

Functions Review

25/02/2025

What we'll cover

- > * Review only what we'll use in this course.
- * There's a mapping from x to $f(x)$

Which numbers

- > * $x \in \mathbb{R}$: x being a real number
- $x \in \mathbb{R}^D$: x " " vector of real numbers
- $f(x) \in \mathbb{R}$: $f(x)$ " " real number

Other ways to write functions

- > * Function as a table of numbers

x	$f(x)$
1	2
2	4
3	9
\vdots	\vdots

this mapping is
→ not useful for calculus

Calculus way of writing functions

- > * In calc. function as equations

$$\underline{ex} \quad h(t) = \frac{-1}{2} g t^2 + v_0 t + h_0$$

$$\underline{ex} \quad A(r) = \pi r^2$$

Why do we use so many different letters?

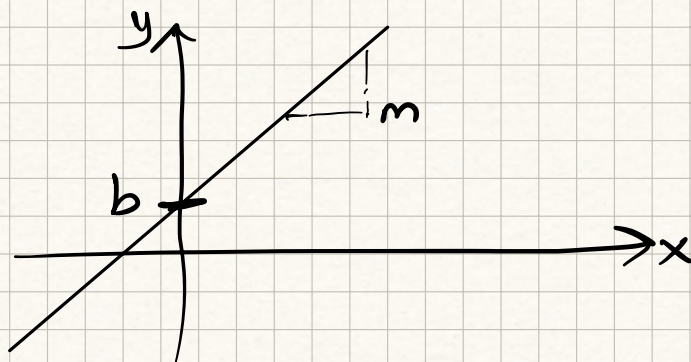
- > * "abstraction" matters, using diff letters don't

Types of functions?

- > * Types of Functions

$$f(x) = mx + b$$

$$y = mx + b$$



Polynomial functions

- > $f(x) = ax^3 + bx^2 + cx + d$
(polynomial)



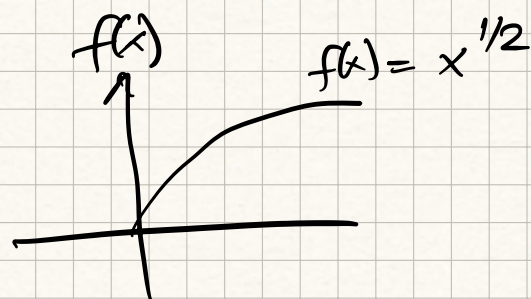
Root functions

$$> f(x) = ax^{1/3} + bx^{1/2} + c$$

(Root functions)

$$\hookrightarrow x^{1/2} = \sqrt{x}$$

$$x^{1/n} = \sqrt[n]{x}$$

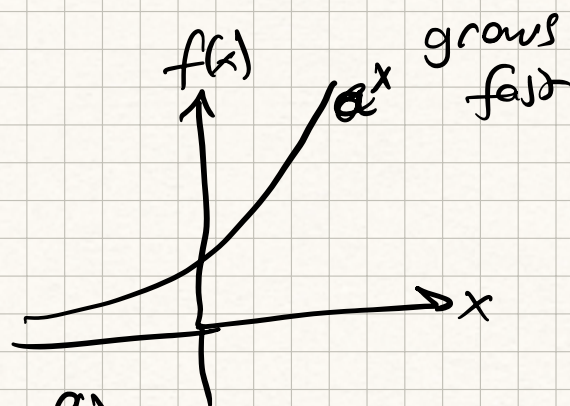


Exponential functions

$$> f(x) = 2^x$$

(exponential func.s)

$$f(x) = e^x$$

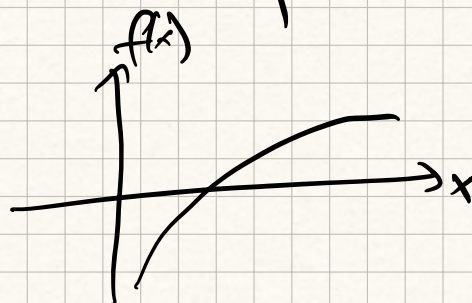


Logarithmic functions

$$> f(x) = \log x$$

$$f(x) = \ln x$$

(logarithmic func.s)



$$\log x = \log_{10} x$$

$$\ln x = \log_e x$$

Trigonometric functions

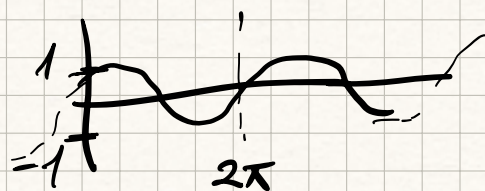
$$y = \sin x$$

$$y = \cos x$$

$$y = \tan x$$

trigonometric func.s

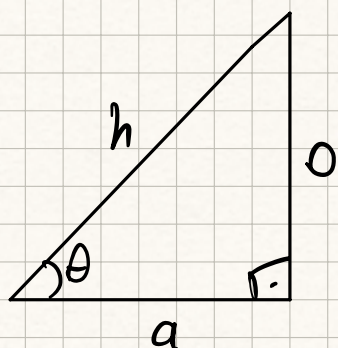
(not too common in ML)



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{o}{a}$$



a = adjacent

o = opposite

h = hypotenuse

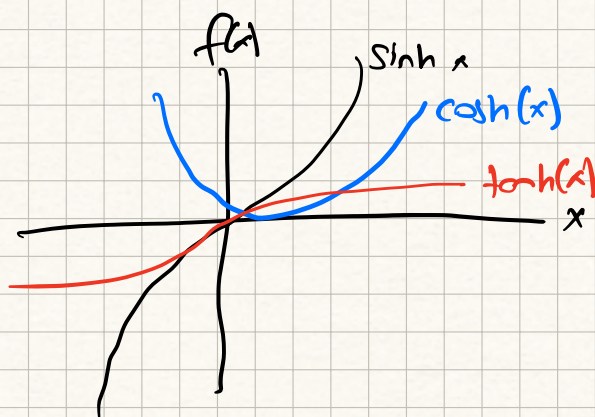
hyperbolic functions

$$y = \sinh(x)$$

$$y = \cosh(x)$$

$$y = \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

(hyperbolic func.)



$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$f(x) = \frac{x^2 - 2x + 3}{x^5 + 2x^3 + 1}$$

$$f(x) = \frac{\sin x}{x} \quad (\text{signal processing})$$

SUMMARY: In calculus, we'll use different types of functions for different types of problems. The most commonly used functions are polynomial, exponential, root, logarithmic, and hyperbolic functions.