Operations on two matrices

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Addition (and subtraction)

* Dimensions have to match!

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

* Addition is commutative and associative.

$A+B = B+A$ $\qquad$ $A+(B+C) = (A+B)+C$

* Subtraction is not commutative or associative.

$A-B \neq B-A$ $\qquad$ $A-(B-C) \neq (A-B)-C$

Scalar multiplication

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} k.a & k.b \\ k.c & k.d \end{bmatrix}$$

Zero matrices

$O_{m,n}$ is a matrix with m number of rows, and n number of columns, and all of its entries are zeros.

Ex

$$O_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad O_{3\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A + O = A$ $\qquad$ $A+(-A) = O$

$A - O = A$ $\qquad$ $-A$ is called "opposite matrix"

Matrix multiplication

$$A_{\underbrace{m,n \cdot B_{n},p}_{\text{must match}}}$$

$$A_{m,n} \cdot B_{n,p} = C_{m,p}$$

must match

$$A = \begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array} \begin{bmatrix} 2 & 6 \\ 3 & -1 \end{bmatrix} \quad B = \begin{array}{cc} C_1 & C_2 \\ \downarrow & \downarrow \end{array} \begin{bmatrix} -4 & -2 \\ 1 & 0 \end{bmatrix} \Rightarrow A \cdot B = \begin{bmatrix} R_1 \cdot C_1 & R_1 \cdot C_2 \\ R_2 \cdot C_1 & R_2 \cdot C_2 \end{bmatrix}$$

Matrix Addition: * Dimensions must be identical. * Add corresponding entries * Is commutative and associative.

Matrix Subtraction: * Dimensions must be identical. * Subtract corresponding entries * Not commutative, not associative

Scalar Multiplication: * Each entry is multiplied by the scalar.

Opposite Matrices: * $A + B = O \Rightarrow B = -A$

Dot product	$R_1 \cdot C_1$ means "dot product of $R_1 \cdot C_1$.
	$R_1 \cdot C_1 = R_{1,1} C_{1,1} + R_{1,2} \cdot C_{2,1}$
	$\Rightarrow A \cdot B = \begin{bmatrix} R_{1,1} \cdot C_{1,1} + R_{1,2} \cdot C_{2,1} & R_{1,1} \cdot C_{1,2} + R_{1,2} C_{2,2} \\ R_{2,1} C_{1,1} + R_{2,2} \cdot C_{2,1} & R_{2,1} \cdot C_{1,2} + R_{2,2} C_{2,2} \end{bmatrix}$
Properties of matrix multiplication	* Not commutative : $A \cdot B \neq B \cdot A$ * Associative : $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ * Distributive : $A(B+C) = A \cdot B + A \cdot C$
Identity matrix	$A \cdot I = A$ ↳ Identity matrix $I \cdot A = A$ ↳ Identity matrix
	$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Elimination Matrix	$E \cdot A = I$ ↳ Elimination Matrix
	E_1 : First row operation E_2 : Second row operation ⋮ E_n : n^{th} row operation
	$E = E_n \cdot \dots \cdot E_2 \cdot E_1$
	* Elimination matrix is also called Inverse Matrix. $E \cdot A = I \Rightarrow E = A^{-1}$

Matrix Multiplication : $A_{m,n} \cdot B_{n,p} = C_{m,p}$

$$A = \begin{bmatrix} R_1 \rightarrow 2 & 6 \\ R_2 \rightarrow 3 & -1 \end{bmatrix} \quad B = \begin{array}{cc} C_1 & C_2 \\ \downarrow & \downarrow \\ \left[\begin{matrix} -4 \\ 1 \end{matrix}\right. & \left.\begin{matrix} -2 \\ 0 \end{matrix}\right] \end{array} \Rightarrow A \cdot B = \begin{bmatrix} R_1 \cdot C_1 & R_1 \cdot C_2 \\ R_2 \cdot C_1 & R_2 \cdot C_2 \end{bmatrix}$$

Dot product

* Associative, distributive, not commutative.

Zero matrix : Matrix with only 0 entries : $O_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Identity matrix : $A \cdot I = A$

* Main diagonal : 1
* Other entries : 0

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$