

A.I. IN AUDIO & SIGNAL PROCESSING

Session 4: Automata and transducers



COURSE STRUCTURE



Quick Summary

Audio processing for Al

- Signal, audio, speech encoding (3h)
- Deep learning for audio processing (4h)

Automata for language modelling

- HMM for speech processing (3h)
- Automata and transducer (3h)

Towards speaking with an Al-bot

- Speech synthesis (4h)
- Automatic speech recognition (4h)
- Speaker and emotion recognition (4h)

SESSION 4: AUTOMATA AND TRANSDUCERS



Quick Summary

1. Boolean automata

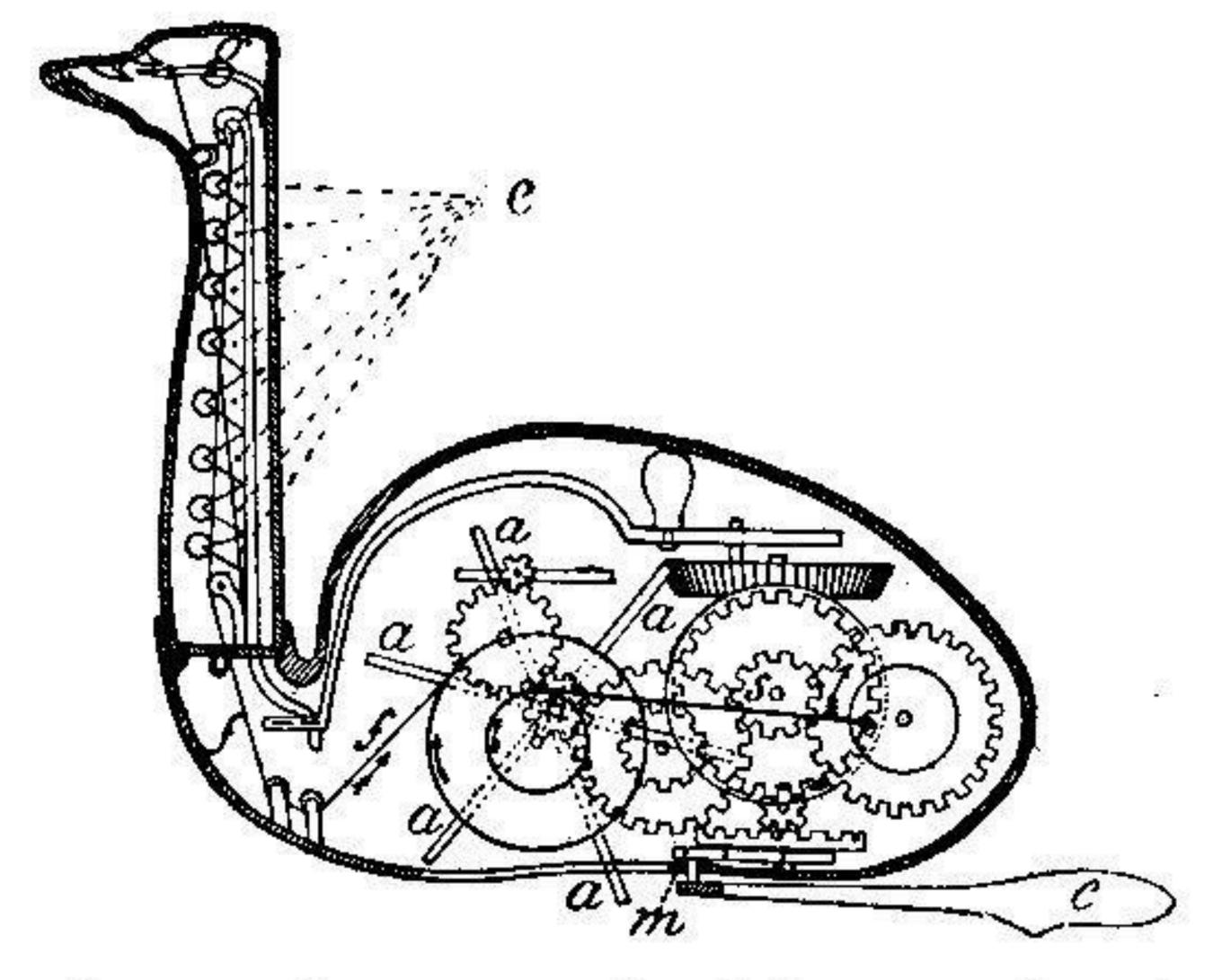
- a) Automata
- b) Transducers
- c) Probabilistic automata

2. Application to Natural Language Processing

- a) Lexicon
- b) Morphology and phonology
- c) Part-of-speech tagging
- d) Syntax
- e) Translation

AUTOMATA AND TRANSDUCERS.

Boolean automata and transducers



Le canard: a, roues à palette; c, patte palmée; e, châssis, f, corde à boyau; g. ressort; m, levier horizontal; s, pignon.

Vaucanson automata (1739)

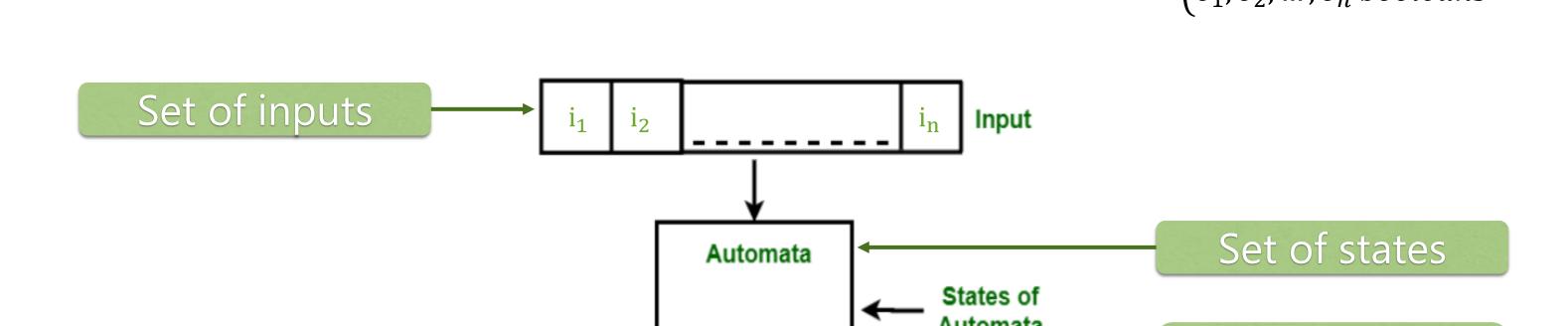
Set of outputs



Deterministic Finite-state Automata

A deterministic finite-state automaton (DFA) is a 5-tuple $(\Sigma, S, \delta, s_0, F)$, where:

- Σ : alphabet, a finite set of symbols
- S: set of states
- δ : transition function δ : $S \times \Sigma \rightarrow S$
- s_0 : initial state
- F: final states, a subset of S



 S_0, S_1, \dots, S_n

Output

Example of simple DFA

Traffic Light

States: Red, Yellow, Green

Transitions: After a given time, Red will change to Green, Green to Yellow, and Yellow to Red

Transitions

function



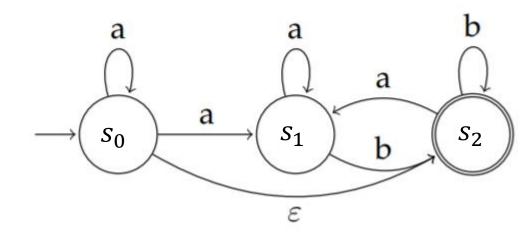
Non-deterministic Finite-state Automata

A non-deterministic finite-state automaton (NFA) is a 5-tuple $(\Sigma, S, \delta, s_0, F)$, with the difference in the definition of δ :

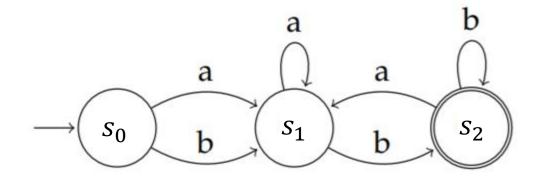
• δ : transition function δ : $S \times \Sigma \to P(S)$

A non-deterministic finite-state automaton with ϵ -transitions (ϵ -NFA) is a quintuple (Σ , S, s_0 , δ , F), with the difference in the definition of δ :

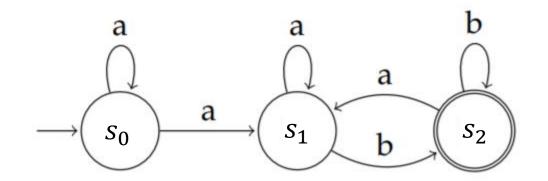
• δ : transition function δ : $S \times (\Sigma \cup \{\epsilon\}) \rightarrow P(S)$



nondeterministic finite automata with ε -transitions.



deterministic finite automaton.



nondeterministic finite automata.

$$\begin{cases} a,b \in \Sigma^2 \\ s_0,s_1,s_2 \in S^3 \\ s_0 \ initial \ state \\ s_2 \in F \\ \epsilon \ empty \ symbol \end{cases}$$

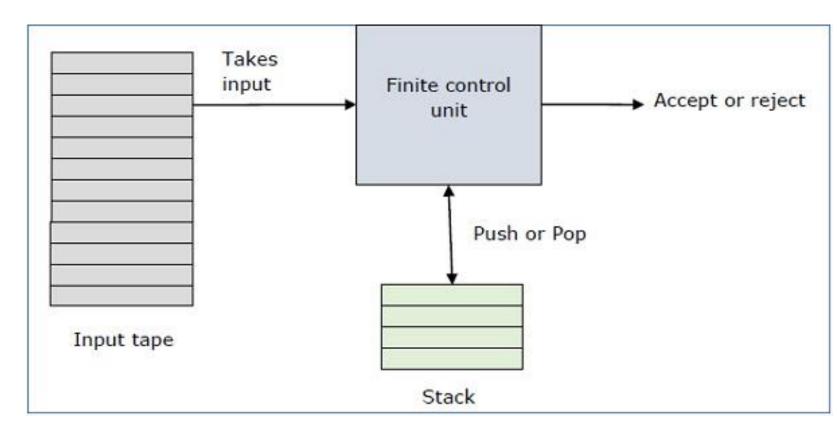


Push-down Automata

A push-down automaton (PDA) is a variation of FSA using a stack. It is a 6-tuple $(\Sigma, S, \delta, s_0, \Gamma, F)$, where:

- Σ : *alphabet*, a finite set of symbols
- S: set of states
- δ : transition function δ : $S \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow S \times \Gamma^*$
- s_0 : initial state
- Γ : stack which can hold symbols of Σ ; Γ^* , string of symbols in stack
- F: accepted states, a subset of S

(an initial stack symbol can be added, otherwise we suppose it is empty; the stack Γ can have its own set of symbols, different from Σ)



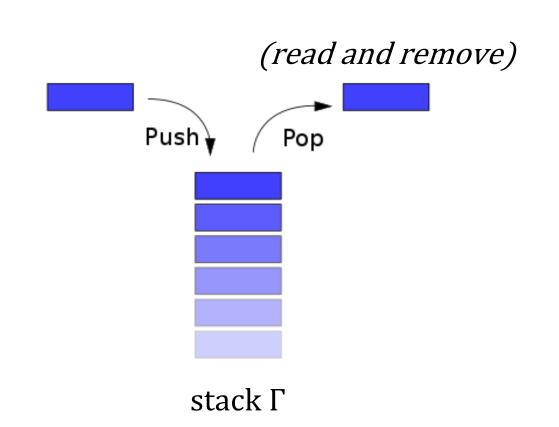
basic idea of PDA principle



Push-down Automata

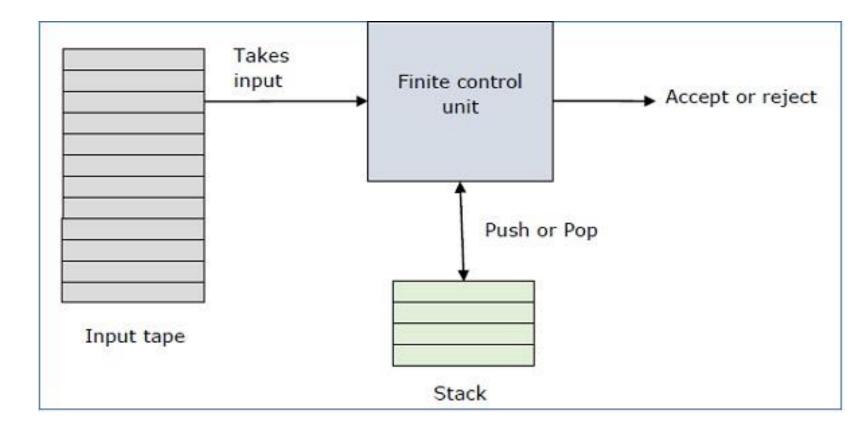
Transition process:

- $\delta(s, i, x) = (s', x'), s'$ new state, x'string of symbols
- If $x' = \epsilon$, the stack is popped
- If x' = x, the stack is unchanged
- If x' = yzy, then x is removed from stack and replaced by yzy

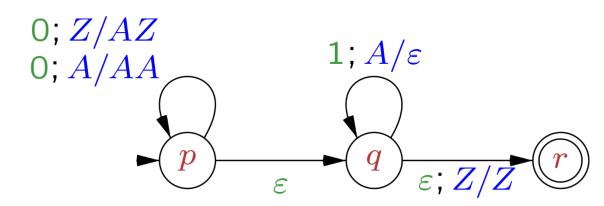


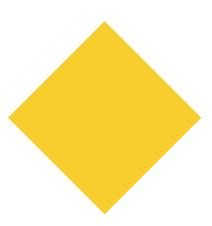
with
$$\begin{cases} s, s' \in S^2 \\ i \in \Sigma \text{ or } i = \epsilon \text{ (empty symbol)} \\ x, x' \in \Gamma^2, \text{strings of elements of } \Sigma \end{cases}$$

(top of stack)



basic idea of PDA principle

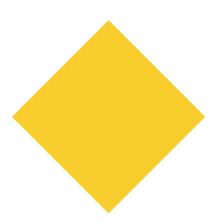




Automata optimization

- Determinazation
 - → transfrom a non-derterministic automata into a deterministic one
- Minimization
 - → reduce the size of an automata

- J. Sakarovitch. Eléments of automata theory. 2003.
- J. E. Hopcroft, J. D. Ullman. Introduction to Automata Theory, Languages, and Computation. 1979.



Finite-state transducers

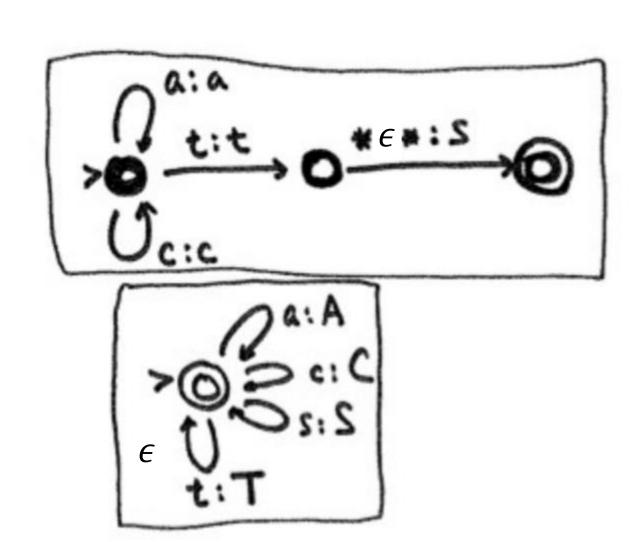
A sequential string-to-string transducer (FST) is 6-tuple $(\Sigma, \Delta, S, \delta, s_0, F)$, where:

- Σ : input alphabet, a set of symbols
- Δ : output alphabet, a set of symbols
- S: set of states
- δ : transition function δ : $S \times (\Sigma \cup \{\epsilon\}) \rightarrow S \times (\Delta \cup \{\epsilon\})$
- s_0 : initial state $\in S$
- *F* : *accepted states*, a subset of *S*

 ϵ is the empty character

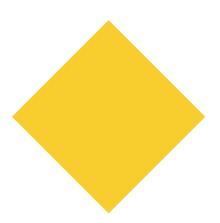


- Text-to-sound
- Translator
- Part-of-speech tagging



M. Mohri. On Some Applications of Finite-State Automata Theory to Natural Language Processing. 1996.

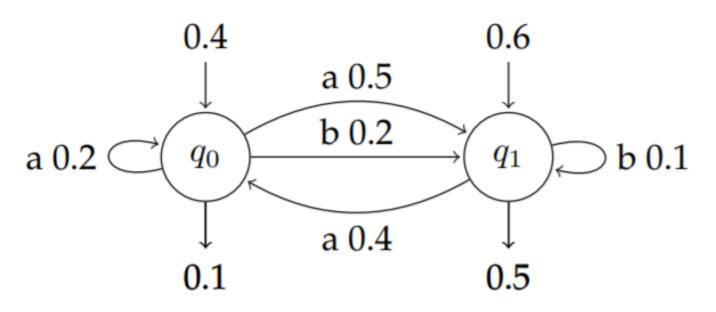
M. Mohri. Minimization of sequential transducers. 1996.



Probabilistic automata

A probabilistic automaton (PA) is defined by a 5-tuple $(\Sigma, S, \phi, \iota, \tau)$, where:

- Σ : *alphabet*, a set of input symbols
- S: set of states
- ϕ : a mapping defining the transition probability function, $\phi: S \times \Sigma \times S \rightarrow [0,1]$
- ι : a mapping defining the initial probability of each state, $\iota: S \to [0,1]$
- τ : a mapping defining the final probability of each state, $\tau: S \to [0,1]$



probabilistic automaton

Constraints to be verified:

- $\sum_{s \in S} \iota(s) = 1$
- $\forall s \in S$, $\tau(s) + \sum_{x \in \Sigma} \sum_{s' \in S} \phi(s, x, s') = 1$

Differences between HMM and probabilistic automata:

 \rightarrow It has been proven that probabilistic automata with no final probabilities are equivalent to hidden Markov models.

Dupont, P., Fran c. D., and Esposito, Y. (2005). Links between probabilistic automata and hidden Markov models: probability distributions, learning models and induction algorithms. Pattern Recognition, 38(9):1349–1371

AUTOMATA AND TRANSDUCERS.

Application to Natural Language Processing

APPLICATION TO NATURAL LANGUAGE PROCESSING



Lexicon

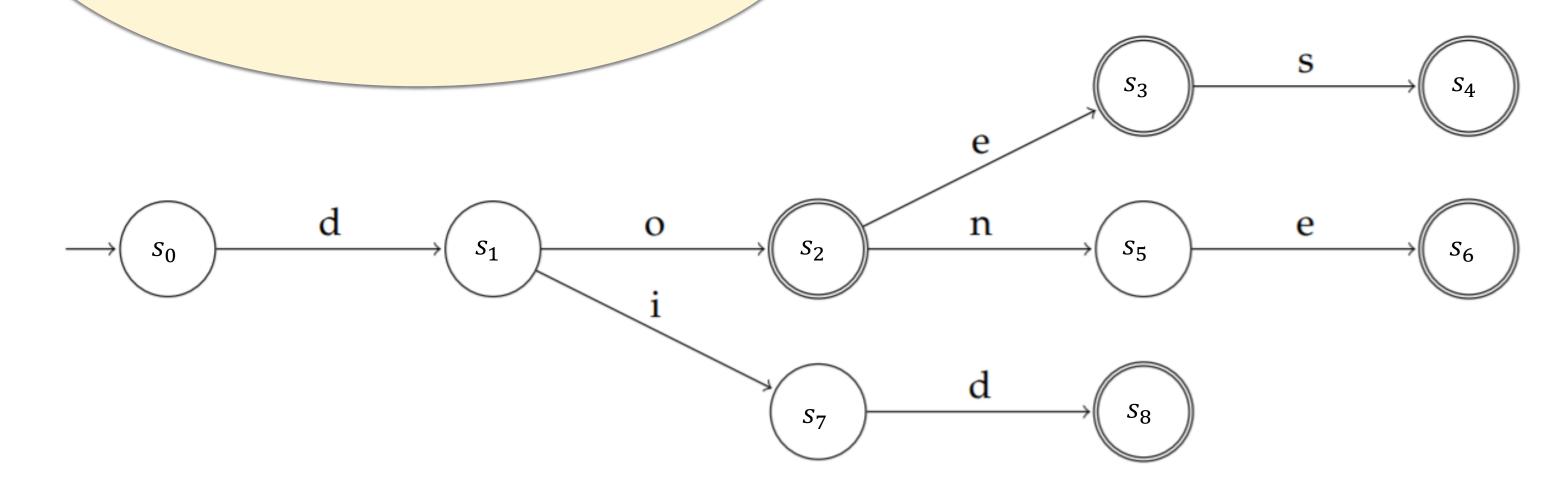
Context

did,V:PRET do,N:s do,V:INF doe,N:s:p does,V:P3s does,N:p done,V:PP done,A

• listings to automata

Lexicology

Study of words, their nature, their function as symbols, their meaning, the relationship of their meaning to epistemology



lexicon represented by an automaton.

Lexicon	FDELAF	GDELAF	EDELAF
Initial size	21.2Mb	27.9Mb	3.6Mb
Automata size	1.2Mb	3.1Mb	470Kb

Lexicon representations: memory consumption. (Mohri, 1996b)

APPLICATION TO NATURAL LANGUAGE PROCESSING



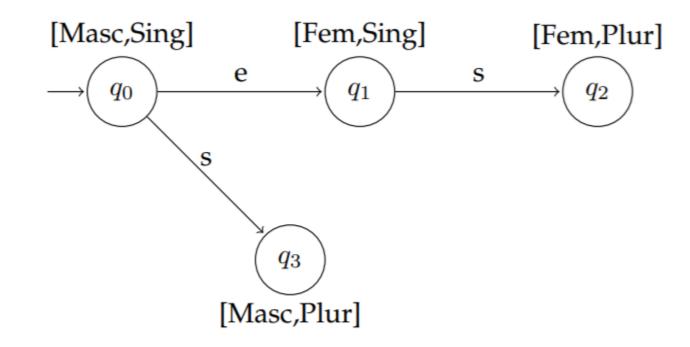
Morphology and phonology

- N+"s" for the plural (dog, dogs),
- V+"s" for the third form of the present (read, reads),
- V+"ed" for the past participle of most verbs (match, matched),
- V+"er" to designate the one who (to hunt, the hunter),
- Adj+"ly" to qualify a way of doing things (slow, slowly)
- Political figure+"ism" to refer the political party of this figure (Bush, bushism)

Phonology

syllable, onset and rime, articulatory gestures, articulatory features, mora,... sound are structured to convey linguistic meaning

Morphological rules need to be implemented in the automaton to increase its efficience.



Morphology

Field of linguistics that focuses on the study of the internal structure of a word. It is the smallest units of syntax.

Sample of morphological inflection rules of adjectives in French.

AUTOMATA AND TRANSDUCERS.

Thank you for your attention.

References:

Jimmy Ma