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CS443

Digital Image Processing

Assignment #3

1. The 7×7 image with eight gray levels is given below, where each gray level value is represented in normalized form from 0 (black pixel) to 1 (white pixel).

$$\begin{bmatrix} 0 & 3/7 & 2/7 & 2/7 & 1/7 & 1/7 & 4/7 \\ 3/7 & 2/7 & 1/7 & 1/7 & 1/7 & 1/7 & 4/7 \\ 2/7 & 0 & 1 & 1/7 & 3/7 & 0 & 0 \\ 0 & 5/7 & 1/7 & 0 & 6/7 & 0 & 1/7 \\ 1/7 & 1/7 & 1/7 & 3/7 & 6/7 & 6/7 & 5/7 \\ 1/7 & 1/7 & 1/7 & 1/7 & 5/7 & 6/7 & 4/7 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4/7 \end{bmatrix}$$

Figure 1: Provided matrix

a. Calculate the probabilities of each gray level and plot the image's histogram.

$$h(r_k) = n_k$$

$$P(r_k) = \frac{h(r_k)}{n} = \frac{n_k}{n}$$ For $k = 1, 2, \dots, L$ where n is total number of pixels i.e. $7 \times 7 = 49$

where

n_k is the number of pixels in the image whose intensity level is r_k.

$P(r_k)$ is the estimation of the probability of occurrence of intensity level r_k.

For *Gray Level* 0:

$$n_0 = 12$$

$$P(r_0) = \frac{n_0}{n} = \frac{12}{49} = 0.24489$$

For *Gray Level* 1:

$$n_1 = 16$$

$$P(r_1) = \frac{n_1}{n} = \frac{16}{49} = 0.32653$$

...

For *Gray Level* 7:

$$n_7 = 2$$

$$P(r_7) = \frac{n_7}{n} = \frac{2}{49} = 0.04081$$

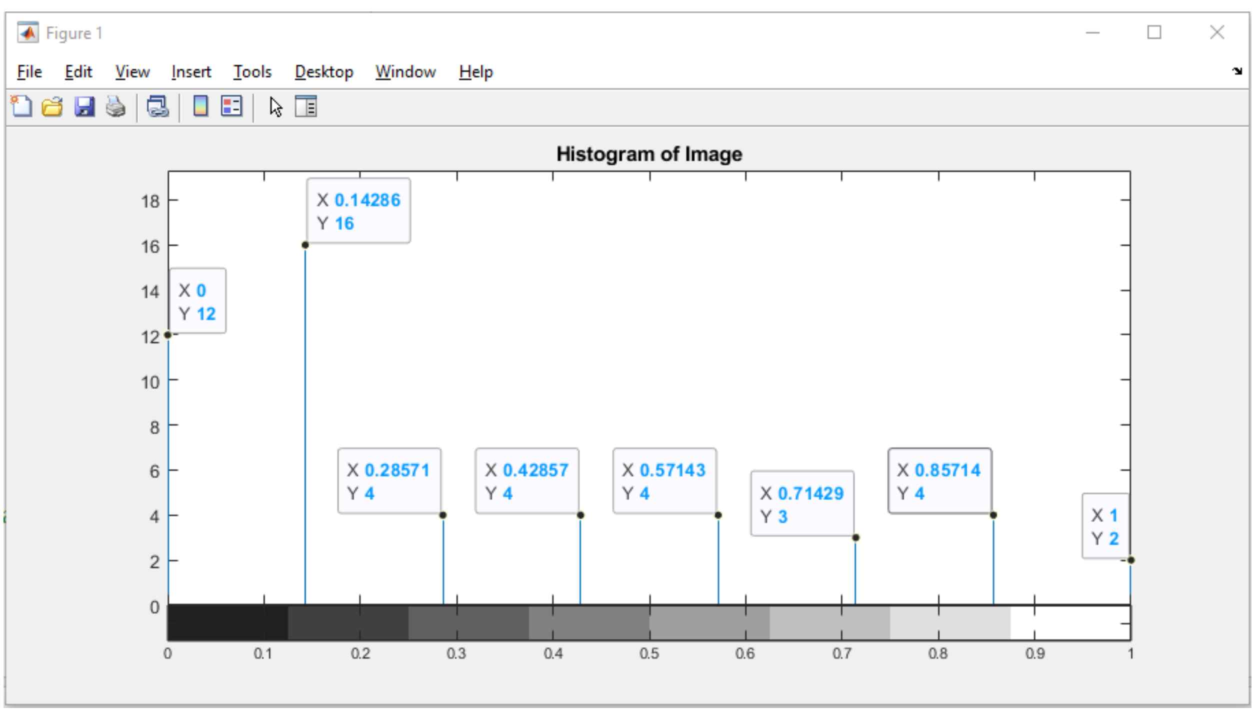
Editor - C:\Users\ibrah\Desktop\question1a.m

question1a.m

```
% Question 1.a
% Matrix defition
I = [0  3/7 2/7 2/7 1/7 1/7 4/7;...
    3/7 2/7 1/7 1/7 1/7 1/7 4/7;...
    2/7 0   1   1/7 3/7 0   0  ;...
    0   5/7 1/7 0   6/7 0   1/7;...
    1/7 1/7 1/7 3/7 6/7 6/7 5/7;...
    1/7 1/7 1/7 1/7 5/7 6/7 4/7;...
    0   1   0   0   0   0   4/7];

% Plot of histogram
imhist(I, 8), title('Histogram of Image');
```

Code Fragment 1: Provided matrix definition and **imhist**



Graph 1: Histogram graph of provided matrix

Gray Level	n_k	$\boldsymbol{P(r_k)}$
0	12	0.2449
1	16	0.3265
2	4	0.0816
3	4	0.0816
4	4	0.0816
5	3	0.0612
6	4	0.0816
7	2	0.0408

Table 1: $\boldsymbol{n_k}$ and $P(r_k)$ for all *Gray Level* values

b. Which pixels are predominant in the original image, dark or bright?

Number of pixels in such that $0 \leq r_k \leq {}^3/_7$ is 33. (darker pixels)

Number of pixels in such that ${}^4/_7 \leq r_k \leq 1$ is 16. (brighter pixels)

Since $33 > 16$, there are more darker pixels than brighter pixels. Therefore, dark pixels are predominant.

This result can be also verified by showing the given matrix image.

Editor - C:\Users\ibrah\Desktop\question1b.m

question1b.m

```
% Question 1.b
% Matrix defition
I = [0   3/7 2/7 2/7 1/7 1/7 4/7;...
     3/7 2/7 1/7 1/7 1/7 1/7 4/7;...
     2/7 0   1   1/7 3/7 0   0  ;...
     0   5/7 1/7 0   6/7 0   1/7;...
     1/7 1/7 1/7 3/7 6/7 6/7 5/7;...
     1/7 1/7 1/7 1/7 5/7 6/7 4/7;...
     0   1   0   0   0   0   4/7];

% Showing the matrix
figure, imshow(I,'InitialMagnification',5000),...
    title('Visualization of Matrix');
```

Code Fragment 2: Provided matrix definition and magnification in the **`imshow`**

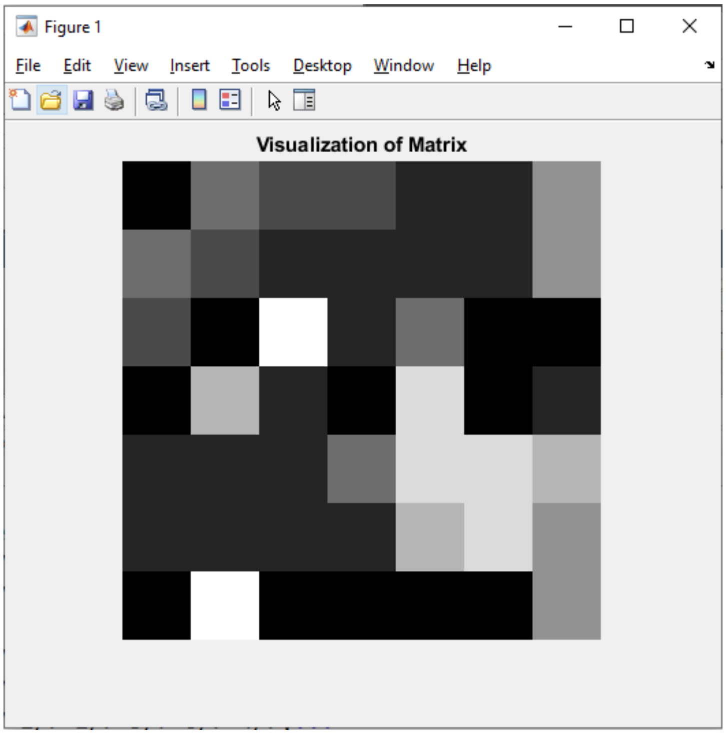


Figure 2: Visualization of provided matrix

From Figure 2, it is obvious that the predominance of image of the matrix is in dark side.

c. Using the cumulative distribution function, equalize the histogram calculated in part (a) and plot the resulting (equalized) histogram.

Editor - C:\Users\ibrah\Desktop\question1c.m

question1b.m | question1c.m | backupQ1c.m

```
% Question 1.c

% Matrix defition
I = [0   3/7 2/7 2/7 1/7 1/7 4/7;...
     3/7 2/7 1/7 1/7 1/7 1/7 4/7;...
     2/7 0   1   1/7 3/7 0   0  ;...
     0   5/7 1/7 0   6/7 0   1/7;...
     1/7 1/7 1/7 3/7 6/7 6/7 5/7;...
     1/7 1/7 1/7 1/7 5/7 6/7 4/7;...
     0   1   0   0   0   0   4/7];

% Obtaining histogram using the cumulative distribution,
histogram(I, 8,'Normalization','cdf'),...
    title('Histogram of Cumulative Distribution Function');
```

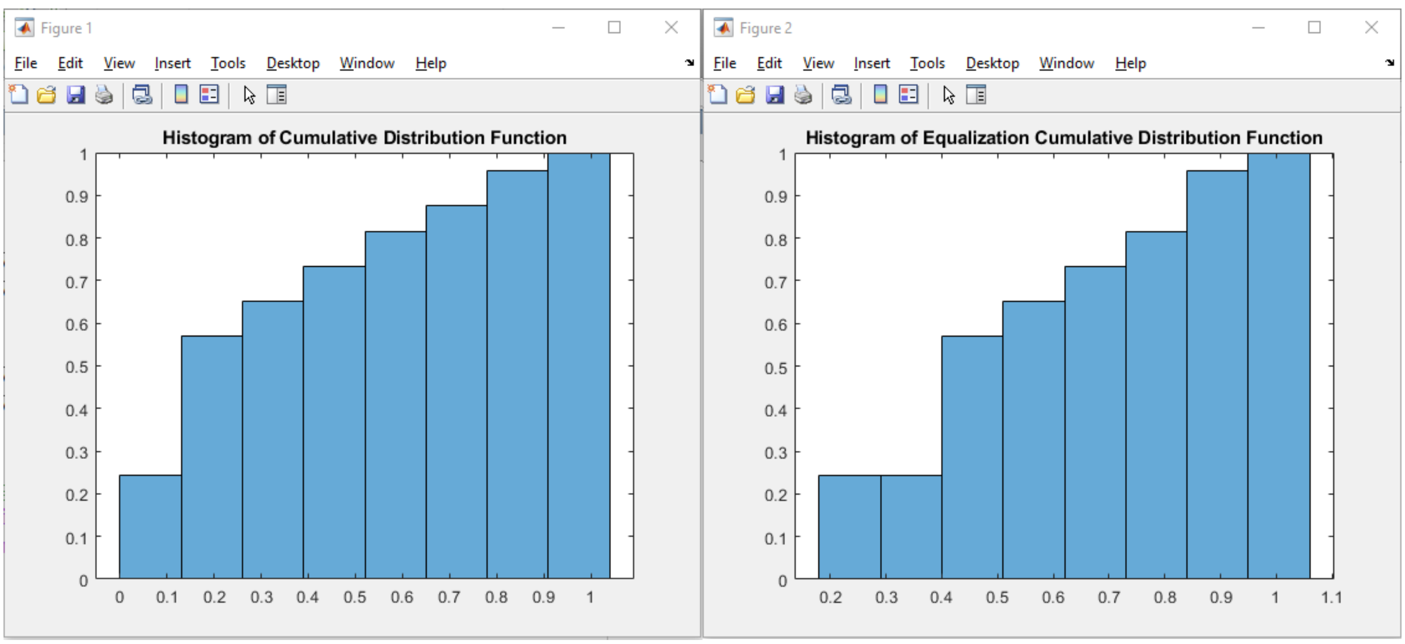
(a)

```
% Equalization of I
I2 = histeq(I);

% After equalization
figure, histogram(I2, 8,'Normalization','cdf'),...
    title('Histogram of Equalization Cumulative Distribution Function');
```

(b)

Code Fragment 3: Use of *Cumulative Distribution Function* (a) and (b)



(a) **(b)**

Graph 2: Histograms before equalization (a) and after equalization (b)

d. Show the resulting 7×7 image after histogram equalization.

Showing the image that created from provided matrix before and after the histogram equalization, the following figures Figure 3 and Figure 4 are obtained.

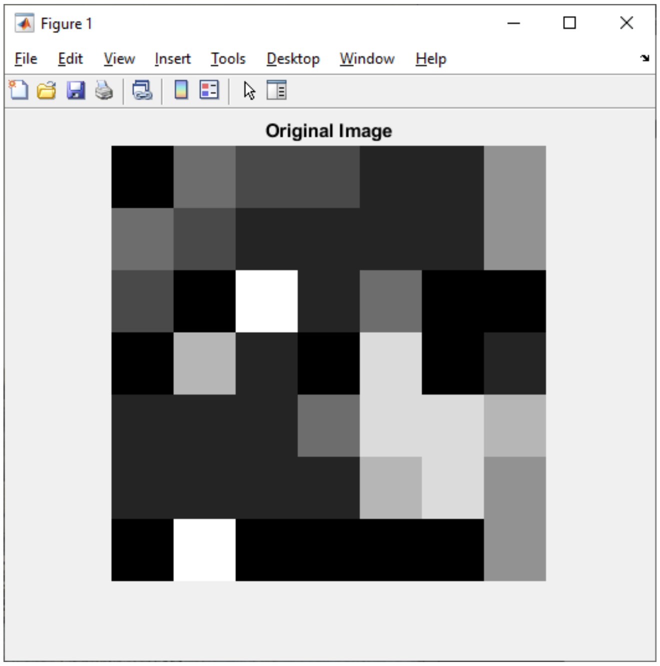


Figure 2: Image before histogram equalization using CDF

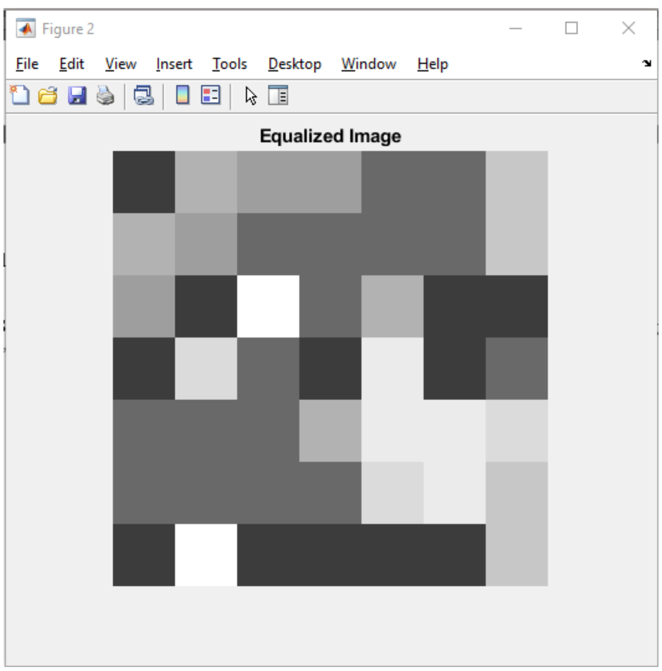


Figure 3: Image after the histogram equalization using CDF

Gray Level	n_k	$\boldsymbol{P(r_k)}$	S_k
0	12	0.2449	0.2449
1	16	0.3265	0.5714
2	4	0.0816	0.6531
3	4	0.0816	0.7347
4	4	0.0816	0.8163
5	3	0.0612	0.8776
6	4	0.0816	0.9592
7	2	0.0408	1

$$\sum \boldsymbol{n_k} = 49 \qquad \sum P(r_k) = 1$$

Table 2: $\boldsymbol{n_k}$ and $P(r_k)$ for all *Gray Level* values – updated with $\boldsymbol{S_k}$

The *Cumulative distribution function* (CDF) can be found as follows,

$$S_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p(r_j)$$

2. Given a 256×256 pixels image with eight gray levels, whose gray-level distribution is given in the following table.

   a. Which pixels predominate in the original image, dark or bright? Explain.

Gray Level	n_k	$\boldsymbol{P(r_k)}$
0	2621	0.04
1/7	0	0.00
2/7	0	0.00
3/7	5243	0.08
4/7	7209	0.11
5/7	12,452	0.19
6/7	24,904	0.38
1	13,107	0.20

Table 3: $\boldsymbol{n_k}$ and $P(r_k)$ for all *Gray Level* values, higher gray levels are highlighted

Table 3 shows that the number of pixels in such that $0 \leq r_k \leq {}^{3}/_{7}$ outnumber and there are less pixels in such that ${}^{4}/_{7} \leq r_k \leq 1$. This indicates that image contains much more bright pixels than the dark ones. Therefore, bright pixels are predominant.

b. Assuming the histogram modification will be successful, what will be the probable effect of this modification on the original image?

Gray Level	$\boldsymbol{P(r_k)}$	$\hat{P}(z_k)$
0	0.04	0.27
1/7	0.00	0.16
2/7	0.00	0.19
3/7	0.08	0.16
4/7	0.11	0.11
5/7	0.19	0.06
6/7	0.38	0.03
1	0.20	0.02

Table 4: $P(r_k)$ and $\boldsymbol{\hat{P}(z_k)}$ for all *Gray Level* values, where $\boldsymbol{\hat{P}(z_k)}$ is the histogram modification assumption

As seen from the Table 3, there is a correlation between the $\boldsymbol{n_k}$ and the $P(r_k)$ that corresponds to it – the higher number of $\boldsymbol{n_k}$ results in the higher $P(r_k)$ value as well. In other words, higher number of pixels which have the same gray level, will lead $\boldsymbol{\hat{P}(z_k)}$ to be higher.

In the Table 4, however, the behavior of $\boldsymbol{\hat{P}(z_k)}$ looks like the inverse of $P(r_k)$. It implies that with the histogram modification, the brighter pixels which were in majority turn to be minority and vice versa for dark pixels before the modification. Therefore, after the histogram modification, predominance is taken over by dark pixels.

The change of the values of $P(r_k)$ may happened due to the *taking inverse of the image*. *Inverse effect* changes the pixels into bright if they are dark and into dark if they are bright.

c. Equalize the original histogram using the function $s = T(r)$.

(a)

Gray Level	$P(r_k)$	CDF
0	0.04	0.04
1/7	0.00	0.04
2/7	0.00	0.04
3/7	0.08	0.12
4/7	0.11	0.23
5/7	0.19	0.42
6/7	0.38	0.80
1	0.20	1

(b)

Gray Level	CDF	CDF $\times$ (Total level $-$ 1)
0	0.04	S_0 $\quad 0$
1/7	0.04	S_1 $\quad 0.04 \times 7 = 0.28 \approx 0$
2/7	0.04	S_2 $\quad 0.04 \times 7 = 0.28 \approx 0$
3/7	0.12	S_3 $\quad 0.12 \times 7 = 0.84 \approx 0$
4/7	0.23	S_4 $\quad 0.23 \times 7 = 1.61 \approx 1$
5/7	0.42	S_5 $\quad 0.42 \times 7 = 2.94 \approx 2$
6/7	0.80	S_6 $\quad 0.80 \times 7 = 5.60 \approx 5$
1	1	S_7 $\quad 1.00 \times 7 = 7.00 \approx 7$

(c)

Gray Level	**New Gray Level**	n_k
0	0	2621
1/7	0	0
2/7	0	0
3/7	0	5243
4/7	1/7	7209
5/7	2/7	12,452
6/7	5/7	24,904
1	1	13,107

Table 5: Equalization of given histogram: Calculation of CDF **(a)**, Calculation of S_k **(b)** and New gray levels with their n_k **(c)**

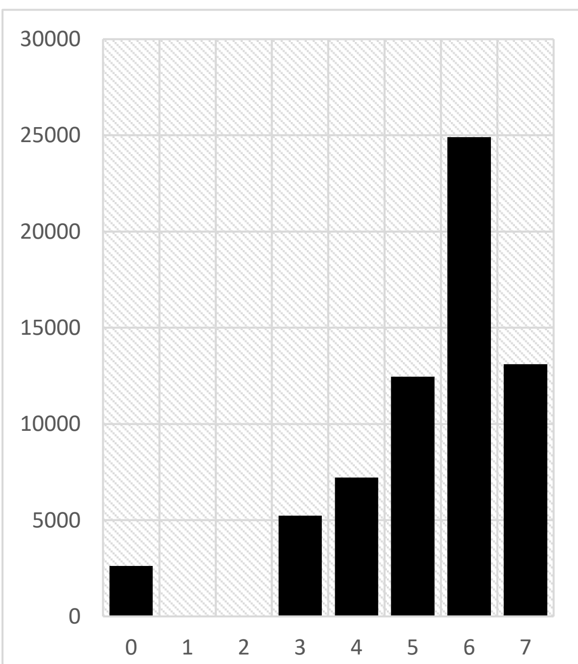
d. Obtain the function $v = G(z)$ and its inverse.

k	Gray Level	$\boldsymbol{P(r_k)}$	S_k	$v_{\boldsymbol{k}}$	$G(z)$	$G^{-1}(v_{\boldsymbol{k}})$
0	0	0.04	0	0.04	0.04	z_0
1	$1/7 \approx 0.1428$	0.00	0	0.04	0.12	z_3
2	$2/7 \approx 0.2857$	0.00	0	0.04	0.23	z_4
3	$3/7 \approx 0.4285$	0.08	0	0.12	0.42	z_5
4	$4/7 \approx 0.5714$	0.11	1	0.23	0.42	z_5
5	$5/7 \approx 0.7142$	0.19	2	0.42	0.80	z_7
6	$6/7 \approx 0.8571$	0.38	5	0.80	0.80	z_7
7	1	0.20	7	1	1	z_8

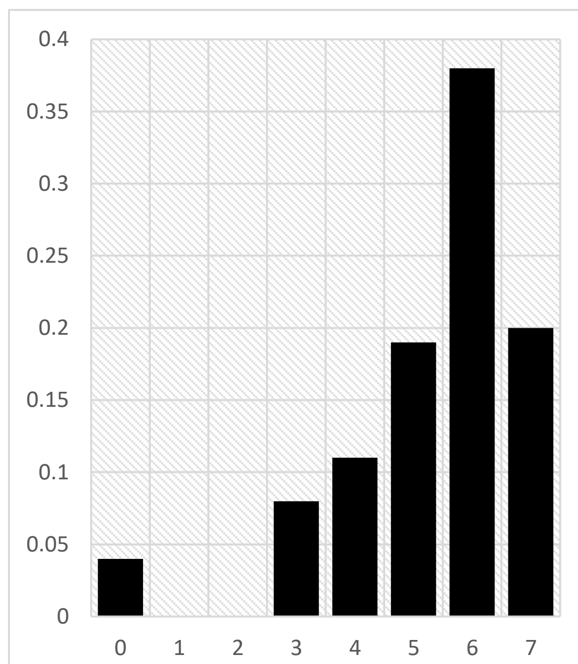
Table 6: Function of $v = G(z)$ and its inverse $\boldsymbol{G^{-1}}(\boldsymbol{v}_k)$

For each *gray level* value, the closest $\boldsymbol{v}_k$ becomes the $G(z)$ value of that corresponding $\boldsymbol{k}$ row.

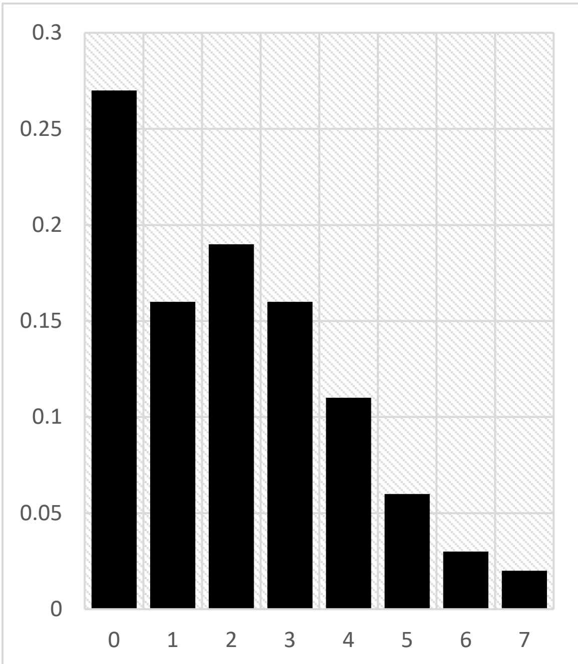
e. Plot the most relevant histograms: original, desired, equalized, and resulting.



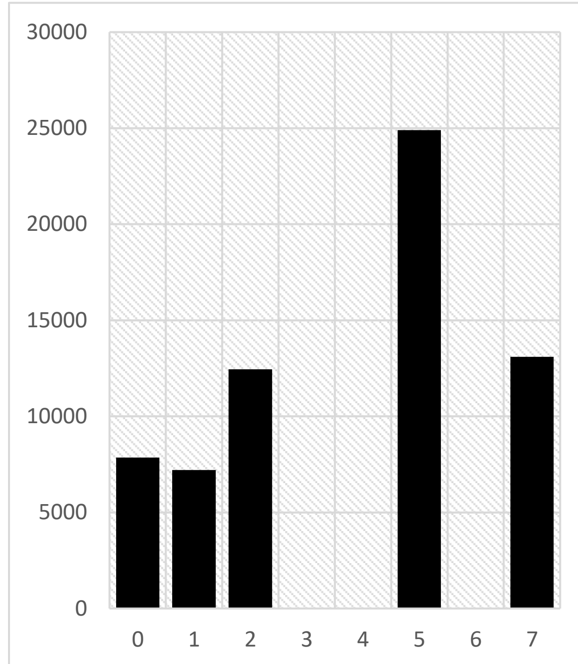
(a)



(b)



(c)



(d)

Table 6: Histogram graphs of **(a)** Original, **(b)** Desired,
(c) Resulting, **(d)** Equalizer

f. Fill out the table below with the final values for $\boldsymbol{n}_k$ and $\widehat{\boldsymbol{P}}(\boldsymbol{z}_{\boldsymbol{k}})$ for the eight values of, $\boldsymbol{z}_{\boldsymbol{k}}$ comparing with the desired values and explaining possible differences.

$z_{\boldsymbol{k}}$	$n_{\boldsymbol{k}}$	$\widehat{P}(z_k)$	$P(r_k)$
0	7864	0.27	0.04
1/7	7209	0.16	0.00
2/7	12452	0.19	0.00
3/7	0	0.16	0.08
4/7	0	0.11	0.11
5/7	24904	0.06	0.19
6/7	0	0.03	0.38
1	13107	0.02	0.20

Table 7: Final values of $\boldsymbol{n}_{k,}$ $\widehat{\boldsymbol{P}}(\boldsymbol{z}_{\boldsymbol{k}})$ and $\boldsymbol{P}(\boldsymbol{r}_{\boldsymbol{k}})$