Adaptive Feedback Controller Design based on Gradient and Stability Approach For a Chemical Process

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Abstract— **Adaptive control refers to modification of the control law used by the controller to cope with the drastic change in parameters of the system being controlled due to changes in environmental conditions and in system itself. The main aim of adaptive control process is to generate an actuating signal in such a way that optimal performance can be maintained regardless of system changes. This paper deals with application of model reference adaptive control scheme and the system performance is compared with Gradient and Stability approach. A first order reference model and chemical plant has been taken as the system for analysis with a PI controller. Comparison is done between different values of time constant and time delay in the system in both MIT rule and Lyapunov method to evaluate the adaptation performance of the designed controller.**

Keywords — Model reference adaptive control, gradient approach, stability approach, PI Controller, Adaptive feedback controller.

I. INTRODUCTION

A control system is in the broadest sense, an interconnection of the physical components to provide a desired function, involving some type of control action with it. The requirement of high performance control system for industrial applications has produced great research efforts for the application of modern control theory and, in particular, adaptive control. As compared to fixed gain PID controllers Adaptive Controllers are very effective to handle the situations where the parameter variations and environmental changes are frequent. The controller parameters are adjusted to give a desired closed-loop performance. The adaptive controller maintains constant dynamic performance in the presence of unpredictable and immeasurable variations. Adaptive control changes the control algorithm coefficients in real time to compensate for variations in the environment or in the system itself. It also varies the system transfer function according to situation.

In this paper we focus on design of a PI controller with adjustable parameters and a mechanism for adjusting the parameters by using the gradient approach (MIT) and the stability approach (Lyapunov Method).

The aim of this assignment is to design two controllers for a chemical process based on the gradient and stability approaches. The performance of both will be compared via Matlab Simulink simulation to show the robustness and the adaptation ability of the controllers. Series of analysis have been done by varying the time constant, τ and time delay, T_D and also the amplitude of the reference input signal, r.

The paper is organized as follows where the basic theory and formula of PI Controller is briefly described in section II, MRAC theory and its structure in section III whereas gradient and stability approaches are described in section IV and V respectively. The adaptive feedback controller design scheme as well as time delay approximation are described in section VI. The simulation results and analysis are as shown in section VII. Finally, the results are concluded at the end of the paper.

II. PI CONTROLLER

An adaptive Proportional-Integral (PI) controller is required to control a chemical process. The general formula adopted for the PI-controller is given as below:

$$u(t) = K\left(e(t) + \frac{1}{T_I}\int_0^t e(t)dt\right)$$

where T_I is integration time constant. If the controller is tuned to be slow and T_I is large, the controller will first act like a Proportional controller, however the steady-state deviation will slowly go to zero after that.

On the other hand, as the integration starts to take effect, In PI- control, the steady-state deviation will finally go to zero. If the controller is tuned to be fast and T_I is small, then both terms (P and I) will affect the control signal all the way from the beginning. The system becomes faster, but the output signal might oscillate and eventually overshoot.

Thus, a proper P and I values need to be selected and adjust properly for better transient performance. This can be done through adaptation method as described in section V.

III. MODEL REFERENCE ADAPTIVE CONTROL

The general structure of the Model Reference Adaptive Control (MRAC) system is shown in figure 1 below. The basic MRAC system consists of 4 main components:

i) Plant to be controlled
ii) Reference model to generate desired closed loop output response
iii) Controller that is time-varying and whose coefficients are adjusted by adaptive mechanism
iv) Adaptive mechanism that uses 'error' (the difference between the plant and the desired model output) to produce controller coefficient

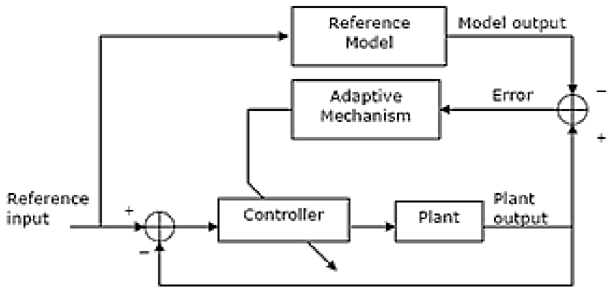

Fig. 1: General Structure of MRAC system

Regardless of the actual process parameters, adaptation in MRAC takes the form of adjustment of some or all of the controller coefficients so as to force the response of the resulting closed-loop control system to that of the reference model. Therefore, the actual parameter values of the controlled system do not really matter. Each components in the MRAC structure is as described below:

Reference Model: It is used to specify the ideal response of the adaptive control system to external command. It should reflect the performance specifications in control tasks. The ideal behavior specified by the reference model should be achievable for the adaptive control system.

Controller: It is usually parameterized by a number of adjustable parameters. In this paper two parameters θ1 and θ2 are used to define the controller law.

Adaptation Mechanism: It is used to adjust the parameters in the control law. Adaptation law searches for the parameters such that the response of the plant which should be same as the reference model.

IV. GRADIENT METHOD (MIT RULE)

This rule is developed in Massachusetts Institute of technology and is used to apply the MRAC approach to any practical system. In this rule the cost function or loss function is defined as $J(\theta) = {}^{1}\!/_{2} \cdot e^2$ where e is the error (y-y_m) and θ is the adjustable parameter. We aim to minimize the error (e=y-y_m) by designing a controller that has one or more adjustable parameters such that a certain cost function is minimized so that the output of the closed-loop system (y) to follow the output of the reference model (y_m).

V. STABILITY METHOD (LYAPUNOV)

The Lyapunov stability theory can be used to describe the algorithms for adjusting parameters in Model Reference Adaptive control system. MRAC can be designed such that the globally asymptotic stability of the equilibrium point of the error difference equation is guaranteed. To do this, the Lyapunov Second Method is used. It requires an appropriate Lyapunov function to be chosen, which could be difficult. This approach has stability consideration in mind and is also known as the Lyapunov Method.

VI. ADAPTIVE FEEDBACK CONTROLLER DESIGN SCHEME

The chemical process is described by the transfer functions and a controller by the form as indicated below where the time delay T_D and the time constant τ, of the process are unknown.

Plant: $$G(s) = e^{-T_D s}\frac{1}{\tau s+1} \quad (1)$$

Ref. Model: $$G_m(s) = e^{-s}\frac{1}{5s+1} \quad (2)$$

Controller: $$u = -ky + \frac{k_i}{s}(r-y) \quad (3)$$

A. Time Delay Approximation

A time delay element is represented by a non-linear transfer function. Since many control design methods require linear systems, it is often necessary to approximate a time delay by a linear transfer function. Therefore, a suitable approximation is needed to represent the time delay term in the system.

There are many approximation methods available but for this assignment, we have selected Padé approximation for the time delay approximation as shown below:

$$e^{-T_D s} = \frac{-T_D s+2}{T_D s+2} \quad (4)$$

$$e^{-s} = e^{-s}\frac{-S+2}{S+2} \quad (5)$$

The above approximations will be used throughout this paper in designing the controller.

B. Gradient Approach Design

The control objective is to adjust the controller parameters, θ1 and θ2, so that e(t) is minimized. To do this, a cost function, J(θ) is chosen and minimized.

$$J(\theta) = \frac{1}{2}e^2 \quad (6)$$

Let $\theta_1 = \frac{k_i}{s}$ and $\theta_2 = \frac{sk+k_i}{s}$. Substitute into the Equation (3), we obtain the control law

$$u = \theta_1 - \theta_2 \tag{7}$$

After the time delay approximation using Eq.(4) and (5), rewriting the Eq.(1) and (2) as follows:

Plant: $$G(s) = \frac{-T_D s+2}{\tau T_D s^2+(2\tau+T_D)s+2} \tag{8}$$

Ref. Model: $$G_m(s) = \frac{-s+2}{5s^2+11s+2} \tag{9}$$

Rewriting the Eq.(8) in y and u,

$$y = \frac{-T_D s+2}{\tau T_D s^2+(2\tau+T_D)s+2}u \tag{10}$$

Substituting the u from Eq.(7) into Eq.(10), we get

$$y = \frac{(-T_D s+2)\theta_1 r}{\tau T_D s^2+(2\tau+T_D)s+2+(-T_D s+2)\theta_2} \tag{11}$$

Similarly, for the reference model, we get

$$y_m = \frac{(-s+2)r}{5s^2+11s+2} \tag{12}$$

Computing the error, $e = y - y_m$,

$$e = \frac{(-T_D s+2)\theta_1 r}{\tau T_D s^2+(2\tau+T_D)s+2+((-T_D s+2)\theta_2} - \frac{(-s+2)r}{5s^2+11s+2} \tag{13}$$

Thus, by taking the derivative of both θ_1 and θ_2 ,we have

$$\frac{d\theta_1}{dt} = -\gamma\cdot e\cdot \frac{(-T_D s+2)r}{\tau T_D s^2+(2\tau+T_D)s+2+(-T_D s+2)\theta_2} \tag{14}$$

$$\frac{d\theta_2}{dt} = \gamma\cdot e\cdot \frac{(-T_D s+2)y}{\tau T_D s^2+(2\tau+T_D)s+2+(-T_D s+2)\theta_2} \tag{15}$$

In this case we need to do some approximation: i.e. perfect model following, y = y_m. Therefore, we then have,

$$\frac{d\theta_1}{dt} = -\gamma\cdot e\cdot r\left[\frac{-T_D s+2}{5s^2+11s+2}\right] \tag{16}$$

$$\theta_1 = -\frac{\gamma}{s}\cdot e\cdot r\left[\frac{-T_D s+2}{5s^2+11s+2}\right] = \frac{k_i}{s} \tag{17}$$

and

$$\frac{d\theta_2}{dt} = \gamma\cdot e\cdot y\left[\frac{-T_D s+2}{5s^2+11s+2}\right] \tag{18}$$

$$\theta_1 = \frac{\gamma}{s}\cdot e\cdot y\left[\frac{-T_D s+2}{5s^2+11s+2}\right] = k + \frac{k_i}{s} \tag{19}$$

Finally, we get

$$k = \frac{\gamma}{s}\cdot e\cdot (r+y)\left[\frac{-T_D s+2}{5s^2+11s+2}\right] \tag{20}$$

and

$$\frac{k_i}{s} = -\frac{\gamma}{s}\cdot e\cdot r\left[\frac{-T_D s+2}{5s^2+11s+2}\right] \tag{21}$$

The MRAC gradient approach is then simulated using Mathlab Simulink block with the reference input of a square wave signal with amplitude 1 and 3 with frequency of 0.0033Hz as well as $\theta_1 = 0.02$ and $\theta_2 = 0.0015$. Both the output of the system responses (y and y_m) as well as (θ_1 and θ_2) are displayed in section VII.

C. *Stability Approach Design*

For the design using Lyapunov method, sequences of steps have been established as shown below:

Step 1:

Derive a differential equation for error, $e = y - y_m$ into $\dot{e}, \ddot{e}$containing the parameter, θ.
From Eq.(11), replacing A= $-T_D$, B= τT_D, C= $2\tau + T_D$ and $d = \frac{1}{B}$, the equation now becomes,

$$\ddot{y} = -(A\theta_2 + C)d\dot{y} - (2+2\theta_2)dy + A\theta_1 d\dot{r} + 2\theta_1 dr \tag{22}$$

$$\ddot{y_m} = -2.2d\dot{y_m} - 0.4y_m - 0.2\dot{r} + 0.4r \tag{23}$$

Then, substituting Eq.(22) and (23) into $\ddot{e} = \ddot{y} - \ddot{y_m}$, we get

$$\ddot{e} = -2.2\dot{e} - 0.4e - \left(\theta_2 + \frac{C}{A} - \frac{2.2}{Ad}\right)Ad\dot{y} - \left(\theta_2 + 1 - \frac{0.2}{d}\right)2dy + \left(\frac{0.2}{Ad} + \theta_1\right)Ad\dot{r} + \left(\theta_1 - \frac{0.4}{2d}\right)2dr \tag{24}$$

Step 2:

Find a suitable Lyapunov function, V (e, θ) - usually in a quadratic form (to ensure positive definiteness).

The Lyapunov function, V ($\dot{e}, e, X_1, X_2, X_3, X_4$) is based on Eq.(24) above where V becomes,

$$V = \lambda_1 \dot{e}^2 + \lambda_2 e^2 + \lambda_3 {X_1}^2 + \lambda_4 {X_2}^2 + \lambda_5 {X_3}^2 + \lambda_6 {X_4}^2$$

Where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 > 0$ is positive definite. The derivative of V then becomes,

$$\begin{aligned}\dot{V} = -4.4\lambda_1\dot{e}^2 + (\lambda_2 - 0.4\lambda_1)2e\dot{e} + \left(\lambda_3\dot{X_1} - \lambda_1 Ad\dot{e}\dot{y}\right)2X_1 \\ + \left(\lambda_4\dot{X_2} - 2\lambda_1\dot{e}dy\right)2X_2 + (\lambda_5\dot{X_3} \\ + \lambda_1 Ad\dot{e}\dot{r})2X_3 + (\lambda_6\dot{X_4} + 2\lambda_1 d\dot{e}r)2X_4\end{aligned}$$

where for stability $\dot{V} < 0$.

Step 3:

Derive an adaptation mechanism based on V(e,θ) such that e goes to zero.

$$\dot{X_1} = \frac{2.5\lambda_2 Ad\dot{e}\dot{y}}{\lambda_3} = \dot{\theta}_2 \tag{25}$$

$$\dot{X}_2 = \frac{5\lambda_2 d\dot{e}y}{\lambda_4} = \dot{\theta}_2 \qquad (26)$$

$$\dot{X}_3 = \frac{-2.5\lambda_2 Ad\dot{e}\dot{r}}{\lambda_5} = \dot{\theta}_1 \qquad (27)$$

$$\dot{X}_4 = \frac{-5\lambda_2 d\dot{e}r}{\lambda_6} = \dot{\theta}_1 \qquad (28)$$

Summing Eq.(25) and (26) we get

$$\dot{\theta}_2 = 1.25d\dot{e}\left(\frac{\lambda_2 A\dot{y}}{\lambda_3} + \frac{2\lambda_2 y}{\lambda_4}\right)$$

$$\theta_2 = k + \frac{k_i}{s} = \left(e\dot{y} + \frac{2\lambda_3}{A\lambda_4}ey\right)\left(\frac{1.25\lambda_2 Ad}{\lambda_3}\right)$$

Similarly, summing Eq.(27) and (28) we get

$$\dot{\theta}_1 = -(2.5\lambda_2\lambda_6 Ad\dot{e}\dot{r} + 5\lambda_2\lambda_5 d\dot{e}r)\left(\frac{1}{2\lambda_5\lambda_6}\right)$$

$$\theta_1 = \frac{k_i}{s} = -\left(e\dot{r} + \frac{2\lambda_5}{A\lambda_6}er\right)\left(\frac{1.25\lambda_2 Ad}{\lambda_5}\right)$$

Thus, rewriting the equation for k and $\frac{k_i}{s}$, we get

$$k = -\gamma_1 e(\dot{y} - 4y) - \gamma_2 e(\dot{r} - 4r)$$

$$\frac{k_i}{s} = \gamma_2 e(\dot{r} - 4r)$$

VII. RESULT AND DISCUSSION

A. *Gradient Approach Result*

The Simulink block diagram for simulation of chemical plant using Gradient method is as shown in figure 2 below.

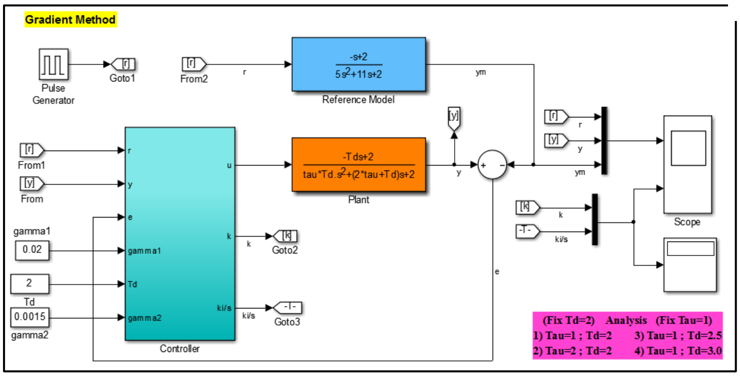

Fig. 2: Simulink Block Diagram for plant simulation using Gradient method

The simulation is done with reference input of square wave signal with amplitude of 1 with period of 300s and pulse width of 80%. The adaptation gain $\gamma_1$and $\gamma_2$is set to be 0.02 and 0.0015 respectively.The analysis is done by fixing τ = 1 and 2 while varying the T_D and vice versa as highlighted in Fig.2 above.

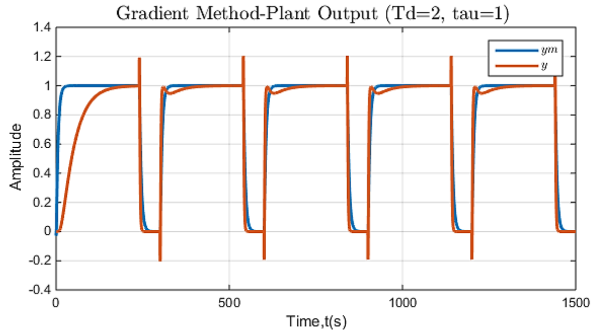

Fig. 3: System Output (y_m ,y) with τ =1 and $T_D = 2$

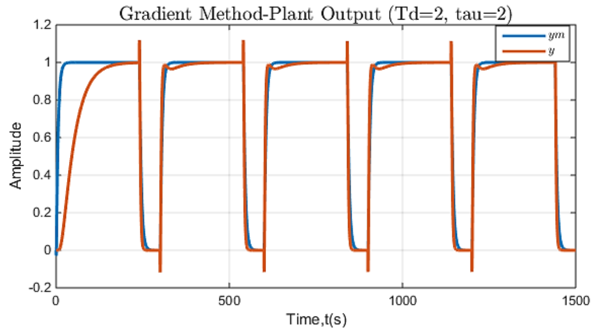

Fig. 4: System Output (y_m ,y) with τ=2 and $T_D = 2$.

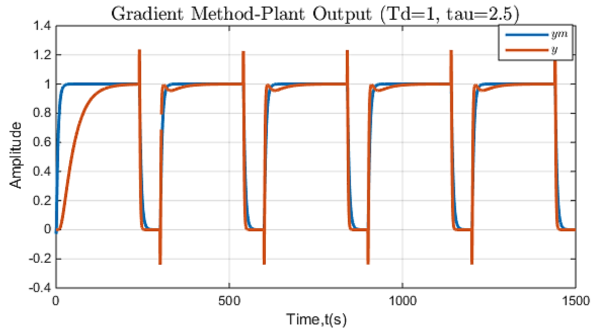

Fig. 5: System Output (y_m ,y) and with τ=1 and $T_D = 2.5$

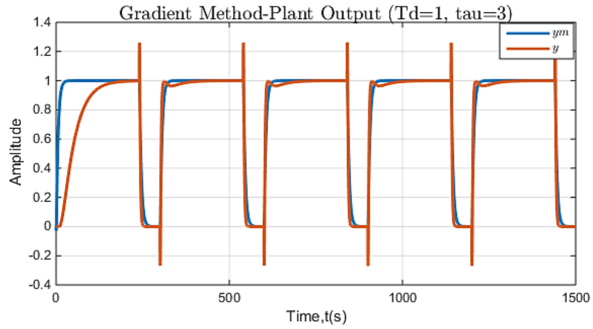

Fig. 6: System Output (y_m ,y) with τ=1 and $T_D = 3.0$

From Fig.3 to 6, we can clearly see that the controller designed using MIT Rule is able to adapt to any changes made to the system by varying the time delay, τ and constant, T_D. The controller has successfully forces the system output to track the reference output y_m closely as shown but with a small overshoot.

In addition, the controller parameters (θ_1, θ_2) designed using Gradient Method is also able to converge to a steady value after some time but with small fluctuations as shown in Fig.7 below.

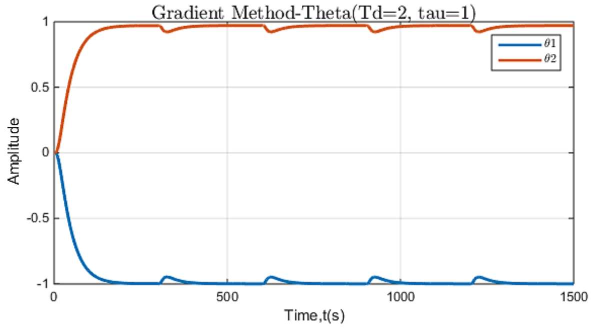

Fig. 7: Controller parameters (θ_1, θ_2) with τ=1 and $T_D = 2.0$

B. Stability Approach Result

The Simulink block diagram for simulation of chemical plant using Lyapunov method is as shown in figure 8 below.

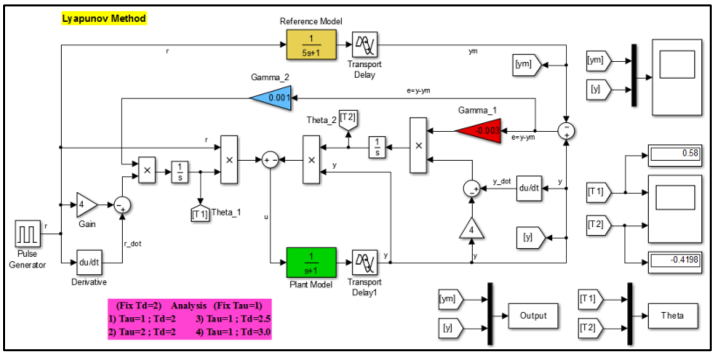

Fig. 8: Simulink Block Diagram for plant simulation using Lyapunov method

The simulation is done with reference input of square wave signal with amplitude of 3 with period of 1200s and pulse width of 80%. The adaptation gain $\gamma_1$and $\gamma_2$is set to be 0.003 and 0.001 respectively.The analysis is done by fixing τ = 1 and 2 while varying the T_D and vice versa as highlighted in Fig.8 above.

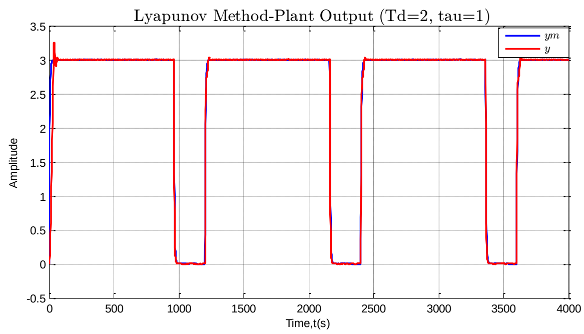

Fig. 9: System Output (y_m ,y) with τ=1 and $T_D = 2.$

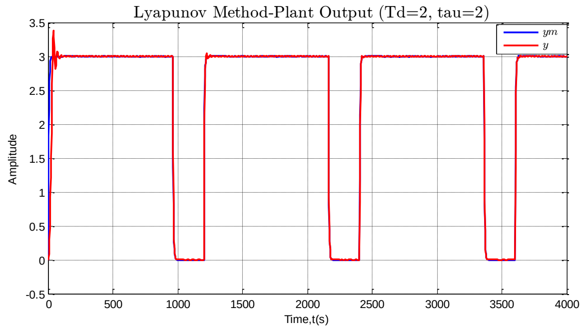

Fig. 10: System Output (y_m ,y) with τ=2 and $T_D = 2.$

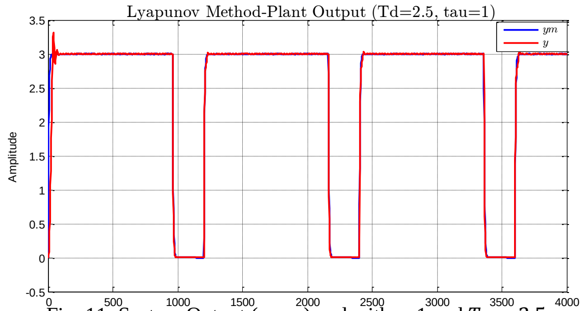

Fig. 11: System Output (y_m ,y) and with τ=1 and $T_D = 2.5$

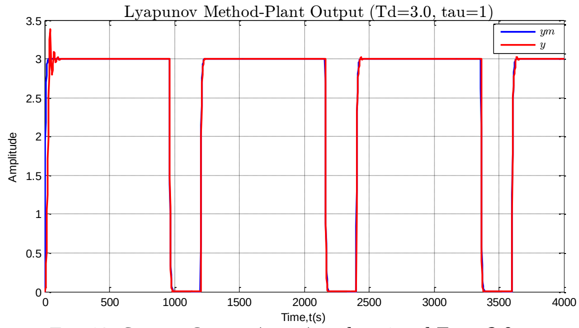

Fig. 12: System Output (y_m ,y) with τ=1 and $T_D = 3.0$

From Fig.9 to 12, we can clearly see that the controller designed using Lyapunov is able to adapt to any changes made to the system by varying the time delay and constant. The controller has successfully forces the system output to track the reference output $y_m$perfectly as shown.

In addition, the controller parameters (θ_1, θ_2) designed using Lyapunov Method is also able to converge quickly to a steady value as shown in Fig.13 below.

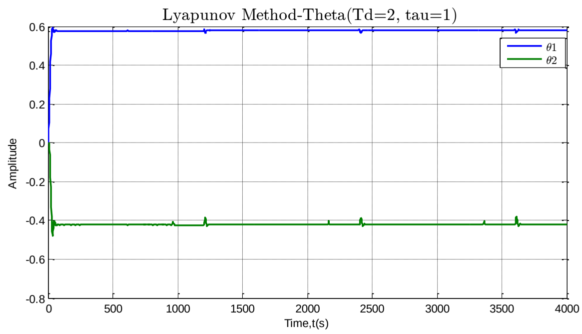

Fig. 13: Controller parameters (θ_1, θ_2) with τ=1 and $T_D = 2.0$

C. Comparison between Gradient and Stability Approach

The MRAC designed using the Gradient Method does not guarantee stability to the resulting closed-loop system. In contrast, the Stability approach/ Lyapunov method has proven to be more stable compared to MIT Rule as shown in Fig.14 below where the comparisons between their responses are plotted.

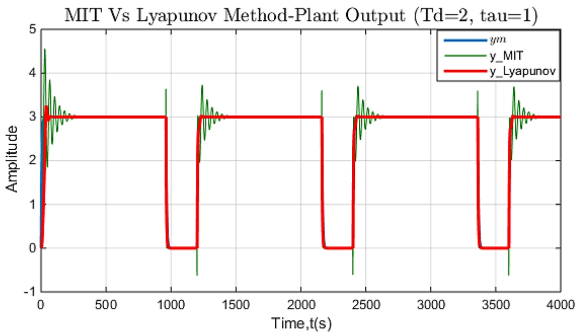

Fig. 14: Comparisons between performance of MIT and Lyapunov method.

Based on the observations from above designs, we can deduce the differences of each design as below:

i) The adjustable controller parameter θ_1 *and* θ_2 for Lyapunov method are simpler as compared to Gradient Method as summarized in table 1 below.

Table 1: Comparison of adjustable parameters for both design approaches.

Lyapunov Method	Gradient Method
$\frac{d\theta_1}{dt} = -\gamma_1 er$	$\frac{d\theta_1}{dt} = \gamma_1 e\left[\frac{(As+2)r}{5s^2+11s+2}\right]$
$\frac{d\theta_2}{dt} = \gamma_2 ey$	$\frac{d\theta_2}{dt} = -\gamma_2 e\left[\frac{(As+2)y}{5s^2+11s+2}\right]$

ii) The performance of MRAC designs is also depended on the amplitude of the reference input signal where increase in amplitude will affect the result of the system. Simulation with amplitude of 1 with good response is as shown in Fig.15 below with output of MIT rules plotted against the Lyapunov Method.

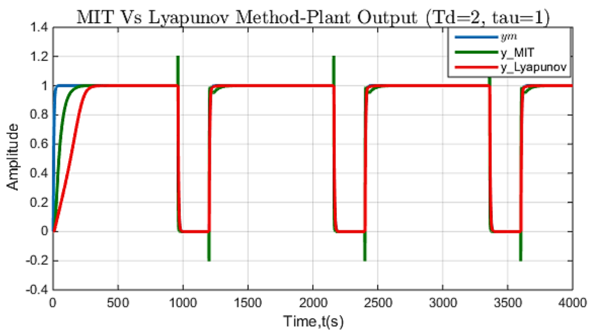

Fig. 15: System Output with Amplitude =1.

This shows that when the system is injected with a reference signal of amplitude = 1, both controllers are able to give a stable output with slow transient response at beginning.

However, when the amplitude is increased to 3 MIT rules does not give satisfactory result with a high overshoot as compared to Lyapunov method. This is shown in Fig. 14 earlier where the comparisons between the two controllers are being made. MIT rule shows high overshoot while Lyapunov Method is still able to track perfectly.

In terms of electronics and control point of view, an accumulation or saturation of error could result in the control actuator which might eventually spoil the actuator in long term operations. Thus, it is concluded that controller design using Gradient Method is said to be less stable as compared to Lyapunov method as expected.

iii) As shown in Fig.7 and 13 earlier, the controller parameters θ_1 and θ_2 of Lyapunov converge quickly as compared to MIT rule with small fluctuations which does not converge when amplitude of reference signal is increased.

VIII. CONCLUSION

In conclusion, this paper has examined the comparisons in performance between both Gradient and Stability approaches for Model Reference Adaptive Control Design and it is proven that Lyapunov method is more preferable since it ensures a stable closed-loop system while gradient approach doesn't take any consideration on the stability of the system.

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