① Orientation to Linear Algebra

28.04.2025

What is algebra?

* Algebra is arithmetic that include non-numeric entities like x.
* $2x + 5 = 25 \Rightarrow x = ?$

What is linear algebra?

* If it has an exponential term, it is NOT linear algebra.

$2x^2 + 5 = 25$ ✗

$2\sqrt{x} + 5 = 25$ ✗

$2x + 5 = 25$ ✓

What is a system of linear equations?

* It's a collection of linear equations. We try to solve the system for all unknowns. We can do it with algebra, with methods like elimination, substitution, and graphing, but using linear algebra makes it so much easier as the system gets larger. (when there's many equations, and many unknowns in each equation)

How can we use linear algebra to solve a regression problem in ML?

house price

distance to school

There could be n features (many)!

$$y = a + bx_1 + cx_2 + \dots + mx_n$$

"y-intercept"

number of bedrooms

* Our goal is to solve this equation for $a, b, c, \dots, m$.

$$y = a + bx_1 + cx_2 + \dots + mx_m$$

$$\left[\begin{array}{c|cccc} y_1 & a + & bx_{1,1} + & cx_{1,2} + & \dots + mx_{1,m} \\ y_2 & a + & bx_{2,1} + & cx_{2,2} + & \dots + mx_{2,m} \\ \vdots & \vdots & \vdots & \vdots & \\ y_n & a + & bx_{n,1} + & cx_{n,2} + & \dots + mx_{n,m} \end{array}\right]$$

For any house i in the dataset, y_i = price and $x_{i,1}$ to $x_{i,m}$ are its features. We solve the system for parameters a, b, c to m.

List all the contemporary applications of LA in ML.

- Solving for unknowns in ML algos, including deep learning.
- Reducing dimensionality (e.g., principal component analysis)
- Ranking results (e.g. with eigenvector)
- Recommenders (e.g. singular value decomposition - SVD)
- Natural language processing (e.g. SVD, matrix factorization)
  - Topic modeling
  - Semantic analysis

Exercise Jill designs solar panels. On April 1st, Jill's "Mark I" design begins generating power: 1kJ/day. On May 1st, her "Mark II" design begins generating 4kJ/day.

① What day is it when Mark II has generated as much total energy as the Mark I?

② How much total energy have both generated by that day?

③ What would be the solutions to ① and ② if Mark II generated 1 kJ/day?

* $(30+d)(1) = d.(4)$

$30 + d = 4d$

$d = 10$ = 10 days after M_2 = 10 May

* $(30+d).1 = 40$ kJ

* $(30+d).1 = d.(1)$

$30+d = d$

$30 = 0$ ⇒ NO SOLUTION!