

University of Adelaide

Thesis

by

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degree of Master

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Declaration of Authorship

I, Junjie Zhang, declare that this thesis titled, ‘Weather Classification’ and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Abstract

Scene classification is an important field in computer vision. For similar weather condition, there are some obstacles for extracting features from outdoor images. In this paper, we present a novel approach to classify cloudy and sunny weather. Inspired by recent study of deep multi-layer neural network and spatial pyramid polling, generate a model based on imangenet dataset. Starting with parameters trained from more than 1 million images, we fine-tune the parameters. Experiment demonstrates that our classifier can achieve the state of the art accuracy.

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

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Abbreviations

LAH List Abbreviations Here

Physical Constants

Speed of Light c = $2.997\ 924\ 58 \times 10^8$ ms⁻¹ (exact)

For/Dedicated to/To my...

Part I

Weather Classification

Chapter 1

Introduction

1.1 Overview

The computer is one of the most significant inventions in history. It provides huge power in the data processing field, like scientific computation. Also, computer systems can aid people in daily life, for example driverless vehicles. However, the human brain still has compelling advantage in some fields, like identifying our keys in our pocket by feel. The complex processes of taking in raw data and taking action based on the pattern are regarded as pattern recognition. Pattern recognition has been important for people in daily life for a long period and the human brain has developed an advanced neural and cognitive system for such tasks.

Weather classification is one of the most important pattern recognition tasks which relates to our work and lives. There are several major kinds of weather conditions, like sunny, cloudy, rainy. And people can classify them easily through their eyes. However, it is not an easy job for machines, especially in computer vision literacy.

In this thesis, I describe an approach to this problem of weather classification based upon the major big trend in machine learning, and more precisely, deep learning. It differs from the feature detectors method, that extracts features manually, and then trains a model to do classification.

Compared to shadow learning which includes decision trees, SVM and naive bayes, deep learning passes input data through several non-linearity functions to generate features and does classification based on those features. It generates a mapping and finds the optimal solution.

1.2 Statistical Pattern Recognition

In the statistical approach, the pattern is represented in terms of d dimensional feature vectors and each element of the vector describes some subjects of the example. In brief, three components are essential to do statistical pattern recognition. First is a representation of the model. Second is an objective function used to evaluate the accuracy of the model. Third is an optimization method for learning a model with minimum errors.

1.3 Artificial Neural Networks

Artificial neural networks(ANNs) were proposed in the mid-20th century. The term is inspired by the structure of neural networks in the brain. It is one of the most successful statistical learning models used to approximate functions. Learning with ANNs yields an effective learning paradigm and has achieved cutting-edge performance in computer vision.

The single-layer perceptron has shown great success for a simple model. For increasing complex models and applications, multi-layer perceptron has exhibited the power of features learning. With hardware developing at the same time, the demand for more efficient optimizing and model evaluation methods for BIG DATA increases.

Recent development in ANN approaches have greatly advanced the performance of state-of-the-art visual recognition systems. With implementing deep convolutional neural networks, it achieves top accuracy in ImageNet Challenge. The model has been used in related fields and performs well in pattern recognition.

1.4 Weather Classification

Weather classification is an important job in daily life. In this thesis, we are focused on two classes of weather conditions, sunny and cloudy.

There are some obstacles in front of weather classification. First, because inputting pixels are independent and high dimension, say a 500x500 RPG image means 750000 pixels, computation is huge. Second, some simple middle level image characters are difficult to recognise by machines, like light, shading, sun shine. It is still not easy to detect these characters with high accuracy. Thirdly, there are no decisive features, for example brightness and lightness, to classify scenes. For example, sun shine can be both

found in sunny and cloudy weather. Last but not least, outdoor images have various background.

Chapter 2

Background

2.1 Related Work

There has been much research done on weather classification. Most works use method - extracting features plus classifiers [2–5]. Some work on low-level features, like color [6], texture [7, 8]. Some use filters or segmentation [7, 9] to extract high-level features, such as sky, shadow [10]. Some work using statistical measurement methods [3, 11] to analyse low-level feature distribution.

Generally, traditional methods follow three basic steps[3, 12]. First, some Regions of Interest (ROIs), for example, sky and shadow, are extracted from a weather image. Then, histogram descriptors are used to represent the difference between them. Finally, a classifier, e.g., Support Vector Machine, is built up based on the extracted features.

Most previous jobs are based on a discriminative model. They extract humanly recognizable features to classify scenes. This type of shadow learning depends mainly on quality of features and human's prior knowledge. An image without prior features or poor features is hard to classify. Furthermore, the methods require a lot of work on data pre-processing and are not flexible. And the approaches depend on structural information to categories an image into one of the classes. Structural information is concluded from illumination invariant features, like SIFT. However, previous jobs have limits on classifying diverse images. It is hard to list total factors which determinate the weather condition.

2.2 Single-Layer Networks

Artificial neurons were introduced as information processing devices more than fifty years ago[13]. Following the early work, perceptrons were deployed in layers to do pattern recognition jobs. A lot of resources were invested in research capability of learning perceptrons theoretically and experimentally. As shown in Figure 2.1, a perceptron computes a weighted summation of its n inputs and then thresholds the result to give a binary output y . We can treat n input as an vector with n elements, and represent the vector as either class A(for output +1) or class B(for output -1).

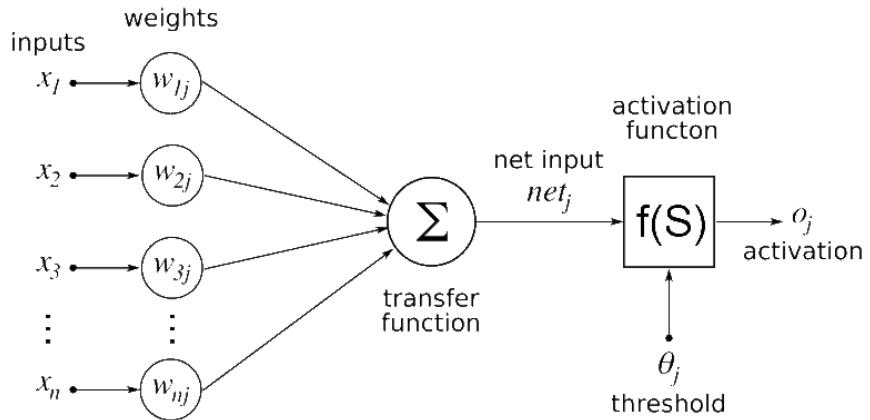


FIGURE 2.1: Diagram of a perceptron.

Each output is updated according to the equation:

$$y_i = f(h_i) = f\left(\sum_j w_{ij}x_j\right) \quad (2.1)$$

where y_i is the i th output, h_i is the net input into node i , The weight w_{ij} connects the i th output and the j th input, and x_j is the j th input. The threshold function $f(h)$ is the activation function and usually makes up the form

$$f(h) = sign(h) = \begin{cases} -1 & h < 0 \\ 1 & h \geq 0 \end{cases} \quad (2.2)$$

and it can be plotted out

Besides the threshold function, there are several deterministic action functions.

- Linear function

$$f(h) = h \quad (2.3)$$

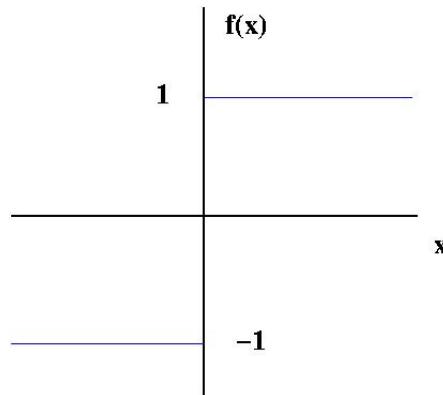


FIGURE 2.2: Threshold function

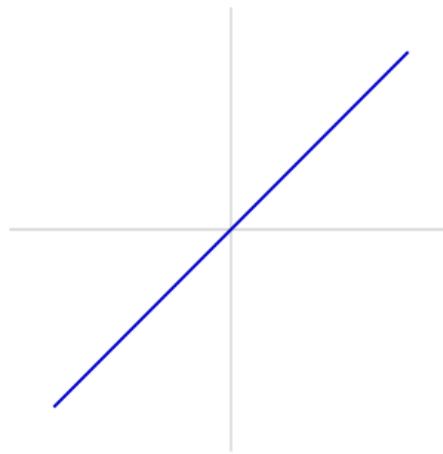


FIGURE 2.3: Linear function

- Sigmoid function

$$f(h) = \frac{1}{1 + e^{-h}} \quad (2.4)$$

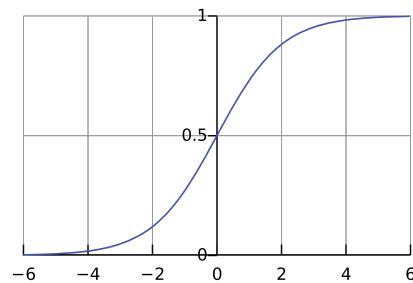


FIGURE 2.4: Sigmoid function

- \tanh function)

$$f(h) = \tanh(h) \quad (2.5)$$

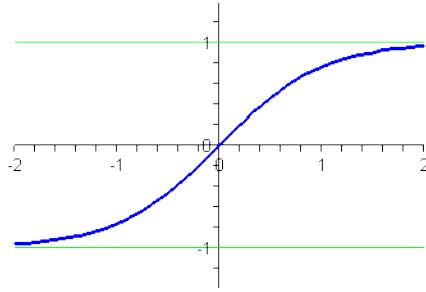


FIGURE 2.5: Tanh function

We can also represent 2.2 in vector notation, as in

$$y = f(h) = f(\mathbf{w} \cdot \mathbf{x}) \quad (2.6)$$

where \mathbf{w} and \mathbf{x} can be regarded as $n \times 1$ dimensional column vectors, and n is the number of input data. The term $w \cdot x$ in 2.6 constructs a $(n - 1)$ -dimension hyperplane which passes the origin. The hyperplane can be shifted by adding an parameter to 2.1, for example

$$y = f(h) = f(\mathbf{w} \cdot \mathbf{x} + \mathbf{b}) \quad (2.7)$$

We can have the same effect by putting a constant value 1 and increase the size of x and w by one. The extra weight w with fixed value 1 is called *bias weight*; it is adaptive like other weights and provides flexibility to hyperplane. Then we get:

$$y = f(h) = f\left(\sum_{i=0}^n w_i x_i\right) \quad (2.8)$$

The aim of learning is to find a set of weights w_i so that:

$$\begin{aligned} y = f\left(\sum_{i=0}^n w_i x_i\right) &= 1 \quad x \in ClassA \\ y = f\left(\sum_{i=0}^n w_i x_i\right) &= 0 \quad x \in ClassB \end{aligned}$$

The single layer neural network classifier is simple to implement, while it can only support a linear decision boundary. We can build a pretty simple neural network to acquire intuition behind mathematical theory. The network has no bias and one neuron which means it has one weight, say w_1 . And we implement a logistic sigmoid activation function on the multiply of input data and weight w_1 . Therefore, the network can map the input data a_0 onto an output a_{out} based on the function

$$a_{out} = f(a_0 w_1) \quad (2.9)$$

where $f()$ is a logistic function. Supposing that an input data 1 maps to an output data 1, we can compute the value of the error function for each possible value of w_1 . Feeding value of w_1 in range $(-10, 10)$, we can plot error surface in Figure 2.6.

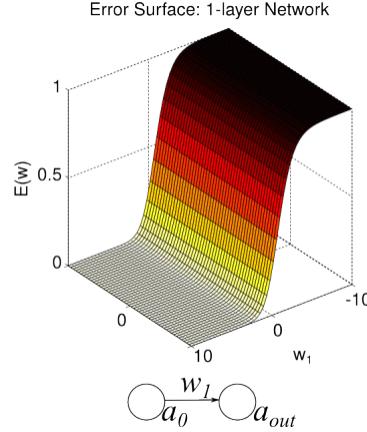


FIGURE 2.6: Error surface for a single layer neural network. Source: Internet

The single layer neural network has a decision boundary which is linear. However, it is clear this is a very limited class of decision boundary. It can be illustrated by two types of dataset.



FIGURE 2.7: Two types of dataset. The left one can be separated by a single layer neural network. The right one cannot be separated by a single neural network.

2.3 Multi-Layer Networks

Single layer networks have some important limitations in terms of the representing range of functions. We are seeking to learn the nonlinearity as the linear discriminant. To improve the representation capability, we can stack layer networks. This is the approach of

multilayer neural networks. Multilayer neural networks implement linear discriminants via mapping input data to a nonlinear space. They can use fairly simple algorithms to learn the form of the nonlinearity from training data.

In the thesis, we limit multi-layer networks in the subset of feedforward networks. Feed-forward networks can provide a general mechanism for representing nonlinear functional mapping between a set of input data and a set of labels. The figure 2.8 is a feedforward network having two layers of adaptive weights.

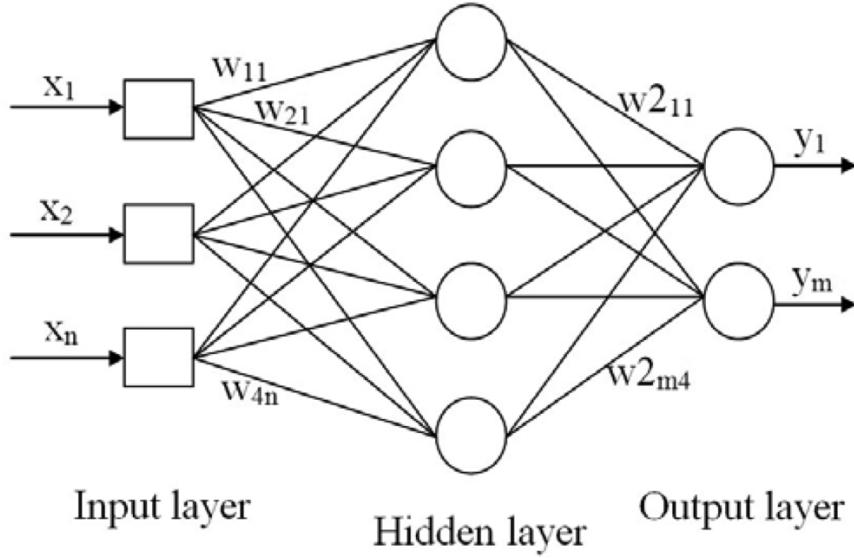


FIGURE 2.8: Diagram of a feedford neural networks.

In the example, the middle column perceptrons act as hidden neurons. The network has n inputs, 4 hidden neurons and m output units. The network diagram represents the function in the form

$$y_m = \hat{f} \left(\sum_{j=0}^m w_{j4}^{(2)} f \left(\sum_{i=0}^n w_{4i}^{(1)} x_i \right) \right) \quad (2.10)$$

In the 4.2, outer activation function could be different with the inner one.

There are some choices for activation functions, sigmoid and tanh are related and can provide high capability with continuous input data. Logistic activation function sigmoid can be represented as

$$f(x) = \frac{1}{1 + e^{-x}} \quad (2.11)$$

Its outputs lie in range $(0, 1)$. We can do a linear transformation $\hat{h}atx = x/2$ on input data and a linear transformation $\hat{h}aty = 2y - 1$ on the output. Then we can get an equivalent activation function tanh which can be represented as

$$f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (2.12)$$

The three layer neural network is capable of approximating any function with enough hidden neurons which means the networks with two layers of weights and sigmoid nonlinearities can provide a high degree of accuracy in classification problems.

Again, we can use a simple example to illustrate the power of multilayer neural network. In this example, we have one input, one hidden neuron and one output. There is no bias in input layer and hidden layer. There are two weights existing in the network, say (w_1, w_2) , and the output can be calculated via

$$a_{out} = f(f(a_0 w_1) w_2) \quad (2.13)$$

where $f()$ is logistic function. With varying w_1 and w_2 , the error surface can be represented in figure 2.10. And the samples which cannot be separated by the single neural

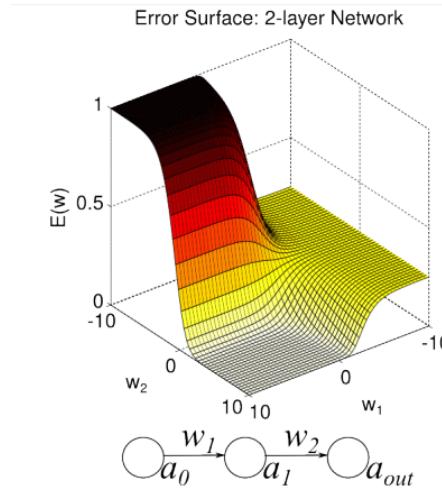


FIGURE 2.9: Error surface for a multilayer neural network. Source: Internet

network can be done by the multilayer neural network.

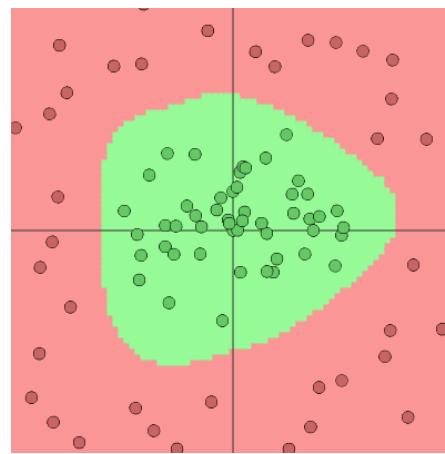


FIGURE 2.10: A multilayer neural network can separate a complicated dataset. Source: Internet

2.4 Stochastic Gradient Descent

Because the weight space in neural networks is continuous, training can reach the optimal weights value through optimization algorithms, which means minimizing loss value of function

$$L(f_w) = \sum L(y, f_w(x)) \quad (2.14)$$

Gradient descent is a method which starts from a random point, then moves to a nearby point that is downhill repeatedly. It converges on a minimum possible loss.

Stochastic gradient descent is a subtype of gradient descent. It only considers a single training point and moves to nearby point based on

$$w = w - \eta \Delta L(w) = w - \eta \sum_{i=1}^n \Delta L_i(w) \quad (2.15)$$

where η is the learning rate and $L_i(w)$ is the value of the loss function at the i^{th} sample. Although stochastic gradient descent does not guarantee convergence, it is fast.

2.5 Backpropagation

Multi-layer neural networks can represent mapping from input data to output classes. How to learn a suitable mapping method from the training dataset? And there is no explicit mapping between output and hidden neurons. This will be resolved by a popular learning algorithm-backpropagation.

Because networks have differentiable activation functions, the activation of output units can be propagated to hidden neurons with regard to weights and bias. If we have an error function, we can evaluate derivatives of the error and update the weights to minimize the error function via some optimization methods.

Backpropagation can be applied to find the derivatives of an error function related to the weights and biases in the network via two stages. First, the derivatives of the error functions, for instance sum-of-squares and Jacobian, with respect to the weights must be evaluated. Second, a variety of optimization schemes can be implemented to compute the adjustment of weights based on derivatives. After putting data forward, for propagation through the network, we can get the output result. It updates weight changes based on gradient descent. Suppose the network has i inputs, h hidden neurons and k outputs. The update equation can be represented as

$$w(j+1) = w(j) + \Delta w(j) \quad (2.16)$$

where $\Delta w(j)$ defined as

$$\Delta w(j) = -\eta \frac{\partial E}{\partial w} \quad (2.17)$$

For the hidden to output layer weights

$$\Delta w(jk) = -\eta \frac{\partial E}{\partial w_{jk}} = -\eta \delta_k y_j \quad (2.18)$$

where

$$\delta_k = \frac{\partial E}{\partial a_k} = (y_k - t_k)y_k(1 - y_k)$$

For the hidden layer weights

$$\Delta w(ij) = -\eta \frac{\partial E}{\partial w_{ij}} = -\eta \delta_j y_i \quad (2.19)$$

where

$$\delta_j = \frac{\partial E}{\partial a_j} = \sum_k \delta_k w_{jk} y_j (1 - y_j)$$

The δ_j of a hidden neuron is based on the δ_k s of output units to which it links. To minimise the error function E with gradient descent, it needs the propagation of errors backwards.

2.5.1 Training protocols

In supervised learning, we have training datasets which are data with labels. We can use the neural networks to find the output of the training data and adjust the weights to optimal values. There are mainly three types of training protocols, stochastic, batch and online training. In stochastic training, we randomly choose samples from the training dataset and update weights every time, depending on output of neural networks. In batch training, we use some samples and pass them through the network, then update weights. In the online training method, each sample of training dataset is used once and weights are updated each time. We usually call one time of passing all training dataset through neural networks, one epoch.

It is worth noting batch learning. In large scale application, the training dataset can be over millions of samples. It is time consuming to compute the loss function over all training datasets, in order to update weights once. It is a practical approach that computes the gradient over a batch of training datasets. Will it be disadvantaged on generalization performance? It depends on the correlations among the training dataset. Consider there are more than one million images in the ILSVRC which is made up of

only 1000 category images. If the images in a batch are selected even from each category, the gradient from the batch is a good enough approximation of the gradient of the full training dataset. Therefore, batch learning leads to faster convergence by evaluating the batch gradients to update weights frequently.

2.6 Overfitting and Regularization

Overfitting is a common phenomenon that the model has a high performance on training datasets, but poor on testing datasets in classification tasks. Thus, a classification problem with two classes and two input variables, (left figure in 2.11), shows decent decision boundaries. With increasing complexity of the model, the decision boundaries become more complex and fit the training dataset extremely well. From the example

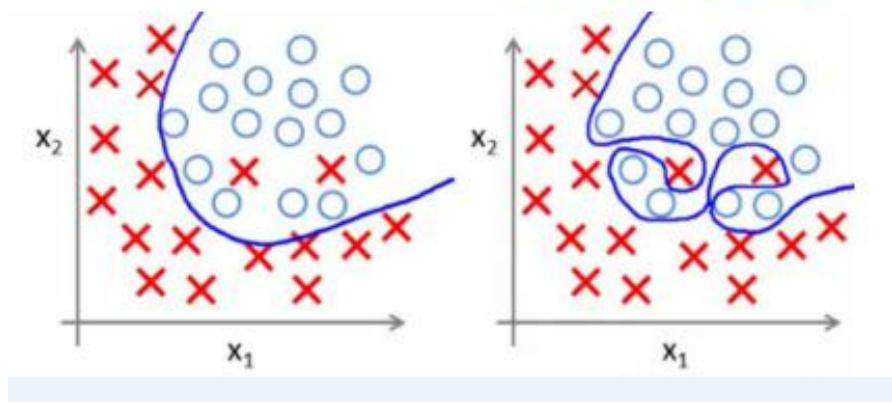


FIGURE 2.11: Overfitting example, the left one has a decent generalization performance and the right one is overfitting.

in Figure 2.11, it is clear that a model, whose complexity is neither too small nor too large, has the best generalization performance.

In order to find the optimal complexity of the neural network, there are mainly two approaches. One is to change the adaptive parameters, like neuron numbers in hidden layers. This is named structural stabilization. It can be approached from two directions. We can start from a small network and increase layer number or neuron number in the learning process and arrive at an effective neural network architecture. The other one is to start from a large network and prune out layers or neurons in the learning process to achieve the optimal network. Another fundamental basic method to control the complexity of the model is use of regularization which includes adding a penalty term in the error function. The degree of regularization can be adjusted by scaling the term via a multiplicative parameter.

Regularization helps to smooth decision boundary surface via introducing a penalty term Ω to the error function

$$\hat{E} = E + \lambda\Omega \quad (2.20)$$

where E is the standard error function and the λ adjustment the extent of the penalty Ω effects on the solution. The task of the learning is to minimize the total error function \hat{E} . It needs to compute the derivatives of Ω with respect to the weights efficiently. A model, which has high accuracy in the training dataset, has a small value for E . At the same time, the smooth error surface of neural networks give a small value for Ω .

A number of regularizers have been performed in applications, like weight decay, early stopping, training with noise, weight sharing etc..

2.6.1 Weight Decay

In order to make decision boundary surface smooth, the weight values should be small. The weight decay regularizer can be represented as

$$\Omega = \frac{1}{2} \sum_i w_i^2 \quad (2.21)$$

where the sum includes all weights and biases. Weight decay of this form leads to major improvements in empirical generalization [14]. Intuitively, in equation 2.21, the smaller the weights are, the better the regularizer is. Usually, the derivatives of the total error function with respect to the weight are used to train the neural network. The network is trained by gradient descent in the continuous time limit. The weights will evolve with time t

$$\frac{w}{t} = -\eta \nabla E = -\eta \lambda w \quad (2.22)$$

where η is learning rate. And the equation has the solution

$$w(t) = w(0) \exp(-\eta \lambda t) \quad (2.23)$$

and the exponential function reduces weights to zero quickly. Weight decay is also named L2 regularization.

2.6.2 Dropout

Neural network with deep layers and a large amount of neurons is a very powerful learning machine. However, the more parameters the network has, the easier it is overfitting. Recently, dropout[1] is a simple and extremely effective regularization technique which

complements the other methods. At the training stage, random neurons are selected with some probability p to update their associated weights, and the others are inactive. In other words, only a reduced network is trained, based on the input data at the training stage. At the testing stage, there is no dropout applied and all neurons are active.

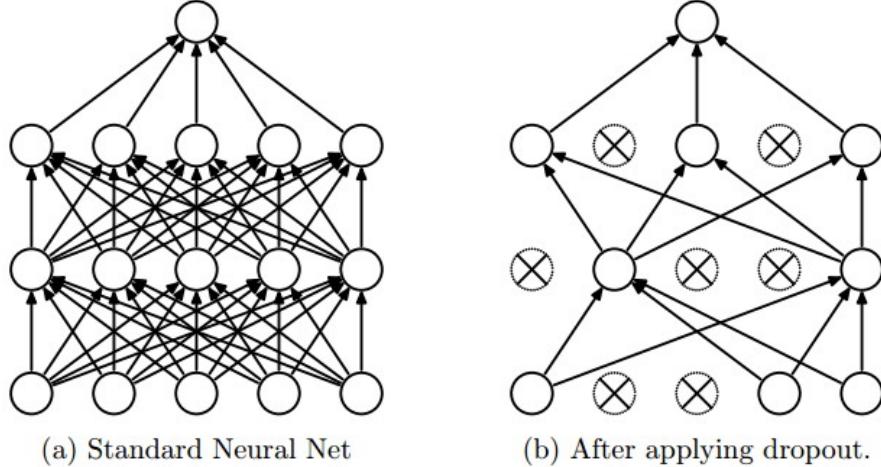


FIGURE 2.12: Illustration of dropout [1].

The dropout method is replaced by performing a scaling of layer outputs by the same probability p . This method can keep the outputs of neurons to be the same in both training and testing stages. For example, if a neuron has p probability to be dropped out in the training stage, the neuron should give an output l without dropout in the testing stage. Then we should apply $p*l + (1-p)*0$ on the output, because the output has $(1-p)$ probability to be 0.

2.7 Softmax classifier

The softmax function, also named normalized exponential, is a generalisation of logistic function which squeezes a d dimensional arbitrary real values vector to a d dimensional vector of real values in the range $(0, 1)$ that add up to 1. Because the softmax function is the gradient log normalizer of categorical probability distribution, it can be used in probabilistic multiclass classification methods.

The softmax function derives from log linear models and interprets the weights in terms of convenient odds ratios. We can constrain the layer input to the output neurons to be positive and divide by the sum.

The softmax layer begins the same way as the normal layer which forms the weighted inputs $z_j^L = \sum_k w_{jk}^L x_k^{L-1} + b_j^L$ where L is the layer number, k is the input data number and j is the output neuron number. It, then, implements a softmax function to the z_j^L

and activates the j output neuron:

$$f_j(z) = \frac{e^{z_j}}{\sum_k e^{z_k}} \quad (2.24)$$

The equation 2.24 implies that the output values are all positive and the sum of all values $\sum_k a_k$ is 1.

The softmax classifier is used to handle a multiclass classification. For a training dataset $(x_1, y_1), \dots, (x_m, y_m), y_i \in \{1, 2, \dots, K\}$ of m labelled examples, the label y can have K different values.

To an unseen sample x , we will use a hypothesis to estimate the probability $P(y = k|x)$ for each value $k = 1, \dots, K$. For example, we want to compute the probability of the class label on each of K different possible values. The neural network will then output a K dimensional vector which represents K estimated probabilities.

$$h_W(x) = \begin{bmatrix} P(y = 1|x; W) \\ P(y = 2|x; W) \\ \vdots \\ P(y = K|x; W) \end{bmatrix} = \frac{1}{\sum_{j=1}^K \exp(W^{(j)\top} x)} \begin{bmatrix} \exp(W^{(1)\top} x) \\ \exp(W^{(2)\top} x) \\ \vdots \\ \exp(W^{(K)\top} x) \end{bmatrix} \quad (2.25)$$

Where W^j are the weights of the model and the normalised distribution ensures that the sum is one.

On the one hand, cross entropy can be used to interpret softmax classifier. The cross entropy between actual distribution p and a predicted distribution q is represented as

$$H(p, q) = - \sum_x p(x) \log q(x) \quad (2.26)$$

Hence, the task of the softmax classifier is to minimise the cross entropy between the actual distribution and the predicted distribution.

On the other hand, the softmax classifier can be interpreted in probability view. Given a sample (x_i, y_i) and parameters W , we can compute the normalised probability:

$$P(y_i | x_i; W) = \frac{e^{f_{y_i}}}{\sum_j e^{f_j}} \quad (2.27)$$

where f_{y_i} is the score predicted by the model with weights W . Therefore the normalised probabilities are computed by exponentiating the values and dividing the sum of all values. We can minimise the negative log likelihood of the ground truth labels which can be regarded as performing the Max Likelihood Estimation(MLE). Aside from MLE,

the Maximum a posteriori(MAP) can also be used to evaluate the performance of the model.

2.7.1 Practical issues

From a numerical view, the exponential computation is easy overflow. Thus, the output of the softmax function is not numerically stable through computing $e^{f_{y_i}}$ and $\sum_j e^{f_{y_j}}$ directly. The implementation requires a normalisation trick. It is mathematically equivalent to multiplying a constant C with both the top and bottom of the fraction.

$$\frac{e^{f_{y_i}}}{\sum_j e^{f_j}} = \frac{Ce^{f_{y_i}}}{C \sum_j e^{f_j}} = \frac{e^{f_{y_i} + \log C}}{\sum_j e^{f_j + \log C}} \quad (2.28)$$

where C can be any positive value. The constant C does not change the output value, but it can improve the numerical stability of the computation. An experienced choice of C is to set $\log C = -\max_j f_j$, and it can shift the vector f to preserve the highest value as 0.

2.7.2 Error function

The error function is used to evaluate the performance of the model. We will generate an error function for softmax regression. An indicator function, $I\{\cdot\}$, is introduced to represent the accuracy for each label. If the predicted result corresponds to the actual label, say $y^{(i)} = k$, the indicator function returns 1, otherwise 0. The error function will be defined as

$$L(W) = - \left[\sum_{i=1}^m \sum_{k=1}^K I\{y^{(i)} = k\} \log \frac{\exp(W^{(k)\top} x^{(i)})}{\sum_{j=1}^K \exp(W^{(j)\top} x^{(i)})} \right] \quad (2.29)$$

where this generates the logistic regression error function

$$\begin{aligned} L(W) &= - \left[\sum_{i=1}^m (1 - y^{(i)}) \log(1 - h_W(x^{(i)})) + y^{(i)} \log h_W(x^{(i)}) \right] \\ &= - \left[\sum_{i=1}^m \sum_{k=0}^1 I\{y^{(i)} = k\} \log P(y^{(i)} = k | x^{(i)}; W) \right] \end{aligned} \quad (2.30)$$

Similar to the logistic regression error function, the softmax error function sum over the predicted different K values of the class value. In the softmax regression, the posterior

probability distribution can be represented as

$$P(y^{(i)} = k|x^{(i)}; W) = \frac{\exp(W^{(k)\top} x^{(i)})}{\sum_{j=1}^K \exp(W^{(j)\top} x^{(i)})} \quad (2.32)$$

It is not easy to solve equation 2.30. Usually an optimization algorithm can help to approximate the optimal value. Taking derivative with respect to weights, we can get the entire gradient

$$\nabla_{W^{(k)}} L(W) = - \sum_{i=1}^m \left[x^{(i)} \left(1\{y^{(i)} = k\} - P(y^{(i)} = k|x^{(i)}; W) \right) \right] \quad (2.33)$$

We can take partial derivative of $L(W)$ with respect to the j th element of $W^{(k)}$.

2.8 Convolutional Neural Networks

Convolutional Neural Networks[15] CNN are widely applied in image understanding and achieve top rank in image classification competition[16]. Compared to regular neural networks, CNN architectures assume that the inputs are images and pixels are related in region. Regular neural networks fully connect layers and the method is computation expensive.

CNN architecture have neurons arranged in 3 dimension, say width, height and depth. For example, there is an image which has dimensions $32 \times 32 \times 3$. The neurons in a layer will connect to a customised region of the previous layer. Moreover, the final output layer has dimensions $1 \times 1 \times d$, where d is the number of class. The dimensions are reduced from 3072 to d . The output is a single vector of class scores.

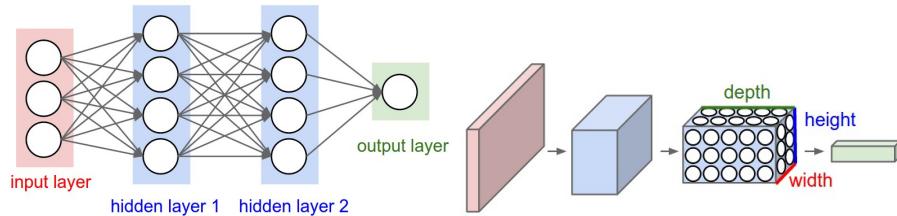


FIGURE 2.13: Left is a fully connect regular neural network. Right is a CNN in 3 dimensions.

2.8.1 Layers in CNN

CNNs have three main type of layers, convolutional layer, pooling layer and fully connected layer. Every layer transforms input to output via a differentiable function.

1. Convolutional Layer
2. ReLU Layer
3. Pooling Layer
4. Fully Connected Layer

Following layer by layer, CNN transforms an image from a set of pixel values to the final class scores.

Convolutional Layer is the key component of CNN and its output can be represented as neurons shaped in 3D volume. A set of learning filters make up the convolutional layer's parameters. Even the size of filter is flexible, it is usually set as a small size. During the feedforward process, each filter slides across the input volume and produces a 2-dimensional feature map. In each slide, the input and the filter have a dot product computation. If there are some specific feature at some spatial position, they can be learnt by filters.

Local connectivity is the key properties of CNN is. Regular neural networks use fully connected method between layers. That is unaffordable for image, even with high capability hardware. Instead, each neuron connects a local region of the input only. There are two main benefits from local connectivity. One is reducing parameters significantly and controlling overfitting. The other one is based on a key image property. Pixels are strongly correlated with nearby pixels. This can be regarded as a local receptive field which can retrieve information from subregions of the image.

To control the output volume arrangement, three parameters are introduced. They are depth, stride and zero-padding. In convolutional layer, depth controls the number of neurons which connect the same subregion of the input volume. All the neurons learn the different features from the input volume. For example, the neurons along the depth in the convolutional layer after input data layer can activate existence of various edges, color, etc.. Stride is another parameter which controls the spatial position of nearby depth column of neurons. The smaller the stride is, the more overlapping receptive fields is

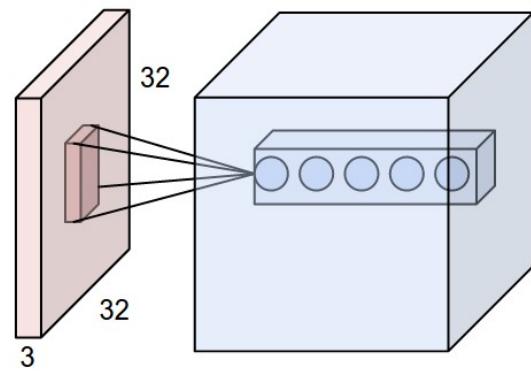


FIGURE 2.14: Diagram for depth in convolutional layer

shared by nearby columns. Last, zero-padding on border makes output volume to keep the same dimension with the input volume.

A scheme, named parameter sharing, is implemented in convolutional layer to limit the number of parameters. The scheme suppose that a filter, which is helpful to compute at some spatial position, should be helpful to compute another position.

ReLU Layer is the abbreviation of Rectified Linear Units. The neurons in the layer operate the non-saturating activation function $f(x) = \max(0, x)$ over the result of dot product in convolutional layers. The layer increase non linearity to network without losing the receptive fields of convolutional layer.

Pooling Layer is a mechanism of downsampling. It is usually appended after the convolutional layer to progressively decrease the spatial size of the feature map. It leads to decrease network parameters and makes the neurons in pooling layer to be relatively insensitive to small shifts of the image.

Fully Connected Layer takes feature map, which has high level representation, from previous layers. Its difference with convolutional layer is that it connects to all neurons in previous layer instead of a receptive field only.

2.9 Spatial Pyramid Matching

The spatial pyramid matching[17] is used to classify high-level semantic attributes, based on low-level features. The method subdivides a image in several different levels of resolution and counts features falling in each spatial bin. It extends bags of features and derives spatial information from the image.

Let two sets of vectors X and Y be in a d -dimensional feature space. The spatial pyramid matching is used to find the approximate correspondence between them. In brief, the spatial pyramid matching places a chain of grids over the feature space and takes the sum of number of matches occurring at each level of resolution. The points falling into the same grid are matched and the match points in finer resolutions have higher weights. Specifically, a chain of grids at resolutions $0, \dots, L$, have $2^0, \dots, 2^L$ cells respectively. H_X^l and H_Y^l are the histograms of X and Y at the l resolution, so that the number of points in the i th cell from X and Y can be represented as $H_X^l(i)$ and $H_Y^l(i)$. The histogram intersection function denotes the number of matches at level l

$$I(H_X^l, H_Y^l) = \sum_{i=1}^D \min(H_X^l(i), H_Y^l(i)) \quad (2.34)$$

Spatial Pyramid Matching (SPM)

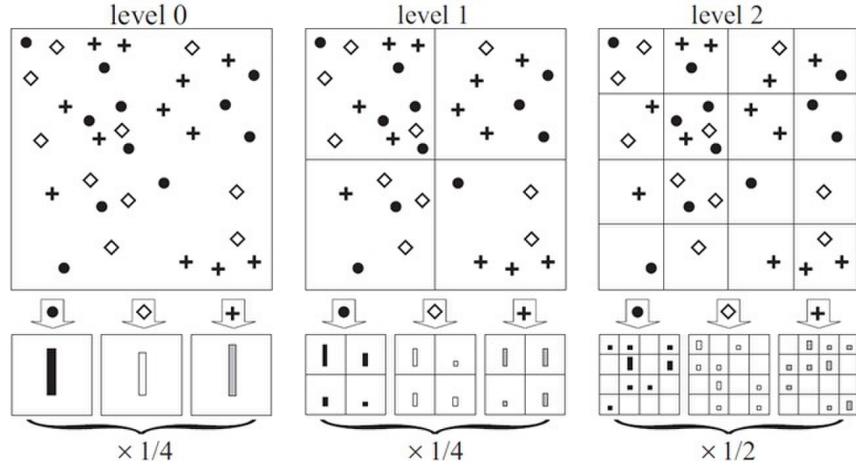


FIGURE 2.15: Diagram of Spatial Pyramid Matching.

Because the match points found in l level include the match points found in the finer level $l + 1$, the number of new match points at level l is $I^l - I^{l+1}$. The weight assigned to level l is $\frac{1}{2^{L-l}}$. Putting all together, the pyramid match kernel can be represented:

$$K^L(X, Y) = I^L + \sum_{l=0}^{L-1} \frac{1}{2^{L-l}} (I^l - I^{l+1}) \quad (2.35)$$

$$= \frac{1}{2^L} I^0 + \sum_{l=1}^L \frac{1}{2^{L-l+1}} I^l \quad (2.36)$$

We perform the spatial pyramid matching in the 2-dimensional image space and apply standard vector quantization in the feature space. All feature vectors are quantized into M discrete types and only the same type of features match to one another. Two sets of 2-dimensional vectors, X_m and Y_m , represent the coordinate of the features of type m in the individual image. The match kernel of two images is the sum of total channel kernels

$$K^L(X, Y) = \sum_{m=1}^M K^L(X_m, Y_m) \quad (2.37)$$

2.10 Transfer Learning

In the literature on machine learning, the transfer learning[18] focuses on storing knowledge from one domain and applying it to a related problem. In other words, the relevant knowledge, learned from previous tasks, can be applied to new tasks. The closer a new

job relates to previous knowledge, the more easily it can be solved. In contrast, other machine learning methods solve problems independently.

The transfer learning has three benefits. One is saving time of preprocessing data. Collecting and processing raw data are time consuming and expensive in each task. It can reduce volume of required data significantly, based on existing knowledge which is extracted from previous learning tasks. The other one is saving time of learning the model from scratch up. Training a model from scratch up, usually, is a long way. The transfer learning can save a lot of time. The last one is to avoid the risk of overfitting. With insufficient training data, a complicated model is easy to overfitting. The transfer learning method can control overfitting because of existing knowledge.

Traditional ML vs. TL

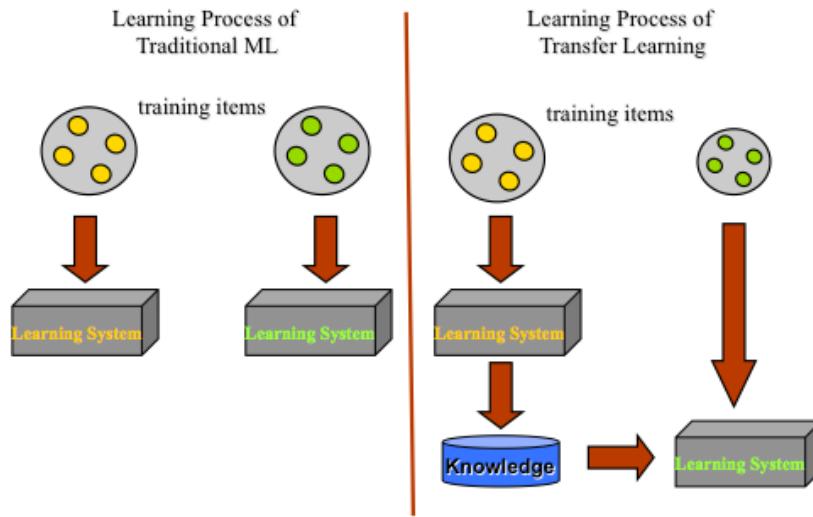


FIGURE 2.16: The left is traditional machine learning method. The right is transfer learning.

A domain can be represented

$$D = \{X, P(X)\} \quad (2.38)$$

where X is the feature space and $P(X)$ is the marginal probability distribution.

In the transfer learning, there are two main challenges:

1. Which part of previous knowledge is useful to the new task?
2. How to represent the existing knowledge in the new model?

The first challenge arise, when we evaluate the relation between the previous knowledge and the current task. The correlated knowledge for the task can improve the performance

of the model. Meanwhile, the unrelated and the negative correlated knowledge are useless, or even harmful for the task. After evaluating the useful knowledge, new learning algorithms should be developed to transfer the knowledge. This leads to the second challenge. Because knowledge can have different representations, the translation process must keep the accuracy and minimise the loss of knowledge.

Chapter 3

Methodology

3.1 Dataset

There are over 15 million labelled images in ImageNet database which has 22,000 categories. The images were collected from the Internet and labelled manually. From 2010, the competition, named the ImageNet Large-Scale Visual REcognition Challenge(ILSVRC), has been held annually. The competition uses a subset of dataset which contains 1000 images in 1000 categories. With 50,000 validation images and 150,000 testing images, there are over 1 million in total.



FIGURE 3.1: 2 Figures from the ImageNet

The weather dataset [10] contains 10,000 images for two categories evenly, cloudy and sunny. They were collected from three sources, Sun Dataset[19], Labelme Dataset[20] and Flickr. They were classified manually and similar images were removed. No unambiguous images exist in the dataset.

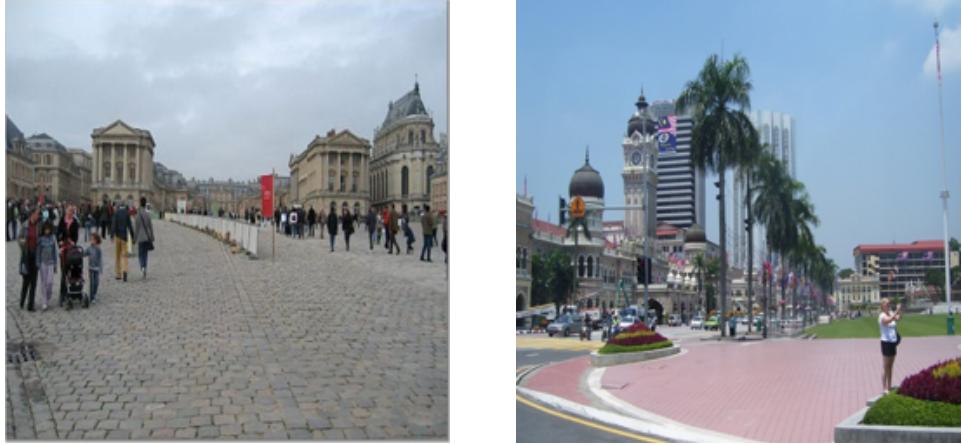


FIGURE 3.2: 2 Figures from the Weather Dataset

In general, the two datasets are different. The ImageNet dataset is used for object classification and the weather dataset is used for scenes classification. The objects of the images are different too. Because the images, of the two datasets, are different size, we have to resize them to a fixed 224×224 to feed into CNN.

3.2 Data Argument

The CNN architecture[16] has about 60 million parameters, and it is essential to avoid overfitting. One of methods is data argument. The dataset is artificially enlarged by cropping and horizontal reflecting images. To each 256×256 image, the network extracts five 224×224 patches from four corners and center and reflects them horizontally. Hence, there are 10 patches for 1 image in total. Figures 3.3 show the method of data argument.

3.3 Spatial Pyramid Pooling

The factors, which have effects on scene classification, can be at any spatial position in the image. This could be a problem to CNN, because it was built up to recognize the objects in the center of images. The spatial pyramid pooling layer can help to solve the problem.

The spatial pyramid pooling layer is deployed behind the fifth convolutional layer. A set of bins are set to discern different local information from the output of the fifth convolutional layer. Assuming dimension of feature maps is $a \times a$ and the bin size is n , each window size is $\lceil a/n \rceil$ and stride size is $\lfloor a/n \rfloor$ where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote the ceiling and flooring operation. For the l level pyramid, there are l spatial pyramid pooling layers.

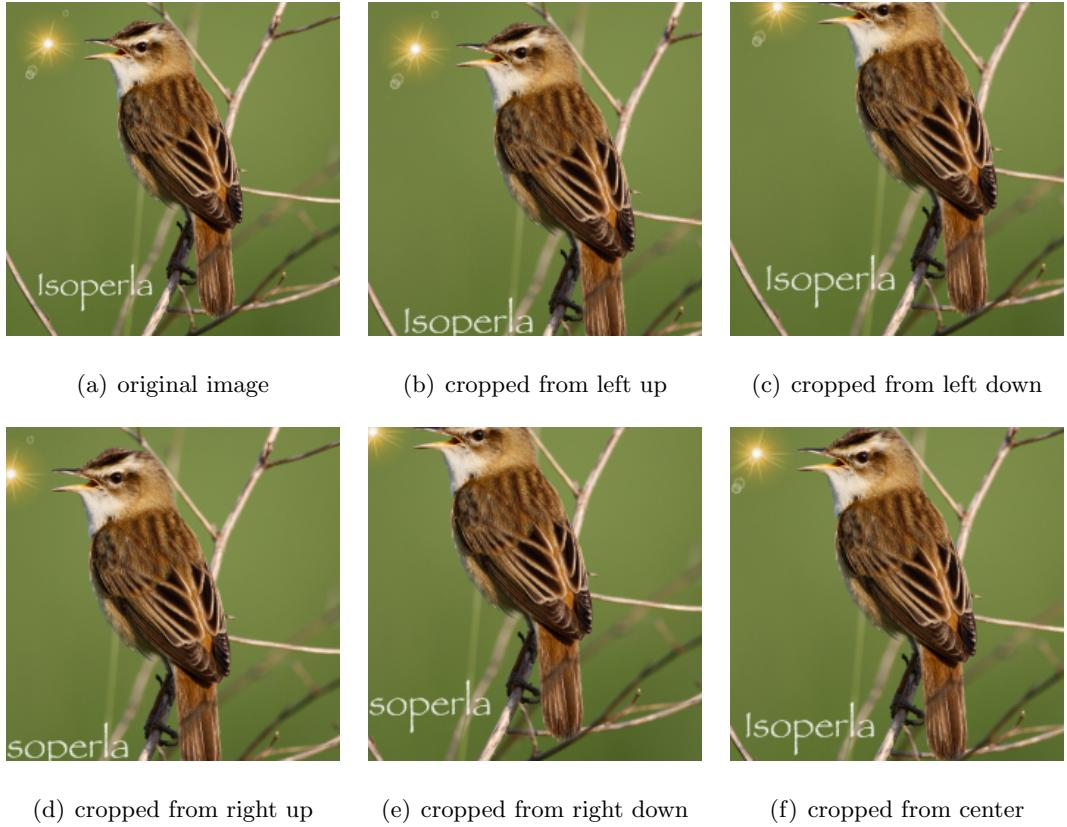


FIGURE 3.3: A set of cropped patches from original image

The l layers will be concatenated into a fully connected layer. The bin sizes can be set to 1, 2, 3 and 6 for the spatial pyramid pooling layers.

With the help of the spatial pyramid pooling layer, most factors, in different scales, are taken into account.

3.4 Convolutional Neural Networks Architecture

Due to the significant performance of the AlexNet[16] which is a deep neural network, we train a model based on the network architecture. The architecture of the network has seven hidden adaptive layers, which are five convolutional layers and two fully connected layers. The network is very deep and has a huge number of parameters. It was implemented on two GPUs in the original experiment. The model is able to reach 62.5% accuracy rates with one prediction and 83% accuracy rates with five predictions in the competition.

The network use the Rectified Linear Units(RELUs)[21] as the activation function, which can achieve a faster learning speed than the $tanh$ function does.

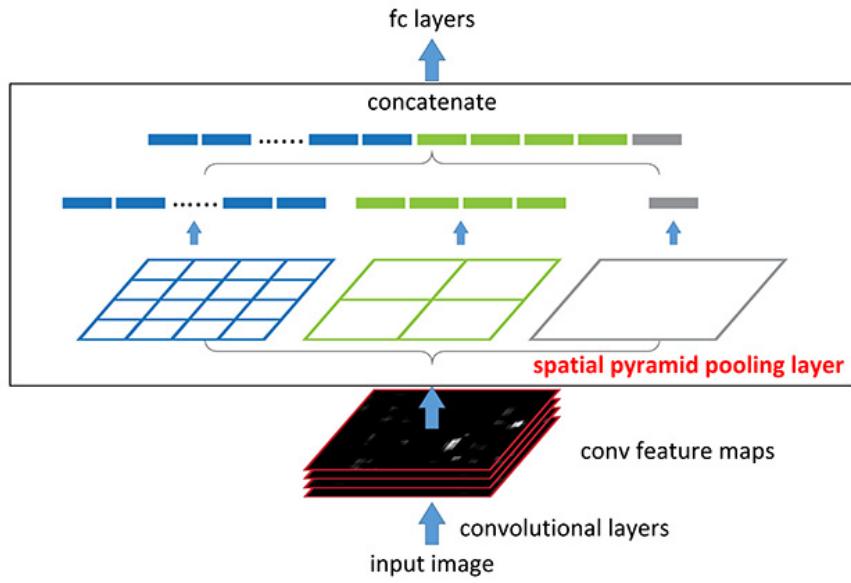


FIGURE 3.4: Diagram of the Spatial Pyramid Pooling layer

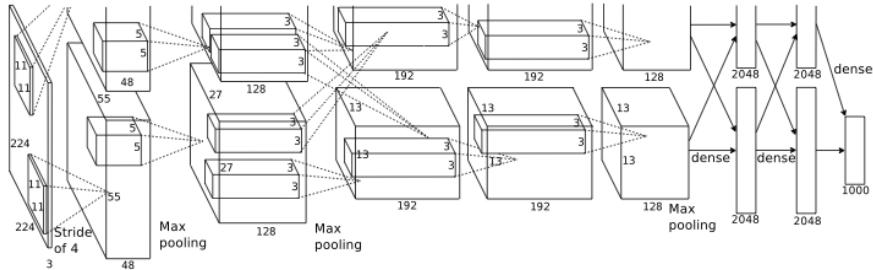


FIGURE 3.5: Architecture of the AlexNet

In the network, there are three types of layers and they play different roles in the model. The input data dimension is $224 \times 224 \times 3$. In the first convolutional layer, there are 96 kernels with size 11×11 . The stride size is 4 and the outputs are 96 neurons. A following max pooling layer downsamples the spatial dimension. The second convolutional layer filters the output of the previous pooling layer with 5×5 kernels. The third one owns 384 3×3 kernels and receives outputs of the second convolutional layer. The fourth convolutional layer has 384 kernels of size 3×3 and the last convolutional layer has 256 kernels of size 3×3 . After those convolutional layers, two fully connected layers have 4096 units each. At the end of network, there is a softmax layer with 1000 outputs.

Layer Name	Layer Description
Input	224×224 RGB image
CONV1	11×11 conv, 96 ReLU neurons, stride 4
POOL1	3×3 max pooling, stride 2
CONV2	5×5 conv 256 ReLU neurons, stride 1
POOL2	3×3 max pooling, stride 2
CONV3	3×3 conv 384 ReLU neurons, stride 1
CONV4	3×3 conv 384 ReLU neurons, stride 1
CONV5	3×3 conv 256 ReLU neurons, stride 1
POOL5	3×3 max pooling, stride 2
SPP	bin size 1,2,3,6
FC6	fully connect, ReLU 4096 neurons
FC7	fully connect, ReLU 4096 neurons
FC8	fully connect, ReLU 1000 neurons
SOFTMAX	1000 way softmax

TABLE 3.1: Architecture of the model

Chapter 4

Experiment

The experiment environment includes hardware, which is i7 CPU, 8G RAM and a GeForce GTX 770, and software, which is Ubuntu Linux 14.04 and Caffe[22], a deep learning software framework. We train the model in Caffe with implemented spatial pyramid pooling layers.

4.1 Training Neural Network

We trained model with the 1000-category ImageNet2012 dataset. We use the network architecture of the model [23] which achieved an excellent result in the 2013 ImageNet Competition. We implement SPP with CUDA C language and deploy it before the first fully-connected layer.

In the experiment, a subset of ImageNet dataset, which contains 1.2 million labelled high-resolution images depicting 1000 object categories for training and 50,000 validation images, is used as training dataset. Most images of the dataset are multi-scale and some are grey. To feed them into the network conveniently, it is better to convert them with a constant dimensionality. The images are resized to 256×256 . And for grey images, they are combined triple times to simulate RGB images. The model is trained on raw RGB values of pixels. Activation function is the Rectified Linear Units(ReLUs), which ensures fast training of the neural network.

$$f(x) = \max(0, x) \tag{4.1}$$

The model 3.5 is trained with a descent optimisation method, stochastic gradient descent. The batch size is 128, and the momentum is 0.9. In each layer, the weights are

initialised with a Gaussian distribution which has a 0 mean, and 0.01 standard deviation. The neuron biases, in the *Conv₂*, *Conv₄*, *Conv₅* and fully connected layers, are initialized with constant 1, while other biases are initialised with constant 0.

The learning rates are set equally for all layers. They are initialised at 0.01 and decrease with the stepdown policy which means that they would drop by a factor of 10 after each 100000 iteration. In total, the learning rate drops 3 times and the accuracy keeps stable after 370K iterations.

The training is regularised via techniques, dropout and weight decay. The dropout regularisation is implemented at the two fully-connected layers and the dropout ratio is set to 0.5. The neurons, which are dropped out, output zero and do not participate in the backpropagation process. Therefore, the neural network samples different architectures each time. It, significantly, decreases complex co-adaptations of neurons because a neuron does not depend on the existence of all other neurons. The weight decay, ϵ , is set to 0.0005 which means the new weight is shrunk according to

$$w^{new} = w^{old}(1 - \epsilon) \quad (4.2)$$

after each weight update.

4.2 Fine-tuning Model

There are two challenges. First, the CNN model is trained to classify objects in images. However, the current task is to classify weather scenes. This can be solved by changing the output of the model from 1000 to 2. Second, the CNN architecture contains more than 60 million parameters which are too many for the weather classification dataset. The target dataset has only 10000 images in total. It is insufficient and the model is easy to overfit. It can be solved by training on the pre-trained CNN model. Because the learned model is close to optimism, it needs tiny adjustment only.

The fine-tuning transfers weights of each layers from the previous model to the new model except for the last fully connected layer. The last layer is taken over by a new layer which contains the same amount of neurons equally to the class number, say two. The weights in the replaced layer are initialized with random values. One advantage of fine-tune is keeping minimum risk of overfitting. The other one is that weights reach quickly optimal values.

In the task, we want to classify sunny and cloudy images. The last layer of the network, *FC8*, is taken place by a new layer with two neurons according to two types of weather

conditions. The weights of the pre-trained model are used to initialise the weights in the new neural network.

To minimise the risk of overfitting, data augmentation is utilised in the experiment. New patches are generated from each image via transformations, say flipping and cropping. The job is done by increasing the spatial generalisation capability of the model. Five patches are cropped from four corners and center of the image, and then they are flipped. In total, ten new images are generated with the same label.

1000 images are kept out to evaluate the model. One iteration is a process of feeding a batch of images into the network. There are 10000 iterations in the full training process, which means that total training image number is 128×10000 . One epoch is that the total training images are fed into the network once. Dividing the size of training images , 9000, the number of epochs is 142. The initial base learning rate is 0.001 and the rate is divided by 10 every 10 epochs. Because the weights in *FC8* are randomly initialised which means they are not close to final optimization value, the learning rate of *FC8* is 10 times of the base learning rate in order to converge quickly.

4.3 Companion Experiments

To compare the performance of fine-tuned model, we do extra experiments with extracting feature methods. We use a pre-trained model with the AlexNet architecture and extract features from the layer *FC7*. We train a SVM classifier based on the features and do weather classification.

4.4 Experimental Results

The results in table 4.1 show that the models, trained by the neural networks, have better performance than the SVM models.

Methods	CNN+SVM	CNN+SPP+SVM	Finetune on CNN	Finetune on CNN + SPP
Accuracy	84.8%	82.1%	93.1%	93.98%

TABLE 4.1: The first test is extracting features from the pre-trained CNN model and training a SVM classifier. The second test is similar with the first except for extracting features from the CNN model with a SPP layer. The third test is fine-tuning the model with AlexNet architecture. The fourth test is fine-tuning the CNN model with a SPP layer

The fine-tuning process is very quick and the accuracy rate is over 90%. After 12 epoch, the accuracy rate exceeds 90%.

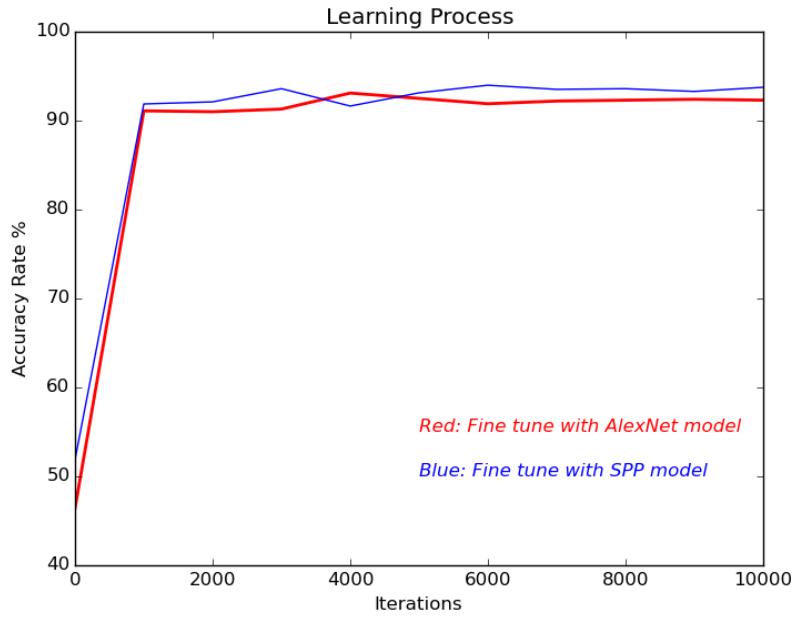


FIGURE 4.1: Learning Process

Learning process may have overfitting which definitely will damage model generalization capability. From figure 4.1, we find that there is no overfitting in the fine-tuning process. Meanwhile, the accuracy increases sharply and keeps stable.

To have a more detailed information of fine-tuning process, we investigate the training loss values. We plot the curve of first 200 iterations and loss value.

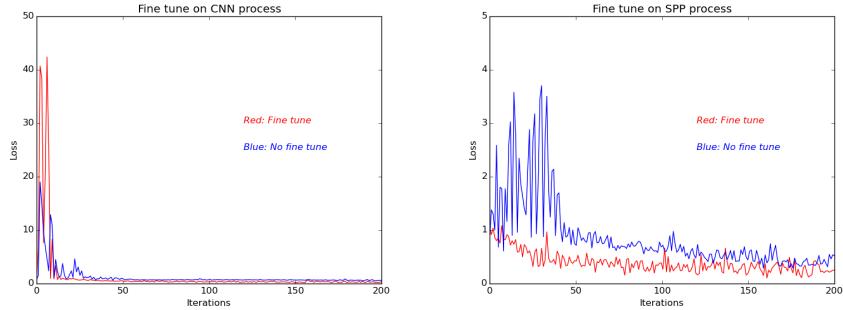


FIGURE 4.2: Finetune Process

From the figure 4.2, we can find that the fine-tuning procedure produces a smoother loss function curve and ceases at a higher accuracy rate. In the left figure, the loss value of the fine-tuning procedure is higher than the value of the no fine-tuning process at the beginning. It then shrinks sharply and keeps lower than the value of the no fine-tuning process. In the right figure, we can find that starting loss values are both less than those in the left figure. And the loss value of the fine-tuning process is less than the value in the no fine-tuning process. We can conclude that generally the fine-tuning is an effective

approach to reduce loss value and the fine-tuning on the SPP model can achieve a better result than the CNN model.

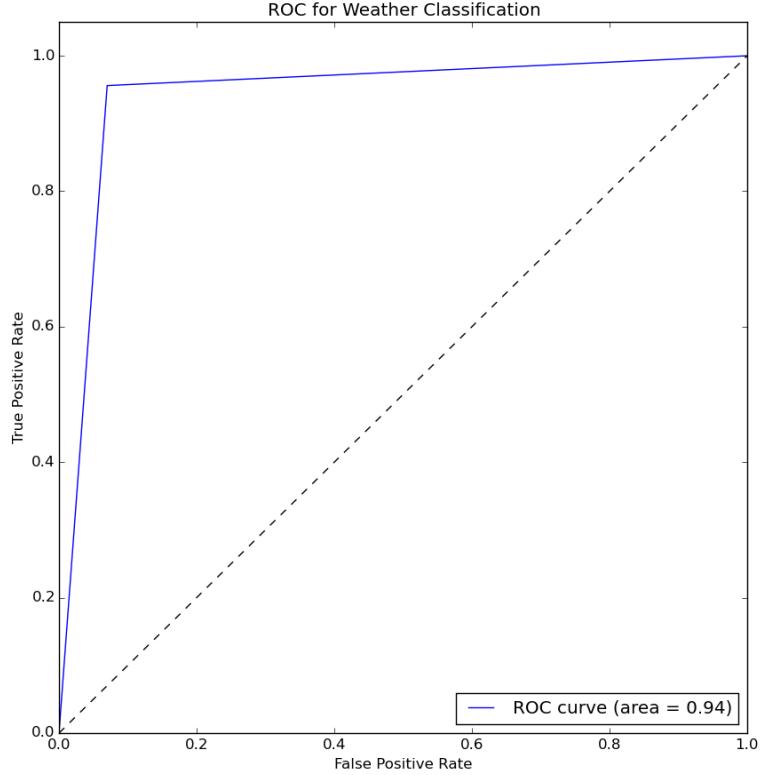


FIGURE 4.3: ROC Curve

4.5 Architecture Analysis

The convolutional neural networks have achieved excellent performance in object classification field, although a full understanding of the mechanism is ambiguous. In order to have some intuition about the CNN, we will do some analysis on the network.

The outputs of each layers have been treated as visual descriptors [24], and the vectors can present some information from the outputs of previous layers. The front layers encode low-level features, and the rear layers are able to capture high-level information. In other words, the image is retrieved heuristically when it passes the convolutional layers.

Fully-connected layers can be represented as a d -dimensional vector. The layer takes all outputs of the previous layer whose feature maps are multidimensional, and flats the feature maps into a d -dimensional vector. The outputs of the second fully connected

layer are feed into a classifier. Two types of classifiers can be used, which are the Support Vector Machine and the softmax.

If a cloudy image is fed into the CNN, Figure 4.4 shows the feature maps of convolutional layers. Because the filter size is too many, only parts of the filters are plotted in images b-d.

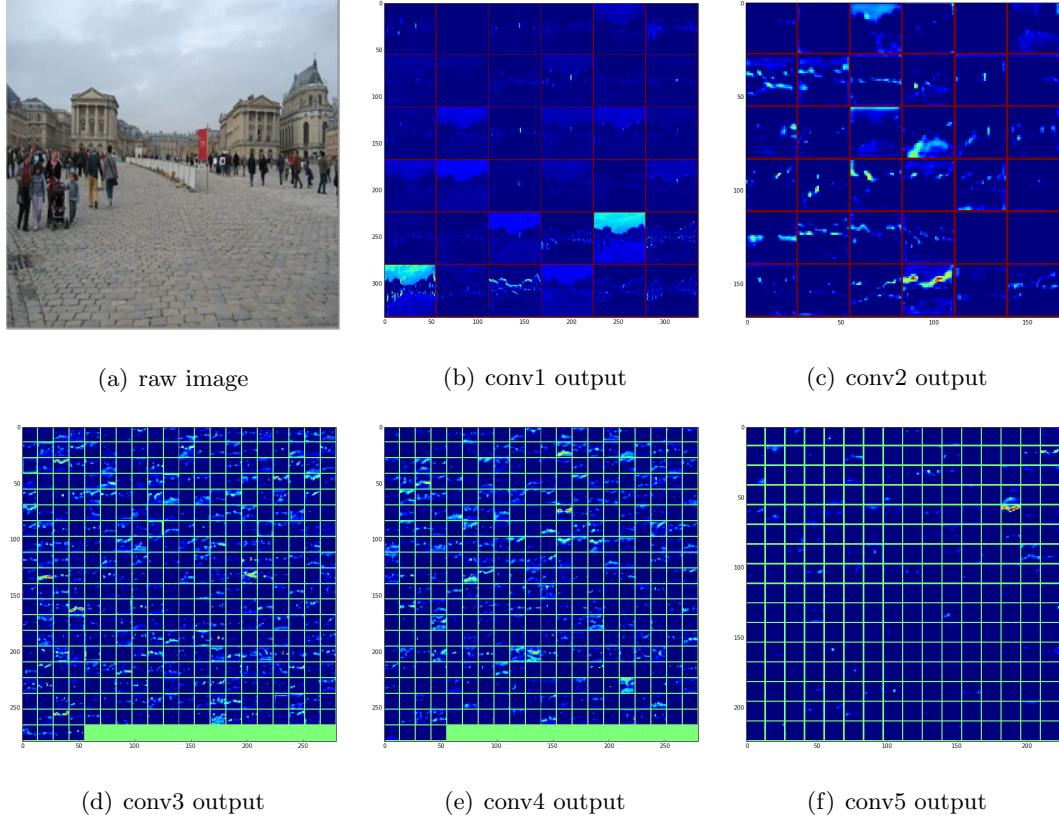


FIGURE 4.4: A cloudy image and the feature maps from convolutional layers

In Figure 4.5, a sunny image is fed into CNN and the according outputs are plotted.

The outputs of the *CONV1* and *CONV2* are still partly recognisable by people. The outputs of the *CONV3* and *CONV4* are quite unrecognisable from a people view. From the output of *CONV5* for the sunny image, the position of sun light is highlighted by several filters. However, the according filters show no signs about the cloudy image.

4.6 Effects of SPP

Because the output of the SPP cannot be represented as visual descriptors, the feature maps of the convolutional layers are compared. The outputs of the *CONV1* and *CONV2* show that more features are recognisable in the SPP model. It is probably that the SPP model has stronger representation capability than the CNN model.

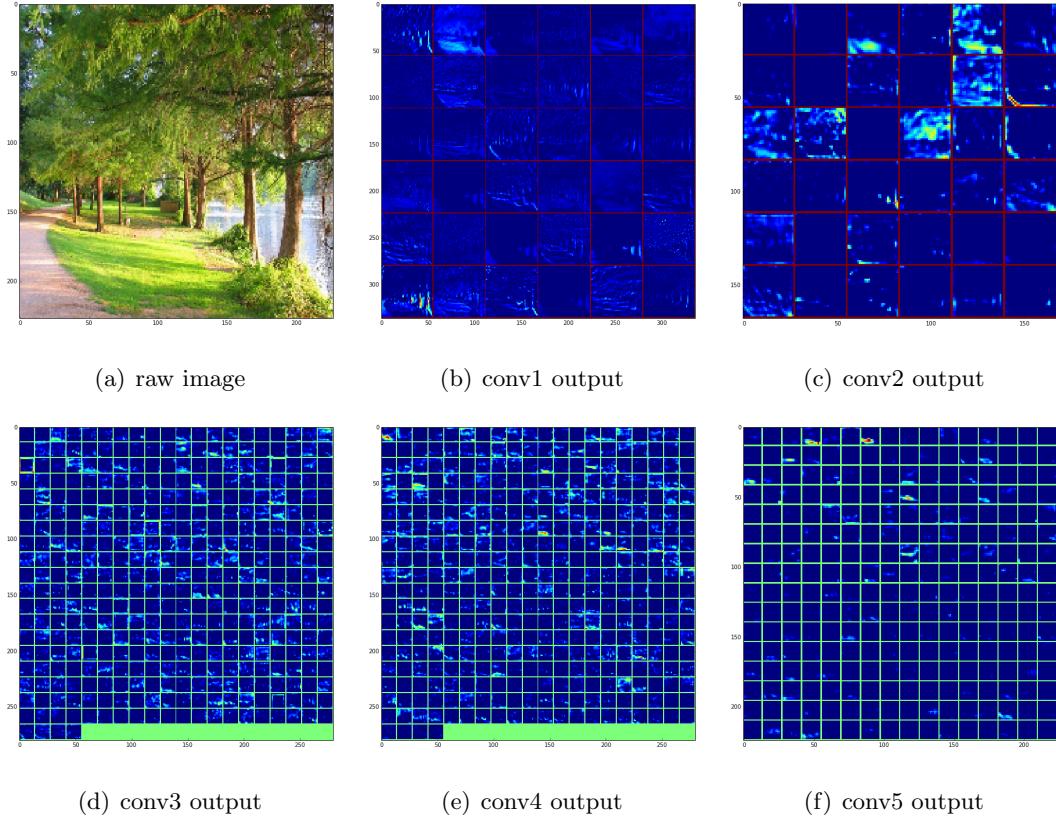


FIGURE 4.5: A sunny image and the feature maps from convolutional layers

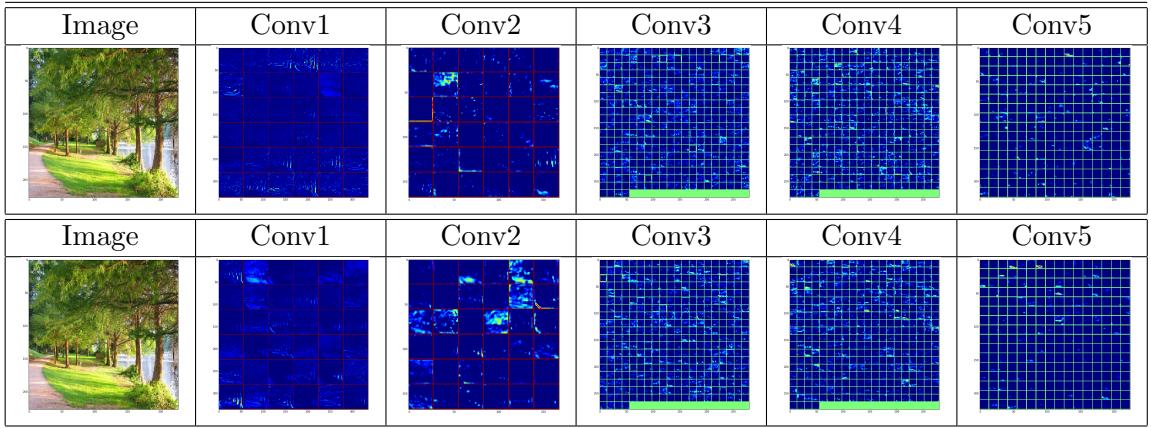


FIGURE 4.6: Visualisation of feature maps from CNN model and SPP model. The upper images are from the CNN model and the lower images are from the fine-tuned model.

In figure 4.7, the outputs of the layer $FC7$ are displayed by histogram method. Comparing histogram difference, the generated features of the SPP model is more divisible than those of the CNN model.

From the feature maps and histogram distributions, it is clear that the SPP model has strong representing capability and can generate divisible features to classifiers.

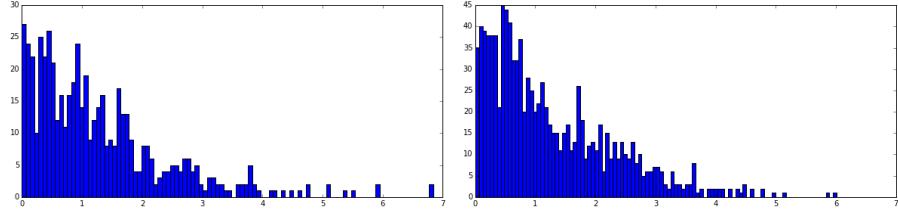


FIGURE 4.7: Histogram distribution of vectors from FC7. The left is from CNN model and the right is from the SPP model.

4.7 Error Results

The images 4.8 are misclassified. They are hard to judge sunny or cloudy, even for human being.



FIGURE 4.8: Misclassified images

4.8 Conclusion and Future Work

In this project, we present an approach to do weather classification through the convolutional neural network and transfer learning. The results shows the strong capacity of the convolutional neural networks and the convenience of the transfer learning. Compared

with the traditional method, which uses extracting features and training a classifier based on the features, the CNN does not depend on specific feature detectors and achieve high accuracy.

In the future work, the full understanding of the CNN mechanism is a demanding work. And the work can be extended to multi-class weather classification and implemented in industry widely.

Part II

Multilabel Learning

Chapter 5

Introduction

5.1 Overview

In the machine learning literature, multi-label classification is a variant of the classification problem where each instance has several labels. In general, the task of multi-label learning is to find a model that can map inputs \mathbf{x} to a set of binary vectors \mathbf{y} . It is different with multi-class classification in terms of the output label space.



FIGURE 5.1: Example Image

The difference between single-label classification and multi-label classification is the number of output labels. For figure 5.1, we can classify it as a picture of a beach in single-label classification problem $\in \{Yes, No\}$. In multi-label classification literature, we can tag beach, sea, chairs, sky, cloud for the picture $\in \{beach, sea, chairs, sand\}$.

X_1	X_2	X_3	X_4	X_5	Y_1	Y_2	Y_3	Y_4
1	0.4	0.2	0	1	0	1	1	0
0	0.2	0.6	1	1	1	0	1	0
0	0.5	0.8	1	1	0	0	1	0
1	0.1	0.4	0	1	1	1	1	0
1	0.8	0.2	0	1	0	1	1	0
0	0.5	0.3	1	1	?	?	?	?

TABLE 5.1: Multilabel $Y_1, \dots, Y_L \in 2^L$

In general, there are two main approaches of tackling the multi-label classification problem. One method is to transfer a multi-label problem into a set of binary classification problems which can be handled by a set of binary classifiers. The other one applies algorithms to complete multi-label classification directly.

Several problem transformation methods can be applied to multi-label classification. A baseline method, named the binary relevance method[25], trains one binary classifier for each label independently. Depending on the results of the classifiers, the combined model predicts all labels for a test sample. The method divides the problem into multiple binary works which has something in common with the one-vs-all method for multi-class classification. Problem transformation methods benefit from scalability and flexibility because single-label classifier can be implemented to job. Support Vector Machine, Naive Bayes, k Nearest Neighbor have been used in the method[25].

There are some other transformation methods, for instance label powerset transformation. The method builds up one binary classifier for each label combination verified in the training dataset[26]. The random k -labelsets algorithm[27] utilizes multiple label powerset classifier which is trained on a random sub dataset of the labels. At last, a voting scheme is used to predict test samples via an ensemble method.

5.2 Multi-Label Learning

Let $X = R^d$ represent the domain of dataset and let $Y = 1, 2, \dots, L$ be the finite set of labels. Given that we have a set of training dataset $T = (x_1, Y_1), (x_2, Y_2), \dots, (x_l, Y_l)$ ($x_i \in X, Y_i \subset Y$) which are extracted from an unknown distribution D . Our target of task is to learn a multi-label classifier $h : X \rightarrow 2^y$ via optimising specific evaluation metric. However, instead of learning a multi-label classifier, we will learn a function f while $f(X) \rightarrow R^d$. Supposing that a high performance classifier can output a closer subset for labels in Y_i than those missing or exceeding in Y_i , which means $f(x_i, y_1) > f(x_i, y_2)$ if $y_1 \in Y_i$ and $y_2 \notin Y_i$. We can transfer real valued function $f(\cdot, \cdot)$ to a ranking function $r(\cdot, \cdot)$ that maps the outputs of $f(x_i, y_1)$ to any $y \in Y$ if $f(x_i, y_1) > f(x_i, y_2)$. It is worth

noting that the multi-label classifier $h(\cdot)$ can be derived from the function $f(\cdot, \cdot)$, where $h(x_i) = y | f(x_i, y) > t(x_i), y \in Y$, and $t(\cdot)$ is a threshold function.

Single-label and multi-class classification can be regarded as two degenerated variants of multi-label learning problem if each sample has only one single label. However, multi-label problem is much more difficult than traditional single-label problems because of high dimensional output space. For example, the number of label sets increases exponentially with increasing number of class labels. If there are 10 class labels for dataset, there are 2^{10} possible label sets maximum.

The challenge of huge combination of output labels is hard to overcome. One of methods is to investigate dependency among labels to reduce label space. For example, if an image labelled with *castle*, it would be highly labelled with *brick* and *mountain*. A movie which is labelled with *comedy* is unlikely related to a *documentary*. Therefore, successful exploitation of the label correlations is regarded as an effective approach to high accuracy multi-label learning process. There are three strategies, depending on the order of the label correlation, to find the relation between labels. They are first order strategy, second order strategy and high-order strategy.

The first order strategy treats the label by label independently and ignores correlation between labels. It can be regarded as decomposing a multi-label learning problem into a set of binary classification problems based on each label. The method benefits from simple computation and high efficiency. However, accuracy could be suboptimal because of ignoring the correlations of labels.

The second order strategy considers pairwise relations between labels. For example, interaction between any pair of labels, or the ranking between related labels and unrelated labels. The method can achieve good generalization performance because the label correlations are investigated in some extent. In real world, there are higher order correlations than second order assumption in many applications.

The high order strategy investigates more than 2 order correlations among labels which can be the influences on each label or addressing connections among sub space of output labels. It is obvious that the high order strategy is more capable than previous two strategies on the cost of complexity and intensive computation.

Chapter 6

Background

Compared to general classification, multi-label classification has different evaluation metrics and learning algorithms.

6.1 Evaluation Metrics

In supervised learning, different metrics, like accuracy and area under the ROC curve, are used to evaluate the generalization performance of model. In the multi-label learning, evaluating performance is more complicated than single-label problems because of increasing number of labels simultaneously. Therefore, two main types of evaluation methods are implemented in multi-label learning, say example-based metrics[28] and label-metrics[27].

The two types of metrics evaluate outputs of classifiers from different perspectives. Given that $S = (x_i, Y_i)$ is a test sample and $h(\cdot)$ is the learned multi-label classifier. The concept of example-based metrics is achieve all class labels of each test example, and then compute the mean value of test sets to evaluate generalization performance. Compared with considering all class labels simultaneously, label-based metrics evaluate performance by treating each class label separately and computing macro/micro-averaged value of all class labels.

In a supervised classification problem, there are a ground truth output and a predicted output for a test example. So the results of each test example can be assigned to one of the four categories:

- True Positive (TP) - label is positive and prediction is also positive
- True Negative (TN) - label is negative and prediction is also negative

- False Positive (FP) - label is negative but prediction is positive
- False Negative (FN) - label is positive but prediction is negative

Here we define a set D of N examples and Y_i to be a family of ground truth label sets and $P_i = h(x_i)$ to be a family of predicted label set. The set of all unique labels is

$$L = \bigcup_{i=0}^{N-1} L_i \quad (6.1)$$

While the definition of indicator function I_A on a set A is presented:

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases} \quad (6.2)$$

6.1.1 Example-based Metrics

Hamming Loss evaluates performance via counting the number of misclassification labels. The smaller value of hamming loss, the better performance.

$$\frac{1}{N \cdot |L|} \sum_{i=0}^{N-1} |L_i| + |P_i| - 2 |L_i \cap P_i| \quad (6.3)$$

Subset Accuracy evaluates the fraction of correctly predicted example while the predicted label set is identical to the ground truth label set. It is equivalent to traditional accuracy metric.

$$\frac{1}{N} \sum_{i=0}^{N-1} I_{\{L_i\}}(P_i) \quad (6.4)$$

Precision is defined as

$$\frac{1}{N} \sum_{i=0}^{N-1} \frac{|L_i \cap P_i|}{|P_i|} \quad (6.5)$$

Recall is defined as

$$\frac{1}{N} \sum_{i=0}^{N-1} \frac{|L_i \cap P_i|}{|L_i|} \quad (6.6)$$

Accuracy is defined as

$$\frac{1}{N} \sum_{i=0}^{N-1} \frac{|L_i \cap P_i|}{|L_i| + |P_i| - |L_i \cap P_i|} \quad (6.7)$$

F1 Measure is an integrated version combined by harmonic mean of **Precision** and **Recall**.

$$\frac{1}{N} \sum_{i=0}^{N-1} 2 \frac{|P_i \cap L_i|}{|P_i| \cdot |L_i|} \quad (6.8)$$

6.1.2 Label-based Metrics

Macro Precision (precision averaged across all labels) is defined as

$$PPV(\ell) = \frac{TP}{TP + FP} = \frac{\sum_{i=0}^{N-1} I_{P_i}(\ell) \cdot I_{L_i}(\ell)}{\sum_{i=0}^{N-1} I_{P_i}(\ell)} \quad (6.9)$$

Macro Recall (recall averaged across all labels) is defined as

$$TPR(\ell) = \frac{TP}{P} = \frac{\sum_{i=0}^{N-1} I_{P_i}(\ell) \cdot I_{L_i}(\ell)}{\sum_{i=0}^{N-1} I_{L_i}(\ell)} \quad (6.10)$$

F1 Measure by label is the harmonic mean between **Precision** and **Recall**.

$$F1(\ell) = 2 \cdot \left(\frac{PPV(\ell) \cdot TPR(\ell)}{PPV(\ell) + TPR(\ell)} \right) \quad (6.11)$$

Micro Precision (precision averaged over all example/label pairs) is defined as

$$\frac{TP}{TP + FP} = \frac{\sum_{i=0}^{N-1} |P_i \cap L_i|}{\sum_{i=0}^{N-1} |P_i \cap L_i| + \sum_{i=0}^{N-1} |P_i - L_i|} \quad (6.12)$$

Micro Recall (recall averaged over all the example/label pairs) is defined as

$$\frac{TP}{TP + FN} = \frac{\sum_{i=0}^{N-1} |P_i \cap L_i|}{\sum_{i=0}^{N-1} |P_i \cap L_i| + \sum_{i=0}^{N-1} |L_i - P_i|} \quad (6.13)$$

Micro F1 Measure by label is the harmonic mean between **Micro Precision** and **Micro Recall**.

$$2 \cdot \frac{TP}{2 \cdot TP + FP + FN} = 2 \cdot \frac{\sum_{i=0}^{N-1} |P_i \cap L_i|}{2 \cdot \sum_{i=0}^{N-1} |P_i \cap L_i| + \sum_{i=0}^{N-1} |L_i - P_i| + \sum_{i=0}^{N-1} |P_i - L_i|} \quad (6.14)$$

For the label-based metrics, the larger the metrics value means the higher generalization performance.

With the previous metrics, there are diverse methods to evaluate model generalization performance. In most multi-label training, learning algorithms optimise one of the metrics. To make evaluation fair and precise, learning algorithms should be tested on different metrics to evaluate performance.

Most metrics are non-convex and discrete, most algorithms turn to optimise alternative multi-label metrics. Recently, some researcher study the consistency of multi-label learning[29]. It is ad hot to figure out if the loss of classifier converges to the Bayes loss with increasing training datasets.

6.2 Learning Algorithms

Algorithms play a key role in the machine learning literature, and there is no exception in multi-label learning. The capability of representation is an important criterion to evaluate performance of a algorithm. Moreover, some relating criteria is important to performance. First, broad spectrum should be considered. A algorithm should cover a range of algorithmic design strategies with unique characteristics. Second, it is reasonable to evaluate the impact which the algorithm poses on the multi-label learning settings. Last, algorithm complexity is a critical factor that the algorithm is practical.

6.2.1 Problem Transformation Methods

6.2.1.1 Binary Relevance

The Binary Relevance is a elementary algorithm which decomposes a multi-label learning problem into a set of independent binary classification problems, and each problem harmonises one label in the set $L = y_1, y_2, \dots, y_q$. The approach initially transforms multi-label dataset into q binary datasets $D_{y_i}(i = 1, 2, \dots, q)$, where each D_{Y_I} includes all samples of the multi-label dataset and has a single binary label to instruct if the dataset has or has no attribute to the relevant label. For example, if a sample has a positive value means that the sample owns the correlation label, otherwise, it does not own it. After transforming the dataset, a set of q binary classifiers, say $H_i(E)(i = 1, 2, \dots, q)$, has been built up using the respective training dataset D_{y_i} .

$$H = \{C_{y_i}(x, y_i) \rightarrow y'_i \in \{0, 1\} | y \in L, i = 1, 2, \dots, q\} \quad (6.15)$$

To classify a new multi-label sample, the Binary Relevance outputs the collection of labels which are predicted with positive value by independent binary classifiers.

The Binary Relevance method combines computational efficiency and simple implementation. With a constant number of samples, the algorithm scales with size q of label set L . Supposing that each classifier has complexity $f(|X|, |D|)$, The Binary Relevance has complexity $O(qxf(|X|, |D|))$. With limit number of q , Binary Relevance can be regarded as a simple and effective method.

One of the disadvantages of Binary Relevance is the limitation of label relationship information. Except that all labels are independent, the Binary Relevance losses the information among labels.

6.2.1.2 Classifier Chains

To improve the performance of Binary Relevance, classifier chains has been introduced in multi-label classification literature[25]. The algorithm transform multi-label learning problems into a chain of binary classifiers based on label dependence.

For a set of q labels L , Classifier Chains model learns q classifiers as Binary Relevance does. However, it links all classifiers through feature space. Given that there are a set of samples $D(x, y_i)(i = 1, 2, \dots, q)$ where y_i is a subset of labels L and x is domain of dataset. We transform the dataset into q datasets in which the j -th sample is based on datasets and previous labels

$$H = \{C_{y_i}(x, y_1, y_2, \dots, y_{i-1}) \rightarrow y'_i \in \{0, 1\} | y \in L, i = 1, 2, \dots, q\} \quad (6.16)$$

Thus it forms a train of binary classifier C_1, C_2, \dots, C_q . Each classifier C_i learns and predicts the binary association of label y_i based on the dataset x and all prior binary relevance predictions $y_j, j = 1, 2, \dots, i - 1$. The learning process starts from C_1 and follows the china step by step. Therefore, C_1 determines $p(y_1|x)$ and each classifier, say C_2, \dots, C_q , determines $p(y_i|x_i, y_1, \dots, y_{i-1})$.

The Classifier Chains method propagates label information through classifiers, and post classifier takes into account previous predictions. This can overcome issues of ignorant label correlation in some degree. At the same time, the method keep benefits of Binary Relevance including computation efficiency and memory efficiency. The computational complexity of Classifier Chains can be close to Binary Relevance depending on the size of labels and complexity of elemental classifiers C_i . As stated in previous, the complexity of Binary Relevance is $O(qxf(|X|, |D|))$ where $f(|X|, |D|)$ is the complexity of elemental classifier. With extra computation introduced by previous labels, the complexity of Classifier Chains is $O(qxf(|X| + q, |D|))$, while the complexity will be worse in case $q \gg |X|$.

6.2.2 Algorithm Adaptation Methods

6.2.2.1 Multi-label k-Nearest Neighbor(ML-kNN)

The algorithm adapts k-nearest neighbor techniques to propose multi-label data and utilize maximum a posteriori (MAP) to make prediction through reasoning embodied labelling information among neighbour [30].

Given dataset X and its label set Y , y represents output vector for x where i -th element $y(i)$ is positive if $i \in Y$, otherwise it is negative. Suppose that $N(x)$ is the set of neighbour of x in training set, we can compute a membership counting vector which represents the number of neighbours of x owning l -th label.

$$C_x(i) = \sum_{a \in N(x)} y_a(i), i \in Y \quad (6.17)$$

To test a new sample t , the algorithm sorts out its category in training dataset by kNN $N(t)$. Let H_1^l denote that t has label l and H_0^l denote that t do not have label l . There are j samples have label l , say $E_j^l (j \in 0, 1, \dots, k)$. Then the category vector y_t is determined via the MAP principle:

$$y_t(l) = \arg \max_{b \in \{0,1\}} P(H_b^l | E_{C_t(i)}^l) l \in Y \quad (6.18)$$

With the Bayesian rule, 6.18 can be represented as

$$y_t(l) = \arg \max_{b \in \{0,1\}} P(H_b^l) P(E_{C_t(i)}^l | H_b^l) l \in Y \quad (6.19)$$

As stated in 6.19, the output of test sample t is $y_t(l)$. It can be computed via prior probability $P(H_b^l) l \in Y, b \in \{0, 1\}$ and the posterior probability $P(E_j^l | H_b^l) (j \in \{0, 1, \dots, k\})$.

6.2.2.2 Collective Multi-label Classifier(CML)

Supposed that labels are highly interdependent in some scenarios, CML explores multi-label conditional random field (CRF) classification models which learn parameters for each pair of labels directly[28].

In dataset (x, Y) , each sample has a corresponding random variables representation (x, y) in which $y = (y_1, y_2, \dots, y_q) y_i \in \{-1, 1\}$ is binary label vector. If the sample contains

j -th label, the j -th element of y is 1, otherwise -1 . Then the aim of multi-label learning is to learn the joint probability distribution $p(x, y)$.

The entropy of (x, y) is represented as $H_p(x, y)$ and gives the distribution $p(\cdot, \cdot)$ of (x, y) . The principle of maximum entropy can be achieved by maximizing $H_p(x, y)$. The fact is expressed with constraints on expectation of function $f(x, y)$. The expected value can be estimated from training dataset

$$F_k = \frac{1}{m} \sum_{(x,y) \in D} f_k(x, y) \quad (6.20)$$

According to normalization constraint on $p(\cdot, \cdot)$, the optimal solution can be represented as:

$$p(y|x) = \frac{1}{Z(x)} \exp\left(\sum_{k \in K} \lambda_k \cdot f_k(x, y)\right) \quad (6.21)$$

Yielding to Gaussian distribution, parameters can be found in a convex log-posterior probability function. For a test sample x , the predicted label set follows to:

$$y_x = \arg \max_y p(y|x) \quad (6.22)$$

It is notable that the exact inference is only suitable for small label space via $\arg \max$, because it needs to reduce the search space of $\arg \max$ significantly via pruning strategies. CML is a second-order approach which considers correlations between every label pair constrains in k_2 .

Chapter 7

Methodology

7.1 Artificial Dataset

To demonstrate the approach, the dataset, named color detector, is created artificially. To each image, it has three labels for representing colour, *red*, *green* and *blue*. If an image is red hue, it has a label for *red* and label values of *green* and *blue* are negative. The representation of colour combination follows colour wheels. If an image has a hue close to *purple*, it has positive labels for *red* and *blue*, and negative label for *green*, and so on.

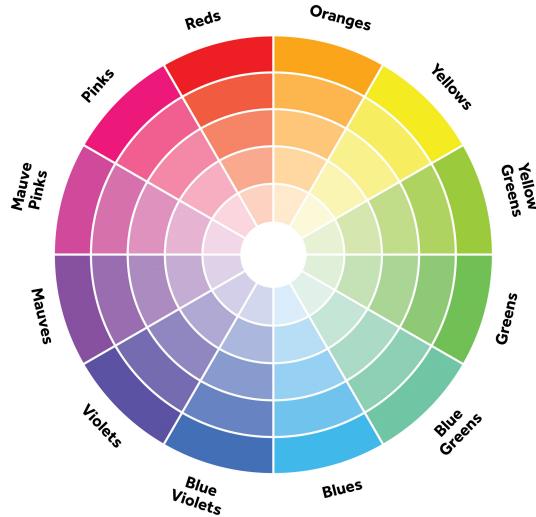


FIGURE 7.1: Colour Wheel Diagram

For each sample x_i , it owns 3 labels $y_0, y_1, y_2 \in \{-1, 1\}$ which represent *red*, *green* and *blue* respectively.

7.1.1 Manufacturing data

The RGB and HSV coordinate systems represent a geometric shape in the color space. Therefore, the distance among colors contains little meaning. However, corresponding distances make some intuitive sense and enable conversion possible between RGB coordinates and HSV coordinates.

To generate dataset artificially, we create 16×16 size images and each image has 256 pixels. For each pixel, we generate a random floating point number h in the range $[0.0, 1.0)$ and use it as value for Hue via formulation.

$$H = h + r * 0.4 - 0.2(r \in [0.0, 1.0)) \quad (7.1)$$

Where r is another random floating point number. Then we generate 2 random floating point numbers as Saturation(S) and Value(V) values. We transform HSV values to RGB values and time 255 for each pixel. Repeating the steps, we can get a 256 pixels image. Because we get a RGB image via converting a HSV image, we compute RGB label value basing on previous h .

The dataset contains 1000 images in total. 960 images are training samples and 40 are test samples.

7.2 Artificial Neural Networks

Artificial Neural networks are a good way to learn a nonlinear function which can be used to map samples to multi labels. At the first layer of a neural networks, neurons take raw dataset while the neurons in the last layer produce outputs. Among the first and the last layers, the middle layers are called hidden layers because they do not connect to external world. The 3-layer neural networks can represent any bounded degree polynomial under certain conditions[31]. The weights of neural networks are learned by algorithms deployed over training dataset. One of successful learning algorithms is the Backpropagation algorithm which updates weights by propagating errors caused by comparing network's prediction for each sample with actual target value.

To adapt classical neural networks, which handle simple-label classification, to classify multi-label samples, we need to modify two factors. One is to design a new error function which is fitting the characteristics of multi-label samples instead of single-label ones. The other is to find a moderate metric according to new designed error function.

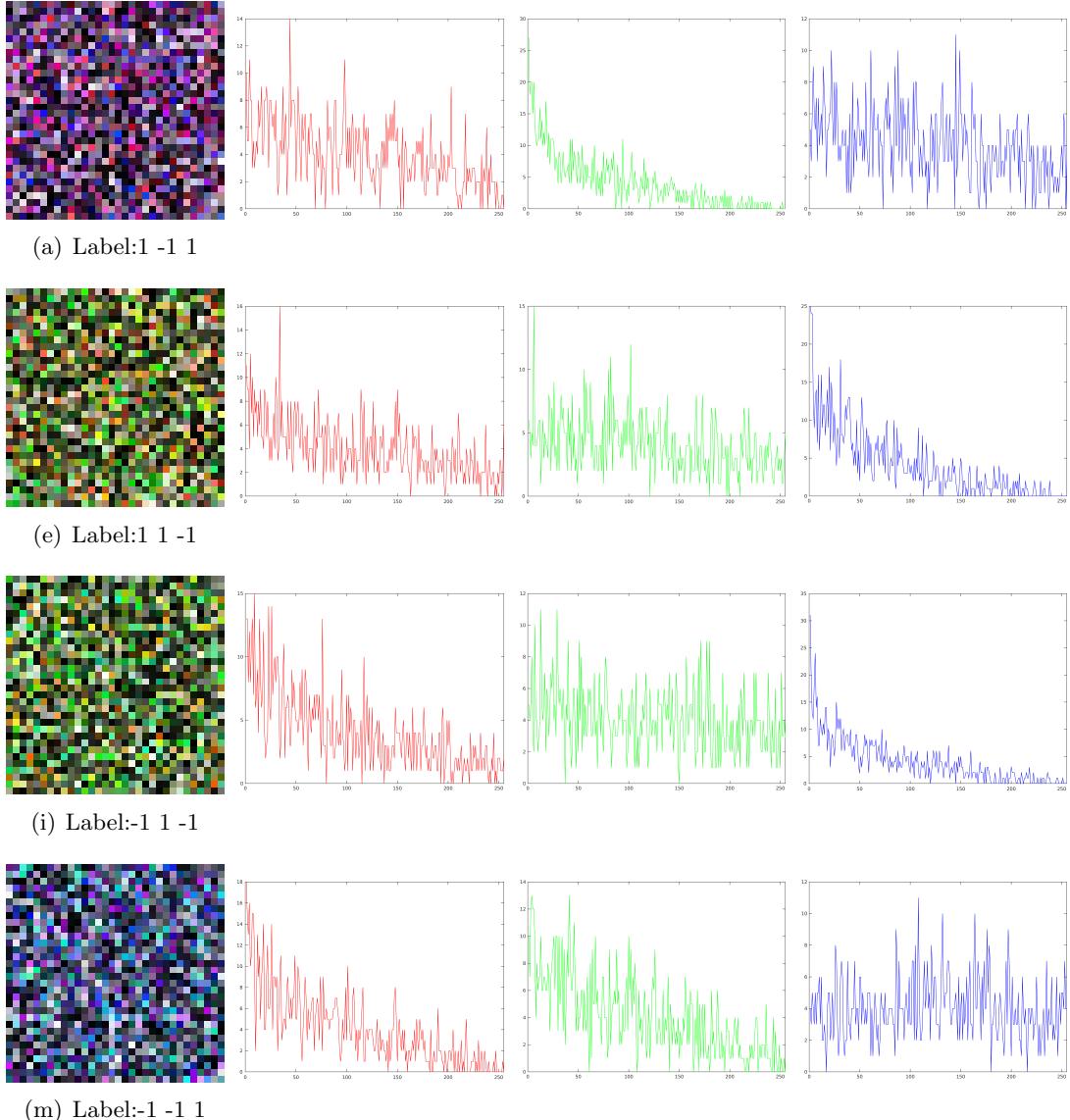


FIGURE 7.2: A multilabel sample and RGB color histograms. Three labels mean red, green and blue.

7.2.1 Network Architecture

Define $\mathcal{X} = \mathbb{R}^d$ as the sample space and $\mathcal{Y} = 1, 2, \dots, Q$ as the set of output labels. Training dataset is composed of m multi-label samples, such as $(x_1, Y_1), ((x_2, Y_2), \dots, ((x_m, Y_m)$, while each sample $x_i \in \mathcal{X}$ is represented as a d -dimensional feature vector and a set of q labels associate with the sample. A neural network can be constructed as figure 7.3 to learn a model from training samples.

The network has d input neurons which corresponds to a d -dimensional feature vector, while last Q neurons represent a combination of output labels. There is a hidden layer in 7.3 which owns n hidden neurons. Input layer is fully connected with hidden layer and the same connecting method between hidden layer and output layer. So there are

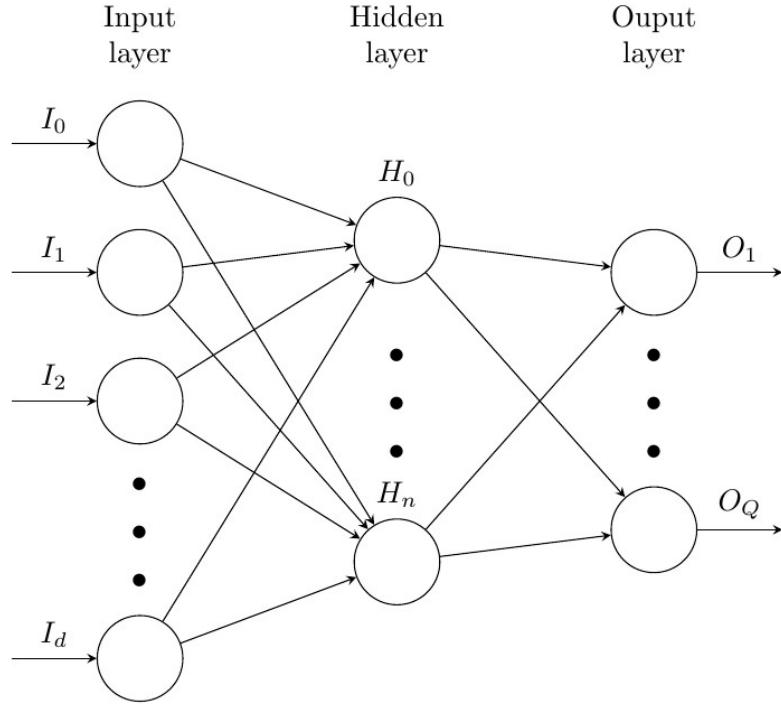


FIGURE 7.3: Network Topology For Multi-lable classification

$d \times n$ weights ($W_{ih}, 1 \leq i \leq d, 1 \leq h \leq n$) between input layer and hidden layer and $n \times q$ weights ($W_{ho}, 1 \leq h \leq n, 1 \leq o \leq q$) between hidden layer and output layer. The bias parameters are represented as I_0 and H_0 .

Because the task of multi-label learning is to predict labels of test samples, it needs to evaluate the global error of model as:

$$E = \sum_{i=1}^m E_i \quad (7.2)$$

E_i is the error on sample x_i which can be defined as:

$$E_i = \sum_{j=1}^Q (c_j^i - d_j^i)^2 \quad (7.3)$$

where $c_j^i = c_j(x_i)$ is predicted j -th label by model on sample x_i , and d_j^i is actual j -th label of sample x_i . The actual label has value of either $+1(j \in \mathcal{Y}_i)$ or $-1(j \notin \mathcal{Y}_i)$.

Various learning algorithms can be used to achieve a model based on training dataset. Backpropagation(BP) algorithm is deployed to learn from errors. However, original BP

algorithm could be improper in multi-label learning because the error function^{7.3} neglects the correlations among labels of a sample. In original BP algorithm, the error function^{7.3} limits on individual label discrimination, such that a specific label $j \in \mathcal{Y}$ belongs to sample x_i or not. It should be taken into consider that labels in Y_i is more important than those outside of Y_i . A new global error function is defined as:

$$E = \sum_{i=1}^m E_i = \sum_{i=1}^m \frac{1}{|Y_i||\hat{Y}_i|} \sum_{(k,l) \in Y_i \times \hat{Y}_i} \exp(-(c_k^i - c_l^i)) \quad (7.4)$$

In error function^{7.4}, i -th errors on the i -th multi-label training sample (x_i, Y_i) have been accumulated. \hat{Y}_i is complementary set of Y_i in \mathcal{Y} and $|\cdot|$ computes the cardinality of a set. Specifically, $c_k^i - c_l^i$ represents the difference between the output labels of network on a label which belongs to sample x_i and a label which does not belong to the same sample. The error function^{7.4} shows that bigger difference leads to better performance. Additionally, the negation of the difference is put into the exponential function to sharply penalize the i -th term if c_k^i is much smaller than c_l^i . The sum of i -th error term accumulates difference between outputs of any pair of labels of which one belongs to the sample and the other does not belong to it. The sum is normalized by the numbers of all pairs, $|Y_i||\hat{Y}_i|$. Then, the correlations between pair labels are computed. In the other words, labels in Y_i should get larger output value than labels in \hat{Y}_i .

As previous statements, error function^{7.4} calculates the difference between output labels of which some belong to a sample and others do not belong to it. The task of learning is minimizing error function^{7.4} via enlarging output values of labels belonging to the training samples and diminishing output values of labels not belonging to it. If training dataset can cover the distribution of whole sample space, neural network model can learn it through minimizing error function by feeding training dataset.

7.2.2 Error Function

The basic goal of regression problems is to figure out the conditional distribution of the output labels, given the input samples. It is common to use a sum-of-squares error function.

The basic goal of classification problems is to figure out the posterior probabilities of class types, given the input samples. Except for sum-of-squares error function, there are some more approximate error functions which can be considered.

The central goal in training neural network is to model the hidden generator of the samples instead of memorizing the training samples. Therefore, the best prediction of

an input sample can be found if the network can present a new value for the sample. Because the general and complete characterization of the generator of the dataset is the probability density $p(x, t)$. It is available to decompose a joint probability density into the product of the conditional density of the labels, the input data and the density of input data,

$$p(x, t) = p(t|x)p(x) \quad (7.5)$$

where $p(t|x)$ represents the probability density of t if x is a distinct label and $p(x)$ represents the density of x

$$p(x) = \int p(t, x)dt \quad (7.6)$$

Most error functions can be obtained from the idea of maximum likelihood. For training dataset,

$$L = \prod_n p(t^n|x^n)p(x^n) \quad (7.7)$$

where each sample is picked out randomly from the same distribution and their probabilities can be multiplied. The error function can be represented by minimizing the negative logarithm of the likelihood since the negative logarithm is a monotonic function.

$$E = -\ln L = -\sum_n \ln p(t^n|x^n) - \sum_n \ln p(x^n) \quad (7.8)$$

where E is notated as error function. The task of feed-forward neural network is to model the conditional probability density $p(t|x)$. And the second term is independent with the parameters in neural networks. We can simplify 7.8 to

$$E = -\ln L = -\sum_n \ln p(t^n|x^n) + C \quad (7.9)$$

It is worth noting that error functions are dependent on different assumptions of the forms of the conditional distribution $p(t|x)$. In this classification task, t represents labels which act as class members or the prediction of the probabilities of class members.

7.2.3 Cross Entropy

The Mean Square Error is a common risk metric comparable to the predicted value of the squared error loss. It is simple to implementation and non-negative. However, it has the disadvantage of heavily weighting outliers[32].

Given that there are two discrete distributions $p(x)$ and $q(x)$ over the same variable x . The relative entropy, relating to cross entropy, is a method to measure the distance

between the distributions:

$$D_{pq}(p(x), q(x)) = \sum_x q(x) \ln \frac{q(x)}{p(x)} \quad (7.10)$$

where the relative entropy is not a true metric because 7.10 could not be symmetric in the interchange $p \leftrightarrow q$ and probably not satisfy the triangle inequality.

Cross entropy loss, also called logistic regression loss, is an alternative method of estimating probability distributions. The measure is commonly used in neural networks. It can be used to evaluate the posterior probabilities of class membership.

Given that training a neuron which has several input data, x_1, x_2, \dots , with weights w_1, w_2, \dots , and a bias, b , the output, for example two classes, can be represented as the weighted sum of the input data.

$$a = f(x) = \sum_i w_i x_i + b \quad (7.11)$$

Then the cross entropy cost function is

$$C = -\frac{1}{n} \sum_x [y \ln a + (1 - y) \ln(1 - a)] \quad (7.12)$$

where n is the number of training data, a is the prediction result and y is actual result.

The cross entropy can act as a cost function because of two properties. The first one is that C is non-negative, because the items of logarithms are in the range $[0, 1]$ and there is a minus sign at the beginning of the sum. The second one, if the prediction output is close to the actual value for all training data, the loss value, C , will be close to 0. Given that $y = 0$ and prediction value $a \approx 0$, the first item in 7.12 is 0 while the second item is $-\ln(1 - a) \approx 0$. The similar situation occurs for situation, $y = 1$ and $a \approx 1$. Therefore, the value of cost function will be small given that prediction value is close to the actual value.

In total, the cross entropy tends to 0 when the neuron acts better at predicting the output y for total training inputs x . They are basic properties for a cost function. Although the quadratic cost function satisfy the properties, the cross entropy has the advantage on avoiding the issue of learning curve slowing down. To illustrate the advantage, the partial derivative of the cross entropy cost function can be computed with the respect

to the weights. Applying the chain rule twice on 7.12

$$\frac{\partial C}{\partial w_j} = -\frac{1}{n} \sum_x \left(\frac{y}{f(x)} - \frac{(1-y)}{1-f(x)} \right) \frac{\partial f}{\partial w_j} \quad (7.13)$$

$$= -\frac{1}{n} \sum_x \left(\frac{y}{f(x)} - \frac{(1-y)}{1-f(x)} \right) f'(x) x_j. \quad (7.14)$$

where the second equation can be represented as

$$\frac{\partial C}{\partial w_j} = \frac{1}{n} \sum_x \frac{f'(x) x_j}{f(x)(1-f(x))} (f(x) - y) \quad (7.15)$$

Because the definition of the sigmoid function is $f(x) = 1/(1+e^{-x})$ and the derivative of sigmoid function is $f'(x) = f(x)(1-f(x))$. Then the items $f'(x)$ and $f(x) = 1/(1+e^{-x})$ cancel in the equation and it becomes

$$\frac{\partial C}{\partial w_j} = \frac{1}{n} \sum_x x_j (f(x) - y) \quad (7.16)$$

The equation shows that rate of learning weights is controlled by $f(x) - y$. The larger the error, the faster the neuron learns. In particular, the property avoids the learning slowdown because the derivative item $f'(x)$ gets canceled out in the quadratic cost.

In the similar way, the bias can be computed by the partial derivative.

$$\frac{\partial C}{\partial b} = \frac{1}{n} \sum_x (f(x) - y) \quad (7.17)$$

7.2.4 Training and Testing

Following previous process, gradient descent is used to minimize the global error function with backpropagation.

To train a sample (x_i, Y_i) , in while x_i is input data and Y_i is associated label, the predicted output labels computed by neural network for is

$$c_k = f(netc_k + \theta_k) \quad (7.18)$$

where θ_k is the bias units of k layer, $f()$ is the activation function of the output neurons which is a $tanh$ function 2.12 in this project. $netc_k$ is the input data of the layer:

$$netc_k = \sum_{s=1}^M b_s w_{sk} \quad (7.19)$$

where w_{sk} is the weights connecting layer s and k , while M is the number of neurons in the hidden layer.

As \tanh function is differentiable, the general error of the k -th output neuron can be defined as:

$$d_k = -\frac{\partial E}{\partial \text{netc}_k} \quad (7.20)$$

combining with 7.18, we can get

$$d_k = -\frac{\partial E_i}{\partial c_j} \frac{\partial c_j}{\partial \text{netc}_k} = -\frac{\partial E_i}{\partial c_j} f'(\text{netc}_k + \theta_k) \quad (7.21)$$

With considering global error function 7.4

$$\frac{\partial E_i}{\partial c_j} = \begin{cases} -\frac{1}{|Y_i||\hat{Y}_i|} \sum_{l \in \hat{Y}_i} \exp(-(c_j - c_l)) & \text{if } j \in Y_i \\ \frac{1}{|Y_i||\hat{Y}_i|} \sum_{k \in Y_i} \exp(-(c_k - c_j)) & \text{if } j \in \hat{Y}_i \end{cases} \quad (7.22)$$

with derivation of \tanh , and substituting it with 7.21 and 7.22 we can get

$$d_k = \begin{cases} \left(-\frac{1}{|Y_i||\hat{Y}_i|} \sum_{l \in \hat{Y}_i} \exp(-(c_j - c_l)) \right) (1 + c_j)(1 - c_j) & \text{if } j \in Y_i \\ \left(\frac{1}{|Y_i||\hat{Y}_i|} \sum_{k \in Y_i} \exp(-(c_k - c_j)) \right) (1 + c_j)(1 - c_j) & \text{if } j \in \hat{Y}_i \end{cases} \quad (7.23)$$

According to previous method, we can define the general error of the s -th hidden neuron as:

$$e_s = -\frac{\partial E_i}{\partial \text{netb}_s} \quad (7.24)$$

with $b_s = f(\text{netb}_s + \lambda_s)$ and chain rule,

$$e_s = -\frac{\partial E_i}{\partial b_s} \frac{\partial b_s}{\partial \text{netb}_s} = -\left(\sum_{j=1}^Q \frac{\partial E_i}{\partial \text{netc}_j} \frac{\partial \text{netc}_j}{\partial b_s} \right) f'(\text{netb}_s + \lambda_s) \quad (7.25)$$

For $d_j = -\frac{\partial E_i}{\partial \text{netc}_j}$ and $\text{netc}_j = \sum_{s=1}^M b_s w_{sj}$, then

$$e_s = \left(\sum_{j=1}^Q d_j \times \frac{\partial (\sum_{s=1}^M b_s w_{sj})}{\partial b_s} \right) f'(\text{netb}_s + \lambda_s) = \left(\sum_{j=1}^Q d_j w_{sj} \right) f'(\text{netb}_s + \lambda_s) \quad (7.26)$$

As function $f()$ is \tanh , we get

$$e_s = \left(\sum_{j=1}^Q d_j w_{sj} \right) (1 + b_s)(1 - b_s) \quad (7.27)$$

Stochastic gradient descent(SGD) method is used to approximate function.

$$\Delta w_{sj} = -\alpha \frac{\partial E_i}{\partial w_{sj}} = \alpha \frac{\partial E_i}{\partial netc_j} \frac{\partial netc_j}{\partial w_{sj}} = \alpha d_j b_s \quad (7.28)$$

$$\Delta v_{hs} = -\alpha \frac{\partial E_i}{\partial v_{hs}} = \alpha \frac{\partial E_i}{\partial netb_s} \frac{\partial netb_s}{\partial v_{hs}} = \alpha e_s a_h \quad (7.29)$$

while bias are updated according to

$$\Delta \theta_j = \alpha d_j \quad \Delta \lambda_s = \alpha e_s \quad (7.30)$$

in previous Equations, α is learning rate in the range of [0.0 1.0].

Based on previous process, a learning algorithm has been set up with backpropagation. More ever, training samples are fed into neural network in each epoch. After all training samples $((x, Y))$ fed into network, weights and bias are updated through equations 7.28 and 7.30. The training samples are fed into the network iteratively while global error value decreases. Finally, the error value converge to a minimum value.

In testing process, network predicts a sample which has a set of actual labels $c_j (j = 1, 2, \dots, Q)$ by ranking the labels. Because the output values of each label is in range $[-1.0, 1.0]$, a threshold function $t(x)$ is used to determine associated label set for the sample x . A large margin ranking system[33] is adopted to generalize the sets. $t(x)$ is a linear function, $t(x) = w^T \cdot c(x) + b$, where $c(x) = (c_1(x), c_2(x), \dots, c_Q(x))$ is a Q -dimensional vector which represent j output labels of a sample. Learning the threshold function $t(x)$ follows the next steps. For every training sample $(x_i, Y_i) (1 \leq i \leq m)$, the relation between $c(x_i)$ and target value $t(x_i)$ is:

$$t(x_i) = \arg \max_t (|\{k|k \in Y_i, c_k^i \leq t\}| + |\{l|l \in \hat{Y}_i, c_l^i \geq t\}|) \quad (7.31)$$

If there are several minimum values and optimal values are in a division, the middle value of the division is chosen. The task of learning the parameters of threshold function is to solve the matrix equation $\Phi \cdot w' = t$. The matrix Φ has dimensions $m \times (Q + 1)$ in which i -th vector is $(c_1^i, c_2^i, \dots, c_Q^i, 1)$, w' is $(Q + 1)$ dimensional vector (w, b) and t is the m dimensional vector $(t(x_1), t(x_2), \dots, t(x_m))$. Linear least squares is used to find the solution of the equation. Given a sample x , the network predicts output label vector $c(x)$ and the threshold value for x is gotten by solving equation $t(x) = w^T \cdot c(x) + b$.

The computation complexity of evaluating derivatives of the error function is linearly with neuron size of network. Three main components are composed of computation, feedforward process, backpropagation process and updating weights process. In feed-forward process to compute b_i and c_j , the computation cost is mainly on evaluation

the sums and activation function. In backpropagation process([7.23](#)and[7.27](#)), the computation complexity of d_k and e_s is $O(Q)$. In updating weights process, the overall computational cost is $O(W)$ where W is the total number of weights .

Chapter 8

Experiment

We use the identical hardware and software environment as described in weather classification project.

8.1 Dataset

We generate 1000 raw RGB images with size 16×16 automatically. 40 images are reserved for testing and 960 images are used for training. Each image has 3 labels $y_i \in (-1, 1)$ representing three attributes, red, green and blue. If the image has the color attribute, the corresponding label is 1, otherwise -1.

8.2 Details of network

We build a network architecture from scratch up. First, we need to determine how many layers needs to construct a multilayer network. In [34], Lippmann has proved that two hidden layers are able to create classification regions of arbitrary desired shape. However, Kolmogorov [35] showed that the superposition of continuous one-dimensional function can represent a continuous function with several variables. Then we set up a neural network with one fully connected hidden layer between input and output layers. Between two fully connected layers, we put a ReLU layer to apply the non-saturating activation function $f(x) = \max(0, x)$ which can help the decision function increase the non-linear properties.

Then we need to determine how many neurons need in the hidden layer. The number of neurons in the hidden layer depends mainly on

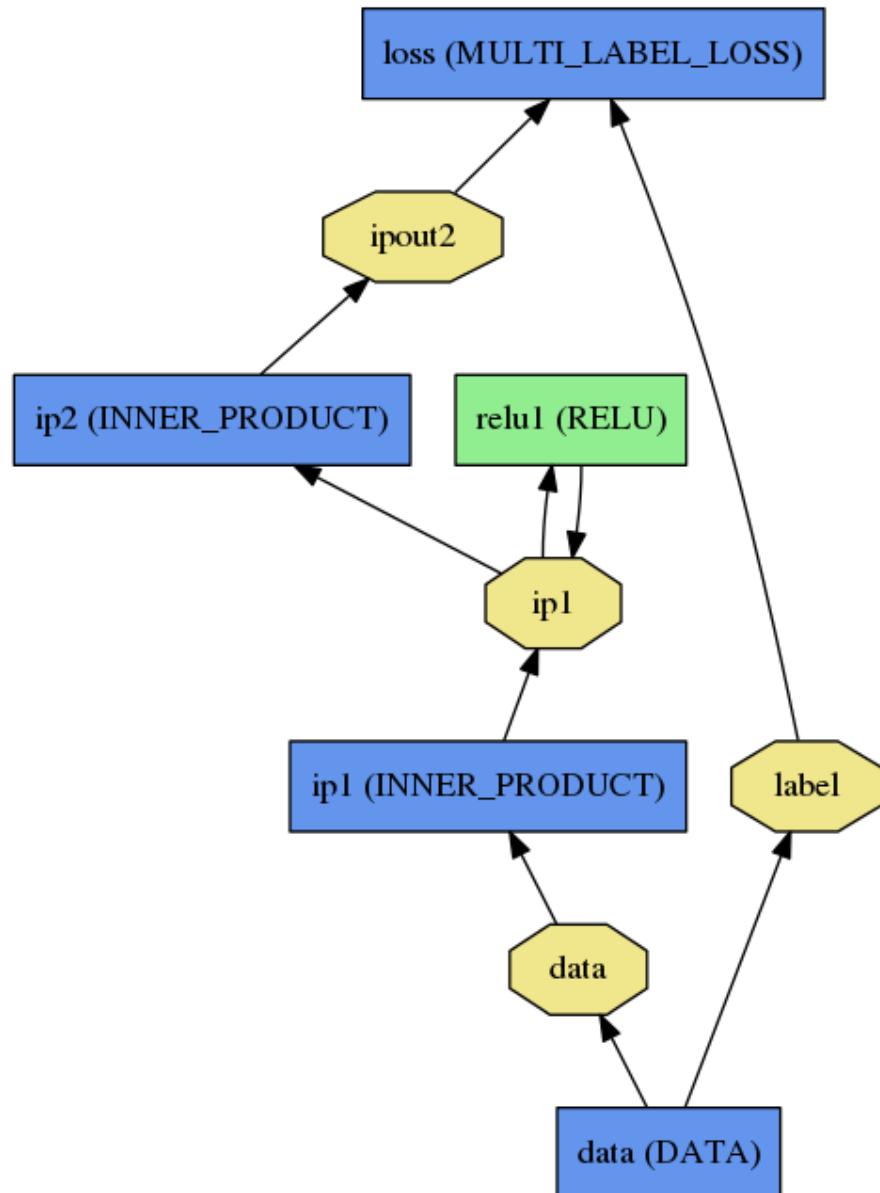


FIGURE 8.1: Network topology for multilable classification.

1. number of input and output neurons
2. number of training samples
3. network architecture
4. training algorithms
5. type of activation function
6. regularization

In general, it is hard to determine the best number of hidden neurons without evaluating several trained models and comparing the generalization error rates among the

models. Lack of hidden neurons will lead to high training error and poor generalization performance because of underfitting and high bias. On the other hand, exceeding neuron number will achieve low training error but still poor generalization performance because of overfitting and high variance. One of crucial considerations is to evaluate hidden neuron effects on the bias/variance trade-off.

There are many rules of thumb to determine the number of hidden neurons [36] such as

1. The number should between the size of input layer and output layer.
2. The number should be $\frac{2}{3}$ of the sum of the input neurons plus the output neurons.
3. The number should be less than twice the size of the input layer.
4. Based on geometric pyramid rule, the number should between the number of input neurons and the output neurons.

It is time-consuming to find the optimal number of hidden neurons. We will start from a number of neurons which is small and increase the number gradually by evaluating the performance of the network.

The connecting method between different layers is fully connecting because we want to find which color label owned by each image in this case. The information of each color distributes evenly in each patch of images. The value of the weights and bias is initialized randomly. For weights, *xavier* algorithm is applied to determine the scale of initialization depending on the number of input and output neurons. The bias is simply initialized as constant, default as 0. The batch size is set to 40 means that there are 40 images feeding into neural network each time.

In training process, the base learning rate is set to 0.0001. The learning rate of weights is equal to base learning rate while the learning rate of bias double the base learning rate. We set momentum with 0.9 to smooth the weight updating across iterations so that the learning process will be stable and fast. Weight decay is 0.0005.

Label-metric is used as evaluation metric. Each label will be counted separately and statistics will be calculated in total. The definitions are described in chapter 5.

8.3 Results

In this section, we present the results of experiments. The test results with different number of hidden neurons are showed in table 8.1 and figure 8.2. Results show that the model with 200 neurons in the hidden layer has the best performance.

Neuron number	Sensitivity	Specificity	Harmonic Mean	Precision	F1 Score
4	0.9245	0.9552	0.9396	0.9423	0.9333
50	0.9245	0.9701	0.9468	0.9608	0.9423
100	0.9623	0.9701	0.9662	0.9623	0.9623
150	0.9057	0.9552	0.9298	0.9412	0.9231
200	0.9811	1.0000	0.9905	1.0000	0.9905
250	0.9434	1.0000	0.9709	1.0000	0.9709
300	0.9811	0.9701	0.9756	0.9630	0.9720
350	0.9434	0.9701	0.9566	0.9615	0.9524
400	0.8868	0.9701	0.9266	0.9592	0.9216
450	0.9245	0.9701	0.9468	0.9608	0.9423
500	0.9622	0.9701	0.9662	0.9623	0.9623

TABLE 8.1: Test results for different number of hidden neurons.

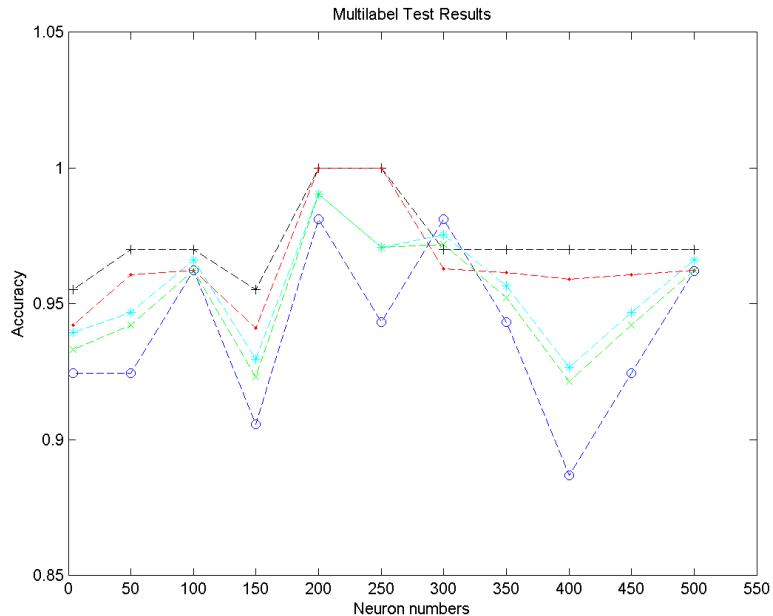


FIGURE 8.2: Test results for different number of hidden neurons. Blue circle for Sensitivity. Black plus for Specificity. Cyan star for Harmonic Mean. Red dot for Precision. Green x for F1 Score.

The table 8.1 and figure 8.2 show that the network with 200 neurons has the best performance.

The learning speed 8.3 is quick. After reaching high accuracy, the model keeps stable.

The ROC curve 8.4 8.5 for multilabel classification shows that average accuracy is high. In details, the micro-accuracy for label 0(red) is the highest, followed by accuracies for label 1(green) and label 2(blue).

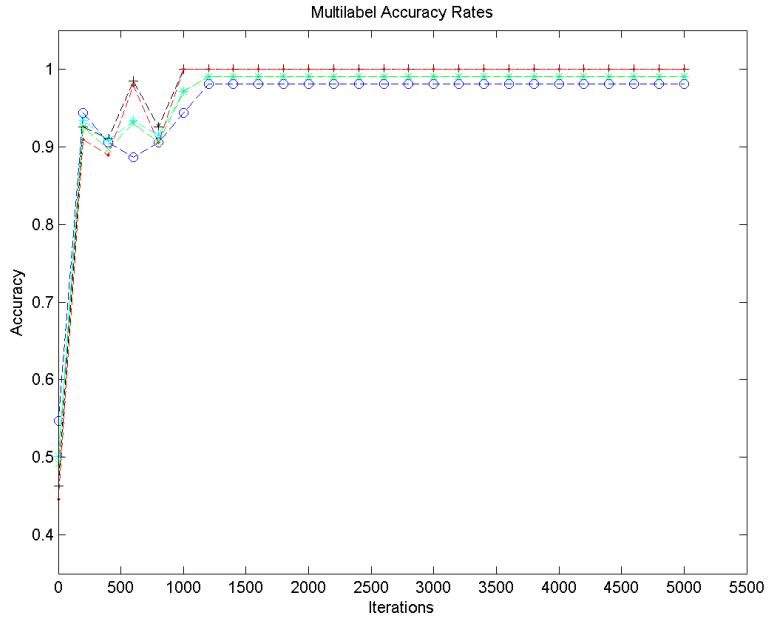


FIGURE 8.3: Learning speed for 200 neurons in the hidden layer. Blue circle for Sensitivity. Black plus for Specificity. Cyan star for Harmonic Mean. Red dot for Precision. Green x for F1 Score.

8.4 Conclusion and Future Work

We have presented a learning approach to classify two types of weather. The simple two type weather classification is not trivial at given diversity of outdoor images. The contribution is that model avoids human being common senses in judging weather conditions, instead of feature maps understood by neural networks. Experimental results proves that this is an effective and inspiring method that can be deployed in related tasks.

The project show high performance of neural network in classifying a single weather image. However, it is far from solving weather classification including other conditions, like rain, fog, storm. And the question why convolutional neural network works is still a puzzle. The two tasks need more research. We hope the project can help attract interest and following work on the subject.

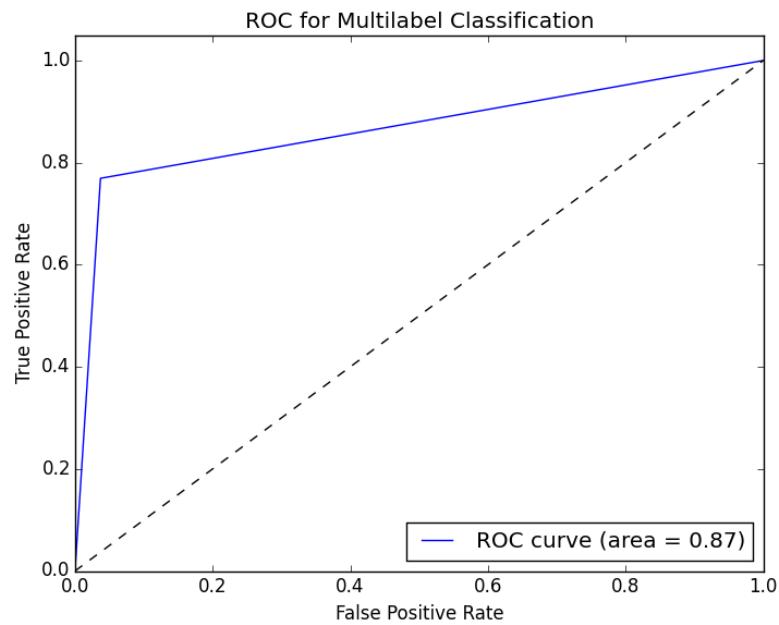


FIGURE 8.4: ROC curve for 200 hidden neurons

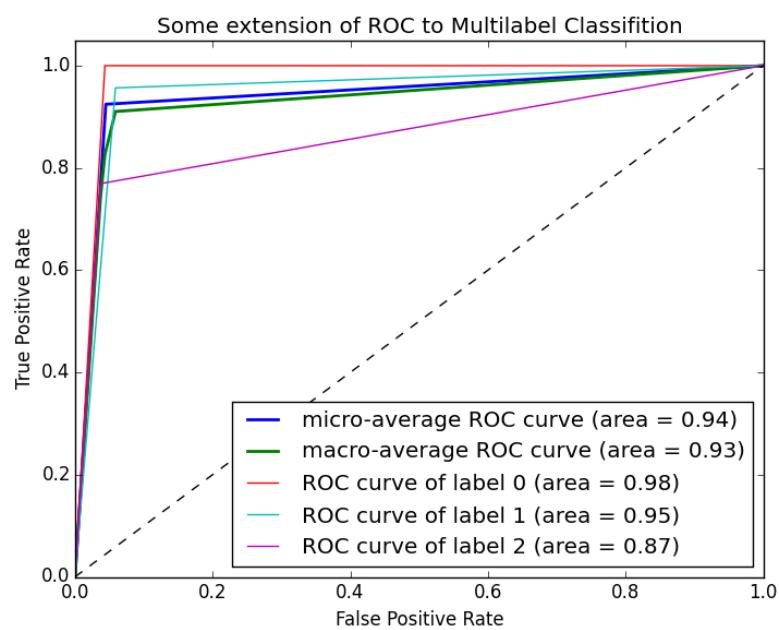


FIGURE 8.5: ROC curve for 200 hidden neurons

Appendix A

Appendix Title Here

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