Ain Shams University
Faculty of Science
Mathematics Department
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Program: Math. Stat. and Comp. Sci.
Course Code: STAT418 (Level 4)
Course Name: Stochastic Processes II
Time Allowed: 2 Hours

Answer all questions (Full Mark:70)

Q1 (Mark: 15)

(A) Discuss the stationarity of each of the following processes

1- The Process $X(t) = A_1 + A_2 t$ where A_1, A_2 are independent random variables with $E(A_i) = a_i$, and $Var(A_i) = \sigma_i^2, i = 1, 2$. (5 marks)

2- The Poisson processes. (5 marks)

(B) Find the probability function for the convolution of two binomial distributions with parameters (n, p) and (m, p). (5 marks)

Q2 (Mark: 20)

(A) Consider a series of Bernoulli trails with probability of success p. Suppose that X denotes the number of failures preceding the first success and Y the number of failures following the first success and preceding the second success. With *joint pdf*

$P_{jk} = Pr(X = j, Y = k) = q^{j+k}p^2, \quad j, k = 0, 1, 2, \ldots$

Find the joint p.g.f of: 1-(X,Y) 2-X+Y. (10 marks)

(B) Use the p.g.f to solve the equations $U_n'(t) = U_{n-1}(t), \ t \geq 0, \ n = 1, 2, \ldots$ with the initial conditions $U_0(t) = 1, t > 0$ and $U_0(0) = 1, U_n(0) = 0$ for $n \neq 0$. (10 marks)

Q3 (Mark: 20)

(A) Define each of the following: (10 marks)

1- The stochastic process and state the 4 classified types of the one dimensional process.

2- The Markov process and Markov chain.

3- Stationary process.

(B) Find the steady state recursive equations for the Birth-Death process and use it in case of M/M/1 Queening system. (10 marks)

Q4 (Mark: 15)

Suppose that the chance of rain tomorrow depends only on previous weather conditions only through whether or not it is raining today and not on the past weather conditions. Suppose also that if it is rains today, then it will rain tomorrow with probability 0.8; and if it does not rain today, then it will rain tomorrow with probability 0.6. Find:

(1) The TPM then calculate the probability that it will rain four days from today given that it is raining today. **(10 marks)**

(2)The limiting distribution (Steady-State) for the given Markov chain. **(5 marks)**

Today \ Tomorrow	rain	not rain
rain	0.8	0.2
not rain	0.6	0.4

$$q = [0.75 \quad 0.25]$$
$$0.75 \quad 0.25$$

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