DSA March 2022

Operations Research 2: Optimization

Assignment

Due date : 27/03/2022

Please try to send a **clean** version of your work with **clear** and **detailed** answers (no one wants to grade something that looks like a draft). Remember that giving just calculations or a "yes/no" answer is, most of the time, not enough. Finally, keep in mind that this a personal work.

PART I Gradient and directional derivative (6 points)

a) Do the partial derivatives of $f(x,y)=e^{x}\cos y$ exist? If yes, what are they? **(1 point)**

b) The function g is defined on $\mathbb{R}^3$ by $g(x,y,z)=\sqrt{x^2+y^2+z^2}$: you are at the point (2,2,2) and you are trying to increase the value of f, which is the best direction to go? **(2 points)**

c) What is the value of the directional derivative of $f(x,y)=3x^2y-4xy$ at the point (x,y) = (1,2) in the direction given by the vector $u=\begin{pmatrix}\frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{pmatrix}$? **(3 points)**

PART II Optimization problems with no constraints (8 points)

a) Is there a local max or a local min at the point (0,0) for the function $f(x,y)=x^3+y^3$? **(2.5 points)**

b) What are the critical points of the function $g(x,y)=y^2-x^2+\frac{x^4}{2}$ and what is their nature? **(2.5 points)**

c) Find the minimum of the function $h(x,y)=x^2-2xy+y^2+6$ **(3 points)**

PART III Optimization problems with a constraint (6 points)

a) Solve $max\, g(x,y)=\frac{1}{x}+\frac{1}{y}$ with the constraint $xy=9$ **(3 points)**

b) Solve the optimization problem $min f(x,y)=x^2-y^2$ for the points (x,y) in the disk centered at the origin and of radius 10 **(3 points)**