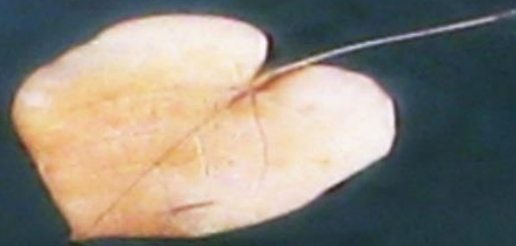


Calculus

The art of measuring
shape and variation!



Practice worksheets for
Mastery of Differentiation

Graeme Henderson

Dear Reader,

It is no secret that, to master any skill, we need to practise it!

School textbooks usually contain sufficient material for you to learn HOW to use certain skills but, due to space restrictions, usually don't contain sufficient examples for you to MASTER those skills.

This publication is intended to fill that gap ... for finding derivatives, at least!

If you are a student, let me suggest that you set time aside regularly to work through a few examples from this booklet. It need not be a great deal of time, but I recommend that, on a weekly, fortnightly, or monthly basis, you spend a few minutes practising the art of finding derivatives. You may find it a useful exercise to do this with friends and to discuss the more difficult examples. To build speed, try calculating the derivatives on the first sheet mentally ... and have a friend or parent check your answers.

If you are a teacher, please note that the sheets have been designed so that they may be laminated back-to-back (questions on one side and answers on the other) and used in a classroom setting.

I have invested a great deal of time in putting this material together. Although I have tried to be very careful, it is quite likely that some mistakes appear in the answers. If you find any, please let me know by visiting my [Crystal Clear Mathematics](http://www.CrystalClearMathematics) website and leaving a message on the "Contact Us" page. If you give permission for me to mention you by name, I will acknowledge your contribution in future (corrected) editions.

Because this took so long to prepare (dozens of hours), and because I am making it available for free, it would be appreciated if you could consider making a donation towards the upkeep of my website ... especially if you represent a school. It costs me many hours and thousands of dollars to maintain the site and add videos and other resources. Any contribution you may be able to make in return for the free use of this material would be greatly appreciated.

I hope this booklet is of help to you!



Graeme Henderson
B.Sc. (Hons), B.Th. (Hons),
Dip.Ed., Dip.Min.

16 April 2015

Copyright

Although I holds the copyright on all the material that I produces (including this 'Mastery of Differentiation' booklet), everything that I produce is available for FREE DOWNLOAD worldwide as long as no commercial gain is made from it. I am determined that people from poorer families and from developing nations have free access to quality training materials in mathematics.

If you can afford to support this ministry, any donation that you can make via the website will be gratefully received.

Speaking of copyright, thank you, [Schristia](#) (on Flickr), for allowing me to use your image for the cover of this booklet.

Derivatives in Your Head!

Differentiate with respect to x:

	A	B	C	D	E	F	G
1.	x^3	$4x^3$	$\frac{8x^2}{3}$	$x^{\frac{1}{2}}$	$3x^{\frac{1}{3}}$	x^{q+2}	\sqrt{x}
2.	x^{10}	$8x$	$\frac{-7x^2}{2}$	$x^{\frac{3}{4}}$	$4x^{\frac{3}{4}}$	x^{3m}	$\sqrt[5]{x}$
3.	x^4	$5x^4$	$\frac{5x^3}{4}$	$x^{\frac{2}{3}}$	$-6x^{\frac{2}{3}}$	x^{p+q}	$-\sqrt[7]{x^2}$
4.	x^5	$-3x^2$	$\frac{2x^5}{3}$	$x^{\frac{5}{4}}$	$-10x^{-\frac{3}{5}}$	$-x^{171}$	$\sqrt[4]{x^3}$
5.	x^6	$2x$	$-\frac{6x^7}{7}$	$x^{\frac{3}{2}}$	$9x^{\frac{2}{3}}$	x^{n+1}	$\sqrt[3]{x^8}$
6.	x^2	8	$\frac{2}{9}x^3$	$x^{1.8}$	$14x^{-\frac{3}{7}}$	x^{m+n+1}	$-\sqrt{x^{1.8}}$
7.	x^7	$-7x^6$	$\frac{11}{5}x^5$	$x^{6.1}$	$-2x^{-\frac{4}{5}}$	$-x^{5d-k+6}$	$\sqrt[3]{x^{2.4}}$
8.	x^9	$9x^5$	$-\frac{3}{4}x^6$	$x^{3.4}$	$4x^{-\frac{5}{3}}$	$x^{\sqrt{2}+1}$	$\sqrt[3]{x^{-5.1}}$
9.	x	$-12x^7$	$\frac{3}{7}x^4$	$x^{-\frac{1}{2}}$	$-7x^{\frac{2}{3}}$	$x^{3\pi+4}$	$\sqrt{\frac{1}{x^{7.6}}}$
10.	x^0	-14	$\frac{5}{8}x^3$	$x^{-\frac{7}{2}}$	$\frac{4x^{\frac{5}{4}}}{5}$	x^{3i-2}	$-\frac{1}{\sqrt[4]{x^3}}$
11.	x^{-2}	$-8x^0$	$\frac{-7x^{-3}}{3}$	$x^{-\frac{4}{3}}$	$-\frac{2}{3}x^{\frac{3}{2}}$	π^3	$\frac{7}{\sqrt[5]{x^2}}$
12.	x^{-1}	$11x^{-7}$	$\frac{2x^{-6}}{9}$	$x^{-\frac{5}{4}}$	$\frac{4}{21}x^{\frac{7}{2}}$	$-5x^{2n+1}$	$-\sqrt[3]{\frac{1}{x^2}}$
13.	x^{-4}	$9x^{-8}$	$-\frac{4x^{-5}}{15}$	$x^{-5.3}$	$-\frac{9}{4}x^{-\frac{4}{3}}$	$3x^{5k+4}$	$\frac{1}{\sqrt{x^3}}$
14.	x^{-3}	$-14x^{-10}$	$\frac{5x^{-14}}{7}$	$x^{-2.6}$	$\frac{-2x^{\frac{5}{4}}}{5}$	$-4x^{\sqrt{3}+7}$	$x\sqrt{x}$
15.	x^{-10}	$4x^{-3}$	$-\frac{2}{9}x^{-3}$	$x^{-8.3}$	$\frac{4}{3}x^{\frac{7}{3}}$	$\sqrt{3}x^{\pi-2}$	$-\frac{5}{3\sqrt{x}}$
16.	x^{-7}	$5x^{-1}$	$\frac{1}{5}x^{-3}$	$x^{\frac{11}{5}}$	$\frac{2}{7}x^{-\frac{2}{5}}$	$-2x^{5i+1}$	$\sqrt[4]{\frac{1}{x^{-1.2}}}$
17.	x^{-16}	$-2x^{-6}$	$\frac{-8}{5}x^{-4}$	$x^{\frac{5}{7}}$	$-\frac{9x^{-\frac{8}{5}}}{4}$	$2.5x^{2e+3}$	$-\frac{3}{7\sqrt{x^{1.4}}}$
18.	x^{-9}	$3x^{-5}$	$\frac{4}{7}x^{-6}$	$x^{-\frac{9}{2}}$	$-\frac{3}{14}x^{\frac{7}{3}}$	$0.4x^{6f}$	$\frac{14}{3\sqrt[7]{x^3}}$
19.	x^{-6}	$-7x^{-4}$	$\frac{5}{8}x^3$	$-7x^{-4}$	$\frac{10}{9}x^{-\frac{3}{5}}$	$2x^{a+b}$	$x^{23}\sqrt{x}$
20.	x^{-11}	$6x^{-2}$	$-\frac{5}{14}x^{-7}$	$6x^{-2}$	$\frac{7x^{-\frac{5}{3}}}{2}$	$3.5x^{4w+2}$	$-\frac{2x^3}{\sqrt[5]{x}}$

Derivatives in Your Head (Answers)!

Solutions:

	A	B	C	D	E	F	G
1.	$3x^2$	$12x^2$	$\frac{16x}{3}$	$\frac{x^{-\frac{1}{2}}}{2}$	$x^{-\frac{2}{3}}$	$(q+2)x^{q+1}$	$\frac{x^{-\frac{1}{2}}}{2} = \frac{1}{2\sqrt{x}}$
2.	$10x^9$	8	$-7x$	$\frac{3x^{-\frac{1}{4}}}{4}$	$3x^{-\frac{1}{4}}$	$3mx^{3m-1}$	$\frac{x^{-\frac{4}{5}}}{5} = \frac{1}{5\sqrt[5]{x^4}}$
3.	$4x^3$	$20x^3$	$\frac{15x^2}{4}$	$\frac{2x^{-\frac{1}{3}}}{3}$	$-4x^{-\frac{1}{3}}$	$(p+q)x^{p+q-1}$	$-\frac{2x^{-\frac{5}{7}}}{7} = -\frac{2}{7\sqrt[7]{x^5}}$
4.	$5x^4$	$-6x$	$\frac{10x^4}{3}$	$\frac{5x^{\frac{1}{4}}}{4}$	$6x^{-\frac{8}{5}}$	$-171x^{170}$	$\frac{3x^{-\frac{1}{4}}}{4} = \frac{3}{4\sqrt[4]{x}}$
5.	$6x^5$	2	$-6x^6$	$\frac{3x^{\frac{1}{2}}}{2}$	$6x^{-\frac{1}{3}}$	$(n+1)x^n$	$\frac{8x^{\frac{5}{3}}}{3} = \frac{8\sqrt[3]{x^5}}{3}$
6.	$2x$	0	$\frac{2}{3}x^2$	$1.8x^{0.8}$	$-6x^{-\frac{10}{7}}$	$(m+n+1)x^{m+n}$	$-0.9x^{-0.1}$
7.	$7x^6$	$-42x^5$	$11x^4$	$6.1x^{5.1}$	$\frac{8x^{-\frac{9}{5}}}{5}$	$-(5d-k+6)x^{5d-k+5}$	$0.8x^{-0.2}$
8.	$9x^8$	$45x^4$	$-\frac{9}{2}x^5$	$3.4x^{2.4}$	$-\frac{20x^{-\frac{8}{3}}}{3}$	$(\sqrt{2}+1)x^{\sqrt{2}+1}$	$-1.7x^{-2.7}$
9.	1	$-84x^6$	$\frac{12}{7}x^3$	$-\frac{x^{-\frac{3}{2}}}{2}$	$-\frac{14x^{-\frac{1}{3}}}{3}$	$(3\pi+4)x^{3\pi+3}$	$-3.8x^{-4.8}$
10.	0	0	$\frac{15}{8}x^2$	$-\frac{7x^{-\frac{9}{2}}}{2}$	$\frac{4x^{\frac{5}{4}}}{5}$	$(3i-2)x^{3i-3}$	$\frac{3x^{-\frac{7}{4}}}{4} = \frac{3}{4\sqrt[4]{x^7}}$
11.	$-2x^{-3}$	0	$7x^{-4}$	$-\frac{4x^{-\frac{7}{3}}}{3}$	$-x^{-\frac{5}{2}}$	0	$\frac{14x^{-\frac{7}{5}}}{5} = \frac{14}{5\sqrt[5]{x^7}}$
12.	$-x^{-2}$	$-77x^{-8}$	$-\frac{4x^{-7}}{3}$	$-\frac{5x^{-\frac{9}{4}}}{4}$	$\frac{2}{3}x^{\frac{5}{2}}$	$-5(2n+1)x^{2n}$	$\frac{2x^{-\frac{5}{3}}}{3} = \frac{2}{3\sqrt[3]{x^5}}$
13.	$-4x^{-5}$	$-72x^{-9}$	$\frac{4x^{-6}}{3}$	$-5.3x^{-6.3}$	$3x^{-\frac{7}{3}}$	$3(5k+4)x^{5k+3}$	$-\frac{3x^{-\frac{5}{2}}}{2} = -\frac{3}{2\sqrt{x^5}}$
14.	$-3x^{-4}$	$140x^{-11}$	$-10x^{-15}$	$-2.6x^{-3.6}$	$-\frac{x^{\frac{1}{4}}}{2}$	$-4(\sqrt{3}+7)x^{\sqrt{3}+6}$	$\frac{3x^{\frac{1}{2}}}{2} = \frac{3\sqrt{x}}{2}$
15.	$-10x^{-11}$	$-12x^{-4}$	$\frac{2}{3}x^{-4}$	$-8.3x^{-9.3}$	$\frac{28}{9}x^{\frac{4}{3}}$	$\sqrt{3}(\pi-2)x^{\pi-3}$	$\frac{5x^{-\frac{3}{2}}}{6} = \frac{5}{6\sqrt{x^3}}$
16.	$-7x^{-8}$	$-5x^{-2}$	$-\frac{3}{5}x^{-4}$	$\frac{11x^{\frac{6}{5}}}{5}$	$-\frac{4}{35}x^{-\frac{7}{5}}$	$-2(5i+1)x^{5i}$	$0.3x^{-0.7}$
17.	$-16x^{-17}$	$12x^{-7}$	$\frac{32}{5}x^{-5}$	$\frac{5x^{-\frac{2}{7}}}{7}$	$\frac{18x^{-\frac{13}{5}}}{45}$	$2.5(2e+3)x^{2e+2}$	$0.3x^{-1.7}$
18.	$-9x^{-10}$	$-15x^{-6}$	$-\frac{24}{7}x^{-7}$	$-\frac{9x^{-\frac{11}{2}}}{2}$	$-\frac{1}{2}x^{\frac{4}{3}}$	$2.4fx^{6f-1}$	$-2x^{-\frac{10}{7}} = -\frac{2}{\sqrt[7]{x^{10}}}$
19.	$-6x^{-7}$	$28x^{-5}$	$\frac{15}{8}x^2$	$28x^{-5}$	$-\frac{2}{3}x^{-\frac{8}{5}}$	$2(a+b)x^{a+b-1}$	$\frac{7x^{\frac{4}{3}}}{3} = \frac{7\sqrt[3]{x^4}}{3}$
20.	$-11x^{-12}$	$-12x^{-3}$	$\frac{5}{2}x^{-8}$	$-12x^{-3}$	$-\frac{35x^{-\frac{8}{3}}}{6}$	$(14w+7)x^{4w+1}$	$\frac{28x^{\frac{9}{5}}}{5} = \frac{28\sqrt[5]{x^9}}{5}$

Derivatives of Powers

Find $\frac{dy}{dx}$ if:

- | | | |
|---|---------------------------------------|--|
| 1. $y = x + \sqrt{x}$ | 2. $y = x^5 - 3\sqrt{x}$ | 3. $y = x^{\frac{5}{2}} - \frac{2}{x}$ |
| 4. $y = 3x^4 - \frac{2}{x} + \frac{6}{x^2}$ | 5. $y = (x + 5)(x + 2)$ | 6. $y = (3x + 1)(5x - 3)$ |
| 7. $y = (5x^2 - 3)(4x^3 + x)$ | 8. $y = (x^3 + 1)(2x + 3)$ | 9. $y = (x^5 - 2x)^2$ |
| 10. $y = (x - 2)(x + 1)(3x + 1)$ | 11. $y = (x - a)^3$ | 12. $y = (2x + 3)^3$ |
| 13. $y = 2x(3x^2 - 7x + 8)$ | 14. $y = 3x^2(x + 1)(x - 2)$ | 15. $y = (x + \frac{1}{x})^2$ |
| 16. $y = \frac{2x + 5}{x}$ | 17. $y = \frac{x^3 - 2}{x}$ | 18. $y = \frac{x^2 - 4x + 7}{x}$ |
| 19. $y = \frac{x^3 - 4x^2 + 3x - 2}{x^2}$ | 20. $y = \frac{3x^7 - 7x + 11}{2x^3}$ | 21. $y = \frac{(2x + 3)(2x - 3)}{x}$ |
| 22. $y = \frac{x + 6}{x^3}$ | 23. $y = \frac{2x^3 + x + 4}{2x^5}$ | 24. $y = \frac{x - 3}{\sqrt{x}}$ |

Find the derivative if:

- | | | |
|-----------------------------------|---|-------------------------------------|
| 25. $f(x) = ax^3 + bx^2 + cx + d$ | 26. $k = \frac{1}{a}(x^2 + \frac{b}{x} + c)$ | 27. $b = -3m^{-8} + 3\sqrt{7}$ |
| 28. $f = ax^4 + bx^2 + c$ | 29. $r = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + 4$ | 30. $g = -8h^5 + 3h^{-2} + h^{1.6}$ |

Find the derivative with respect to the variable indicated:

- | | | |
|---|--|---|
| 31. $C = 2\pi r$ [r] | 32. $y = mx + b$ [x] | 33. $v = u + at$ [t] |
| 34. $A = \pi r^2$ [r] | 35. $E = \frac{1}{2}mv^2$ [v] | 36. $P = \frac{V^2}{R}$ [V] |
| 37. $s = ut + \frac{1}{2}at^2$ [t] | 38. $P = RI^2$ [I] | 39. $V = \frac{4\pi r^3}{3}$ [r] |
| 40. $F = \frac{\pi r^4 P}{8\eta l}$ [r] | 41. $T = 2\pi\sqrt{\frac{l}{g}}$ [l] | 42. $v = \sqrt{\frac{2GM}{r}}$ [r] |
| 43. $F = \frac{GMm}{r^2}$ [r] | 44. $E = \frac{q}{4\pi\epsilon_0 r^2}$ [r] | 45. $\Lambda = \sqrt{\frac{h^2}{2\pi m k T}}$ [T] |

Calculate the first, second, third, and fourth derivatives of:

- | | | |
|---------------------------|-------------------------|--------------------------|
| 46. $y = 4x^4 + 2x^3 + 3$ | 47. $k = x^3 + 7x - 11$ | 48. $b = -m^{-2} + 3m^3$ |
|---------------------------|-------------------------|--------------------------|

Find the gradient of the tangent to the curve at the point indicated:

- | | | |
|---------------------------|---------------------------|--|
| 49. $y = x^2 + 3x$ (2,10) | 50. $y = 2x^3 - 4$ (3,50) | 51. $y = -x^2 + \frac{1}{x}$ (-2,-4.5) |
|---------------------------|---------------------------|--|

Find the equation of the tangent to the curve at the point indicated:

- | | | |
|--------------------------|----------------------------|-----------------------------------|
| 52. $y = 3x^2 - x$ (1,2) | 53. $y = x^3 + 4x$ (-1,-5) | 54. $y = x^2 - \frac{1}{x}$ (1,2) |
|--------------------------|----------------------------|-----------------------------------|

Find the value of x that satisfies the equation given:

- | | |
|---------------------------------|--|
| 55. $y = x^4 + 3x + 1$ | $y^{IV} - y^{III} + \frac{2y^{II}}{x^2} = 0$ |
| 56. $y = \frac{1}{x}, x \neq 0$ | $x^3 y^{II} + x^3 y^I + xy = 0$ |

Derivatives of Powers (Answers)

Answers:

1. $\frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}$
2. $\frac{dy}{dx} = 5x^4 - \frac{3}{2\sqrt{x}}$
3. $\frac{dy}{dx} = \frac{5x^{\frac{3}{2}}}{2} + \frac{2}{x^2}$
4. $y' = 12x^3 + \frac{2}{x^2} - \frac{12}{x^3}$
5. $y' = 2x + 7$
6. $y' = 30x - 4$
7. $y' = 100x^4 - 21x^2 - 3$
8. $y' = 8x^3 + 9x^2 + 2$
9. $y' = 10x^9 - 24x^5 + 8x$
10. $y' = 9x^2 - 4x - 7$
11. $y' = 3x^2 - 6ax + 3a^2$
12. $y' = 24x^2 + 72x + 54$
13. $\frac{dy}{dx} = 18x^2 - 28x + 16$
14. $\frac{dy}{dx} = 12x^3 - 9x^2 - 12x$
15. $\frac{dy}{dx} = 2x - \frac{2}{x^3}$
16. $\frac{dy}{dx} = -\frac{5}{x^2}$
17. $\frac{dy}{dx} = 2x + \frac{2}{x^2}$
18. $\frac{dy}{dx} = 1 - \frac{7}{x^2}$
19. $\frac{dy}{dx} = 1 - \frac{3}{x^2} + \frac{4}{x^3}$
20. $\frac{dy}{dx} = 6x^3 + \frac{7}{x^3} - \frac{33}{2x^4}$
21. $\frac{dy}{dx} = 4 + \frac{9}{x^2}$
22. $\frac{dy}{dx} = -\frac{2}{x^3} - \frac{18}{x^4}$
23. $\frac{dy}{dx} = -\frac{2}{x^3} - \frac{2}{x^5} - \frac{10}{x^6}$
24. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{3}{2\sqrt{x^3}}$
25. $f'(x) = 3ax^2 + 2bx + c$
26. $\frac{dk}{dx} = \frac{1}{a}(2x - \frac{b}{x^2})$
27. $\frac{db}{dm} = 24m^{-9}$
28. $\frac{df}{dx} = 4ax^3 + 2bx$
29. $\frac{dr}{dt} = t^3 - t^2 + t - 1$
30. $\frac{dg}{dh} = -40h^4 - \frac{6}{h^3} + 1.6h^{0.6}$
31. $\frac{dc}{dr} = 2\pi$
32. $\frac{dy}{dx} = m$
33. $\frac{dv}{dt} = a$
34. $\frac{dA}{dr} = 2\pi r$
35. $\frac{dE}{dv} = mv$
36. $\frac{dP}{dV} = \frac{2V}{R}$
37. $\frac{ds}{dt} = u + at$
38. $\frac{dP}{dI} = 2RI$
39. $\frac{dV}{dr} = 4\pi r^2$
40. $\frac{dF}{dr} = \frac{\pi r^3 P}{2\eta l}$
41. $\frac{dT}{dt} = \frac{\pi}{\sqrt{gl}}$
42. $\frac{dv}{dr} = -\sqrt{\frac{GM}{2r^3}}$
43. $\frac{dF}{dr} = -\frac{2GMm}{r^3}$
44. $\frac{dE}{dr} = -\frac{q}{2\pi\epsilon_0 r^3}$
45. $\frac{d\lambda}{dT} = -\frac{h}{\sqrt{8\pi m k T^3}}$
46. $y' = 16x^3 + 6x^2$
 $y'' = 48x^2 + 12x$
 $y''' = 96x + 12$
 $y^{IV} = 96$
47. $\frac{dk}{dx} = 3x^2 + 7$
 $\frac{d^2k}{dx^2} = 6x$
 $\frac{d^3k}{dx^3} = 6$
 $\frac{d^4k}{dx^4} = 0$
48. $\frac{db}{dm} = 2m^{-3} + 9m^2$
 $\frac{d^2b}{dm^2} = -6m^{-4} + 18m$
 $\frac{d^3b}{dm^3} = 24m^{-5} + 18$
 $\frac{d^4b}{dm^4} = -120m^{-6}$
49. $\frac{dy}{dx} = 2x + 3 = 7$ at (2,1)
50. $\frac{dy}{dx} = 6x^2 = 54$ at (3,-1)
51. $y' = -2x - \frac{1}{x^2} = \frac{15}{4}$ at (-2,3)
52. $y = 5x - 3$
53. $y = 7x + 2$
54. $y = 3x - 1$
55. $y' = 4x^3 + 3, y'' = 12x^2, y''' = 24x, y^{IV} = 24$ and
 $x = 2$
56. $y' = -\frac{1}{x^2}, y'' = \frac{2}{x^3}$ and
 $x = 3$

Chain Rule

Differentiate:

- | | | |
|-------------------------------------|---------------------------------------|---|
| 1. $(x + 3)^4$ | 2. $(2x + 5)^3$ | 3. $(1 - x)^7$ |
| 4. $(7x - 2)^6$ | 5. $(x^2 + 1)^3$ | 6. $(x^6 + x^3)^{20}$ |
| 7. $(x^2 - 1)^{100}$ | 8. $(3x^2 - 2x)^2$ | 9. $2(5x - 3)^8$ |
| 10. $3(x + 13)^2$ | 11. $(3x^2 + 7x)^4$ | 12. $(x^2 + 7x - 1)^8$ |
| 13. $2(x^7 + 3x^2 - 1)^6$ | 14. $3(x^5 - 2x)^2$ | 15. $(5x^2 + 4)^{11}$ |
| 16. $\frac{(x - x^2 - x^4)^5}{5}$ | 17. $\frac{(5 - x)^{-2}}{2}$ | 18. $(2x + 1)^{-1}$ |
| 19. $(9 - 4x)^{-3}$ | 20. $(4x^2 - 3x^3 + x)^{-2}$ | 21. $5(x^2 - 9)^{-3}$ |
| 22. $(x^{-1} - 2x^{-2})^{-3}$ | 23. $(3x + 1)^{\frac{1}{2}}$ | 24. $(6x + 1)^{\frac{1}{3}}$ |
| 25. $(5x + 7)^{\frac{3}{2}}$ | 26. $(x^3 - 5x^2 + x)^{\frac{3}{4}}$ | 27. $(x^5 - 5x)^{\frac{1}{5}}$ |
| 28. $(4x^2 - 6x + 1)^{\frac{7}{3}}$ | 29. $(2x^3 - 9x + 12)^{-\frac{2}{3}}$ | 30. $(x^{-4} + 7x^{-2} + 8)^{-\frac{5}{2}}$ |

Find y' :

- | | | |
|--|-------------------------------------|---------------------------------------|
| 31. $y = \frac{1}{3x - 1}$ | 32. $y = \frac{2}{3x^2 - x + 5}$ | 33. $y = \frac{1}{x^3 + x^2 + x + 1}$ |
| 34. $y = \frac{3}{(7x^2 - 3x + 7)^{10}}$ | 35. $y = \frac{1}{9 - x^2}$ | 36. $y = \frac{7}{3(5x^2 + 2)^3}$ |
| 37. $y = \frac{3}{4(2x - 5)^8}$ | 38. $y = \frac{1}{x^4 + 5x^3 - 2x}$ | 39. $y = \frac{1}{12(4x - 1)^3}$ |
| 40. $y = \sqrt{2x + 5}$ | 41. $y = \sqrt{x^2 - 3}$ | 42. $y = \sqrt[3]{9x - 4}$ |
| 43. $y = \sqrt[3]{x^3 - 3x}$ | 44. $y = \sqrt{4 - x^2}$ | 45. $y = \sqrt[3]{8x^3 + 27}$ |
| 46. $y = \sqrt[3]{(11 - 3x)^2}$ | 47. $y = \sqrt[3]{(4x - 1)^4}$ | 48. $y = \sqrt[5]{7x^3 - 2x^2 + 5}$ |

Find the derivative:

- | | | |
|---------------------------------------|---|---|
| 49. $y = \sqrt[4]{7m^3 - 4m^2 + 2}$ | 50. $c = \sqrt[3]{(4k^2 + 3)^2}$ | 51. $r = \sqrt[4]{(4w + 3)^5}$ |
| 52. $d = \frac{3}{\sqrt{x + 2}}$ | 53. $f = \frac{2}{\sqrt{4e + 5}}$ | 54. $g(x) = \frac{1}{\sqrt{1 - 2x}}$ |
| 55. $k(n) = \frac{4}{\sqrt{n^2 + 6}}$ | 56. $p(r) = \frac{12}{\sqrt[4]{(7 - r)^5}}$ | 57. $q(z) = \frac{5}{\sqrt[5]{z^5 - 32}}$ |

Calculate $\frac{dy}{dx}$ for the following functions:

- | | | |
|--|---|---|
| 58. $y = (x - 3)\sqrt{(x - 3)}$ | 59. $y = \sqrt{4 - \sqrt{x + 2}}$ | 60. $y = [(2x + 1)^{10} + 1]^{10}$ |
| 61. $y = (x - \frac{1}{x})^4$ | 62. $y = (x^2 + \frac{1}{x^2})^3$ | 63. $y = (3x - 1)^{2k+1}$ |
| 64. $y = (\frac{x^3}{3} + \frac{x^2}{2} + x)^{-k}$ | 65. $y = [(2x + 1)^2 + (x + 1)^2]^3$ | 66. $y = (7x + \sqrt{x^2 + 3})^6$ |
| 67. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ | 68. $y = [(1 + \frac{1}{x})^{-1} + 1]^{-1}$ | 69. $y = [x^2 + (x^2 + 9)^{\frac{1}{2}}]^{\frac{1}{2}}$ |

Chain Rule (Answers)

1. $4(x+3)^3$
2. $6(2x+5)^2$
3. $-7(1-x)^6$
4. $42(7x-2)^5$
5. $6x(x^2+1)^2$
6. $60x^{59}(2x^3+1)(x^3+1)^{19}$
7. $200x(x^2-1)^{99}$
8. $4x(3x-1)(3x-2)$
9. $80(5x-3)^7$
10. $6(x+13)$
11. $4x^3(6x+7)(3x+7)^3$
12. $8(x^2+7)(x^2+7x-1)^7$
13. $12x(7x^5+6)(x^7+3x^2-1)^5$
14. $6x(x^4-2)(5x^4-2)$
15. $110x(5x^2+4)^{10}$
16. $(1-2x-4x^3)(x-x^2-x^4)^4$
17. $(5-x)^{-3}$
18. $-2(2x+1)^{-2}$
19. $12(9-4x)^{-4}$
20. $-\frac{2(8x-9x^2+1)}{(4x^2-3x^3+x)^3}$
21. $-30x(x^2-9)^{-4}$
22. $-3x^5(4-x)(x-2)^{-4}$
23. $\frac{3}{2\sqrt{3x+1}}$
24. $2(6x+1)^{-\frac{2}{3}}$
25. $\frac{15(5x+7)^{\frac{1}{2}}}{2}$
26. $\frac{3(3x^2-10x+1)}{4\sqrt[4]{x^3-5x^2+x}}$
27. $(x^4-1)(x^5-5x)^{-\frac{4}{5}}$
28. $\frac{14(4x-3)(4x^2-6x+1)^{\frac{4}{3}}}{3}$
29. $-\frac{2(2x^2-3)}{\sqrt[3]{(2x^3-9x+12)^5}}$
30. $\frac{5(2-7x^2)}{x^5\sqrt{(x^{-4}+7x^{-2}+8)^7}}$
31. $y' = -\frac{3}{3x-1^2}$
32. $y' = -\frac{2(6x-1)}{3x^2-x+5^2}$
33. $y' = -\frac{(3x^2+2x+1)}{x^3+x^2+x+1^2}$
34. $y' = -\frac{30(14x-3)}{(7x^2-3x+7)^{11}}$
35. $y' = \frac{2x}{(9-x^2)^2}$
36. $y' = -\frac{70x}{(5x^2+2)^4}$
37. $y' = -\frac{12}{(2x-5)^9}$
38. $y' = \frac{(4x^3+15x-2)}{(x^4+5x^3-2x)^2}$
39. $y' = -\frac{1}{(4x-1)^4}$
40. $y' = \frac{1}{\sqrt{2x+5}}$
41. $y' = \frac{x}{\sqrt{x^2-3}}$
42. $y' = \frac{3}{\sqrt[3]{(9x-4)^2}}$
43. $y' = \frac{x^2-1}{\sqrt[3]{(x^3-3x)^2}}$
44. $y' = -\frac{x}{\sqrt{4-x^2}}$
45. $y' = \frac{8x^2}{\sqrt[3]{(8x^3+27)^2}}$
46. $y' = -\frac{2}{\sqrt[3]{11-3x}}$
47. $y' = \frac{16\sqrt[3]{4x-1}}{3}$
48. $y' = \frac{x(21x-4)}{5\sqrt[5]{(7x^3-2x^2+5)^4}}$
49. $\frac{dy}{dm} = \frac{21m^2-8m}{4\sqrt[4]{(7m^3-4m^2+2)^3}}$
50. $\frac{dc}{dk} = \frac{16k}{3\sqrt[3]{4k^2+3}}$
51. $\frac{dr}{dw} = 5\sqrt[4]{4w+3}$
52. $\frac{dd}{dx} = -\frac{3}{2\sqrt{(x+2)^3}}$
53. $\frac{df}{de} = -\frac{4}{\sqrt{(4e+5)^3}}$
54. $g'(x) = \frac{1}{\sqrt{(1-2x)^3}}$
55. $k'(n) = -\frac{4n}{\sqrt{(n^2+6)^3}}$
56. $p'(r) = \frac{15}{\sqrt[4]{(7-r)^9}}$
57. $q'(z) = -\frac{5z^4}{\sqrt[5]{(z^5-32)^6}}$
58. $\frac{dy}{dx} = \frac{3\sqrt{(x-3)}}{2}$
59. $\frac{dy}{dx} = \frac{1}{4\sqrt{4-\sqrt{x+2}}\sqrt{x+2}}$
60. $\frac{dy}{dx} = 200(2x+1)^9 \cdot [(2x+1)^{10}+1]^9$
61. $\frac{dy}{dx} = 4(1+\frac{1}{x^2})(x-\frac{1}{x})^3$
62. $\frac{dy}{dx} = 6(x-\frac{1}{x^3})(x^2+\frac{1}{x^2})^2$
63. $\frac{dy}{dx} = 3(2k+1)(3x-1)^{2k}$
64. $\frac{dy}{dx} = -\frac{k(x^2+x+1)}{(\frac{x^3}{3}+\frac{x^2}{2}+x)^{k+1}}$
65. $\frac{dy}{dx} = 6(5x+3) \cdot [(2x+1)^2+(x+1)^2]^2$
66. $\frac{dy}{dx} = 6(7x+\sqrt{x^2+3})^5 \cdot [7+\frac{x}{\sqrt{x^2+3}}]$
67. $\frac{dy}{dx} = \frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}$
68. $\frac{dy}{dx} = \frac{1}{x^2(1+\frac{1}{x})^2[(1+\frac{1}{x})^{-1}+1]^2}$
69. $\frac{dy}{dx} = \frac{2x+\frac{x}{\sqrt{x^2+9}}}{\sqrt{x^2+(x^2+9)^{\frac{1}{2}}}}$

Product Rule

Expand and differentiate, and compare by differentiating using the product rule:

- | | | |
|-----------------------|-------------------|----------------------------|
| 1. $(3x - 1)(4x + 3)$ | 2. $5x(6x - 1)$ | 3. $x^3(3x + 2)$ |
| 4. $4x^2(2x^3 - 1)$ | 5. $3x^4(7x - 2)$ | 6. $(2x^2 - 5)(3x^2 + 8x)$ |

Differentiate mentally without simplification (check your answers below):

- | | | |
|----------------------------|------------------------------------|-------------------------------------|
| 7. $(3x - 2)(5x + 7)$ | 8. $(4 - x^2)(3x + 5)$ | 9. $(5x - 2)(x - 3)$ |
| 10. $(5x - 2)(x - 1)$ | 11. $(x^2 - 1)(x + 7)$ | 12. $(x^4 + 8)(6 - 5x)$ |
| 13. $(2x + 9)(x^2 - 4)$ | 14. $(3x + 5)(8x - 1)$ | 15. $(9x^2 - 5)(3x - 8)$ |
| 16. $5x^2(3x - 8)$ | 17. $4x^5(2x^2 - 5x + 3)$ | 18. $(x^2 - 7x + 1)(3x - 4)$ |
| 19. $(5x^3 + 2)(4x - x^3)$ | 20. $(x^2 + 3x + 1)(x^3 - 4x + 5)$ | 21. $(x^{100} - 5)(5x^8 - 11x + 1)$ |

Differentiate on paper without simplification:

- | | | |
|------------------------------|------------------------------------|--------------------------------|
| 22. $(x + 2)(x^2 - 2x + 7)$ | 23. $(1 - x^3)(7x + 4)$ | 24. $(3x - 5)(x^3 + 2x^2 - 8)$ |
| 25. $(x^2 - 2)(5x - x^3)$ | 26. $(x^2 + 3x - 1)(x^3 - 4x + 7)$ | 27. $(x^3 - 2x + 8)(6 - 5x)$ |
| 28. $(8x^2 - 5x)(13x^2 - 4)$ | 29. $(x^5 - 2x^3)(7x^2 + x - 8)$ | 30. $(3 - x^3)(8x + 1)$ |
| 31. $(x + 1)(x + 2)(x + 3)$ | 32. $(x + 1)(x^2 + 2)(x^3 + 3)$ | 33. $4x(x - 1)(2x - 3)$ |

Differentiate (using an embedded chain rule):

- | | | |
|-----------------------------------|--------------------------------------|--|
| 34. $x^2(x + 1)^3$ | 35. $8x(3x - 2)^5$ | 36. $2x^4(3 - x)^3$ |
| 37. $(x + 1)(2x - 5)^4$ | 38. $(x^3 - 4x^2 + 1)(x^2 + 3)^5$ | 39. $(3x^2 - 2x - 1)(x^2 + 5x - 2)^2$ |
| 40. $4x^3(2x - 1)^{-3}$ | 41. $2x^8(11 - x)^{-7}$ | 42. $(4x - 3)(5x + 3)^{-2}$ |
| 43. $2x(x + 3)^{\frac{1}{2}}$ | 44. $(3x - 1)(6 - x)^{-\frac{3}{4}}$ | 45. $4x^3(8x - 1)^{\frac{4}{5}}$ |
| 46. $(2x + 9)\sqrt{x^2 - 4}$ | 47. $x\sqrt{11 - x}$ | 48. $x^2\sqrt[3]{4x - 7}$ |
| 49. $(3x + 8)^3(x + 1)^4$ | 50. $(2x^2 - 3x + 1)^2(5x - 1)^4$ | 51. $(5x + 3)^4(x - 2)^{-\frac{1}{2}}$ |
| 52. $(8x + 1)^3\sqrt[4]{9 - x^3}$ | 53. $(2x + 7)^8\sqrt[3]{(x + 4)^5}$ | 54. $(6x + 1)^8(3x - 7)^{-5}$ |
| 55. $\frac{x}{(2x - 9)^5}$ | 56. $(x + x^{-1})(x - x^{-2})$ | 57. $(x + \sqrt{x})(1 + \sqrt{x})$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | |
|-------------------------------|-----------------------------|---------------------------------------|
| 58. $y = x^2(x + 1)^3$ (-1,0) | 59. $y = x(3x - 2)^2$ (1,1) | 60. $y = (x + 1)\sqrt{x + 3}$ (-2,-1) |
|-------------------------------|-----------------------------|---------------------------------------|

Solutions to the Mental Calculations Above

- | | | |
|--|---|---|
| 7. $3(5x + 7) + (3x - 2)5$ | 8. $-2x(3x + 5) + (4 - x^2)3$ | 9. $5(x - 3) + (5x - 2)1$ |
| 10. $5(x - 1) + (5x - 2)1$ | 11. $2x(x + 7) + (x^2 - 1)1$ | 12. $4x^3(6 - 5x) + (x^4 + 8)(-5)$ |
| 13. $2(x^2 - 4) + (2x + 9)(2x)$ | 14. $3(8x - 1) + (3x + 5)8$ | 15. $18x(3x - 8) + (9x^2 - 5)3$ |
| 16. $10x(3x - 8) + 5x^2(3)$ | 17. $20x^4(2x^2 - 5x + 3) + 4x^5(4x - 5)$ | 18. $(2x - 7)(3x - 4) + (x^2 - 7x + 1)3$ |
| 19. $15x^2(4x - x^3) + (5x^3 + 2)(4 - 3x^2)$ | 20. $(2x + 3)(x^3 - 4x + 5) + (x^2 + 3x + 1)(3x^2 - 4)$ | 21. $100x^{99}(5x^8 - 11x + 1) + (x^{100} - 5)(40x^7 - 11)$ |

Product Rule (Answers)

Simplified answers:

- | | | |
|-----------------|---------------------|-------------------------------|
| 1. $24x + 5$ | 2. $60x - 5$ | 3. $12x^3 + 6x^2$ |
| 4. $40x^4 - 8x$ | 5. $105x^5 - 24x^3$ | 6. $24x^3 + 48x^2 - 30x - 40$ |

Answers with structure, but no simplification:

- | | | |
|---|---|---|
| 22. $1(x^2 - 2x + 7) + (x + 2)(2x - 2)$ | 23. $-3x^2(7x + 4) + (1 - x^3)(7)$ | 24. $3(x^3 + 2x^2 - 8) + (3x - 5)(3x^2 + 4x)$ |
| 25. $2x(5x - x^3) + (x^2 - 2)(5 - 3x^2)$ | 26. $(2x + 3)(x^3 - 4x + 7) + (x^2 + 3x - 1)(3x^2 - 4)$ | 27. $(3x^2 - 2)(6 - 5x) + (x^3 - 2x + 8)(-5)$ |
| 28. $(16x - 5)(13x^2 - 4) + (8x^2 - 5x)(26x)$ | 29. $(5x^4 - 6x^2)(7x^2 + x - 8) + (x^5 - 2x^3)(14x + 1)$ | 30. $-3x^2(8x + 1) + (3 - x^3)(8)$ |
| 31. $(1)(x + 2)(x + 3) + (x + 1)(1)(x + 3) + (x + 1)(x + 2)(1)$ | 32. $(1)(x^2 + 2)(x^3 + 3) + (x + 1)(2x)(x^3 + 3) + (x + 1)(x^2 + 2)(3x^2)$ | 33. $4(x - 1)(2x - 3) + 4x(1)(2x - 3) + 4x(x - 1)(2)$ |

Simplified answers (equivalent expressions use or remove radicals or negative indices):

- | | | |
|--|---|--|
| 34. $x(5x + 2)(x + 1)^2$ | 35. $16(9x - 1)(3x - 2)^4$ | 36. $-2x^3(7x - 12)(3 - x)^2$ |
| 37. $(10x + 3)(2x - 5)^3$ | 38. $x(13x^3 - 48x^2 + 9x - 14) \cdot (x^2 + 3)^4$ | 39. $2(x^2 + 5x - 2) \cdot (5x^3 + 29x^2 + 10x - 8)$ |
| 40. $-12x^2(2x - 1)^{-4}$ | 41. $-2x^7(x - 88)(11 - x)^{-8}$ | 42. $-2(10x - 21)(5x + 3)^{-3}$ |
| 43. $3(x + 2)(x + 3)^{-\frac{1}{2}}$ | 44. $-\frac{3(4x^2 - 27x + 1)(6 - x)^{-\frac{7}{4}}}{4}$ | 45. $\frac{4x^2(152x - 15)(8x - 1)^{-\frac{1}{5}}}{5}$ |
| 46. $\frac{4x^2 + 9x - 8}{\sqrt{x^2 - 4}}$ | 47. $-\frac{(3x - 22)}{2\sqrt{11 - x}}$ | 48. $\frac{14x(2x - 3)}{3^3\sqrt{(4x - 7)^2}}$ |
| 49. $\frac{(21x + 41)(3x + 8)^2}{(x + 1)^3}$ | 50. $\frac{2(2x^2 - 3x + 1)}{(40x^2 - 49x + 13)(5x - 1)^3}$ | 51. $\frac{(35x - 43)(5x + 3)^3}{2\sqrt{(x - 1)^3}}$ |
| 52. $-\frac{(8x + 1)^2(120x^3 + 3x^2 - 864)}{4^4\sqrt{(9 - x^3)^3}}$ | 53. $\frac{1}{3} \frac{(58x + 227)(2x + 7)^7}{\sqrt[3]{(x + 4)^2}}$ | 54. $\frac{3(18x - 117)(6x + 1)^7}{(3x - 7)^6}$ |
| 55. $-\frac{8x + 9}{(2x - 9)^6}$ | 56. $2x + \frac{1}{x^2} + \frac{3}{x^4}$ | 57. $2 + \frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}$ |

The equations of the tangents are:

- | | | |
|-------------|------------------|-----------------------|
| 58. $y = 0$ | 59. $y = 7x - 6$ | 60. $y = \frac{x}{2}$ |
|-------------|------------------|-----------------------|

Quotient Rule

Divide each term of the numerator by the denominator before differentiating. Compare this with the result you get by using the quotient rule:

1. $\frac{x+6}{x}$

2. $\frac{x^3-1}{x^2}$

3. $\frac{3x-1}{x^2}$

4. $\frac{x^3-3x^2}{x}$

5. $\frac{x^4+2x^3}{x^2}$

6. $\frac{2x^2+5x-1}{x}$

Using negative indices, differentiate by using the product rule. Compare this with the result you get by using the quotient rule:

7. $\frac{x+5}{2x+1}$

8. $\frac{x-3}{5x+2}$

9. $\frac{3x+8}{x-5}$

10. $\frac{x^2-2}{x^2+9}$

11. $\frac{x^3}{x+4}$

12. $\frac{x+1}{3x^2-7}$

Differentiate mentally without simplification (check your answers on the following page):

13. $\frac{1}{2x-1}$

14. $\frac{x^3}{x^2-4}$

15. $\frac{x+4}{x-6}$

16. $\frac{2x+5}{4x-3}$

17. $\frac{x}{2x^2-8}$

18. $\frac{x-7}{x^2}$

19. $\frac{x^2+4x-1}{x+3}$

20. $\frac{x^2-9x+11}{2x+5}$

21. $\frac{3x-1}{x^2+12}$

22. $\frac{6x+7}{x^2-x+3}$

23. $\frac{x^3+x}{x^2-x-1}$

24. $\frac{5x^2-2x}{3x+1}$

Differentiate (using an embedded chain rule):

25. $\frac{2x}{(x+1)^{\frac{1}{2}}}$

26. $\frac{(2x+7)^3}{4x-1}$

27. $\frac{x-1}{(7x+3)^4}$

28. $\frac{(3x-4)^5}{(2x+1)^3}$

29. $\frac{2x-5}{\sqrt{x+1}}$

30. $\frac{\sqrt{x-1}}{4x+1}$

31. $\frac{\sqrt{x^2+1}}{(x-8)^2}$

32. $\frac{x-4}{\sqrt[3]{x}}$

33. $\frac{(x+3)^4}{x^2}$

Find the derivative of:

34. $y = \frac{x+3}{\sqrt{x}+2}$

35. $f = \frac{p^{\frac{2}{3}}}{2p+1}$

36. $b = \frac{\sqrt[3]{w}}{w^2+5}$

37. $m = \frac{h^3-1}{h^3+1}$

38. $g = \frac{7t^4+11}{t+8}$

39. $e = \left(\frac{4y+3}{5y-1}\right)^3$

40. $k = \frac{8n^2-5n+11}{n+2}$

41. $r = \frac{v-6}{\sqrt{(v+1)^5}}$

42. $z = \frac{4a-9}{(a+5)^{\frac{3}{4}}}$

Find the equation of the tangent to the following curves at the point indicated):

43. $y = \frac{x+3}{x-1}$

(2,5)

44. $y = \frac{x+3}{3x-2}$

(1,4)

45. $y = \frac{x}{x-2}$

(3,3)

46. $y = \frac{x^2}{x-1}$

(-1,-1/2)

47. $y = \frac{x^2+1}{x^2-3}$

(2,5)

48. $y = \frac{x-7}{x^2}$

(1,-6)

Quotient Rule (Answers)

You should discover that the results are the same and that it is advisable to simplify expressions first!

$$\begin{array}{lll} 1. & -\frac{6}{x^2} & 2. & \frac{x^3 + 2}{x^3} = 1 + \frac{2}{x^3} & 3. & -\frac{3x - 2}{x^3} \\ 4. & 2x - 3 & 5. & 2x + 2 & 6. & \frac{2x^2 + 1}{x^2} = 2 + \frac{1}{x^2} \end{array}$$

You should discover that the results are the same and that the quotient rule is (usually) simpler to use.

$$\begin{array}{lll} 7. & -\frac{9}{(2x + 1)^2} & 8. & \frac{17}{(5x + 2)^2} & 9. & -\frac{23}{(x - 5)^2} \\ 10. & \frac{22x}{(x^2 + 9)^2} & 11. & \frac{2x^2(x + 6)}{(x + 4)^2} & 12. & -\frac{3x^2 + 6x + 7}{(3x^2 - 7)^2} \end{array}$$

You should have found the following structures:

$$\begin{array}{lll} 13. & \frac{0(2x - 1) - 1(2)}{(2x - 1)^2} & 14. & \frac{3x^2(x^2 - 4) - x^3(2x)}{(x^2 - 4)^2} & 15. & \frac{1(x - 6) - (x + 4)1}{(x - 6)^2} \\ 16. & \frac{2(4x - 3) - (2x + 5)4}{(4x - 3)^2} & 17. & \frac{1(2x^2 - 8) - x(4x)}{(2x^2 - 8)^2} & 18. & \frac{1(x^2) - (x - 7)(2x)}{x^4} \\ 19. & \frac{(2x + 4)(x + 3) - (x^2 + 4x - 1)1}{(x + 3)^2} & 20. & \frac{(2x - 9)(2x + 5) - (x^2 - 9x + 11)2}{(2x + 5)^2} & 21. & \frac{3(x^2 + 12) - (3x - 1)(2x)}{(x^2 + 12)^2} \\ 22. & \frac{6(x^2 - x - 3) - (6x + 7)(2x - 1)}{(x^2 - x + 3)^2} & 23. & \frac{(3x^2 + 1)(x^2 - x - 1) - (x^3 + x)(2x - 1)}{(x^2 - x - 1)^2} & 24. & \frac{(10x - 2)(3x + 1) - (5x^2 - 2x)3}{(3x + 1)^2} \end{array}$$

The derivatives are:

$$\begin{array}{lll} 25. & \frac{x + 2}{(x + 1)^{\frac{3}{2}}} & 26. & \frac{2(8x - 17)(2x + 7)^2}{(4x - 1)^2} & 27. & -\frac{21x - 31}{(7x + 3)^5} \\ 28. & \frac{3(4x + 13)(3x - 4)^4}{(2x + 1)^4} & 29. & \frac{2x + 9}{2\sqrt{(x + 1)^3}} & 30. & -\frac{4x - 9}{2\sqrt{x - 1}(4x + 1)^2} \\ 31. & -\frac{x^2 + 8x + 2}{\sqrt{x^2 + 1}(x - 8)^3} & 32. & \frac{2(x + 2)}{3\sqrt[3]{x^4}} & 33. & \frac{2(x - 3)(x + 3)^3}{x^3} \end{array}$$

The derivatives are:

$$\begin{array}{lll} 34. & \frac{dy}{dx} = \frac{x + 4\sqrt{x} - 3}{2\sqrt{x}(\sqrt{x} + 2)^2} & 35. & \frac{df}{dp} = -\frac{2(p - 1)}{3\sqrt[3]{p}(2p + 1)^2} & 36. & \frac{db}{dw} = -\frac{5(w^2 - 1)}{3w^{\frac{2}{3}}(w^2 + 5)^2} \\ 37. & \frac{dm}{dh} = \frac{6h^2}{(h^3 + 1)^2} & 38. & \frac{dg}{dt} = \frac{21t^4 + 224t^3 - 11}{(t + 8)^2} & 39. & \frac{de}{dy} = -\frac{57(4y + 3)^2}{(5y - 1)^4} \\ 40. & \frac{dk}{dn} = \frac{8n^2 + 32n - 21}{(n + 2)^2} & 41. & \frac{dr}{dv} = -\frac{3v - 32}{2\sqrt{(v + 1)^7}} & 42. & \frac{dz}{da} = \frac{4a + 107}{4(a + 5)^{\frac{7}{4}}} \end{array}$$

The equations of the tangents (in gradient-intercept form) are:

$$\begin{array}{lll} 43. & y = -4x + 13 & 44. & y = -11x + 15 & 45. & y = -2x + 9 \\ 46. & y = \frac{3x}{4} + \frac{1}{4} & 47. & y = -16x + 37 & 48. & y = 13x - 19 \end{array}$$

Exponential Functions

Differentiate with respect to x:

- | | | |
|------------------------------|--------------------------------------|------------------------------|
| 1. $y = e^x$ | 2. $y = 3e^x$ | 3. $y = e^{4x}$ |
| 4. $y = 2e^{5x}$ | 5. $y = e^{5x} - e^{2x}$ | 6. $y = 2e^{3x} + e^{-x}$ |
| 7. $y = e^{3.5x} + e^{1.9x}$ | 8. $y = 6e^{2x} - \frac{e^{-2x}}{2}$ | 9. $y = e^{2x} \cdot e^{7x}$ |
| 10. $y = e^{x^2}$ | 11. $y = e^{x^2-2x+7}$ | 12. $y = 3e^{-x^4}$ |
| 13. $y = 8e^{5x-1}$ | 14. $y = x^3 e^{2x}$ | 15. $y = (2x + 1)e^{-x}$ |

Find the derivative function:

- | | | |
|------------------------------|------------------------|-----------------------------------|
| 16. $y = xe^{-2x}$ | 17. $y = x^3 e^{-x}$ | 18. $y = x^3 - xe^{4x}$ |
| 19. $y = (x^2 - 6)e^{8x}$ | 20. $y = \sqrt{x}e^x$ | 21. $y = 4e^{2x^2}$ |
| 22. $y = xe^{x^2}$ | 23. $y = e^{(e^x)}$ | 24. $y = \frac{e^{2x+1}}{2x+7}$ |
| 25. $y = \frac{e^{3x}}{x^2}$ | 26. $y = e^{\sqrt{x}}$ | 27. $y = \frac{e^x + 1}{e^x - 1}$ |

Calculate $\frac{dy}{dx}$:

- | | | |
|--|---------------------------------|---|
| 28. $y = \frac{x}{e^{-x}}$ | 29. $y = (e^x + 2)^8$ | 30. $y = e^{\sqrt[4]{x}} - e^{\frac{1}{x}}$ |
| 31. $y = e^{\sqrt{x}} + e^{\frac{5}{4}}$ | 32. $y = 4x^3 + 3x^2 - e^{-2x}$ | 33. $y = 2e^{1-x}$ |
| 34. $y = (e^x + x)^{10}$ | 35. $y = e^{x^3+1}$ | 36. $y = x^e e^x$ |

Find the derivative of:

- | | | |
|----------------------------------|---------------------------|----------------------------------|
| 37. $y = x^e e^{x-e}$ | 38. $p = m^2 e^{-\pi m}$ | 39. $a = \frac{k-1}{e^{2k}-1}$ |
| 40. $v = (t^2 - 3t)e^{8t}$ | 41. $j = \sqrt{d}e^{d+4}$ | 42. $b = \frac{e^{2q}}{e^q + 4}$ |
| 43. $f = \frac{e^g - e^{-g}}{2}$ | 44. $h = A + Be^{-6w}$ | 45. $l = \frac{1-n^2}{2e^n}$ |

Given the function on the left, demonstrate that the relationship on the right is true:

- | | |
|------------------------------|------------------------|
| 46. $y = e^x + e^{-x}$ | $y'' = y$ |
| 47. $y = 4e^{-x} + 5e^{-3x}$ | $y'' + 4y' + 3y = 0$ |
| 48. $y = e^{2x} + e^{8x}$ | $y'' - 10y' + 16y = 0$ |
| 49. $y = e^{2x} + e^{4x}$ | $y'' - 6y' + 8y = 0$ |
| 50. $y = (x+1)e^{5x}$ | $y'' - 10y' + 25y = 0$ |
| 51. $y = A + Be^{-4x}$ | $y'' + 4y' = 0$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | | | | |
|---------------|-------|------------------|-------|-------------------|-------|
| 52. $y = e^x$ | (0,1) | 53. $y = e^{-x}$ | (0,1) | 54. $y = e^{x-2}$ | (3,e) |
|---------------|-------|------------------|-------|-------------------|-------|

Find the minimum value of each function (and its location) given:

- | | | |
|--------------------|---------------------|--------------------|
| 55. $y = (x-2)e^x$ | 56. $y = -e^{-x^2}$ | 57. $y = xe^{x-1}$ |
|--------------------|---------------------|--------------------|

Exponential Functions (Answers)

The required derivatives are:

- | | | |
|--|---|--|
| 1. $y' = e^x$ | 2. $y' = 3e^x$ | 3. $y' = 4e^{4x}$ |
| 4. $y' = 10e^{5x}$ | 5. $y' = 5e^{5x} - 2e^{2x}$ | 6. $y' = 6e^{3x} - e^{-x}$ |
| 7. $y' = 3.5e^{3.5x} + 1.9e^{1.9x}$ | 8. $y' = 12e^{2x} + e^{-2x}$ | 9. $y' = 9e^{9x}$ |
| 10. $y' = 2xe^{x^2}$ | 11. $y' = (2x - 2)e^{x^2 - 2x + 7}$ | 12. $y' = -12x^3e^{-x^4}$ |
| 13. $y' = 40e^{5x-1}$ | 14. $y' = 3x^2e^{2x} + 2x^3e^{2x}$ | 15. $y' = e^{-x} - 2xe^{-x}$ |
| 16. $y' = e^{-2x} - 2xe^{-2x}$ | 17. $y' = 3x^2e^{-x} - x^3e^{-x}$ | 18. $y' = 3x^2 - e^{4x} - 4xe^{4x}$ |
| 19. $y' = 2(4x^2 + x - 24)e^{8x}$ | 20. $y' = \frac{e^x}{2\sqrt{x}} + \sqrt{x}e^x$ | 21. $y' = 16xe^{2x^2}$ |
| 22. $y' = e^{x^2} + 2x^2e^{x^2}$ | 23. $y' = e^xe^{(e^x)}$ | 24. $y' = \frac{4(x+3)e^{2x+1}}{2x+7^2}$ |
| 25. $y' = \frac{(3x-2)e^{3x}}{x^3}$ | 26. $y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 27. $y' = -\frac{2e^x}{(e^x-1)^2}$ |
| 28. $\frac{dy}{dx} = e^x + xe^x$ | 29. $\frac{dy}{dx} = 8e^x(e^x + 2)^7$ | 30. $\frac{dy}{dx} = \frac{e^{\frac{4}{\sqrt{x}}}}{4\sqrt[4]{x^3}} - \frac{e^{-\frac{1}{x}}}{x^2}$ |
| 31. $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 32. $\frac{dy}{dx} = 12x^2 + 6x + 2e^{-2x}$ | 33. $\frac{dy}{dx} = -2e^{1-x}$ |
| 34. $\frac{dy}{dx} = 10(e^x + 1)(e^x + x)^9$ | 35. $\frac{dy}{dx} = 3x^2e^{x^3+1}$ | 36. $\frac{dy}{dx} = (x + e)x^{e-1}e^x$ |
| 37. $\frac{dy}{dx} = (x + e)x^{e-1}e^{x-e}$ | 38. $\frac{dp}{dm} = (2 - \pi m)me^{-\pi m}$ | 39. $\frac{da}{dk} = \frac{3e^{2k} - 2ke^{2k} - 1}{(e^{2k} - 1)^2}$ |
| 40. $\frac{dv}{dt} = (8t^2 - 22t - 3)e^{8t}$ | 41. $\frac{dj}{dd} = \frac{(2d+1)e^{d+4}}{2\sqrt{d}}$ | 42. $\frac{db}{dq} = \frac{e^{3q} + 8e^{2q}}{(e^q + 4)^2}$ |
| 43. $\frac{df}{dg} = \frac{e^g + e^{-g}}{2}$ | 44. $\frac{dh}{dw} = -6Be^{-6w}$ | 45. $\frac{dl}{dn} = \frac{n^2 - 2n - 1}{2e^n}$ |

The derivatives required in order to demonstrate the relationships are:

- | | | |
|--|-----------------------------|------------------------------|
| 46. $y = e^x + e^{-x}$ | $y' = e^x - e^{-x}$ | $y'' = e^x + e^{-x}$ |
| 47. $y = 4e^{-x} + 5e^{-3x}$ | $y' = -4e^{-x} - 15e^{-3x}$ | $y'' = 4e^{-x} + 45e^{-3x}$ |
| 48. $y = e^{2x} + e^{8x}$ | $y' = 2e^{2x} + 8e^{8x}$ | $y'' = 4e^{2x} + 64e^{8x}$ |
| 49. $y = e^{2x} + e^{4x}$ | $y' = 2e^{2x} + 4e^{4x}$ | $y'' = 4e^{2x} + 16e^{4x}$ |
| 50. $y = (x+1)e^{5x} = e^{5x} + xe^{5x}$ | $y' = 6e^{5x} + 5xe^{5x}$ | $y'' = 35e^{5x} + 25xe^{5x}$ |
| 51. $y = A + Be^{-4x}$ | $y' = -4Be^{-4x}$ | $y'' = 16Be^{-4x}$ |

The equations of the tangents (in gradient-intercept form) are:

- | | | |
|-----------------|------------------|-------------------|
| 52. $y = x + 1$ | 53. $y = -x + 1$ | 54. $y = ex - 2e$ |
|-----------------|------------------|-------------------|

The minimum values of the functions are:

- | | | |
|----------------------------|----------------------------|---|
| 55. $y = -e$ (at $x = 1$) | 56. $y = -1$ (at $x = 0$) | 57. $y = -\frac{1}{e^2}$ (at $x = -1$) |
|----------------------------|----------------------------|---|

Logarithmic Functions

Differentiate with respect to x:

- | | | |
|------------------------|-----------------------------|--------------------------|
| 1. $y = \log_e x$ | 2. $y = \log_e(3x)$ | 3. $y = 2\log_e x$ |
| 4. $y = \ln(3x + 1)$ | 5. $y = \ln 3x + 1$ | 6. $y = \ln x + 2x$ |
| 7. $y = 5\ln(3x)$ | 8. $y = \ln(x^2)$ | 9. $y = \ln(x^5)$ |
| 10. $y = \ln(x^2 - 5)$ | 11. $y = 2\ln x + 5\ln(2x)$ | 12. $y = 7x - \ln(4x^3)$ |

Simplify, using logarithmic laws, before finding the derivative function:

- | | | |
|-------------------------|-------------------------------|-------------------------|
| 13. $y = \ln\sqrt{x+9}$ | 14. $y = \ln \frac{x+1}{x+3}$ | 15. $y = \ln(x-5)(x+8)$ |
|-------------------------|-------------------------------|-------------------------|

Calculate the derivative:

- | | | |
|-----------------------------|------------------------------|--|
| 16. $y = \ln(2x-1)(x+8)$ | 17. $y = \ln(x+6)^4$ | 18. $y = \ln \frac{(x+1)(x+2)}{(x+3)}$ |
| 19. $y = \ln \frac{x}{x-2}$ | 20. $y = \ln\sqrt{x+4}$ | 21. $y = \ln\sqrt{(x+1)^3}$ |
| 22. $y = \ln \frac{1}{x}$ | 23. $y = \ln \frac{1}{3x+2}$ | 24. $y = \ln \frac{1}{2x^5}$ |

Calculate $\frac{dy}{dx}$:

- | | | |
|--------------------------------|-------------------------|--|
| 25. $y = x \ln x$ | 26. $y = 2x^3 \ln(x+4)$ | 27. $y = x \ln x - 3x$ |
| 28. $y = \ln(x^2)$ | 29. $y = (\ln x)^2$ | 30. $y = \ln(\ln x)$ |
| 31. $y = (1 + \ln x)^5$ | 32. $y = (\ln x - x)^9$ | 33. $y = (x^2 + \ln x)^6$ |
| 34. $y = \frac{\ln x}{x-2}$ | 35. $y = (2x+1)\ln x$ | 36. $y = x^3 \ln(x+1)$ |
| 37. $y = \log(x)$ | 38. $y = \log_7(5x)$ | 39. $y = \log\left(\frac{2x^2-1}{\sqrt{x}}\right)$ |
| 40. $y = \frac{e^{2x}}{\ln x}$ | 41. $y = e^x \ln x$ | 42. $y = \ln \frac{e^x + 1}{e^x - 1}$ |

Given the function on the left, solve the equation on the right:

- | | |
|------------------------|---------------------------|
| 43. $y = \ln x$ | $xy'' + (y')^2 = 2$ |
| 44. $y = (\ln x)^2$ | $xy'' + y' = 1$ |
| 45. $y = x \ln x$ | $y'' + xy' - y = 2$ |
| 46. $y = x^2 \ln x$ | $xy'' - y' = 8$ |
| 47. $y = (x+3)\ln x$ | $xy'' + y' = 3$ |
| 48. $y = \ln(x^2 - 1)$ | $2(x^2 - 1)y'' + 5y' = 0$ |
| 49. $y = e^x \ln x$ | $y'' - y' = 0$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | | | | |
|-------------------------|-------|-------------------|-------|-----------------------|-------|
| 50. $y = \ln\sqrt{2-x}$ | (1,0) | 51. $y = x \ln x$ | (e,e) | 52. $y = e^x + \ln x$ | (1,e) |
|-------------------------|-------|-------------------|-------|-----------------------|-------|

Find the minimum value of each function (and its location) given:

- | | | |
|---------------------|-----------------------|-------------------------------|
| 53. $y = (\ln x)^2$ | 54. $y = x^2 - \ln x$ | 55. $y = \frac{1}{x} + \ln x$ |
|---------------------|-----------------------|-------------------------------|

Logarithmic Functions (Answers)

The derivatives are:

- | | | |
|--|---|--|
| 1. $y' = \frac{1}{x}$ | 2. $y' = \frac{1}{x}$ | 3. $y' = \frac{2}{x}$ |
| 4. $y' = \frac{3}{3x+1}$ | 5. $y' = \frac{1}{x} + 1$ | 6. $y' = \frac{1}{x} + 2$ |
| 7. $y' = \frac{5}{x}$ | 8. $y' = \frac{2}{x}$ | 9. $y' = \frac{5}{x}$ |
| 10. $y' = \frac{2x}{x^2-5}$ | 11. $y' = \frac{7}{x}$ | 12. $y' = 7 - \frac{3}{x}$ |
| 13. $y' = \frac{1}{2(x+9)}$ | 14. $y' = \frac{1}{x+1} - \frac{1}{x+3}$ | 15. $y' = \frac{1}{x-5} + \frac{1}{x+8}$ |
| 16. $y' = \frac{2}{2x-1} + \frac{1}{x+8}$ | 17. $y' = \frac{4}{x+6}$ | 18. $y' = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3}$ |
| 19. $y' = \frac{1}{x} - \frac{1}{x-2}$ | 20. $y' = \frac{1}{2(x+4)}$ | 21. $y' = \frac{3}{2(x+1)}$ |
| 22. $y' = -\frac{1}{x}$ | 23. $y' = -\frac{3}{3x+2}$ | 24. $y' = -\frac{5}{x}$ |
| 25. $\frac{dy}{dx} = \ln x + 1$ | 26. $\frac{dy}{dx} = 6x^2 \ln(x+4) + \frac{2x^3}{x+4}$ | 27. $\frac{dy}{dx} = \ln x - 2$ |
| 28. $\frac{dy}{dx} = \frac{2}{x}$ | 29. $\frac{dy}{dx} = \frac{2 \ln x}{x}$ | 30. $\frac{dy}{dx} = \frac{1}{x \ln x}$ |
| 31. $\frac{dy}{dx} = \frac{5(1 + \ln x)^4}{x}$ | 32. $\frac{dy}{dx} = 9(\ln x - x)^8 \left(\frac{1}{x} - 1\right)$ | 33. $\frac{dy}{dx} = 6(x^2 + \ln x)^5 \left(2x + \frac{1}{x}\right)$ |
| 34. $\frac{dy}{dx} = \frac{(x-2) - x \ln x}{x(x-2)^2}$ | 35. $\frac{dy}{dx} = 2 \ln x + \frac{2x+1}{x}$ | 36. $\frac{dy}{dx} = 3x^2 \ln(x+1) + \frac{x^3}{x+1}$ |
| 37. $\frac{dy}{dx} = \frac{1}{x \ln 10}$ | 38. $\frac{dy}{dx} = \frac{1}{x \ln 7}$ | 39. $\frac{dy}{dx} = \frac{6x^2 + 1}{2 \ln 10 \cdot x(2x^2 - 1)}$ |
| 40. $\frac{dy}{dx} = \frac{(2x \ln x - 1)e^{2x}}{x \ln^2 x}$ | 41. $\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$ | 42. $\frac{dy}{dx} = -\frac{2e^x}{e^{2x} - 1}$ |

The solutions to the equations are:

- | | | |
|-------------------------------|---------------------------|-------------------------|
| 43. $y = \ln x$ | $xy'' + (y')^2 = 2$ | $(\frac{1}{2}, -\ln 2)$ |
| 44. $y = (\ln x)^2 = \ln^2 x$ | $xy'' + y' = 1$ | $(2, \ln^2 2)$ |
| 45. $y = x \ln x$ | $y'' + xy' - y = 2$ | $(1, 0)$ |
| 46. $y = x^2 \ln x$ | $xy'' - y' = 8$ | $(4, 16 \ln 4)$ |
| 47. $y = (x+3) \ln x$ | $xy'' + y' = 3$ | $(e, e+3)$ |
| 48. $y = \ln(x^2 - 1)$ | $2(x^2 - 1)y'' + 5y' = 0$ | $(2, \ln 3)$ |
| 49. $y = e^x \ln x$ | $y'' - y' = 0$ | $(1, 0)$ |

The equations of the tangents (in gradient-intercept form) are:

- | | | |
|--------------------------------------|------------------|----------------------|
| 50. $y = -\frac{x}{2} + \frac{1}{2}$ | 51. $y = 2x - e$ | 52. $y = (e+1)x - 1$ |
|--------------------------------------|------------------|----------------------|

The minimum values of the functions are:

- | | | |
|---------------------------|---|---------------------------|
| 53. $y = 0$ (at $x = 1$) | 54. $y = \frac{1}{2}(1 + \ln 2)$ (at $x = \frac{1}{\sqrt{2}}$) | 55. $y = 1$ (at $x = 1$) |
|---------------------------|---|---------------------------|

Trigonometric Functions

Differentiate with respect to x :

- | | | |
|--|--------------------------------------|--------------------------------|
| 1. $y = \tan x^2$ | 2. $y = \tan^2 x$ | 3. $y = \sin 3x$ |
| 4. $y = 2 \cos x$ | 5. $y = \sin x \cos x$ | 6. $y = \sin x + 5 \cos x$ |
| 7. $y = \tan 2x$ | 8. $y = x \sin x$ | 9. $y = \sin 2x \tan 3x$ |
| 10. $y = \frac{\sin x}{x}$ | 11. $y = \frac{x}{\cos x}$ | 12. $y = \frac{\sin x}{x^2}$ |
| 13. $y = \sin\left(x + \frac{\pi}{4}\right)$ | 14. $y = x^2 + \tan \frac{x}{2}$ | 15. $y = \cos \frac{\pi x}{3}$ |
| 16. $y = \frac{\cos x}{1 + \sin x}$ | 17. $y = \frac{3x + 4}{\sin 5x}$ | 18. $y = x^2 \cos x$ |
| 19. $y = \sin x^3$ | 20. $y = \cos^3 x$ | 21. $y = x^3 \tan 8x$ |
| 22. $y = \sin^2 4x$ | 23. $y = \cos(x^2 + 1)$ | 24. $y = \tan \pi x$ |
| 25. $y = \cos \sqrt{x}$ | 26. $y = \sqrt{\cos 2x}$ | 27. $y = \sin^4 \sqrt{x}$ |
| 28. $y = \tan^3 x$ | 29. $y = \sin^2 x + \cos^2 x$ | 30. $y = (\sin x + \cos x)^2$ |
| 31. $y = \tan(x^2 - 1)$ | 32. $y = \tan(\pi - x)$ | 33. $y = (2x + \tan 7x)^9$ |
| 34. $y = 7 \tan(x^2 + 5)$ | 35. $y = \sin(\cos x)$ | 36. $y = \cos(\sin x)$ |
| 37. $y = \sin x (1 + \cos x)$ | 38. $y = \sqrt[3]{\frac{\tan x}{x}}$ | 39. $y = \tan(\sqrt{\cos x})$ |

Calculate $\frac{dy}{dx}$:

- | | | |
|---------------------|------------------------|----------------------------------|
| 40. $y = \cot x$ | 41. $y = \sec x$ | 42. $y = \operatorname{cosec} x$ |
| 43. $y = \sec^2 4x$ | 44. $y = \tan x^\circ$ | 45. $y = \sin 3x^\circ$ |
| 46. $y = \sin bx$ | 47. $y = \cos(bx + c)$ | 48. $y = \tan(ax^2 + c)$ |

Find the derivative of:

- | | | |
|---------------------------|---------------------------------|--------------------------------|
| 49. $y = e^x \sin x$ | 50. $m = e^{\sin w}$ | 51. $j = e^{-a} \cos a$ |
| 52. $p = e^{4r} \sin 2r$ | 53. $s = \sin(e^t + t)$ | 54. $b = \cos(\ln s)$ |
| 55. $k = \ln(\cos q)$ | 56. $c = \ln(\sin 2n)$ | 57. $f = \ln(\tan h^2)$ |
| 58. $l = \sin(1 - \ln v)$ | 59. $z = \frac{e^{4u}}{\tan u}$ | 60. $q = \frac{\sin m^2}{e^m}$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | | | | |
|--------------------|--------|-------------------------|---|------------------|--|
| 61. $y = x \cos x$ | (0, 0) | 62. $y = \sin x \tan x$ | $\left(\frac{\pi}{6}, \frac{1}{2\sqrt{3}}\right)$ | 63. $y = \sec x$ | $\left(\frac{\pi}{4}, \sqrt{2}\right)$ |
| 64. $y = \sin x$ | (0, 0) | 65. $y = x + \tan x$ | (0, 0) | 66. $y = \csc x$ | $\left(\frac{\pi}{3}, \frac{2}{\sqrt{3}}\right)$ |

Find the (relative) minimum value(s) of each function (and their locations) in the domain $0 \leq x \leq 2\pi$.

- | | | |
|------------------------|------------------------------------|------------------------------------|
| 67. $y = x + 2 \sin x$ | 68. $y = \sqrt{3} \sin x + \cos x$ | 69. $y = 2 \sec x + \tan x$ |
| 70. $y = \sin^2 x$ | 71. $y = \tan x - 2x$ | 72. $y = \cos x - \sqrt{3} \sin x$ |

Trigonometric Functions (Answers)

The derivatives are:

1. $y' = 2x \sec^2 x^2$
2. $y' = 2 \tan x \sec^2 x$
3. $y' = 3 \cos 3x$
4. $y' = -2 \sin x$
5. $y' = \cos^2 x - \sin^2 x$
6. $y' = \cos x - 5 \sin x$
7. $y' = 2 \sec^2 2x$
8. $y' = \sin x + x \cos x$
9. $y' = 2 \cos 2x \tan 3x + 3 \sin 2x \sec^2 3x$
10. $y' = \frac{x \cos x - \sin x}{x^2}$
11. $y' = \frac{\cos x + x \sin x}{\cos^2 x}$
12. $y' = \frac{x \cos x - 2 \sin x}{x^3}$
13. $y' = \cos\left(x + \frac{\pi}{4}\right)$
14. $y' = 2x + \frac{1}{2} \sec^2 \frac{x}{2}$
15. $y' = -\frac{\pi}{3} \sin \frac{\pi x}{3}$
16. $y' = -\frac{1}{1 + \sin x}$
17. $y' = \frac{3 \sin 5x - 5(3x + 4) \cos 5x}{\sin^2 5x}$
18. $y' = 2x \cos x - x^2 \sin x$
19. $y' = 3x^2 \cos x^3$
20. $y' = -3 \sin x \cos^2 x$
21. $y' = 3x^2 \tan 8x + 8x^3 \sec^2 8x$
22. $y' = 8 \sin 4x \cos 4x$
23. $y' = -2x \sin(x^2 + 1)$
24. $y' = \pi \sec^2 \pi x$
25. $y' = -\frac{\sin \sqrt{x}}{2\sqrt{x}}$
26. $y' = -\frac{\sin 2x}{\sqrt{\cos 2x}}$
27. $y' = \frac{2 \sin^3 \sqrt{x} \cos \sqrt{x}}{\sqrt{x}}$
28. $y' = 3 \tan^2 x \sec^2 x$
29. $y' = 0$
30. $y' = 2(\cos^2 x - \sin^2 x)$
31. $y' = 2x \sec^2(x^2 - 1)$
32. $y' = -\sec^2(\pi - x)$
33. $y' = 9(2x + \tan 7x)^8 \cdot (2 + 7 \sec^2 7x)$
34. $y' = 14x \sec^2(x^2 + 5)$
35. $y' = -\sin x \cdot \cos(\cos x)$
36. $y' = -\cos x \cdot \sin(\sin x)$
37. $y' = \cos x + \cos^2 x - \sin^2 x$
38. $y' = \frac{x \sec^2 x - \tan x}{3x^{\frac{4}{3}} \tan^{\frac{2}{3}} x}$
39. $y' = -\frac{\sin x \sec^2(\sqrt{\cos x})}{2\sqrt{\cos x}}$
40. $\frac{dy}{dx} = -\csc^2 x$
41. $\frac{dy}{dx} = \tan x \sec x$
42. $\frac{dy}{dx} = -\cot x \operatorname{cosec} x$
43. $\frac{dy}{dx} = 8 \tan 4x \sec^2 4x$
44. $\frac{dy}{dx} = \frac{\pi}{180} \sec^2 x^\circ$
45. $\frac{dy}{dx} = \frac{\pi}{180} \cos 3x^\circ$
46. $\frac{dy}{dx} = b \cos bx$
47. $\frac{dy}{dx} = -b \sin(bx + c)$
48. $\frac{dy}{dx} = 2ax \sec^2(ax^2 + c)$
49. $\frac{dy}{dx} = e^x(\sin x + \cos x)$
50. $\frac{dm}{dw} = \cos w e^{\sin w}$
51. $\frac{dj}{da} = -\frac{\sin a + \cos a}{e^a}$
52. $\frac{dp}{dr} = 4e^{4r} \sin 2r + 2e^{4r} \cos 2r$
53. $\frac{ds}{dt} = (e^t + 1) \cos(e^t + t)$
54. $\frac{db}{ds} = -\frac{\sin(\ln s)}{s}$
55. $\frac{dk}{dq} = -\tan q$
56. $\frac{dc}{dn} = 2 \cot 2n$
57. $\frac{df}{dh} = \frac{2h}{\cos h^2 \sin h^2}$
58. $\frac{dl}{dv} = -\frac{\cos(1 - \ln v)}{v}$
59. $\frac{dz}{du} = e^{4u}(4 \cot u - \csc^2 u)$
60. $\frac{dq}{dm} = \frac{2m \cos m^2 - \sin m^2}{e^m}$

The equations of the tangents (in gradient-intercept form) are:

61. $y = x$
62. $y = \frac{7x}{6} + \frac{1}{2\sqrt{3}} - \frac{7\pi}{36}$
63. $y = \sqrt{2}x + \sqrt{2} - \frac{\sqrt{2}\pi}{4}$
64. $y = x$
65. $y = 2x$
66. $y = -\frac{2x}{3} + \frac{2}{\sqrt{3}} + \frac{2\pi}{9}$

The (relative) minimum values of the functions are:

67. $y = \frac{4\pi}{3} - \sqrt{3}$ (at $x = \frac{4\pi}{3}$)
68. $y = -2$ (at $x = \frac{4\pi}{3}$)
69. $y = \sqrt{3}$ (at $x = \frac{11\pi}{6}$)
70. $y = 0$ (at $x = 0, \pi, 2\pi$)
71. $y = 1 - \frac{\pi}{2}$ (at $x = \frac{\pi}{4}$) and $y = 1 - \frac{5\pi}{2}$ (at $x = \frac{5\pi}{4}$)
72. $y = -2$ (at $x = \frac{2\pi}{3}$)