

Evaluating Functions

26/02/2025

What is a function?

input → [function] → output

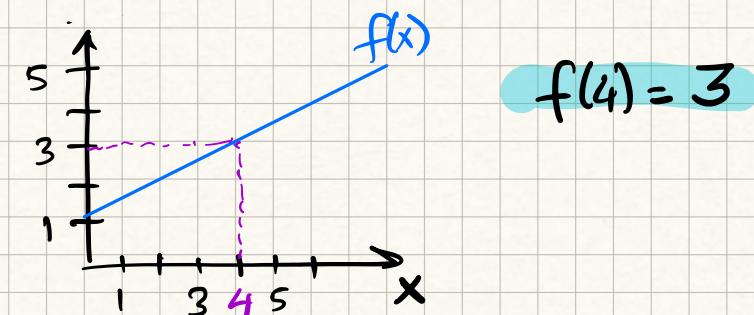
$$f(x) = x + 5$$

$$f(5) = 5 + 5 = 10$$

input ↓
 output

what is NOT
a function?

$x^2 + y^2 = 4$ not a function
b/c more than one output



Evaluate from
graph

Summary: In mathematics, a function from set X to a set Y assigns to each element of X , exactly one element of set Y .

Inputs and Outputs of a Function

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Matching an input to
a function's output

$$f(t) = -2t + 5$$

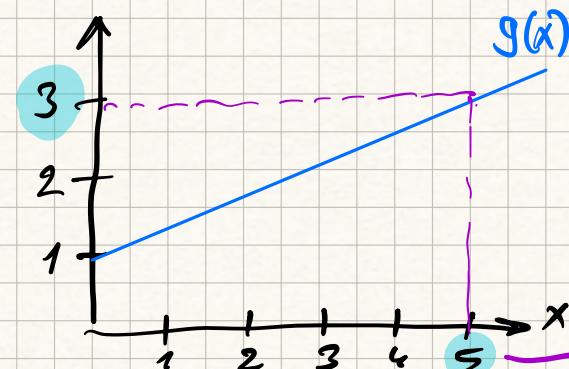
$$f(t) = 13 \Rightarrow t = ?$$

$$13 = -2t + 5$$

$$2t = -8$$

$$t = -4$$

From graph?

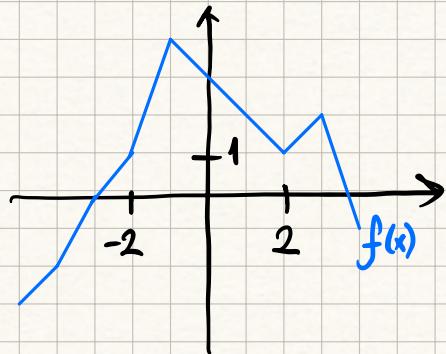


$$g(x) = 3 \Rightarrow$$

$$x = ?$$

$$x = 5$$

Two inputs with the same output



$$f(-2) = -1$$

$$f(2) = 1$$

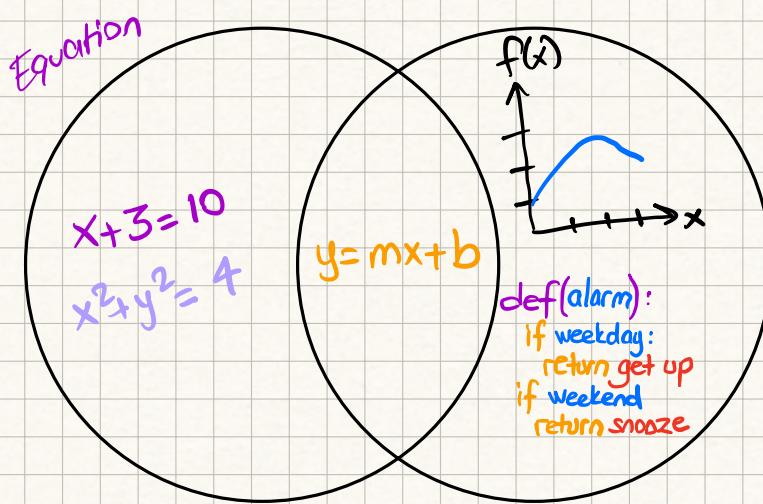
$$f(-2) = f(2)$$

Summary: We can use the graph to match inputs and outputs to each other. A function can have multiple inputs with the same output.

Functions and Equations

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Differences between equations and functions



Obtaining a function from an equation

For a given input value b , the function f outputs a value a to satisfy the following equation.

$$4a + 7b = -52$$

Write a formula for $f(b)$ in terms of b .

$$4a = -52 - 7b$$

$$a = -13 - \frac{7}{4}b = f(b)$$

Summary: Not every equation is a function. Not every function is an equation. There are equations who define a function.

Interpreting Function Notation

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Arjun opened up a savings account last year and put an initial sum in it.

Let $M(t)$ denote the account balance M (measured in dollars), t days since it was opened.

↗ Balance after 30 days

What does the statement $M(30) - M(0) = 100$ mean?

↳ initial balance

✗ 30 days after it was opened, the balance of Arjun's account was equal to \$100. $M(30) = 100$

✗ Arjun had the initial amount of money in his account 30 days after he opened it. $M(0) = M(30)$

✓ Arjun made a profit of \$100 over the first 30 days since the account was opened.

Let $P(t)$ denote the number of people P at the Copacabana beach in Rio de Janeiro, t hours past midnight on a specific day.

↗ num. of people at 9am

What does the statement $P(5) < P(9)$ mean?

↳ num. of ppl at 9am

✗ The time at which there were 5 people at the beach is earlier than the time there were 9 people.

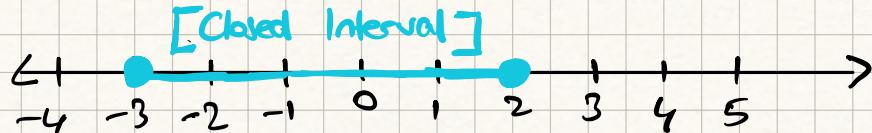
✗ 5:00 a.m. is an earlier time than 9:00 a.m.

✓ There were more people at the beach at 9:00 a.m. than there were at 5:00 a.m.

Introduction to Domain and Range of a function

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Interval notation?



$$\{x \in \mathbb{R} \mid -3 \leq x \leq 2\} = \{x \in \mathbb{R} \mid x \in [-3, 2]\}$$

"x is a member of real numbers

such that -3 is less than or

equal to x, and x is less

than or equal to 2.

(Open Interval)



$$\{x \in \mathbb{R} \mid -1 < x < 4\} = \{x \in \mathbb{R} \mid x \in (-1, 4)\}$$

* (-1, 4) * [-1, 4]



$$\{x \in \mathbb{R} \mid x \neq 1\} = \{x \in \mathbb{R} \mid x \in (-\infty, 1) \text{ or } x \in (1, \infty)\}$$

Domain of a function?

Set of all inputs over which the function has defined outputs.

$$f(x) = \frac{2}{x} \Rightarrow \boxed{f} \stackrel{?}{\rightarrow}$$

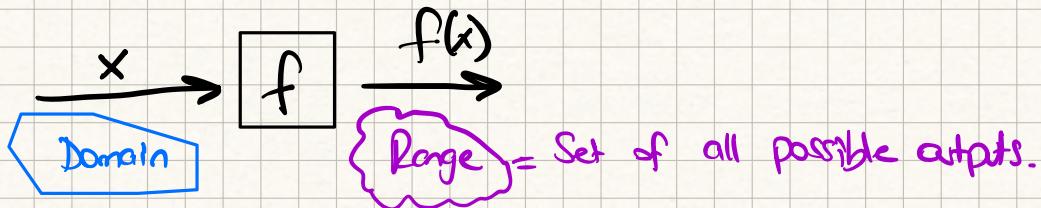
$$\text{Domain: } \{x \in \mathbb{R} \mid x \neq 0\}$$

$$g(y) = \sqrt{y-6} \rightarrow y-6 \text{ can't be negative}$$

$$\text{Domain} = \{y \in \mathbb{R} \mid y \geq 6\}$$

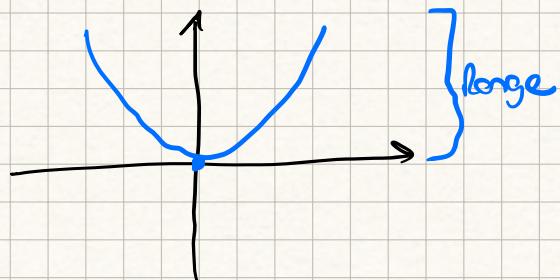
$$h(x) = \begin{cases} 1 & \text{if } x = \pi \\ 0 & \text{if } x = 3 \end{cases} \Rightarrow \text{Domain: } \{3, \pi\}$$

Range of a function?

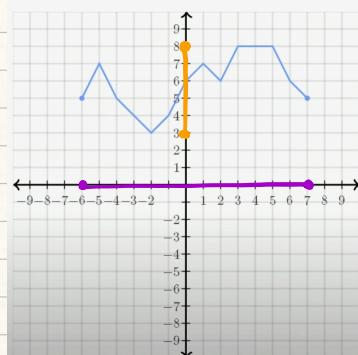


$$f(x) = x^2$$

$$\{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$$



The function $f(x)$ is graphed; what is its domain?



$$\{x \in \mathbb{R} \mid -6 \leq x \leq 7\}$$

$$\{f(x) \in \mathbb{R} \mid 3 \leq f(x) \leq 8\}$$

Domain and range from graph?

Summary: A function maps values from its domain to its range. We use interval notation to define these two sets.

Determining the Domain of the Function

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► $f(x) = \sqrt{2x-8}$

$$2x-8 \geq 0$$

$x \geq 4$

The function is defined when x is greater than or equal to 4.

► $g(x) = \frac{1}{\sqrt{6-|x|}}$

$$6-|x| > 0$$

$$|x| < 6$$

$$-6 < x < 6$$

►
$$h(x) = \begin{cases} \frac{x+10}{(x+10) \cdot (x-9) \cdot (x-5)} & x \neq -10 \\ \pi & x \neq 5 \end{cases}$$

$$\begin{array}{lll} x+10 \neq 0 & x-9 \neq 0 & x-5 \neq 0 \\ x \neq -10 & x \neq 9 & x \neq 5 \end{array}$$

$$\left\{ x \in \mathbb{R} \mid x \neq -10, x \neq 5 \right\}$$

► Pooja had a beautiful plant. The plant began sprouting 2 days before Pooja bought it, and she had it for 98 days before it died. At its tallest, the plant was 30 centimeters tall.
 $+(-2)$
 $+198$

Let $h(t)$ denote the height of Pooja's plant h (measured in centimeters), t days from the time she bought it.

Which number type is more appropriate for the domain of the function?

Integers Real numbers

$$\left\{ t \in \mathbb{R} \mid -2 \leq t \leq 98 \right\}$$

Define the interval of the domain.

► Thomas has 400 candy bars in his shop, and each costs \$0.50.

Let $p(b)$ denote the price p (measured in dollars) of a purchase of b candy bars.

Which number type is more appropriate for the domain of the function?

Integers Real numbers

$$p(b) = p \cdot \frac{1}{2}$$

can't buy 1.25 candies!

$$p \in \mathbb{Z} \mid 0 \leq p \leq 400$$

Define the interval of the domain.

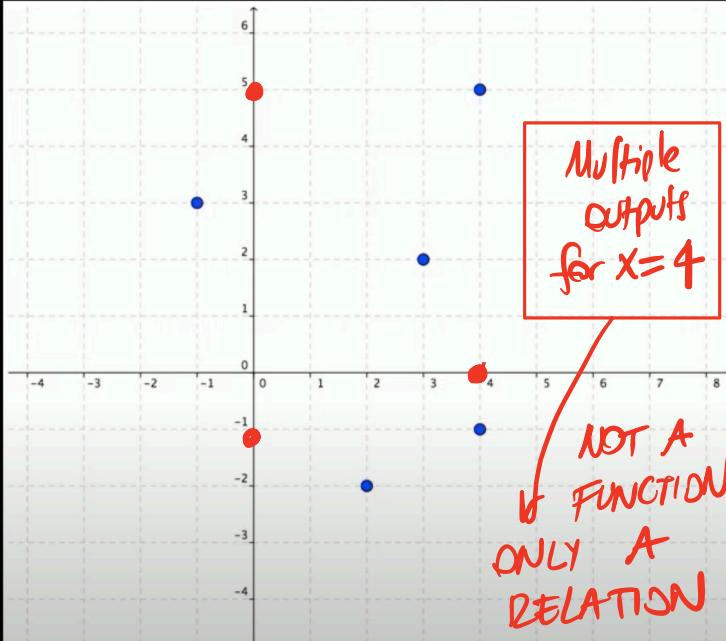
Summary: To find a domain of a function $f(x)$, concentrate on what values x **can not** take, and exclude those values from the domain.

Recognizing Functions

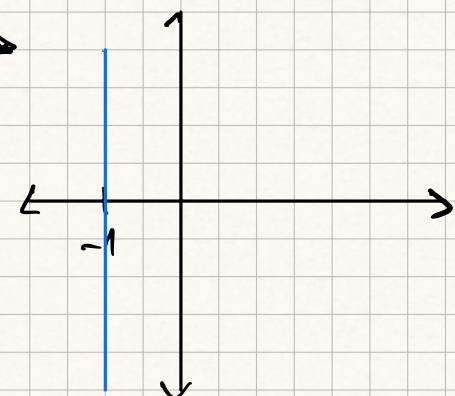
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From graph?

Determine whether the points on this graph represent a function.

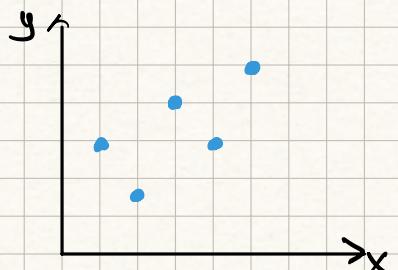


Does a vertical line represent a function?



From tables?

Name	Height
Joelle	5'6"
Nathan	4'11"
Stewart	5'11"
Tariq	5'6"
	6'1"



* No duplicate inputs with different outputs, it's a functional relationship.

From verbal description? "The value of y is always three more than twice x . Is y a function or not?

$y = 2x + 3$ = no multiple outputs for any input, " y is a function of x "

Attention to Phrasing!

Jada is ordering Mother's Day gifts online. The shipping costs are based on the dollar amount of the order. For orders less than \$20, shipping costs \$4. For orders of \$20 or more, shipping is \$7.

- Can the dollar amount of the order be represented as a function of shipping cost? amt. of order (shipping cost) ?



- Multiple outputs for the same input.
Therefore **NO!**

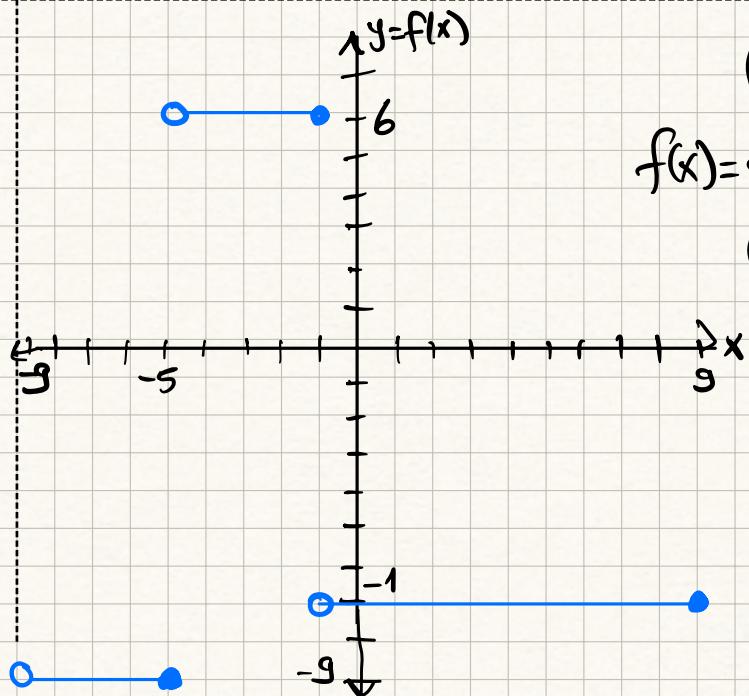
!! The shipping cost can be represented as a function of the amount of the order.

Summary:

Piecewise Functions

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Definition



$$f(x) = \begin{cases} -9, & -9 < x \leq -5 \\ 6, & -5 < x \leq -1 \\ -7, & -1 < x \leq 9 \end{cases}$$

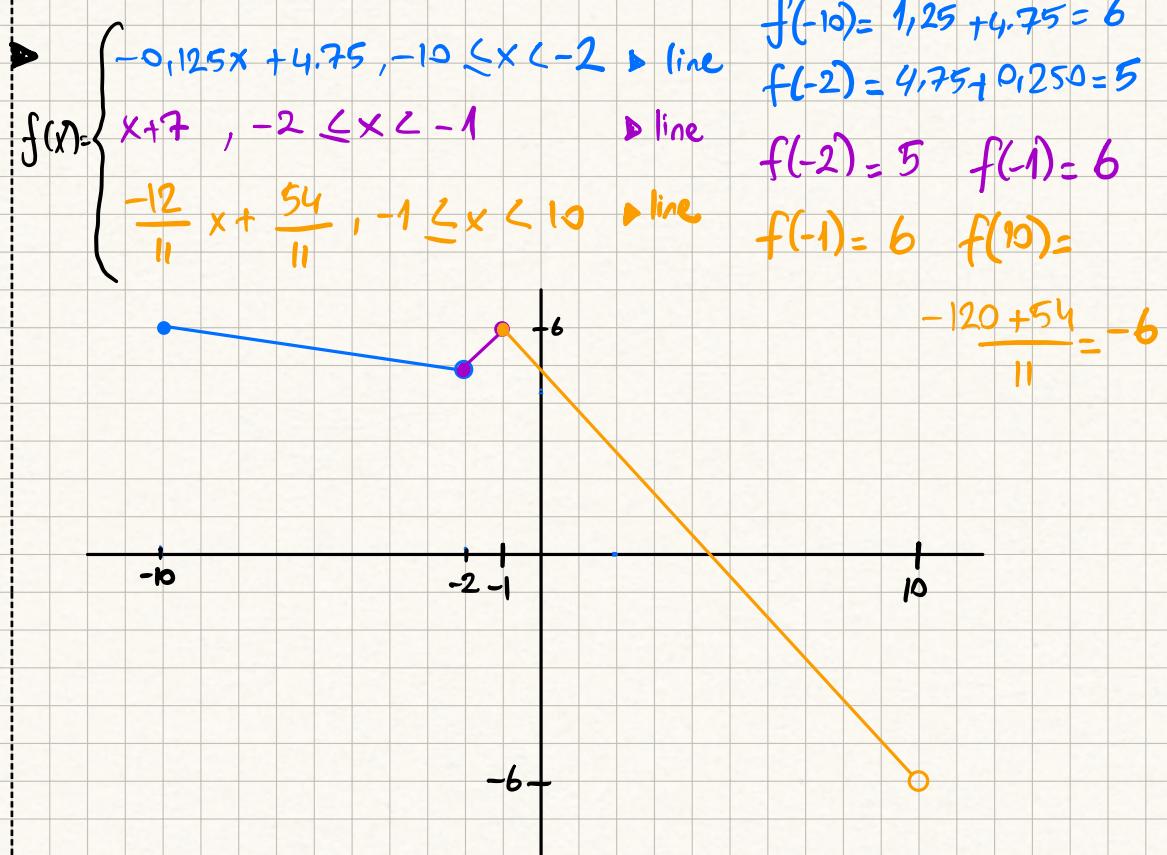
How to evaluate?

$$f(x) = \begin{cases} t^2 - 5t, & t \leq -10 \\ t + 19, & -10 < t < -2 \\ \frac{t^3}{t+9}, & t \geq -2 \end{cases}$$

$\Rightarrow f(-10) = ?$

$$\begin{aligned} f(-10) &= (-10)^2 - 5(-10) \\ &= 100 + 50 \\ &= 150 \end{aligned}$$

How to graph?



Domain & Range
of step functions

$f(x) = \begin{cases} 1, & 0 < x \leq 2 \\ 5, & 2 < x \leq 6 \\ -7, & 6 \leq x \leq 11 \end{cases}$

$\{x \in \mathbb{R} \mid 0 < x \leq 11\}$

$\{f(x) \in \mathbb{R} \mid f(x) \in \{1, 5, -7\}\}$

$g(x) = \begin{cases} x + 7, & -6 < x \leq -3 \\ 1 - x, & -3 < x < 4 \\ 2x - 11, & 4 \leq x \leq 6 \end{cases}$

$\{x \in \mathbb{R} \mid -6 < x \leq 6\}$

$g(-6) = 1, g(-3) = 4, -1 < g(x) \leq 4$

$g(-3) = 4, g(4) = -3, -3 < g(x) < 4$

$g(4) = -3, g(6) = 1, -3 \leq g(x) \leq 1$

$\{g(x) \in \mathbb{R} \mid -3 \leq g(x) \leq 4\}$

Summary:

Maximum and Minimum Points

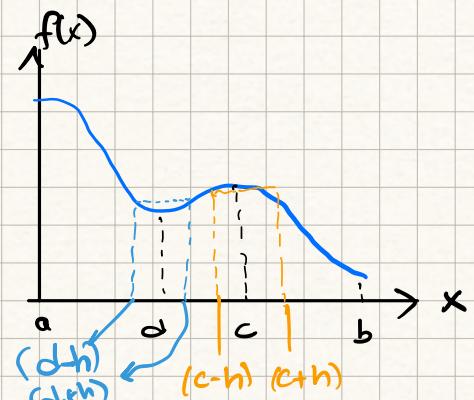
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► Absolute max/min values $f(a) / f(b)$

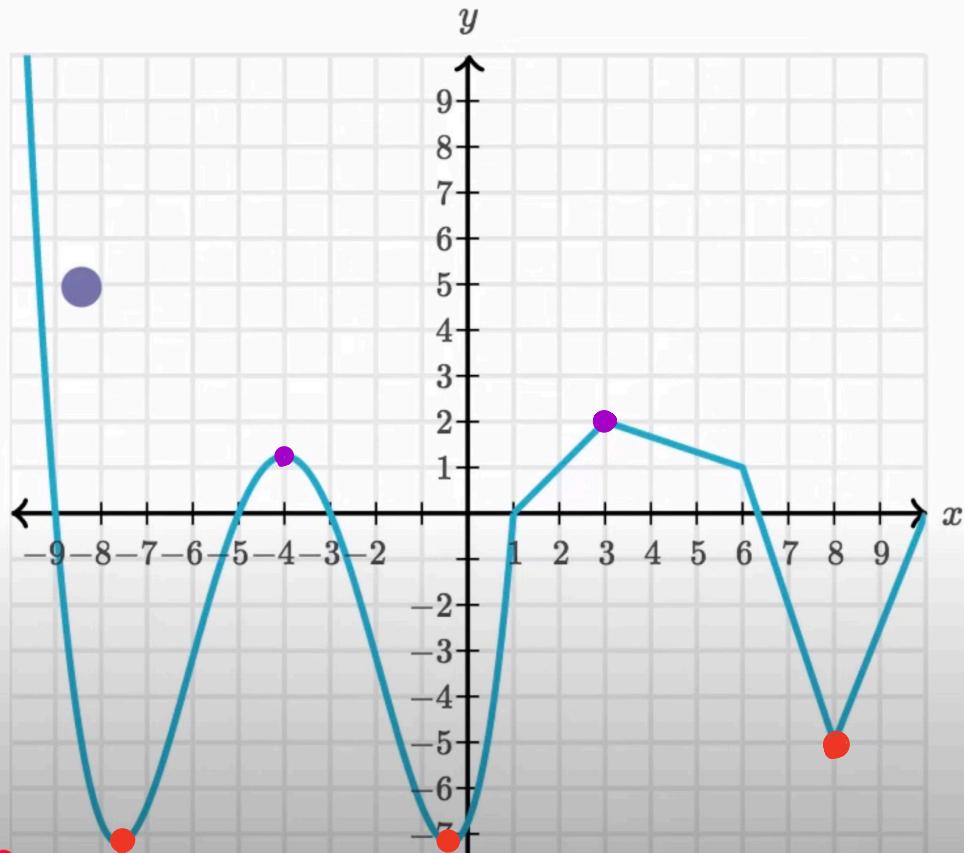
► Relative max/min values $f(c) / f(d)$

" $f(c)$ is a relative maximum if $f(c) \geq f(x)$ for all $x \in (c-h, c+h)$ for $h > 0$ "

" $f(d)$ is a relative minimum if $f(d) \leq f(x)$ for all $x \in (d-h, d+h)$



Mark all the relative extremum points in the graph below.

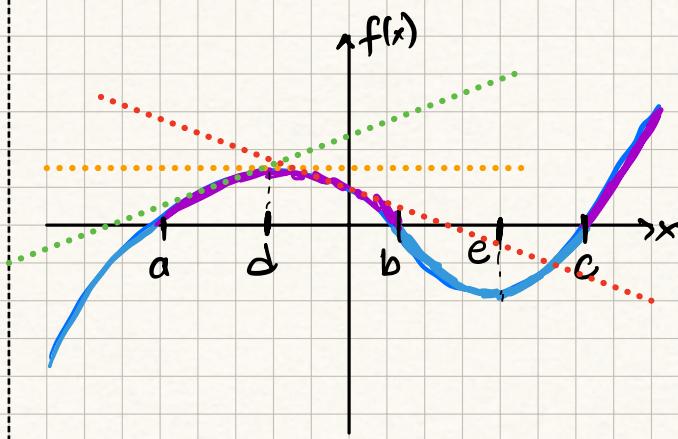


► If line horizontal \Rightarrow many points rel min/max at the same time

Summary:

Intervals Where a Function is Positive, Negative, Increasing, Decreasing

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Positive / Negative
Relative to the x -axis

Increasing / Decreasing
Slope of the
tangent line

$$m > 0, m = 0, m < 0$$

Summary: