PRECALCULUS

01 - COMPOSITE AND INVERSE FUNCTIONS

* Function composition is the action of combining two functions in such a way so that the outputs of one function becomes the input of the other.

$$f(g(x)) = (f \circ g)(x) = \text{"f composed with g"}$$

* We can evaluate a composite function either by "inside out evaluation" or by finding the composite function by substitution."
* We can use "The last output as a function of the first input" expression to express a composit function in terms of the model it generates.
* When we compose functions, we must make sure that it makes sense to plug the output of the inner function as an input for the outer function.
* Inverse functions reverse each other: $f(a) = b \Rightarrow f^{-1}(b) = a$
* A function is invertible only if there is a one-to-one relationship between its domain and range.
* If we can draw a horizontal line that intercepts the function on multiple points on the graph, then the function is invertible.
* If the function has both increasing and decreasing intervals, it is not invertible within a domain that includes a minima or maxima. In this case, we can restrict its domain to make it invertible.
* The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ across the line $y = x$.
* $f(g(x)) = g(f(x)) = x \Rightarrow f(x)$ and $g(x)$ are inverses of each other.
* We can use specific values to prove that two continuous functions are not inverses of each other, but we can not use specific values to prove that two continuous functions are inverses of each other.

02 TRIGONOMETRY

* The (x,y) coordinate on the unit circle corresponding to an angle of θ degrees gives us $(\cos\theta, \sin\theta)$
* $\vec{S} = [0\ 1\ 2\ 3\ 4]$, $\vec{C} = [4\ 3\ 2\ 1\ 0]$, $\vec{\theta} = [0\ 30\ 45\ 60\ 90] \Rightarrow$
  * $\sin(\theta_i) = \dfrac{\sqrt{S_i}}{2}$ * $\cos(\theta_i) = \dfrac{\sqrt{C_i}}{2}$
* $\sin(\theta) = x \Rightarrow \sin^{-1}(x) = \theta$; Domain: $[-1,1]$, Range: $[\frac{-\pi}{2}, \frac{\pi}{2}]$ * where $\cos(\theta) \geqslant 0$
* $\cos(\theta) = x \Rightarrow \cos^{-1}(x) = \theta$; Domain: $[-1,1]$, Range: $[0, \pi]$ * where $\sin(\theta) \geqslant 0$
* $\tan(\theta) = x \Rightarrow \tan^{-1}(x) = \theta$; Domain: $(-\infty, \infty)$, Range: $(\frac{-\pi}{2}, \frac{\pi}{2})$ * where $\cos(\theta) > 0$
* Law of Sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$
* Law of Cosines: $c^2 = a^2 + b^2 - 2ab(\cos(\gamma))$
* A sinusoidal equation is a mathematical expression that describes a smooth periodic oscillation resembling the sine or cosine function. It's used to model repetitive phenomenia such as sound waves, light waves, tides, etc.
  * $y = A\sin(B(x-C)) + D$
  * $y = A\cos(B(x-C)) + D$
  * A: Amplitude (The peak deviation from the center)
  * B: Frequency (The period of wave)
  * C: Phase Shift ($C>0 \Rightarrow$ shifts right, $C<0 \Rightarrow$ shifts left)
  * D: Vertical Shift (Moves the entire graph up $(+)$ or down $(-)$)
* When solving sinusoidal equations, take into account that the trigonometric identities on the unit circle as well as the repetitive nature (periodicity) of the trigonometric functions.
* We can derive all trig. angle addition identities using:
  1. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
  2. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
* $\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x) + \tan(y)}$

03 COMPLEX NUMBERS ($\mathbb{C}$)

* A complex number is any number that can be written as $a+bi$ (a.k.a. the rectangular form), where i is the imaginary unit ($i^2=-1$, $i=\sqrt{-1}$), and $a, b \in \mathbb{R}$. "a" is called the "real part", "b" is called the "imaginary part" of the complex number.
* The complex plane consists of real axis (horizontal) and imaginary axis (vertical) which intercept at zero.
* Distance between two complex numbers z and w is:

$$|z-w| = \sqrt{(Re(z)-Re(w))^2 + (Im(z)-Im(w))^2}$$

* Midpoint of z and w is $\dfrac{Re(z)+Re(w)}{2} + i\,\dfrac{Im(z)+Im(w)}{2}$
* Conjugate of the $z=a+bi$ = $\bar{z}=a-bi$
  * $z+\bar{z} = 2\,Re(z)$
  * $z\cdot\bar{z} = |z|^2 = Re(z)^2 + Im(z)^2$
* To divide a complex number by another, we multiply both numbers by the conjugate of the denominator.
* Modulus of a complex number ($r=|z|$) is its absolute value, its distance from the origin: $z=a+bi \Rightarrow r=|z|=\sqrt{a^2+b^2}$
* Argument of a complex number is its angle: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
* If we're given r and θ:
  * $z = r(\cos(\theta)+\sin(\theta))$ (a.k.a the polar form)
  * $z = r.e^{i\theta}$ (a.k.a the exponential form)
* Complex multiplication: When we multiply z with w, we scale it by $|w|$ and rotate it by θ_w. Therefore, it helps to express w in its polar form to easily visualize the rotation.
* Complex division: When we divide z by w, we scale it by $\frac{1}{|w|}$ and rotate it by $-\theta_w$. It's same as multiplying by $\frac{\bar{w}}{|w|^2}$.

* $w_1 \cdot w_2 = |w_1| \cdot |w_2| \left[\cos(\theta_{w_1} + \theta_{w_2}) + i \sin(\theta_{w_1} + \theta_{w_2})\right]$

* $\dfrac{w_1}{w_2} = \dfrac{|w_1|}{|w_2|} \cdot \left[\cos(\theta_{w_1} - \theta_{w_2}) + i \sin(\theta_{w_1} - \theta_{w_2})\right]$

* $w^n = |w|^n \cdot \left[\cos(\theta_w \cdot n) + i \sin(\theta_w \cdot n)\right]$

* $z^n = w \Rightarrow z = \sqrt[n]{|w|} \left[\cos\left(\dfrac{\theta + 2k\pi}{n}\right) + i \sin\left(\dfrac{\theta + 2k\pi}{n}\right)\right], k = 0, 1, 2, \dots, n-1$

* $f(x) = ax^2 + bx + c \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$