Exercises in Machine Learning

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Overview

- Exercise 12
- Exercise 13

Exercise 12: Probabilities

We consider a medical diagnosis task. We have knowledge that over the entire population of people 0.8% have cancer. There exists a (binary) laboratory test that represents an imperfect indicator of this disease. That test returns a correct positive result in 98% of the cases in which the disease is present, and a correct negative results in 97% of the cases where the disease is not present.

(a) Suppose we observe a patient for whom the laboratory test returns a positive result. Calculate the a posteriori probability that this patient truly suffers from cancer.

(b) Knowing that the lab test is an imperfect one, a second test (which is assumed to be independent of the former one) is conducted. Calculate the a posteriori probabilities for $cancer$ and $\neg cancer$ given that the second test has returned a positive result as well.

Exercise 12: Probabilities

(a) Suppose we observe a patient for whom the laboratory test returns a positive result. Calculate the a posteriori probability that this patient truly suffers from cancer.

- We know $P(cancer) = 0.008$ and $P(\neg cancer) = 0.992$.
- For the laboratory test, we know that
  $P(\oplus|cancer) = 0.98 \Rightarrow P(\ominus|cancer) = 0.02$
  $P(\oplus|\neg cancer) = 0.03 \Rightarrow P(\ominus|\neg cancer) = 0.97$
- We are looking for the maximum a posteriori (MAP) hypothesis after the laboratory test.
- We obtain
  $P(\oplus|cancer) \cdot P(cancer) = 0.98 \cdot 0.008 = 0.0078$
  $P(\oplus|\neg cancer) \cdot P(\neg cancer) = 0.03 \cdot 0.992 = 0.0298$

Exercise 12: Probabilities

(a) Suppose we observe a patient for whom the laboratory test returns a positive result. Calculate the a posteriori probability that this patient truly suffers from cancer.

- Thus, $h_{MAP} = \arg\max_{h \in H} P(D|h) \cdot P(h) = \neg cancer$.
- In particular, we get (after normalization) $P(cancer|\oplus) = \frac{0.0078}{0.0078+0.0298} = 0.21$ and $P(\neg cancer|\oplus) = 0.79$

Exercise 12: Probabilities

(b) Knowing that the lab test is an imperfect one, a second test (which is assumed to be independent of the former one) is conducted. Calculate the a posteriori probabilities for $cancer$ and $\neg cancer$ given that the second test has returned a positive result as well.

- We know $P(cancer) = 0.008$ and $P(\neg cancer) = 0.992$.
- For the laboratory test, we know that
  $P(\oplus|cancer) = 0.98 \Rightarrow P(\ominus|cancer) = 0.02$
  $P(\oplus|\neg cancer) = 0.03 \Rightarrow P(\ominus|\neg cancer) = 0.97$
- We are looking for the maximum a posteriori (MAP) hypothesis after the second laboratory test which is assumed to be independent of the former one.

Exercise 12: Probabilities

(b) Knowing that the lab test is an imperfect one, a second test (which is assumed to be independent of the former one) is conducted. Calculate the a posteriori probabilities for $cancer$ and $\neg cancer$ given that the second test has returned a positive result as well.

- Therefore, we have
  $P(\oplus\oplus|cancer) \cdot P(cancer) = P(\oplus|cancer) \cdot P(\oplus|cancer) \cdot P(cancer) = 0.98 \cdot 0.98 \cdot 0.008 = 0.007644$
  $P(\oplus\oplus|\neg cancer) \cdot P(\neg cancer) = P(\oplus|\neg cancer) \cdot P(\oplus|\neg cancer) \cdot P(\neg cancer) = 0.03 \cdot 0.03 \cdot 0.992 = 0.000894$
- Thus, $h_{MAP} = \arg\max_{h \in H} P(D|h) \cdot P(h) = cancer$.
- In particular, we get (after normalization)
  $P(cancer|\oplus\oplus) = \frac{0.007644}{0.007644+0.000894} = 0.895$ and
  $P(\neg cancer|\oplus\oplus) = 0.105$.

Exercise 12: Probabilities

We turn to politics. For the upcoming mayor election, 1000000 people are allowed to vote either for candidate A or candidate B. There were 1000 registered voters who have already voted by postal voting.

(c) Assume that all postal voters have voted for candidate A. Moreover, we assume that all remaining voters decide by flipping a (non-manipulated) coin. What is the probability that candidate A wins the election?

- It's higher than most people would guess. ;-)
  $\longrightarrow$ Blackboard

Overview

- Exercise 12
- Exercise 13

Exercise 13: Naive Bayes Classifier

In the following, we consider the data set introduced in Assignment 1 where the task is to describe whether a person is *ill*. We use a representation based on four features per subject to describe an individual person. These features are “running nose”, “coughing”, “reddened skin”, and “fever”, each of which can take the value true (‘$+$’) or false (‘–’).

Training Example	N (running nose)	C (coughing)	R (reddened skin)	F (fever)	**Classification**
d_1	+	+	+	–	positive (ill)
d_2	+	+	–	–	positive (ill)
d_3	–	–	+	+	positive (ill)
d_4	+	–	–	–	negative (healthy)
d_5	–	–	–	–	negative (healthy)
d_6	–	+	+	–	negative (healthy)

Exercise 13: Naive Bayes Classifier

(a) Given the data set in the table on the previous slide, determine all probabilities required to apply the naive Bayes classifier for predicting whether a new person is ill or not. Use the m-estimate of probability with an equivalent sample size $m = 4$ and a uniform prior p.

- We describe instances as tuples $d_i = \langle a_1, \ldots, a_n \rangle$.
- In our case it holds $n = 4$ and $V = \{ill, healthy\}$.
- In the end, the naive Bayes classifier searches for that $v_j \in V$ such that

$$v_{MAP} = \arg\max_{v_j \in V} P(v_j | a_1, \ldots, a_n)$$

- Using the Bayes rule this gives rise to

$$v_{MAP} = \arg\max_{v_j \in V} \frac{P(a_1, \ldots, a_n | v_j) \cdot P(v_j)}{P(a_1, \ldots, a_n)}$$

Exercise 13: Naive Bayes Classifier

- $P(a_1, \ldots, a_n | v_j)$ is hard to assess which is why the naive Bayes classifier assumes that the attributes are independent of one another, i.e.

$$P(a_1, \ldots, a_n | v_j) = \Pi_{i=1}^{n} p(a_i | v_j)$$

- Thus, in contrast to the maximuma a-posteriori estimate, the naive Bayes estimate is given by

$$v_{NB} = \arg\max_{v_j \in V} p(v_j) \cdot \Pi_{i=1}^{n} P(a_i | v_j)$$

- With the data given and $V = \{ill, healthy\}$ we have $P(ill) = P(healthy) = \frac{3}{6}$.

Exercise 13: Naive Bayes Classifier

- Accordingly, for the conditional probabilities for the presence of the four different symptoms we obtain the following.
- running Nose (N): $P(N|ill) = \frac{2}{3}$, $P(N|healthy) = \frac{1}{3}$
- Coughing (C): $P(C|ill) = \frac{2}{3}$, $P(C|healthy) = \frac{1}{3}$
- Reddened skin (R): $P(R|ill) = \frac{2}{3}$, $P(R|healthy) = \frac{1}{3}$
- Fever (F): $P(F|ill) = \frac{1}{3}$, $P(F|healthy) = \frac{0}{3}$
- Next, we make use of the m-estimate of probability using an equivalent sample size $m = 4$ and a uniform prior p.
- This, of course, changes all probabilities recently computed.

Exercise 13: Naive Bayes Classifier

- We obtain:
- running Nose (N): $P(N|ill) = \frac{2+m\cdot 0.5}{3+m} = \frac{4}{7}$,
  $P(N|healthy) = \frac{1+m\cdot 0.5}{3+m} = \frac{3}{7}$
- Coughing (C): $P(C|ill) = \frac{4}{7}$, $P(C|healthy) = \frac{3}{7}$
- Reddened skin (R): $P(R|ill) = \frac{4}{7}$, $P(R|healthy) = \frac{3}{7}$
- Fever (F): $P(F|ill) = \frac{3}{7}$, $P(F|healthy) = \frac{2}{7}$

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \dots, d_6)$ correctly.

- Consider $d_1 = \langle N, C, R, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(N, C, R, \overline{F}|ill) = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = \frac{256}{7^4}$
  - $P(N, C, R, \overline{F}|healthy) = \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{5}{7} = \frac{135}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^{n} P(a_i|v_j) = ill$
    $\Rightarrow$ correctly classified

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \dots, d_6)$ correctly.

- Consider $d_2 = \langle N, C, \overline{R}, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(N, C, \overline{R}, \overline{F}|ill) = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} = \frac{16 \cdot 12}{7^4}$
  - $P(N, C, \overline{R}, \overline{F}|healthy) = \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{5}{7} = \frac{15 \cdot 12}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^{n} P(a_i|v_j) = ill$
    $\Rightarrow$ correctly classified

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \ldots, d_6)$ correctly.

- Consider $d_3 = \langle \overline{N}, \overline{C}, R, F \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(\overline{N}, \overline{C}, R, F | ill) = \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} = \frac{108}{7^4}$
  - $P(\overline{N}, \overline{C}, R, F | healthy) = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{2}{7} = \frac{96}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^n P(a_i | v_j) = ill$
    $\Rightarrow$ correctly classified

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \ldots, d_6)$ correctly.

- Consider $d_4 = \langle N, \overline{C}, \overline{R}, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(N, \overline{C}, \overline{R}, \overline{F} | ill) = \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} = \frac{12 \cdot 12}{7^4}$
  - $P(N, \overline{C}, \overline{R}, \overline{F} | healthy) = \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{5}{7} = \frac{12 \cdot 20}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^n P(a_i | v_j) = healthy$
    $\Rightarrow$ correctly classified

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \ldots, d_6)$ correctly.

- Consider $d_5 = \langle \overline{N}, \overline{C}, \overline{R}, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(\overline{N}, \overline{C}, \overline{R}, \overline{F} | ill) = \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} = \frac{9 \cdot 12}{7^4}$
  - $P(\overline{N}, \overline{C}, \overline{R}, \overline{F} | healthy) = \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{5}{7} = \frac{16 \cdot 20}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^n P(a_i | v_j) = healthy$
    $\Rightarrow$ correctly classified

Exercise 13: Naive Bayes Classifier

(b) Verify whether the naive Bayes classifier classifies all training examples $(d_1, \ldots, d_6)$ correctly.

- Consider $d_6 = \langle \overline{N}, C, R, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(\overline{N}, C, R, \overline{F} | ill) = \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = \frac{12 \cdot 16}{7^4}$
  - $P(\overline{N}, C, R, \overline{F} | healthy) = \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{5}{7} = \frac{12 \cdot 15}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^n P(a_i | v_j) = ill$
    $\Rightarrow$ wrongly classified

Exercise 13: Naive Bayes Classifier

(c) Apply your naive Bayes classifier to the test patterns corresponding to the following subjects: a person who is coughing and has fever, a person whose nose is running and who suffers from fever, and a person with a running nose and reddened skin ($d_7 = (\overline{N}, C, \overline{R}, F)$, $d_8 = (N, \overline{C}, \overline{R}, F)$, and $d_9 = (N, \overline{C}, R, \overline{F})$).

- Consider $d_7 = \langle \overline{N}, C, \overline{R}, F \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(\overline{N}, C, \overline{R}, F | ill) = \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} = \frac{12 \cdot 9}{7^4}$
  - $P(\overline{N}, C, \overline{R}, F | healthy) = \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{2}{7} = \frac{12 \cdot 8}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^{n} P(a_i | v_j) = ill$
    $\Rightarrow$ A person that is coughing and has fever is classified to be ill.

Exercise 13: Naive Bayes Classifier

(c) Apply your naive Bayes classifier to the test patterns corresponding to the following subjects: a person who is coughing and has fever, a person whose nose is running and who suffers from fever, and a person with a running nose and reddened skin ($d_7 = (\overline{N}, C, \overline{R}, F)$, $d_8 = (N, \overline{C}, \overline{R}, F)$, and $d_9 = (N, \overline{C}, R, \overline{F})$).

- Consider $d_8 = \langle N, \overline{C}, \overline{R}, F \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(N, \overline{C}, \overline{R}, F | ill) = \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} = \frac{12 \cdot 9}{7^4}$
  - $P(N, \overline{C}, \overline{R}, F | healthy) = \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{2}{7} = \frac{12 \cdot 8}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^{n} P(a_i | v_j) = ill$
    $\Rightarrow$ A person that has a running nose as well as fever is classified to be ill.

Exercise 13: Naive Bayes Classifier

(c) Apply your naive Bayes classifier to the test patterns corresponding to the following subjects: a person who is coughing and has fever, a person whose nose is running and who suffers from fever, and a person with a running nose and reddened skin ($d_7 = (\overline{N}, C, \overline{R}, F)$, $d_8 = (N, \overline{C}, \overline{R}, F)$, and $d_9 = (N, \overline{C}, R, \overline{F})$).

- Consider $d_9 = \langle N, \overline{C}, R, \overline{F} \rangle$.
  - $P(ill) = P(healthy) = \frac{1}{2}$
  - $P(N, \overline{C}, R, \overline{F} | ill) = \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = \frac{12 \cdot 16}{7^4}$
  - $P(N, \overline{C}, R, \overline{F} | healthy) = \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{5}{7} = \frac{12 \cdot 15}{7^4}$
  - $v_{NB} = \arg\max_{v_j \in V} P(v_j) \cdot \Pi_{i=1}^{n} P(a_i | v_j) = ill$
    $\Rightarrow$ A person that has a running nose as well as reddened skin is classified to be ill.

Exercise 13: Naive Bayes Classifier

(d) Now, we no longer distinguish between positive and negative training examples, but each instance is assigned one out of k classes. The corresponding training data is provided in the table below. Calculate all probabilities required for the application of a naive Bayes classifier, i.e. $P(v)$ and $P((a_F, a_V, a_D, a_{Sh}) | v)$ for $v \in \{H, I, S, B\}$ and $a_F \in \{no, average, high\}$ and $a_V, a_D, a_{Sh} \in \{yes, no\}$. Again, use the m-estimate of probability method with $m = 6$ and p uniform.

Training	Fever	Vomiting	Diarrhea	Shivering	**Classification**
d_1	no	no	no	no	healty (H)
d_2	average	no	no	no	influenza (I)
d_3	high	no	no	yes	influenza (I)
d_4	high	yes	yes	no	salmonella poisoning (S)
d_5	average	no	yes	no	salmonella poisoning (S)
d_6	no	yes	yes	no	bowel inflammation (B)
d_7	average	yes	yes	no	bowel inflammation (B)

Exercise 13: Naive Bayes Classifier

Training	Fever	Vomiting	Diarrhea	Shivering	**Classification**
d_1	no	no	no	no	healty (H)
d_2	average	no	no	no	influenza (I)
d_3	high	no	no	yes	influenza (I)
d_4	high	yes	yes	no	salmonella poisoning (S)
d_5	average	no	yes	no	salmonella poisoning (S)
d_6	no	yes	yes	no	bowel inflammation (B)
d_7	average	yes	yes	no	bowel inflammation (B)

- Obviously, it holds $P(H) = \frac{1}{7}$, $P(I) = \frac{2}{7}$, $P(S) = \frac{2}{7}$, and $P(B) = \frac{2}{7}$.
- Conditional probabilities for the Vomiting attribute using the m-estimate of probability:
  $P(V|H) = \frac{0+3}{1+6} = \frac{3}{7}, P(\overline{V}|H) = \frac{1+3}{1+6} = \frac{4}{7},$
  $P(V|I) = \frac{0+3}{2+6} = \frac{3}{8}, P(\overline{V}|I) = \frac{2+3}{2+6} = \frac{5}{8},$

Exercise 13: Naive Bayes Classifier

- Conditional probabilities for the Vomiting attribute using the m-estimate of probability (continued):
  $P(V|H) = \frac{0+3}{1+6} = \frac{3}{7}, P(\overline{V}|H) = \frac{1+3}{1+6} = \frac{4}{7},$
  $P(V|I) = \frac{0+3}{2+6} = \frac{3}{8}, P(\overline{V}|I) = \frac{2+3}{2+6} = \frac{5}{8},$
  $P(V|S) = \frac{1+3}{2+6} = \frac{4}{8}, P(\overline{V}|S) = \frac{1+3}{2+6} = \frac{4}{8},$
  $P(V|B) = \frac{2+3}{2+6} = \frac{5}{8}, P(\overline{V}|B) = \frac{0+3}{2+6} = \frac{3}{8}$
- Conditional probabilities for the Diarrhea attribute using the m-estimate of probability:
  $P(D|H) = \frac{0+3}{1+6} = \frac{3}{7}, P(\overline{D}|H) = \frac{1+3}{1+6} = \frac{4}{7},$
  $P(D|I) = \frac{0+3}{2+6} = \frac{3}{8}, P(\overline{D}|I) = \frac{2+3}{2+6} = \frac{5}{8},$
  $P(D|S) = \frac{2+3}{2+6} = \frac{5}{8}, P(\overline{D}|S) = \frac{0+3}{2+6} = \frac{3}{8},$
  $P(D|B) = \frac{2+3}{2+6} = \frac{5}{8}, P(\overline{D}|B) = \frac{0+3}{2+6} = \frac{3}{8}$

Exercise 13: Naive Bayes Classifier

- Conditional probabilities for the Shivering attribute using the m-estimate of probability:
  $P(Sh|H) = \frac{0+3}{1+6} = \frac{3}{7}, P(\overline{Sh}|H) = \frac{1+3}{1+6} = \frac{4}{7},$
  $P(Sh|I) = \frac{1+3}{2+6} = \frac{4}{8}, P(\overline{Sh}|I) = \frac{1+3}{2+6} = \frac{4}{8},$
  $P(Sh|S) = \frac{0+3}{2+6} = \frac{3}{8}, P(\overline{Sh}|S) = \frac{2+3}{2+6} = \frac{5}{8},$
  $P(Sh|B) = \frac{0+3}{2+6} = \frac{3}{8}, P(\overline{Sh}|B) = \frac{2+3}{2+6} = \frac{5}{8}$
- Conditional probabilities for the Fever attribute using the m-estimate of probability:
  $P(F_{no}|H) = \frac{1+2}{1+6} = \frac{3}{7}, P(F_{avg}|H) = \frac{0+2}{1+6} = \frac{2}{7}, P(F_{high}|H) = \frac{0+2}{1+6} = \frac{2}{7},$
  $P(F_{no}|I) = \frac{0+2}{2+6} = \frac{2}{8}, P(F_{avg}|I) = \frac{1+2}{2+6} = \frac{3}{8}, P(F_{high}|I) = \frac{1+2}{2+6} = \frac{3}{8},$
  $P(F_{no}|S) = \frac{0+2}{2+6} = \frac{2}{8}, P(F_{avg}|S) = \frac{1+2}{2+6} = \frac{3}{8}, P(F_{high}|I) = \frac{1+2}{2+6} = \frac{3}{8},$
  $P(F_{no}|B) = \frac{1+2}{2+6} = \frac{3}{8}, P(F_{avg}|B) = \frac{0+2}{2+6} = \frac{2}{8}, P(F_{high}|I) = \frac{1+2}{2+6} = \frac{3}{8}$

Exercise 13: Naive Bayes Classifier

(e) Apply your recently constructed naive Bayes classifier to a person with high fever, i.e. to $(high, no, no, no)$, as well as to a person who suffers from vomitting and shivering, i.e. to (no, yes, no, yes).

- Consider $d = (high, no, no, no)$, i.e. $\langle F_{high}, \overline{V}, \overline{D}, \overline{Sh} \rangle$.
- Calculate $P(v|x) = P(v) \cdot \Pi_{i=1}^{4} P(a_i|v)$ for all $v \in \{H, I, S, B\}$.
- $P(H) \cdot \Pi_{i=1}^{4} P(a_i|H) = \frac{1}{7} \cdot \frac{2}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} \cdot \frac{4}{7} = \frac{1}{7} \cdot \frac{8 \cdot 16}{7^4} \approx 0.0076$
- $P(I) \cdot \Pi_{i=1}^{4} P(a_i|I) = \frac{2}{7} \cdot \frac{3}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{4}{8} = \frac{2}{7} \cdot \frac{15 \cdot 20}{8^4} \approx 0.0209$
- $P(S) \cdot \Pi_{i=1}^{4} P(a_i|S) = \frac{2}{7} \cdot \frac{3}{8} \cdot \frac{4}{8} \cdot \frac{3}{8} \cdot \frac{5}{8} = \frac{2}{7} \cdot \frac{9 \cdot 20}{8^4} \approx 0.0126$
- $P(B) \cdot \Pi_{i=1}^{4} P(a_i|B) = \frac{2}{7} \cdot \frac{3}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} \cdot \frac{5}{8} = \frac{2}{7} \cdot \frac{9 \cdot 15}{8^4} \approx 0.0094$
- Thus, after normalization, the naive Bayes classifier concludes that person d is healthy with a probability of 15.0%, suffers from influenza with 41.4%, from salmonella poisoning with 25.0%, and from bowel inflammation with 18.6%.

Exercise 13: Naive Bayes Classifier

(e) Apply your recently constructed naive Bayes classifier to a person with high fever, i.e. to $(high, no, no, no)$, as well as to a person who suffers from vomitting and shivering, i.e. to (no, yes, no, yes).

- Consider $d = (no, yes, no, yes)$, i.e. $\langle F_{no}, V, \overline{D}, Sh \rangle$.
- Calculate $P(v|x) = P(v) \cdot \Pi_{i=1}^{4} P(a_i|v)$ for all $v \in \{H, I, S, B\}$.
- $P(H) \cdot \Pi_{i=1}^{4} P(a_i|H) = \frac{1}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} = \frac{1}{7} \cdot \frac{9 \cdot 12}{7^4} \approx 0.0064$
- $P(I) \cdot \Pi_{i=1}^{4} P(a_i|I) = \frac{2}{7} \cdot \frac{2}{8} \cdot \frac{3}{8} \cdot \frac{5}{8} \cdot \frac{4}{8} = \frac{2}{7} \cdot \frac{12 \cdot 10}{8^4} \approx 0.0084$
- $P(S) \cdot \Pi_{i=1}^{4} P(a_i|S) = \frac{2}{7} \cdot \frac{2}{8} \cdot \frac{4}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} = \frac{2}{7} \cdot \frac{12 \cdot 6}{8^4} \approx 0.0050$
- $P(B) \cdot \Pi_{i=1}^{4} P(a_i|B) = \frac{2}{7} \cdot \frac{3}{8} \cdot \frac{5}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} = \frac{2}{7} \cdot \frac{9 \cdot 15}{8^4} \approx 0.0094$
- Thus, after normalization, the naive Bayes classifier concludes that person d is healthy with a probability of 21.9%, suffers from influenza with 28.8%, from salmonella poisoning with 17.1%, and from bowel inflammation with 32.2%.