Composing Functions (Bileşke Fonksiyonlar)

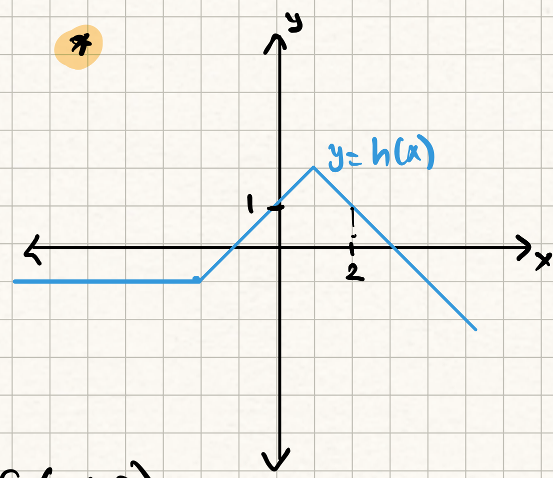
07.05.2025

Introduction

* $f(x) = x^2 - 1$

*

t	$g(t)$
1	3
2	-3
3	4
4	-1

* 

* $f(g(2)) = ?$

$$2 \rightarrow \boxed{g} \rightarrow g(2) \rightarrow \boxed{f} \rightarrow f(g(2))$$

$$-3 \rightarrow f \rightarrow (-3)^2 - 1 = 8$$

* $f(h(2)) = ? = f(1) = 1^2 - 1 = 0$

* $\underbrace{h(\underbrace{g(\underbrace{f(2)}_{3})}_{4})}_{-1} = ?$

* $f(g(x)) = (f \circ g)(x)$ = "f composed with g"

What's a real-world example for composite functions?

* Cam is a farmer. Each year he plants seed that turn into corn. The function below gives the amount of corn, C, in kilograms (kg), that he expects to produce if he plants corn on a acres of land:

$$C(a) = 7500a - 1500$$

---

* Function composition is the action of combining two functions in such a way so that the outputs of one function becomes the input of the other.

$$f(g(x)) = (f \circ g)(x) = \text{"f composed with g"}$$

The function belows predicts how much money, M, in dollars, he will earn from selling c kilograms of corn:

$$M(c) = 0.9c - 50$$

How much money does he expect to make if he plants corn seed on a a acres of land?

$$M(C(a)) = (M \circ C)(a) = ?$$

$$\begin{aligned} M(C(A)) &= 0.9\,C(a) - 50 \\ &= 0.9\,(7500\,a - 1500) - 50 \\ &= 6750a - 1350 - 50 \\ &= 6750\,a - 1400 = (M \circ C)(a) \end{aligned}$$

* How much money can Cau expect to earn if he sells corn produced on 1.5 acres?

$$\begin{aligned} (M \circ C)(1.5) &= 6750(1.5) - 1400 \\ &= \$8725 \end{aligned}$$

How to evaluate composite functions?

* If $f(x) = 3x - 1$ and $g(x) = x^3 + 2$, what is $f(g(3)) = ?$

1. "Inside out" evaluation:

$g(3) = 3^3 + 2 = 11 \qquad f(11) = 3(11) - 1 = 32$

2. Finding the composite function

$$\begin{aligned} (f \circ g)(x) &= 3(x^2 + 2) - 1 \\ &= 3x^2 + 5 \end{aligned} \qquad \Rightarrow (f \circ g)(3) = 3(3)^2 + 5 = 32$$

* We can evaluate a composite function either by "inside out evaluation" or by "finding the composite function by substitution."

Modeling with Composite Functions

07.05.2025

How can we express a composit function in terms of the model it creates?

* Carter has noticed a few quantitative relationships related to the success of his football team and has modeled them with the following functions:

Function	Input	Output
N	Winning percentage, w	Average number of fans per game, $N(w)$
W	Average daily practice time, x	Winning percentage, $W(x)$
P	Number of rainy days, r	Average daily practice time $P(r)$

What does the expression $N(W(x))$ represents?

	input		output
① $W(x) \Rightarrow$	"average daily practice time" (first input)	→	"winning percentage"
② $N(W(x)) \Rightarrow$	"winning percentage"	→	"average number of fans per game" (last output)

$N(W(x))$ = "The average number of fans per game, as a function of the average practice time."

* "The last output as a function of the first input."

* We can use "The last output as a function of the first input" expression to express a composit function in terms of the model it generates

Not every model makes sense!

* Jaylen modeled the following relationships about their bus ride:

- $P(b)$: Probability that Jaylen gets to work on time as a function of the time bus arrives (b).
- $N(k)$: Number of people at the bus stop when the bus arrives as a function of the time the bus arrives (k)
- $T(x)$: Time the bus arrives as a function of centimeters of precipitation (yagis) per hour.

? Which **two** of the following composed functions make sense?

(A) $P(T(x))$ (B) $P(N(k))$ (C) $N(T(x))$ (D) $T(P(b))$

(A) Precipitation $\xrightarrow{T}$ arrival time $\xrightarrow{P}$ ✓ prob. on time

(B) arrival time $\xrightarrow{N}$ # of people at the stop $\xrightarrow{P}$ ✗ prob. on time

(C) precipitation $\xrightarrow{T}$ arrival time $\xrightarrow{N}$ ✓ # people at the stop

(D) arrival time $\xrightarrow{P}$ prob. on time $\xrightarrow{T}$ ✗ arrival time

* When we compose functions, we must make sure that it makes sense to plug the output of the inner function to the outer function.

* When we compose functions, we must make sure that it makes sense to plug the output of the inner function as an input for the outer function.

Invertible Functions

07.05.2025

What are inverse functions?

* Inverse functions reverse each other:

$$f(a)=b \Rightarrow f^{-1}(b)=a$$

* For a function to be invertible, its inverse must be a function too.

x	1	2	3	4
$h(x)$	2	1	2	5

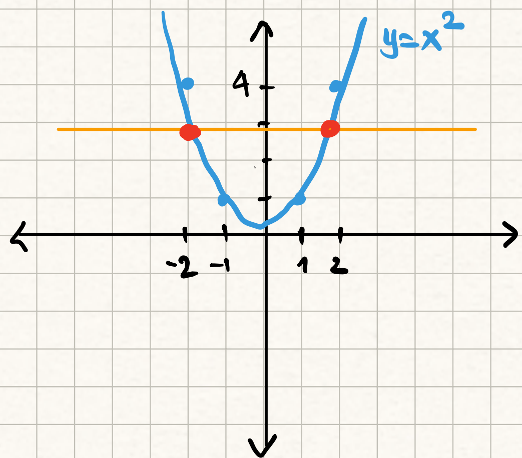
$\Rightarrow h^{-1}(2) = \{1, 3\}$ ✗ NOT A FUNC!

h is non-invertible

* A function is invertible only if each input has a unique output (one-to-one mapping).

What is a horizontal line test?

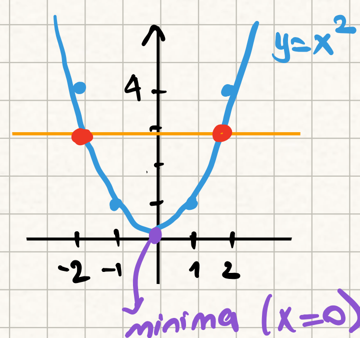
* If we can draw a horizontal line that intercepts the function on multiple points, the function is non-invertible.



$f(x)=x^2$ is non-invertible!

Restricting domains of functions to make them invertible

* If the function is not strictly monotonic (so it has both increasing and decreasing intervals) it is not invertible within a domain which includes a minima or maxima



* If we restrict our domaint to $-\infty < x \leq 0$ or $0 \leq x < \infty$, the function becomes invertible.

---

* Inverse functions reverse each other: $f(a)=b \Rightarrow f^{-1}(b)=a$
* A function is invertible only if there's a one-to-one relationship between its domain and its range
* If we can draw a horizontal line that intercepts the function on multiple points, the function is non-invertible.
* If the function has both increasing and decreasing intervals, it is not invertible within a domain that includes a minima or maxima.

Reading Inverse Values from a Graph

08.05.2025