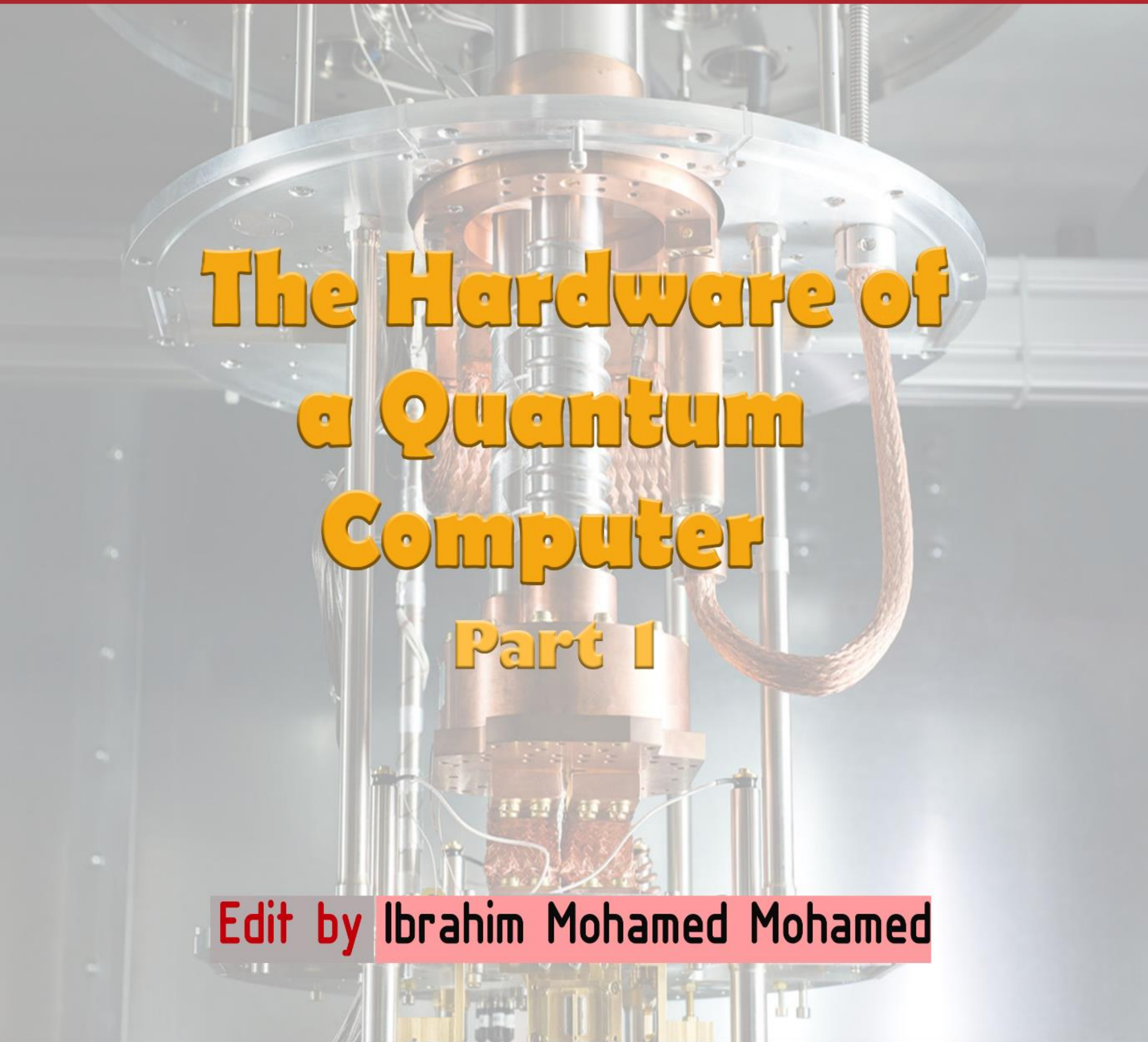
The Hardware of a Quantum Computer

Part 1



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Welcome

Welcome to the first part of the Quantum Computing & Quantum Internet. "Handout"Usually, lectures, articles and Videos about quantum computation mostly talk about qubits themselves, and all the quantum phenomena that make qubits exciting, futuristic, and almost impossible to fully understand. Qubits on their own, however, don't form a working quantum computer. The qubit might be the most spectacular element of the computer, but many more layers are needed to actually work with the quantum phenomena that give the quantum computer its great powers.

This Book will touch on all of these layers: from the devices which bridge the gap between the quantum chip and the classical control hardware, to the mathematical aspects of some quantum algorithms.

What you will learn

In this Book, you will learn:

How all the building blocks of a quantum computer work, and deeper information on the qubits which lie at the heart of a quantum computer and internet. How qubits can be used and controlled efficiently, and the workings of the four most promising types of solid-state qubits: Silicon Spin Qubit, Diamond NV Center Qubit, Superconducting Transmon Qubit and Topological Qubit.

Get the most out of this book!

This book aims to give you an understanding of the scientific basis behind the quantum computer and a quantum internet; and of four different qubits. Our challenge to you is to really understand the working principles of qubits and, at the same time, the working principles of a computer made of these qubits.

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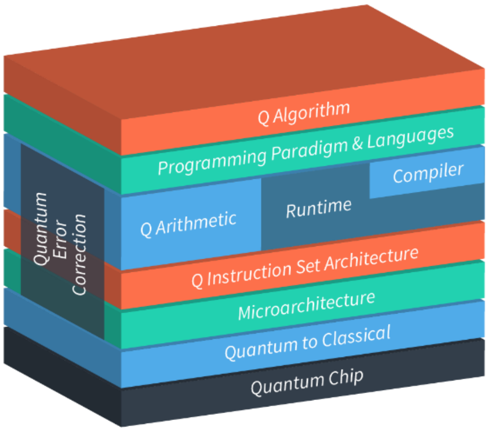
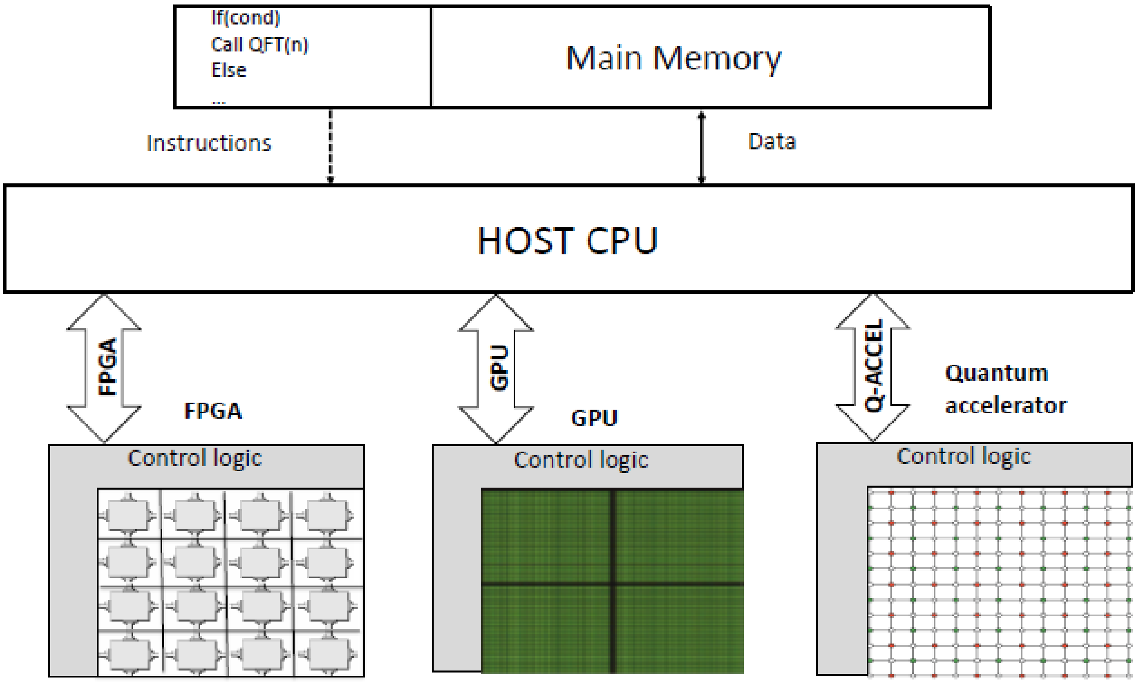
M o d u l e   1

When we build a quantum computer, we need to integrate different kinds of technologies to be able to interact with the machine. The quantum computer contains the following 'building blocks’



In this first module we will give you an overview of the whole system. Koen Bertels, Professor in Computer Engineering, takes you on a journey through all layers. In the next 4 modules we will dive deeper in the layer of the quantum chip (qubits).
Each qubit needs its own specific material. These materials need to contain one of the particles that we need to create the qubits and not be contaminated with other particles. Therefore we need to 'grow' materials. Giordano Scappucci will explain how this works.
To fully understand the topics of this course, you will need some mathematical tools. In the lectures on Ket notation, Postdoctoral researchers Ben Criger will explain the basic mathematical principles you will encounter during this course.
This module contains 3 lectures and 3 quizzes.
Good luck and enjoy!

Introducing the building blocks of a quantum computer



Koen Bertels, Building Blocks of a Quantum Computer

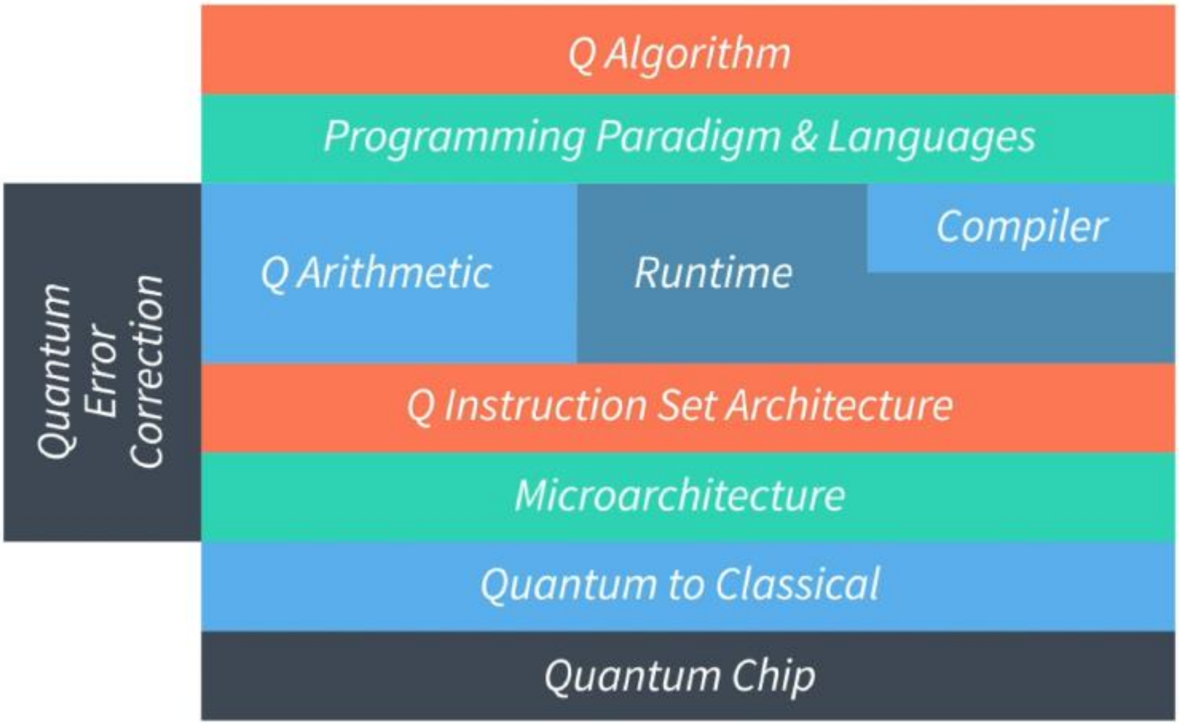
When we build a computer, we have to integrate different kinds of technologies such as those for the memory, bus interconnect, processor chips but also the peripheral devices such as a keyboard or a even screen to be able to interact with the machine. This lesson discusses the big picture of a quantum computer: namely from how to program it, to reading out the result.

Now the first word we want to emphasize is the word "compute". Even though QuTech is created to build a quantum computer together, we are actually not really building a quantum computer, but we are building a quantum accelerator. Namely, a computational devise that can be connected to a classical processor that will provide the performance for a series of applications that we can never reach classically.

Here you see the global view of what we currently understand as a heterogeneous multicore architecture. Heterogeneous because we have different kind of accelerator technologies. We have an FPGA which stands for the field programmable gate array. We have GPU's, for Graphic Programming Unit. These are the vector processors we are using to produce graphics computation. The third alternative accelerator technology will be a quantum co-processor which has the quantum properties that will provide a substantial increase in the compute power.

This is basically what we see. That also means that when you write any application, you will most likely end up using different kinds of accelerators including the FPGA, the GPU and also the quantum accelerator and therefore your application has to be compiled for four different instruction sets; namely, your Intel processor for instance on the classical machine, your FPGA instruction set, the GPU instruction set, and also the quantum instruction set.

That is very important to understand, so whatever we are going to discuss in the following talks is only going to focus on the quantum accelerator device or the application processes that we need classically as well as quantumly to be provided.
Whenever you talk about a computer, classically we have divided that in different layers; the lowest layer is always let's say the hardware, the chip that has been designed. It is never a single chip or a single processor. No, it has memory, it has interconnections, so a bus that allows the processor to communicate with the memory to retrieve instructions and to retrieve or produce the data.
This is shown here. And then it goes up to the microarchitecture that we need, up to the application level.
I will now go in detail on each of those layers in a quantum context.
Because we are basically adopting the same kind of layer view of what a quantum accelerator or a quantum computer would be, and we simply put the Q of quantum in front of every layer. And that is our research and working program.
The highest level is the quantum algorithm that we know of. We don't even know yet what they will be. We have examples such as factorization that is used in cryptography in securing communication between machines. But quantum algorithms can as well be designing a new molecule for personalized medicine. It can be that you might want to have a climate module that you want to run on your quantum accelerator that take all kind of mechanisms into account that we currently are unable to compute or even model on a classical machine. So that is the quantum algorithm layer, and that is where the biggest opportunity lies worldwide. Where many companies and organizations can start developing their own quantum application. Because every company or every end-user can think of how they can use that computational aspects of such a quantum device.
One layer lower is that when you have an application that you need to program. You do that classically. You have a programming language like C++, Fortran, Cobol or any language that you can think of. Those languages can produce the code for a classical processor. But we also have to develop our own quantum language for the quantum accelerator.
There are a couple of languages that have been developed so far. There is ScaffCC, and ProjectQ. And here at QuTech we are developing our own programming language called OpenQL, inspired by OpenCL, which is a language developed for GPU programming. We are now shifting it to the quantum infrastructure, so making the changes to that language.So that is the programming language layer.



Koen Bertels, Building Blocks of a Quantum Computer

For every programming language we need a compiler. A compiler takes the input of your algorithmic logic and compiles it into a lower level language that is classically called an assembly language. Here we are working on a quantum assembly language which we call QASM, which was originally developed for a book 'Quantum computation and information' by Nielsen and Chuang. To generate the figures in their book of the quantum circuits. We just expanded that language into a full-fledged quantum assembly that is being generated by our OpenQL compiler, which is the programming language that we also developed. So that you can express your quantum logic in such a way.

We are internationally collaborating with other partners working on similar things such that we standardize this quantum assembly language. Because for now everybody has its own local version, and that is not very good that everybody has his own variation, but if we all agree that this is what we assume to be QASM, then progress will come much faster.

The next layer is quantum arithmetic's because the mathematics of what you need to do is completely different than classically. The quantum gates operate quite differently, that's why you need to develop the quantum arithmetic; how to do a quantum operation.

I will not say anything about run-time, which is another part that we need in a quantum accelerator. Because we will need it, but what kind of functionality it should have is a bit unclear right now. That is where there is the tension between the compiler development

because we still can develop a lot of things in the compiler and maybe at a later stage we will develop that in a run-time support.

All of this QASM basically maps very well one on one on with the quantum instruction set.

This quantum instruction set basically describes what the operations are that your quantum device is capable of executing. That is why we have to think of what these instructions are. We know that classically we use an assignment like A = B + C. We should be able to do something similar in a quantum device. But it is not as simple as retrieving data from a memory location and perform the addition and writing back the result, because in quantum we use qubits.
A qubit is a quantum bit. Classically, we reason in terms of bits, zeros and ones. And as you know we are now using qubits, quantum bits. These can also be zero or one too, but they can also be zero and one at the same time. And that is the famous superposition that we are exploiting in a quantum device.
We can also combine two qubits so we don't have two different states but we have a combination of those states; namely 4 states at the same time.
If I combine 3 qubits, I have 2^3 = 8 states.
Now what is very nice about quantum is that the quantum mechanics gives us parallelism for free. Because I can apply quantum gates on those 2^n different states
You will come to understand in the upcoming lectures that nothing comes for free however. There is still a lot of challenges that need to be resolved. But that is ultimately the big challenge and the big opportunity that quantum offers.That is why you have to understand what this instruction set is and the corresponding architecture should be.And that immediately brings us to the layer of the micro-architecture.
Just like any classical processors we have also a quantum micro architecture for my quantum device, which contains the processor, the memory and also the interconnects of how the processor will communicate with the qubits. It has local registers in the processor and an ALU, an arithmetic logical unit so that it can compute logical and arithmetical operations, and write back the result to memory so that the user gets an idea of what the algorithm has computed as a result.
So in the quantum case we have a micro-architecture which has a similar kind of functionality. And that is the one we are also currently implementing in a real device that already controls a number of the physical qubits; a superconducting as well as a spin qubit that we are developing at QuTech.
For now we are working on a 17 qubit micro-architecture, so that in principle we can go up to 2^17 parallel executions on the combination of those 17 qubits.
A layer lower is the quantum to classical layer. Because whatever you perform on the quantum level is always an analog phenomenon.
Now you say; "analog? I thought we were building a computer? Which should be digital..". Well, everything up to the micro-architecture is clearly digital, but ultimately what you send down to the quantum chip is a for example a microwave, it can be other things, but let's say it is a microwave and we control an individual electron at the atomic level. The individual electron is important to understand. So if I have 17 qubits, I basically have 17 electrons, not hundred thousands, not millions, but 17. And there are ways to combine

those 17 electrons and their spins in the way that they interact and move that indicates that they are doing a particular calculation.
So that means that this layer is necessary for translating all of the logical steps that you need to do in your algorithm into the appropriate microwave or the physical signal that you want to send to this electron and to the qubit.
And then ultimately it enters into the quantum chip. Which consists of these qubits which are connected to each other. And then we hope of course that we get a meaningful result. Now it is never the less important to understand for a quantum accelerator for any computational device is that it is a non-deterministic way of computing. That means that it is not like in a classical machine that you run a thousand times the same algorithm and you will get a thousand times exactly the same result. Quantumly this is absolutely not true, because when you want to read out the result several things happen. The most important is that any entangled superposition that exists, actually is going to get destroyed.
So if I have for example 2^17 possibilities, I am only going to get one of those possibilities back as a result. And all the others will disappear. And that is why you maybe have to do a computation 10 times, a 100 times, we don't even know how many times we need to do that. And then you can make a histogram of what has been computed, and the readout that has the highest frequency of occurrence has a high probability of being read by our micro-architecture and that is what we can report back to the end user.
So that is something that you should not forget. A quantum device is a very powerful device, it gives massive parallelism in principle. But we need multiple runs and average out what those calculations of those results are. And the one with the highest frequency is the most likely result of your quantum device.

Main takeaways

- **Current efforts are put into building a quantum accelerator, a computational add-on to the classical computers to exceed current performance.**
- **Heterogeneous multicore architecture entails a Field Programmable Gate Array, a Graphics Programming Unit and a quantum co-processor.**
- **The building blocks for a quantum computer are a quantum algorithm, a quantum language, a compiler, arithmetic, an instruction set, a micro-architecture, a quantum to classical conversion and a quantum chip.**

Practice Quiz 1

Question 1: Advantage

When N classical bits are read out after a computation they can be found in one of 2N different states (e.g. if you read out 2 classical bits from your computer's memory you will find them to be 00, 01, 10, or 11). When N qubits are read out after a computation they can also be found in one of 2N different states, just like classical bits. Where, then, does the advantage of using a quantum accelerator for some tasks come from?

- There is no advantage to using a quantum accelerator.

- Unlike the classical bits, which can be in only one of the 2N states before the read-out, the qubits can be in a superposition of the 2Nstates (i.e. they can be in all 2N states at the same time) before the read-out.

- A quantum accelerator uses the spin of electrons as the unit of computation. Since electrons are smaller than the transistors used in classical computers, quantum accelerators are faster.

Question 2: Parallelism

"Quantum parallelism" is a term that is sometimes simplistically used to refer to the fact that when using N qubits in superposition to do computation it is as if we are doing 2N computations at the same time. What is the main reason this can be misleading?

- While we can do operations on all the constituent states of a superposition simultaneously, we can only ever read out a single one of those states.

- Qubits are much more difficult to work with than classical bits, and so we can't really claim to be able to do computations on them.

- It should be "N computations at the same time" instead of "2N computations at the same time" because we only have N qubits.

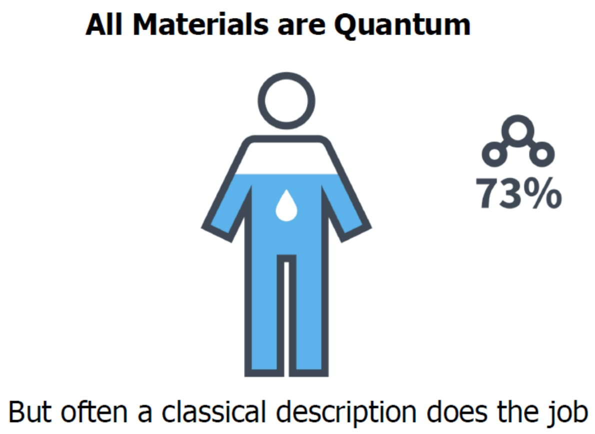
Quiz 1: Introducing the building blocks of a quantum computer

A programming language and compiler

A programming language is used a lot in classical computing, and it will likely be just as needed in quantum computing. The concept of a compiler is often mentioned together with a programming language. So what are they?

Quantum materials

QUANTUM MATERIALS ARE AN ESSENTIAL FEATURE IF ONE WOULD LIKE TO BUILD A QUANTUM COMPUTER OR USE THE QUANTUM INTERNET. ON A DAILY BASIS, GIORDANO SCAPPUCCI IS WORKING ON OPTIMIZING QUANTUM MATERIALS AT QUTECH. HE WILL INTRODUCE YOU TO THIS FIELD OF QUANTUM MECHANICS.



GIORDANO SCAPPUCCI ,QUANTUM MATERIALS

Quantum mechanics describes how atoms bind and electrons interact at a fundamental level. Naively speaking, all materials are quantum because all matter, in the end, must be explained by quantum mechanics. For example, our body is largely made up of water. In water, chemical reactions bind hydrogen and oxygen atoms together. Although this can be explained by quantum mechanics, we usually don’t think about water in these terms, and a high-school-level chemistry description does the job. This is, because we can often approximate the quantum behaviour in materials by a classical description. It turns out that a classical description is in fact suitable for most of the phenomena that we experience in the macroscopic world around us. Sometimes, however, to fully understand the behaviour of certain materials, it is necessary to keep quantum in view: there is no classical explanation to help our understanding.

A feature of the broad category of Quantum Materials is that their behaviour is generally rooted in the quantum world.

And while behaviours observed in quantum materials cover a wide spectrum, there are some common features that recur.

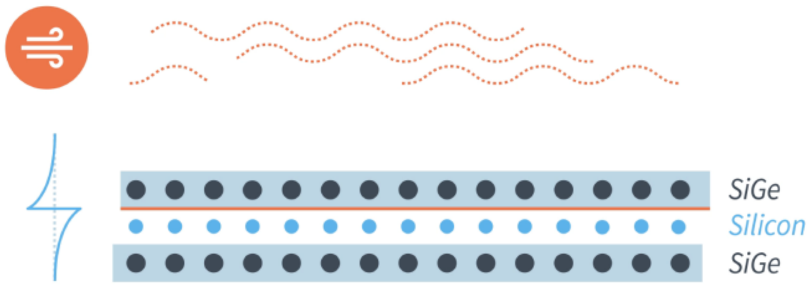
For example, many quantum materials derive their properties from reduced dimensionality. Electrons trapped in 2, 1, or 0 dimensions have different characteristics than electrons in 3D. Emergence is another recurring theme across quantum materials. Phenomena emerge due

Making of Quantum Materials

We quantum engineer the physical properties of materials
by using quantum confinement, strain and material composition

GIORDANO SCAPPUCCI ,QUANTUM MATERIALS

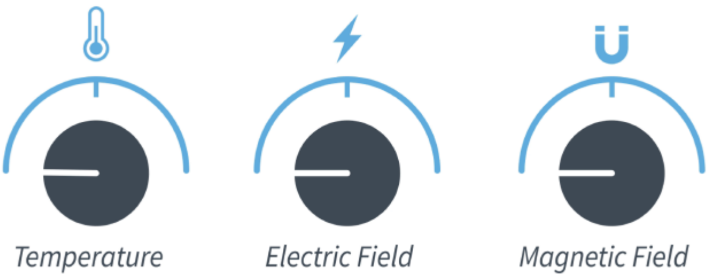
to the collective behaviour of original constituents. In superconductor materials, the interaction between electrons, mediated by vibrations of the crystal lattice, creates the so-called Cooper pairs. Differently from individual electrons, Cooper pairs can share the same quantum state. Superconductivity emerges from the concerted state of pairs that propagates without electrical resistance through the crystal. Certainly, you need quantum mechanics to understand this! The first quantum revolution, with the development of quantum mechanics, provided a microscopic understanding of nature. It had a tremendous impact in our life. Once you understand the periodic table and electronic wave functions, you understand semiconductors and how transistors are built and work. We now carry around billions of transistors in our pockets to perform complex tasks. It is clear that we need the concept of the photon to understand the laser. We are currently in the midst of a second quantum revolution, in which quantum matter is engineered to develop disruptive technologies beyond the reach of classical understanding. Quantum computing is the man on the moon goal of such second quantum revolution. By working in a fundamentally different way, quantum computers have the promise of solving complex problems that classical computers cannot handle. Quantum materials provide the environment where qubits, the elemental unit of quantum information processing, are defined and live. Therefore, quantum materials are at the basis of a quantum computer. To have a working quantum computer you need to couple together many qubits, while maintaining their long coherence time. These requirements are often conflicting. Qubits that are hard to couple together have long coherence times, because they tend to be isolated. On the other hand, systems that are easy to couple together tend to lose coherence quickly, because they can be perturbed easily by the environment. In all cases, the precision of the qubit materials is crucial to solve these two challenging requirements. And when we say “precision”, we mean precision in terms of: materials



uniformity, chemical composition, and electrical properties Ultimately, to make the phenomenally large number of qubits necessary to build a quantum computer, we would like to use the same manufacturing techniques of the microelectronic industry. Chemical vapour deposition, or CVD, is an industrial process that uses high purity gases to make high quality materials, with desired physical and electronic properties.

For example, by depositing layers of silicon and silicon-germanium, we can tune the lattice parameter of silicon to be larger than usual and match that one of silicon germanium. As a result, the electronic properties of such hetero-structure, which is made of different materials, make it possible to form a 2D electron gas at the interface between silicon and silicon-germanium. Qubits can then be made by isolating with electric fields one electron spin at that interface. If you want to isolate one single electron spin and make it a qubit, you need to know the properties of the host material very well. Once we make our crystal into a silicon heterostructure, we usually break it, and have a close look at it from the side with a high-resolution electron microscope. We call this microscope a transmission electron microscope, or TEM. Its working principle is based on electronic diffraction, another quantum effect, of an electron beam passing through a thin piece of material. The spatial resolution is so high that we are able to see atomic arrangements in the lattice. In addition, a TEM is capable of distinguishing different elements from each other by their atomic weight. Therefore, it is a useful tool to analyze materials composition. Light elements appear darker than heavy elements. This allows us to check precisely to what extent, the material we made matches our expectations. For example, you see here the perfect atomic arrangement of silicon and germanium atoms at the critical interface between silicon and silicon-germanium. The lattice spacing is exactly the same at both sides of the sharp interface, indicating that

Measuring of quantum materials

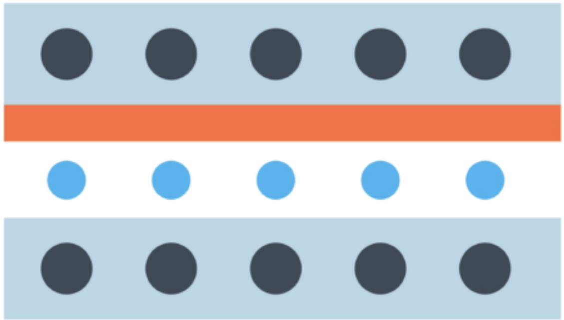


GIORDANO SCAPPUCCI ,QUANTUM MATERIALS

the CVD process successfully strained silicon to match the underlying silicon-germanium. The fact that we are able to make crystals with such precision and very little imperfections, is a key asset to then make good qubits. Once we know how the material we made looks like, we usually study its electronic properties, by modifying them with external parameters.Temperature, electric fields, and magnetic fields are the few knobs that we turn in our labs to probe quantum materials.

These studies provide useful feedback to make the material a better environment for the qubits. Take, for example, the Si heterostructure mentioned earlier. If we cool it to the very low temperatures at which qubits usually operate, say below one degree above absolute zero, it would be insulating. However, we can make the material conducting by imposing a vertical electric field. The electric field forms a metallic channel at the interface between silicon and silicongermanium, which is then populated by electrons. The higher the electric field, the more the electrons in the channel. By studying how the electrical resistance of such channel responds to a magnetic field, we are able to measure the number of electrons in the channel and their mobility. The mobility tells us how fast electrons can travel in such channel, and is an indication of the disorder in the system. The higher the mobility, the lower the disorder, and there will be a better chance of fabricating many qubits with similar properties. Materials homogeneity is one requirement to scale up the number of qubits into a quantum computer. Think of the billions of transistors in your laptop, if they were all behaving differently, there would be no chance of having them to work for you. And while we are currently optimizing quantum materials to build a quantum computer, ultimately, the hope is that when such a machine will exist, one of the first uses will be to efficiently simulate quantum systems, fulfilling the vision put forward by Richard Feynmann more than 30 years ago. In turn, this will help us understand and build even more complex quantum materials with extraordinary properties that today we cannot even predict.

Ultimately, quantum materials will be simulated by quantum computers



Giordano Scappucci ,Quantum Materials

Main takeaways

- **Quantum materials provide the environment where qubits, the elemental unit of quantum information processing, are defined and live.**
- **Precision in material uniformity, chemical composition and electrical properties are crucial for the requirements of having both many qubits and long decoherence times.**
- **Chemical Vapour Deposition is an industrial process that uses high purity gases to make high quality materials, with desired physical and electronic properties.**
- **Transmission electron microscopy is a process to inspect the fabricated heterostructures with high resolution.**
- **Temperature, electric fields and magnetic fields are useful parameters to determine properties of quantum materials such as mobility and electron density.**

Practice Quiz 2

Material Criteria

A quantum computer should have some necessary properties to be called a quantum computer. These are called the DiVincenzo criteria:

- The system must be scalable with well defined qubits;

- The computer must have the ability to initialize the state of qubits;

- Qubits must have long decoherence times;

- The computer must be able to perform a universal set of quantum gates on the qubits;

- It should be possible to measure the qubits.

We have to keep those criteria in mind when we choose the material for our qubits. For example, having homogeneous materials makes it possible to scale the system easily.

What property should the material have to ensure long decoherence times?

- ○ **Use materials with low density where qubits can easily be accessed for error correction.**
- ○ **Use materials with high density to reduce the interaction of qubits with neighbouring qubits.**
- ○ **Use materials that can be cooled down quickly.**
- ○ **Avoid materials with a complex structure, as it is more likely to interact with the qubits.**

Introduction to Ket notation

We would like to control quantum degrees of freedom in order to implement quantum computing, among other goals that we have. To talk about these degrees of freedom, we're going to use some notation that you might not be familiar with, but it's all very close to linear algebra.

Linear Algebra

column & row vectors: $\vec{x} = \mathbf{x} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}, \quad \vec{y}^{\top} = \mathbf{y}^{\top} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

inner products: $\vec{y}^{\top}\vec{x} = \mathbf{y} \cdot \mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} = 68$

matrices: $\mathbf{A}\vec{x} = \hat{A}\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 68 \\ 167 \\ 266 \end{bmatrix}$

Ben Criger! ,Ket Notation

You are likely already familiar with linear algebra, which has column vectors denoted with an arrow, or with boldface type. It also has a transpose operation that maps column vectors to row vectors, and vice versa. We can calculate the inner product of two such vectors, either by transposing one of them and multiplying, or by writing the dot product explicitly. Either way, we get a scalar by taking the inner product of these two vectors. In addition, we can transform one vector into another using a matrix, which can be written using boldface type or a hat, your preference. There is a slightly different notation that we use for vectors and matrices in quantum computing, which we often call ket notation.

Ket Notation

column vectors $\mapsto$ *kets*: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

dual vectors $\equiv$ *bras*: $\langle\psi| = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix}$

inner products $\equiv$ *brackets* $\langle\psi|\phi\rangle = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} \begin{bmatrix} \gamma \\ \delta \end{bmatrix} = \alpha^*\gamma + \beta^*\delta$

$|0\rangle/|1\rangle$ measurement yields $|0\rangle$ with probability $|\langle\psi|0\rangle|^2 = |\alpha|^2$

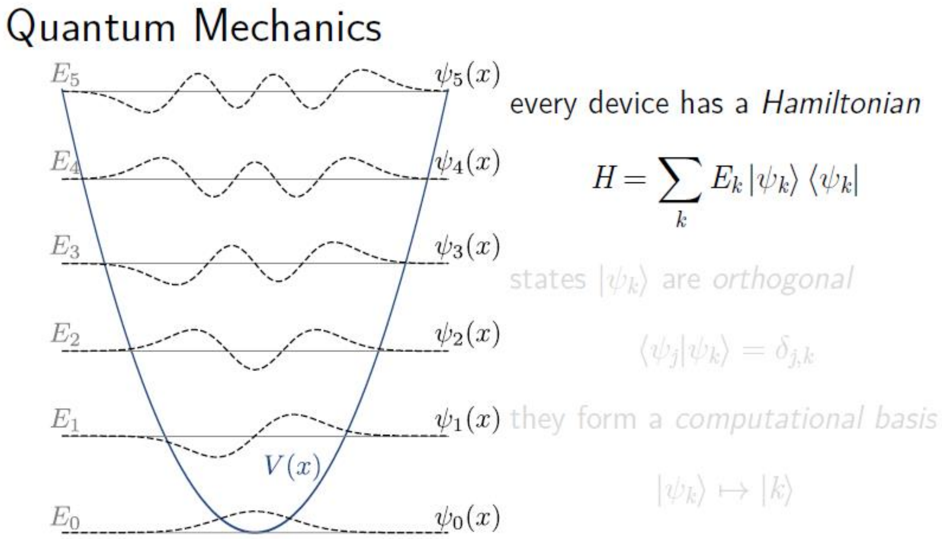
normalization: $\langle\psi|\psi\rangle = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\alpha|^2 + |\beta|^2 = 1$

states can be expressed in different bases

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \qquad \alpha|0\rangle + \beta|1\rangle = \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle$$

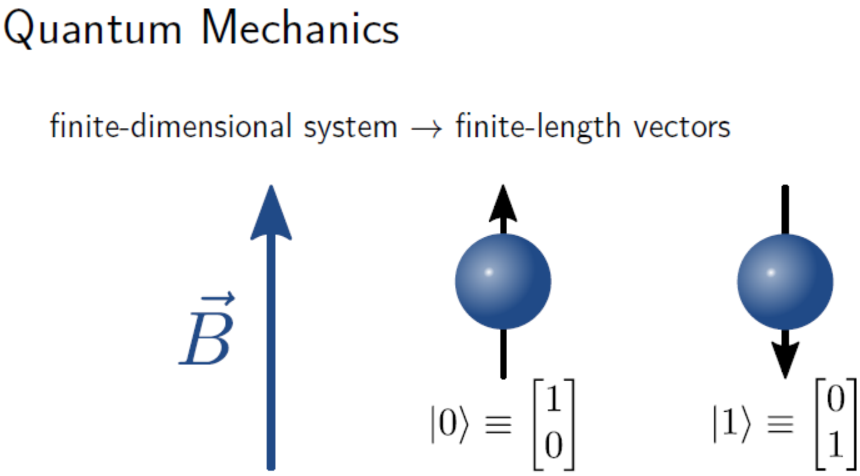
Ben Criger! ,Ket Notation

In ket notation, a quantum state is expressed using a column vector with complex coefficients. For example, we can express the state of a qubit, a two-level system, as a linear combination of two basis vectors, which we call 0 and 1. In a departure from regular linear algebra, there is a well-defined dual vector for each quantum state, called a bra, which we can obtain by taking the complex conjugate and the transpose of the ket vector. This is important for calculating inner products, which we always do by multiplying the bra for one state by the ket of the other, forming a bra-ket.



Ben Criger! ,Ket Notation

A central feature of quantum mechanics is that, when we perform a measurement to determine whether a state is 0 or 1, for example, we get a random answer, and the probability of measuring a state to be 0 is given by the squared magnitude of its 0 coefficient.



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One consequence of this is that, since such a measurement on a qubit state must result in 0 or 1, these squared magnitudes must sum to 1, since they are probabilities. This is called Born's rule and the constraint that the probabilities must sum to 1, is called normalization. We can also express qubit states in different bases. Consider the often-used plus-minus basis, which consists of the normalized sum and difference of the 0 and 1 ket vectors. Given a state expressed in the 0/1 basis, we can calculate the coefficients required to express the same state in the plus-minus basis, as I have done here. This leaves us with a small question: if any basis is just as good as any other, is there a basis that we should use as the default? Fortunately, the devices that we use to store and manipulate qubit states provide us with just such a default basis. One such device, which we see quite frequently in quantum mechanics, is the harmonic oscillator, shown on the left. These devices are described by a Hamiltonian, which is a matrix that assigns an energy E_k to each of its eigenstates, or preferred basis states, psi_k. These states are orthogonal and normalized (so we say that they're orthonormal), so the inner product of psi_j with any psi_k other than itself is 0. This allows us to use these states as a computational basis, replacing any detailed knowledge of the wavefunction psi_k with a simple label k that indicates which state we're talking about. Some devices have finite-dimensional state spaces, unlike the harmonic oscillator. This allows us to express the basis states as column vectors without using an infinite amount of space, which is very nice. Such devices include the spin-1/2, which is shown in this figure. Now that's it for the representation of single-qubit states. Let's take a look at how ket notation can help us describe the operations that we want to perform on these states, and the measurements we would like to make. The logic gate is the smallest classical computing system, and all classical computations can be expressed as a large sequence of these gates. The quantum counterpart to a logic gate is a unitary matrix, which replaces the computational basis with a new basis that depends on the operation that we want to perform. Probabilistic measurements are also described by matrices, but these matrices are hermitian, so in ket notation, they are just weighted sums of these ket-bra terms, where the psi_k states form an orthonormal basis for the space. If a measurement results in the state psi_k, the value r_k becomes known to the experimentalist. The expected value for this measurement is given by sandwiching the measurement operator with the state that we're measuring, and as you can see, the expectation value is just a weighted sum of the r_k terms, with the probabilities given by Born's rule. That's probably a lot to take in all at once, so let's take a look at a few examples. Here, we have a unitary operation that exchanges states in the computational basis, which we call X, or the Pauli X if you're already familiar with Pauli matrices. Here it is decomposed into ket-bra terms, and here's what happens when we use it to transform one of the computational basis states.

Operations & Observables

logic gates $\equiv$ *unitary* matrices $\equiv$ changes of basis

$$U = \sum_k |\psi_k\rangle \langle k| \quad (\langle\psi_j|\psi_k\rangle = \delta_{jk})$$

readout $\equiv$ measurement operators $\equiv$ *hermitian* matrices

$$A = \sum_k r_k |\psi_k\rangle \langle\psi_k| \quad (r_k \text{ real})$$

average experimental outcomes $\equiv$ 'sandwich' products

$$\langle\phi|A|\phi\rangle = \sum_k r_k \langle\phi|\psi_k\rangle \langle\psi_k|\phi\rangle = \sum_k r_k |\langle\phi|\psi_k\rangle|^2$$

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We just get the other state. 0 goes to 1, and 1 goes back to 0. Not so exciting.
Now let's take a look at a more interesting operation, the Hadamard gate, H. As you can see, this changes the basis from the 0/1 basis to the +/- basis that we discussed earlier. Put in a 0, get out a +, put in a 1, get out a -. There is another gate, called the phase gate, which only multiplies the one state by a factor of i, changing the basis in a very subtle way. Now let's take a look at a few measurement operators.

Operations & Observables

unitary operations

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |1\rangle\langle 0| + |0\rangle\langle 1| \quad X|0\rangle = |1\rangle \;\; X|1\rangle = |0\rangle$$

$$H = \tfrac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = |+\rangle\langle 0| + |-\rangle\langle 1| \quad H|0\rangle = |+\rangle \;\; H|1\rangle = |-\rangle$$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = |0\rangle\langle 0| + i|1\rangle\langle 1| \quad P|0\rangle = |0\rangle \;\; P|1\rangle = i|1\rangle$$

Ben Criger! ,Ket Notation

In an interesting coincidence, the Pauli X shows up again, since it's both unitary and hermitian, we can use it both as an operation and a measurement. Here, we have decomposed it into its eigenbasis, so the ket-bra terms are different than before, and we can see that its output values are +/- 1.

Operations & Observables

hermitian observables

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |+\rangle\langle+| - |-\rangle\langle-| \quad \langle+|X|+\rangle = 1 \;\; \langle-|X|-\rangle = -1$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1| \quad \langle 0|Z|0\rangle = 1 \;\; \langle 1|Z|1\rangle = -1$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{matrix} |0\rangle\langle 0| + |1\rangle\langle 1| \\ \text{or} \\ |+\rangle\langle+| + |-\rangle\langle-| \end{matrix} \quad \langle\psi|I|\psi\rangle = 1 \text{ for all } |\psi\rangle$$

<u>Ben Criger! ,Ket Notation</u>

Here's another matrix which is both unitary and hermitian, the pauli Z. It returns the exact same values as X, just for states in the 0/1 basis instead of the +/- basis. And finally we have the identity, which is also unitary and hermitian. We can note that the identity doesn't change the basis at all, so that indeed it can be expressed in any basis. Also when used as a measurement operator, it always returns 1, no matter what state is input. Now that we have a few examples done, I would like to focus a little on a useful geometric representation of qubit states, called the Bloch Sphere.

The Bloch Sphere

coefficients in polar co-ordinates

$$|\psi\rangle = r_0 e^{i\phi_0}|0\rangle + r_1 e^{i(\phi_0+\phi)}|1\rangle$$

global phases don't matter

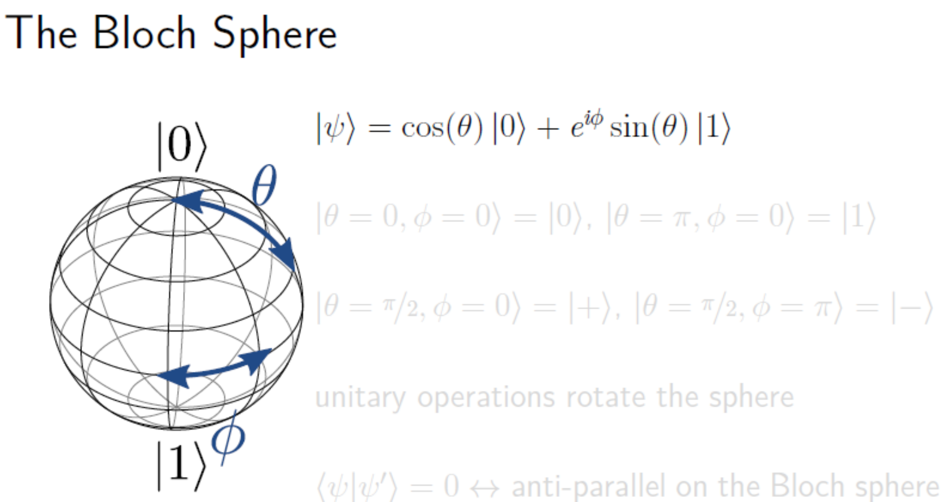
$$|\psi\rangle \mapsto e^{i\alpha}|\psi\rangle$$
$$\langle\psi|A|\psi\rangle \mapsto \langle\psi|e^{-i\alpha}Ae^{i\alpha}|\psi\rangle = \langle\psi|A|\psi\rangle$$
$$|\psi\rangle = r_0|0\rangle + r_1 e^{i\phi}|1\rangle$$

normalization constrains qubits' states further

$$\langle\psi|\psi\rangle = r_0^2 + r_1^2 \equiv 1$$
$$\therefore |\psi\rangle = \cos(\theta)|0\rangle + e^{i\phi}\sin(\theta)|1\rangle$$

<u>Ben Criger! ,Ket Notation</u>

To show how qubit states can be mapped to the surface of a sphere, let's start by expressing the coefficients alpha and beta in polar co-ordinates. Next, we can note that we can get rid of the phase on the alpha coefficient, since a ket multiplied by a phase produces the same measurement results as the ket itself, for any potential experiment, so these are not different in any physical sense. This results in a simpler expression for the state, but it's not yet as simple as it can be. To get it even simpeler, we need to recall that these states have to be normalized, so the sum of the squares of the two radii involved has to be 1. This implies that we can express them as the cosine and sine of an angle theta, since cos(theta)^2 + sin(theta)^2 is always 1. This family of angles theta and phi also describes the set of points on the surface of a sphere of unit radius in 3d space. We can see that, setting theta to 0, and phi to whatever we want (here it's 0), we get cos(theta) = 1, sin(theta) = 0, so the corresponding state is simply the 0 basis state. If we set theta to pi however, we get the 1 state regardless of the setting of phi. Likewise if we set theta to pi/2, and phi to either 0 or pi, we get one of the states from the +/- basis on the equator of the sphere.Unitary operations, which change the basis we are working in, effectively rotate the sphere.



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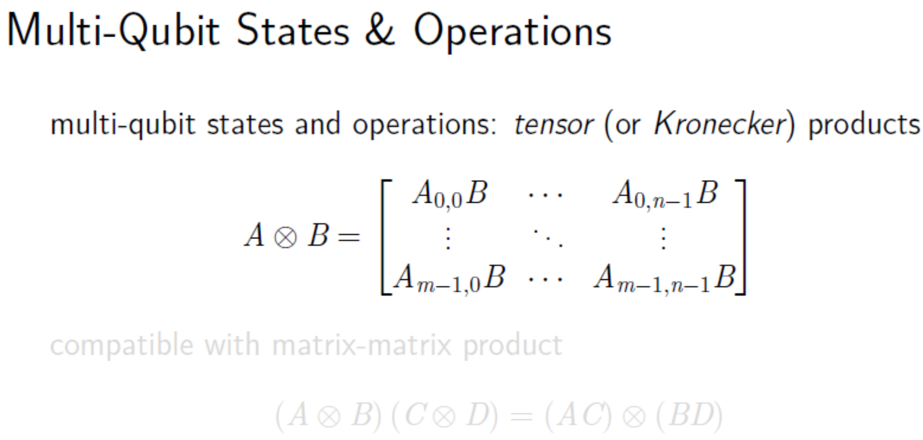
For example, the Hadamard operation from earlier rotates the +/- states on the equator by 90 degrees until they're at the poles. Also note that the states on opposite sides of the sphere are actually orthogonal, so this mapping does not preserve the angle between states, but it's still useful for describing single-qubit states and operations. But how do we describe multi-qubit states and operations?



Ben Criger! ,Ket Notation

Specifically, how do we build them up from operations on smaller subsystems of a many-qubit state? To accomplish this, we use a different kind of matrix product, called the tensor product, or Kronecker product. To take the tensor

product of two matrices A and B, we write out a block matrix where each block is equal to B times the appropriate element of A. The upper left block is B times the upper left element of A, and so on.
The interesting thing about the tensor product is that it's compatible with the regular matrix product.

Multi-Qubit States & Operations

$$|+\rangle \otimes |0\rangle = |+0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$X \otimes I = \begin{bmatrix} 0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ 1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

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That is to say if I take the matrix product of two tensor products, I get the same matrix as if I took the matrix products first, then evaluated the tensor product. This is also easier to understand if we take a look at a few examples.

Example: Bell State Preparation

controlled-NOT: $\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

Bell state: $|\Omega\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Bell states are *entangled*: cannot be written as tensor product

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\text{CNOT} \times (H \otimes I)(|0\rangle \otimes |0\rangle)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$H \otimes I(|0\rangle \otimes |0\rangle) = |+\rangle\, |0\rangle = |+0\rangle = \frac{1}{\sqrt{2}}\, |00\rangle + |10\rangle$$

$$\begin{aligned}
|\Omega\rangle &= \textsc{cnot} \times \left(\tfrac{1}{\sqrt{2}} \left(|00\rangle + |10\rangle \right) \right) \\
&= \tfrac{1}{\sqrt{2}} \big(|0\rangle \langle 0|0\rangle \otimes I |0\rangle + |0\rangle \langle 0|1\rangle \otimes I |0\rangle \\
&\qquad\qquad + |1\rangle \langle 1|0\rangle \otimes X |0\rangle + |1\rangle \langle 1|1\rangle \otimes X |0\rangle \big) \\
&= \tfrac{1}{\sqrt{2}} (|00\rangle + |11\rangle)
\end{aligned}$$

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Here we see the tensor product of two states, + and 0, resulting in a state which we often call +0. Each 2-by-1 block of the state vector we're calculating is proportional to the vector for the zero state, and it's multiplied by one of the elements of the plus state. We can also take tensor products of operations and observables, here we take the tensor product of X and the identity, making a block matrix whose blocks are all equal to the identity, multiplied by one of the elements of X. Now that we have seen the basic formalism and notation, we're ready to calculate the results of a small sequence of quantum operations, which prepares a Bell state. First, we introduce a two-qubit gate which cannot be written as a tensor product of one-qubit gates, the controlled not, or cnot for short. As we can see, it can be decomposed into a sum of two tensor products that says if the first qubit is in the zero state, do the identity, and if the first qubit is in the one state, perform an X on the second qubit. Here we see the Bell state, which we're going to prepare using the cnot. We can write it out using either ket notation, or as a column vector. Note that the Bell state, just like the cnot, cannot be written as a tensor product. The first step in preparing the Bell state is to perform a Hadamard operation on the first qubit, which has been initiliazed in the zero state. So we calculate the tensor product of the Hadamard with the identity, since we're not going to do anything to the second qubit immediately. Now we're ready to set up our sequence of operations. First, we prepare the state 00, then we apply the Hadamard, then the cnot. This results in a Bell state, which is written out here as a column vector. Now this is the correct answer, but it was a little tedious to come to, and the matrices involved are a little large. 3x3 matrices are typically big enough for any phisicist, so 4x4 is overdoing it a little. Let's try and do it the easy way, using some more ket notation. First, we use the compatibility of the Kronecker and regular products to show that the state we get from executing the Hadamard on the first qubit is simply +0, without having to use any matrices. Now, all we have to show is that the cnot will take our state, which is 00 + 10, to the 00 + 11 state that we're after.

If we insert the decomposition of the cnot into tensor products that we saw earlier, we can see that the resulting state has coefficients proportional to these inner products of the 0 and 1 states. Of course, the 0/1 basis is orthonormal, so the products 00 and 11 evaluate to 1, and the products 01 and 10 evaluate to 0, leaving us with two remaining terms. These terms are exactly the 00 and 11 terms that we were after. And that's how ket notation van help us to describe the effects of operations and measurements on quantum states.

Main takeaways

- **In ket notation, a quantum state is expressed using a column vector with complex coefficients. The squared magnitudes of these coefficients are the probabilities of measuring that particular outcome.**
- **Devices used for storing and manipulating qubit states are described by a Hamiltonian, which is a matrix that assigns energies to each of its eigenstates or preferred basis states.**
- **Quantum logic gates are described by unitary matrices. Probabilistic measurements are described by Hermitian matrices.**
- **An entangled state is a multi-qubit state that *cannot* be written as a tensor product of one-qubit states.**
- **Qubit states are geometrically represented using the Bloch sphere.**
- **Multi-qubit states and operations can be described using the tensor product. Note, an actual two-qubit gate *cannot* be described with a tensor product.**

Advantages and disadvantages of Ket notation

In the lecture on ket notation we discussed three ways to represent quantum states: column vectors with complex coefficients, kets themselves and the geometric representation, which uses the Bloch sphere. Each one of these has their own little advantages and disadvantages, which we are going to see in the course of these sections. First, let's take a look at some of the advantages of ket notation. So here's ket notation. Is very nice for sparse states. That is to say: states which don't have that many non-zero entries in

some bases. Take for example this state here, which I will call psi, along with most other states. It has got two nonzero terms. There is a one over root two term for the component in 00000 and a one over root two term for the all-ones as well. That would be very cumbersome to write out in a full vector, because that would require 2^5 or 32 entries, but here you can do it very compactly. Now, if you don't have a compact state like this, if you just have some arbitrary coefficients, it can be very painstaking and time-consuming to write out something like this. Beta times 01 plus gamma times 10 plus delta times 11. Here you would be better of to use something more like a column vector. Alpha, beta, gamma, delta. And in some other times, especially if you're considering operations - transformations on the set of states – it can be beneficial to consider the Bloch sphere, where you have the reference state zero, some other states one, plus, minus and like so.

Synthesising rotations

I'd like to show you an example of something that's a little bit difficult to do when looking at the matrix based picture of quantum mechanics, but if we put everything on the Bloch sphere it makes perfect sense. In many experiments, experimentalists can only rotate along two axes of the Bloch sphere. The x-axis, which goes in this direction and the y-axis, which is going in this way. Many algorithms rely on rotations around the z-axis, which is independent from these other two. And we would like to see how to synthesize the z-rotations out of the tools that we have available: x and y rotations. Now, I happen to know the answer to this question and I can begin by writing down the matrices that describe the operations that we would like to do. So, here is z and I'll say this is z(theta). For those of you who are really in the know, this is e^{i*z*theta}. But if this doesn't make sense to you, then this is just a diagonal matrix with e^{i*theta/2} and e^{-i*theta/2} on the diagonal, zeros off the diagonal. And we can see immediately that x and y rotations on their own aren't going to give us this z-rotation.So we can write down x theta and y theta, which are just rotations around these axes by angles theta. E^{ix*theta/2}, e^{i*y*theta/2}. And this guy is equal to cosine of theta over two minus i sine of theta over two, minus i sine theta over two cosine theta over two. E^{i*y*theta/2} is quite similar. Cosine theta over two, sine theta over two, minus sine theta over two, cosine theta over two. Now, there's a product of these rotations. A y an x and a y, that will excecute one of these sets.
Lets look at those matrix rotations first. What I am going to do is set theta, well, equal to pi over two for a y rotation. And that's going to give me one over root two, one one, minus one, one. Y theta equal to minus pi over 2 will give me operations that look like one, minus one, one, one. You might recognize these to be quite similar to the Hadamard basis that changes from the computational basis to the plus minus basis, and that is by design. Now lets take a look at what happens if we sandwich an x rotations around theta with these two matrices, which we lovingly call y 90 and y -90. Because they are 90 degree rotations. So, we're going to get a, well, y pi by 2 x theta y minus pi 2. Is equal to 1 over root two one, minus one, one, one, cosine theta over two, minus i sine theta over two, minus i sine theta over two, cosine of theta over two. Then we have another one over root two, one, minus one, one, one. We multiply these through. So, first off I can take these two factors of one over square root of two, make them a factor of a half. One minus one one one. Then if I multiply these here, I get cos theta over two plus I sine theta over two, because these two minus signs cancel. So that's cos theta over two plus I sine theta over two. Here I'll get cosine theta over two minus I sine theta over two. Here I'll get minus cosine, minus I sine. So that's minus I cosine theta plus I sine theta over two. And then here I'll get cosine minus I sine. So, cosine theta over two minus I sine theta over two. Continuing on with all this math I can notice that these two elements are equal and opposite. So If I take one and minus one, I'll get cosine plus I sine, minus negative cosine plus I sine. Which is simply

two cosine theta over two plus I sine theta over two. Here I take a number and it's opposite, I get zero. Here I can take these two numbers which are equal and add them. That gives me theta over two minus I sine theta over two. And if I take one and minus one and these two I get zero as well. That's a lot of matrix multiplication. Now, let's also use Eulers identity. We know that this is equal to e^{i*theta/2} and that this is equal to e^{-i*Theta/2}. From arithmetic using complex numbers. And so we have managed to synthesize this z rotation around theta just by using x rotations and y rotations. So by using a little bit of ket notation and some trigonometric identities, we can show that we can obtain universal control using only the tools that are available to the experimentalists. This is nice because it means we don't have to design new hardware. We can just put together operations
that we already have in order to obtain universal control. But this is not that simple as it could be. There is a far easier way to see this, which we can do using the Bloch sphere. The action of this sequence of unitary operations. Y 90, x around some angle theta and then y minus 90 can be expressed in terms of matrices. But it's much easier to see using the Bloch sphere. If we write down the Bloch sphere with the x axis pointing out of the board, we can see the effect of a y 90, a rotation around 90 degrees of the y axis, is just turning the sphere sort of like a steering wheel, so that the x axis now faces down and the z axis is where the x axis used to be. If we now apply a rotation around the x axis, that is giving us a rotation around our old z axis. And once we have that done, all we have to do is reverse the y 90 with a y minus 90 to restore the orientation of our original axes, having picked up a phase on the z. And that's how we can synthesize a diagonal unitary. e^{itheta/2}, e^{-i*theta/2} out of the operations to which the experimentalists have access. A little bit of notations saves you a lot of hardware design.

State decomposition

Often in quantum computation, it is necessary to express one state in terms of a basis of some other states. I am going to go ahead and call this state decomposition, although you can see it under a variety of names and usually it's clear from the context that they mean expressing one state in terms of a basis that is more familiar or convenient to. Now the rule of state decomposition is that any state phi can be expressed as a sum over k terms, where each term consists of an inner product of phi - the trial state – with one of the basis states phi_k, and the states phi_k. Number times vector. And this is the same as decomposing a vector in a basis if you are already familiar with linear algebra. As an example of this, let's take a look at a trial wave function of zero, and a basis which is the Hadamard basis which we reviewed in the lecture. The sum over psi_k, phi, psi_k is then the inner product of plus and zero times plus and the inner product of minus and zero times minus. Now here I figured out those inner product. The inner product of plus zero is one over root two times the dot product of these two vectors, which is just one

over root two. And the calculation for minus zero is very similar, there is a minus sign here, but it gets multiplied by
zero, making no difference, and we end up with one over root two again. Therefore, zero can be decomposed as one over root two, plus + minus, just as the plus state can be decomposed as one over root two, zero + one.

Performing arbitrary measurements

I'd like to talk today about how a little bit of ket-notation can save us a big experimental headache, when it comes time to try measure a qubit in an arbitrary basis. First, we have to review a few of the important facts. The conjugate transpose of a matrix or a vector is given by first taking the transpose of this 2- by-2 matrix that leaves the diagonal elements in the same place, and exchanges the elements being c (and b) here, but also taking the complex conjugate of each element in the matrix. A second important fact, which you can prove yourself using this definition, and I encourage you to do so, is that the dagger or complex conjugate transpose of U- psi (ket) is just the bra for psi times U-dagger. Now, that follows from linear algebra, but it is easy to check for yourself. There's also another construction that I'd like to use, which is that for every state psi, there is another state which I call psi-perp(endicular), which consists of these coefficients. So, if psi is alpha-beta, psi-perp is the conjugate of beta and minus the conjugate of alpha. And these states are by construction orthogonal, which you can also see just by taking the inner product and seeing that it's always zero regardless of what alpha and beta are. And with those three ingredients, we can solve this question. So, experimentalists usually measure a single operator: the Pauli-Z operator, that was perhaps discussed earlier. But we'd like to be able to measure in whatever basis we want. So that if we have a question: "Is this state this or that?", we can answer it without having to restrict ourselves to the 0-1 basis. So, we have a two-step plan to solve this. First, we're going to apply some operator U, and then we're going to measure in the 0-1 basis. Now, if you've got a controllable qubit, you can apply an operator U as you see fit, and as discussed in the statement of the question, we can measure in the 0-1 basis. We can measure the Z-operator. So, the expectation value of this measurement, if we first apply U on psi, that replaces psi with psi-U-dagger in the bra and Upsi in the ket, and then we take the familiar sandwich product to determine the expectation value of the operator, we end up with psi-U-dagger-Z-U-psi.

So, it's as if we had our original state psi, and instead of measuring Z, we measured U-dagger- Z-U. Now, U-dagger-Z-U we can write out like so. And if we label the state U-dagger-0 as psi, then U-dagger-Z-U is equal to psi-psi minus psi-perp-psi-perp. This is a measurement that will return 1 if the state is psi, and -1 if the state is psi-perp, giving us an arbitrary basis to measure in.

A fact about maximally entangled states

Often when doing quantum information, ket-notation can fail you. This happens a lot when you have a densely packed vector full of a bunch of different coefficients and there's no obvious structure. Now, hopefully when that happens to you, you're only going to deal with a few qubits. Otherwise, you are going to writing out coefficients for a very long time. Here, we can see a not too bad example with two qubits where we're going to prove that for any maximally entangled state of the form psi-star-psi plus psi-perp-star-psi-perp, it's always equal to the Bell state that's fully correlated that we know and love: 1 over root 2 (times) 0-0 plus 1-1. And this is regardless of what psi is. So, if we take an arbitrary wavevector psi which is two complex coefficients alpha and beta, we can define psi-star, which is just the complex conjugate of that state, which is also a valid state, psi-perp, which is some state which is orthogonal to psi, which I encourage you to check for yourself by taking the inner product, and psi-perp-star, which is the complex conjugate of the orthogonal state. And if we write out all of our tensor products using the formula that we learned earlier, we obtain a pair of densely packed vector full of those coefficients, which would be very awkward in ket-notation. But we have here alpha-star-alpha, alpha-star-beta, beta-star-alpha and betastar- beta. And we're going to add to that: beta-beta-star, minus beta-alpha-star, minus alphabeta- star and alpha-alpha-star.Now, beta-star-beta plus alpha-alpha-star is just the magnitude of alpha squared plus the magnitude of beta squared, which is 1. And we see that same thing in the top term here,alpha-star-alpha plus beta-beta-star, that's 1. And then these inner terms cancel. Because you have alpha-star-beta minus beta-alpha-star, that has just been flipped here. And (for) betastar-alpha minus alpha-beta-star, if I flipped these two, it becomes obvious that they are equal and opposite, so they cancel.This implies for example, that for example if Alice and Bob are a Bell state, and Alice measure in a 0-1 basis, she can get a state 0 or 1 and she can tell that Bob has the same state. Now she measures instead in the basis psi-star psi-star-perp, she can tell that Bob has the state psi or psi-perp depending on her measurement result.

Experimental and theoretical measurements

I'd like to this opportunity to clarify something that confuses a lot of students when they first learn about quantum mechanics, which is the difference between measurements that occur in a laboratory and measurements that we describe in theory. And also, I'd like to point out hopefully the ways in which these things are similar So, you may have already seen a picture like this one. My apologies to my experimental friends if it's not exactly the same. But, in the laboratories you can measure some currents in nanoamperes, and if there's a spin in the down state pointing parallel to the field, there will be no bump in the current. And if it's in the up state, there will be a bump. And this is a large macroscopic classical signal that they can detect. And this relates to a measurement operator. You can imagine that we take the total area under this curve; the total amount of current, in the case that there's no bump, and that corresponds to the spin down state. And the total current in the case that there is a bump will be slightly higher, and that corresponds to the spin up state. So, that real number: the amount of current times the projector on to the down state plus the total amount of current in the case that there is bump times the projector on to the up state gives us our measurement operator. Now, the important thing about these two signals being distinguishable, but there being a large difference between them, is that if they're equal, we end up with some number times down-down plus some number which is identical, times up-up. So, the measurement operator becomes the identity, which doesn't discern between the up and down states. So as useless is the measurement. But if these numbers are discernably different, then down-down and up-up receive different measurement outcomes, and out measurement operator is more like a Z, which is one we could use to actually distinguish between the computational basis states in quantum computing.

Practice Quiz 3

QUESTION 1: HOW MUCH ENTANGLEMENT IS IN THAT STATE?

Suppose we are given a pure, two-qubit state

$$|\psi\rangle_{AB}=\alpha_{00}|00\rangle+\alpha_{01}|01\rangle+\alpha_{10}|10\rangle+\alpha_{11}|11\rangle,$$

where $|\alpha_{00}|^2+|\alpha_{01}|^2+|\alpha_{10}|^2+|\alpha_{11}|^2=1$. We can compute the amount of entanglement between the two qubits, as quantified by the entanglement measure known as the concurrence, using the following expression:

$$C(|\psi\rangle_{AB})=2|\alpha_{00}\alpha_{11}-\alpha_{01}\alpha_{10}|.$$

If the two qubits are maximally entangled, then $C(|\psi\rangle)=1$; and if the qubits are not entangled at all, or are in a product state, then $C(|\psi\rangle)=0$.

Use the above expression to compute the amount of entanglement between the qubits in each of the following states:

$|\psi\rangle_{AB}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$.

$|\psi\rangle_{AB}=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$.

$|\psi\rangle_{AB}=|00\rangle$.

$|\psi\rangle_{AB}=\sqrt{\frac{48}{100}}|00\rangle+\sqrt{\frac{48}{100}}|11\rangle+\sqrt{\frac{4}{100}}|10\rangle$.

Learn more

BUILDING BLOCKS

MORE ON THE ARCHITECTURE OF A HETEROGENEOUS QUANTUM COMPUTER

HTTPS://DOI.ORG/10.1145/2903150.2906827

KET NOTATION

TO LEARN MORE ABOUT THE KET NOTATION AND TO GET MORE ACQUAINTED WITH IT:

QUANTUM COMPUTATION AND QUANTUM INFORMATION BY NIELSEN AND CHUANG, SECTIONS 1.2, 1.3, 2.1-2.1.4, 2.2.1-2.2.3

Module 2

In the first module we discussed all the basic building blocks of a quantum computer. It is now time to dive deeper into the block at the bottom of the stack: the quantum chip! Qubits can be realized in many different physical platforms, and in the next week's we will introduce you to some of these realizations.

In this module, we will start with an introduction about electron spin qubits realized in semiconductor quantum dots. These qubits highly resemble the classical semiconductor transistors that we use everyday in almost every electronic equipment. This unique characteristic has attracted a lot of attention from the most important semiconductor companies. Check out this article about Intel's vision about spin qubits.

Lieven Vandersypen, Professor in applied physics, will explain how we can control single electrons and isolate them in semiconducting islands called quantum dots. Once we have isolated single electrons, we need to be able to manipulate their spins and make those spins interact with each other in a controllable manner. This will be the topic of the second lecture, given by Menno Veldhorst.

This module contains 2 lectures and 2 quizzes. Good luck and have fun!

Spin qubits

When we look back at the development of semi-conducting technology, a great moment was in 1947 -the demonstration of the first transistor. Shortly after that, people began to put several transistors together on a circuit with capacitors and resistors; and developed the first transistor radio. Even though this technology was quite remarkable, it clearly had its limits; how many components could you really solder together by hand on a board? So a second momentous step was in 1958 -the first demonstration of an integrated circuit, where all of the relevant components were integrated monolithically in the same piece of germanium or silicon, and that's led to the remarkably powerful and complex processors that we use in computers and memory chips today. Now, based on this very same core technology of transistors and integrated circuits, we have found in the last decade how to build some of the highest-quality quantum bits. If we look at how a transistor is built, essentially, it is a switch that controls the flow of electrons between two contacts using the voltage applied to a single gate. If we now imagine that we replace the one gate by multiple gates side-by-side separating the contacts, then with the gates we can locally pull in electrons or push away electrons, depending on the polarity of the voltage that we apply to the gates. In this way it is actually possible to isolate small puddles of electrons from the rest of the world. A quantum dot is the small space where electrons are pulled in below one gate separated by other puddles of electrons. As a community, it's now become routine to go to the extreme limit, where below each of these electrodes, just

one single electron is isolated; and the spin of that electron is going to be our quantum bit. So how in the world can we isolate individual electrons and control them? It starts with the notion of the charging energy, that is the energy that results from the Coulomb repulsion between electrons. As we add charges to the island, this costs energy, and if you go to very small capacitances, it turns out that the charging energy, the energy required to add one single electron charge, can be larger than the thermal energy. To give some examples, for a small island with a radius of a 100 nanometers, the charging energy is 3 meV. To put that number into perspective, the thermal energy at 4K is ten times less than this. So at 4K it is actually possible to control the number of charges on these islands one-by-one. How do we know that individual charges are being added to the island? The standard method is to look at the current that flows between the contacts through the quantum dot –the central island. In this schematic, we see a set of lines, called electrochemical potential lines, that can be pushed up or pulled down by the gate voltage as a ladder, and each line indicates the energy needed to add the next electron to the island. What we see in the configuration on the left is that this energy is larger than the energy of the highest occupied state in the reservoirs, called the Fermi energy, so the electrons in the reservoir don't have enough energy to go into the quantum dot. Furthermore, the line at the dot below it is lower than the Fermi energy in both reservoirs so no electron can leave the
dot. In other words, electrons cannot be removed from the island and electrons cannot be added to the island; current is blocked. We call this Coulomb blockade. In the configuration on the right, we have adjusted the gate voltage in such a way that the ladder comes down, and the electrochemical potential μ(N) lies exactly within the window between the source and drain Fermi energies or electrochemical potentials. In this condition, if an electron can move from the source into the island, then from the island it can move out to the drain. However, before the first electron leaves, no second electron can enter. So individual electrons are really being added one by one as they pass through the quantum dot. But altogether, these many electrons moving through one by one, do produce a measurable current. The current flows whenever the electrochemical potential lies within this biased window between source and drain. We see this beautifully in measurements as sharp peaks in the conductors and current through the quantum dot for specific gate voltages where we have reached an alignment as shown in the schematic on the right, and for the intermediate gate voltages, the current is blocked.A second important method to measure and detect the presence or absence of individual electrons is what we call charge sensing. Basically, if you imagine a single quantum dot and another quantum dot next to it, as you have seen, the current through the second quantum dot sensitively depends on the gate voltage; with a small change in gate voltage we can produce a large current or completely shut off the current. Now imagine a second quantum dot is placed next to the first. It turns out that a single charge

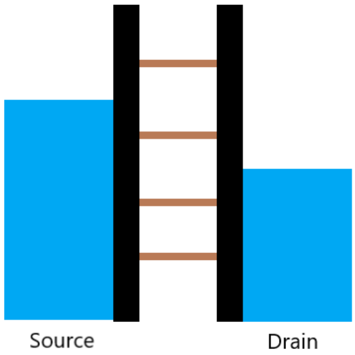
added to the second quantum dot acts like a small shifting gate voltage through capacitive coupling, it shifts the position of the levels in the first quantum dot. So when a single charge is added to one quantum dot, the current through a neighbouring quantum dot is changed in a measurable way.Let’s look at an example of two quantum dots. In this case we need an additional gate in between the two quantum dots to control their coupling. By lowering the voltage on this tunnel barrier gate we can tune the electrical potentialin such way that the two electrons are well isolated from each other. If we draw a gate voltage space where the gate voltage that controls the potential of one quantum dot is plotted on the horizontal axis, and the gate voltage that controls the second quantum dot on the vertical axis; then, if the two quantum dots are uncoupled, for specific voltages on the first gate electrode electrons are added to the first quantum dot. These transitions are represented by the vertical lines in the plot. Similarly, for specific voltages on the second gate electrode, electrons are added one at a time to the second quantum dot. These are the horizontal lines. If we now consider two quantum dots in each other's vicinity, two effects will happen. The first is that crosstalk. The voltage applied to the first gate electrode also affects the potential of the second quantum dot, which sits to its side; and vice versa. That's why, the lines that were vertical and horizontal before, are now aligned at an angle.

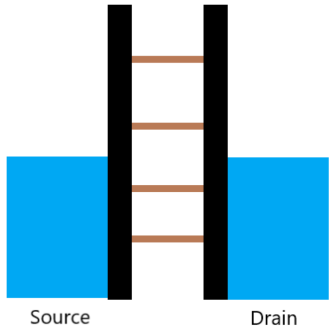
Main takeaways

- **In quantum dots, single electrons can be confined to be used as quantum bits.**
- **Spin qubits have to be operated at sufficiently low temperatures, such that the charging energy exceeds the thermal energy.**
- **The coulomb blockade prevents electrons to enter or leave the dot when the electrochemical potential of the quantum dot does not match the Fermi energy.**
- **Neighbouring dots experience inter-dot capacitance and crosstalk.**

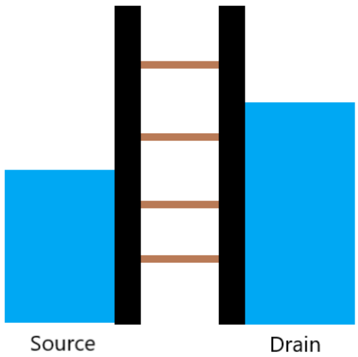
Practice Quiz 4

In one of the sections, Lieven used a “ladder” diagram to describe situations in which electrons from the reservoir can and cannot jump on the quantum dot. This ladder picture is often handy when thinking about how to induce and stop current from going through the quantum dot. Below are a few exercises to help you make sure you are comfortable with this picture.





A) B) C)



Question 1

In which of the situations above will current flow from left (source) to right (drain)? Remember that current flows in the opposite direction as electrons flow.

- ○ A
- ○ B
- ○ C

In which of the following situations will current flow from right (drain) to left (source)?

- ○ A
- ○ B
- ○ C

IN WHICH OF THE FOLLOWING SITUATIONS WILL THERE BE NO CURRENT?

- A
- B
- C

SOME PROBLEMS HAVE OPTIONS SUCH AS SAVE, RESET, HINTS, OR SHOW ANSWER. THESE OPTIONS FOLLOW THE SUBMIT BUTTON.

QUESTION 2: INDUSTRIAL INVOLVEMENT

IF WE WANT TO BUILD LARGE-SCALE QUANTUM COMPUTERS, WE MIGHT NEED MILLIONS OR EVEN BILLIONS OF THESE QUANTUM DOTS. WHY DOES AN INDUSTRY INVOLVEMENT BECOME FUNDAMENTAL?

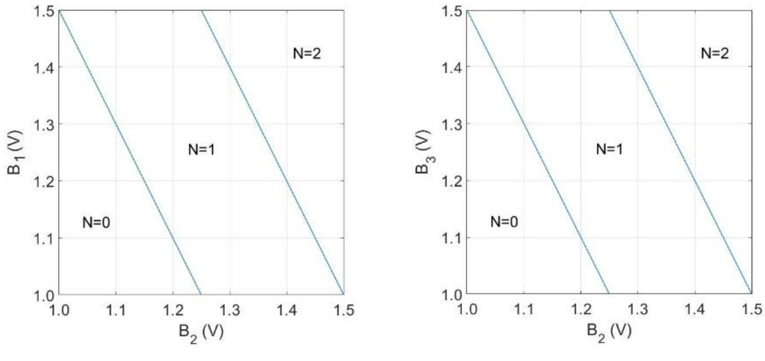
MARK THE 3 BOXES THAT APPLY.

- [ ] TO INCREASE DEVICE YIELD.
- [ ] TO IMPROVE QUANTUM DOT UNIFORMITY.
- [ ] INDUSTRIAL CLEANROOMS GIVE US A GREATER FLEXIBILITY TO EXPLORE NEW IDEAS.
- [ ] TO PRODUCE SUBSTRATES OF HIGHER QUALITY.
- [ ] ISOLATE SINGLE ELECTRONS IS NOT POSSIBLE IN UNIVERSITY CLEANROOMS DUE TO LOW QUALITY SUBSTRATES .

QUIZ 4: INTRODUCING SPIN QUBITS

Charge stability diagrams

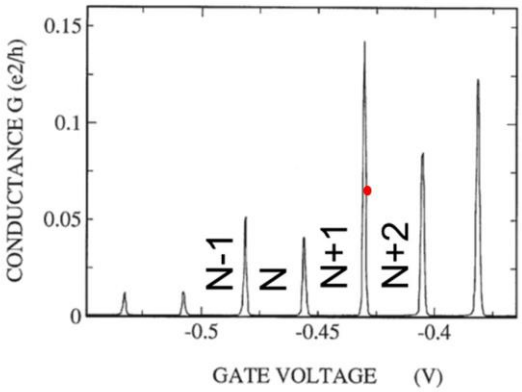
In the section, Lieven explained how the slope of charge transitions in a charge stability diagram depends on the coupling with nearby gates. In the figures below we see a schematic of two charge stability diagrams where we sense transitions originating from a single quantum dot coupled to three nearby gates B1,B2 and B3. The quantum dot is located under the gate with the strongest capacitive coupling.



Charge sensing

A quantum dot is also a very sensitive electrometer. Since electrons carry a negative charge, any movement will change the electrical potential of the environment near the quantum dot and they can therefore be sensed.

Let's suppose that we have two weakly coupled quantum dots close to each other and we want to use the first to sense the charge transitions happening in the second quantum dot. In order to do that, we tune the first quantum dot in a regime of high sensitivity (see red circle in the figure below).



Operations on spin qubits

Operations on spin qubits With standard semiconductor technology, billions of transistors can be integrated on a single chip. This forms one of the key motivations for quantum dot qubits, as these qubit types are fabricated using the same technology, such that one can envision billions of quantum bits on a chip. In this lecture we focus on the operation of these quantum dot qubits. We will discuss how qubits are defined, how they can be initialized and readout, how the qubits can be controlled and how these qubits can be coupled to one another to execute two--qubit logic gates. We start with an empty quantum dot connected to an electron reservoir. The quantum dot energy levels are above the Fermi energy of the reservoir and so no electrons can tunnel from the reservoir to the dot. We define our qubit states on the spin states of the quantum dot. To do so, we apply a magnetic field on the order of a Tesla. Due to the Zeeman energy, the spin states are then split by about hundred micro electron volt, assuming a g--factor around 2. The lowest level corresponds then to the state spin down, while the upper level corresponds to the state spin up. We can initialize our quantum dot in the state spin down, by simply lowering the energy level such that the state spin down is below the Fermi energy, while the state spin up is still above the Fermi energy. At this position, only an electron with state spin down can tunnel from the reservoir to thequantum dot. If we pulse the levels even deeper, the quantum dot will remain in the state spin down. No electrons can tunnel from the reservoir to the quantum dot, since it is already occupied, and thus the state remains spin down. If we want to readout the state, we can simply do the reverse protocol. We align the levels such that the Fermi energy of the reservoir is in between the spin down and spin up level. An electron will only tunnel out if the state is spin up.

This sequence converts a spin in to charge, because there is only charge movement if the state is spin up. As quantum dot systems can also be used as highly sensitive and accurate electrometers, we can readout the spin state by measuring the charge transfer during this protocol. In the graphbelow we see the current through a quantum dot electrometer. We see that that there are different current levels. These differences correspond to the different gate voltages that are applied to align the electron reservoir and quantum dot levels. The middle level corresponds to the alignment for readout. In some cases we see that there is a small bump around 1ms and in some case we don't see it. This corresponds to the spin--to--charge conversion. The experiment shown here is performed in GaAs. Electrons in GaAs have a negative g--factor. A good exercise is to verify yourself that in this particular case a bump corresponds to the state spin down and not to the state spin up, as one may expect. This method of readout is called Elzerman readout and has been one of the central methods for readout of spin qubits. Now that we know how to initialize and readout the spin state, we can start controlling the state and turn it into a qubit. Qubit control can be realized by introducing an alternating magnetic field in a direction perpendicular to the applied magnetic field. This is done by applying an ac current through a small strip close to the quantum dot system. The AC current generates an electromagnetic wave. When the frequency of this wave matches the resonance frequency of the qubit, photons are generated with an energy equal to the Zeeman energy of the spin states and they can flip the spin state. If we introduce this alternating magnetic field then we will observe that the spin starts to rotate as a function of time. These oscillations are called Rabi oscillations and they form the basis of our single qubit rotations.

This mechanism has been demonstrated with quantum dot systems and first in the material GaAs and in the figure you see that as a function of time, coherent oscillations appear with a frequency dependent on the applied electromagnetic power. Unfortunately, these oscillations do not survive until infinity. The oscillations decay due to qubit decoherence. A major source of decoherence for spin qubits in GaAs is the interaction between the electron spin and nuclear spins in the environment. GaAs has many isotopes with a nonzero nuclear spin and these can interact with the qubit through the hyperfine interaction. These interactions vary over time. Consequently, the qubit frequency changes over time and so the emitted photons from the stripline are not always in resonance with the electron spin. This is a major limitation, but fortunately, it can be solved by simply changing the host material. Silicon can be purified to an isotope with zero nuclear spin, Silicon--28. As one can see, in this material one can just simply keep on going with rotating the electron spin. This is of course very favourable for qubit operations. The resulting improvement becomes even more apparent if we compare the coherence time of electron spins in different materials. In this figure, you see three different sequences to determine the quantum coherence of a qubit. The simplest sequence is the Ramsey experiment. It starts by rotating the spin to the equator using a pi/2 pulse. Then, another pi/2 pulse is applied to rotate it to the state spin up. In the experiment, the waiting time in between these two pulses is varied. This results typically in an exponential decay, where the time constant in the exponent defines the coherence time. The Hahn echo and CPMG sequences are more sophisticated sequences that can extend the coherence time and they probe the ultimate quantum coherence.

From the figure it becomes clear that the quantum coherencecan be greatly improved by both using sophisticated sequences and by using materials with zero nuclear spins, resulting in a maximum coherence time of tens of milliseconds. And this is an eternity in the quantum world. Now that we know how to initialize and read and control qubits, we can start to couple qubits together. To do so, we use the exchange interaction. This interaction can be controlled by tuning the electric gates of a quantum dot qubit, which modulates the potential landscape. To turn the interaction on, the electric gates are pulsed such that the energy barrier between the qubits is short and low. Consequently, the wave function of the two electron spins start to overlap and they hybridize. In the figure on the left, there is no interaction. In the figure on the right there is interaction. Now what is crucial here, is that when the interaction is on, the resonance frequency of one qubit is determined by the state of the other qubit. In the left figure, we see that the energy in flipping the red qubit is independent on the state of the blue qubit. It is good to verify this yourself. If we overlay these figures it becomes clear that the scenario is different when interaction is on. When interaction is on, flipping the red qubit costs less energy when the blue qubit is in the state spin down, while it costs more energy when it is in the state spin up. This is very important, as we can now control one qubit depending on the state of the other qubit, and use this, for example to create quantum entanglement. One can also see from the figure that we have two knobs that we can use for control. First, we can directly control the tunnel barrier. Secondly, we can also change the relative energies of the electron spin states. By moving towards the state where one electron spin is almost doubly occupied, the exchange interaction increases.

The first approach where one controls the tunnel barrier may be the preferred method, as operation is here at the so--called symmetry point. At this point the qubit is the least sensitive to noise, as the slope of the energy level is to first order zero with respect to the detuning. However, the second approach is what is typically been exploited in experiments today as this is often easier to do. In the experiment we see here, the second option of controlling the detuning has been adopted and the experiment combines both single and two qubit gates. The two graphs on the right show two experiments. In the top graph, the red qubit is always in the state spin up. In the bottom graph the red qubit is in the state spin down. The blue qubit is initialized using the Elzerman

method by aligning the electron spin to the reservoir. Then a pi/2 pulse is executed by applying an AC current through the stripline. Now the detuning is changed. This detuning brings down the second empty level of the red qubit. The state of the blue qubit hybridizes with this level and the resonance frequency changes accordingly. This change in resonance frequency causes in--plane rotations of the blue qubit. As we can see from the experiment, the blue qubit starts to rotate, but with a frequency that is dependent on the state of the red qubit. A special point is reached after roughly 0.5 a microsecond, as then there is exactly a pi--rotation difference between the two states. The experimental sequence is completed by another pi/2 pulse using the stripline. We now see that at this special point of 0.5 microsecond, the blue qubit returns to the state spin up or spin down, depending on the state of the red qubit. This qubit dependent controlled rotation is called a controlled not gate or CNOT gate and constitutes a very important gate in quantum computation. Now we can combine all aspects, from readout and initialization, to single and two--qubit control, and use this implement elementary quantum algorithms.
For example, here you see an experiment where these operations are combined to demonstrate Grover's search algorithm and the Deutsch--Jozsa algorithm. These experiments demonstrate the feasibility of quantum dot qubits, and now one of the main challenges is to up--scale the qubit number and qubit quality to go to large systems. In the field of quantum dot quantum computation, the vision of a future quantum computer is one where small arrays of quantum dots are coupled together using long--range qubit links. The small arrays operate using the ingredients as we discussed here. The long--range links could be based on quantum buses, where an electron spin is transported over an array of quantum dots. Alternatively, the electronspin can be coupled to photons in superconducting microwave cavities. Such approaches are now actively being pursued and rapid advancements are being made, and these lay the foundation for a future quantum dot quantum computer.

Main take-aways

- Applying a magnetic field on a quantum dots causes Zeeman splitting in the energy level of the qubit states. The g-factor gives a measure for this splitting with regards to the magnetic field strength.
- Qubit control can be realized by introducting an alternating magnetic field in a direction perpendicular to the applied magnetic field, causing Rabi oscillations.
- Exchange interaction is the mechanis behind coupling of two qubits.
- Using either quantum buses or coupling to photons in superconducting microwave cavities, small arrays can be coupled together.

Practice Quiz 5

QUESTION 1

In the absence of a magnetic field, electrons with spin up and electrons with spin down have the same energy. When a magnetic field is introduced, ...

- the energy of both spin up and spin down electrons will increase by the same amount.
- nothing changes.
- electrons with spin up will have a higher energy than electrons with spin down, causing the original energy level to split into two energy levels.

QUESTION 2

In the presence of the external DC magnetic field, a qubit has a certain resonance frequency. By applying an AC magnetic field that has the same frequency as this resonance frequency we can perform quantum gates: a qubit pointing down can be rotated so that it points up, and vice versa.
What is the main reason it is difficult to have the AC field be on resonance with the qubit's frequency?

- The DC magnetic field (which we know the amplitude of) is not the only field on the qubit. Nuclear spins in the background of the material induce an unpredicatable magnetic field that changes the qubit's frequency in an unpredictable way.
- The earth's magnetic field offsets the DC magnetic field applied to the qubit in an unpredictable way.
- The AC magnetic field comes from a transmission line that suffers from fabrication imperfections, which broaden the spectrum of the AC field (i.e. the AC field does not have one frequency; it has many).

Learn more

Electron spins in quantum dots

The article 'Spins in few-electron quantum dots' goes deeper into everything you might want to know about electrons spins in quantum dots.

Spin qubits

In this paper 'Interfacing spin qubits in quantum dots and donors—hot, dense, and coherent' you can learn more about the advantages and potentialities of spin qubits.

Time Evolution and the Schrödinger Equation

An interesting section lecture by Allen Adams (MIT) about Schrodinger equation and Hamiltonian evolution.

Module 3

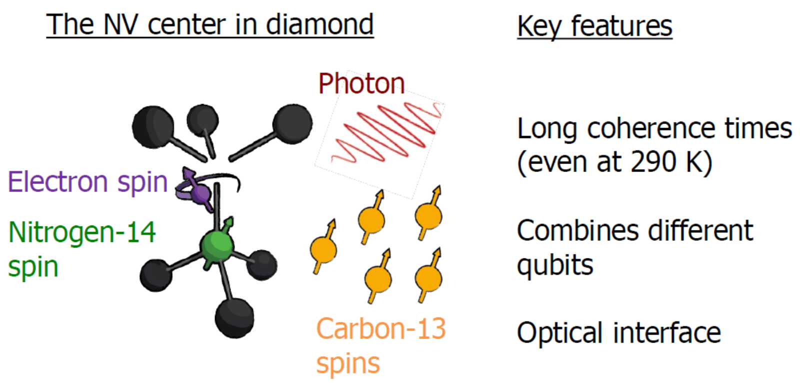
In Module 2, we saw how single electron spins can be used to form qubits. However, spin qubits can be realized in other physical systems in addition to semiconductor quantum dots. This week, we will discuss one of these examples, nitrogen-vacancy centers in diamond.

NV centers, with their long-lived and controllable quantum states, are considered one of the most promising platforms for building the first scalable quantum internet. One of the main requirements of quantum networks is the ability to quickly create robust entanglement links. Check out this recent article published in Nature to discover how this can be realised in these diamond spin qubits.

Tim Taminiau, from TU Delft, will describe the unique features of this new type of qubit platform. Throughout two lectures, he will explain how we can define a spin system in diamond, how we can optically manipulate it, and what we need in order to build quantum networks for quantum computation and quantum communication.

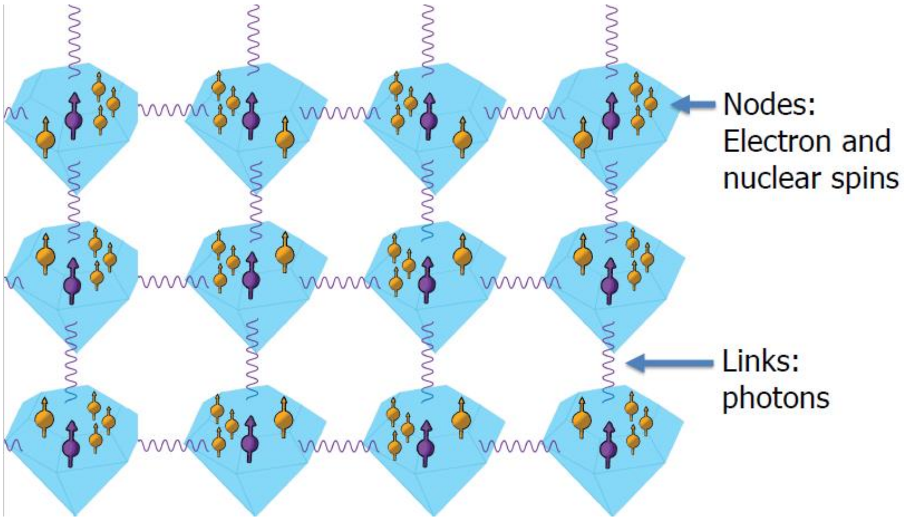
Let's begin Module 3!

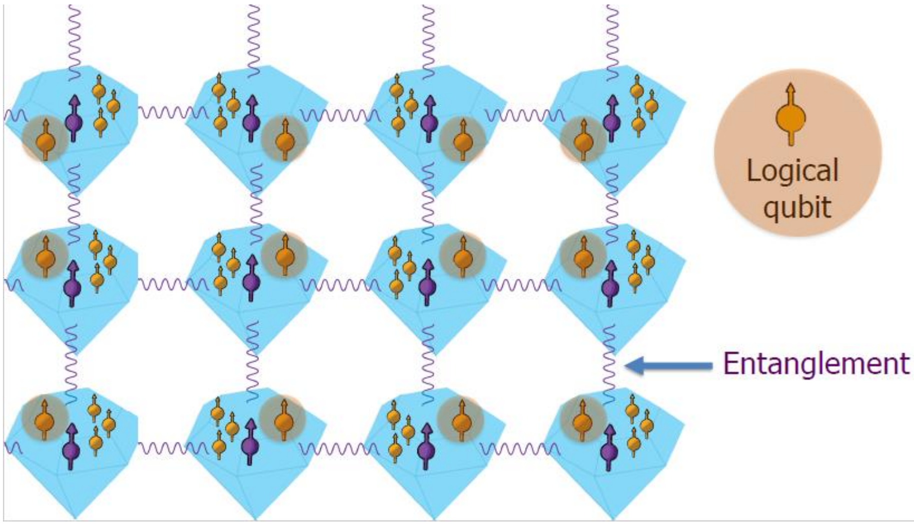
NV center qubits



Tim Taminiau, NV Center Qubit

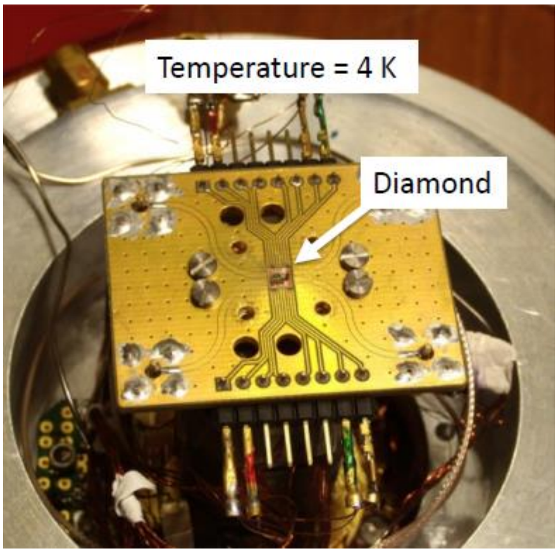
NV center qubits Today we are going to learn about a very promising type of quantum bit, spins associated to the nitrogen vacancy or NV center in diamond. This NV center consists of a substitutional nitrogen atom in the diamond lattice, next to a missing carbon atom, a vacancy. At this vacancy, some electrons are trapped, that form an electron spin that we can use as a quantum bit. These NV centers have several unique features that make them promising for quantum information systems. First, the electron spin has a very long coherence time, even up to seconds; which means that we can control it as a good qubit. Quite remarkably, this qubit can even work in a large range of

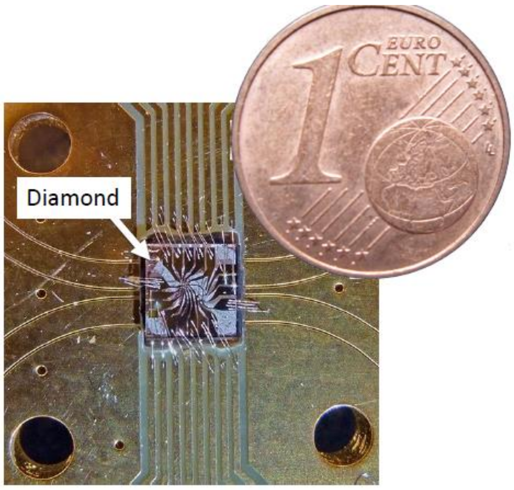




TIM TAMINIAU, NV CENTER QUBIT

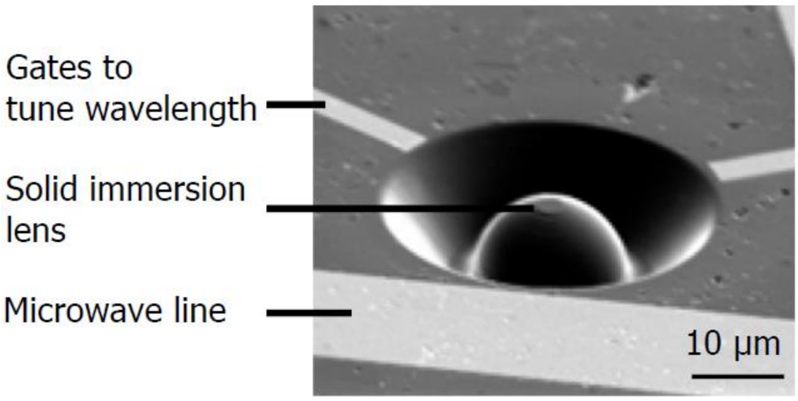
temperatures, all the way up to room temperature.Second, this electron spin is not the only qubit we have in the system. It couples to nuclear spins in the environment, which gives us extra qubits to store and process quantum information. Additionally, the electron spin also interacts with photons, elementary particles of light. This makes it possible to send quantum states far away and to connect and entangle NV centers over a distance. With such a hybrid system of photons and spins we can create quantum networks.In a quantum network we have nodes consisting of multiple spins or qubits that can store and process quantum states and we can then link these together using optics into a network. This is a very exciting approach. In this way we can make a large quantum computer by just connecting many copies of small and simple quantum processors together. This avoids the challenges of making a single chip of ever-increasing complexity. Because these optical connections can also go over long distances, we can use these networks for quantum cryptography as well which will enable fundamentally secure communication . Finally, one can even combine these two ideas and use the network to access a quantum computer remotely. In that way, even the party that hosts the quantum computer cannot know what computation you are performing or what the outcome is. How does such a computation work in a network? The logical qubits, which actually hold the information of the computation, are spread out over the entire network. We use optical links to distribute quantum entanglement and we store and process that entanglement in the restof the qubits. These entangled states are then used as a resource to perform error correction and the quantum computation, and these are also spread out over the entire network. What kind of samples do we use for this?Here you see an image of a





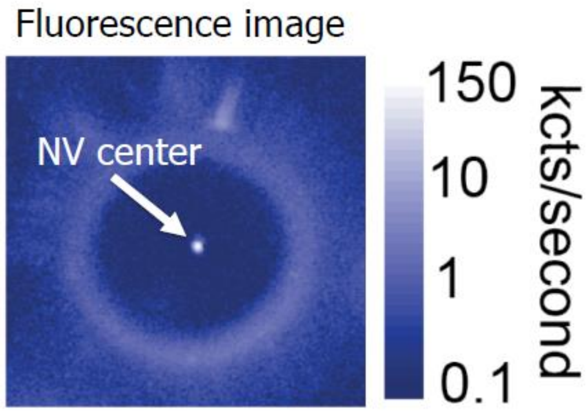
TIM TAMINIAU, NV CENTER QUBIT

little diamond wired up in a chip in a cryostat, which we bring to 4 Kelvin.This is quite a bit a higher temperature than you will typically see for other qubits. If we zoom in, you see here a diamond which is just a few mm big. And if we zoom in even further with an electron microscope, we can see these structures. The grey background here is the surface of the diamond. The electrodes are used to apply electric fields to control the emission wavelength of the NV center, the colour it emits. And the microwave lineis used to apply microwave fields that control the spin state.



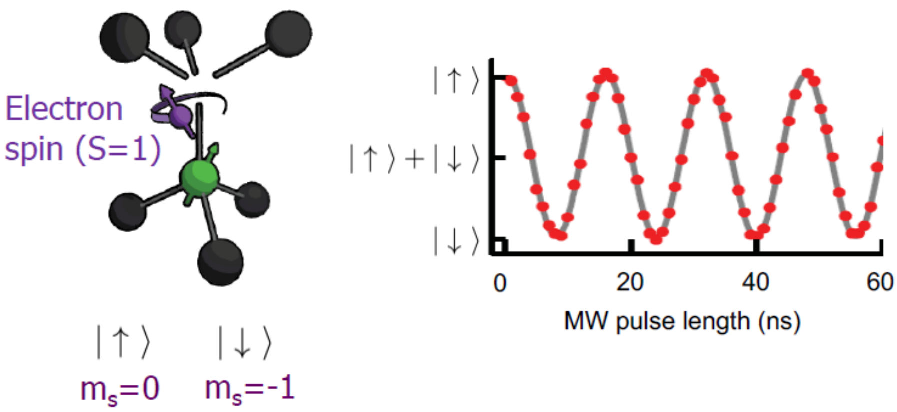
TIM TAMINIAU, NV CENTER QUBIT

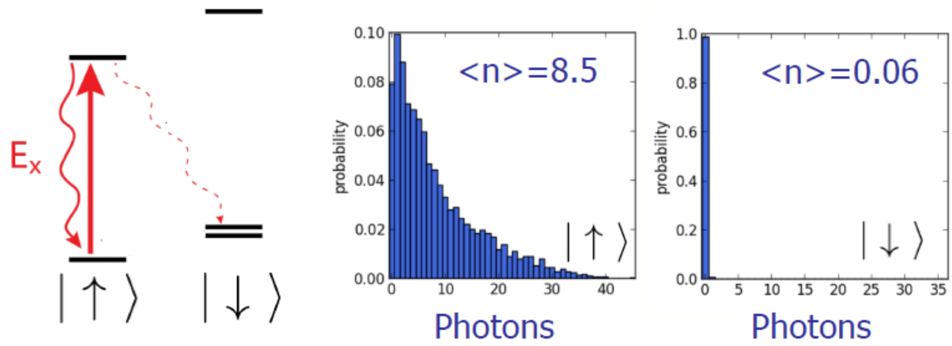
The half sphere is a lens that has been sculptured out of the diamond; we do this to get the photons effectively out of the diamond. Diamond has a very high refractive index. That is one of the reasons they are so shiny! But it also means that if we have flat surface we get a lot of total internal reflection. By curving the surface we can get about 10 to 20 times more light out.In this fluorescence image you can see that we indeed have exactlyone single NV center at the middle of the lens.



TIM TAMINIAU, NV CENTER QUBIT

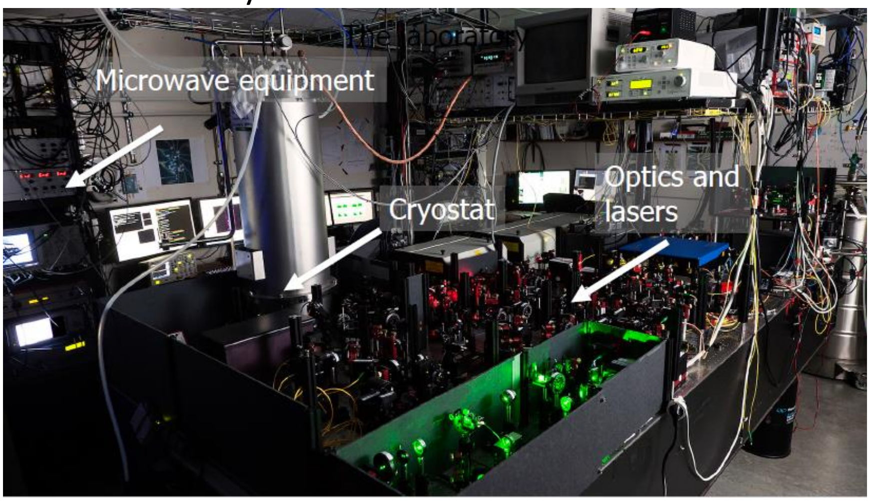
The electronic spin of the NV is actually a spin 1. To use it as a qubit we just select two levels. We can control this spin by applying microwave pulses.
If we start with the spin pointing upwards and we apply a microwave pulse of variable length, we can see that the spin rotates from up to





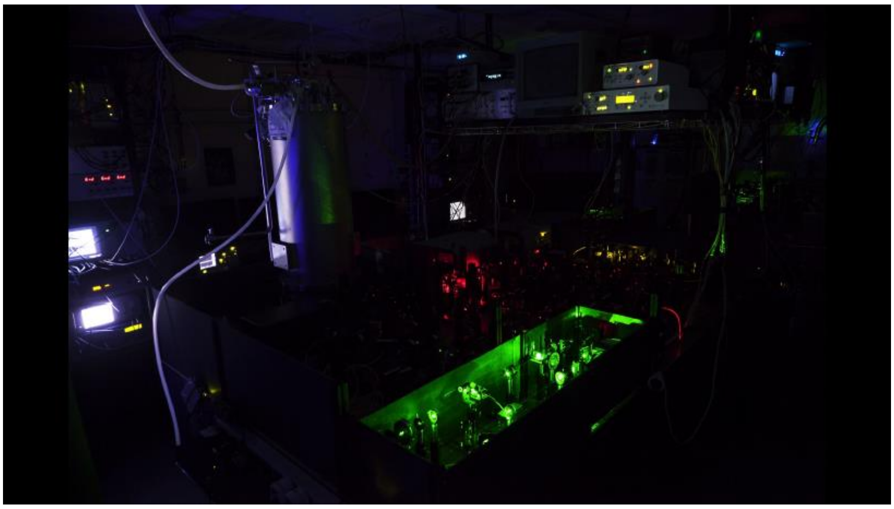
TIM TAMINIAU, NV CENTER QUBIT

down and then back up again in a coherent fashion. Exactly halfway in that rotation you have created quantum superposition of spin-up and down. How can we measure what the state of the spin is? For that, we use optics. The idea is really quite simple. There are different optical transitions in the NV center, which are associated to different spin states. So, if we apply a laser pulse that is only resonant with a transition for spin up, then only when the spin is up we excite the NV center and we detect photons. If the spin is down, it stays dark. So in this way we can read out what the spin state is. This is not perfect, because the game is to catch at least one photon before the spin flips for some other reason. And that is why these solid immersion lenses on the sample that improve the light extraction are so important. Ok, that concludes the basic controls and measurement the electron spin of the NV center. This is what it looks like in an actual laboratory.



TIM TAMINIAU, NV CENTER QUBIT

The tube that you see is a cryostat that contains the diamonds cooled down to 4 Kelvin. And you can also see a lot of optics and lasers to measure the spin and microwave electronics to control it. Of course in reality, when the system is running, it is dark, like this.



TIM TAMINIAU, NV CENTER QUBIT

That is because we have to detect single photons coming from the NV centers. We will leave this running for now and in the next lecture, we will learn how we can control multiple qubits by using the NVcenter to control nuclear spins in the environment and also how we can use photons to link up NV centers into a quantum network. Thank you for joining us and have a great day!.

Main takeaways

- Spins of trapped electrons in nitrogen vacancy centers make promising quantum bits.
- These qubits have long coherence times and can work in a large range of temperatures.
- The electrons couple to surrounding nuclear spins, offering extra information storage possibilities.
- Also, the interaction between electron spins and photons allow the realization of quantum networks.

Practice Quiz 6

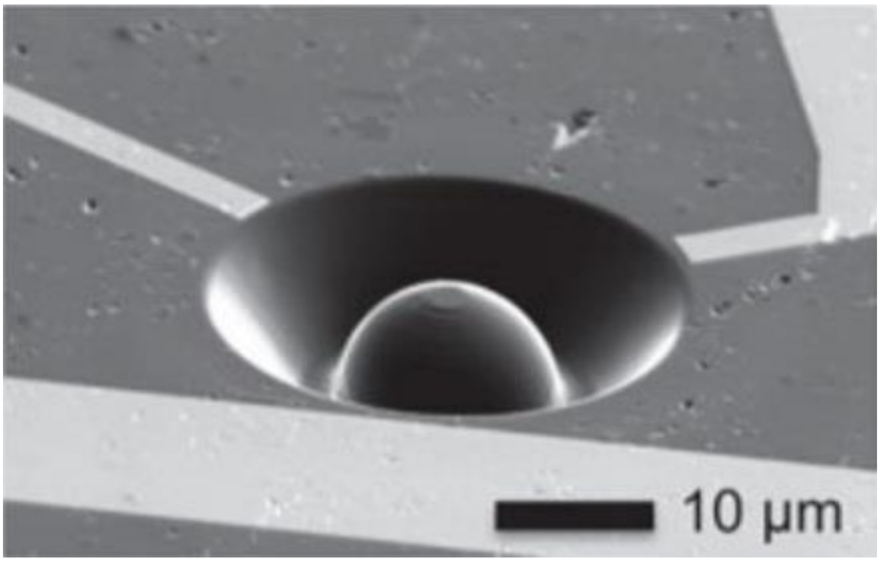


IMAGE FROM CHILDRESS, L., AND HANSON, R. (2013). DIAMOND NV CENTERS FOR QUANTUM COMPUTING AND QUANTUM NETWORKS. MRS BULLETIN, 38(2), 134-138.

QUESTION 1

SHOWN IN THE PICTURE ABOVE IS A NITROGEN-VACANCY CENTER DIAMOND THAT CONTAINS A QUBIT. WHY IS THE DIAMOND MADE IN THE SHAPE OF THE HALF-SPHERE?

- SO THAT IT ACTS AS A LENS: ITS PURPOSE IS TO COUNTER THE EFFECT OF TOTAL INTERNAL REFLECTION AND ALLOW MORE PHOTONS TO ESCAPE THE QUBIT DURING READOUT.
- TO ISOLATE THE QUBIT FROM THE ENVIRONMENT.
- TO MAKE THE QUBIT EASIER TO COOL.

QUESTION 2

HOW CAN SINGLE-QUBIT ROTATIONS BE PERFORMED ON THE ELECTRON SPIN?

- USING ONLY AN EXTERNAL DC MAGNETIC FIELD TO ROTATE THE ELECTRON SPIN.
- USING ONLY AN EXTERNAL DC ELECTRIC FIELD TO ROTATE THE ELECTRON SPIN.
- JUST LIKE QUANTUM DOT QUBITS IN THE PREVIOUS MODULE, A WIRE RUNNING NEXT TO THE QUBIT EMITS AN AC ELECTROMAGNETIC FIELD ON RESONANCE WITH THE QUBIT'S FREQUENCY.

QUESTION 3

WHAT MECHANISM IS DESCRIBED IN THE SECTION FOR READING OUT THE STATE A QUBIT?

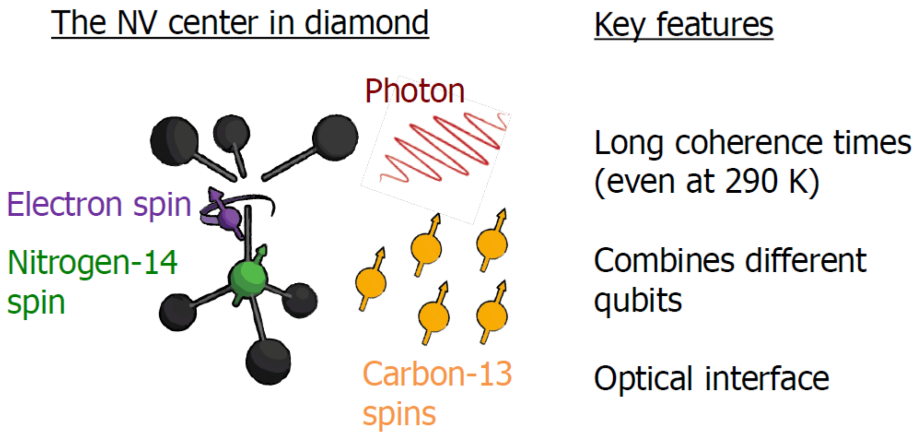
- A SPIN-CHARGE-SENSING QUANTUM DOT POSITIONED NEXT TO THE DIAMOND DETECTS WHETHER THERE IS AN ELECTRON ON THE DIAMOND OR NOT.

- Light with a specific frequency is shined on the electron such that only if it was in the spin-up state it gets excited to the next energy level. If, after waiting, we detect light that means the electron was in spin-up, and if there is no light that means the electron was in the spin-down state.

- The electron is transported from the diamond using a wire and an external magnetometer is used to read its spin.

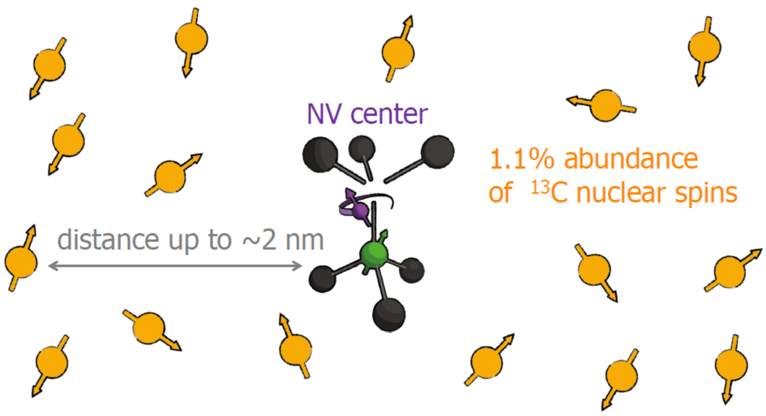
Operations on NV center qubits

Operations on a NV center qubit Previously, we have seen how we can control the electron spin of the NV center in diamond as a qubit. In this Part , we will learn how we can use the NV center to also control multiple nuclear spin qubits, and how we can link NV centers together using photons to create a quantum network.



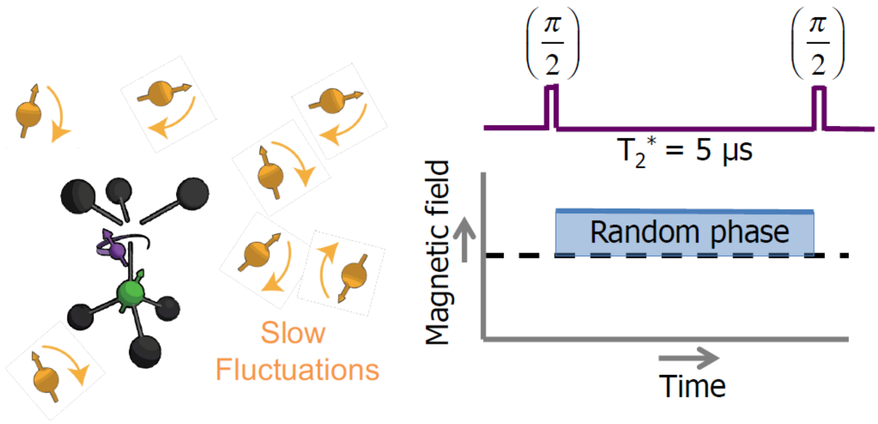
TIM TAMINIAU, NV CENTER QUBIT

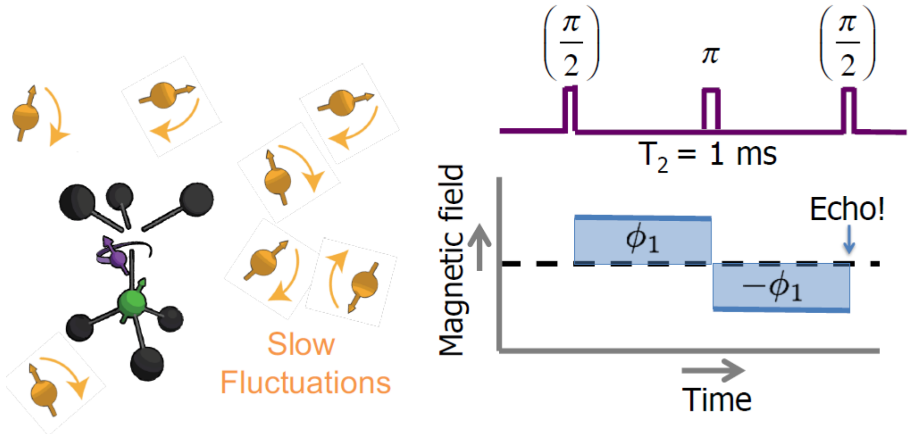
The first thing to realize is that this electron spin is surrounded by an entire cloud of nuclear spins. About 1% of diamond consists of Carbon-13, which is a spin-1/2 system and the rest consists of Carbon-12, which has no spin. Now normally speaking, these nuclear spins are a source of decoherence. They flip-flop around randomly, and create a slowly-varying magnetic field on the NV-center.

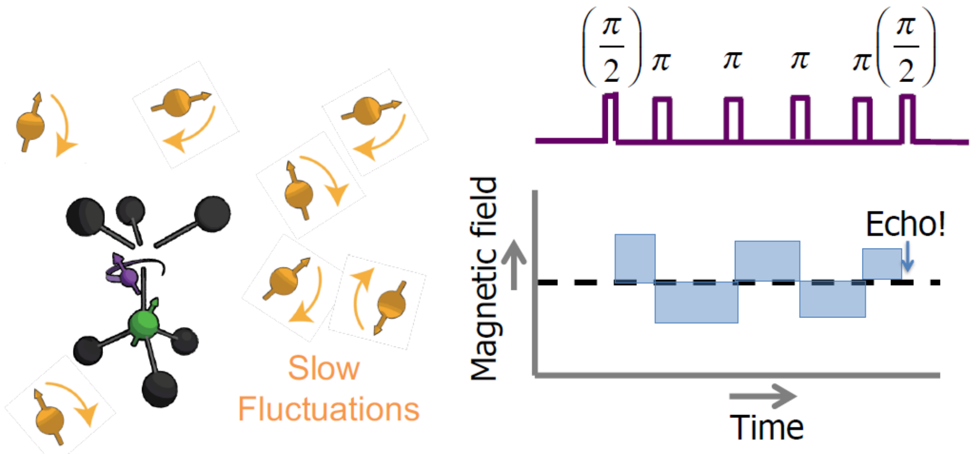


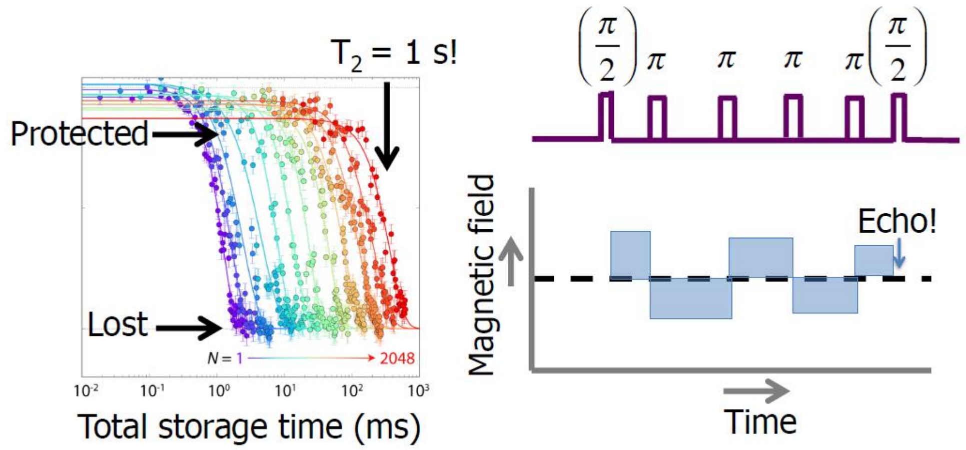
TIM TAMINIAU, NV CENTER QUBIT

If we prepare the NV center in a quantum superposition state, then this fluctuating magnetic field changes the NV energy levels, that causes its phase evolution to become random and the quantum state is lost. This dephasing time is only about 5 microseconds, not that long . Luckily, we can play a trick. We can apply a pulse that inverts the state of the NV.



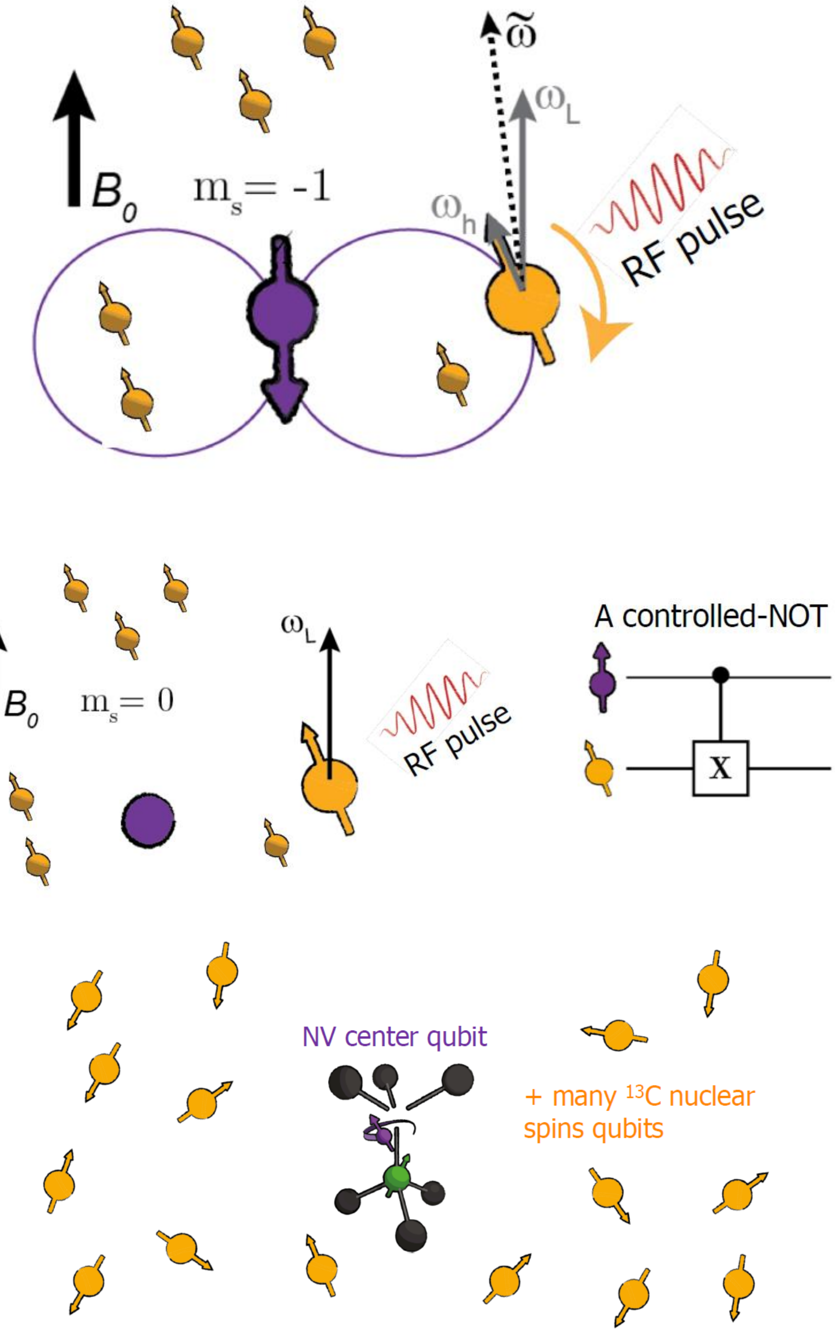






TIM TAMINIAU, NV CENTER QUBIT

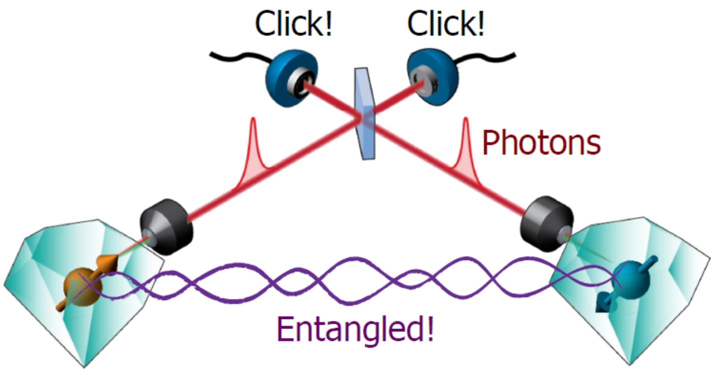
That if we flip it, this also inverts the effect of the magnetic field on the spin. So, if we have the same time before and after this flip, then the effect of the field exactly cancels and the quantum state is protected. We call this a spin echo. Of course this cancelation only works if the magnetic field is constant over time. But don't worry. As long as the field fluctuates slowly, we can just flip the electron spin multiple times, faster and faster, so that everything still averages out. In this graph you can see that this works. We bring the electron spin in a super position and as we apply more and more pulses, we protect the electron spin longer and longer. In this way, we can even protect the quantum state for over a second, so really over macroscopic time scales and about six orders of magnitude better than without flipping the electron. We thus have very good coherence for the electron spins. How can we now use this to control multiple nuclear spins in the environment ?The key here is that if the electron spin is, let's say, in a state pointing downwards, then it creates a dipolar magnetic field. Each nuclear spin has a different position and angle from the electron spin, so that each of them feels a different magnetic field. This gives each nuclear spin a unique frequency, so that we can apply radiofrequency pulses that are resonant only with a particular targeted nuclear spin. We can selectively control the spins. Moreover, if we now flip the electron to its zero state, it does not create a magnetic field.



TIM TAMINIAU, NV CENTER QUBIT

Now the same RF pulse will have no effect on this nuclear spin. The evolution of the nuclear spin thus depends on the state of the electron spin: it rotates if the electron spin is in state 1, but it doesn't rotate for state 0. This means we have a controlled quantum gate between the electron and nuclear spin. A CNOT. That is exactly what we need for quantum computation.

This enables us to control multiple nuclear spins near a single NV center. Each NV center becomes a system of 5 or even more qubits .How do we link these NV centers together into a network? For that we use photons. Consider two NV centers in two different diamonds. We first make each of these NV centers emit a photon that is entangled with the spin state.



TIM TAMINIAU, NV CENTER QUBIT

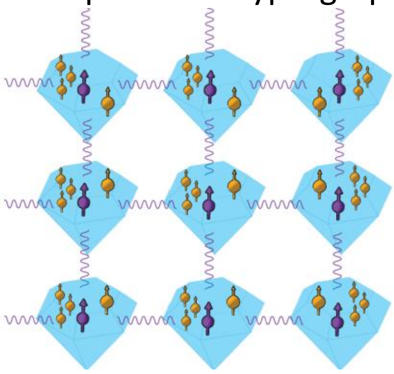
And then we take these two photons and bring them together on a beam splitter. After this beam splitter it is fundamentally impossible to tell which photon came from which NV center. This means if the detectors behind the beam splitter register a certain pattern of photons, then we know, that for example, one of the NV centers is pointing up and the other one is pointing down. Because we fundamentally cannot know which one is up and which one is down, quantum mechanics tells us that we have created an entangled state between two distant NV centers. Note that this does not succeed every time you try. Not all measurement outcomes lead to entanglement and photons are also often lost on their way to the detectors. This is not a problem because we are just trying to create entanglement to use it as a resource to perform quantum computation in our network. So we can just keep trying and trying until we get the right measurement outcome, which then heralds the generation of entangled state, and then we can use it in the network.

Entanglement over 1.3 km

Science nature Top 10 science event of 2015

TIM TAMINIAU, NV CENTER QUBIT

This works. Here you see a state-of-the-art experiment in Delft, where we created entanglement between one NV center in the physics building, all the way on the left and another NV center, all the way on the right. About 1,3 kilometres away, at the other side of the Delft campus. This experiment was done to provide a more stringent test if quantum entanglement really exists. The outcome was yes, quantum mechanics is still correct. So, we can now go and use this entanglement to build quantum networks for quantum technology. The next big step will be to combine the control of multiple qubits in each node with optical links to generate entanglement, in order to build increasingly larger quantum networks. Of course there are still many challenges. We need really good quantum control over all these spins and photons. We need to understand how to efficiently run quantum computations and error correction over these networks, and we need to build the complex electronics and software to control the networks. For this, we need physics, quantum computer science and engineering to come together. It will be really exciting to see if we can really build large-scale quantum networks and make quantum computation and quantum cryptography a reality.



TIM TAMINIAU, NV CENTER QUBIT

Main takeaways

- SPIN ECHO IS A WAY TO PROTECT THE ELECTRON SPINS FROM DECOHERENCE CAUSED BY THE SURROUNDING NUCLEAR SPINS.
- THE DIPOLAR MAGNETIC FIELD OF THE ELECTRON SPIN GIVES A UNIQUE ENERGY SPLITTING TO DIFFERENTLY POSITIONED NUCLEAR SPINS, ALLOWING SELECTIVE CONTROL.
- THE NUCLEAR SPIN'S DEPENDENCE ON THE ELECTRON SPIN STATE FORMS THE BASIS FOR CONTROLLED QUANTUM GATES.
- USING A BEAM SPLITTER, PHOTONS CAN BE USED TO GENERATE ENTANGLEMENT BETWEEN TWO DISTANT NV CENTERS.

Practice Quiz 7

QUESTION 1: A QUANTUM NETWORK

WHAT IS THE PROCEDURE EXPLAINED IN THE LECTURE FOR CONNECTING DIFFERENT NV CENTERS TO BUILD A QUANTUM NETWORK? WE SHINE A LASER ONTO AN NV CENTER AND:

○ THE NV CENTER REFLECTS THE LIGHT. THIS REFLECTED LIGHT CARRIES INFORMATION ABOUT THE QUANTUM STATE, AND IS SUBSEQUENTLY SENT TO A DIFFERENT NV. THE COLLISION OF THE LIGHT WITH THE OTHER NV CAUSES IT TO CARRY THAT INFORMATION TOO.

○ MEASURE THE NV CENTER TO LEARN ITS STATE. THIS INFORMATION IS SEND TO A DIFFERENT NV. USING THIS INFORMATION WE CAN MANIPULATE THE NV TO REPRODUCE THE STATE AT THE FIRST NODE.

○ EXCITE THE ELECTRON. AS THE ELECTRON FALLS BACK, A PHOTON IS EMITTED. THIS PHOTON IS SENT TO A DIFFERENT NV AND SUBSEQUENTLY COLLECTED BY ITS ELECTRON, AFTER WHICH THE ELECTRON HAS REPRODUCED THE STATE OF THE FIRST EXCITED ELECTRON.

SIMILARLY SHINE A LASER ON THE OTHER NV CENTER. WE LET THE PHOTONS, WHICH ARE EMITTED DUE TO ELECTRONS FALLING BACK, INTERFERE. BY MEASURING THEM, WE EFFECTIVELY ENTANGLE THE TWO NV CENTERS.

Quiz 7: Operations on NV center qubits

DYNAMICAL DECOUPLING (DD)

In the Part , Tim told us all about the trick we can apply to protect superpositions against decoherence from random phase rotations. This process is termed dynamical decoupling (DD). In the following questions, we will together mathematically investigate the beautiful features of dynamical decoupling.

Learn more

LOOPHOLE-FREE BELL TEST

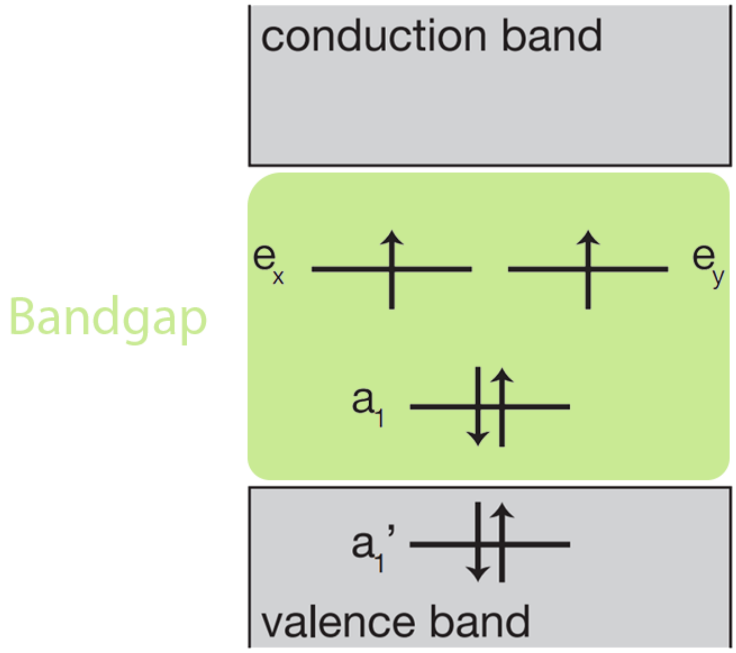
BACK IN 2015, THE GROUP OF PROFESSOR HANSON FROM QUTECH MANAGED TO PROVE ENTANGLEMENT BY PROVING THE BELL INEQUALITY. READ MORE ABOUT THIS HERE:
PAPER ON LOOPHOLE-FREE BELL TESTHTTPS://WWW.NATURE.COM/ARTICLES/NATURE15759

NEW: ENTANGLEMENT ON DEMAND!

SCIENTISTS FROM DELFT MANAGED TO CREATE SUPERPOSITION FASTER THAN IT WOULD DECOHERE, ALLOWING DETERMINISTIC ENTANGLEMENT.
PAPER ON ENTANGLEMENT ON DEMAND

OR LOOK AT THIS SECTION!
SECTION ON ENTANGLEMENT ON DEMAND

BONUS QUESTION: BANDGAP OF A NV CENTER



IN THE FIGURE, YOU CAN SEE THE SO-CALLED BANDGAP DIAGRAM OF AN NV CENTER. THE Y-AXIS REPRESENTS ENERGY AND THE HORIZONTAL LINES ARE THE ENERGY STATES. EACH ELECTRON IS REPRESENTED BY AN UP- OR DOWN-ARROW.

IN THIS PARTICULAR BANDGAP DIAGRAM YOU CAN SEE SIX TRAPPED ELECTRONS. THIS SIXTH ELECTRON IS ADDED BY EXCITATION OF ONE OF THE INITIALLY TRAPPED ELECTRONS FROM THE VALENCE BAND TO A HIGHER ENERGY STATE. THIS VACANCY IS EASILY FILLED UP WITH ONE OF THE MANY ELECTRONS PRESENT IN THE VALENCE BAND.

THIS NEW SYSTEM, WITH 6 TRAPPED ELECTRONS IS CALLED THE NV MINUS STATE.

BONUS QUESTION: BANDGAP OF A NV CENTER

LOOKING AT THE FIGURE ABOVE. WHY DO WE ONLY USE THE 4 ELECTRONS THAT ARE TRAPPED IN THE BANDGAP?

- STATES IN THE BANDGAP ARE PROTECTED AGAINST THE ELECTRON TRANSFERS TO AND FROM THE DIAMOND.
- THE NUMBER OF POSSIBLE QUBIT STATES FOR 6 TRAPPED ELECTRONS IS TOO LARGE, MAKING COMPUTATION IMPOSSIBLE.
- THE DEGENERATE EXCITED STATES ARE ONLY SYMMETRIC WITH RESPECT TO THE BANDGAP STATES.

Module 4

In module 2 and 3, we discussed spin qubits in semiconductor quantum dots and nitrogen-vacancy centers. Despite the differences, in both cases qubits were realized with the spin degree of freedom of the electron. It is now time to discover a totally new platform: superconducting qubits.

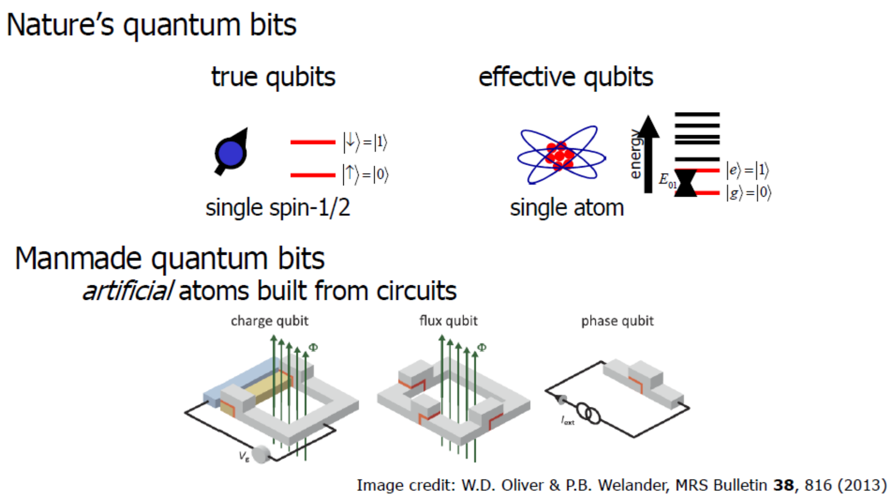
If a quantum computer is able to perform complex quantum algorithms it might need millions or even billions of qubits. Superconducting qubits are now days one of the most advanced platforms in the quantum community regarding the number of qubits, and many large companies, such as Google, Intel and IBM, are now investing in them. IBM Q experience is an online platform where users from the general public can already access a five-qubit processor and run small algorithms.

In the first three Part , Leonardo DiCarlo, Professor in applied physics, will describe a transmon, one of the basic building blocks of superconducting qubits. Later, Niels Bultink, Brian Tarasinski and Adriaan Rol will explain how to perform the operations required by universal quantum computation: qubit measurements, single-qubit gate and two-qubit gates.

The transmon qubit

In this three-part series on the introduction to superconducting qubits, we start by introducing the basic concepts of one kind of superconducting qubit, and then gradually proceed to materials about a full-fledged quantum processor comprising of many qubits.

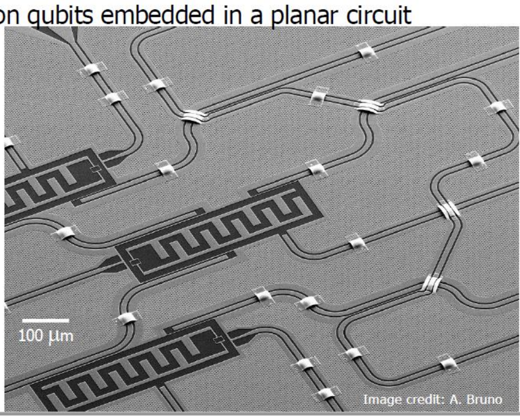
In this first Part, Professor Leo DiCarlo talks about the entailing physics behind a transmon qubit, which is an example of a superconducting qubit. Remember that the elaborate expressions and notations in this section are not meant to intimidate the avid learners. It will suffice to grasp the core concepts from these sections. If however, all those notations have got you enthused, then we recommend you to go through all the different nuances and enhance your learning; because courses like these are not just meant to be passed and forgotten, are they?



Leonardo DiCarlo, Superconducting quantum circuits:The transmon qubit

Superconducting qubit: The transmon-Leonardo DiCarlo Today I will give you an introduction to quantum computing with superconducting quantum circuits; more specifically, quantum computing with transmon quantum bits or qubitsin the circuit quantum electrodynamics architecture. Superconducting qubits differ in two important ways from truly quantum two-level systems provided to us by nature, such as the spin of an electron or the spin of certain nuclei, like Carbon 13 or Silicon 29. First, they are multi-level systems, not unlike the electronic levels in atoms. Multi-level quantum systems can be used effectively as qubits by confining all dynamics to two quantum levels, usually the ground state and the first excited state of the system. Second, these qubits are circuits that we fabricate ourselves! This has advantages and disadvantages. On the bright side, we have freedom to design! In a sense, we can play 'god' designing artificial atoms. However, fabrication uncertainty keeps us from making any two qubits the same! For this reason, we like to say that qubits have individual personality! While the form of the Hamiltonian describing them quantum mechanically is well known, the parameters of this Hamiltonian cannot be perfectly targeted in fabrication. This has interesting implications for scaling up our quantum processors, which I will discuss in a later Part. But back to design. Superconducting qubits generally consist of superconducting electrodes, or islands, that are interconnected by Josephson junctions. The Hamiltonian typically consists of two non-commuting contributions, one capacitive and one inductive. The first tends to localize Cooper pairs. The second favors their tunnelling from one island to another, in other words, their hopping. There exist numerous varieties of superconducting qubits: the charge qubit, the flux qubit, the phase qubit, the fluxonium, and many more. These qubit types differ in terms of the number of superconducting islands, the number of junctions, and, also importantly, the relative energy scales of the capacitive and inductive terms. Typically, superconducting qubits work in the frequency range between 4 and 8 GHz, approximately. This frequency, let's call it f_{01}, is related via Planck's constant to the energy difference E_{01} between the quantum levels that we assign as states 0 and 1.f_{01} is also the frequency of the microwave pulses that we need to induce coherent transitions between these levels.In this Book, we will focus on the transmon, a derivative of the charge qubit.

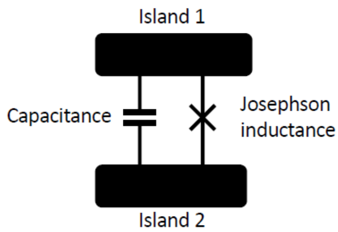
Transmon qubits embedded in a planar circuit



Leonardo DiCarlo, Superconducting quantum circuits:The transmon qubit

The transmon is the superconducting qubit that we specialize on in QuTech.

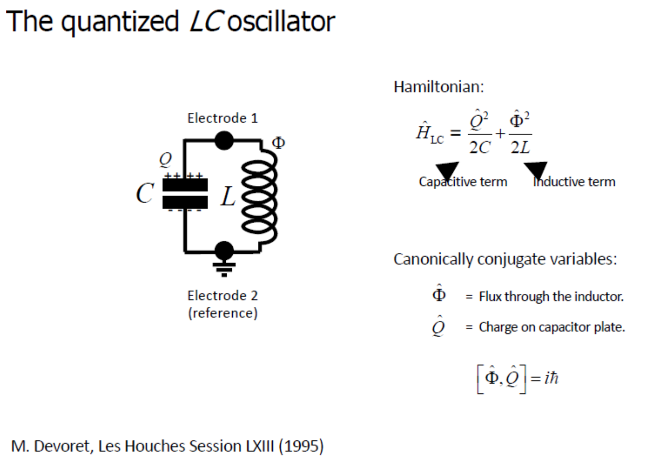
The Transmon qubit



Theory of the transmon: J. Koch *et al.*, Phys Rev. A **76**, 042319 (2007)

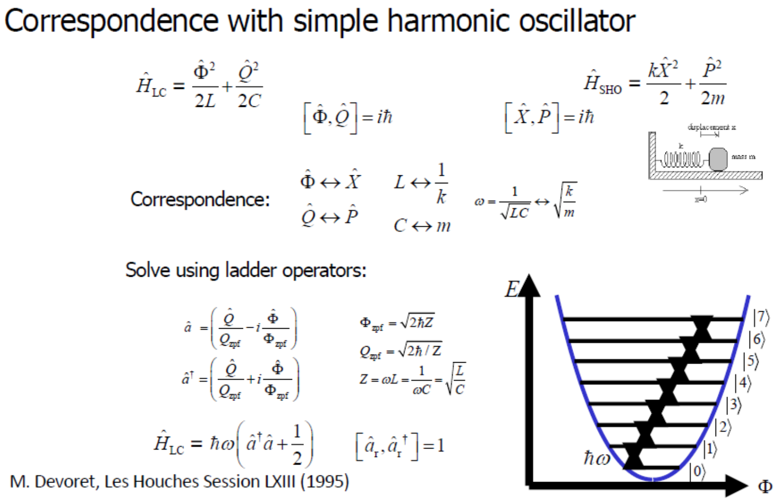
Leonardo DiCarlo ,Superconducting quantum circuits:The transmon qubit

In its simplest form, the transmon consists of two islands interconnected by one junction. Those of you familiar with circuits, particularly the electrical engineers, will recognize that the transmon looks, well, just like a parallel combination of one capacitor and one inductor. In other words, an LC oscillator! This gets us most of the way there, so let's take a look.The Hamiltonian forthe LC oscillator consists of two terms that are each quadratic with respect to one variable:



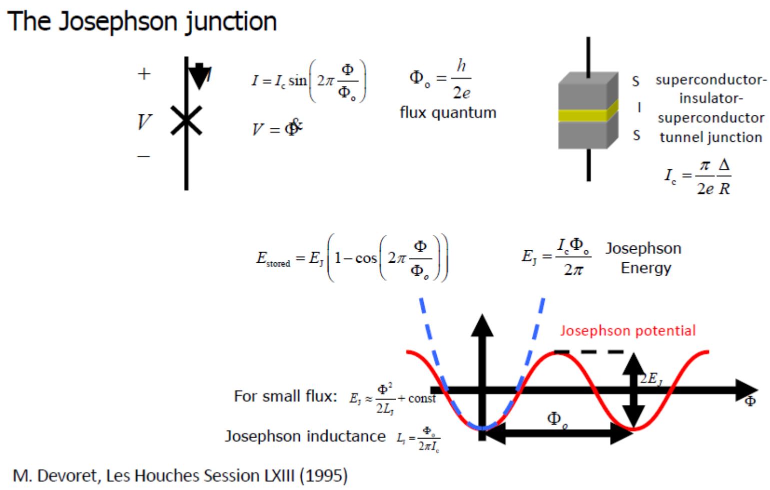
Leonardo DiCarlo ,Superconducting quantum circuits:The transmon qubit

The capacitive term is quadratic on the charge accumulated on one island (the opposite charge is accumulated in the other). The inductive term is quadratic on the flux through the inductor. This charge and this flux do not commute. In fact, they are canonically conjugate variables.A direct correspondence between this Hamiltonian can be made to that of the simple harmonic oscillator by mapping flux to the position of the mass and charge to the mass' momentum.



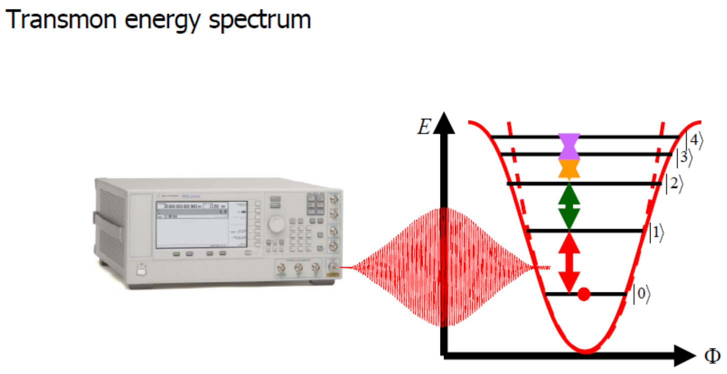
Leonardo DiCarlo, Superconducting quantum circuits:The transmon qubit

Physics students will thus not be surprised to learn that the spectrum of the quantized LC oscillator is perfectly harmonic: levels are equally spaced in energy! This equal spacing is given by a familiar formula: one over root LC.But unfortunately, a harmonic spectrum does not a good qubit make! It is very difficult to confine the dynamics to just two levels, so 'leakage' out of the qubit subspace is a permanent threat.That is why the transmon differs from the LC oscillator in a fundamental way.



Leonardo DiCarlo, Superconducting quantum circuits:The transmon qubit

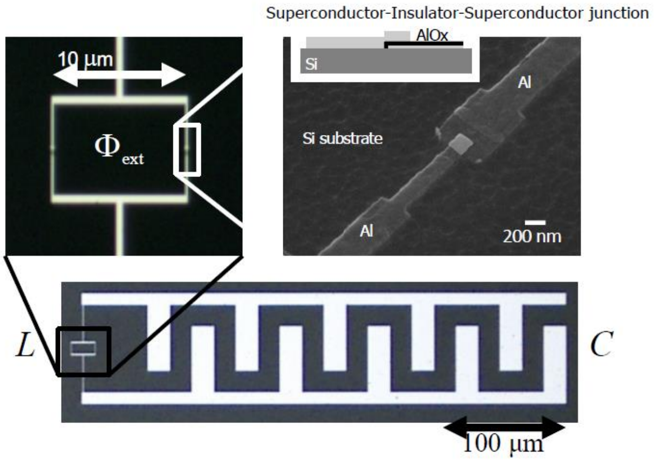
In the transmon, the inductance is provided by a Josephson junction and not by a typical coil inductor. The inductive energy for the junction is not quadratic, but a cosine function of the generalized flux through it. This difference has important consequences for the spectrum: crucially, it disrupts the harmonic spectrum in ways that are very



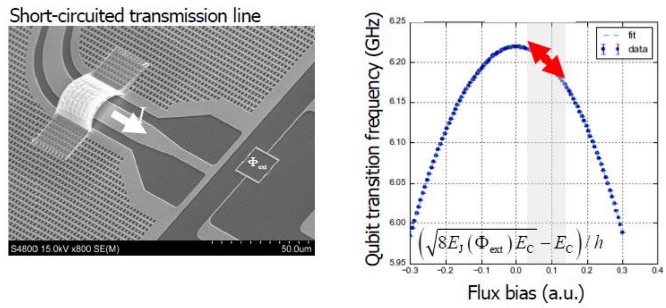
Leonardo DiCarlo ,Superconducting quantum circuits:The transmon qubit

practical. At typical parameter values deep in the so-called transmon regime, where the energy scale of the inductive term is much larger than that of the charging term and let's say, for the frequency f_{01} of about 6 GHz, the transition from the first to the second-excited state is lower by approximately 300 MHz. This difference is sufficient in practice to confine the dynamics to the lowest two levels, our qubit subspace, when performing single-qubit gates with pulses of duration about 20 ns. Brian will discuss the implementation of single-qubit gates in detail.As a final note, we build our transmon not from one but two Josephson junctions in parallel.

Two-junction transmon



Flux control of transmon frequency



Leonardo DiCarlo ,Superconducting quantum circuits:The transmon qubit

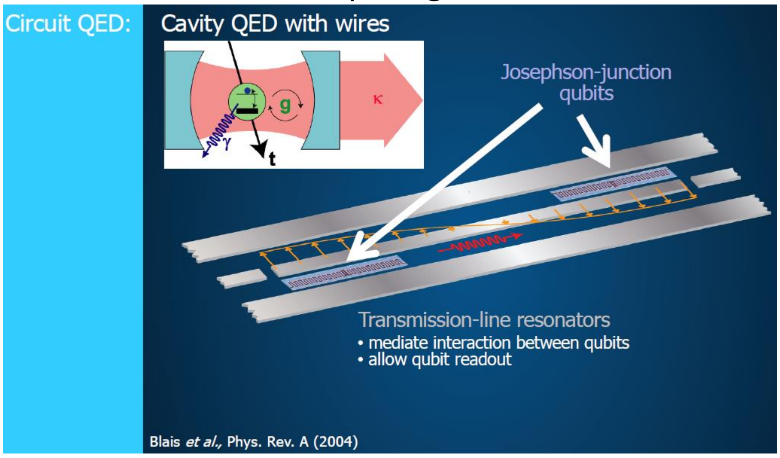
This gives us the possibility to tune the inductive element and thereby the qubit transition frequency, by threading a magnetic flux through the loop defined by the two junctions.We can do this independently for each qubit and on nanosecond timescales. This capability is the workhorse enabling two-qubit gates. Adriaan will present these in a later Part.

Main takeaways

- **It is possible to create superconducting qubits by restricting all dynamics to (generally, the lowest) two levels of the multiple levels of the system.**
- **Transmon qubits are an example of one of the many different kinds of superconducting qubits.**
- **In a transmon, the inductance is provided by a Josephson junction and not by a typical coil inductor. This disrupts the harmonic energy spectrum and helps confine the system to two levels.**

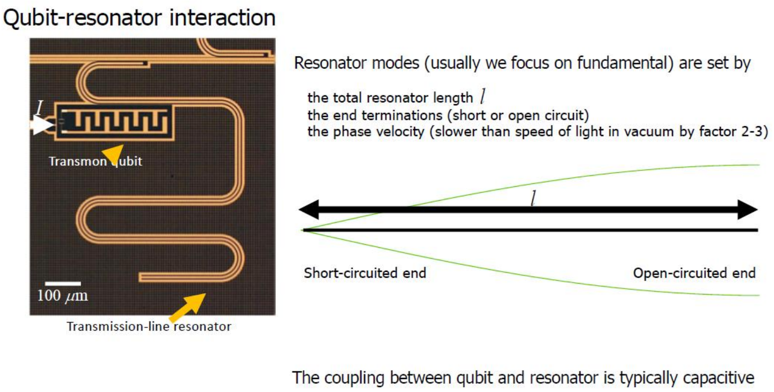
Circuit QED

Superconducting qubit: Circuit Quantum Electrodynamics -Leonardo DiCarlo
Hi! Last time, we introduced our superconducting qubit of choice in QuTech, the transmon. We focused on a transmon in isolation, studying its weakly anharmonicspectrum. Today, we will add the remaining ingredients that we need to perform qubit readout and two-qubit gates.



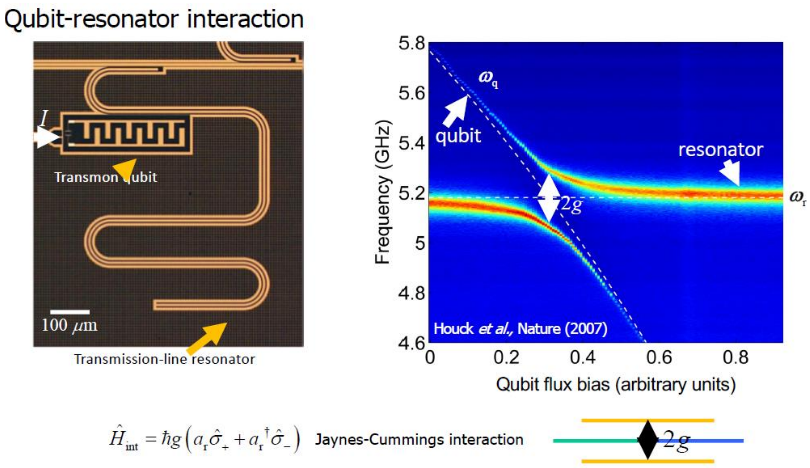
Leonardo DiCarlo ,Superconducting quantum circuits: Circuit QED

These ingredients are transmission-line resonators. The resulting architecture for quantum hardware, combining qubits and resonators, goes by the name of circuit quantum electrodynamics, or circuit QED. As this name suggests, circuit QED is a solid-state version of cavity QED, an older field of physics that studies the interaction between light and matter at its most fundamental, with flying atoms and single photons trapped inside three dimensional cavities.



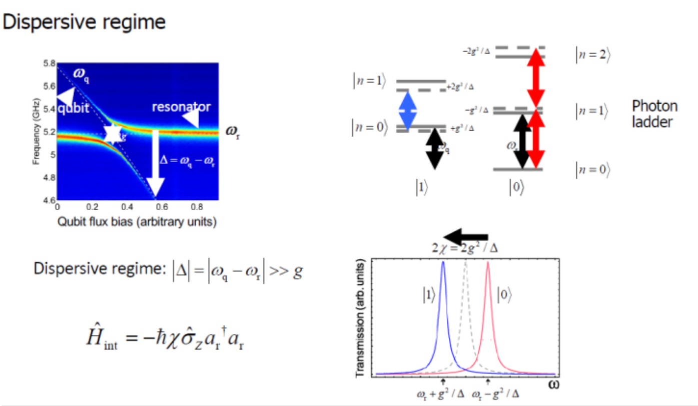
Leonardo DiCarlo, Superconducting quantum circuits: Circuit QED

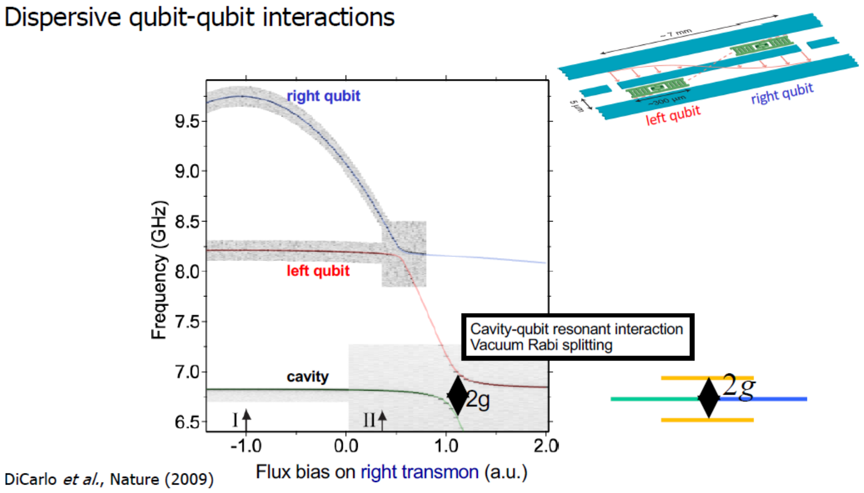
In circuit QED, qubits play the role of atoms and transmission-line resonators play the role of cavities. We build this resonators from coplanar waveguide transmission lines that are terminated with either open-or short circuits.

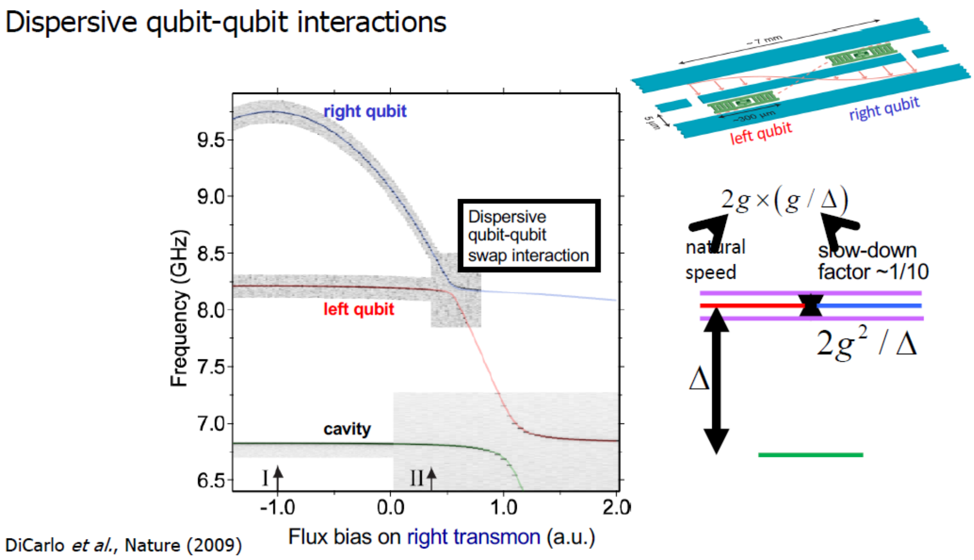


Leonardo DiCarlo, Superconducting quantum circuits: Circuit QED

For example, the resonator I show you here has a short at the far end, and an open termination at the close end. Well, it's not exactly open. The resonator at this end couples capacitively to a feedline that will be used later to interrogate the scattering properties of the resonator near its fundamental frequency. This fundamental mode corresponds to the length matching approximately one quarter of the wavelength. For this reason, we call it a quarter-wave or lambda over four resonator. Resonators have higher modes of resonance as well, but most of the time, we focus exclusively on the fundamental. The capacitive coupling between the qubit and the resonator is most clearly visualized by performing spectroscopy of the combined system. As this typical image shows, as we tune the qubit through resonance with the resonator, we observe the emergence of an avoided crossing. This avoided crossing is known as the vacuum Rabi splitting. The minimum splitting is equal to twice the coupling constant, g, in the celebrated Jaynes-Cummings Hamiltonian describing the system quantum mechanically. Most of the time, however, we avoid this resonant regime. We tend to focus on the so-called dispersive regime in which the detuning Delta of the qubit with respect to the resonator is several times larger than g in magnitude.





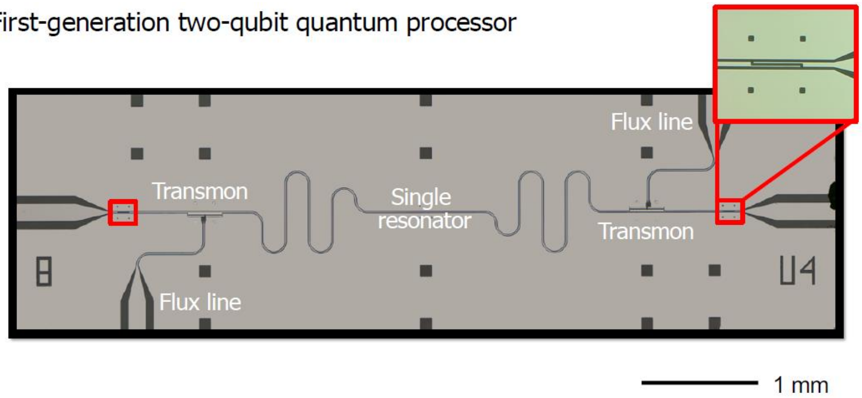


Leonardo DiCarlo, Superconducting quantum circuits: Circuit QED

In this dispersive regime, there is a small but finite remnant of the avoided crossing. If we consider the avoided crossings arising between qubit-resonator levels with equal total number of excitations, we learn something very useful: the ladder of photon excitations remains harmonic, but the resonance frequency depends on the state, |0> or |1> of the qubit. This dependence of the resonator frequency on qubit state is the key ingredient allowing qubit readout. We can interrogate the resonator, and thereby also the qubit, by measuring the

transmission properties of a microwave pulse applied to the feedline with a frequency close the bare fundamental. Nielswill cover this in greater detail in his Part .Moving on, when two qubits couple to a common resonator, an avoided crossing is also observed when one qubit is tuned through resonance with the other qubit. Compared to the vacuum Rabi splitting, which you can see here, the qubit-qubit avoided crossing is smaller, by a factor (g/Delta) These qubit-qubit interactions mediated by a dispersively coupled common 'bus' resonator are the key to doing two-qubit gates. Adriaan will discuss the specific avoided crossing that we use to implement two-qubit conditional-phase gates. Hint: it is not this one!So, let's now see examples of how transmons and resonators can be combined to build simple quantum processors.

First-generation two-qubit quantum processor



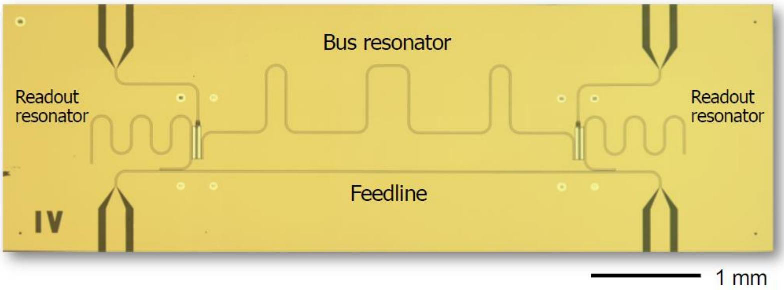
One resonator for both qubit-qubit coupling and for readout

DiCarlo *et al.*, Nature (2009)

Leonardo DiCarlo ,Superconducting quantum circuits: Circuit QED

This is the first one I built, back in 2008, together with collaborators at Yale. It has two transmon qubits and only one resonator. This resonator is used for both qubit-qubit coupling and readout functions.The resonator has capacitive terminations at both ends, so it's a half-wave or lambda over two resonator. The optimal parameters of a resonator for readout and for coupling are often in conflict with one another. For this reason, the next generation of processors that we built in Delft uses different resonators for these functions. Here's an example two-qubit processor from 2013.

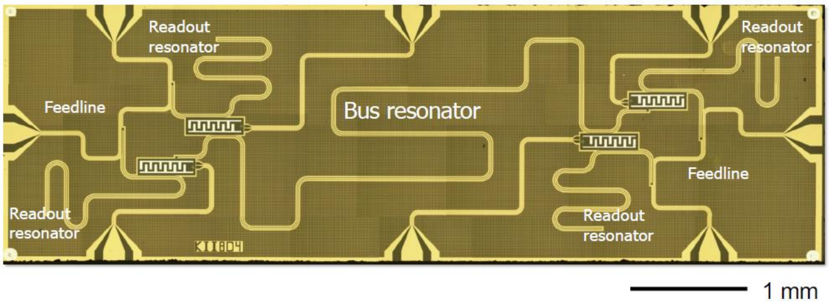
Second-generation two-qubit processor



Different resonators for qubit-qubit coupling and for readout of each qubit

Groen *et al.*, Phys. Rev. Lett. (2013)

Second-generation four-qubit processor

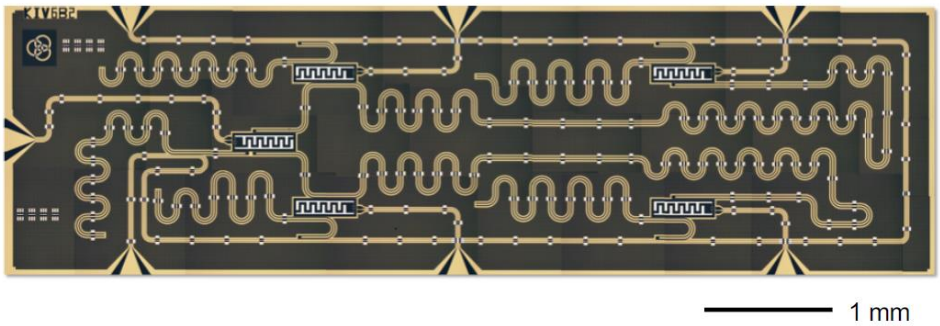


Different resonators for qubit-qubit coupling and for readout of each qubit

Saira *et al.*, Phys. Rev. Lett. (2014)

Second-generation five-qubit processor

Challenge: identify the readout resonators and bus resonators

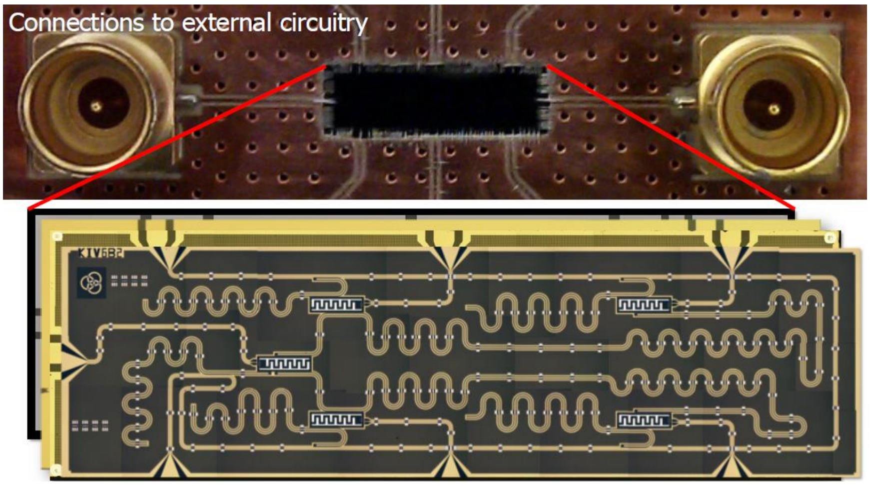


Different resonators for qubit-qubit coupling and for readout of each qubit

Riste *et al.*, Nature Communications (2015)

Leonardo DiCarlo, Superconducting quantum circuits: Circuit QED

Note the common lambda over two bus resonator in the middle, and the lambda over four readout resonators, one per qubit, coupled to the common feedline. Here we see the same concept in a four-qubit processor. The planar constraints required us to have two feedlines. Hopefully, you can easily recognize the bus resonator common to all 4 qubits, and the dedicated readout resonators, one for each qubit. So, as a simple exercise, I challenge you to identify the bus resonators and readout resonators in this five-qubit processor from 2015. How many of each are there? In this more complex processor, we could get away with one feedline for all readout resonators! You will see the trick or the astucein the next Part !To motivate this next Part on scalable circuit QED, note that all of these processors have a common form factor (roughly 2 mm by 7 mm) and a common location of the up to eight ports connecting them to the external world.



Leonardo DiCarlo, Superconducting quantum circuits: Circuit QED

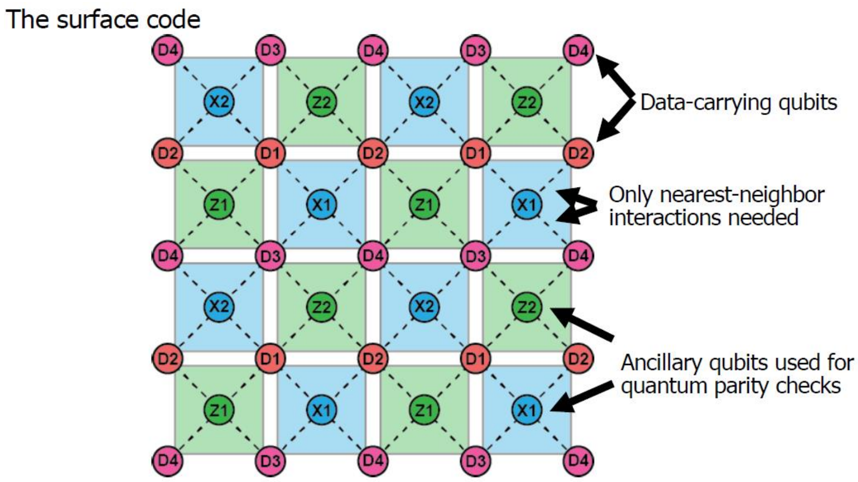
This reflects the use of a common printed circuit board throughout all these years! Surely a new approach must be taken to increase the qubit count! And indeed, the third generation of quantum processors completely rethinks the approach, with extensibility to hundred qubits as the top priority! I can't wait to show you!.

Main takeaways

- **CIRCUIT QUANTUM ELECTRODYNAMICS (QED) INVOLVES THE COMBINING OF QUANTUM HARDWARE WITH RESONATORS.**
- **IN THE SECOND-GENERATION PROCESSORS FOR SUPERCONDUCTING QUBITS, THERE ARE DEDICATED RESONATORS FOR READOUT OF EACH QUBIT AND A COMMMON 'BUS' RESONATOR CONNECTING TO ALL THE QUBITS.**
- **THE FREQUENCY OF A READOUT RESONATOR DEPENDS ON THE QUBIT STATE; AND THIS DEPENDENCE IS THE KEY INGREDIENT THAT FACILITIATES THE MEASUREMENT OF QUBITS.**
- **BY MEDIATING THE INTER-QUBIT INTERACTION ON THE COMMON 'BUS' RESONATOR, IT IS POSSIBLE TO PERFORM TWO-QUBIT OPERATIONS.**

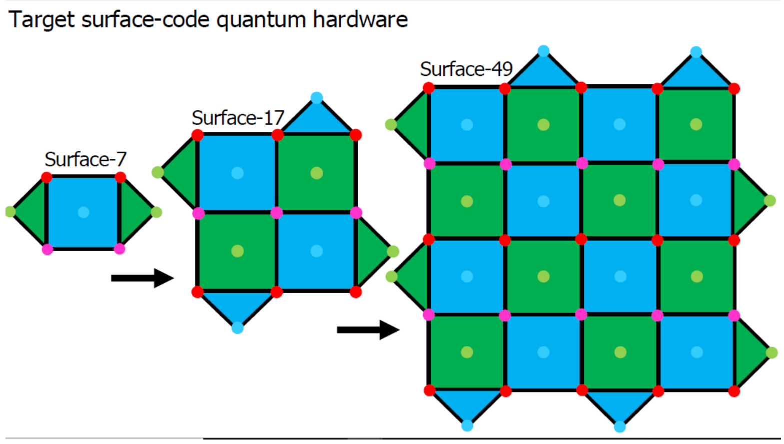
Assembling a quantum processor

Superconducting qubit : Assembling a quantum processor -Leonardo DiCarlo Hi! Today I will show you how we can assemble a quantum processor from circuit QED quantum hardware. I will focus on the approach to fault-tolerant quantum computing named Surface Code, which we pursue within QuTech.Surface code calls for a 2-dimensional square lattice of qubits with only nearest-neighbor interactions In addition, these qubits must be individually addressable both for single-qubit gating and for measurement.



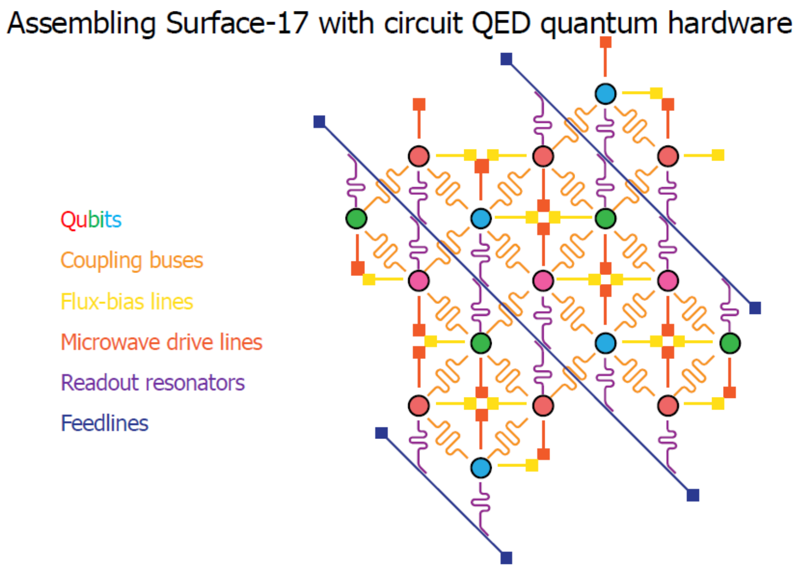
Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

Currently, we are testing surface-code chips with 7, 17 and 49 qubits. We call these Surface-7, Surface-17, and Surface-49. We assemble all of these using a common approach that we believe scales to larger surfaces. I will now describe this approach using the specific example of Surface-17.



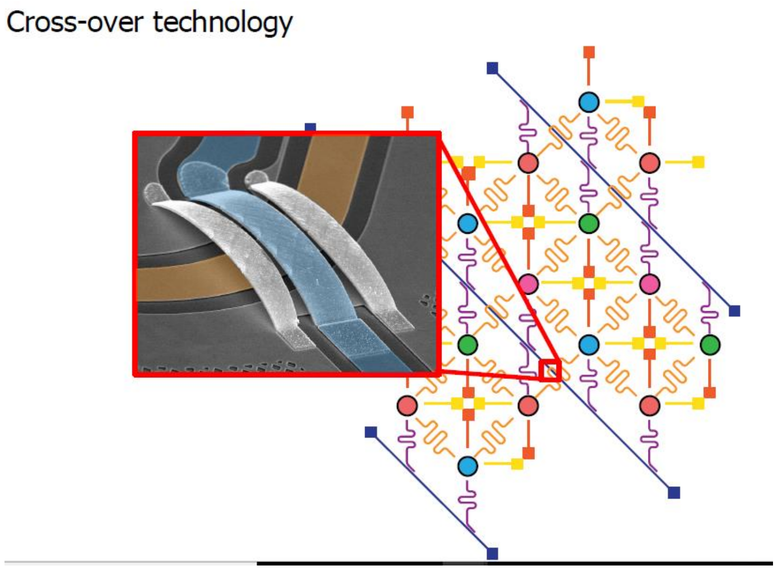
Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

First we layout the square lattice of qubits. Let me symbolize the qubits by circles. Please disregard their assigned color, for now. To perform two-qubit conditional-phase gates between nearest neighbors, as presented by Adriaan, we add, first, a coupling bus resonator to interconnect them; and second, a dedicated flux-bias control line to each qubit.



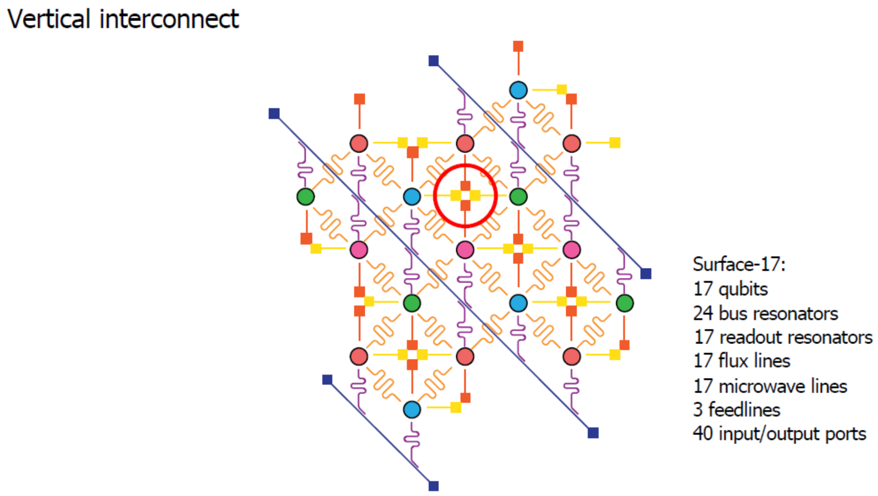
Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

To perform single-qubit gates, we follow the approach introduced by Brian, adding a dedicated microwave drive line to each qubit. Finally, in order to measure each qubit individually, we add dedicated readout resonators.



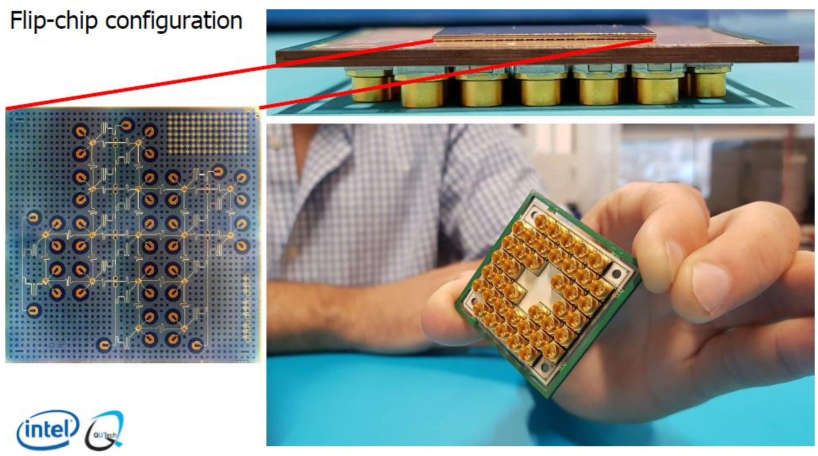
Leonardo DiCarlo ,Superconducting quantum circuits: Assembling a quantum processor

These readout resonators are coupled to diagonally running feedlines and probed independently using frequency division multiplexing as described by Niels.You may have already noticed the crossing of transmission lines on the chip, which is not possible in a truly planar structure. For this, we make use of the third dimension, in the form of air-bridge crossovers. Cool stuff, isn't it? Beautiful. So let's look at the totals for Surface-17: 17 qubits, 24 buses, 17 readout resonators, 17flux lines, 17 microwave drive lines, and 3 feedlines.



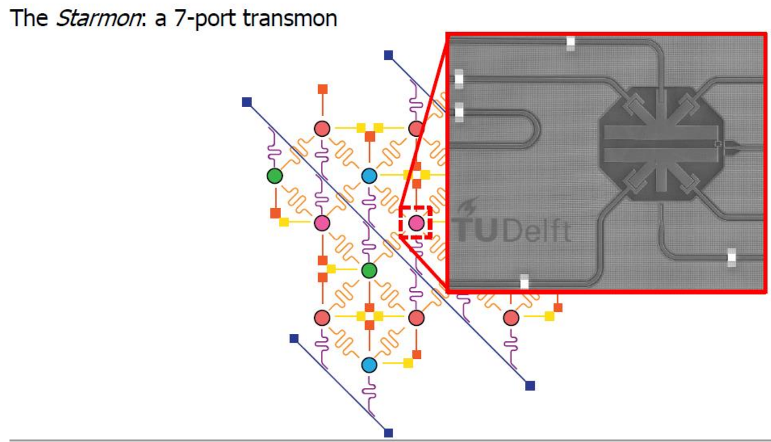
Leonardo DiCarlo ,Superconducting quantum circuits: Assembling a quantum processor

The grand total of ports connecting the quantum chip to the outside world is 40. We call these ports vertical I/O ports because they connect to the outside world not via the edges of the planar chip, as traditionally done, but vertically. In one approach, which we purse together with Intel, the quantum chip is flipped and the ports connect directly to a multi-layer printed circuit board using a ball-grid array.



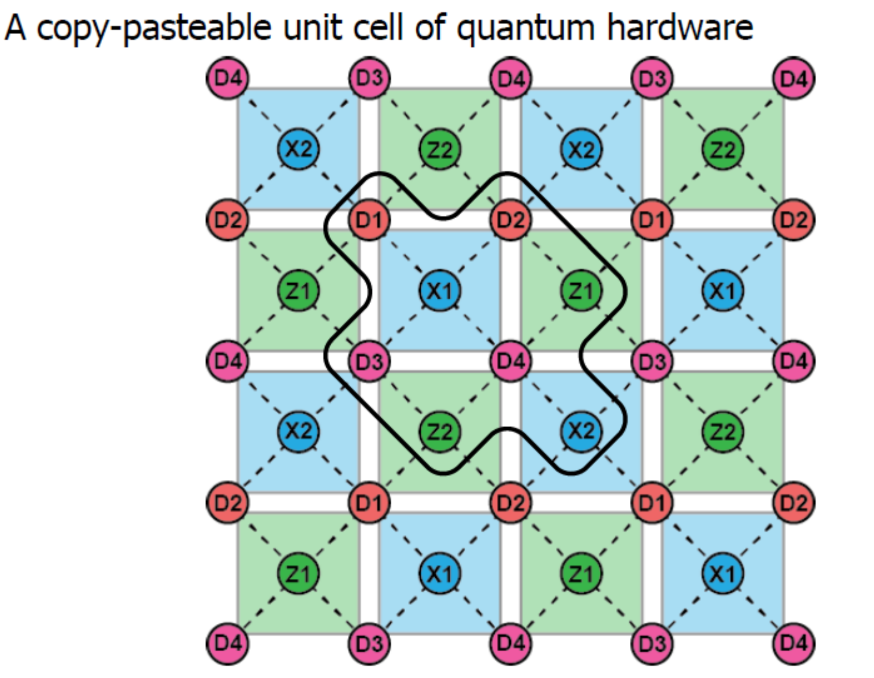
Leonardo DiCarlo ,Superconducting quantum circuits: Assembling a quantum processor

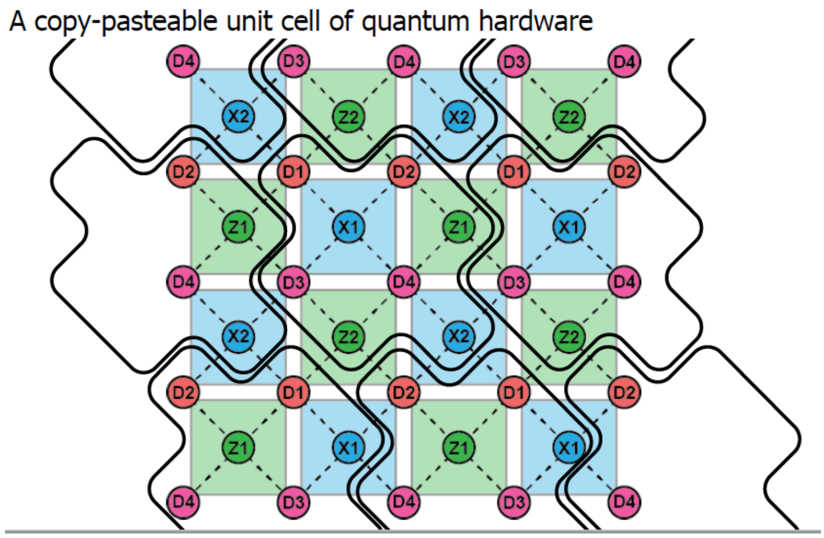
Achieving reliable vertical interconnect has been a key pursuit in our field over the last few years! Now, let's go back to the colored circles representing our transmon qubits. Except on the edges, qubits couple to 7 objects: 4 buses, 1 readout resonator, 1 flux line, and 1 microwave
line. These interconnectivity requirements give our transmons a characteristic shape and nickmane, Starmon.



Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

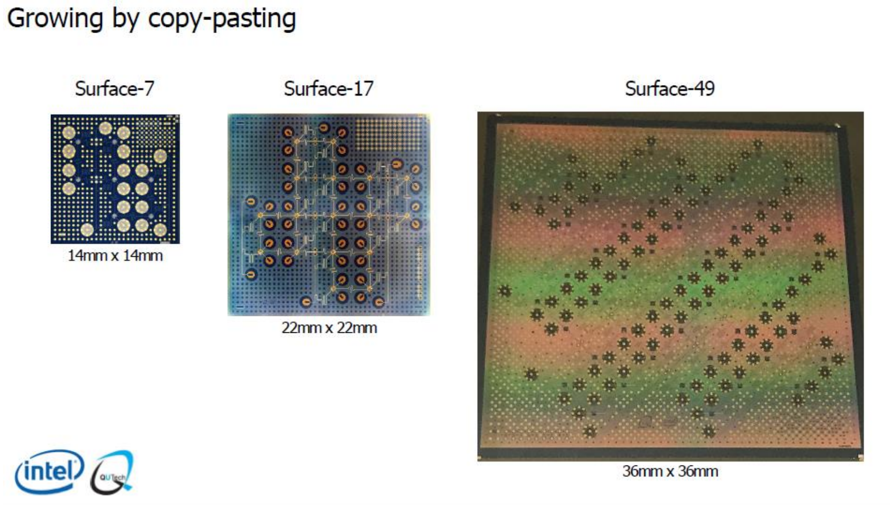
The colors we assign to the circles denote the qubit operating frequency, at which single-qubit gates are performed.

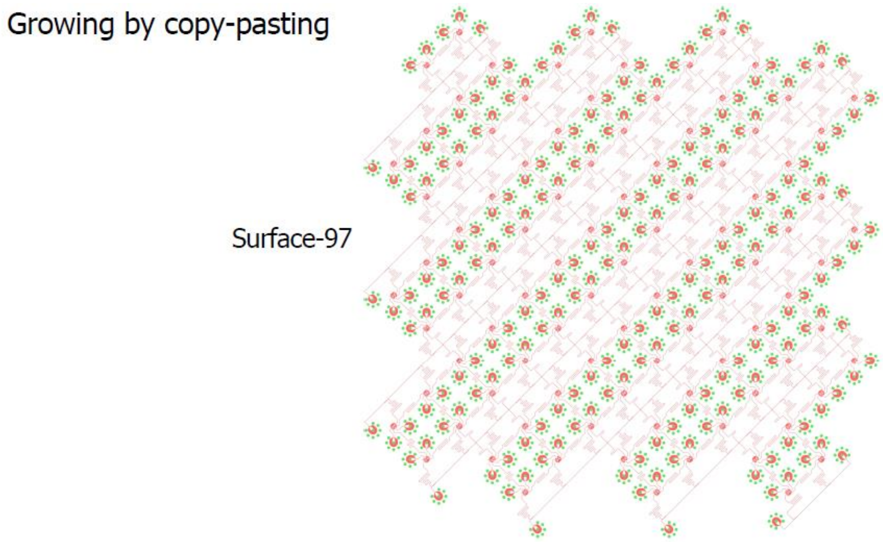




Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

In total, four frequencies suffice to control a surface-code of any size! This affords us significant savings in the microwave-frequency control electronics (which unfortunately, I don't have time to discuss here).





Leonardo DiCarlo, Superconducting quantum circuits: Assembling a quantum processor

But with regards to the quantum hardware, this repetition of qubit frequencies allows us critically to define an 8-qubit unit cell. By exactly replicating this 8-qubit unit cell and performing the necessary truncation at the boundaries, we can build a surface code of arbitrary size! In fact, we put together Surface-49 test chips in this very way, literally copy-pasting and truncating. We believe this approach will scale to even larger surface codes, such as Surface-97. I leave you with the CAD drawing of such a future chip!.

Main takeaways

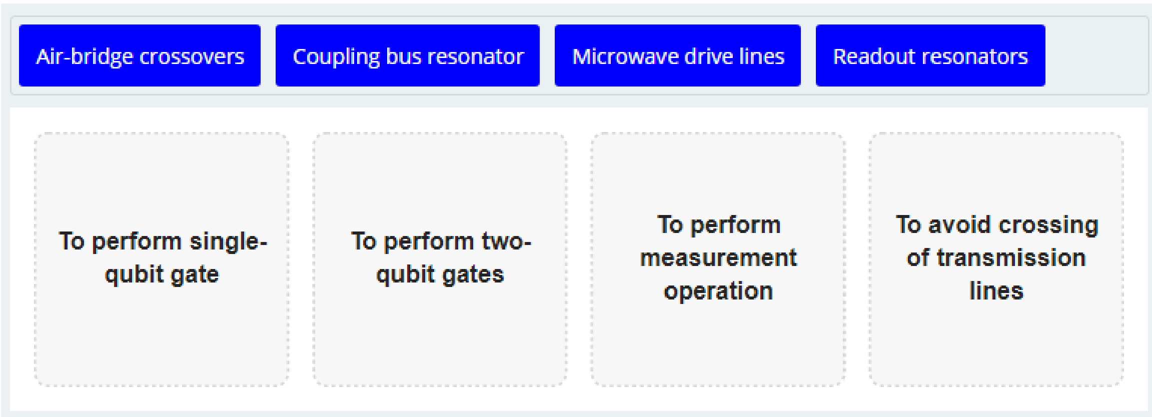
- Surface Code is a fault-tolerant approach of assembling a quantum processor from the hardware of circuit quantum electrodynamics.
- Surface Code is a 2-dimensional square layout of qubits facilitating nearest-neighbour interactions and individual qubit address-ability.
- Four frequencies are sufficient to control a Surface Code of any size. This leads to an arbitrary-sized Surface Code being composed of repetitive 8-qubit unit cells.

Practice Quiz 8

In this quiz we will be assembling a quantum processor.

QUESTION 1: ASSEMBLING A SURFACE CODE CHIP

In the Part you must have come across the purposes of various components used while assembling a Surface Code chip. In this problem, each of the four blue boxes have a component name on them. Below these boxes, there is a white rectangle with four square shaped zones placed in a horizontal manner. Each of these four zones denotes a purpose of the components in the blue boxes. Your job is to drag and drop each of the four blue boxes to the corresponding zone below. This means that you are expected to map a component on the chip to its correct purpose. Note that there is a strict one-to-one correspondence between the components and the purposes, meaning that there is a purpose (i.e. zone) for every component (i.e. blue box); and each zone can have one and only one blue box.



QUESTION 2: I/O PORTS ON SURFACE-17 CHIP

HOW DO THE INPUT-OUTPUT (I/O) PORTS ON THE SURFACE-17 CHIP DIFFER FROM THE I/O PORTS ON CONVENTIONAL CHIPS?

- THE I/O PORTS ON THE SURFACE-17 CHIP ARE MUCH THICKER THAN CONVENTIONAL I/O PORTS.
- THE SURFACE-17 CHIP IS WIRELESS.
- THERE IS NO NEED FOR INPUT OR OUTPUT.
- THERE ARE MUCH LESS WIRES NEEDED DUE TO MULTIPLEXING ALL THE SIGNALS INPUT SIGNALS.
- WE CAN CREATE A SUPERPOSITION OF INPUT AND OUTPUT SIGNALS ON THESE WIRES. THIS IS DUE TO THE FACT THAT 17 IS A PRIME NUMBER.
- THE I/O PORTS ON SURFACE-17 CHIP CONNECT VERTICALLY TO THE OUTSIDE WORLD; WHEREAS THE I/O PORTS ON CONVENTIONAL CHIPS CONNECT HORIZONTALLY TO THE OUTSIDE WORLD.
- THE I/O PORTS ON SURFACE-17 CHIP ARE JUST LIKE THE ONES ON CONVENTIONAL CHIPS.

QUIZ 8: SUPERCONDUCTING QUBIT

In this quiz, you will be tested on the content from the three sections on the Introduction to Superconducting Qubits series. Questions 1 to 7 are based on the section on the transmon qubit; question 8 briefly delves into concepts introduced in the Part on circuit QED.

The transmon qubit and LC oscillator

Having started on a lighter (and perhaps, an easier) note, let us now inspect how similar or different a transmon qubit is compared to an LC oscillator. For this purpose, we will first focus on a quantised LC circuit.

In the section, it has been mentioned that the angular frequency (ω) to transition from one level to the next in a quantised LC circuit is related to the inductance (L) and capacitance (C) values as follows:

ω=$1\sqrt{LC}$.

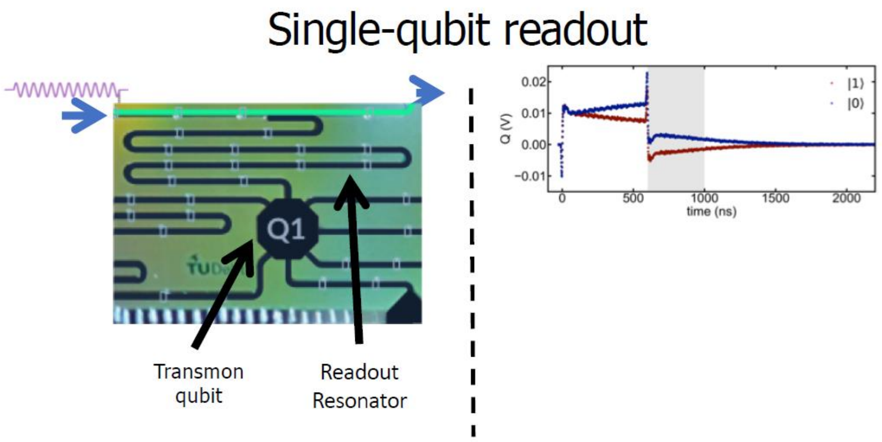
Now, the transition frequency (*f*) of that circuit is related to the angular frequency (ω) as follows:

ω=$2\pi\cdot f$.

Use this information to answer the following question.

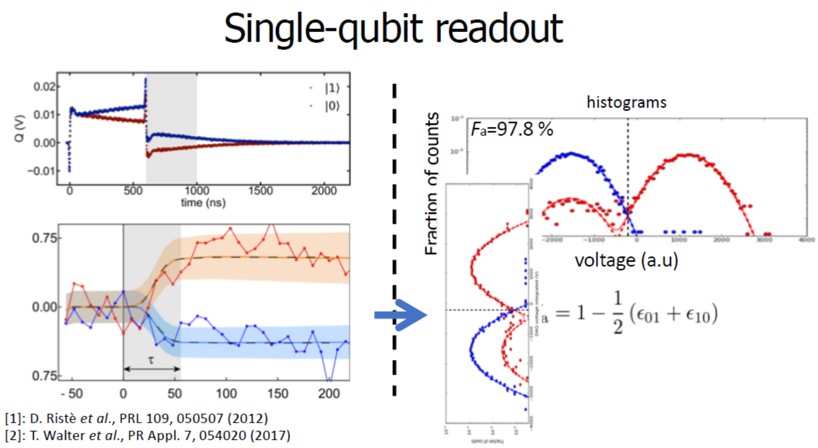
Measurement

Operations in superconducting qubits: Measurements -Niels BultinkIn this Part I will explain how we can perform measurement of superconducting qubits .



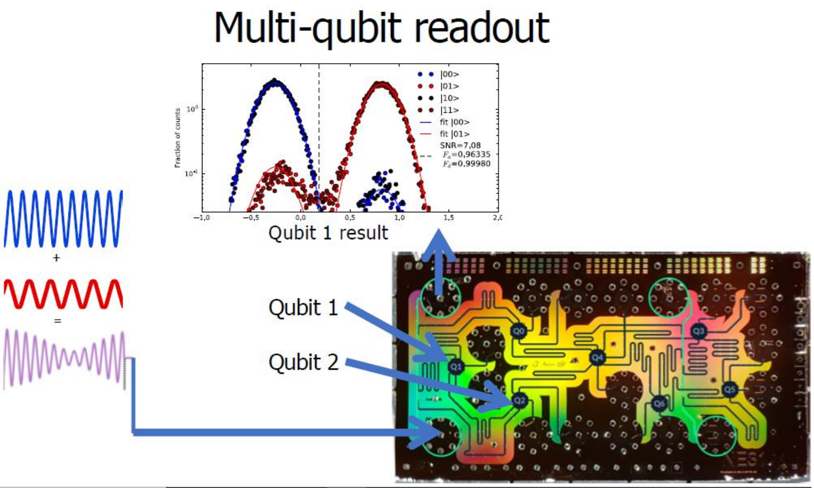
Niels Bultink, Operations in superconducting qubits:Measurements

At the end of every quantum algorithm, the computation result has to be obtained by performing measurement of the qubits. And, as dictated by the laws of quantum mechanics, any superposition states are projected into well-defined zeros and ones. A transmon qubit, as depicted here, can be measured via a readout resonator that is coupled to it. The resonance frequency of the resonator is quite far away from the qubit transition frequency, on the order of GHz. However, due to the coupling, there is a shift in the resonator's frequency depending on the qubit state.



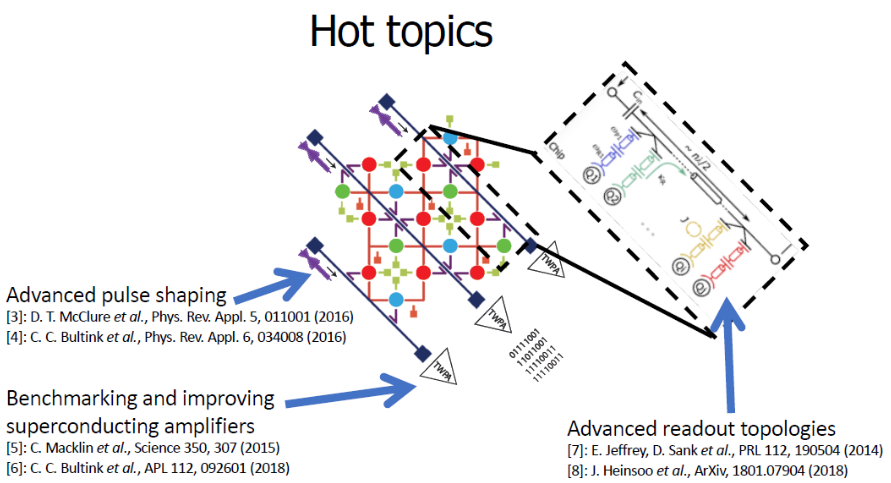
Niels Bultink, Operations in superconducting qubits:Measurements

This shift is evidenced here by the measured resonator transmission dips of the qubit in the ground state and the excited state. This shift is typically on the order of a few megahertz, three orders of magnitude smaller than the detuning. We can observe this shift (and thereby the qubit state) by injecting the resonator with a pulse near the resonator frequency. The pulse, as shown here, is reflected by the resonator. Here, we show the output voltage as a function of time for the qubit in the ground state and the excited state, which clearly look different. Hence, we are able to distinguish and measure the two different qubit states. The traces I've just shown you, look very clean and distinguishable as they are averaged over thousands of measurements. However, for most real quantum protocols, we require to discern the qubit state in a just single run. But to avoid disturbing our precious quantum system, we can only use readout pulses that consist of a couple of photons. At these power levels, in the order of just a few femto Watts, we are struggling to distinguish the different quantum states as the signal is hampered by quantum noise, as depicted here.To quantify how well qubit measurement is performing in the presence of this noise, we record the integrated voltage of thousands of individual traces. By plotting these individual shots in the histograms, we extract the fidelity of the measurement. The fidelity expresses the probability that the measurement returnsthe right outcome averaged over the two possible qubit input states. Epsilon01 expresses the probability for erroneously getting outcome 1 for a ground state input and Epsilon10 vice versa. In this case, aided by the world's lowest noise, superconducting amplifiers we can achieve a fidelity close to 98%.I've just shown you the readout of one qubit. Of course, a full quantum computer consists of many quantum bits and these quantum bits all have to be read out at the same time. To allow this simultaneous readout, we couple each qubit to its own readout resonator and choose the lengths of the resonators such that they resonate all at a distinct frequency. Just like the different strings in a piano.



Niels Bultink, Operations in superconducting qubits:Measurements

By next sending down a pulse that is the sum of multiple components, in this case two, each tuned to its own readout resonator, we can probe multiple readout resonators at the same time. Each component will only be picked up by the targeted resonator. In the analysis of the output signals, we again separate the two different frequency components. Here, we show the readout results for qubit 1 for each of the four possible two-qubit basis states, indicating that the signal practically only depends on the state of the targeted qubit, separating 00 and 10 from 00 and 11.Dually, we here show the readout results for the other qubit, which separate 00 and 10 from 00 and 11.You might now be able to imagine how this readout scheme can be extended to measure ever increasing numbers of qubits. Here, we depicted a seventeen-qubit device where we play the simultaneous readout trick using seventeen readout resonators divided over three different input and output lines.



Niels Bultink, Operations in superconducting qubits:Measurements

This might sound to you like a finished story. However, there are many improvements to be made still to get to afully scalable quantum computer. Exemplary topics that are currently being addressed in research, and which might also be attractive for additional reading are: Advanced readout pulse shaping to quickly populate and depopulate the readout resonators to speed up the readout. Second, superconducting amplifiers to suppress noise in the readout amplification chain and methods to quantify their performance. And lastly we work on advanced readout topologies. In this case, using multiple cascaded readout resonators per qubit, to increase the flux of photons and further speed up the readout.

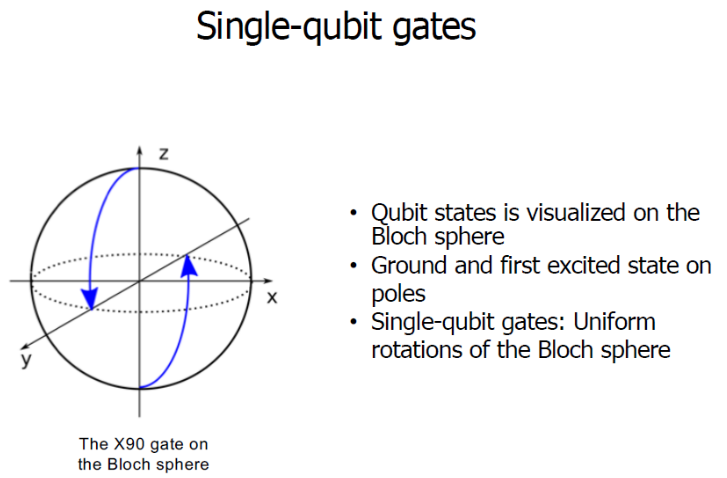
Main takeaways

- A transmon qubit can be measured via a readout resonator that is coupled to it. Depending on the qubit state, there will be an observable shift in the resonator frequency.
- To quantify how well qubit measurement is performing in the presence of this noise, we record the integrated voltage of thousands of individual traces. By plotting these individual shots in the histograms, we extract the fidelity of the measurement.
- The highest achievable fidelity aided by the world's lowest noise, superconducting amplifiers is 99%.
- To allow simultaneous readout for multiple qubits, each qubit will be coupled to its own resonator with unique lengths and thus readout frequency.

Single qubit gates

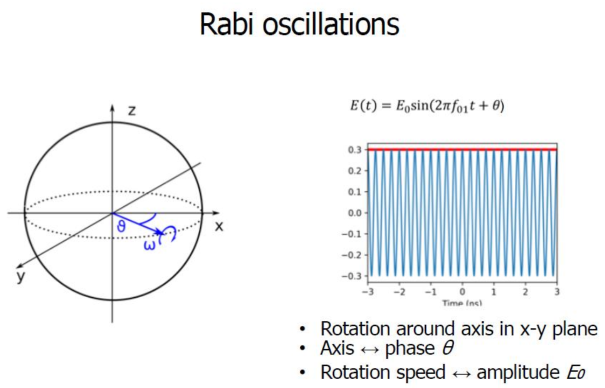
Having learnt about measurement, we will now dive deeper into single qubit gates.

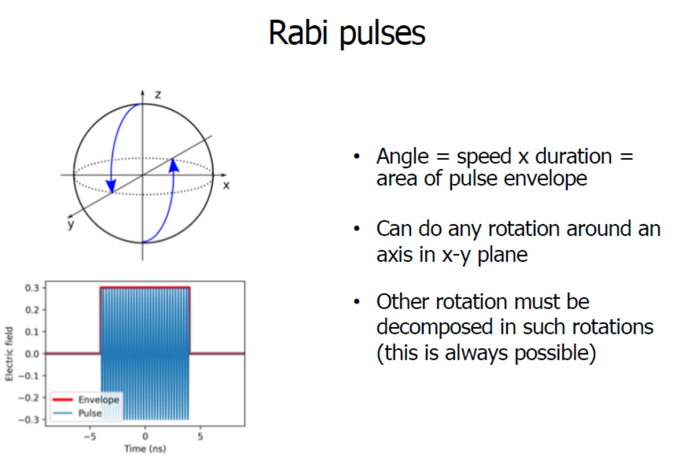
Operations in superconducting qubits: Single qubit gates-Brian TarasinskiIn this Part , I want to show you how we control the state of a single superconducting transmon qubits, that is, doing so-called single-qubit gates.



Brian Tarasinski ,Operations in superconducting qubits: Single-qubit gates

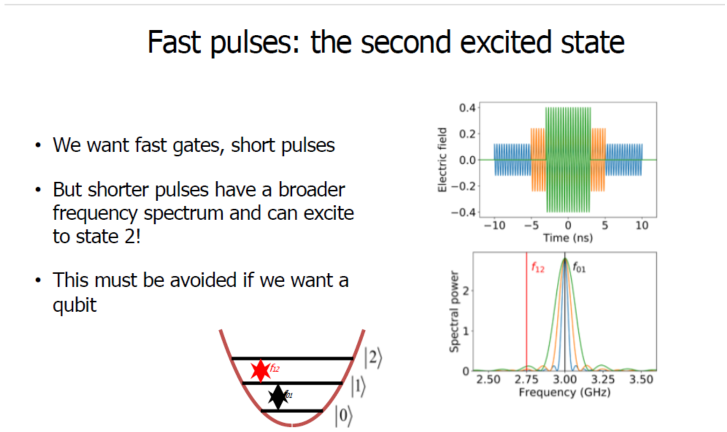
The state of a single qubit can be visualized on the Bloch sphere, with the ground and excited state on the poles and the other points on the surface corresponding to quantum superpositions. Single-qubit gates are then rotations of the Bloch sphere, for instance here the X90 gate, which converts the ground and excited states into superpositions.We use the physical effect called Rabi oscillation:





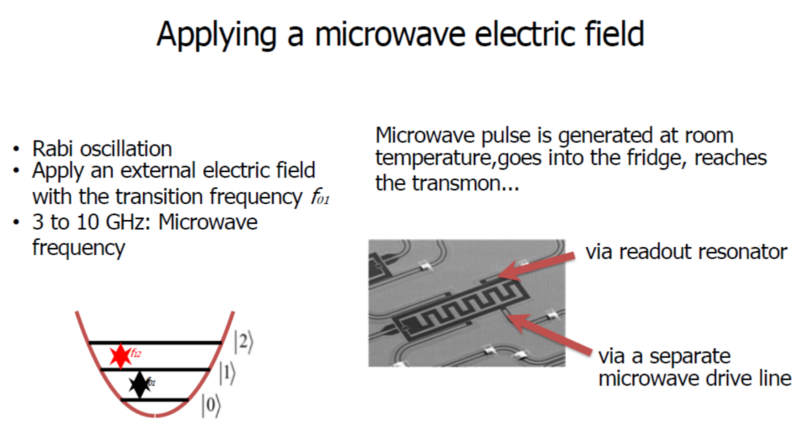
Brian Tarasinski ,Operations in superconducting qubits: Single-qubit gates

We change the qubit state by applying an external oscillating electric field at the qubit frequency, corresponding to the energy difference between the ground and excited state.



Brian Tarasinski, Operations in superconducting qubits: Single-qubit gates

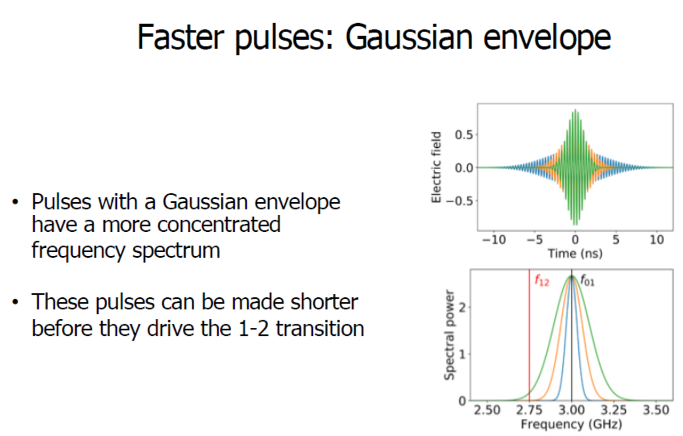
That frequency is usually in the microwave range, between 3 and 10 GHz. To apply the electric field, we need a microwave line that ends next to the transmon. We can use the same line that is also used for measurements, about which you will hear more in the next Part. Or, on larger devices with many qubits, we often make a “dedicated drive line” for each qubit. With this, we can generate the electric field close to only one selected qubit. Let’s see how the oscillating field affects the qubit state. When we apply it, it drives the qubit from the ground to the first excited state and back. On the Bloch sphere, this looks like a rotation with constant speed. The axis of rotation always liesin the x-y plane. We can control where exactly it lies by changing the phase of the applied field: A sine wave leads to a rotation around the x-axis, while a cosine with 90 degrees phase offset will induce a rotation around the y-axis.On the other hand, the speed of the rotation is proportional to the amplitude of the electric field.



Brian Tarasinski ,Operations in superconducting qubits: Single-qubit gates

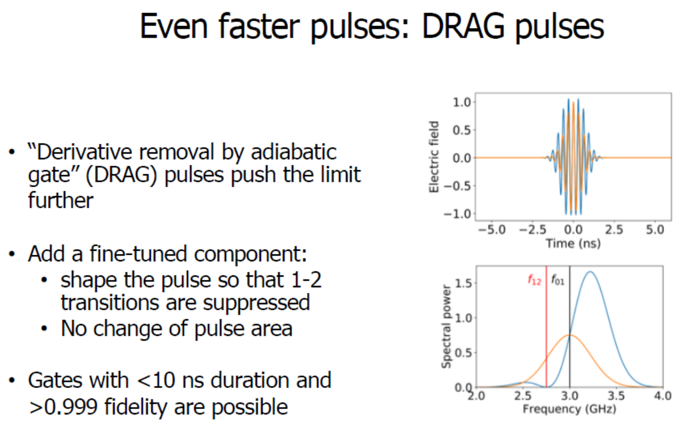
In order to perform a desired gate, for instance a rotation by 90 degrees around the x-axis, we thus need to apply a short pulse with the correct phase, amplitude and length.The rotation angle is determined by the product of length and amplitude, that is, by the area under the pulse envelope .This way, we can perform any rotation with the axis in the x-y plane. If we want to rotate around another axis, which does not lie in the x-y plane, we have to decompose it. It turns out that any single-qubit gate can be performed using no more than three microwave pulses in sequence. Of course, we want to make the pulses as short and strong as possible, to be able to do quantum computations quickly. However, the shorter the pulse, the more frequency components it has besides the transition frequency f01. We can see this by looking at the spectral decomposition of the pulse. Unfortunately, that means that if we make the pulses shorter and shorter, at some point, the spectrum is so broad that the pulse drives oscillations not only between the states 0 and 1, but also 1 and 2!

Then, the transmon does not work as a qubit anymore, since three transmon states are involved, and the nice Bloch sphere rotation picture breaks down. We must avoid this situation, and this limits us in making the pulses too short. To push the limit of how short we can make the pulse, instead of a square pulse, we need a pulse which is well localized in time and also frequency. A much better choice is a pulse with a Gaussian shaped envelope.



Brian Tarasinski ,Operations in superconducting qubits: Single-qubit gates

It has the same area, but affects the 1-2 transition much, much less. Using a technique called “derivative removal by adiabatic gate”, or DRAG, we can push the limit further: By superimposing a fine-tuned out-of-phase component with an envelope proportional to the derivative of the original pulse, the rotation angle is unchanged, but the transitions to the 2 state are actively suppressed.



Brian Tarasinski ,Operations in superconducting qubits: Single-qubit gates

Using this technique, pulse lengths can be reduced to well below 5 ns without the second transmon state being a concern. This concludes our quick look at how we control the state of individual transmon qubits using microwave pulses. In the next Part , Niels and Adriaan will show you how we measure the transmon state, and how we change the state of a transmon depending on the state of another transmon, to perform larger calculations.

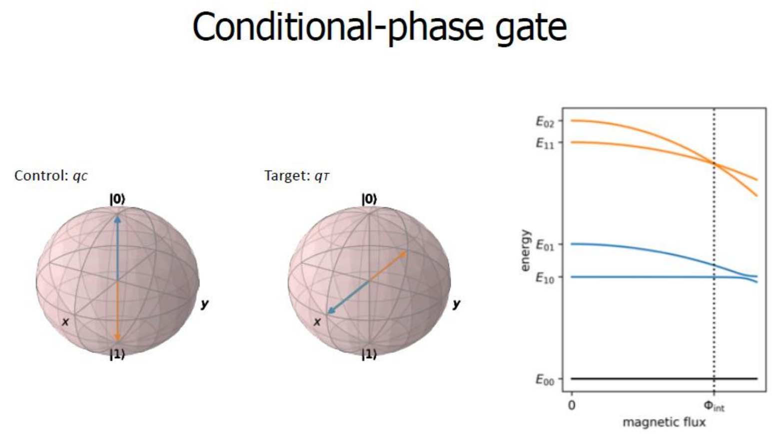
Main takeaways

- Qubit operations are done using Rabi oscillation by applying an external oscillating electric field at the transmons qubit.
- Performing desired gates require a short pulse with the correct phase, amplitude and length. This way any rotation within the x-y plane can be performed.
- The minimum duration of a pulse is limited due to the broadening of the frequency spectrum as pulses get shorter.
- In case the frequency spectrum is to broad, not only oscillations between the 0 and 1 state occur, but also between 1 and 2.
- To push the limit of this short pulse duration, a Gaussian shaped envelope is used in combination with a technique called Derivative Removal by Adiabatic Gate (DRAG).

Two qubit gates

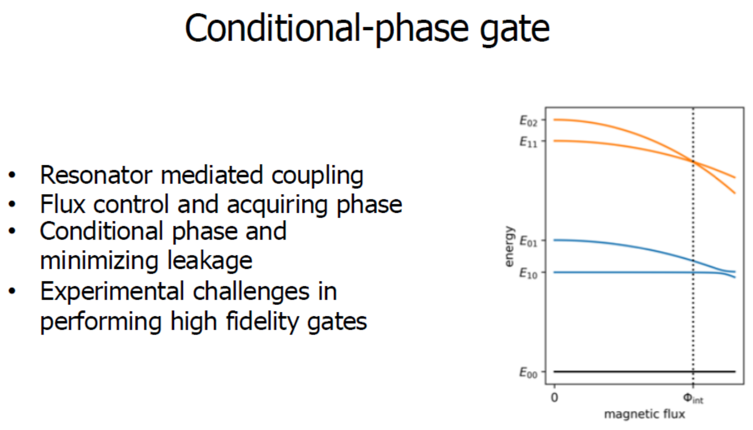
Closing the Part series of operations on superconducting qubits, Adriaan Rol will tell you more about two qubit gates.

Operations in superconducting qubits: Two-qubit gates-Adriaan Rol Interactions between qubits are controlled using two-qubit gates.



Adriaan Rol, Operations in superconducting qubits: Two-qubit gates

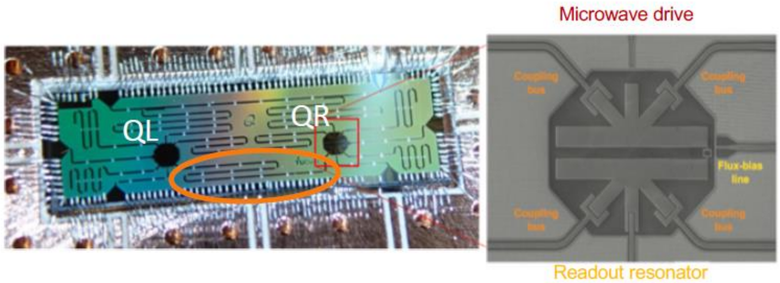
In transmon qubits the two-qubitgate that is used to form the universal gate set is the conditional-phase gate, also known as a CPhase-or CZ-gate. Applying a conditional-phase gate to two qubits causes the target qubit to acquire \pi radians of phase based on the state of the control qubit.



Adriaan Rol, Operations in superconducting qubits: Two-qubit gates

As an example let us consider two qubits, a control qubit, qC starting in either the ground or the excited state, and a target qubit, qT starting the in |+> state. When the control qubit is in the ground state and we apply the C-Phase gate there is no effect on the target qubit, however, when the control qubit is in the excited state the target qubit will acquire \pi radians of phase. In transmon qubits, the C-Phase gate is implemented by tuning in and out of resonance with an interaction in the two-excitation manifold. To understand how to perform such a C-Phase gate, we will look at •what mediates the interaction; • how qubits can be tuned in and out of resonance with an interaction and why this causes the qubits to accumulate phase; • how to use this interaction to perform a C-Phase gate while minimizing leakage out of the computational subspace, and finally; • what the experimental challenges are when implementing high fidelity gates. In superconducting transmon qubits two-qubit gates are based on a transversal qubit-qubit coupling. This coupling is mediated through a coupling resonator.

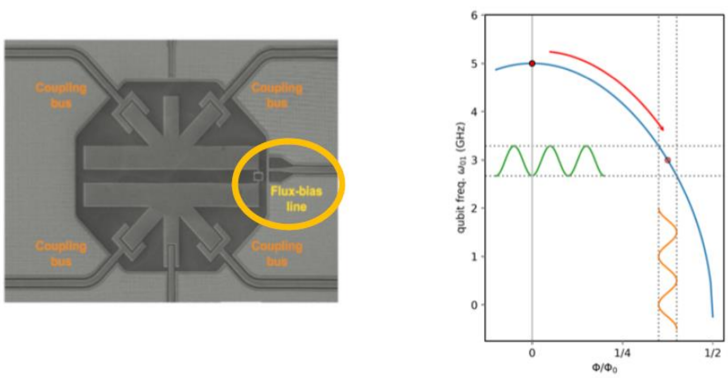
Resonator mediated coupling



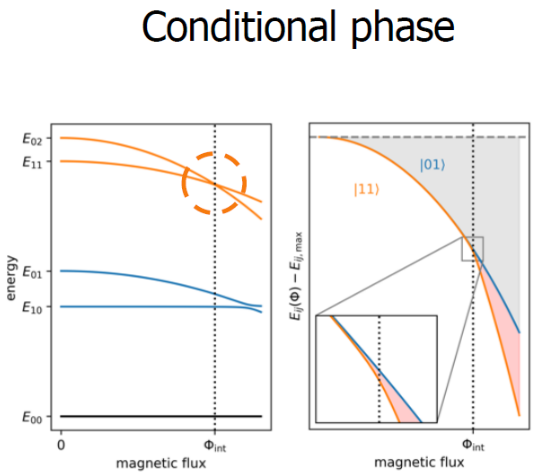
Adriaan Rol ,Operations in superconducting qubits: Two-qubit gates

Because the interaction is mediated by a coupling resonator, it is possible to couple physically separated qubits, making room for other components on the chip, such as readout resonators. To be able to tune in and out of resonance with interaction the transition frequency of a transmon qubit must be controlled. By applying a current to the flux-bias line, the amount of flux through the SQUID loop of the transmon changes, making it is possible to control the qubit's frequency. When a pulse is applied to the flux-bias line, the qubit is detuned from it's operating frequency for the duration of the pulse, while the qubit is detuned, phase accumulates. To understand how this flux control can be used to perform a C-Phase gate, let us take a look at the level diagram of two transmon-qubits as a function of the flux through the target qubit.

Flux control

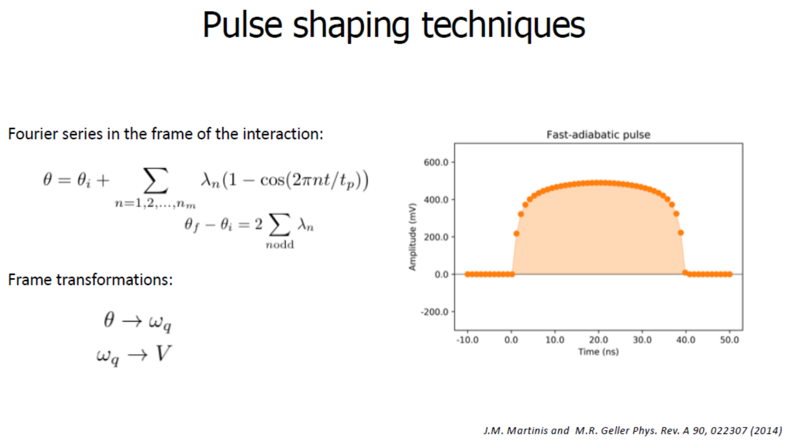


In this level diagram, the subscripts denote the number of excitationsin the control-and target-qubit respectively. The interaction that is used to perform a C-Phase gate is an avoided crossing between the |11> and |02> state. The amount of phase that the target qubitpicks up is most evident when the |11> and |01> level diagrams are overlaid, by expressing them in terms of detuning with respect to their maximum energy. It can be seen that the detuning of the target qubit is different based on the state of the target qubit. This difference is marked in red over here. By detuning the qubit into the red region, it is possible to make the target qubit acquire a phase conditional on the state of the control qubit. However, the interaction we are using is an interaction between the |11> and |02> state. As the |02> state is not part of the computational subspace, we want to avoid any energy transfer to this state.



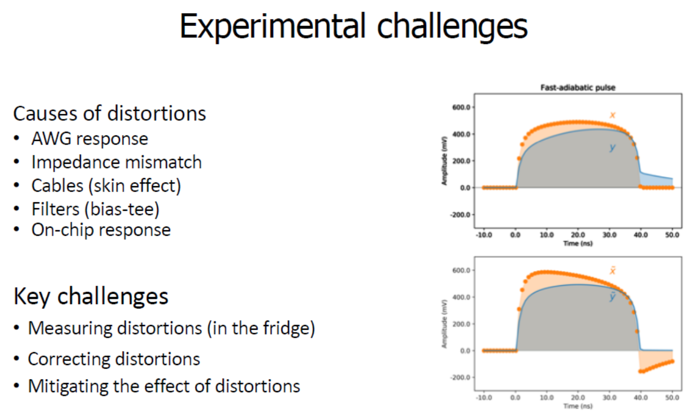
Adriaan Rol, Operations in superconducting qubits: Two-qubit gates

This is typically done using a special pulse-shape known as a fast-adiabatic pulse that minimizes the leakage.



Adriaan Rol ,Operations in superconducting qubits: Two-qubit gates

This pulse shape can be expressed as a Fourier series in the frame of the interaction, which, with the knowledge of the system parameters can be converted first into a frequency and then into a voltage to be applied to the flux-bias line.



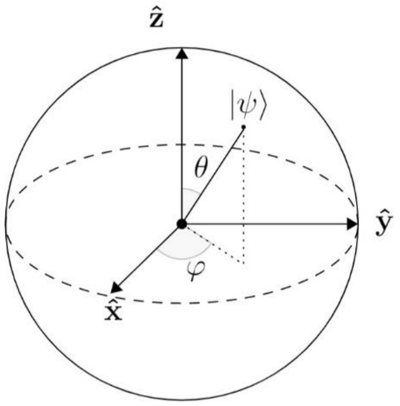
Adriaan Rol ,Operations in superconducting qubits: Two-qubit gates

Because the conditional phase and the leakage depend on the exact trajectory of the qubit, flux-pulsing-based two-qubit gates are highly sensitive to distortions of the pulse shape.Distortions can be caused by electrical components in the signal path between the waveform generator and the qubit such as filters and cables but even the on-chip response causes distortions to the signal the qubit experiences. These effects are typically corrected by pre-distorting the waveform “x” with a filter designed to invert the distortions, turning it into “x-tilde”, so that the qubit experiences, not a pulse “y”, but a pulse “y-tilde” that is equal to the intended waveform. The key challenge in flux-pulsing based CZ gates is to correct for these distortions. This requires characterizing the distortions that the qubit experiences when cooled down and correcting these with sufficiently high precision. At the same time, efforts are under way to become more resilient against these effects, both by exploring new pulsing shaping techniques and through innovations in hardware.

Main takeaways

- In transmon qubits, a the two qubit CPhase- or CZ-gate is used to form the universal gate set, which is implemented by tuning in and out of resonance with an interaction in the two-excitation manifold.
- The qubit-qubit coupling in transmons is mediated through a coupling resonator allowing convenient physical separation between qubits.
- Applying a current to the flux-bias line, the flux through the SQUID loop of the transmon changes allowing control of the qubit's frequency.
- To avoid energy transfer to undesired states, a fast-adiabatic pulse that minimizes leakage is used.
- The key challenge in flux-pulsing based CZ-gates is correcting for distortions caused by electrical components in the signal path between the wave generator and the qubits.

Practice Quiz 9



QUESTION 1

The Bloch sphere is a useful tool to visualize the state of qubit and the effect of single-qubit operations. What state (\ket{\psi}\) corresponds to the state shown on the Bloch sphere below?

- cosφ2|0⟩+eiφsinθ2|1⟩.
- cosφ2|0⟩+eiφsinφ2|1⟩.
- cosθ2|0⟩+eiφsinθ2|1⟩.
- cosφ|0⟩+eiθsinφ|1⟩.

QUIZ 9: OPERATIONS ON SUPERCONDUCTING QUBITS

BELL PAIRS

The Bell pairs are a certain set of entangled two qubit states. There are four of them, often denoted by $|\Phi+\rangle,|\Phi-\rangle,|\Psi+\rangle\&|\Psi-\rangle$. All of these play a vital part in quantum information and networking, but $|\Phi+\rangle$ is used the most. It is the state $|00\rangle+|11\rangle 2\surd$ and it is exactly this state that is often meant when scientist say ‘a Bell pair’.
In this quiz, we will investigate how to create this Bell pair on a transmon qubit chip.

Creating the Bell state from the 0 state
In quantum computing, we always initialize qubits in the $|0\rangle$ state. So when creating a Bell pair, we start with two qubits in that state: $|\psi init\rangle=|0\rangle\otimes|0\rangle=[1000]$. If we perform a Hadamard transform on the first qubit, we get the state $|\psi 2\rangle=(H\otimes I)|0\rangle\otimes|0\rangle=H|0\rangle\otimes I|0\rangle=|+\rangle\otimes|0\rangle=1\surd 2(|0\rangle\otimes|0\rangle+|1\rangle\otimes|0\rangle)$.
In matrix form, this is $|\psi 2\rangle=\surd 12[1010]$.
Now that we are able to do a CNOT, we can create a Bell pair by applying the CNOT to the state $|\psi 2\rangle$. The second qubit is bit flipped only if the first qubit is in the $|1\rangle$ state, so we see that only the $|10\rangle$ part is 'touched' by the CNOT. We get: $CNOT|\psi 2\rangle=CNOT\surd 12(|00\rangle+|10\rangle)=\surd 12(|00\rangle+|11\rangle)=|\Phi+\rangle$.
Or in matrix form: $CNOT|\psi 2\rangle=\surd 12[1000010000010010][1010]=\surd 12[1001]=|\Phi+\rangle$
Conclusion: to create a Bell pair on a set of transmon qubits, we need to perform one single-qubit operation (the Hadamard) and one multi-qubit operation (the CZ).

Now that we have a method of preparing Bell pairs, we might want to analyse these Bell pairs by measuring some of them, and checking if we indeed have the desired correlations. If we want to measure the qubits we need to couple them to readout-resonators, as explained in the Part.
To measure the qubit, we pulse it with an alternating electric signal. This signal is reflected, but also influenced by the state that the qubit is in. Therefore, by measuring the reflected signal, we can know in what state the qubit is in.

Learn more

To go more in depth into the working principles of transmon qubits, we invite you to read the following references:

The standard theory reference
J. Koch, et al., Charge-insensitive qubit design derived from the Cooper pair box, Physical Review A 76, 042319 (2007).

The standard experimental reference
J. A. Schreier, et al., Suppressing charge noise decoherence in superconducting charge qubits, Physical Review B 77, 180502 (2008).

Two very accessible blog articles by C. Dickel
How to make artificial atoms out of electrical circuits - part 1
How to make artificial atoms out of electrical circuits - part 2

Individually gating same-frequency qubits
S. Asaad, C. Dickel, N. K. Langford, S. Poletto, A. Bruno, M. A. Rol, D. Deurloo, and L. DiCarlo, Independent, extensible control of same-frequency superconducting qubits by selective broadcasting, NPJ Quantum Information 2, 16029 (2016).

Circuit QED architecture
A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation, Physical Review A 69, 062320 (2004).

Surface coding
A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, Surface codes: Towards practical large-scale quantum computation, Physical Review A 86, 032324 (2012).

For further details into the approach used in Professor Dicarlo's lab for **building surface-code quantum harware using an 8-qubit unit cell**
R. Versluis, S. Poletto, N. Khammassi, B. Tarasinski, N. Haider, D. J. Michalak, A. Bruno, K. Bertels, and L. DiCarlo, Scalable quantum circuit and control for a superconducting surface code, Physical Review Applied 8, 034021 (2017).

Module 5

In the previous modules we have discussed qubits realized with electron spins and superconducting circuits. Despite the differences in their hardware implementation, these platforms have something very important in common. The qubits are imperfect and subjected to noise. For example, fluctuations in magnetic fields can perturb an electron spin such that its phase is lost. This is why we need quantum error correction.

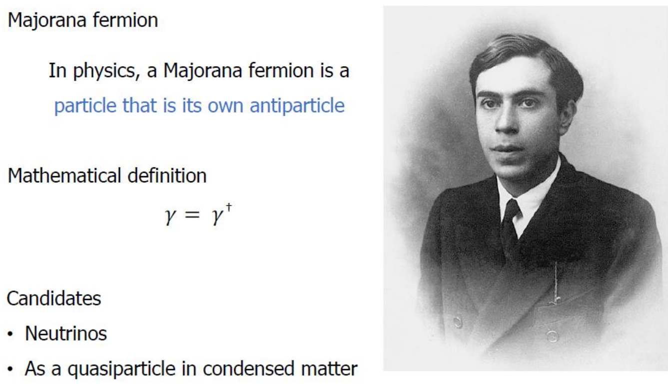
This week we will introduce you a completely new kind of qubit, which can potentially alleviate the need of quantum error correction in a future quantum computer. This qubit is based on a quasi-particle called Majorana fermion, which is halfway between being an electron and not being an electron. When you fuse two Majoranas together you end up having one or no electrons. The fact the that the information (having or not having an electron) is delocalized in two Majoranas far from each other, makes harder for the environment to perturb its state.

The promise of a qubit much less sensitive to noise than other implementations has attracted Microsoft's interest. Check out this section to discover what is, according to Microsoft, the path towards a scalable quantum system with Majoranas.

In the first three Section , Michael Wimmer will introduce the concept of topology and Majorana bound states. Attila Geresdi will then explain, in an other three Section , how we can realize quantum gates and in which kind of physical system we can find this new type of quasi-particles.

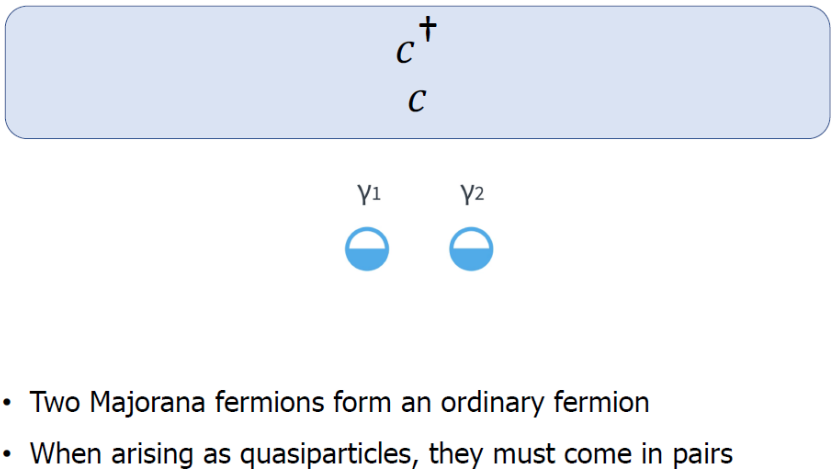
Majorana fermions and where to find them

In the next few Section. We want to introduce you to topological quantum computing. To understand what topological quantum computing is, we first need to introduce you to the basic building blocks which are the so called Majorana fermions or Majorana bound states.



Michael Wimmer, Majorana fermions and where to find them

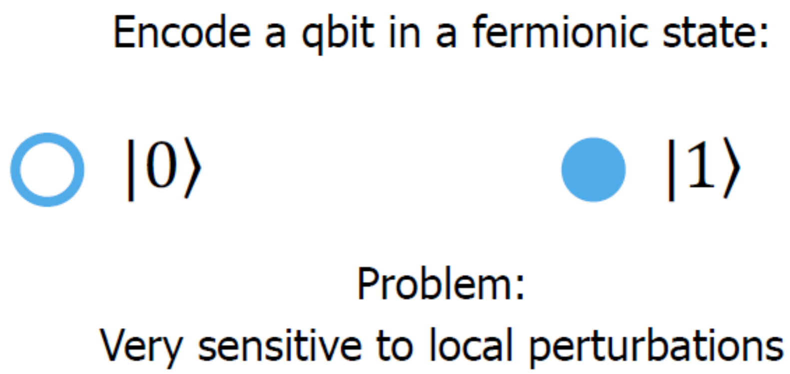
To understand Majorana fermions it is useful to first have a quick glimpse at high energy physics. In high energy physics we know that for every particle there is always an antiparticle. For example, there is an electron and positron, a proton and antiproton and so forth. These are examples of what we call Dirac fermions. However, there is also a separate kind of fermion in high energy physics which is called a Majorana fermion. It is special because it is a fermion which is its own antiparticle. The concept was actually first introduced by Ettore Majorana whose picture you can see here. Interestingly, to this date there are no Majorana fermions known in nature as elementary particles. This is actually very similar to Majorana's life story itself.



Michael Wimmer, Majorana fermions and where to find them

He himself vanished during traveling on a ship, nobody knows for sure what happened to him. Majorana fermions also could exist in principle, but nobody knows for sure if they do. The definition of a Majorana fermion is that it is a particle that is its own antiparticle. In physical terms this means that the creation operator equals the annihilation operator. There are some candidates for these Majorana fermions. For example, in high energy physics neutrinos are generally described as Dirac fermions, but there is an extension in which they could be Majorana fermions.

But people have also thought about how one could effectively realize Majorana fermions as quasi-particles in condensed matter systems. This will be the topic of this lecture. There is a little caveat here ; what we will find in condensed matter are not quite the Majorana Fermions that high energy physicists are looking for. What we find are states, and it’s actually more appropriate to call them Majorana bound states, which is the term I will be using from now on –or I will simply refer to them as Majorana.

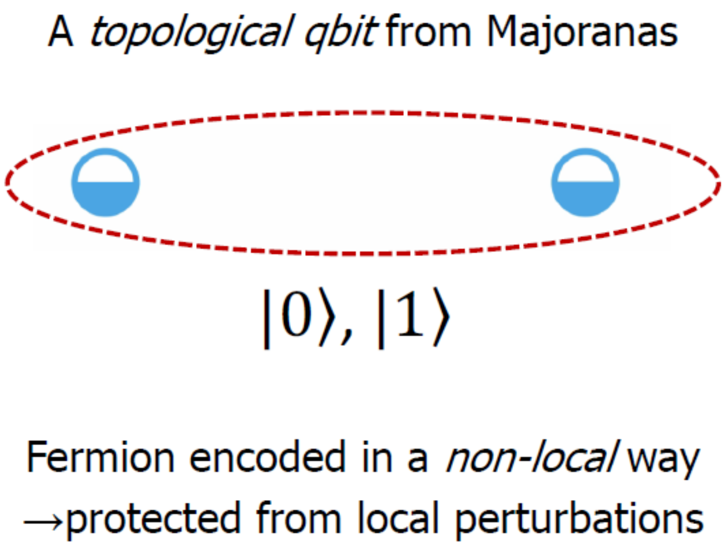


Michael Wimmer, Majorana fermions and where to find them

To introduce Majorana bound states, we can first do a simple mathematical trick. In condensed matter physics, a state can be filled, this is an electron, or it can be empty, in which case we have a hole. These are the equivalent of particle and antiparticle ,respectively. They are described by fermionic operators: creation operator c dagger for the electron and annihilation operator c for the hole. Now, I can do a simple linear superposition of operators: an equal superposition of a creation and annihilation operator. It is very easy to see, I urge you to do the mathematics yourself, that those operators are Majorana operators: Gamma 1 equals gamma 1 dagger, and gamma 2 equals gamma 2 dagger. Graphically, you can see that these states are at the same time occupied and unoccupied. Of course, this was just a mathematical trick: a transformation to go from one basis to another one. What you see here is that still, one ordinary Fermionic operator can always be described by two Majorana operators. However, once you can separate the two Majorana bound states, and they become distinguishable from each other, then things become interesting. We can summarize at this point: First, two Majoranas form one fermionic state.

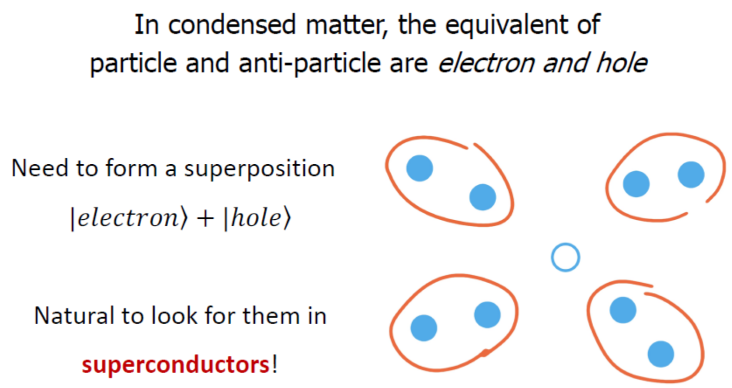
Second, as quasiparticles in condensed matter systems, they always come in pairs –in condensed matter the building blocks are always ordinary fermions.

Now, what is the connection to qubits? With fermionic states, we can encode qubits. If the state is empty this will be the state 0 of the qubit, and if it is filled itis the state 1. An example for this is charge qubits. The problem is that these are usually very sensitive to local perturbations. But now Majoranas come to the rescue! As I just told you, 2 Majorana's correspond to 1 fermion. But if I have 2 spatially separated Majoranas I can encode one fermionic degree of freedom in a very nonlocal way, protected from any local perturbation.



Michael Wimmer, Majorana fermions and where to find them

This is our topological qubit: two Majorana bound states form one topoloigical qubits, which is protected against almost all sorts of perturbations and is expected to have a very long coherence time. For that reasons it is very interesting to look for Majorana Bound states in condensed matter.



Michael Wimmer ,Majorana fermions and where to find them

Where should we look for Majoranas in condensed matter? We can again start from the concept of particle and antiparticle. As I told you before, the

equivalent of particles and antiparticles in condensed matter are electrons and holes. A Majorana is a superposition of those two things. The problem is however that electron and hole have opposite charge. It is usually impossible to form a superposition of them. It however turns out that a good system for Majoranas are superconductors. In superconductors we have a sea of Cooper pairs. Cooper pairs are states consisting of 2 electrons.
Suppose now we have a single hole in our system. If I take just one Cooper pair out of the vast sea of Cooper pairs, then this Cooper pair together with the hole just looks like a single electron. The distinction between electron and a hole are thus effectively blurred in a superconductor. Hence, we can form a linear superposition of electrons and holes there! It is thus very natural to look for Majorana states in superconductors.

Main takeaways

- A Majorana fermion is a fermion which is its own antiparticle. In principle they could exist, but nobody knows for sure if they do.
- In condensed matter, Majoranas regard Majorana bound states, rather than actual Majorana fermions.
- Two Majoranas together form one fermionic state and they always come in pairs.
- The possibility of spatially separating Majoranas protect against local perturbations, laying the basis of topological qubits.
- Superconductors are a good system for Majoranas, as linear superpositions of electrons and holes can be formed.

Majorana bound states in superconductors

Majorana bound states can appear quite naturally in superconductors. But, how can we find the Marjoranas? Michael will explain more about that in this Section.

Let us now look a little bit more in detail, how we can find Majorana bound states in superconductors. I showed you in the last Section that Majorana bound states can appear quite naturally there.

Superconductors: particle-hole symmetry

$$\boxed{\gamma(E) = \gamma^{\dagger}(-E)}$$

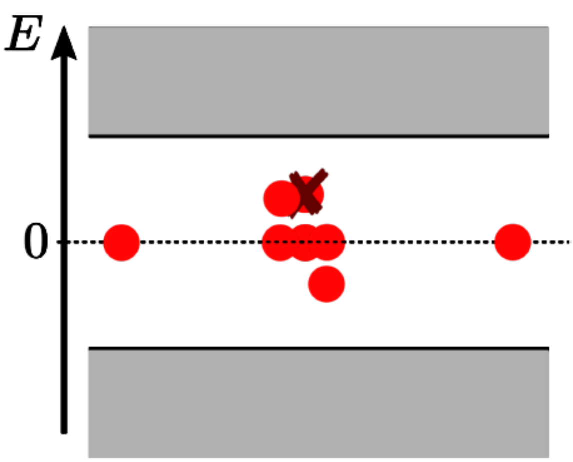
At $E = 0$: Majorana bound states

$$\gamma = \gamma^{\dagger}$$

Michael Wimmer, Majorana bound states in superconductors

The blurring of electron and hole can be described mathematically as a so-called particle-hole symmetry.

Consequences of particle-hole symmetry

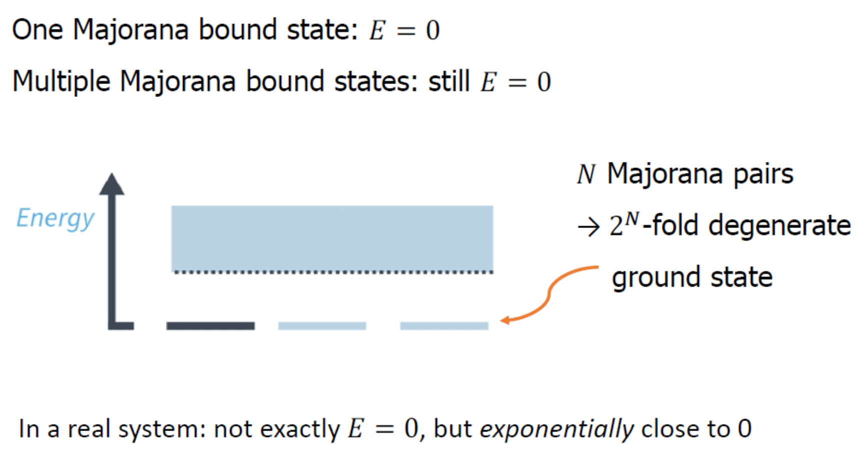


Trivial superconductor: no Majoranas

Topogical superconductor: with Majoranas

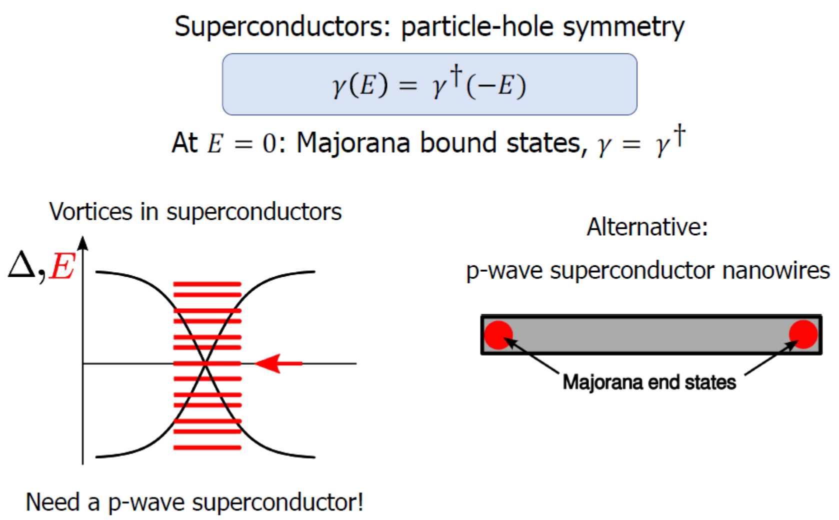
Michael Wimmer, Majorana bound states in superconductors

For a particle at energy E you must have an antiparticle at energy minus E. That is given by this formula here, with the annihilation operator gamma at energy E equals the creation operator at energy minus E. For zero energy we then immediately get Majorana bound states! In this case the Majorana creation operator, gamma dagger, equals the annihilation operator gamma. So, all we need to do is look for states in superconductors with zero energy! Interestingly, the particle hole symmetry also helps to protect these Majorana bound states. We say that they are topologically protected.



Michael Wimmer, Majorana bound states in superconductors

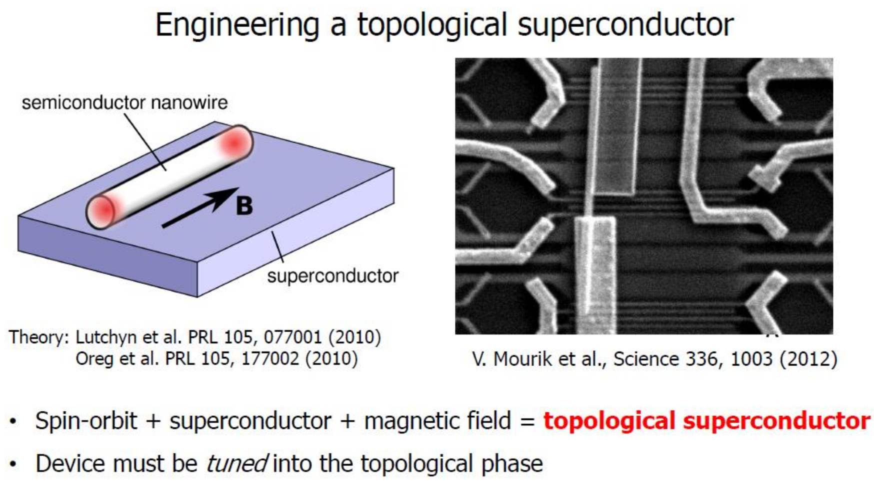
Let me show you now in a simple picture what this actually means. Particle-hole symmetry means that the energy spectrum must be symmetric around zero energy. This spectrum has a superconducting gap, this is shown as a white region in the Image.



Michael Wimmer, Majorana bound states in superconductors

If you have one state at zero energy, one Majorana bound state, then this state is protected and has to remain at zero energy -regardless of what kind of perturbations you do to your systems. If the Majorana state were to move

away from 0 energy, the system would not have particle hole symmetry anymore. That symmetry is fundamental, so this is not allowed. This way the particle hole symmetry protects the Majorana bound states. We call these states symmetry protected topological states. Now I told you before, that in real condensed matter systems Majorana bound states always come in pairs.
Any normal fermionic state can be described with two Majorana states. In this case a perturbation can actually move them symmetrically in the spectrum. So that of course is not protected. However, true Majorana bound states are spatially separated, far apart and cannot talk to each other. Each of those is then again protected by the particle hole symmetry. We can thus distinguish two kinds of superconductors: either there are Majorana bound states -and then they are protected). Or there are no Majoranas at all. A superconductor that has Majoranas we call a topological superconductor. A superconductor without Majorana fermions we call trivial superconductor. Now, there is actually an interesting aspect about Majorana bound states being at 0 energy. You can have multiple Majorana pairs and the states that you can make out of these Majorana pairs all have zero energy, too. So, with N Majorana bound state pairs, you actually have a 2^N fold degenerate ground state, because each pair can be occupied or not occupied.



Michael Wimmer ,Majorana bound states in superconductors

In a topological superconductors we thus generally have a gap, and at 0 energy a 2^N fold degenerate ground state. This will be important in a later stage, as this allows for topologically protected operations on Majorana bound states. This will be covered in a separate Section.
In reality, we cannot separate the Majorana bound states infinitely far from each other. Hence there is a small overlap left over. But this overlap is then exponentially small, so the states will be exponentially close to zero energy, which is good enough. At this point it might seem easy to find Majoranas: we just have to find states in superconductors with zero energy. But this is actually not as easy as it seems. Because, how could one get states at small energies in a superconductor? After all, there is the superconducting gap. We could

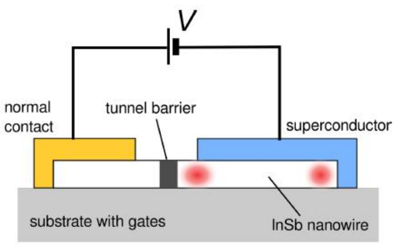
consider though the situation where there is a vortex in the superconductor. In a vortex, magnetic flux penetrates the superconductor and locally suppresses the superconducting gap Delta. The suppressed gap is shown here by a black line. Still, if you calculate the bound states of this system, you find that there is only a state at a finite energy. The reason for this, is the quantum mechanical zero-point motion. To get rid of the zero-point motion one needs to consider unconventional superconductor, such as so called P-wave superconductors. In that case, there is an additional Berry phase of Pi that can cancel the zero-point motion. We get exactly then one state at zero energy, which is a Majorana bound state. It turns out that instead of going to vortices, which are actually hard to control, we can go to one dimensional systems : nanowires. If we make a nanowire out of P-wave superconductor, you will also get Majorana states at the ends of the wire. So, a P-wave superconductor would be nice to have, but it turns out that all the superconductors in nature that we know of are just trivial superconductors. There are some candidates that might be P-wave superconductors, but nobody knows for sure. The most promising approach is thus to engineer the P-wave superconductor out of normal, ordinary trivial materials. I want to focus here on one particular example. It was shown that a semiconducting nanowire with spin-orbit interaction in proximity to a s-wave superconductor in a finite magnetic field can support Majoranas. Now one has to put all of these ingredients together, but this isnot enough, one also has to tune some parameters to get to the topological phase. In particular, in this case we need to tune the magnetic field so that the Zeeman splitting exceeds the superconducting gap. Additionally, we also have to tune the chemical potential into the Zeeman gap. If we can do this, for example, with a gate, then Majorana bound states will appear at the right gate settings and magnetic field. This is now a system you can make in the lab. All the ingredients are in principle known experimentally. This was done for the first time in 2012 in Delft, and this is what in practice the experimental system looks like.

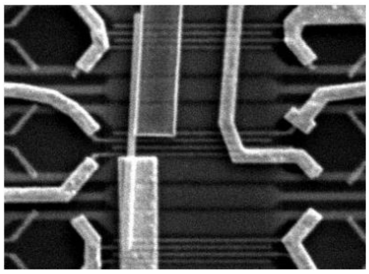
Main takeaways

- Particle-hole symmetry is fundamental to Majoranas. For a particle at energy E must have an antiparticle(hole) at energy –E.
- At zero energy, we then get immediately Majorana Bounds states. We thus have to look for superconductors with zero energy.
- Quantum mechanical zero-point motion makes it difficult to find states with zero energy in conventional superconductors.
- Experimentalists attempt to engineer a nontrivial superconductor out of ordinary materials to find Majoranas.

Majorana experiments

Now that we know how to make Majorana systems in principle, lets look at the current experimental status.



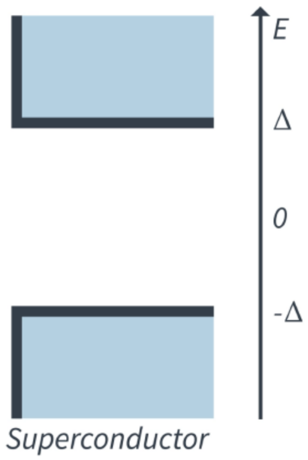


V. Mourik et al., Science 336, 1003 (2012)

- Majorana bound states appear at finite magnetic field
- Measurement: current-voltage characteristics

Michael Wimmer, Majorana experiments

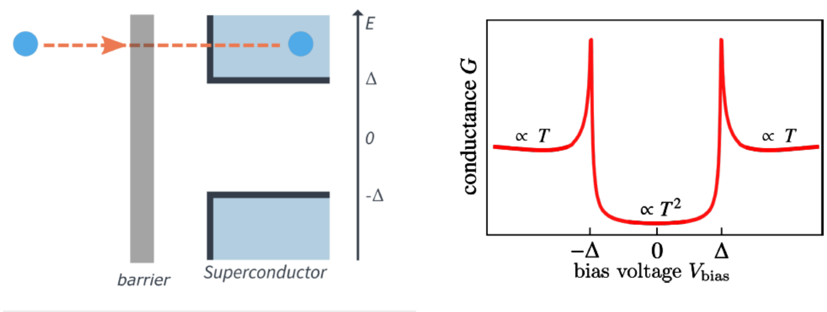
We ended the previous Section with the picture of the Delft experiment where Majorana bound states were observed for the first time. Remember that Majoranas only appeared by tuning the system into a topological phase. In particular, they only appear at a finite magnetic field.



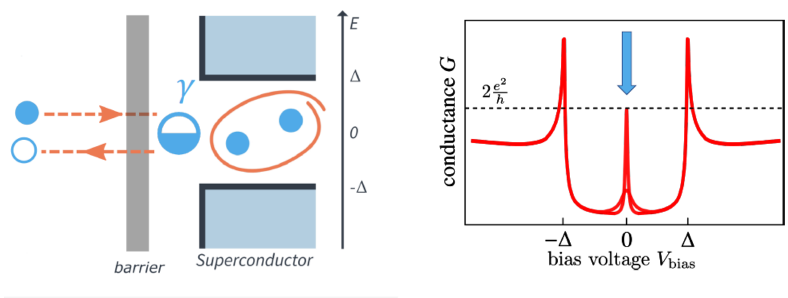
Michael Wimmer, Majorana experiments

In order to observe Majorana bound states, one needs to measure them. How do you actually measure Majoranas? It turns out that a standard current-voltage measurement is enough. Apply a voltage between a normal contact and the superconducting part –where the Majoranas are located -and measure the current that flows across a tunnel barrier. This so-called conductance spectroscopy is one of the standard techniques to actually find Majorana bound states. I want to spend a little bit of time to explain how this works. Let us first understand how current flows in normal-superconducting junctions in general. We have a normal metal in contact with a superconductor.

Andreev spectroscopy at zero B-field (no Majoranas)



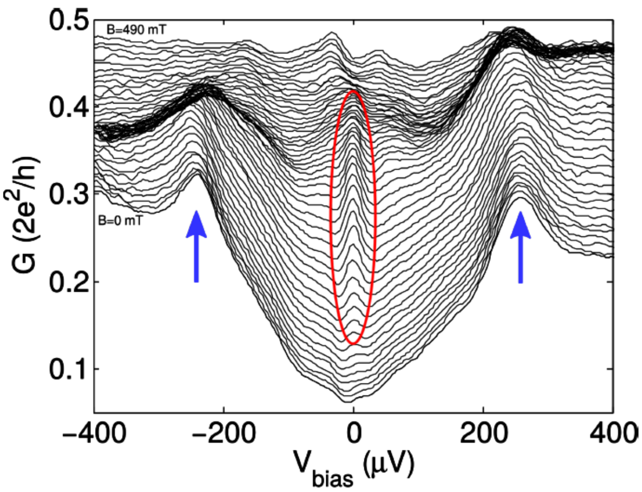
Andreev spectroscopy at finite B-field (with Majoranas)



Michael Wimmer, Majorana experiments

We want current to flow from the normal metal to the superconductor. This should be possible, as the superconductor has no resistance. However, the transport processes turn out to be a little bit more involved. Let us first consider the case of a normal superconductor and forget about Majoranas for a while. We are interested in current flowing from the normal metal to the superconductor. Current in the normal metal is carried by electrons. However, when an electron comes to the superconductor something funny happens. The superconductor has a quasiparticle gap, which doesn't allow single electrons in the system. If an electron comes there, naïvely one would say the electron is just reflected, but this would result in no current through the superconductor.
We now have to remember that current in a superconductor is carried by Cooper pairs, which are pairs of 2 electrons. So, when a single electron arrives at the superconductor it can actually go into the superconductor as a Cooper pair, but it needs a second electron. The second electron comes from the Fermi sea, leaving a hole in the normal metal. This hole then travels backwards in the metal. This process carries then a current, because the electron is negatively charged and travels to the right, the hole is positively charged and travels to the left. Itis called Andreev reflection, and is the fundamental

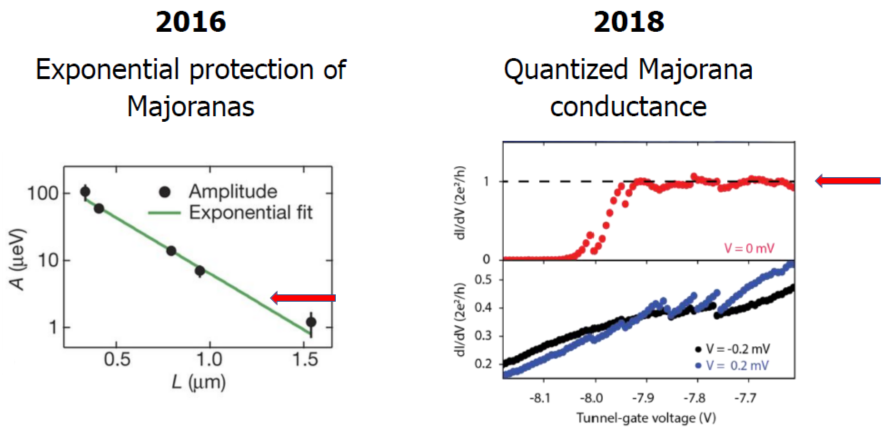
process of current flowing in a normal-superconducting junction. Conductance spectroscopy is usually done in the presence of a tunnel barrier in the junction.



V. Mourik et al., Science **336**, 1003 (2012)

Michael Wimmer, Majorana experiments

If the bias voltage is small, then current is carried by Andreev reflection. In this case the electron has to go through the barrier and the hole has to come back, so there are two tunnelling events. Each process has a small tunnel probability T. The total probability of this process is thus proportional to T^2. For a bias voltage larger than the superconducting gap, the electron can just enter the superconductor as an electron. It thus has to go only once through the barrier, so the conductance is then just proportional to the tunnelling probability T. Since Tis smaller than 1, this probability is higher than the previous case. Inside the gap the conductance is thus lower than outside the gap.

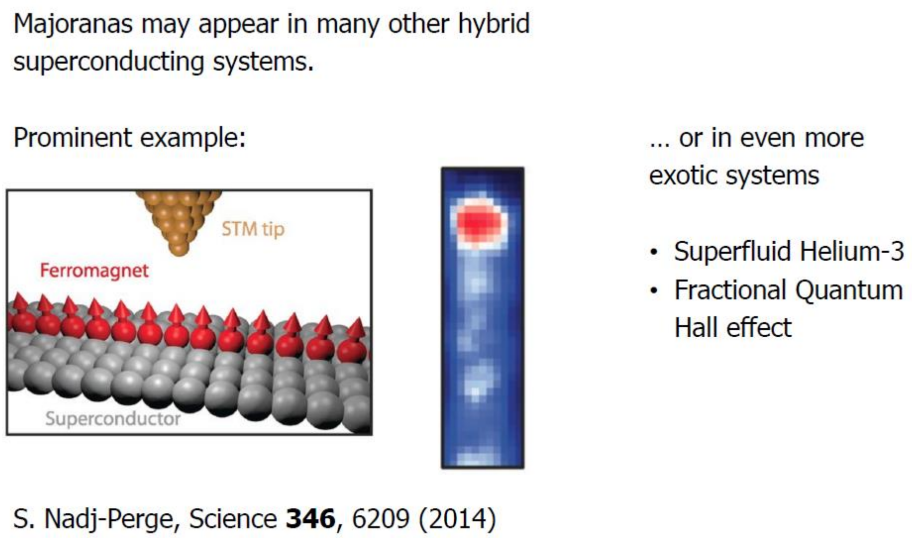


S. Albrecht *et al.,* Nature **531**, 206 (2016) H. Zhang *et al.,* Nature **556**, 74 (2018)

Michael Wimmer, Majorana experiments

Right at the gap there is a resonant behavior, so that in total the conductance has this particular shape as shown on the slide. Note that one can directly read off the superconducting gap Delta from such a measurement. What happens if you have a Majorana bound state there? A Majorana bound state always sits at zero energy, and gives rise to a resonant process at zero bias voltage. In the conductance, we thus observe a peak at zero bias voltage.

Since this is a resonant process, the conductance is actually quantized at the value of 2E^2/h–at least for zero temperature. At finite temperature, this peak can actually be lower, if thermal broadening exceeds the tunnel coupling. Common to both scenarios is that we expect to see a peak at zero bias voltage inside a superconducting gap, as a signature of Majorana bound states. This particular measurement was done 2012 in Delft. I show here the conductance at zero magnetic field. We see the gap structure that I explained to you before, a suppressed conductance inside the gap, and conductance peaks at the value of the induced superconducting gap. This was first a zero magnetic field, so no Majoranas. The measurements for finite magnetic field are all the additional curves that are shown here in this plot. The magnetic field increases as you go further to the top. We observe that as a magnetic field is increased, suddenly apeak at zero bias voltage is emerging. This is the signature of a Majorana bound state. Now, this was already six years ago. The signatures then were non-ideal: the peak is not quantized as we might have hoped for. Also, there are other effects that might induce a peak at zero bias, besides Majoranas. However, since 2012 experimental is to have very worked hard one stablishing Majoranas more in their system. I want to highlight 2 milestones in this field. The first one was the experiment in 2016 in the group of Charlie Marcus in Copenhagen. They showed that the states in these nanowires have an energy that is exponentially close to zeroas a function of the length of the nanowire. This shows the exponential protection of Majoranas experimentally. Another important milestone was again in Delft in the group of Leo Kouwenhoven. After optimizing the nanowires, they found a quantized conductance peak of the Majorana bound states.



Michael Wimmer, Majorana experiments

If you take all of these experiments together, there is actually very strong evidence that we have Majorana bound states in these nanowire systems. I told you in detail about the nanowire platform for Majorana bound states. Before I end, let me emphasize that there are more possible systems hosting them. I show one other particular example here. It was predicted that a chain of magnetic atoms on top of a superconductor would also host Majoranas. The group of Ali Yazdani built this system using a STM, and indeed found states localized at the ends of these magnetics chains. I also want to mention that actually Majorana particles do not only exist in superconducting systems as I showed you, but they can also appear in interacting systems, for example superfluid Helium or fraction quantum Hall effect. So far, nanowire-based systems remain the most technologically advanced. Experimentalists are working now hard on realizing the first topological qubit there. How such a qubit would work is the topic of the next Section.

Main takeaways

- Majoranas only appear at a finite magnetic field.
- Measurement of Majoranas is done with a standard current-voltage measurement called conductance spectroscopy.
- The fundamental process of current flowing in a junction between a normal conductor and a superconductor is called Andreev reflection.
- Majorana Bound states always sit at zero energy and give rise to a resonant process at zero bias voltage.

Practice Quiz 10

QUESTION 1

What is the antiparticle of each of the following particles? (Capitalize the first letter of each word in your answer)

Electron

Majorana Fermion

QUESTION 2

A Majorana bound state in a superconductor has ...

- positive energy.
- zero energy.
- negative energy.

QUESTION 3: CONDENSED MATTER SYSTEMS

There are a lot of people working on realizing Majorana's in condensed matter systems.

What do we refer to when we are talking about Majorana's in condensed matter systems?

- Dirac fermions which, with an extra extension, are described as Majorana's
- Majorana bound states
- Majorana fermions

QUIZ 10: TOPOLOGICAL QUBITS

This quiz is closely related to the Section introducing topological qubits. We recommend you to Read this Section carefully because the questions are intended to check that you understood (parts of) the Section .

QUESTION 2: A PROOF

In the section by Michael Wimmer it is told, that you can create the Majorana operators from a superposition of creation and annihilation operators. Do this for yourself and show yourself that indeed the corresponding operators are Majorana operators. Furthermore, show that the creation and annihilation operators for normal fermions are not Majorana operators.

Did you do the proof for yourself?

- ○ Yes
- ○ No

What are anyons?

Anyons are an essential ingredient if you want to do quantum computing with topological qubits. In this first Section Attila Geresdi will explain more about these anyons.

To understand the importance of anyons in quantum computation, let's take two identical elementary particles.

Wavefunction of two identical quantum particles:

$$\Psi(1,2)$$

Attila Geresdi , What are anyons

Quantum mechanics dictates that their state is described by their joint wavefunction. If we now exchange these two particles, then this wavefunction picks up a phase, denoted by α. If we now exchange them again, then we pick up the same phase, α, once again. In our three dimensional world, we are now back to our original wavefunction. It then follows that we only have two types of elementary particles: $\alpha$is either zero and after two exchanges we also have zero rotation, the wavefunction that we started with. This α defines bosons, such as photons. Alternatively, α equals to π, and after two exchanges, we rotated the wavefunction with 2π, which is a full circle. In this case, we have fermions, such as electrons, protons, neutrinos. In two dimensions, this picture changes dramatically, because the particles can keep track of how many times they were exchanged around each other.

After one exchange:

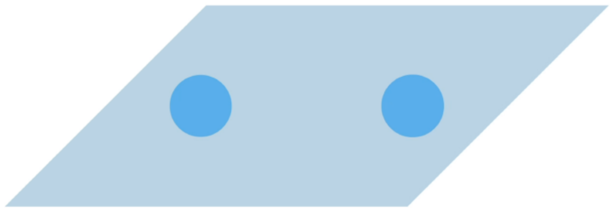
$$\Psi(2,1) = e^{i\alpha}\Psi(1,2)$$

After two exchanges:

$$\Psi(2,1) = e^{i\alpha}\Psi(1,2)$$
$$\Psi(1,2) = e^{2i\alpha}\Psi(1,2)$$

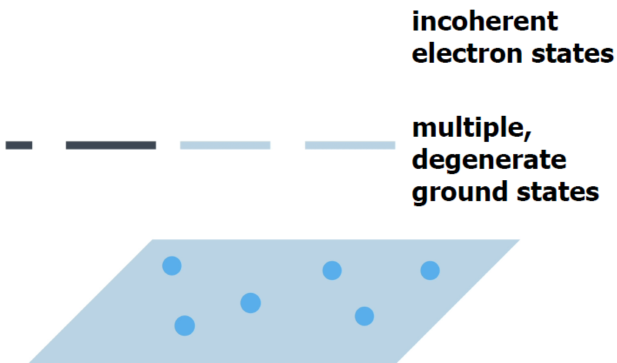
if $\alpha = 0$, the particles are bosons

if $\alpha = \pi$, the particles are fermions



Non-Abelian Anyons

$$\Psi(1,2) = \mathcal{B}_{12}\Psi(2,1)$$



Attila Geresdi , What are anyons

Therefore we lose the previous restriction of α, which only allowed for two types of elementary particles. In two dimensions, the α phase can be anything, and we have a different set of particles for every α. These particles are called anyons. More generally, the exchange operation may be a non-trivial unitary operation, which brings the system to a new quantum mechanical state after each exchange. As these operations generally don't commute, we call the associated anyons non-Abelian.

As a side-note, I mention that this expression comes from group theory in mathematics. Now we have a connection to quantum computation: every quantum gate is a unitary operation, so we will some how have to harness this non-trivial exchange operation to execute quantum algorithms with these particles. We will discuss this in the next Section. Now let's take a set of these non-Abelian anyons. A key property of this system is that it has multiple quantum mechanical states with same, lowest energy. In other words, it has a degenerate ground state. The ground state is separated from the higher, incoherent energy levels by an energy gap. To perform coherent operations, we want to stay in the ground state of the system. Our exchange operation then moves the system from one ground state to another. This means, that these states define a quantum bit, which is free from relaxation: since it is in its ground state, it cannot lose energy to its environment. It also cannot gain energy from its environment, because of the energy gap above the ground state. We created a qubit which is protected from noise or thermal fluctuations from its environment as long as those are smaller than the energy gap. The size of the gap depends on the physical implementation of the qubit, and it is one of the most important parameters to optimize. Furthermore, small changes in the exchange path don't matter; if we exchange the same set of particles, we always do the same quantum operation. And as a result, we can now understand why operating on these quantum particles can lead to perfect quantum gates: if we slightly change the exchange path because of external noise, or control infidelity, our quantum operation remains the same.

This property is usually referred to as the topological equivalence of the exchange paths. In the next Section , we will see how the non-Abelian anyons fulfill the requirements of building a quantum computer.

Main takeaways

- Particles of which the phase *[Math Processing Error]* can be anything are called anyons.
- As operations on the anyons generally do not commute, the particles are so-called non-Abelian.
- To perform coherent operations, we want to stay in the ground state of the system.
- The qubit is protected from noise or thermal fluctuations from its environment in case energies of these are smaller than the gap.

Quantum computation on anyons

Now let’s do quantum computation with anyons.

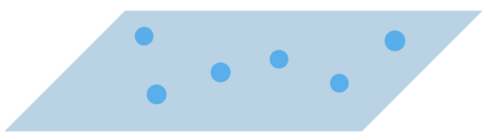
Reminder: DiVincenzo criteria

- **Scalable system of quantum objects that can be entangled**
- **Ability of state initialization**
- **Several gate operations before the system loses coherence**
- **Universal set of gates**
- **Ability of qubit state measurement**

Attila Geresdi ,Quantum Computation on Anyons

If we want to develop a scalable quantum computer, we have to consider the DiVincenzo criteria: We need a scalable system of quantum objects on which we operate. These will be our qubits. We first have to initialize our qubits. Then, we have to able to do several gate operations before this system loses coherence. We need a universal set of quantum gates and, finally, we have to measure the quantum state of each qubit at the end of the quantum algorithm. So let’s see how these criteria are fulfilled for a set of anyons. As Michael has shown in his lecture, we create the anyons from actual electrons. Specifically, the Ising anyons we will discuss in detail later on, are created pairwise. Scalability is then provided by the ensemble of anyons we can create in our physical device. The quantum gates, the unitary operations are linked to the exchange of these anyons. As we discussed in the previous Section, we need the non-Abelian property of the anyons to perform quantum gates. Let’s see how the exchange of two anyons happens as a function of time. If we follow the path of the particles, it now looks like a pair of braided looms.

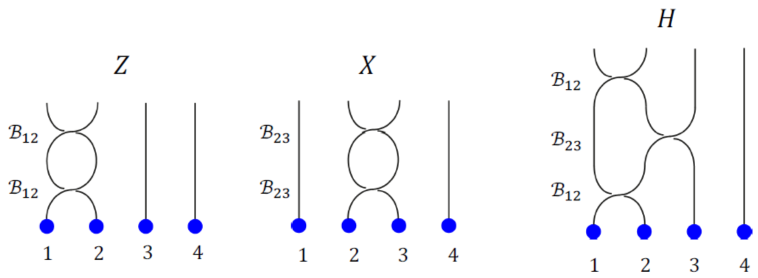
Quantum gates: Braiding



Attila Geresdi ,Quantum Computation on Anyons

This is why the quantum operations on topological qubits are called braiding. Exchanging another pair of anyons leads to a different quantum operation.Let’s see a few examples, specifically for the Majorana bound states, which form Ising anyons: We can create a Z gate by exchanging a pair of anyons twice. On the same system, the exchange of another pair corresponds to an X gate. Or the Hadamard gate can be performed by sequential braiding operations on these anyons. It is important that these braiding operations are always discrete; they either happen or don’t happen.

Example single qubit gates



Zero gate error:
Braids are discrete, therefore the qubit rotation is always precise

Caveat: Phase gate is missing from the universal gate set.

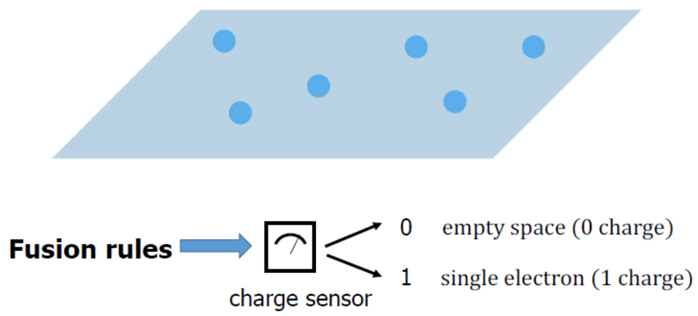
Attila Geresdi ,Quantum Computation on Anyons

As a result, the quantum gates that we create here are always perfect; their fidelity is 100%.There is however a catch: with discrete braiding operations we cannot reach the entire Bloch sphere of a qubit so some quantum gates required for universal quantum computation will be missing from the set that we can do with braiding. These additional gates can be supplemented by topologically not protected operations on the qubit, but with a less-than-100% gate fidelity.
And finally after all the quantum operations we have to measure the state of the qubit.

Qubit state measurement:
Fusion

Attila Geresdi ,Quantum Computation on Anyons

Qubit state measurement:
Fusion



Attila Geresdi ,Quantum Computation on Anyons

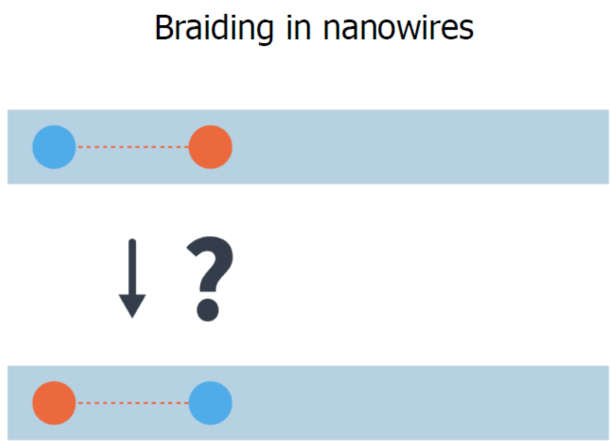
This operation is called fusion, which happens when we merge, or fuse these particles. This behavior of the anyons is described by their so-called fusion rules. In our case, that is the Ising anyons, this can result in a single electron (that is one elementary charge) or no electron (zero charge). We can then distinguish between these two states with charge sensors, as it is done for instance for spin qubits. In the next section, we will see this happening in a physical system.

Main takeaways

- The quantum gates, the unitary operations, are linked to the exchange of the anyons.
- Quantum operations on topological qubits is done via braiding.
- Braiding operations are always discrete. They either happen or don’t happen. As a result, the quantum gates have a fidelity of 100%.
- Braiding operations cannot reach the entire Bloch sphere. Additional gates can be supplemented with unprotected operations.

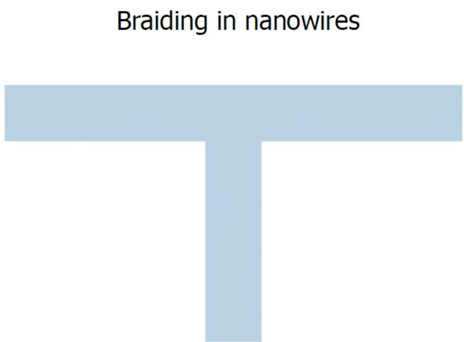
Qubit implementation in nanowire networks

Let’s discuss some real-life implementations of these topological quantum bits.



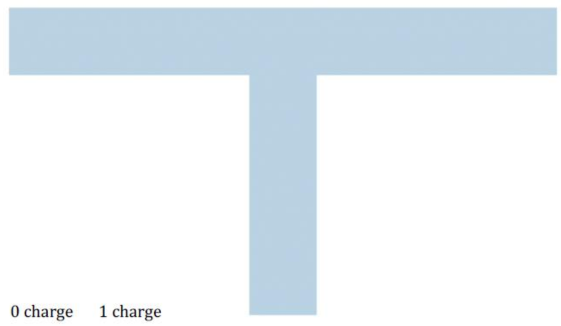
Attila Geresdi ,Qubit implementation in nanowire networks

The most pursued platform consists of semiconductor nanowires attached to superconducting leads where Majorana bound states emerge, and have the braiding statistics of the Ising anyons. Immediately when we envision a pair of these states in a one-dimensional nanowire, we encounter a problem. We cannot exchange these two particles without colliding them. This would lead to fusing them, which then measures the quantum state, and hence loses quantum coherence. The way to perform braiding is a clever workaround: if we attach an extra segment to the nanowire, we can move this topological object around, just like you do a Y-turn with your car.



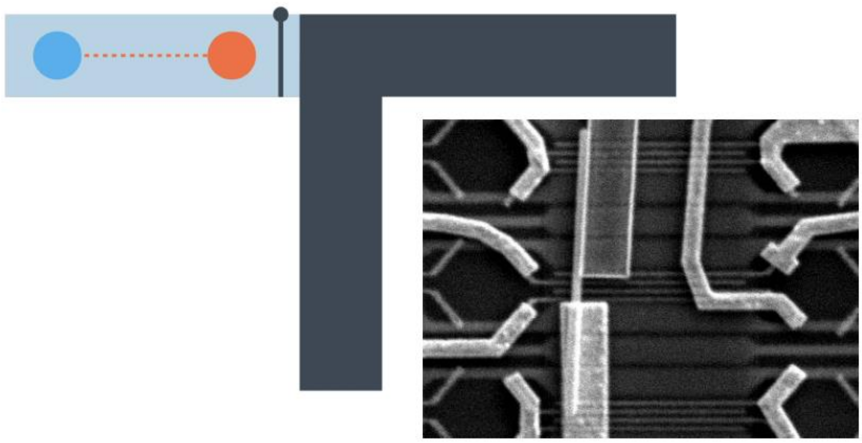
Attila Geresdi ,Qubit implementation in nanowire networks

Quantum state measurement



Attila Geresdi ,Qubit implementation in nanowire networks

Attila Geresdi ,Qubit implementation in nanowire networks

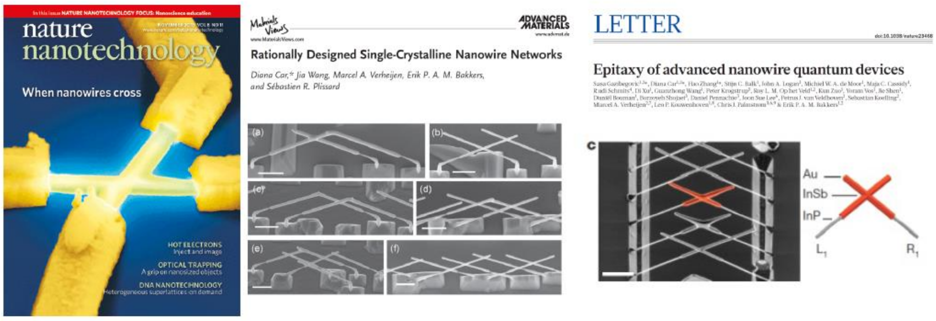


Mourik, Zuo et al, Science **336**, 1003 (2012).

Attila Geresdi ,Qubit implementation in nanowire networks

This allows for braiding of the two particles, orange and blue, as shown in the Image. Since the presence of this additional arm is crucial for this research direction, there is actually a lot of development effort going into creating these structures.

Nanowire networks for braiding



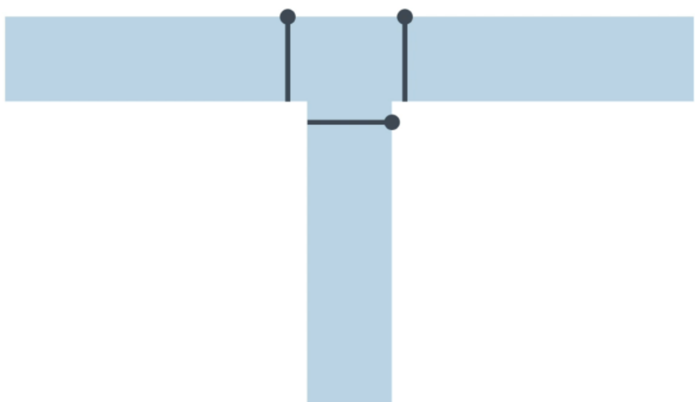
Plissard, van Weperen et al, Nature Nanotechnology **8**, 859 (2013).

Car et al, Advanced Materials **26**, 4875 (2014).

Gazibegovic et al, Nature **548**, 434 (2017).

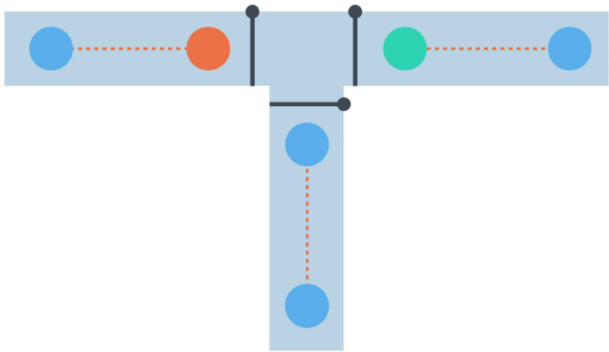
Attila Geresdi ,Qubit implementation in nanowire networks

A few of these, published in scientific literature, are shown here. Let's go on to the quantum state measurement in this scheme. Remember that a pair of Majorana states encode either zero electron or one electron charge, and these states are degenerate, that is they are the same in energy level. If we want to distinguish between them, we have to break this degeneracy. If we close off the rest of the nanowire, the two states split off because of the charging energy of the system, just like for quantum dots built for spin quantum bits. Similarly, the two charge states can then be measured by charge detectors that are put close to the nanowire. This readout scheme, relying on the interaction between electrons in the nanowire, is called interaction-based operation.



Interaction-based braiding

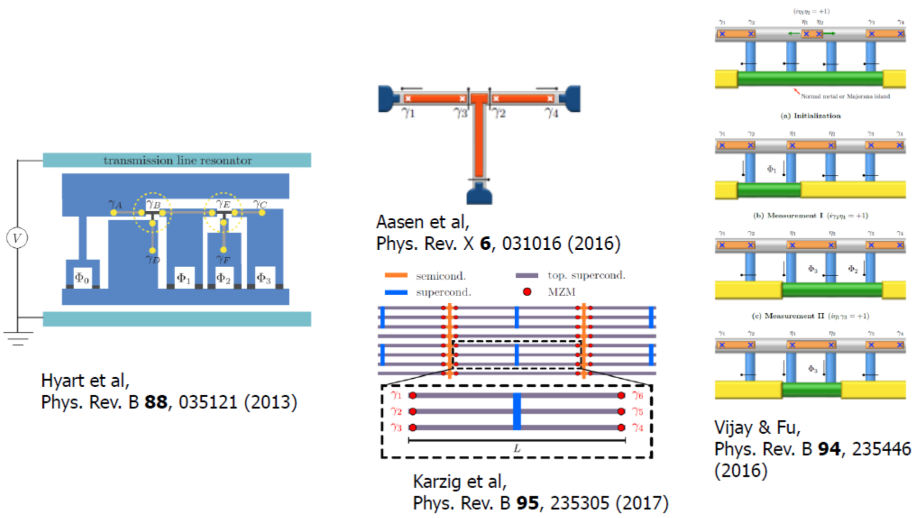
Attila Geresdi ,Qubit implementation in nanowire networks



Interaction-based braiding

Attilla Geresdi ,Qubit implementation in nanowire networks

What we need for this operation, is a set of valves, which we can open and close at will. In real devices, these valves are electrostatic gates, which can locally control the flow of electrons inside the nanowire. When we apply a negative voltage, we remove the electrons and hence close the valve. On the other hand, when we apply a positive voltage on one of these gates, we open the valve, and let the electrons flow. We can also use this concept to perform braiding, so that we need not to physically move the Majorana states, as we did before. We start from a T-device with a pair of Majorana's in each segment. With three valves, we can control the coupling between each pair of topological segments, enabling interaction-based braiding. The Image shows how to exchange, or braid states 2 and 3. Importantly, each Majorana state is always located at the end of each segment.



Attilla Geresdi ,Qubit implementation in nanowire networks

When opening a valve, we create a longer topological segment, which shuttles the quantum state of the Majorana to the very end of this longer segment. With this device, we can perform braiding operations, and we can also perform state measurement. Therefore we have just sketched a prototype topological qubit. As an outlook, let's see a few proposed geometries of topological qubits, similar to the one we have just discussed. As you can see, there are many different schemes, showing that this is a very active research topic. It remains to be seen which geometry will lead to the very first experimental demonstration of a topological qubit, and, eventually, a scalable system of these intriguing qubits.

Main takeaways

- The way to performing braiding is using a Y-junction.
- In order to measure the states, degeneracy has to be broken by splitting off states.
- It remains to be seen which geometry will lead to the very first experimental demonstration of a topological qubit.

Practice Quiz 11

QUESTION 1

Why is a Majorana bound state immune against thermal fluctuations in its environment?

- Because the ground state is degenerate.
- It cannot lose energy because it is in the ground state; and it cannot gain energy because of the energy gap.
- Because it is insensitive to imperfections in the exchange path.

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Submit
Some problems have options such as save, reset, hints, or show answer. These options follow the Submit button.

QUESTION 2

Consider two Majorana bound states in a nanowire. Due to the 1D nature of the nanowire it is not possible to braid the bound states without fusing them. Attila mentioned a solution to that in the section. What was it?

- Fuse the Majorana bound states then unfuse them in the opposite direction.
- Apply a magnetic field to temporarily change the energy of one of the bound states so that they don’t fuse when their positions are exchanged.
- Attach an extra segment to the nanowire that allows us to perform the braiding without the bound states crossing paths.

QUIZ 11: OPERATIONS ON TOPOLOGICAL QUBITS

Introduction to the next questions
The next questions will involve matrix computations and complex conjugation. For this we want to remind you the following. Given a matrix M=[abcd] where a,b,c,d are complex numbers. The conjugate transpose (denoted by M† is given by M†=[¯a¯c¯b¯d]. Where ¯a is the complex conjugate of the complex number a. For example, if a=2+3i, then ¯a=2−3i. And for M given as below: M=[1i−ii]
we haveM†=[1i−i−i]

[SEE WIKIPEDIA FOR MORE INFORMATION ON CONJUGATING COMPLEX NUMBERS]()

Learn more
This section grants more insight in what topological quantum computation is.
https://youtu.be/igPXzKjqrNg

Advanced introduction of topological quantum computing
This paper gives an elaborate introduction to topological quantum computing where also the underlying mathematics are explained. This material is mainly suited for participants with a background in quantum mechanics, do not be discouraged if it is hard to follow.
https://arxiv.org/pdf/1705.04103.pdf

Module 6

Welcome to Module 6!

A few sections ago "Koen Bertels" started this Book with a description of the basic building blocks of a quantum computer. In five modules we went through the first layer of the stack, the quantum chip. We introduced you to many kinds of qubits, and highlighted their potentialities. Spin qubits in quantum dots can be very dense and coherent systems, NV centers in diamond which can be efficiently coupled and manipulated via photons, topological qubits which offer the promise of a much lower sensitivity to noise, and superconducting qubits which represents one of the most advanced platforms today. These are, of course, only some examples of physical realizations of qubits. In the 'quantum' community groups are also working with donors, trapped ions, NMR (Nuclear Magnetic Resonance) and many other platforms.

In this closing Section , Menno Veldhorst will start by looking back to what we have seen and discuss how these qubit implementations could be turned into a large scale quantum computer. Finally he will look forward to the next part of the Book, where we will go through all the remaining layers of the quantum stack, up to the quantum algorithms.

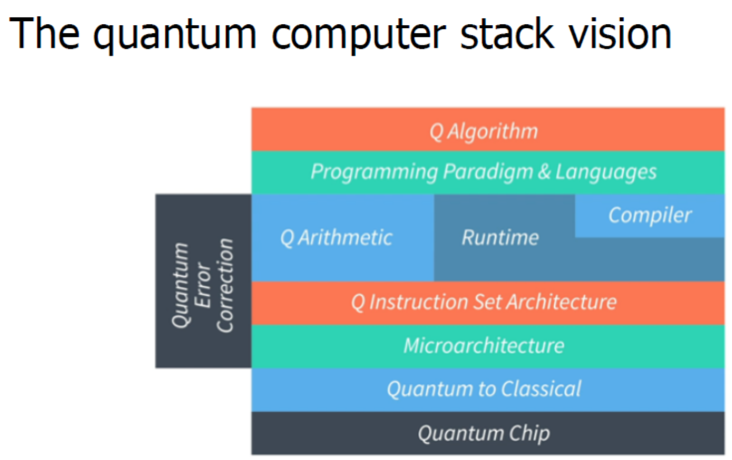
This module contains the final exam of this course. We have added a practice exam as warming up for the final test. This gives you the opportunity to dive into the topics that will be dealt with in the final exam.

Enjoy the final Section and good luck with the final exam!

Round up: Building blocks of quantum computer

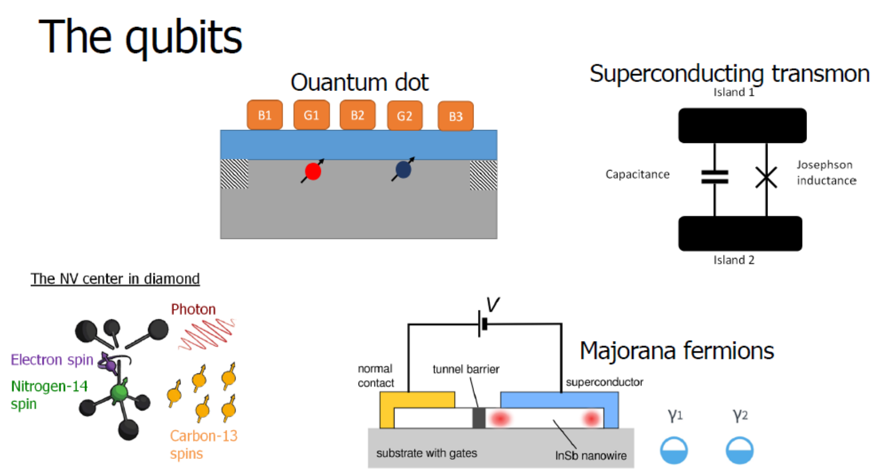
In this final Section of this Book we look back at the the key building block of the quantum computer: the qubit. Menno will set out the features of the four types of qubit platforms that have been discussed. What are their similarities, and how do these qubits differ from each other? Menno will also look forward to the second part of this Book, where the other layers of the quantum computer will be discussed. What is needed to develop a fully functional quantum computer? And what is the approach to the development of a quantum internet?

Closing Section Building Blocks of a Quantum Computer –part 1This is the closing Section of the first part of our series of Sections on the building blocks of a future quantum computer. We started by introducing a stacked layer approach toward a future quantum computer.



Menno Veldhorst,Closing lecture

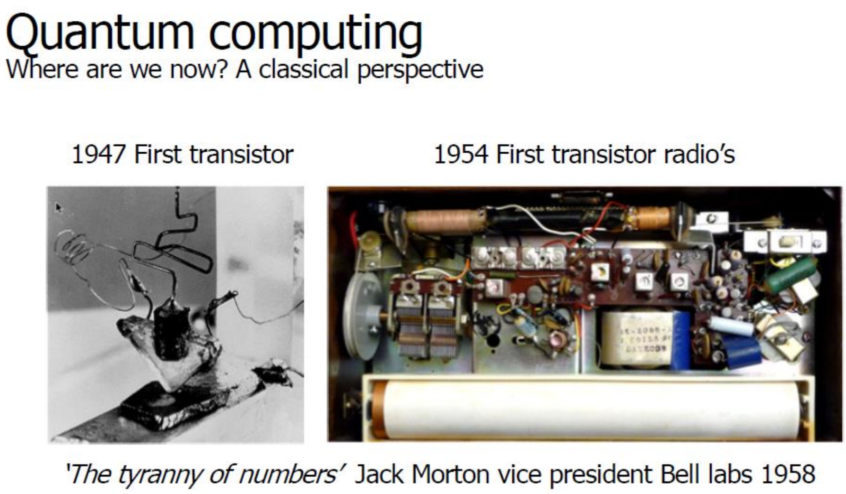
In part 2 of this Book we will focus on the higher layers of this stack. In addition we will also discuss the path toward a quantum internet. A quantum internet allows for communication between nodes where the security of the communication is dictated by the laws of quantum mechanics. In the first part of this Book, we mainly focussed on the lowest level, the quantum chip that forms the foundation.The quantum chip is the quantum circuitry hosting the physical qubits. Physical qubits can be controlled to a superposition state and coupled together to create entanglement. These principles are on the basis of a quantum computer and quantum internet. The quantum mechanical state of a qubit is usually fragile and can suffer from environmental interactions and decohere over time. However, in order to do operations on these qubits it is essential that many operations can be done within the qubit coherence time.



Menno Veldhorst,Closing lecture

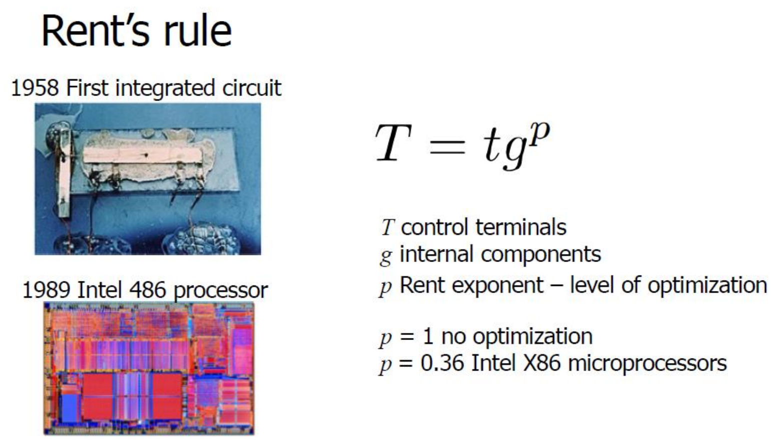
Theoretical predictions state that for practical fault tolerant quantum computing it will be necessary to perform thousands of operations within the qubit coherence time. While more sophisticated quantum error correction protocols are actively being studied, it is clear that good qubits are essential and that qubits need to be of high-fidelity. Today, various qubits are being explored and investigated to reach this goal.

In this series of Sections we have introduced to you several of the most promising qubit systems. Qubits based on the spin states of electrons or nuclei associated with defects or donors in silicon or diamond can reach very long coherence times. In materials with low net nuclear spins, detrimental magnetic interactions with the environment are small. In addition, the strong confinement leads to small overlap with other states. NV centers in diamond are particularly interesting, as they can be coupled to photons, providing an optical link between spin qubits that are distant from each other. Spin qubits in quantum dots also exhibit very long coherence times, but they also offer the advantage of being manmade. This allows to realize qubits at a predefined location, which is clearly beneficial for scaling up the number of qubits. Superconducting transmons offer this advantage as well. In addition, they are larger and so, at least in the few-qubit regime, it is even easier to fabricate these types of qubits. Large efforts are devoted to improving the qubit environment leading to qubits that become more and more isolated and thus to qubits with extended coherence. This is achieved by for example removing magnetic noise from nearby nuclear spins or electric noise stemming from charge defects in the substrate. In addition, clever qubit designs enable to decrease a qubit sensitivity to noise. For spin and superconducting qubits, sweet spots exist where to first order qubits are insensitive to certain noise. A special class of qubits exists that can be made intrinsically insensitive to some noise. These are called topological qubits and are qubits such as the ones based on emergent Majorana fermion states. These qubits could become exponentially less sensitive to local noise with increasing system size. That is, by increasing the separation between Majorana fermions, the qubits become more and more protected against local noise and can thus hold their coherence for a longer time. Today, we do not know what the best qubit platform will be and so there is an active race going on between all these different platforms. We also often find out that developments made in one platform can be implemented into other platforms and so all these activities strongly benefit from each other. Where are we now with quantum computing? To answer this question, let’s take a classical perspective.

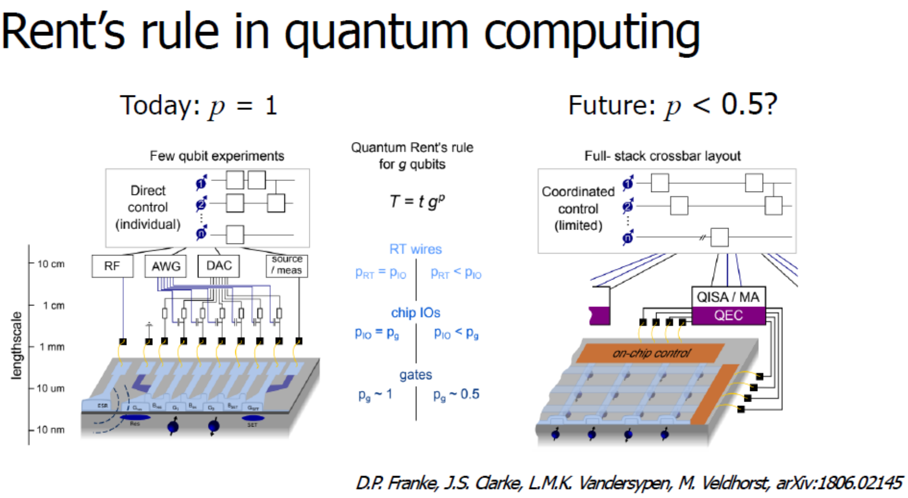


Menno Veldhorst,Closing lecture

In the 1950s, electrical systems contained many components, each requiring to solder to numerous others. This clearly prevented to go to large numbers. Jack Morton, at the time vice president of device development at Bell labs, referred to this situation as the 'tyranny of numbers'. Quantum devices are now at a similar stage of maturity. For classical electronics the solutions was the invention of the integrated circuit. The power and beauty of these systems become apparent if we compare the numbers of connectors at the outside of the chip to the number of active components inside the chip. Large grid arrays have a couple of thousand connectors, but these can address billions of transistors on a classical processor.

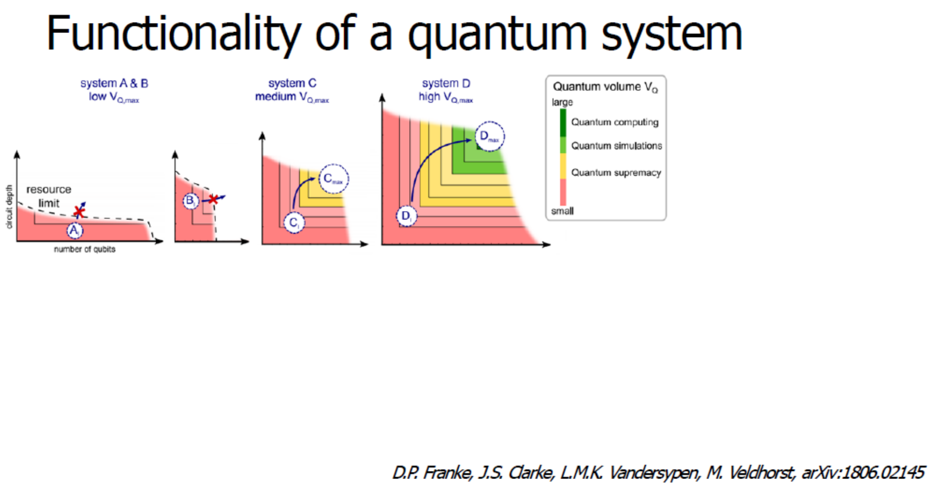


Menno Veldhorst,Closing lecture



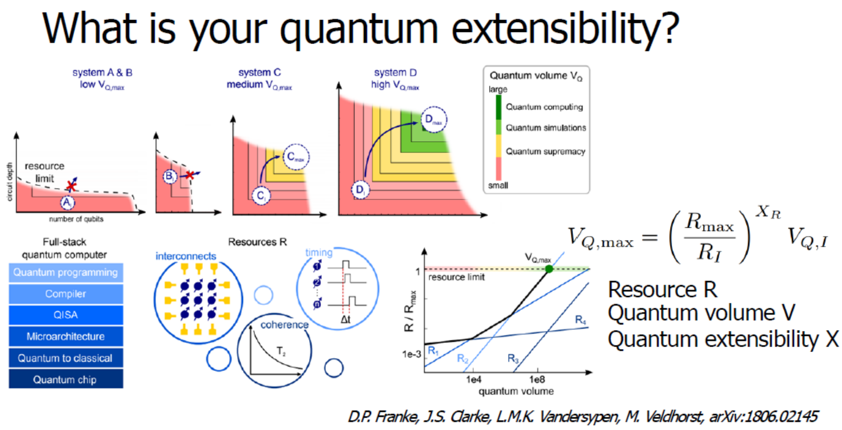
Menno Veldhorst,Closing lecture

This huge ratio between components and connectors is described by Rent's rule and this rule has been one of the main drivers behind Moore's law. It is thus most likely that we will require for practical quantum information applications the development of a 'quantum integrated circuit'. Today's quantum devices require individual electronics and connects for each and every qubit. Increasing these systems to large numbers will clearly require a different scaling law. Concepts from classical memory technology, such as crossbar layouts where signals only have to come from the sides to address a large array, are now being proposed as an efficient way forward for quantum systems.



Menno Veldhorst,Closing lecture

These approaches come with their own challenges, but do provide a powerful method to avoid an interconnect bottleneck. Interconnects are not the only challenge that need to be addressed. Every critical resource must be evaluated and optimized. In the figure here you see some fictitious qubit platforms. The functionality of each qubit platform is described according to its quantum volume. The usefulness of a platform will depend on the number of qubits available, but also on the amount of operations that can be executed on them. A system with only a few qubits is not powerful, even if they are of very high-fidelity. Likewise, a system containing many qubits of very low quality is also practically useless. The quantum volume takes this into account and is defined as the square of the lowest number of these two quantities. Increasing the quantum volume will enable to reach quantum supremacy, the era where quantum systems become too complex to be efficiently simulated with classical computers. Further increasing the quantum volume will enable to perform useful practical quantum simulations and finally to reach practical quantum computing. Nonetheless, while this is certainly a useful metric, the quantum volume does not provide the information whether a system is capable of reaching these applications.



Menno Veldhorst,Closing lecture

This scalability aspect is described by the quantum extensibility of a system. Depending on all the resources, required at every position of the stack, a platform has the capability to scale up toward practical quantum information. Defining these resources is a crucial aspect for quantum information research today. We have learned the aspects of some of these resources in the first part of this course, such as qubit decoherence. In part 2 of this Book we will introduce the other building blocks of a quantum computer and show you the vision of a full-stack future quantum computer. In addition, we will also introduce to you the foundations of a quantum internet. What is required and where are we now? While building a quantum computer and quantum internet is perhaps the greatest scientific challenge of this century, you will also learn in part 2 why it is completely worth it and you will hear all the opportunities that arise once this becomes a reality.

Main takeaways

- Physical qubits can be controlled to a superposition state and coupled together to create entanglement. These principles are on the basis of a quantum computer and quantum internet.
- The qubit approaches you have been taught in this course are the spin qubit, the NV center qubit, the transmon qubit and the topological qubit. Which qubit platform is the best is not yet clear.
- Today's quantum devices require individual electronics and connects for each and every qubit. Increasing these systems to large numbers will clearly require a different scaling law.
- This scalability aspect is described by the quantum extensibility of a system. Depending on all the resources, required at every position of the stack, a platform has the capability to scale up toward practical quantum information.
- In part 2 of this course we will introduce the other building blocks of a quantum computer and show you the vision of a full-stack future quantum computer. In addition, we will also introduce to you the foundations of a quantum internet. We hope to see you there!

Practice exam

This is a practice exam. It does not count for grades, however the questions are of a similar level to the Final exam (next page). Use this to test your knowledge before starting the Final exam.

Good luck!

Throughout the course, four types of qubits have been discussed: quantum dots, NV centres, superconducting circuits and Majorana fermions. While all of these implementations are quite different in many respects, they also have many similarities, which allows for expertise and equipment to be shared between different areas of research. The following questions highlight the similarities and differences between the four implementations that we've discussed.

PRACTICE QUESTION 1: SPIN IN QUANTUM COMPUTING

Which of the implementations introduced in this course use spin degrees of freedom to store quantum information?

Select the two that apply.

- ☐ Quantum dots
- ☐ Nitrogen-Vacancy centres
- ☐ Superconducting circuits
- ☐ Majorana fermions

You have used 0 of 2 attempts Some problems have options such as save, reset, hints, or show answer..

PRACTICE QUESTION 2: TEMPERATURE IN QUANTUM COMPUTING

Which of the implementations introduced in this book cannot function at high temperatures (above 4 Kelvin)?

Select the three that apply.

- ☐ Quantum dots
- ☐ Nitrogen-Vacancy centres
- ☐ Superconducting circuits
- ☐ Majorana fermions

PRACTICE QUESTION 3: VOLTAGE PULSES IN QUANTUM COMPUTING

Which of the implementations introduced in this book use electric voltage pulses (also known as DC pulses) for qubit control and/or measurement?
Select the three that apply.

- [ ] Quantum dots
- [ ] Nitrogen-Vacancy centres
- [ ] Superconducting circuits
- [ ] Majorana fermions

KET NOTATION

In the first module of this book we've introduced you to Ket notation. The next question is all about this!

PRACTICE QUESTION 4: EXPECTATION VALUE

Let α & β be real and positive. We have the state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$, and $\alpha=1/3$.What should β than be?
What should β than be?
Enter your answer with two decimals accuracy.

SPIN QUBITS

Now we will have a closer look at the first qubit that we have introduced to you: **the spin qubit**.

QUESTION 5 AND 6 REFER TO THIS QUBIT, WHICH WAS EXPLAINED IN MODULE 2.

PRACTICE QUESTION 5: RADIUS OF A DOT

When an electron tunnels onto the quantum dot, the energy of this quantum dot increases by an amount equal to $E_c=e^2/C$. Approximating the quantum dot as a disc, of which the capacitance is given by $C=8\epsilon_r\epsilon_0R$, we can estimate the size of the quantum dot necessary to see tunnelling events.
For a temperature of 4K, which of the following is the maximum radius R the quantum dot can have to be able to confine single electrons? Assume that the thermal energy should be one order of magnitude smaller than the charging energy, such that $10k_BT=E_C$.
$e=1.602\times10^{-19}C$, $\epsilon_r=3.9$, $\epsilon_0=8.854\times10^{-12}F/m$, $k_B=1.38064852\times10^{-23}J/K$.

- ( ) 100 nm
- ( ) 130 nm
- ( ) 160 nm
- ( ) 200 nm

Practice question 6: Charge sensing

How does charge sensing of a spin qubit work?

- A quantum dot nearby is able to detect changes in electric field as an electron jumps on and off the qubit quantum dot.
- A sensing quantum dot connected to the qubit quantum dot is used to measure the current as an electron jumps between the two dots.
- By increasing the bias voltage over the qubit quantum dot, the top gate of the qubit quantum dot is used to sense the change in electric field.
- The change in current due to an electron jumping on and off the quantum dot is measured with a electrometer connected in series.

NV center qubits

Let's have closer look at **the NV center qubit**. **Question 7** is about the electrons in NV center qubits and 8 refers to the dynamical decoupling, which was discussed in the section of module 3.

Practice question 7: Electrons in NV centers

What is the main purpose for an electron in an NV center?

- To function as a data qubit which is controlled via nuclei.
- To function as a bus for communication with nuclei.
- To tunnel through the diamond to distant NV centers to entangle these.
- The electron is irrelevant in quantum computing with NV centers.

Practice question 8: Dynamical decoupling

Applying a π pulse to our qubit state in an NV center results in a bit flip. In dynamical decoupling in NV center qubits, these pulses are applied in order to counter noise picked up due to coupling with the environment. For every time period τ, a phase of N=eiφ will be picked up.
Our initial qubit state is:
|Ψ⟩=12√(|0⟩+|1⟩)
The random sequence applied to this qubit is as follows:
τ−π−π−τ−π−τ−τ−τ−π
The phase picked up by the |1⟩-component of the state during the protocol is a power of N (e.g. if the final state is 12√(|0⟩+N2|1⟩), the power is 2). Which power of N describes the final state produced by the given protocol?
Multiples of τ indicate waiting periods where noise will occur, and π indicates a pi pulse, or bit flip. Ignore global phases in your response.

Superconducting qubits

The next three questions are based on superconducting or transmon qubits, which were introduced in module 4. Practice question 9 is based on performing two-qubit operations, practice question 10 focuses on single-qubit operations, and practice question 11 tests your knowledge on the assembly of a processor.

Practice question 9: Two-qubit gates on transmon qubits

Which of the following constructs is important to be able to perform two-qubit gates for transmon qubits?

- Removing the coupling resonators between the qubits.
- Replacing the Josephson junction in the circuit for each qubit with an inductor.
- Replacing the Josephson junction with a loop containing two parallel junctions.
- It is impossible to perform two-qubit gates on transmon qubits.

Practice question 10: Single-qubit gates on transmon qubits

We have seen that using a single microwave pulse, it is possible to perform a single-qubit operation on a transmon qubit when the operation involves a rotation around an axis in the XY plane. If the single-qubit operation involves a rotation around an axis $\vec{\eta}$ that is not in the XY plane, then a sequence of microwave pulses is required to perform the operation. What is the maximum number m of pulses in this sequence for achieving a rotation about this arbitrary axis $\vec{\eta}$?

While this has already been discussed in the lecture on single-qubit gates on transmon qubits, you could have a look at the hints in case you are stuck.

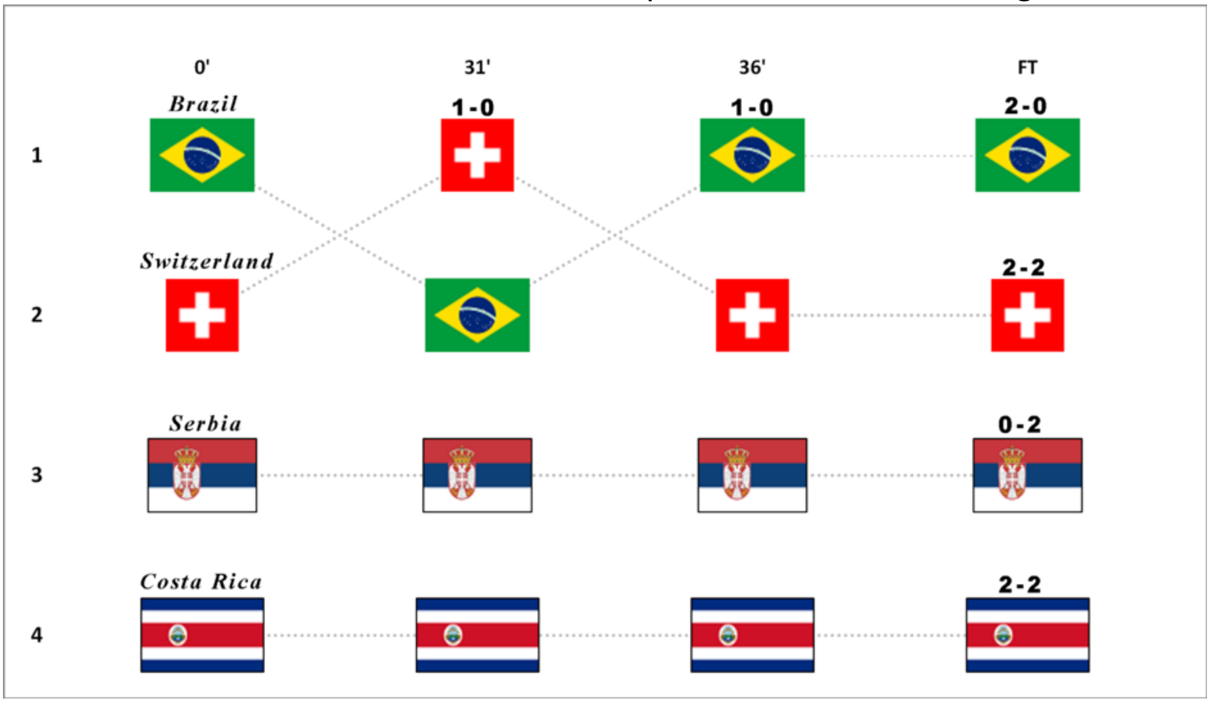
Practice question 11: Assembling the processor

Which of the following facilitates the sharing of feedlines among qubits in a surface-17 chip?

- Amplitude multiplexing
- Frequency multiplexing
- Phase multiplexing
- Time multiplexing

Topological qubits

The last qubit discussed in this book was **the topological qubit**. In practice question 12 and 13 we test your knowledge of the concept of braiding, which you've learned about in module 5.
During the group phase of the football World Cup in Russia in 2018 the standings of group E evolved as follows during the final matches of Serbia-Brazil and Switzerland-Costa Rica.
Brazil and Switzerland battled for the first place, with Brazil winning that race.



Credits to the picture go to u/Meladroit1 who created the graphs for all groups and posted them on Reddit

Practice question 12: Topological qubits

For this question we use the ranking of the World Cup in group E during the last group phase matches. In the 31st minute Switerland overtook Brazil in the ranking, but Brazil quickly recoverred and took the first spot again in the 36th minute. In the module on topological qubits we have seen a similar pattern.
What operation on qubit(s) can you relate to this figure?

- Entanglement swapping, swapping two qubits to perform a entanglement generating gate
- Braiding of two qubits to perform a single qubit gate on the first qubit
- Braiding of two particles to perform a single qubit gate
- Entanglement swapping between qubit 1 and 2, which creates entanglement between the two other qubits 3 and 4.

Practice question 13: Topological qubits

This question will be more mathematical, and won't require the World Cup figure. Given is the braiding operator $B_{12}=e^{-i\pi/8}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$.
What can you say about the operator $B_{12}B_{12}B^{\dagger}_{12}B^{\dagger}_{12}$?

- The result is a 2 by 2 identity matrix, so $B_{12}B_{12}B^{\dagger}_{12}B^{\dagger}_{12}$ is a single qubit operation.
- The result is a 2 by 2 identity matrix, so $B_{12}B_{12}B^{\dagger}_{12}B^{\dagger}_{12}$ has a small effect on the two qubits
- $B_{12}B_{12}B^{\dagger}_{12}B^{\dagger}_{12}$ is a Z-gate on both qubits
- $B_{12}B_{12}B^{\dagger}_{12}B^{\dagger}_{12}$ is a Z-gate on one qubit

Q u a n t u m   L i b r a r y

How to use this library?

The Quantum Library is a separate module in the book where you can find Sections explaining various quantum principles. When reading a section, you can always refer back to this library to gain a better understanding. These topics appear in the library (*each separate hyperlink below takes you directly to that topic*):

entanglement - different ways of entanglement - teleportation - measurement - measurement in superposition - repeaters

Like a regular library, these Sections are here for your reference. These same Section are repeated and used in the book at different moments where they are required. However, if you would like to "binge read" all the of the content at once, or treat it like a snack-bar, please feel free to use these Sections as you wish.
Each Section is accompanied by a practice exercise, which is not graded. Feel free to complete these exercises at your own pace. If you don't understand one of the exercises, simply return to it after completing the section concerning ket notation in Module 1. If, later in the book, you'd still like to know more about one of the exercises, post a question in the forum, and we'll do our best to explain.

Entanglement

It's almost romantic to think that two particles can be entangled even when they are millions of kilometres apart. Learn more about quantum entanglement. How is it generated and what are the implications?

Quantum entanglement is a special connection between two qubits. There are many ways to generate entanglement. One way to produce it is by bringing two qubits close together, perform an operation to entangle them and then move them apart again. When they are entangled, you can move them arbitrarily far apart from each other and they will remain entangled. This entanglement will manifest itself in the outcomes of measurements on these qubits. When measured these qubits will always yield zero or one perfectly at random, but no matter how far away they are from each other, they will always yield the same outcome. Entanglement has two very special properties that enable all the applications derived from it. The first property is that entanglement cannot be shared. If two qubits are maximally entangled with each other, then no other party in the universe can have a share of this entanglement. This property is called the monogamy of entanglement. The second property of entanglement, which makes it so powerful, is called maximal coordination. This property manifests itself when measuring the qubits. When two qubits that are entangled are measured in the same basis, no matter how far away they are from each other, they will always yield the same outcome. This outcome is not decided beforehand, but it is completely random and is decided when the measurement happens.

Entanglement further explained

Hi, so now I'd like to explain to you the phenomenon of entanglement. So what is entanglement precisely? Imagine we have 2 particles: we have particle A and particle B. And these particles can be either full, which is the filled have here, empty, or a superposition of the two. Now let's say that particle A and B are entangled. The weird thing about this entanglement is that when we would measure one of the particles, say we'd like to measure particle A, and we get the outcome/result full. Instantaneously the particle at B collapses into the full state as well. This happens instantaneously, so even faster than the speed of light. However, particle B, or an observer at particle B would never know if Alice, the observer at A has already measured her particle. In order for him to know if Alice measured her particle, Alice needs to send a signal over a classical internet, which cannot exceed the speed of light, to notify Bob, who is at particle B, if the particle has been measured. Only then they can compare their results and see if their particles were indeed entangled. The particles can also be entangled in a different way. So, this corresponds to when the particle A would be full when we measure it, particle B would be empty, and vice versa: if A would result in empty, B would result in full. If we do not know beforehand which kind of entanglement we have, we can also not know what the state of B will be after measuring A.

QUESTION 1: ENTANGLEMENT FROM RELATIVE PHASES

In Module 1, we note that entangled states are those that cannot be written as *tensor products*.
The tensor product of two single-qubit states looks like this:
$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\otimes\begin{bmatrix}\gamma\\ \delta\end{bmatrix}=\begin{bmatrix}\alpha\gamma\\ \alpha\delta\\ \beta\gamma\\ \beta\delta\end{bmatrix}$
As an example, note that the state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\\ 0\\ 1\end{bmatrix}$ cannot be written as a tensor product, since $(\alpha\gamma)(\beta\delta)$ must be equal to $(\alpha\delta)(\beta\gamma)$ arithmetically, but this cannot be true if we multiply the coefficients of the given state.
Is the state $\frac{1}{2}\begin{bmatrix}1\\ 1\\ 1\\ -1\end{bmatrix}$ entangled?

- Yes
- No

Different ways of entanglement

Do two entangled qubits always give the same measurement outcome?

Entanglement can be generated in different ways. This difference will manifest itself in the outcome statistics when performing measurements. In this example you can see an entangled pair of qubits that always produces the opposite answers, rather than the same answers as we saw before.

QUESTION 1: ENTANGLEMENT WITH GENERAL CORRELATIONS

Correlated and anti-correlated states aren't the only maximally entangled states. In fact, any state which can be written as

$12\surd(|\psi\rangle\otimes|\varphi\rangle+||\psi\perp\rangle\otimes||\varphi\perp\rangle)$

is maximally entangled, where the symbol $||\psi\perp\rangle$ is a state which is orthogonal to (has zero inner product with) $|\psi\rangle$.

For example, if we set $|\psi\rangle=|\varphi\rangle=|0\rangle$, then $||\psi\perp\rangle=||\varphi\perp\rangle=|1\rangle$, and we produce the state $12\surd(|00\rangle+|11\rangle)$, which is the familiar fully-correlated entangled state we saw earlier.

Which of the following states are maximally entangled?

- [ ] $12\surd(|00\rangle-|11\rangle)$
- [ ] $12\surd(|00\rangle+|01\rangle)$
- [ ] $13\surd(|0+\rangle+|-1\rangle)$
- [ ] $12\surd(|0+\rangle+|1-\rangle)$

Measurement

Let's see how we transform the quantum information in a qubit to classical information.

Measurement is the act of observing a quantum state. This observation will yield classical information such as a bit. It is important to note that this measurement process will change the quantum state. For instance if the state is in superposition, this measurement will 'collapse' it into a classical state; zero or one. This collapse will happen randomly. Before we do the measurement we have no way of knowing what the outcome will be. What we can do however is to calculate the probability of each outcome. This probability is a prediction about the quantum state, a prediction that we can test by preparing the state many times, measuring it and then counting the fraction of each outcome.

QUESTION 1: PROBABILITY OF MEASUREMENT OUTCOME

A measurement defines a basis, with the measurement operator ∑krk|ψk⟩⟨ψk| being a weighted sum of 'outer products' of these basis vectors. Once such a measurement is applied to an input state |φ⟩, one of the states from the measurement's pre-defined basis is returned, with a probability given by Born's rule: pk=|⟨φ|ψk⟩|2. For example, if we perform a measurement of the Z operator (|0⟩⟨0|−|1⟩⟨1|) on the state |+⟩=|0⟩+|1⟩2√, we can calculate the probability of the |0⟩ state being output:

p0=|⟨+|0⟩|2

We write out the inner product:

⟨+|0⟩=(⟨0|+⟨1|2√)|0⟩=12√(⟨0|0⟩+⟨1|0⟩)

We know that ⟨0|0⟩=1 and ⟨1|0⟩=0, since the 0/1 basis is *orthonormal* (Read the Section on ket notation for more information). This allows us to simplify the inner product:

(⟨0|+⟨1|2√)|0⟩=12√(1+0)=12√.

Therefore, p0=12, since it is the squared magnitude of the inner product which we have calculated.

Suppose that we measure the operator X=|+⟩⟨+|−|−⟩⟨−| with the state |0⟩ as input. What is the probability that the state |+⟩ is produced?

It is not necessary to replicate the calculation above.

- 0
- 14
- 12
- 1

Measurement in superposition

Jonas is going to explain what happens when we measure a system in superposition.

Measurement in superposition When performing measurement we do not necessarily have to collapse into the zero or the one state. We can also choose to collapse into a pair of superposition states. This has important consequences when measuring entangled pairs. When both parties perform the same type of measurement the outcomes will always agree. But if one party performs one type and the other party performs the other type the answers will no longer agree. In fact they will be completely uncorrelated.

QUESTION 1: PARTIAL MEASUREMENT OF AN ENTANGLED STATE

We can take a closer look at what happens when a maximally entangled state is measured using two bases which are not identical. Suppose that the initial state is |00⟩+|11⟩2√, and we measure the operator Z⊗I (this corresponds to Alice measuring Z and Bob measuring nothing):

Z⊗I=1000010000−10000−1

Which of the eigenstates of Z⊗I may be output from this measurement, and with what probabilities?

See also the exercise in "Measurement".

- |00⟩ with probability 12, |10⟩ with probability 12
- |00⟩, |01⟩, |10⟩, or |11⟩, each with probability 14
- |00⟩ with probability 12, |11⟩ with probability 12

QUESTION 2: PARTIAL MEASUREMENT OF AN ENTANGLED STATE - CONTINUED

Suppose that Alice and Bob have prepared some entangled state, and Alice has performed the partial measurement described above, resulting in the basis state |00⟩.

What happens if Bob measures Z or X on his qubit?

See the ket notation section from Module 1, and/or previous exercises for definitions.

- If Bob measures Z, he receives |0⟩, but if Bob measures X, he receives a random state from the basis defined by X.
- If Bob measures Z, he receives he receives a random state from the basis defined by Z, and if Bob measures X, he receives |+⟩.
- Bob's state will be random, and uncorrelated with Alice's, regardless of the operator measured.

Teleportation

Let's listen to Jonas to learn about quantum teleportation!

Quantum teleportation is a method to send qubits using entanglement. Teleportation works as follows: First Alice and Bob need to establish an entangled pair of qubits between them. Alice then takes the qubit that she wants to send and the qubit that is entangled with Bob's qubit and performs a measurement on them. This measurement collapses the qubits and destroys the entanglement, but gives her two classical outcomes in the form of 2 classical bits. She takes this two classical bits and sends them over the classical internet to Bob. Bob then applies a correction operation that depends on these two classical bits on his qubit. This allows him to recover the qubit that was originally in Alice's possession. Note that we have now transmitted a qubit without really using a physical carrier that is capable of transmitting qubits. But of book you already need entanglement to do this. It is also important to note that quantum teleportation does not allow for faster than light communication. This is so because Bob cannot make sense of the qubit in her possession before he gets the classical measurement outcomes from Alice. These classical measurement outcomes must take a certain amount of time to be transmitted. And this time is lower bounded by the speed of light.

Teleportation further explained

Feedback -Teleportation . So we've talked a lot about entanglement, but now we will see how we can use this entanglement to teleport a certain quantum state. Solet's take a look at how this works. First, we have two stations: we have station Alice and we have station Bob. Alice and Bob share an entangled state, which means Bob has one qubit of the entangled state and Alice has the other one of the entangled state. What Alice also has is another qubit which is not entangled with any of those which has a state A. Alice and Bob's goal is to teleport Alice's state A to Bob's qubit .The first thing they do is to perform some operations which we again denote as a black box. After the operations have been done, Alice will measure her qubits. Since Alice has 2 qubits the total outcome can be 4 different situations. For example: both the qubits could be measured into the state empty, the first qubit might have been full and the second empty and vice versa. And there is also the situation where both of the qubits are measured to be full. And these four situation also results in 4 different states for Bob. So in the empty-empty case we get the full state A back, but as you can see not all the measurement outcomes result in this nice state A. They do resemble A, but they are rotated or flipped a bit. So Bob needs to do something in order to correctly retrieve the state A. And for this he needs Alice's help. What does Alice do? Alice uses her information she had on the measured qubits to send Bob instructions. If her first bit was measured to be full, she says to Bob: rotate your qubit 180 degrees clockwise. If it's empty, do

nothing. Almost the same goes for the second qubit, only now the rotation is 90 degrees clockwise. So if it's full, Bob has to rotate the bit, if it's empty Bob has to do nothing. So let's take a look at what this would result in. So let's take a look at the first case: Alice measured empty-empty. She knows now: I measured empty-empty, and because of my clever operations in the black box I know that Bob must already be in the state A. So I order him to do nothing. She sends Bob this information over a classical internet, so she cannot exceed the speed of light on this one. Then we go to the second possibility, so she measured full on the first qubit and empty on the second qubit. Remember: the instruction on this one was: rotate your qubit 180 degrees clockwise. So let's take a look. We see we have an A upside down here, and we have to rotate it 180 degrees. And we see that we get the correct state A. So it works for this outcome. But now we take a look at what happened for the third possibility. What is the first qubit was empty and the second qubit was full. This corresponds to a 90 degrees clockwise rotation. So she sends this information to Bob, and Bob says: OK, right, 90 degrees, I can do this! So we see we have this sort of flipped A here, and were going to rotate it 90 degrees clockwise, again resulting in the state A. And you can already a little bit check for the final case, when both the qubits are full, we have to rotate 180 degrees clockwise and also rotate
90 degrees clockwise. So this would again result in: first the 180 degrees clockwise and now the 90 degrees clockwise, which is again the state A. So if Bob follows the orders of Alice correctly, and Alice of course sends the correct information, Bob will always retrieve the quantum state A that Alice initially had. So the quantum state has been teleported from Alice's lab to Bob's lab.

QUESTION 1: ONE-BIT "TELEPORTATION"

There's a simpler protocol, called "one-bit teleportation", which does not consume an entangled state, but requires Alice and Bob to be close together, such that they can perform the CNOT gate on their qubits:
$\mathrm{CNOT}:|00\rangle\mapsto|00\rangle,|01\rangle\mapsto|01\rangle,|10\rangle\mapsto|11\rangle,|11\rangle\mapsto|10\rangle$
Alice begins with the state $|+\rangle=12\surd(|0\rangle+|1\rangle)$, and Bob begins with the state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$. They then perform the CNOT.
What is the resulting state?

- $12\surd(\alpha|0\rangle+\beta|1\rangle)\otimes(|0\rangle+|1\rangle)$
- $12\surd[(\alpha|0\rangle+\beta|1\rangle)\otimes|0\rangle+(\beta|0\rangle+\alpha|1\rangle)\otimes|1\rangle]$
- $(\alpha|0\rangle+\beta|1\rangle)\otimes|0\rangle+(\beta|0\rangle+\alpha|1\rangle)\otimes|1\rangle$

Repeaters

WHAT IS A QUANTUM REPEATER?

Repeaters enable long distance communication over a quantum network. An optical fiber can transmit a qubit over roughly 100 kilometers. If you want to send qubits over an very long distance just a fiber is not enough. To send information over this long distances we need quantum repeaters. Quantum repeaters can be thought of as a series of short entangled links connecting the two points. The quantum information can then be teleported through these links and arrive safely at its destination.

Repeaters further explained

Feedback -Quantum Repeaters, so now I'd like to explain to you the quantum repeater. First of all, why do we need a quantum repeater? Well, suppose you want to create entanglement between to places which are very far apart. That problem is, is that when you send a qubit it might lose its information during its travels. So the longer the distance the qubit has to travel, the more information might be lost. So we would like to minimize these distances as much as possible. And for this we can use repeaters .So how do these repeaters actually work? Let's take a look. First of all we start with the two separate stations: we have station Alice and we have station Bob. Alice has two qubits which are entangled and Bob also has two qubits which are entangled, but the two stations are not entangled with each other. This is where the repeater comes in, which is shown here in the middle. So for the next step Alice and Bob both send one of their qubits to the repeater, which looks like this. Now important to note here is that there is still no entanglement between Alice and Bob, because these two particles are not entangled. To create this entanglement, we need to do operations. These clever operations are performed here, in this black box. This black box performs some operations on the qubits and eventually measures them, which results in the qubits in the repeater to collapse and generate an entangled state between Alice and Bob. So we can see now that the two qubits which were there in the middle initially are now gone and there is a pure entangled state between Alice and Bob. Now I know I might have been a little bit vague about what happened in the black box. But this is because what happens in the black box might be a little bit too technical. If you're still interested in the details, I'd like to refer you to the link below which explains what happens in the repeaters in more detail.

QUESTION 1: DEFINITION OF A QUANTUM REPEATER

True or False: a quantum repeater simply teleports half of an entangled state from a position between Alice and Bob to Bob.

- True
- False

Reference

https://www.edx.org/es/course/hardware-of-quantum-computer