IE 3094 MDP Project

Dynamic Airline Pricing

Ibrahim El Shar & Boyuan Lai

1 Introduction

Suppose that an airline manager is responsible for setting the price for airline tickets at the beginning of each period in a finite planning horizon consisting of N periods of equal length. To make the problem more interesting we assume that customers can return their tickets for a certain p_{rt}, (To avoid losses we can restrict p_{rt} on the lowest price sold so far. However, it might be better to leave it without restriction since it might encourage returning tickets allowing thus the opportunity to possibly resell them at a higher price in the future. We also think that since we are maximizing the revenue than the losses from the difference between the sale and return price will be automatically delt with/ avoided. Note that the return price will always be less than the sale price at any period.) The goal is to optimize the prices to set for flight tickets p_t and the value of returned tickets p_{rt} at the beginning of period t. Demands in each period are non-negative, independent and depends on the ticket fare according to a stochastic demand function

$$D_t(p_t, p_{rt}, \epsilon_t) := \kappa_t(p_t, p_{rt}) + \epsilon_t,$$

where $D_t(p_t, p_{rt}, \epsilon_t)$ is the demand in period t, ϵ_t are random perturbations and $\kappa_t(p_t, p_{rt})$ is a deterministic demand function of the price p_t and the returned price p_{rt}. The random variables ϵ_t are independent over time with $E[\epsilon_t] = 0$ without loss of generality. Furthermore, we assume that the expected demand $E[D_t(p_t, p_{rt}, \epsilon_t)] = \kappa_t(p_t, p_{rt}) < \infty$ is strictly decreasing in the ticket price p_t and strictly increasing in the returned ticket price p_{rt}, both p_t and p_{rt} are restricted to a set of feasible price levels $[\underline{p}_t, \overline{p}_t]$ and $[\underline{p}_{rt}, \overline{p}_{rt}]$ resoectively, where $\underline{p}_t, \overline{p}_t$ are the minimum and the maximum prices and $\underline{p}_{rt}, \overline{p}_{rt}$ are the minimum and the maximum returned prices that can be set during time period t. The problem can be formulated as a Markov Decision Process (MDP) with state s_t which is a scalar representing the number of remaining seats, at beginning of period t. Note that there is an obvious trade-off in setting the prices for the demand. Setting low fares at the beginning of the planning horizon may result in a full booking but may be less profit than following a strategic policy that sets the price to a certain threshold and then increase the price in each period. Our intuition tells us that the optimal policy will be a function of the state (number of remaining seats). Similarly, there is a trade-off between the number of returned tickets and the returned tickets' value in the sense that one may want to have as much less number of returns as possible however that may not be optimal profit-wise since the returned tickets' price may be less than sale price itself.

2 Model

- Decision epochs: $T = \{1, \dots, N\}$

- The state $s_t \in \{0, \dots, C\}$, is the number of remaining seats at beginning period t, where C is the maximum number of seats.

- The actions are the prices $p_t \in [\underline{p}_t, \overline{p}_t]$ and the returned value $p_{rt} \in [\underline{p}_{rt}, \overline{p}_{rt}]$ to set in each period.

- Rewards at each period t, $r_t(s_t, p_t, p_{rt}) = p_t E[\min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\}] - p_{rt} E[R_t(s_t, p_{rt})]$, where $R_t(s_t, p_{rt})$ is a discrete random variable representing the number of returned tickets during period t, taking values in $\{0, 1, \dots, C - s_t\}$. Furthermore, it is increasing in p_{rt}.

- Transition probabilities: for all $0 \le s_t, s_{t+1} \le C, \forall t$

$$P_t(s_{t+1}|s_t, p_t, p_{rt}) = P(R_t(s_t, p_{rt}) - \min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\} = s_{t+1} - s_t)$$

- Note that to avoid negative demands we constrain $|\epsilon_t| \le \kappa_t(\overline{p}_t, \underline{p}_{rt})$

Let $v_t^*(s_t)$ be the revenue-to-go function at the beginning of period t with number of available seats s_t. Let there be no end-of-horizon profit, i.e., $v_{N+1}^*(s) = 0$ for all s. Then, for each $t = 1, 2, \dots, N$, we have the Bellman recursion,

$$\begin{aligned} v_t^*(s_t) &= \max_{p_t, p_{rt}} \mathbf{E}\Big[p_t E[\min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\}] - p_{rt} E[R_t(s_t, p_{rt})] + \gamma v_{t+1}(s_{t+1})\Big] \\ s_{t+1} &= s_t + R_t(s_t, p_{rt}) - \min\{s_t, \kappa_t(p_t, p_{rt}) + \epsilon_t\} \quad \forall\, t \in T \end{aligned} \tag{2.1}$$

where $\gamma \in (0, 1)$ is a discount factor.

3 Literature Review

Our line of research problem goes under the broad title of yield management. Herein we focus on reviewing the papers that deal mainly with the airline industry where the main focus is maximizing revenue. Alderighi et al. [2015] and Escobari [2012] show that airlines face stochastic demand and that prices are related to remaining capacity. Their findings support the theoretical results of Gallego and Van Ryzin [1994] and other research work that studies optimal pricing under stochastic demand, limited capacity, and finite time selling window. Broadly speaking the literature on airline pricing can be divided into two categories. Models that assume customers are willing to pay a certain price and then the firm decision is to either accept their request or not. An example of such models are the ones by Van Slyke and Young [2000], Sawaki [2003], and Lautenbacher and Stidham Jr [1999]. On the other hand, a more realistic model considers customers willingness to purchase tickets at a given price is unknown. Our model falls into this category. Gallego and Van Ryzin [1994] study the problem of dynamic pricing inventories where demand is price sensitive and stochastic. The price dependent stochastic demand is modeled as a Poisson process. For the case where the demand function is exponential they found a closed form solution. They also used a deterministic heuristic to find an upper-bound on the revenue under general demand functions. They further showed the monotonicity of the optimal price policy and they extend their results to allow time-dependent demand, compound Poisson demand, discrete prices, variable initial capacity, discounting, overbooking, resupply and cancellations. Similarly, Feng and Xiao [2000] model the demand as a Poisson process that depends on the price. They studied the problem of switching between predetermined finite set of prices at certain time thresholds that depends on the remaining stock and time. They also find the optimal time switching policy. When demand is a nonhomogenous Poisson process and the price set is compact, the structural properties of the optimal price-switching policy is shown by Zhao and Zheng [2000]. To maximize revenue, some authors have considered rationing the capacity with price classes instead of changing prices dynamically to ensure that high-paying customers are served. Ladany [1996], developed a dynamic program with a deterministic nonlinear demand and solve for the optimal number of price classes, optimal capacity for each price class, and the optimal price for each class. Other modeling approaches including using graph theory for pricing and seat allocation for example Garcia-Diaz [1997] and Kuyumcu and Garcia-Diaz [2000].

Compare to existing approach, we consider customers willingness to purchase tickets at a given price is unknown with one price class. We further generalize the problem by considering the return ticket price and its effect on the demand

4 Numerical Experiments

In this section, we study the effect of dynamic returned tickets price by considering two cases. We assume a linear deterministic demand function. In the first case, we assume that the deterministic demand function is independent of the returned ticket price,

$$\kappa_t(p_t) = 60.2 - 10p_t.$$

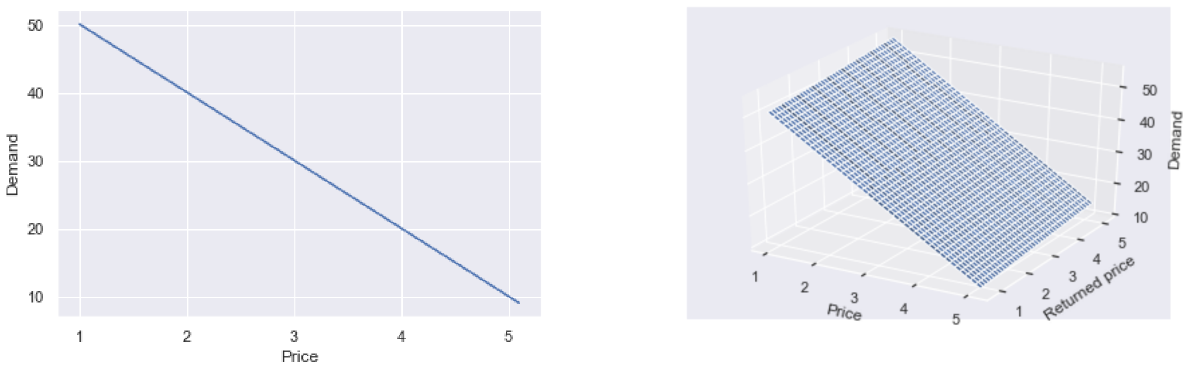
The second numerical example however considers the case where the demand is slightly affected by the returned ticket price,

$$\kappa_t(p_t, p_{rt}) = 60.2 - 10p_t + p_{rt}.$$

In both cases, we restrict p_t and p_{rt} to a set of discrete price levels, $p_t \in \mathcal{P}_t = \{1, 1.1, 1.2, \ldots, 5.1\}$ and $p_{rt} \in \mathcal{P}_{rt} = \{0.8, 0.9, 1.1, \ldots, 5.1\}$. Fig. 1 shows the demand functions in both cases as function of p_t and p_{rt}. Furthermore, we assume that the demand perturbations ϵ_t are independent, have zero mean, and follows the distribution shown in Fig. 2. The maximum number of seats considered is $C = 150$. The number of returned tickets $R_t(s_t, p_{rt}) \in \{0, \ldots, 150 - s_t\}$ follows a binomial distribution with parameter $f(p_{rt})$ such that

$$f(p_{rt}) = \frac{p_{rt} - 0.9\underline{p}_{rt}}{\overline{p}_{rt} - 0.8\underline{p}_{rt}} \times 0.9.$$

Note that $f(p_{rt})$, the probability that a customer returns his ticket, is decreasing in p_{rt} as illustrated in Fig. 3. We solve the DP for $N = 13$ stages, where in the last stage no action is required.



(a) Case 1: $\kappa_t(p_t) = 60.2 - 10p_t$.

(b) Case 2: $\kappa_t(p_t, p_{rt}) = 60.2 - 10p_t + p_{rt}$.

Figure 1: Deterministic demand function of the price, p_t, and returned ticket price, p_{rt}.

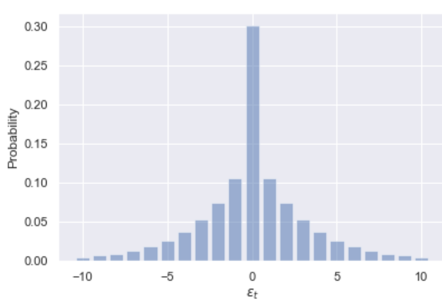


Figure 2: Demand perturbations ϵ_t distribution.

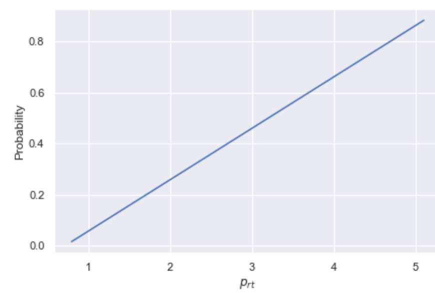


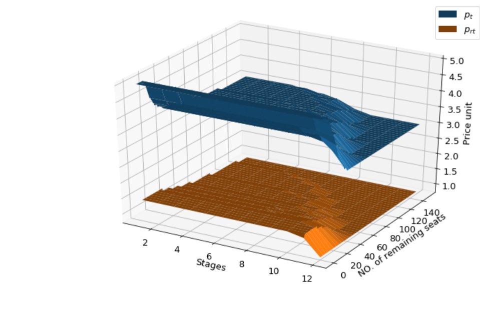
Figure 3: The binomial parameter $f(p_{rt})$ as function of the returned ticket price, p_{rt}.

The results of the numerical experiments of both cases are shown in Fig. 4. For case 1, the optimal policy is decreasing in both the price and the returned ticket price as shown in Fig. 4a. However, the monotonicity result does not hold for case 2 as seem in Fig. 4b. This is because the demand is affected by the return ticket price for case 2. This effect is clearly reflected by the optimal policy in the sense that for large states (number of remaining seats), the optimal action is to set a high return ticket price in order to increase the demand[1]. To check the average price set by the optimal policy at each stage, we simulate 5000 sample path, starting from 150 remaining seat, under the optimal policy for both cases. The results of the simulations are shown in Fig. 4c and Fig. 4d for case 1 and case 2 respectively. Furthermore, the average number of remaining seats and average percentage of returned tickets obtained from the simulations are shown in Fig. 4e to Fig. 4h. Note that the shaded region shown in these plots represent one standard deviation form the sample mean. For both cases, the average number of remaining seats at stage 13 are 3.4 and the average ratio of returned tickets are under 15%. The highest ratio of returned tickets recorded is 25% for case 1 and 40% for case 2. Nonetheless, the expected discounted value at stage 1 starting from 150 remaining seats is 525 for case 1 and 565 for case 2.

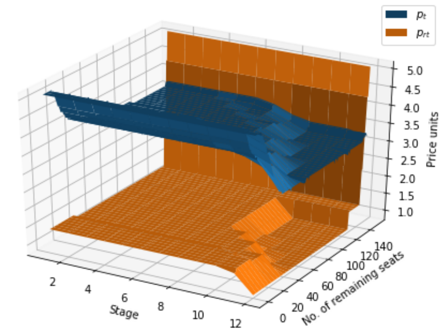
5 Conclusion

In this project, we study a new version of the airline pricing problem by optimizing over both ticket price and the return ticket price. Our numerical examples show that, in the case where demand is affected by the flexible returned tickets price even by a very small amount, the added value of having dynamic returned ticket price is significant and the optimal policy is no longer monotone decreasing. Otherwise, it is always optimal to set the sale price higher than the return price.

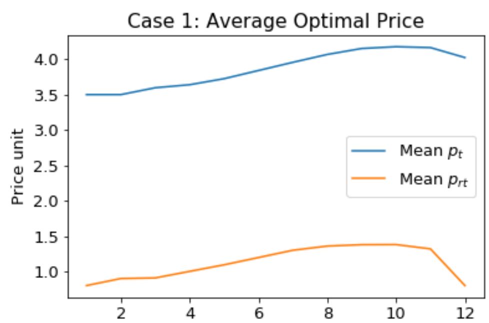
[1] The ticket price and the return ticket price are both declared to the arriving customers.



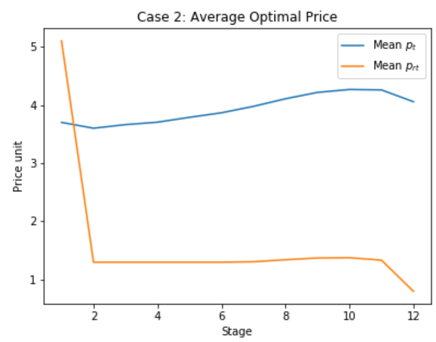
(a) Case 1: Optimal Policy



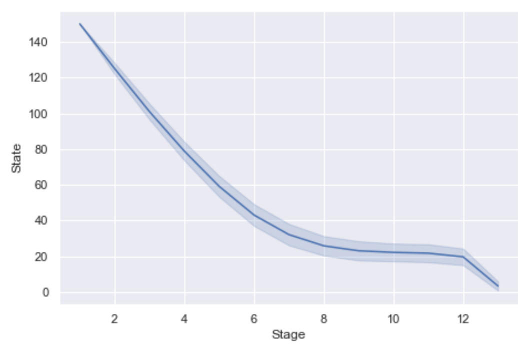
(b) Case 2: Optimal Policy



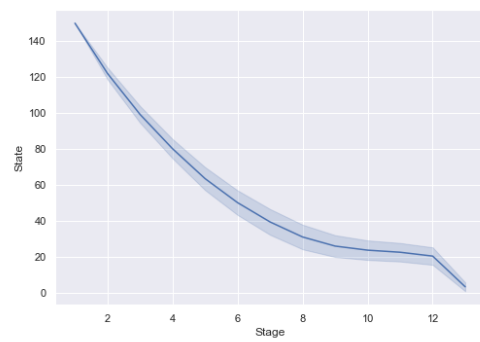
(c) Case 1: Average Optimal Price



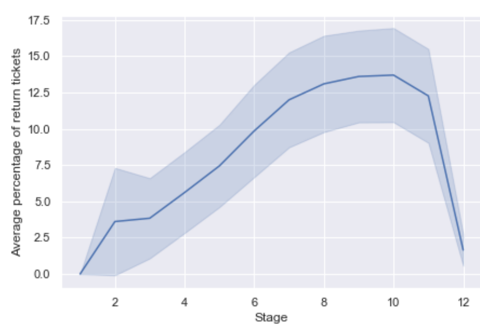
(d) Case 2: Average Optimal Price



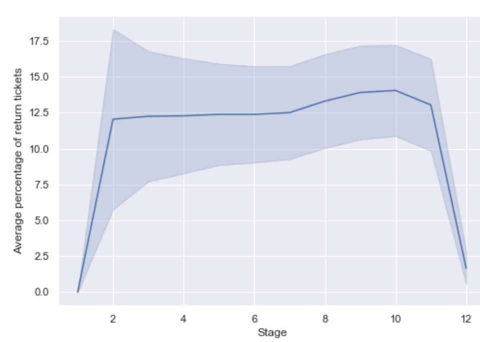
(e) Case 1: Average State



(f) Case 2: Average State



(g) Case 1: Average Percentage of Return Tickets



(h) Case 2: Average Percentage of Return Tickets

Figure 4: Case 1 vs. Case 2 comparison plots.

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