Revised Simplex Report

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Motivation

One of the challenging problems in Linear programming optimization is the time required to find the optimum.simplex method is one of the algorithms to solve optimization problems.
Original simplex method requires computation of new tableau in each iteration.However, a lot of information in this tableau is not needed.This is not an efficient way to do it in terms of computation time and resources.Revised simplex method tackle these problems and introduce more efficient way to implement the algorithm.

Theory

In original simplex algorithm,computing and storing new tableau in each iteration is done.However,most of its information is not needed and following items are just needed:

1. Relative cost function Cj to determine the entering nonbasic variable to the basic set.
2. non basic variable column corresponding to most negative Cj
3. b:values of the basic variable

Revised simplex targets these problems by generating Cj and corresponding entering variables from the original data itself making use of the inverse of the current basis matrix.

Problem definition :

Minimize

$$f(X) = c1x1 + c2x2 + \cdots + cnxn \quad \textbf{(1)}$$

S.T

$$AX = A1x1 + A2x2 + \cdots + Anxn = b \ \textbf{(2)}$$

$$Xn\times 1 \geq 0n\times 1 \quad \textbf{(3)}$$

Let's assume the problem has a solution and :

$$B = [Aj1\ Aj2 \cdots Ajm] \quad \text{is a basis matrix}$$

$$XBm\times 1 = [xj1\ \ xj2\ldots\ \ xjm]$$

$$cBm\times 1 = [cj1\ cj2\ldots\ cjm]$$

XB is vector of basic variables corresponding to B
cB is the cost coefficient.Hence ,

$$XB = B^{-1}b = bhat\ ,\ b \geq 0 \quad \text{if } XB \text{ is feasible}$$

By including the objective function as an equation ,the system could be written as follows :

$$\Sigma j = Pj\,xj + Pn + 1(-f) = q \quad \textbf{,j=1,2....n} \quad \textbf{(4)}$$

$$Pj = [a1j\ a2j ...\ amj\ cj],\ Pn + 1 = [0\ 0 ...\ 0\ 1]\ and\ q = [b1\ b2 ...\ bm\ 0]$$

Now,defining a feasible basis for the system defined by (2)

$D\,m + 1 \times m + 1 = [Pj1\ Pj2 \cdots Pjm\ Pn + 1] = [B^{-1}\ 0\ ; cTB\ 1]$, $the\ ;\ means\ new\ column$

So D inverse equal $D^{-1} = [B^{-1}\ 0; -\ cTB^{-1}\ 1]$

Introducing new variable called simplex multiplier :

$$\pi^{T} = cTB^{-1} = [\pi 1\ \pi 2 ...\ \pi m] \quad \textbf{(5)}$$

Multiplying each column of Eq (4) by D^{-1} the following canonical form obtained :

$$\begin{aligned} x_{j1} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad & \bar{b}_1 \\ x_{j2} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad & \bar{b}_2 \\ \vdots \quad + \sum_{j\,\text{nonbasic}} \bar{\mathbf{A}}_j x_j = & \vdots \\ x_{jm} \qquad\qquad\qquad\qquad\quad & b_m \\ -f + \sum_{j\,\text{nonbasic}} \bar{c}_j x_j = & -f_0 \end{aligned}$$

where

$$\begin{Bmatrix} \bar{\mathbf{A}}_j \\ \bar{c}_j \end{Bmatrix} = \mathbf{D}^{-1}\mathbf{P}_j = \begin{bmatrix} \mathbf{B}^{-1} & \mathbf{0} \\ -\pi^{\mathrm{T}} & 1 \end{bmatrix} \begin{Bmatrix} \mathbf{A}_j \\ c_j \end{Bmatrix} \quad \textbf{(6)}$$

Hence,the updated column form (6) is :

$$\bar{A}j = B^{-1}Aj \qquad \textbf{(7)}$$

Now ,cost coefficients is : $\hat{C}j = cj - \pi TAj$ **(8)**

Using Eq (7),(8) ,we can now generate $\bar{A}$, $\hat{C}j$ from the original data of the problem and the pivot element can now obtained be min($\hat{C}j$) of $\hat{C}j$<0.Also the ration can be obtained by bhat/a(pivot column).

Computational procedure matrix :

$$\left[\begin{array}{cc|c} \underbrace{\mathbf{P}_{j1}\ \mathbf{P}_{j2}\ \cdots\ \mathbf{P}_{jm}\ \mathbf{P}_{n+1}}_{\mathbf{D}} & \underbrace{\mathbf{e}_1\ \mathbf{e}_2\ \cdots\ \mathbf{e}_{m+1}}_{\mathbf{I}} & \overbrace{\begin{matrix} a_{1s} \\ a_{2s} \\ \vdots \\ a_{ms} \\ c_s \end{matrix}}^{\mathbf{P}_s} \\ m+1 \times m+1 & m+1 \times m+1 & \end{array}\right] \quad \textbf{(9)}$$

Multiplying by D^{-1} gives :

$$\left[\begin{array}{cc} \mathbf{D}^{-1} & \begin{matrix} \bar{a}_{1s} \\ \bar{a}_{2s} \\ \vdots \\ \boxed{\bar{a}_{rs}} \\ \vdots \\ \bar{a}_{ms} \\ \bar{c}_s \end{matrix} \end{array}\right]$$

By pivot reduction on ars :

$$[[e1\ e2 \cdots er - 1\ \beta\ er + 1 \cdots em + 1]\ D^{-1} new\ er\ ]$$

- **Algorithm and Implementation**
  - Construct initial feasible basis B then computing B^{-1}
  - Compute $\hat{C}j = cj - \pi TAj$ by multiplying the last row of the basis with each column of A

```
c=canform(end,1:end-1)*A(1:end,:);
d=canform(end-1,1:end-1)*A(1:end,:);
```

  - If all $\hat{C}j$ >0 then optimum found ,else find the most negative c :$\hat{C}s$

```
[v,minindex]=min(d);
minindex=minindex(1);
```

- Calculate the entering variable values as $Anew = B^{-1}$*Ajs

```
enteringvar=B*A(1:end-va,minindex);
```

Where B is B inverse.

- Calculate the ratio bhat=bj/$Anew$

```
ratio=canform(1:mt,end)./enteringvar;
```

- Taking the minimum positive ratio index along with the s column in the basis matrix to be the pivot element.

```
pivot_row =minratio;
```

- Performing pivot reduction on that element

```
canform=rref(canformtemp);
```

- Iterate until all $\hat{C}j > 0$,then report the value of the (last row,last column) of the basis matrix as optimum value for the objective function along with variable values.
- Note : the above algorithm is for phase 2. However, phase 1 will be exact the same but with adding w row to the basis and minimizing it until it become 0
- In addition for computing $\hat{C}j = cj - \pi TAj$ for the objective function
- $dhatj = dj - \sigma Aj$ by multiplying the **w** row of the basis with each column of A
- then ,by choosing most negative $dhatj$ and continue with the same steps in phase 2
- Until w=0 then move to phase 2 or all d>0 then no feasible solution found

Application

Revised simplex method is an efficient way to solve LP problems.LP problems appear on a lot of real life problem for example, maximize profit or reduce the cost in industry.Also,it appears a lot in logistics where the shortest path is required.Hence,revised simplex method is very efficient way to solve these problems along with other LP problems.

Comparison with Matlab linprog function

Comparison done with 3 examples as follows

- Example 1

```
################Matlab Linprog result#################

Optimal solution found.

     0
     5

Optimal solutin found = 20
collapsed time for matlab = 0.005641
################ my Revised simplex#################
Optimal solutin found = 20
X values
     5

x corresponding index (X corresponding to not displayed indecies=0) :
     2

collapsed time for my revised simplex = 0.002998
```

- Example 2

```
################Matlab Linprog result###############

No feasible solution found.

Linprog stopped because no point satisfies the const

collapsed time for matlab = 149.1194
################ my Revised simplex################
no feasible solution found
    <missing>
X values
   NaN

x corresponding index (X corresponding to not displa
   NaN

collapsed time for my revised simplex = 0.007172
>>
```

- Example 3

```
################Matlab Linprog result##################

Optimal solution found.

     0
     2

   -0.6667

collapsed time 0.005668
################ my Revised simplex##################
Optimal solutin found = -0.66667
X values
    0.0000
    2.0000

x corresponding index (X corresponding to not displayed indecies=0) :
     1     2

collapsed time 0.003538
```

From the above examples,my algorithm implementation is more efficient than the matlab algorithm.