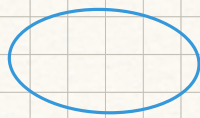
Introduction to Conic Sections

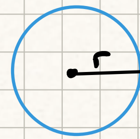
20.05.2025

What are the 3 types of conic sections?

① Ellipse

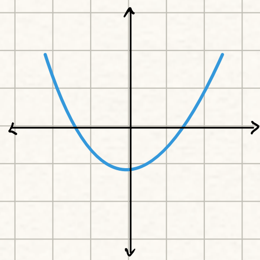


1a Circle*

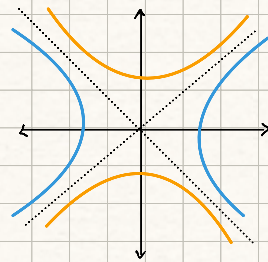


* Circles are also ellipses

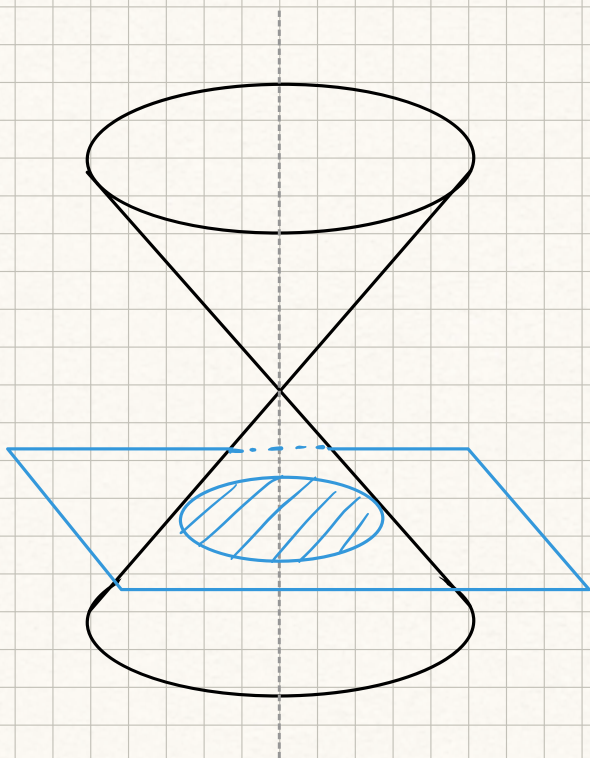
② Parabola



③ Hyperbola



Why called "conic?"

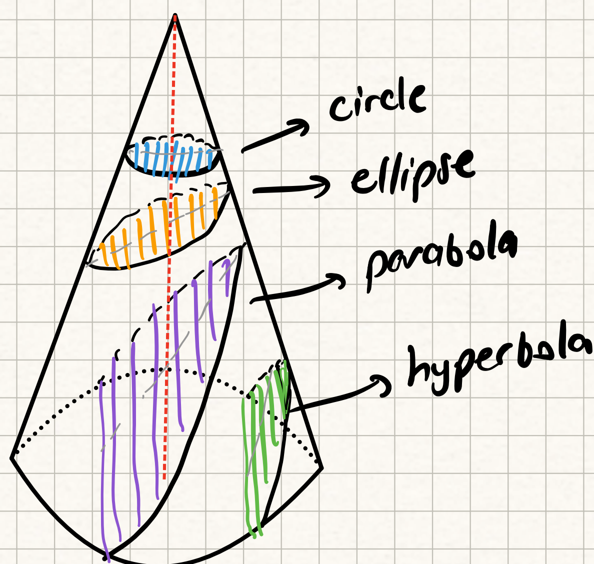


* A conic section is a curve obtained from a cone's surface intersecting a plane.

* Plane ⊥ Cone → Circle

Plane

* Ellipse
* Parabola
* Hyperbola



* A conic section is a curve obtained from a cone's surface intersecting a plane.
* The 3 main types of conic sections are ellipses, parabolas, and hyperbolas. Circles are a special kind of ellipses, so they're conic sections as well.

Center and Radii of an Ellipse

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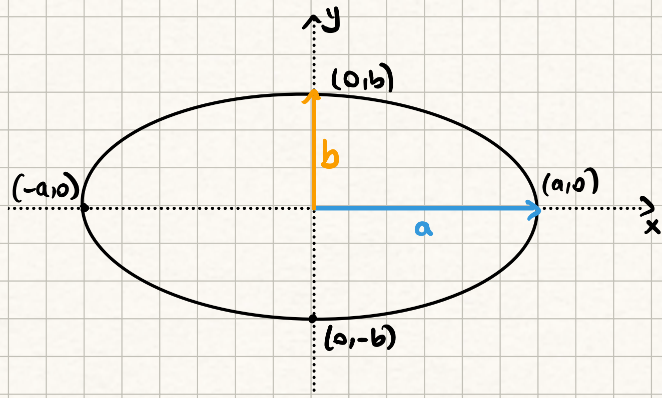
Intro to ellipses



- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
- $b =$ semi minor axis (semi = half, minor = shorter)

  $=$ minor radius
- $a =$ semi major axis (semi = half, major = longer)

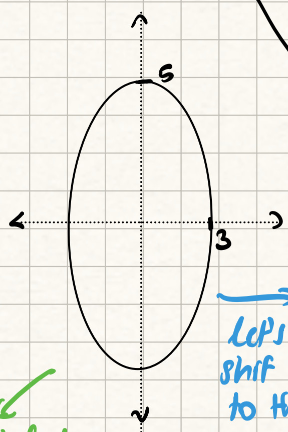
  $=$ major radius
- $a = b \Rightarrow$ A circle!

  $\Rightarrow \dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = 1$

- $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

  $\Rightarrow a = 3$

  $b = 5$



let's shif 5 to the right!

- $\dfrac{(x-5)^2}{a} + \dfrac{y^2}{b} = 1$

let's shif down by 2

- $\dfrac{x}{a^2} + \dfrac{(y+2)^2}{b^2} = 1$

- Notice that if we shift one direction, we use the opposite sign of that direction!

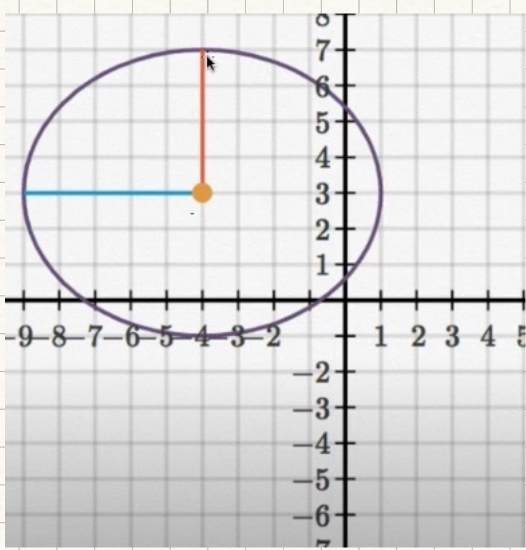
- $\dfrac{(y-1)^2}{4} + \dfrac{(x+2)}{9} = 1 \Rightarrow$ $a = ?$, $b = ?$, origin $= ?$

  $a = \sqrt{9} = 3$

  $b = \sqrt{4} = 2$

  $O = (-2, 1)$

Ellipse standard equation from graph



* $O = (-4, 3)$

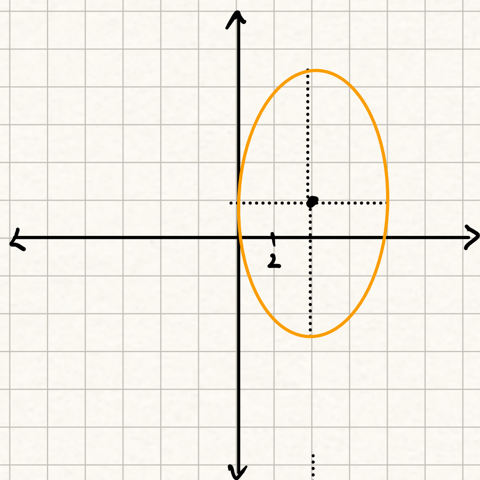
$a = 5$

$b = 4$

$$\rightarrow \frac{(x+4)^2}{25} + \frac{(y-3)^2}{16} = 1$$

Ellipse graph from standard equation

* Draw the graph for $\dfrac{(x-4)^2}{16} + \dfrac{(y-1)^2}{49} = 1$?



$a = 4$

$b = 7$

$O = (4, 1)$

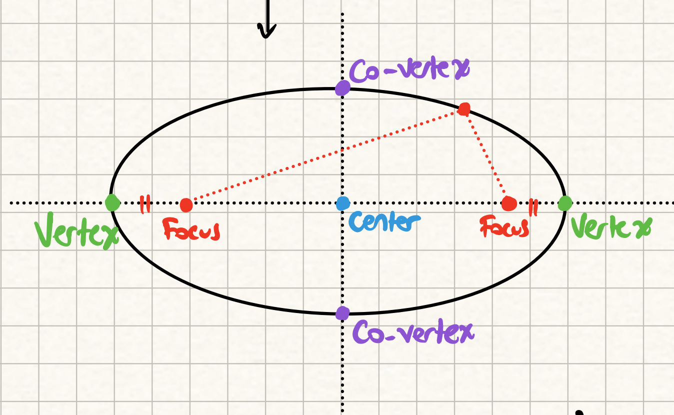
Review: Features and equation of an ellipse

* An ellipse has two radii of unequal size: The major radius is longer than the minor radius.

* Major axis connects the vertices, minor axis connects the co-vertices.

* The vertices and the co-vertices intersect at the center of the ellipse

* The foci lie on the major radius. The sum of the distances from the foci to each point on the ellipse is always the same



* $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, is the equation of the ellipse centered at (h, k), whose horizontal radius is a and vertical radius is b.

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* An ellipse has two radii of equal size: The major radius is longer than the minor radius.
* Major axis connects the vertices, minor axis connects the co-vertices.
* The vertices and the co-vertices intersect at the center of the ellipse.

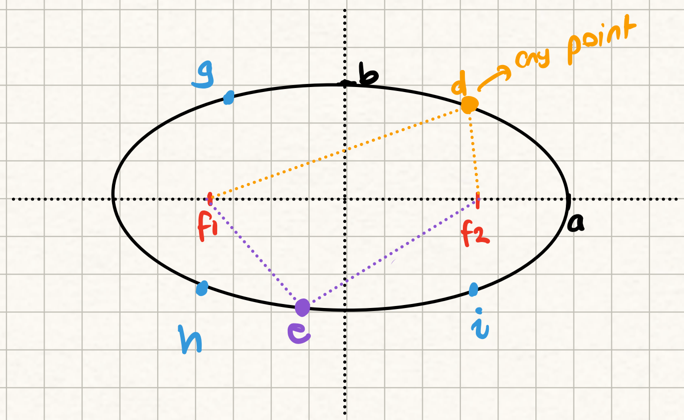
* $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ is the equation of the ellipse centered at (h,k). whose horizontal radius is "a" and vertical radius is "b".

* The foci lie on the major radius. The sum of the distances from the foci to each point on the ellipse is always the same. The equation of the foci is $f^2 = |a^2 - b^2|$

Foci of an Ellipse

From equation

* $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a > b$ $\Rightarrow$



* $|d-f_1| + |d-f_2| = 2a$

$|e-f_1| + |e-f_2| = 2a$

g

h

i

* $f^2 = |a^2 - b^2|$

* $\dfrac{(x-1)^2}{9} + \dfrac{(y+2)}{4} = 1$ $\Rightarrow$

$a = 3$

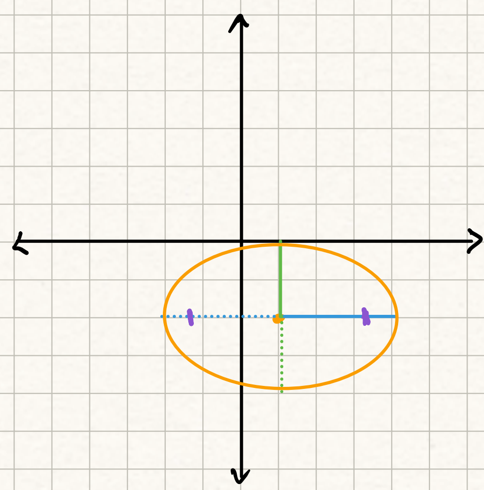
$b = 2$

$O = (1,-2)$

$f = \sqrt{9-4} = \sqrt{5}$

$\Rightarrow f_1 = (1-\sqrt{5}, -2)$

$f_2 = (1+\sqrt{5}, -2)$



Introduction to Hyperbolas

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$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Let's solve for y:

$$\frac{-y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$(-b^2)$ $(-b^2)$

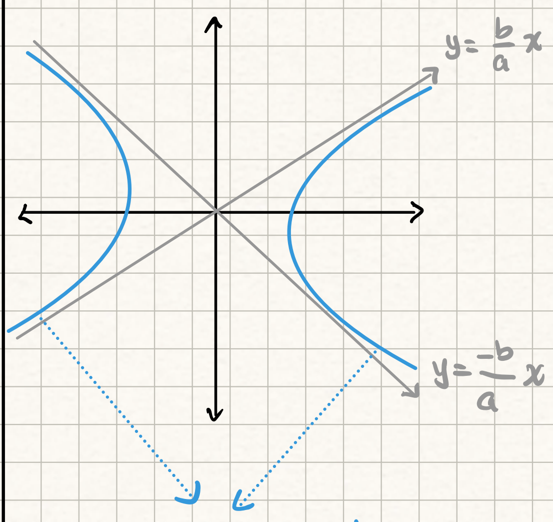
$$y^2 = -b^2 + \frac{b^2}{a^2}x^2$$

$$y^2 = \frac{b^2}{a^2}x^2 - b^2$$

$$y = \mp\sqrt{\frac{b^2}{a^2}x^2 - b^2}$$

$$x \to \pm\infty$$

$$y \approx \pm\sqrt{\frac{b^2}{a^2}x^2} = \pm\frac{b}{a}x$$



B/C $x \neq 0$!

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

Let's solve for y

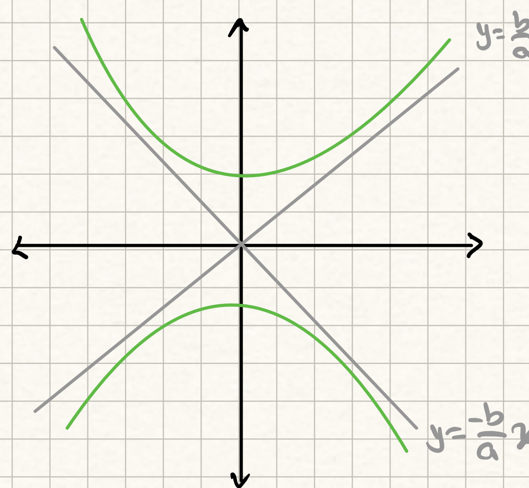
$$\frac{y^2}{b^2} = 1 + \frac{x^2}{a^2}$$

$$y^2 = b^2 + \frac{b^2}{a^2}x^2$$

$$y = \mp\sqrt{\frac{b^2}{a^2}x^2 + b^2}$$

$$x \to \pm\infty$$

$$y \approx \mp\sqrt{\frac{b^2}{a^2}x^2} = \mp\frac{b}{a}x$$

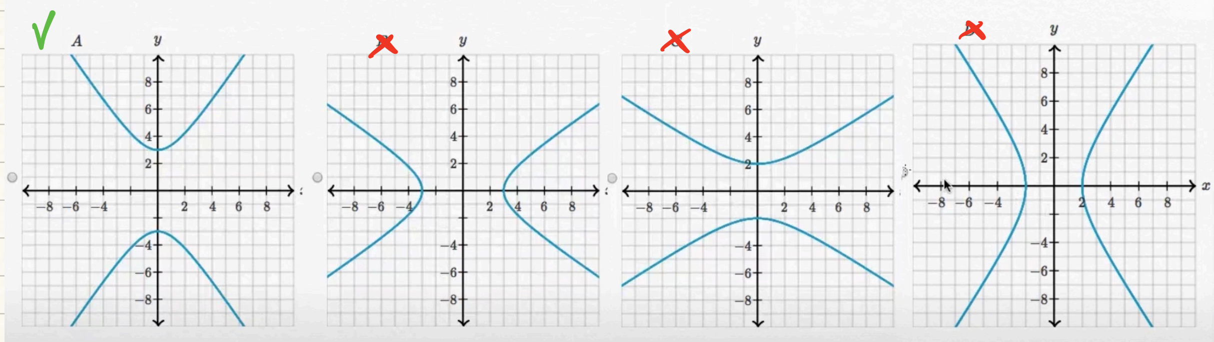


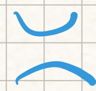
B/C $y \neq 0$!

* A hyperbola is made up two distinct branches or curves that extend away from each other.
* Center is the point equidistant from the two branches, around which the hyperbola is symmetrical
* Transverse axis is the line segment connecting the two closest points on the two branches.
* Conjugate axis is the line segment perpendicular to the transverse axis, passing through the center.

Vertices and direction of a hyperbola

Which of the following graphs can represent the hyperbola $\frac{y^2}{9} - \frac{x^2}{4} = 1$?



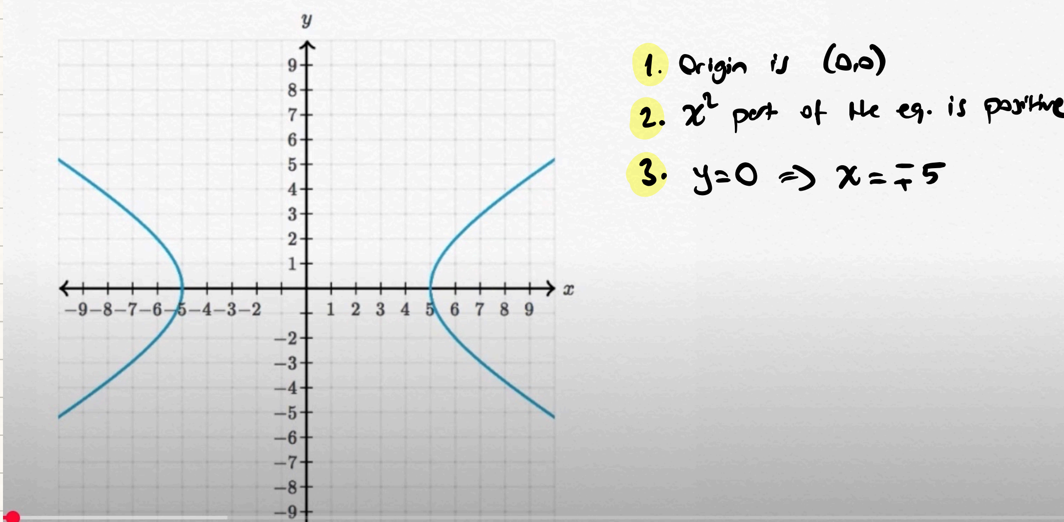
- The origin is (0,0)
- The positive part of the equation is the $\frac{y^2}{9}$, so 
- $f(0) = \mp 3$

Example 2:

Choose the equation that can represent the hyperbola graphed below.

- $\frac{x^2}{25} - \frac{y^2}{9} = 1$ ✓
- $\frac{y^2}{9} - \frac{x^2}{25} = 1$
- $\frac{x^2}{9} - \frac{y^2}{25} = 1$

  $y=0 \Rightarrow x = \mp 3$
- $\frac{y^2}{25} - \frac{x^2}{9} = 1$



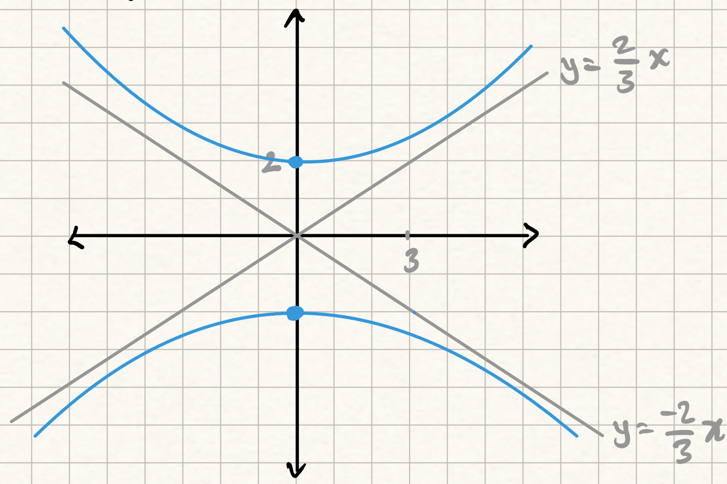
1. Origin is (0,0)
2. x^2 part of the eq. is positive
3. $y=0 \Rightarrow x = \mp 5$

* $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ and $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2}$ are the equations for the hyperbolas, where the origin is (h,k).

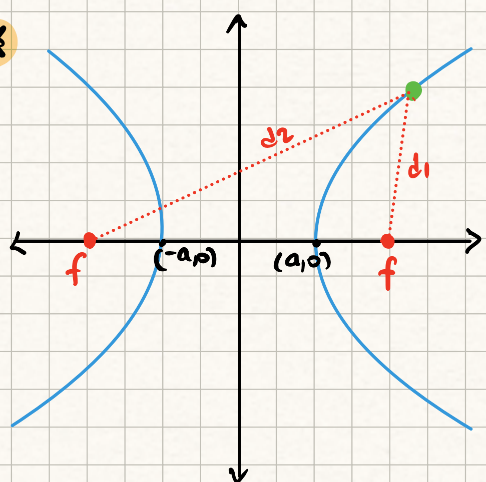
* Asymptotes are the two lines that the branches of the hyperbola will approach as they extend further and further away from the center. These asymptotes intersect at the center and their formula is $y = \mp \frac{b}{a}$

Example with asymptotes

* $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1 \Rightarrow y = \mp \dfrac{b}{a}x = \mp \dfrac{4}{9}x = \mp \dfrac{2}{3}x$



Foci of a hyperbola from equation

* 

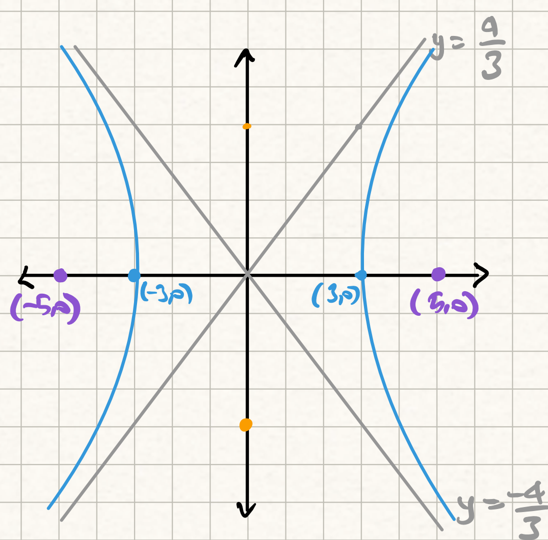
* $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

* $|d_1 - d_2| = 2a$

* $f^2 = a^2 + b^2$

* $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$\Rightarrow a = 3$
$b = 4$
$y = \mp \dfrac{4}{3}$
$f = \sqrt{16+9} = 5$



* Foci are the two fixed points located inside each curve of the hyperbola such that, for any point on the hyperbola, the difference of distances to the foci is a constant. The formula for the foci is: $f^2 = a^2 + b^2$

Hyperbolas Not Centered at the Origin

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Equation of a hyperbola not centered at the origin

$$\frac{(x-1)^2}{16} - \frac{(y+1)^2}{4} = 1$$

$a = 4$

$b = 2$

$O = (1, -1)$

$y = \frac{1}{2}x$

$|f| = 2\sqrt{5}$

$f_1 = (1-2\sqrt{5}, -1)$

$f_2 = (1+2\sqrt{5}, -1)$