Evaluating Functions

26/02/2025

What is a function?

input → [function] → output

$f(x) = x + 5$

$f(5) = 5 + 5 = 10$

(5: input; 10: output)

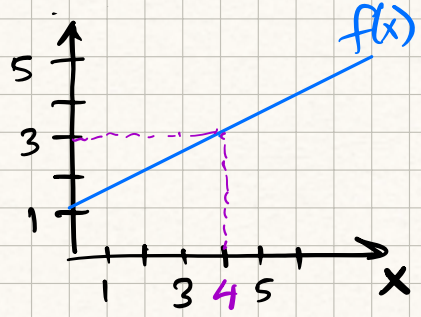
What is NOT a function?

$x^2 + y^2 = 4$ not a function b/c more than one output

$x = 1 \Rightarrow y^2 = 3$

$y = \mp\sqrt{3}$

Evaluate from graph



$f(4) = 3$

Summary: In mathematics, a function from set X to a set Y assigns to each element of X, exactly one element of set Y.

Inputs and Outputs of a Function

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Matching an input to a function's output

$f(t) = -2t + 5$

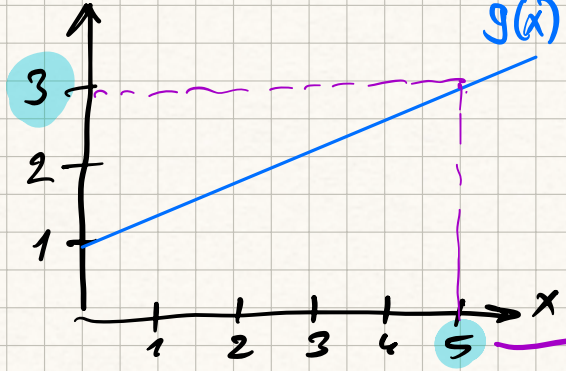
$f(t) = 13 \Rightarrow t = ?$

$13 = -2t + 5$

$2t = -8$

$t = -4$

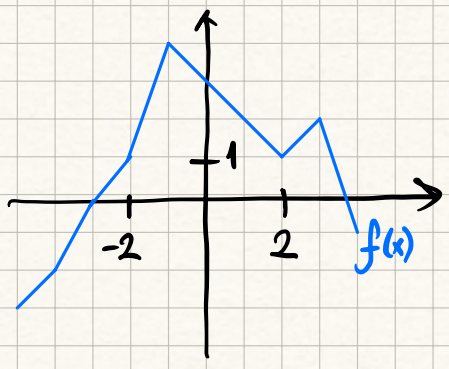
From graph?



$g(x) = 3 \Rightarrow x = ?$

$x = 5$

Two inputs with the same output



$f(-2) = -1$

$f(2) = 1$

$f(-2) = f(2)$

Summary: We can use the graph to match inputs and outputs to each other. A function can have multiple inputs with the same output.

Functions and Equations

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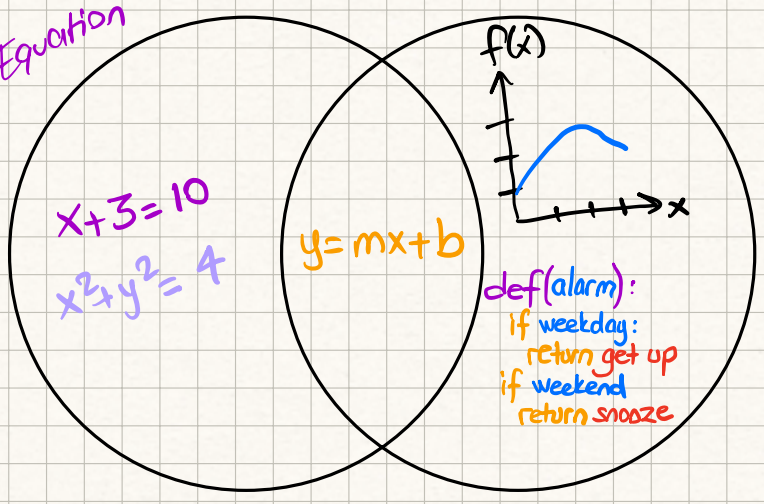
Differences between equations and functions



Obtaining a function from an equation

For a given input value b, the function f outputs a value a to satisfy the following equation.

$$4a + 7b = -52$$

Write a formula for $f(b)$ in terms of b.

$4a = -52 - 7b$

$a = -13 - \frac{7}{4}b = f(b)$

Summary: Not every equation is a function. Not every function is an equation. There are equations who define a function.

Interpreting Function Notation

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Arjun opened up a savings account last year and put an initial sum in it.

Let $M(t)$ denote the account balance M (measured in dollars), t days since it was opened.

↱ Balance after 30 days

What does the statement $M(30) - M(0) = 100$ mean?

↳ initial balance

- ✗ 30 days after it was opened, the balance of Arjun's account was equal to $\$100$. $M(30)=100$
- ✗ Arjun had the initial amount of money in his account 30 days after he opened it. $M(0)=M(30)$
- ✓ Arjun made a profit of $\$100$ over the first 30 days since the account was opened.

Let $P(t)$ denote the number of people P at the Copacabana beach in Rio de Janeiro, t hours past midnight on a specific day.

↗ num. of people at 5am

What does the statement $P(5) < P(9)$ mean?

↳ num. of ppl at 9am

- ✗ The time at which there were 5 people at the beach is earlier than the time there were 9 people.
- ✗ 5:00 a.m. is an earlier time than 9:00 a.m.
- ✓ There were more people at the beach at 9:00 a.m. than there were at 5:00 a.m.

Introduction to Domain and Range of a function

26/02/2025

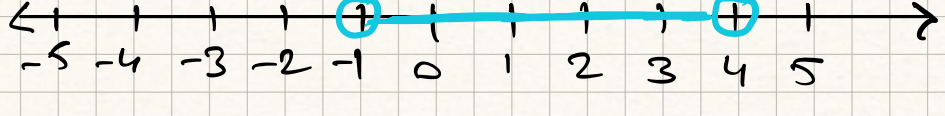
Interval notation?

[Closed Interval]

$$\{x \in \mathbb{R} \mid -3 \leq x \leq 2\} = \{x \in \mathbb{R} \mid x \in [-3, 2]\}$$

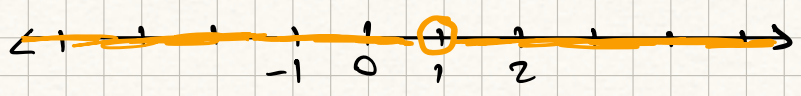
"x is a member of real numbers such that -3 is less than or equal to x, and x is less than or equal to 2.

(Open Interval)



$$\{x \in \mathbb{R} \mid -1 < x < 4\} = \{x \in \mathbb{R} \mid x \in (-1, 4)\}$$

* $(-4, 1]$ * $[-2, 1)$



$\{x \in \mathbb{R} \mid x \neq 1\} = \{x \in \mathbb{R} \mid x \in (-\infty, 1) \text{ or } x \in (1, \infty)\}$

Domain of a function?

Set of all inputs over which the function has defined outputs.

$f(x) = \frac{2}{x} \Rightarrow$ 0 → [f] → ?

Domain: $\{x \in \mathbb{R} \mid x \neq 0\}$

$g(y) = \sqrt{y-6}$ → y-6 can't be negative

Domain = $\{y \in \mathbb{R} \mid y \geq 6\}$

$h(x) = \begin{cases} 1 & \text{if } x = \pi \\ 0 & \text{if } x = 3 \end{cases} \Rightarrow$ Domain: $\{3, \pi\}$

Range of a function?

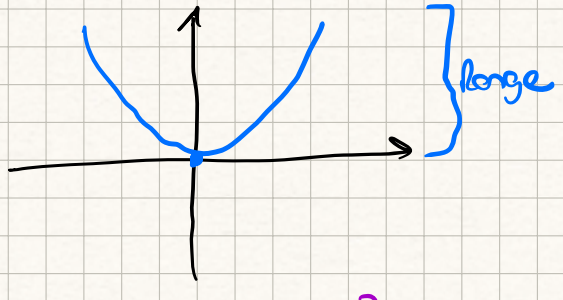
x → [f] → f(x)

Domain

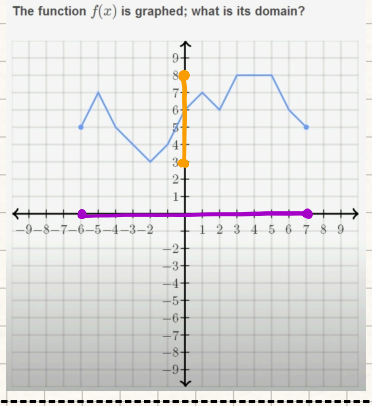
Range = Set of all possible outputs.

$f(x) = x^2$

$\{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$



Domain and range from graph?



$\{x \in \mathbb{R} \mid -6 \leq x \leq 7\}$

$\{f(x) \in \mathbb{R} \mid 3 \leq f(x) \leq 8\}$

Summary: A function maps values from its domain to its range. We use interval notation to define these two sets.

Determining the Domain of the Function

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- $f(x) = \sqrt{2x-8}$

$2x - 8 \geq 0$

$x \geq 4$

"The function is defined when x is greater than or equal to 4."

- $g(x) = \dfrac{1}{\sqrt{6-|x|}}$

$6 - |x| > 0$

$|x| < 6$

$-6 < x < 6$

- $h(x) = \begin{cases} \dfrac{x+10}{(x+10)\cdot(x-9)\cdot(x-5)} & x=5 \\ x & x \neq 5 \end{cases}$

$x + 10 \neq 0 \quad x - 9 \neq 0 \quad x - 5 \neq 0$

$x \neq -10 \quad x \neq 9 \quad x \neq 5$

$\{x \in \mathbb{R} \mid x \neq , x \neq -10\}$

- Pooja had a beatiful plant. The plant began sprouting 2 days *before* Pooja bought it, and she had it for 98 days before it died. At its tallest, the plant was 30 centimeters tall.

$+(98)$ $t(-2)$

Let $h(t)$ denote the height of Pooja's plant h (measured in centimeters), t days from the time she bought it.

Which number type is more appropriate for the domain of the function?

○ Integers ✓ Real numbers

$\{t \in \mathbb{R} \mid -2 \leq t \leq 98\}$

Define the interval of the domain.

- Thomas has 400 candy bars in his shop, and each costs \$0.50.

Let $p(b)$ denote the price p (measured in dollars) of a purchase of b candy bars.

Which number type is more appropriate for the domain of the function?

$p(b) = p \cdot \frac{1}{2}$

✓ Integers ○ Real numbers

can't buy 1,25 candies! $p \in \mathbb{Z} \mid 0 \leq p \leq 400$

Define the interval of the domain.

Summary: To find a domain of a function $f(x)$, concentrate on what values x can not take, and exclude those values from the domain

Recognizing Functions

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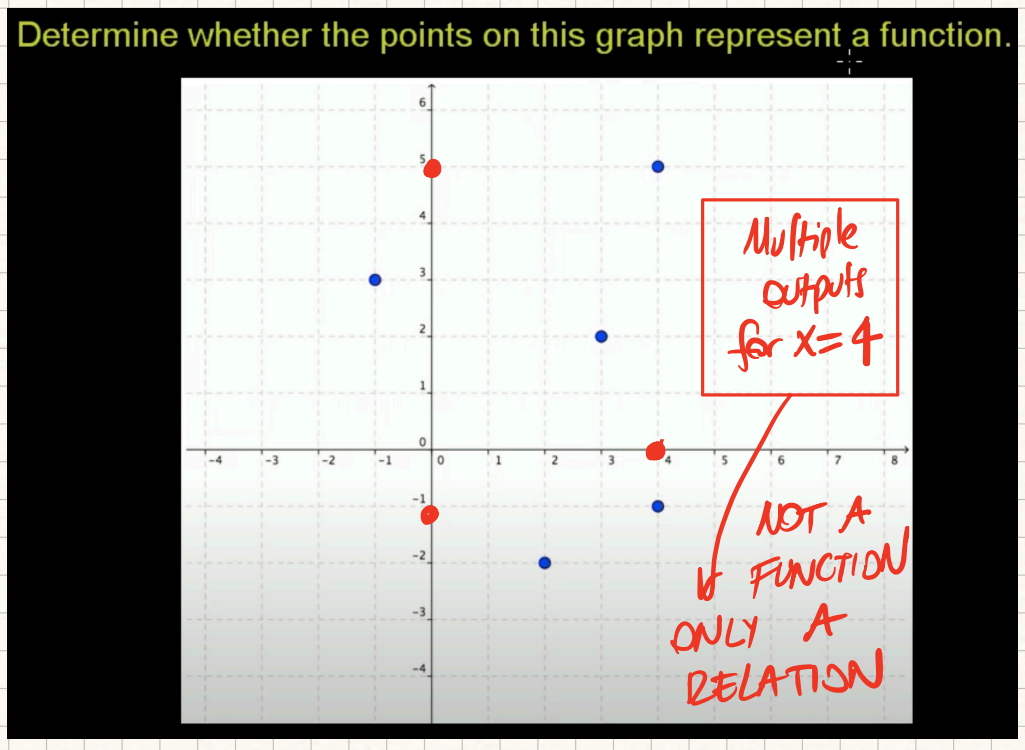
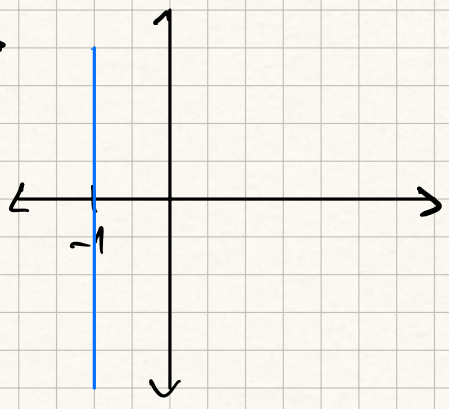
From graph?



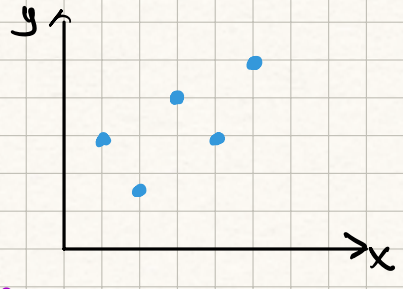
Does a vertical line represent a function?



$x=-1$ has infinite outputs

From tables?

Name	Height
Joelle	5'6"
Nathan	4'11"
Stewart	5'11"
LJ	5'6"
Tariq	6'1"



* No duplicate inputs with different outputs, it's a functional relationship.

From verbal description?

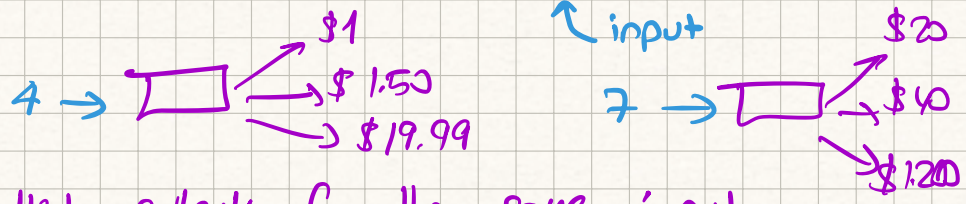
"The value of y is always three more than twice x."
Is y a function or not?

$y = 2x + 3$ = no multiple outputs for any input, "y is a function of x"

Attention to phrasing!

Jada is ordering Mother's Day gifts online. The shipping costs are based on the dollar amount of the order. For orders less than $20, shipping costs $4. For orders of $20 or more, shipping is $7.

- Can the dollar amount of the order be represented as a function of shipping cost? amt. of order (shipping cost) ?



- Multiple outputs for the same input. Therefore NO!

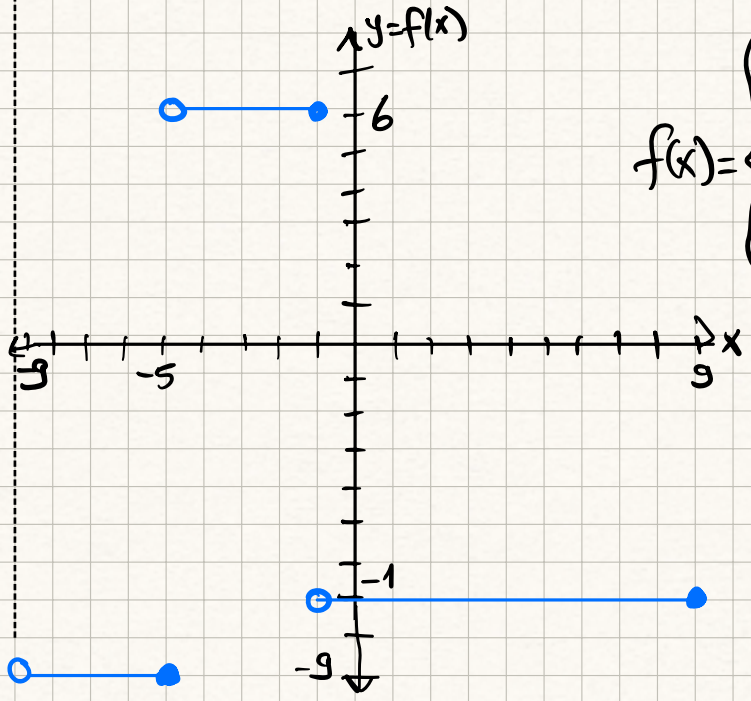
!! The shipping cost can be represented as a function of the amount of the order.

Summary: To recognize a graph|equation|verbal expression, we confirm that no inputs can be mapped to more than one output.

Piecewise Functions

27/02/2025

Definition



$$f(x)=\begin{cases} -9, & -9<x\leq -5 \\ 6, & -5<x\leq -1 \\ -7, & -1<x\leq 9 \end{cases}$$

How to evaluate?

$$f(x)\begin{cases} t^2-5t, & t \le -10 \\ t+19, & -10 < t < -2 \\ \frac{t^3}{t+9}, & t \ge -2 \end{cases}$$

$\Rightarrow f(-10) = ?$

$f(-10) = (-10)^2 - 5(-10)$

$= 100 + 50$

$= 150$

How to graph?

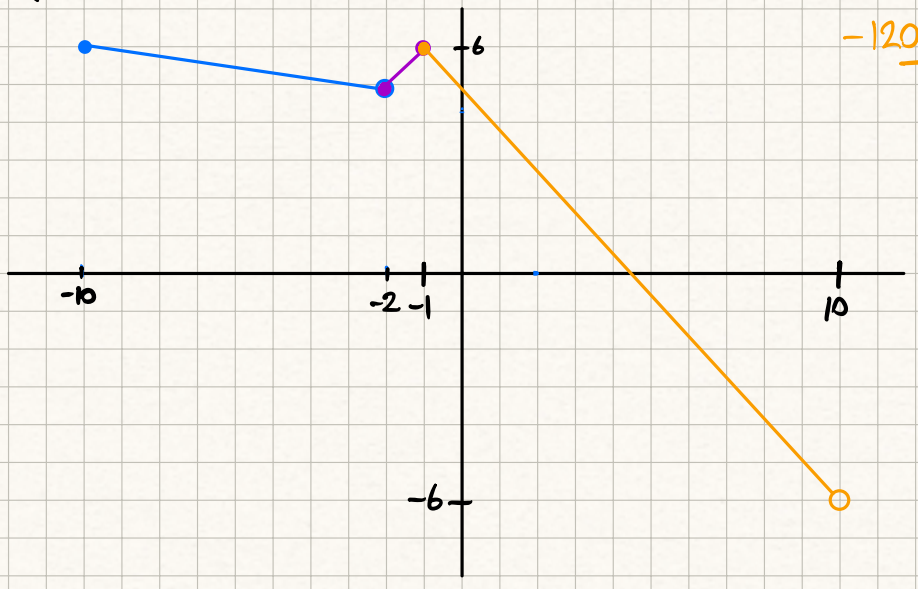
$$f(x)\begin{cases} -0.125x + 4.75, & -10 \le x < -2 \quad \text{line} \\ x+7, & -2 \le x < -1 \quad \text{line} \\ \frac{-12}{11}x + \frac{54}{11}, & -1 \le x < 10 \quad \text{line} \end{cases}$$

$f(-10) = 1.25 + 4.75 = 6$

$f(-2) = 4.75 + 0.250 = 5$

$f(-2) = 5 \quad f(-1) = 6$

$f(-1) = 6 \quad f(10) = \frac{-120+54}{11} = -6$



Domain & Range of step functions

$$f(x)\begin{cases} 1, & 0 < x \le 2 \\ 5, & 2 < x < 6 \\ -7, & 6 \le x < 11 \end{cases}$$

$\{x \in \mathbb{R} \mid 0 < x \le 11\}$

$\{f(x) \in \mathbb{R} \mid f(x) \in \{1, 5, 7\}\}$

$$g(x)\begin{cases} x+7, & -6 < x \le -3 \\ 1-x, & -3 < x < 4 \\ 2x-11, & 4 \le x \le 6 \end{cases}$$

$\{x \in \mathbb{R} \mid -6 < x \le 6\}$

$g(-6) = 1, \; g(-3) = 4, \; -1 < g(x) \le 4$

$g(-3) = 4, \; g(4) = -3 \quad -3 < g(x) < 4$

$g(4) = -3, \; g(6) = 1 \quad -3 \le g(x) \le 1$

$\{g(x) \in \mathbb{R} \mid -3 \le g(x) \le 4\}$

Summary: Piecewise (or step) functions declare the domains in multiple expressions.

Maximum and Minimum Points

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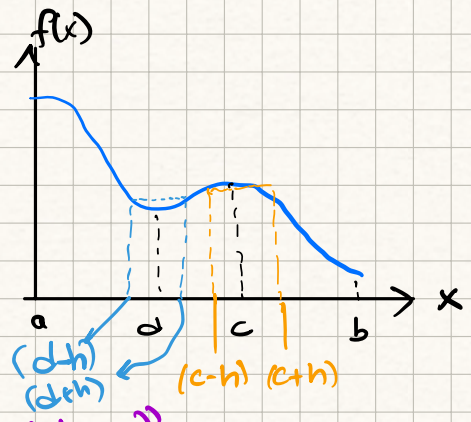
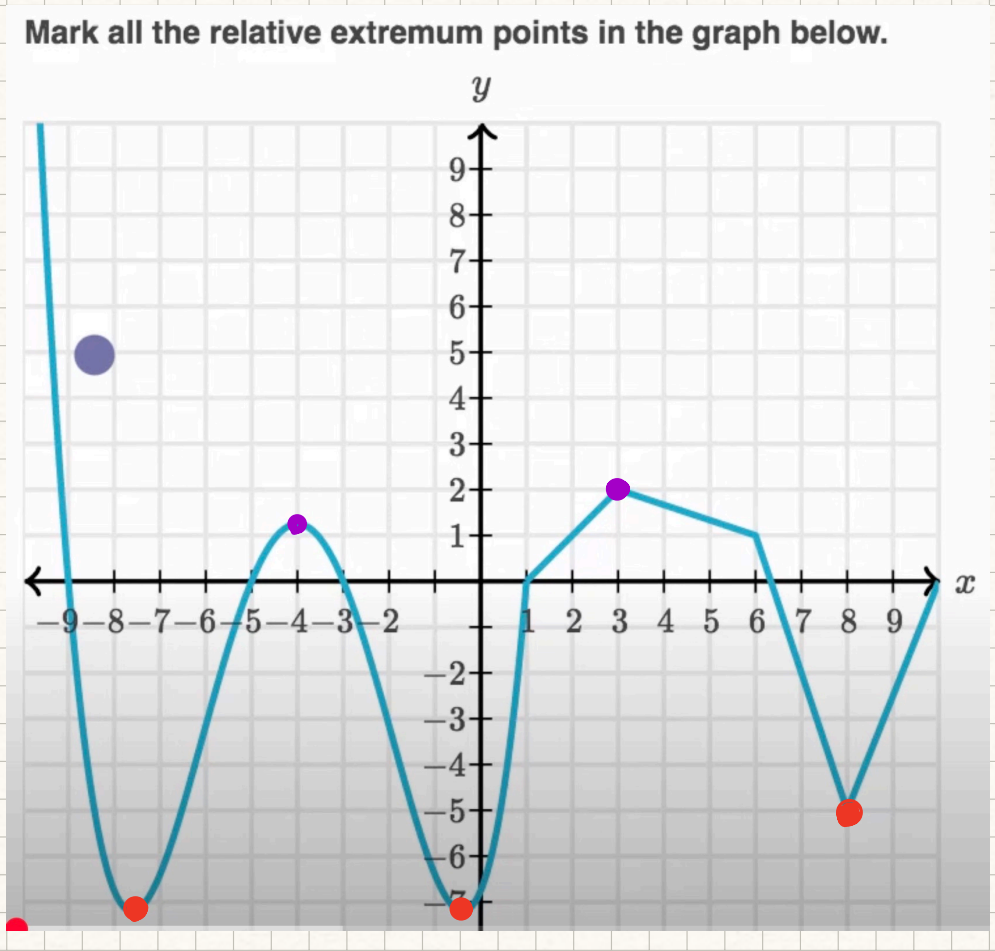
- Absolute max / min values $f(a)$ / $f(b)$
- Relative max / min values $f(c)$ / $f(d)$



"$f(c)$ is a relative maximum if $f(c) \geq f(x)$ for all $x \in (c-h, c+h)$ for $h>0$"

"$f(d)$ is a relative minimum if $f(d) \leq f(x)$ for all $x \in (d-h, d+h)$



- If line horizontal ⇒ many points rel min/max at the same time

Summary: Absolute maximums and minimums are the overall max & min. values of a function, while relative max & mins are local " " " .

Intervals Where a Function is Positive, Negative, Increasing, Decreasing

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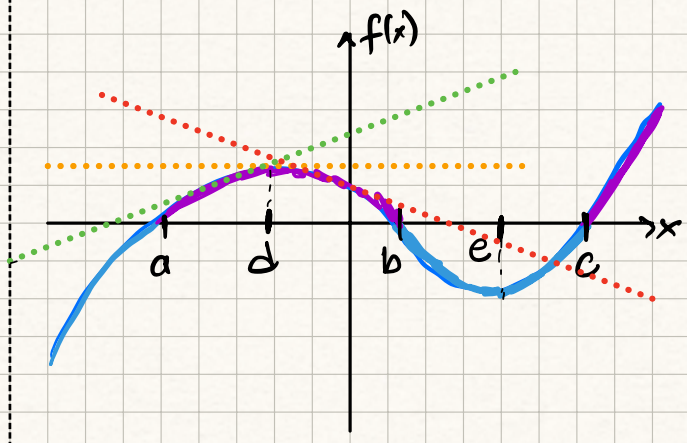


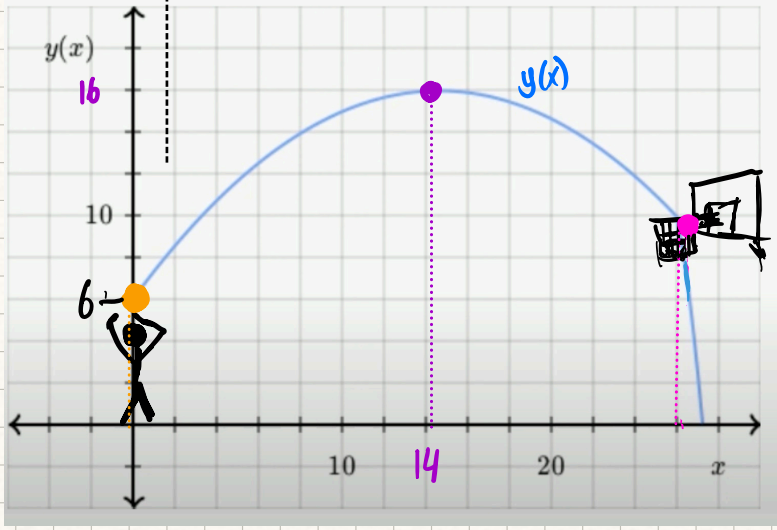
Positive / Negative
Relative to the x-axis
$f(x)>0$, $f(x)<0$

Increasing / Decreasing
Slope of the tangent line
$m>0$, $m=0$, $m<0$

Real-world example

Mr. Theisen is honing his deadly three-point precision on the basketball court. For one of his shots, the height of the ball (in feet) as a function of horizontal distance (in feet), $y(x)$, is plotted below. Mr. Theisen is standing at $x = 0$.

What is the significance of the y-intercept of this function?



(0) : y-intercept
● : absolute max value
● : relative max value

+ ax. max.

Summary: An interval of a func is positive or negative relative to the x-axis. Increasing func. intervals have positive slopes. Decreasing func. intervals have negative slopes.

Average Rate of Change

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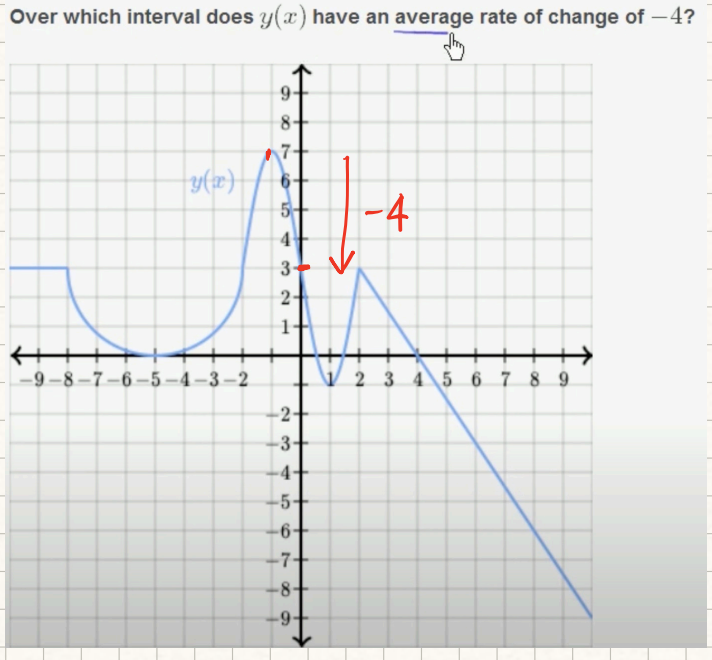
- Linear function ⇒ slope = average rate of change
- Polynomial func ⇒ slope of the line connecting two data points → average rate of change between those data points.

From graph?



a) $-1 < x < 1$

b) $-5 < x < 1$

c) $-2 < x < 4$

d) $-8 < x < 2$

From table?

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y(x)	6	4	2	0	-1	-2	-3	0	3	6	9

$-5 < x < -2 \Rightarrow \frac{\Delta y}{\Delta x} = \frac{-6}{3} = -2$

From equation?

$y = \frac{1}{8}x^3 - x^2$, over which interval avg rate of chg $= \frac{1}{2}$

$a = \frac{\Delta y}{\Delta x} = \frac{-3-(-5)}{4} = \frac{2}{4} = \frac{1}{2}$

a) $-2 < x < 2$

b) $0 < x < 4$

c) $-3 < x < 2$

d) $-4 < x < 1$

Summary: The average rate of change of function f over the interval $a \leq x \leq b$ $= \frac{f(b)-f(a)}{b-a}$

Combining Functions