_t) = 0.01 \cdot 0.5 = 0.005 \\
\Pr(\mathrm{N}_{t+1} \text{ and } \mathrm{M}_t) &= \Pr(\mathrm{N}_{t+1}|\mathrm{M}_t)\Pr(\mathrm{M}_t) = 0.01 \cdot 0.5 = 0.005 \\
\Pr(\mathrm{N}_{t+1} \text{ and } \mathrm{N}_t) &= \Pr(\mathrm{N}_{t+1}|\mathrm{N}_t)\Pr(\mathrm{N}_t) = 0.99 \cdot 0.5 = 0.495.
\end{aligned}$$

In the form of a joint distribution,

	M_t	N_t
M_{t+1}	0.495	0.005
N_{t+1}	0.005	0.495

7.1.23. With 0 mites,

$$\begin{aligned}
\Pr(L=0|M=0) &= \frac{\Pr(L=0 \text{ and } M=0)}{\Pr(M=0)} = \frac{0.21}{0.5} = 0.42 \\
\Pr(L=1|M=0) &= \frac{\Pr(L=1 \text{ and } M=0)}{\Pr(M=0)} = \frac{0.13}{0.5} = 0.26 \\
\Pr(L=2|M=0) &= \frac{\Pr(L=2 \text{ and } M=0)}{\Pr(M=0)} = \frac{0.16}{0.5} = 0.32.
\end{aligned}$$

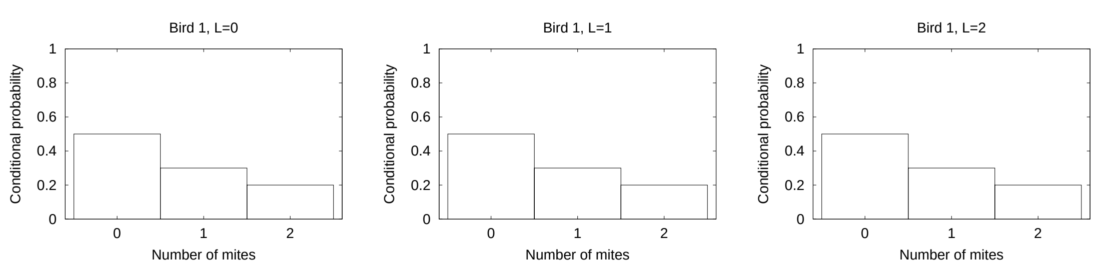
With 1 mite,

$$\begin{aligned}
\Pr(L=0|M=1) &= \frac{\Pr(L=0 \text{ and } M=1)}{\Pr(M=1)} = \frac{0.13}{0.3} = 0.43 \\
\Pr(L=1|M=1) &= \frac{\Pr(L=1 \text{ and } M=1)}{\Pr(M=1)} = \frac{0.07}{0.3} = 0.23 \\
\Pr(L=2|M=1) &= \frac{\Pr(L=2 \text{ and } M=1)}{\Pr(M=1)} = \frac{0.10}{0.3} = 0.33.
\end{aligned}$$

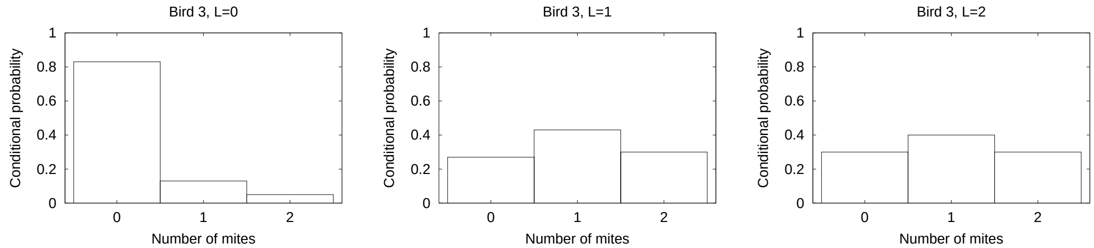
With 2 mites,

$$\begin{aligned}
\Pr(L=0|M=2) &= \frac{\Pr(L=0 \text{ and } M=2)}{\Pr(M=2)} = \frac{0.06}{0.2} = 0.30 \\
\Pr(L=1|M=2) &= \frac{\Pr(L=1 \text{ and } M=2)}{\Pr(M=2)} = \frac{0.10}{0.2} = 0.50 \\
\Pr(L=2|M=2) &= \frac{\Pr(L=2 \text{ and } M=2)}{\Pr(M=2)} = \frac{0.04}{0.2} = 0.20.
\end{aligned}$$

The first two are rather similar, meaning that louse number is not affected by the first mite. With two mites, however, it becomes more likely that the bird has 1 louse and less likely that it has 0 or 2.
7.1.25.



7.1.27.



7.1.29. The joint distribution is

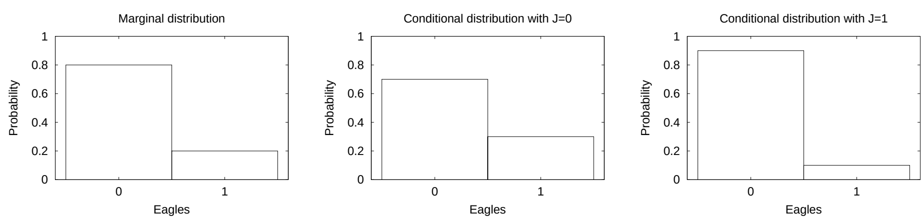
	$E=0$	$E=1$
$J=0$	0.35	0.15
$J=1$	0.45	0.05

The marginal distributions are $\Pr(J=0)=\Pr(J=1)=0.5$ and $\Pr(E=0)=0.8$ and $\Pr(E=1)=0.2$. The conditional distributions for the eagle are

$$\begin{aligned}\Pr(E=0|J=0) &= \frac{\Pr(E=0 \text{ and } J=0)}{\Pr(J=0)} = \frac{0.35}{0.5} = 0.7\\ \Pr(E=1|J=0) &= \frac{\Pr(E=1 \text{ and } J=0)}{\Pr(J=0)} = \frac{0.15}{0.5} = 0.3.\end{aligned}$$

and

$$\begin{aligned}\Pr(E=0|J=1) &= \frac{\Pr(E=0 \text{ and } J=1)}{\Pr(J=1)} = \frac{0.45}{0.5} = 0.9\\ \Pr(E=1|J=1) &= \frac{\Pr(E=1 \text{ and } J=1)}{\Pr(J=1)} = \frac{0.05}{0.5} = 0.1.\end{aligned}$$

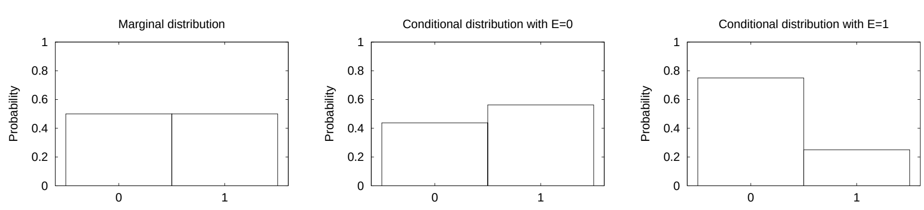


For the rabbit, the conditional distributions are

$$\begin{aligned}\Pr(J=0|E=0) &= \frac{\Pr(J=0 \text{ and } E=0)}{\Pr(E=0)} = \frac{0.35}{0.8} = 0.4375\\ \Pr(J=1|E=0) &= \frac{\Pr(J=1 \text{ and } E=0)}{\Pr(E=0)} = \frac{0.45}{0.2} = 0.5625.\end{aligned}$$

and

$$\begin{aligned}\Pr(J=0|E=1) &= \frac{\Pr(J=0 \text{ and } E=1)}{\Pr(E=1)} = \frac{0.15}{0.2} = 0.75\\ \Pr(J=1|E=1) &= \frac{\Pr(J=1 \text{ and } E=1)}{\Pr(E=1)} = \frac{0.05}{0.2} = 0.25.\end{aligned}$$



It is more likely that she will see an eagle when no rabbits are seen, and more likely that she will see a rabbit when no eagles are seen. One must be avoiding the other.

7.1.31. We used the law of total probability to find $\Pr(\mathrm{P}) = 0.24$. We can then find

$$\Pr(\mathrm{P} \cap \mathrm{D}) = \Pr(\mathrm{P}|\mathrm{D})\Pr(\mathrm{D}) = 1.0 \cdot 0.2 = 0.2.$$

We can then fill in the rest of the joint distribution as

	D	N
P	0.2	0.04
P^c	0.0	0.76

Therefore, $\Pr(\mathrm{D}|\mathrm{P}) = \dfrac{\Pr(\mathrm{D} \cap \mathrm{P})}{\Pr(\mathrm{P})} = 0.2/0.24 = 0.833.$

7.1.33. We used the law of total probability to find $\Pr(\mathrm{P}) = 0.82$. We can then find

$$\Pr(\mathrm{P} \cap \mathrm{D}) = \Pr(\mathrm{P}|\mathrm{D})\Pr(\mathrm{D}) = 1.0 \cdot 0.8 = 0.8.$$

We can then fill in the rest of the joint distribution as

	D	N
P	0.8	0.02
P^c	0.0	0.18

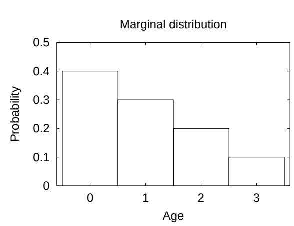
Therefore, $\Pr(\mathrm{D}|\mathrm{P}) = \dfrac{\Pr(\mathrm{D} \cap \mathrm{P})}{\Pr(\mathrm{P})} = 0.8/0.82 = 0.976.$

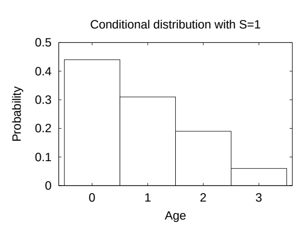
7.1.35. The random variables are age A, taking on values 0, 1, 2, and 3, and stain S, taking on value 1 for proper staining and 0 otherwise. The joint distribution is

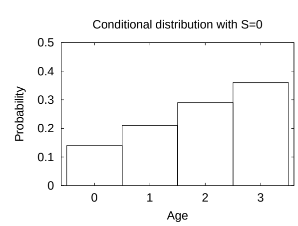
	$S = 1$	$S = 0$
$A = 0$	0.38	0.02
$A = 1$	0.27	0.03
$A = 2$	0.16	0.04
$A = 3$	0.05	0.05

The marginal distribution for cell age is given in the problem, and the marginal distribution for staining is $\Pr(S = 1) = 0.86$ and $\Pr(S = 0) = 0.14$. The conditional distributions for age are given in the following columns.

	$S = 1$	$S = 0$
$A = 0$	0.44	0.14
$A = 1$	0.31	0.21
$A = 2$	0.19	0.29
$A = 3$	0.06	0.36







The conditional distribution for age when $S = 1$ looks a lot like the marginal distribution, but with $S = 0$ it is very different. Cells that do not stain properly tend to be much older.

7.1.37. Multiplying the marginal probabilities gives

	$I_a = -1$	$I_a = 0$	$I_a = 1$	$I_a = 2$
$I_b = -1$	0.04	0.02	0.03	0.01
$I_b = 0$	0.12	0.06	0.09	0.03
$I_b = 1$	0.08	0.04	0.06	0.02
$I_b = 2$	0.16	0.08	0.12	0.04

7.1.39. With meiotic drive, the marginal probabilities of an **A** from either the pollen or ovule is 0.6. With independence, the joint probabilities are the product of the marginal probabilities

		Ovule	
		A	a
Pollen	A	0.36	0.24
	a	0.24	0.16

The probability of a heterozygote is 0.48. With non-independent assortment, we know that the ovule provides A with probability 0.5. The pollen provides A with probability 0.5 also, from the law of total probability. Furthermore, $\Pr(aA) = \Pr(a|A)\Pr(A) = 0.4 \cdot 0.5 = 0.2$. The joint distribution is then, filling in the remaining values to match the marginal distributions.

		Ovule	
		A	a
Pollen	A	0.3	0.2
	a	0.2	0.3

7.1.41. The marginal distribution for females is R with probability 0.5, B with probability 0.3 and G with probability 0.2. For males it is R with probability 0.44, B with probability 0.324 and G with probability 0.236. There seem to be slightly fewer of the red males than expected, perhaps they are vulnerable to attack by hawks. The conditional distribution are: with R females 0.25 R, 0.39 B, 0.36 G, with B females 0.75 R, 0.09 B, 0.16 G. with G females 0.45 R, 0.51 B, 0.04 G. The females seem to prefer to mate with dissimilar males.