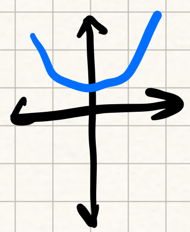
Functions and Algebra Review

01/03/2025

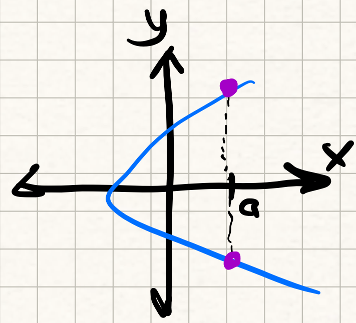
"One output" rule

$f(x) = x^2 + 1$

input → $\boxed{\text{function}}$ → output ↗ * only one output for an input



→ function



Not a function! b/c $f(a) \nearrow y_1$, $f(a) \searrow y_2$

Domain and range

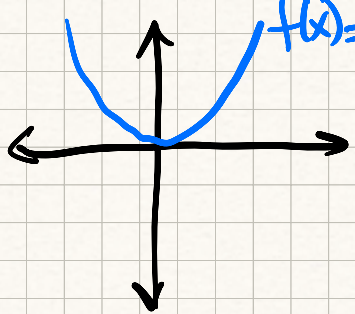
$f(x) = \frac{1}{x} \Rightarrow$

x can't be 0 $\Rightarrow$ $\underline{\text{Domain}}$: $x \in \mathbb{R}, x \neq 0$

Domain = What's allowed in the box.
Range = All the possible outputs

$f(x) = \frac{1}{x} \Rightarrow f(x)$ can not be 0

Range: $f(x) \in \mathbb{R}, f(x) \neq 0$



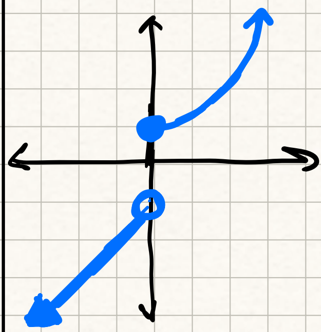
$x \in \mathbb{R}$
$f(x) \geq 0$

A function is a relation where each member of the domain set (all possible inputs) is mapped to exactly one member of the range set (all possible outputs).

Piecewise func.

$$f(x)=\begin{cases} x^2+1, & x>0 \\ x-1, & x\leq 0 \end{cases}$$

$f(0) = -1$
$f(1) = 2$



Multiply by 1, add 0

$$\frac{1}{3}\cdot\underbrace{\frac{3}{3}}_{1} + \frac{1}{2}\cdot\underbrace{\frac{2}{2}}_{1} = ?$$

multiplying by 1

$x+3 = 8 \Rightarrow$

$x+3\underbrace{-3 = 8+3}$

adding zero

$(x+2)(x-1) = x^2 - x + 2x - 2$
$= x^2 + x - 2$

$(x^2+x-2)(x+1)$
$x^3+x^2+x^2+x-2x-2$
x^3+2x^2-x-2

0 1 0
0 1 1 0
0 1 2 1 0
0 1 3 3 1 0
1 4 6 4 1

0 $(x+y)^0 = 1$
1 $(x+y)^1 = 1x + 1y$
2 $(x+y)^2 = 1x^2 + 2xy + 1y^2$
3 $(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$
4 $(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

Piecewise functions are functions defined by multiple sub-functions, each applying to a specific interval of the main functions domain.

Rationalizing denominators

$$\frac{x-2}{\sqrt{x+3}} \cdot \frac{\sqrt{x+3}}{\sqrt{x+3}} = \frac{(x-2)\sqrt{x+3}}{x+3}$$

$x+3>0$
$x>-3$
$x \in \mathbb{R}, x>-3$

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$$

$$\frac{x+4}{\sqrt{(x+6)} \cdot \sqrt{x+5}} = \frac{(x+4) \cdot \sqrt{(x+6)} \cdot \sqrt{x+5}}{(x+6)(x+5)} = \frac{}{x^2+11x+30}$$

Exponent Rules

$2^3 = 2 \cdot 2 \cdot 2$

$x^2 \cdot x^3 = x^5 \Rightarrow x^a \cdot x^b = x^{(a+b)}$

$x \cdot x \cdot x \cdot x \cdot x = x^5$

$(x^2)^3$

$x^2 \cdot x^2 \cdot x^2$

$x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$

$\Rightarrow (x^a)^b = x^{a \cdot b}$

$x^0 = 1$

$x^{-a} = \frac{1}{x^a}$

$(\sqrt{x})^2 = x^1$

$(x^{1/2})^2 = x^1$

$\sqrt[m]{x^n} = x^{n/m}$

Logarithm Rules

$3^x = 10 \Rightarrow$

$x = \log_3 10$

BASE remains BASE

$\log_5 x = 2 \Rightarrow$

$x = 5^2$

11 RULES

① $\log_x x = 1$ — $\log_5 5 = 1$, $\log_2 2 = 1$

② $\log_x 1 = 0$ — $\log_5 1 = 0$, $\log_{62} 1 = 0$

Exponential functions are of the form $f(x)=a^x$, where a is a positive constant (the base) and x is the exponent. They model growth or decay. Logarithmic functions are of the form $f(x) = \log_a(x)$. They measure the power to which the base must be raise to obtain x.

③ $\log 5 = \log_{10} 5$

$\log 10 = \log_{10} 10 = 1$

$\log 1 = \log_{10} 1 = 0$

④ $\log_x y^a = a.\log_x y$

$\log_3 5^2 = 2.\log_3 5$

$\log_2 8 = \log_2 2^3 = 3.\log_2 2 = 3$

⑤ $\log_{x^b} y = \frac{1}{b}.\log_x y$

$\log_9 27 = \log_{3^2} 3^3 = \frac{3}{2}\log_3 3 = \frac{3}{2}$

$\log 1000 = \log_{10} 10^3 = 3$

⑥ $\log_x y = \frac{\log y}{\log x}$

$\log_3 5 . \log_5 9 = \frac{\cancel{\log 5}}{\cancel{\log 3}} . \frac{2.\cancel{\log 3}}{\cancel{\log 5}} = 2$

⑦ $\log_x y = \frac{1}{\log_y x}$

$\log_3 2 = a \Rightarrow \log_2 9 = ?$

$\Rightarrow 2.\log_2 3 = \frac{2}{a}$

⑧ $\log_x (a.b) = \log_x a + \log_x b$

$\log_2 10 = \log_2 2 + \log_2 5$

$= \log_2 5 + 1$

$\log_3 36 = \log_3 3 + \log_3 3 + \log_3 2^2$

$= 2.\log_3 2 + 2$

$= 2(\log_3 2 + 1) = 2(a+1)$

⑨ $\log_x (a/b) = \log_x a - \log_x b$

$\log_3 (27/4) = \log_3 3^3 - \log_3 2^3$

$= 3 - 3\log_3 2$

$= 3(1-a)$

N/A. Memorize the rules!

⑩ $a^{\log_b c} = c^{\log_b a}$

$3^{\log_3 5} = 5^{\log_3 3} = 5$

$5^{2.\log_5 2} = 25^{\log_5 2}$
$= 2^{\log_5 25} = 2^2 = 4$

⑪ $\ln x = \log_e x$

$\ln e = \log_e e = 1$

$\ln e^3 = 3 . \log_e e = 3$

What's the use case for e and logarithm?

- $P_0 e^{kt}$ = population of P_0 after t times with a growth rate of k per time.

Let's say it doubles in 5 years, what's the growth rate per year? (k)

$2 \times \cancel{P_0} = \cancel{P_0} . e^{5k}$

$2 = e^{5k}$

$\log_e 2 = 5k$

$\frac{\log_e 2}{5} = k$

$\Rightarrow k = \frac{\ln 2}{5}$

- Populations, investments, growth/decay

$A = P.e^{r.t}$ where, A = final amount, P = initial principal, t = time, r = growth rate (decay rate if negative), e = natural base of the natural algorithm (approx. 2.71828).

Induction Proofs, Limits, and Continuity