IE3094 MDP
DYNAMIC AIRLINE PRICING

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INTRODUCTION

- A standard definition of dynamic pricing in airline markets typically focuses on how fares evolve over the booking period that precedes a flight take-off.
- In this study, we optimize over both:
  1. Ticket price
  2. Returned tickets price

Related Literature

- Broadly speaking the literature on airline pricing can be divided into two categories:

  1. Models that assume customers are willing to pay a certain price and then the firm decision is to either accept their request or not. (Van Slyke & Young 2000, Sawaki 2003)
  2. A more realistic model considers customers willingness to purchase tickets at a given price is unknown. (Gallego & Van Ryzin 1994, Feng & Xiao 2000)

Demand Model

- Demand in each period depends on :
  1. Ticket price $p_t \in [\underline{p}_t, \overline{p}_t]$
  2. Returned ticket price $p_{rt} \in [\underline{p}_{rt}, \overline{p}_{rt}]$

$$D_t(p_t, \epsilon_t) = \kappa_t(p_t, p_{rt}) + \epsilon_t$$

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Deterministic function of the price & returned price

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Noise perturbations that are independent over time & have zero mean.

Model

- Decision epochs: $T = \{1, \ldots, N\}$
- States: $s_t \in \{0, \ldots, C\}$

  where s_t is the number of remaining seats in period t and C is the maximum number of seats.

- Rewards: $r_t(s_t, (p_t, p_{rt}))$

$$= p_t E[\min(s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t)]$$ Revenue from tickets sold

$$- p_{rt} E[R_t(s_t, p_{rt})]$$ Cost of returned tickets

Model

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Revenue from tickets sold

Cost of returned tickets

Random variable representing the number of returned tickets during period t. It depends on the number of remaining seats and return price.

Model

- Transition probabilities:

$$P_t(s_{t+1}|s_t, p_t, p_{rt}) = P(R_t(s_t, p_{rt}) - \min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\} = s_{t+1} - s_t)$$

- For $t = 1, 2, \ldots, T$ we have the Bellman recursion

$$v_t^*(s_t) = \max_{p_t, p_{rt}} \left\{ p_t E[\min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\}] - p_{rt} E[R_t(s_t, p_{rt})] + \gamma E[v_{t+1}(s_{t+1})] \right\}$$

$$v_N(s) = 0 \ \forall s \in \mathcal{S}$$

where γ is a discount factor.

Numerical Experiments

- Linear deterministic demand function:

$$\kappa_t\left(p_t\, , p_{rt}\right) = a - bp_t + cp_{rt}$$

Parameters	*a*	*b*	*c*
Case 1	60.2	10	0
Case 2	60.2	10	1

$$p_t \in \{1, 1.1, 1.2, \ldots, 5.1\}$$

$$p_{rt} \in \{0.8, 1.1, 1.2, \ldots, 5.1\}$$

Numerical Experiments

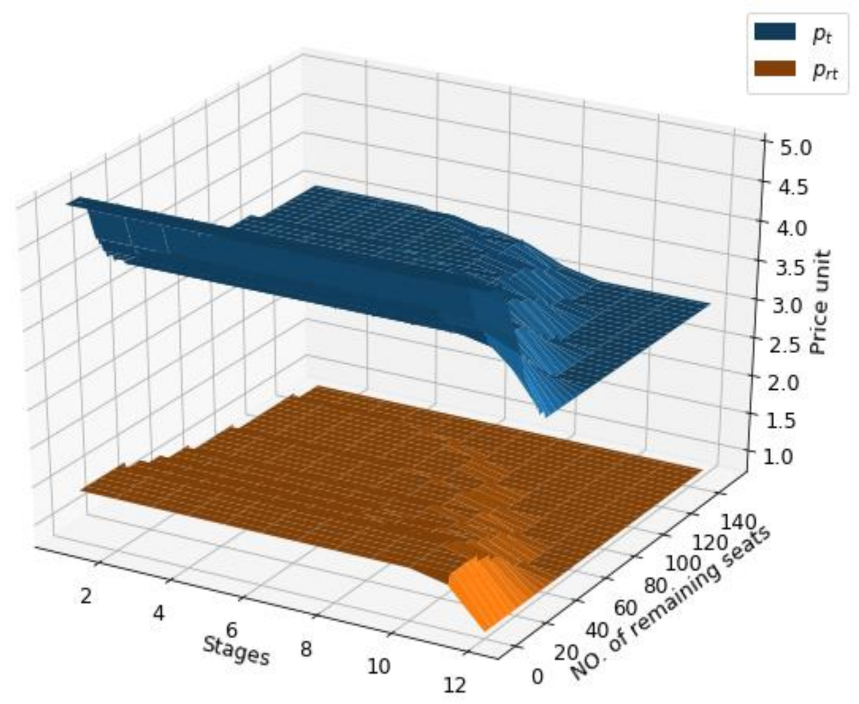
- Max number of seats $C=150$
- Finite horizon $N=12$
- Discount factor $\gamma=0.9$
- Noise $\epsilon_t \in \{-10, \ldots, 10\}$ & $E[\epsilon_t] = 0$
- The number of returned tickets $R_t(s_t, p_{rt}) \in \{0, \ldots, C - s_t\}$

  follows a binomial distribution with parameter p, where

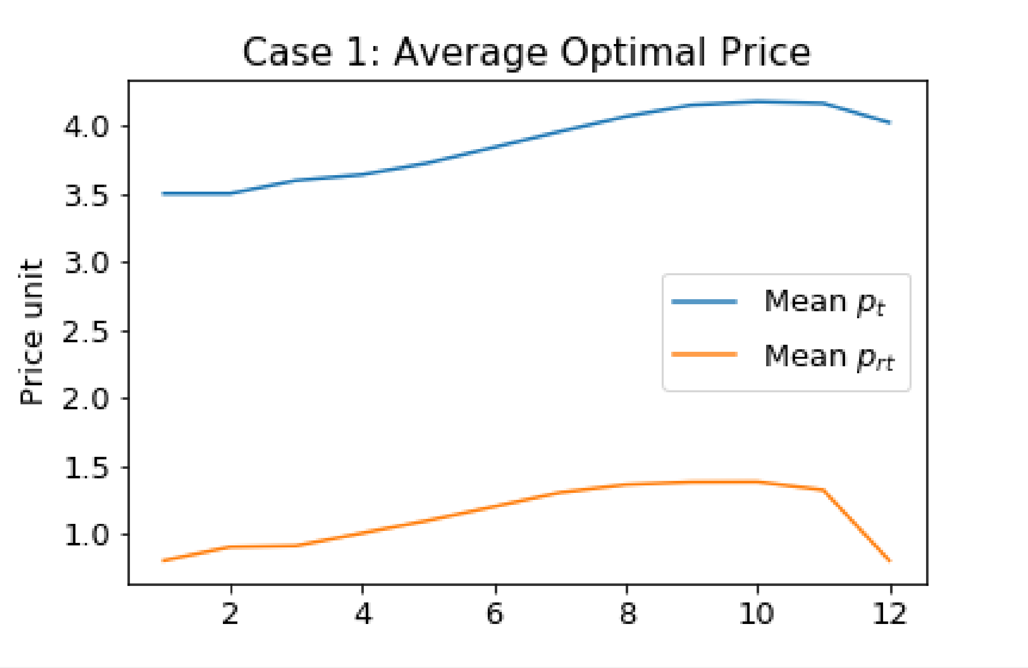
$$p = \frac{p_{rt} - 0.9 \underline{p}_{rt}}{\bar{p}_{rt} - 0.8 \underline{p}_{rt}} \times 0.9$$

Numerical Experiments

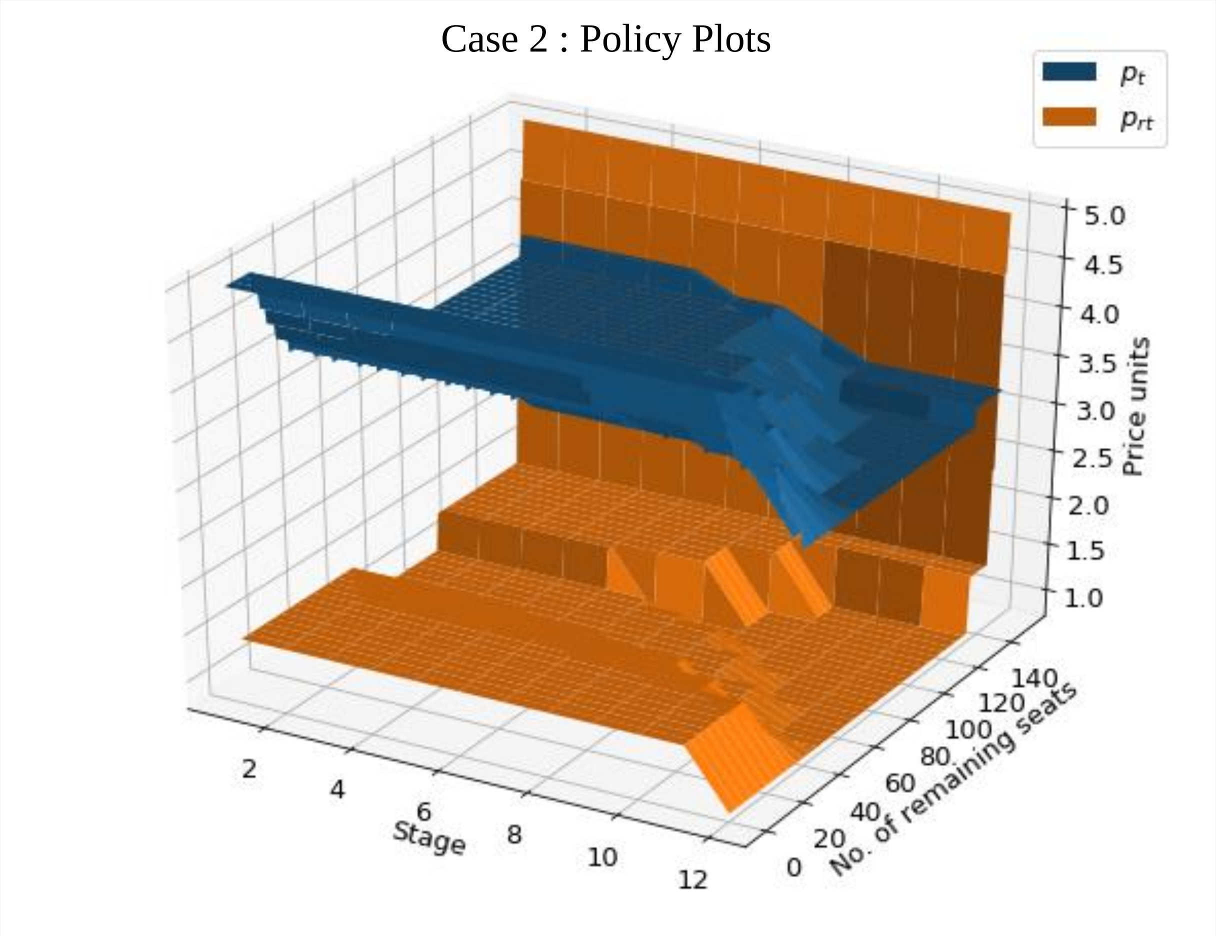
Case 1 : Policy Plots



Numerical Experiments



Numerical Experiments



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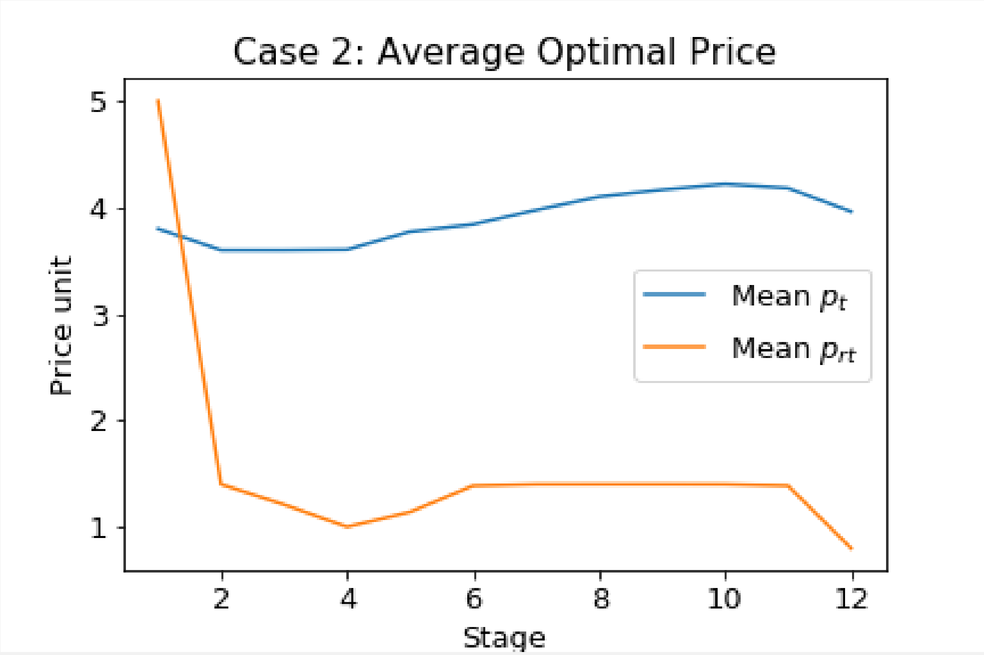

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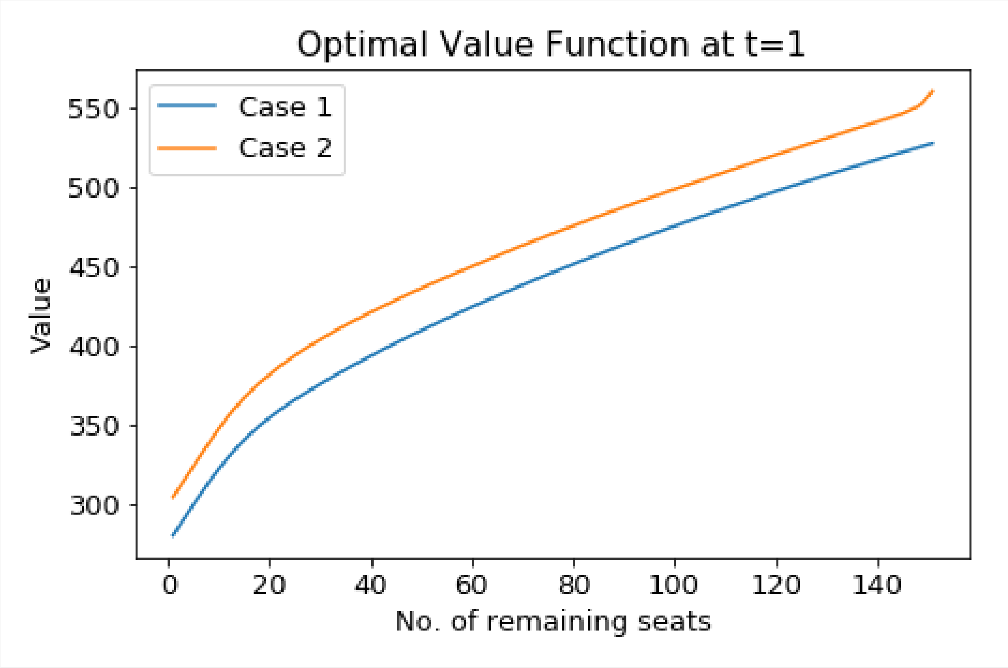

CONCLUSION

- In the case where demand is affected by the flexible returned tickets price even by a very small amount:
  - The added value of having dynamic returned ticket price is significant
  - The optimal policy is no longer monotone decreasing.

- Otherwise, it always optimal to set the sale price higher than the return price.

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Questions?