Evaluating Functions

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What is a function?

input → [function] → output

$f(x) = x + 5$

$f(5) = 5 + 5 = 10$

(5: input; 10: output)

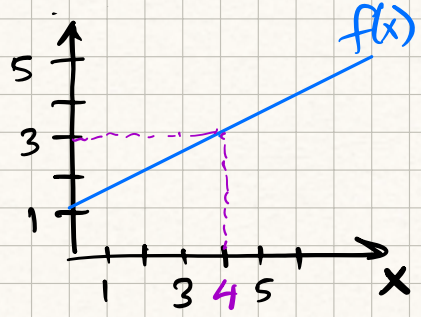
What is NOT a function?

$x^2 + y^2 = 4$ not a function b/c more than one output

$x = 1 \Rightarrow y^2 = 3$

$y = \mp\sqrt{3}$

Evaluate from graph



$f(4) = 3$

Summary: In mathematics, a function from set X to a set Y assigns to each element of X, exactly one element of set Y.

Inputs and Outputs of a Function

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Matching an input to a function's output

$f(t) = -2t + 5$

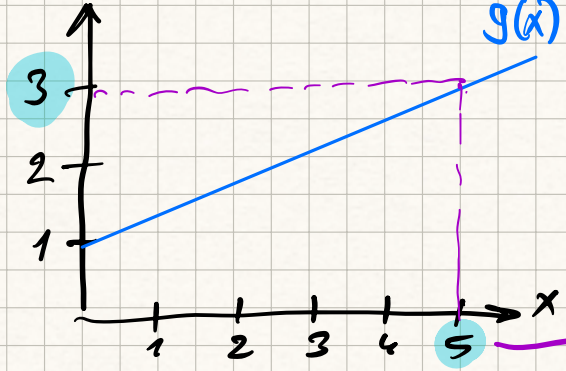
$f(t) = 13 \Rightarrow t = ?$

$13 = -2t + 5$

$2t = -8$

$t = -4$

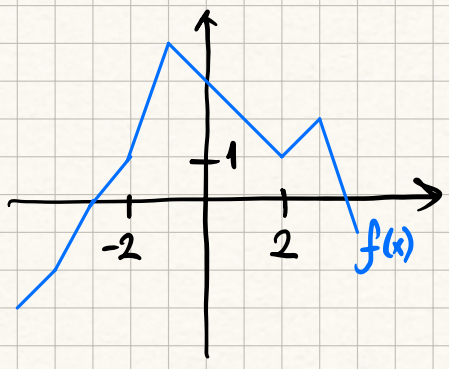
From graph?



$g(x) = 3 \Rightarrow x = ?$

$x = 5$

Two inputs with the same output



$f(-2) = -1$

$f(2) = 1$

$f(-2) = f(2)$

Summary: We can use the graph to match inputs and outputs to each other. A function can have multiple inputs with the same output.

Functions and Equations

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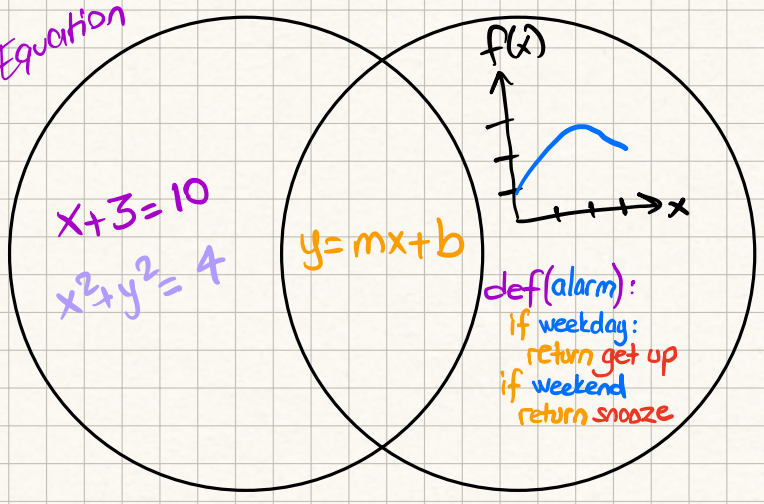
Differences between equations and functions



Obtaining a function from an equation

For a given input value b, the function f outputs a value a to satisfy the following equation.

$$4a + 7b = -52$$

Write a formula for $f(b)$ in terms of b.

$4a = -52 - 7b$

$a = -13 - \frac{7}{4}b = f(b)$

Summary: Not every equation is a function. Not every function is an equation. There are equations who define a function.

Interpreting Function Notation

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Arjun opened up a savings account last year and put an initial sum in it.

Let $M(t)$ denote the account balance M (measured in dollars), t days since it was opened.

↱ Balance after 30 days

What does the statement $M(30) - M(0) = 100$ mean?

↳ initial balance

- ✗ 30 days after it was opened, the balance of Arjun's account was equal to $\$100$. $M(30)=100$
- ✗ Arjun had the initial amount of money in his account 30 days after he opened it. $M(0)=M(30)$
- ✓ Arjun made a profit of $\$100$ over the first 30 days since the account was opened.

Let $P(t)$ denote the number of people P at the Copacabana beach in Rio de Janeiro, t hours past midnight on a specific day.

↗ num. of people at 5am

What does the statement $P(5) < P(9)$ mean?

↳ num. of ppl at 9am

- ✗ The time at which there were 5 people at the beach is earlier than the time there were 9 people.
- ✗ 5:00 a.m. is an earlier time than 9:00 a.m.
- ✓ There were more people at the beach at 9:00 a.m. than there were at 5:00 a.m.

Introduction to Domain and Range of a function

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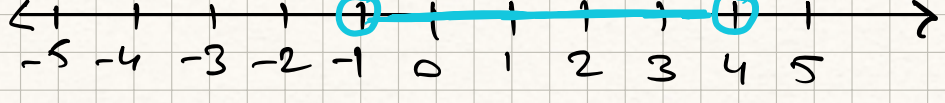
Interval notation?

[Closed Interval]

$$\{x \in \mathbb{R} \mid -3 \leq x \leq 2\} = \{x \in \mathbb{R} \mid x \in [-3, 2]\}$$

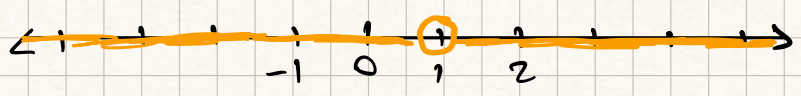
"x is a member of real numbers such that -3 is less than or equal to x, and x is less than or equal to 2.

(Open Interval)



$$\{x \in \mathbb{R} \mid -1 < x < 4\} = \{x \in \mathbb{R} \mid x \in (-1, 4)\}$$

* $(-4, 1]$ * $[-2, 1)$



$$\{x \in \mathbb{R} \mid x \neq 1\} = \{x \in \mathbb{R} \mid x \in (-\infty, 1) \text{ or } x \in (1, \infty)\}$$

Domain of a function?

Set of all inputs over which the function has defined outputs.

$$f(x) = \frac{2}{x} \Rightarrow \text{0} \rightarrow \boxed{f} \rightarrow \text{?}$$

$$\text{Domain}: \{x \in \mathbb{R} \mid x \neq 0\}$$

$$g(y) = \sqrt{y-6} \rightarrow y-6 \text{ can't be negative}$$

$$\text{Domain} = \{y \in \mathbb{R} \mid y \geq 6\}$$

$$h(x) = \begin{cases} 1 \text{ if } x = \pi \\ 0 \text{ if } x = 3 \end{cases} \Rightarrow \text{Domain}: \{3, \pi\}$$

Range of a function?