# AP CALCULUS BC

## Of LIMITS AND CONTINUITY

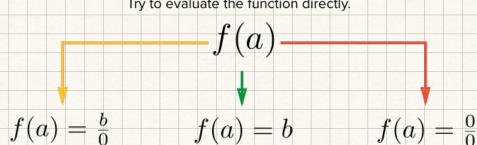
- \* A limit is the value that a function (or sequence) approaches as the argument (or index) approaches some value.
- \* When a limit doesn't approach the same value from both sides, then the limit doesn't exist: lim  $f(x) = \lim_{x \to c^+} f(x) = \lim_{x \to c^+} f(x)$
- No limit. On the other hand, just because a function is defined for some x-value doesn't mean that limit exists.
- \* Let lim f(x) = L, lim g(x) = M, x>c
  - \* lim (f(x)+g(x)) = L+M \* lim (k.f(x)) = k. L
  - # lim (f(x)-g(x)) = L-M # lim  $(\frac{f(x)}{g(x)}) = \frac{L}{M}$
  - with (f(x).g(x)) = L.M with  $(f(x)^{\frac{c}{5}}) = L^{\frac{c}{5}}$
- \* when evaluating the limits of combined functions, we must verify whether
  the left-hand and right-hand limits are equal. If they are, the overall
  limit exist, even if the individual limit of one of the component
  functions also not.
- \*  $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$  if and only if:
  - 1)  $\lim_{x \to a} g(x) = L$  (exists)
  - @ AND f(x) is continuous at 2.

# \* Selecting procedures for determining limits:

# Calculating $\lim_{x \to a} f(x)$

#### A. Direct substitution

Try to evaluate the function directly.



## C. Limit found (probably)

example:

## D. Indeterminate form

example:

 $\lim_{x \to 1} \frac{1}{x-1}$ 

$$\lim_{x \to 3} x^2 = (3)^2 = 9$$

example: 
$$\lim_{x \to 3} x^2 = (3)^2 = 9$$
 
$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$

Inspect with a graph or table to learn more about the function at **x=a**.



Try rewriting the limit in an equivalent form.



### E. Factoring

example:

$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$$

F. Conjugates

example:

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

example:

$$\lim_{x \to 0} \frac{\sin(x)}{\sin(2x)}$$

can be reduced to

can be rewritten as

$$\lim_{x \to -1} \frac{x-2}{x-3}$$

by factoring and cancelling

$$\lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$

using conjugates and cancelling.

$$\lim_{x \to 0} \frac{1}{2\cos(x)}$$

using a trig identity.



Try evaluating the limit in its new form.

H. Approximation

When all else fails, graphs and tables can help approximate limits.

- \* If  $g(x) \leq f(x) \leq h(x)$  and  $\lim_{x \to 0} g(x) = \lim_{x \to 0} h(x) = L$ , then  $\lim_{x \to 0} f(x) = L$
- ore 3 types of discontinuity:
  - (Penovable (Point) Discontinuity: Two-sided limit exists, which means one-sided limits exist and are equal to each other. However, two-sided limit is not equal to the function's value at that point.
  - 2) Jump Discontinuity: Two-sided limit does not crist because one-sided limits are not equal to each other, even though they exist.
  - 3 Asymptotic Discontinuity: Two-sided limit does not exist because one-sided limits are unbounded, therefore they don't exist.
- \* f is continuous at x=c  $\iff$  UM f(x) = f(c)
- \* f is continuous over (a,16) <=> f is continuous over every point in the interval
- f is continuous over  $[a,b] \iff f$  is continuous over (a,b) AND  $\lim_{x\to 0^+} f(x) = f(b)$ AND  $\lim_{x\to b^-} f(x) = f(b)$
- + f is continuous on all real numbers = it has no types of discontinuity (renovable, jump, or asymptotic).
- say that limit is going to infinity in that direction.
- # A function can not cross its vertical asymptote, but it can cross its horizontal asymptote (even multiple times).
- \* Functions with horizontal asymptotes have one-sided limits for a approaches a and -00. These limits one finite real numbers.
- \* The intermediate value theorem states that for any function of that is continuous over the interval [a,b], the function will take any value between fla) and flb) over the interval. More formally, for any value L between fla) and f(b), there's a value c in [a,b] for which f(c) = L.