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BMS College of Engineering, Bangalore-560019

(Autonomous Institute, Affiliated to VTU, Belgaum)

July / August 2017 Supplementary Semester Examinations

Course: Engineering Mathematics -1 — Duration: **3 hrs**

Course Code:**15MA1ICMAT** — Max Marks: **100**

Date: 26.07.2017

Instructions: **Answer five full questions choosing one from each unit.**

UNIT 1

1 a) If $y^{1/m}+y^{-1/m}=2x$, then show that $\left(x^2-1\right)y_{n+2}+(2n+1)x\,y_{n+1}+\left(n^2-m^2\right)y_n=0.$ **6**

b) Expand $f(x)=\sin x$ in powers of $(x-\pi/2)$. Hence find the value of $\sin 91^\circ$ correct to four decimal places. **7**

c) Show that the radii of curvature for the polar curve $r^n=a^n\sin n\theta$ is directly proportional to $\dfrac{1}{r^{n+1}}$. **7**

UNIT 2

2 a) If $x^x y^y z^z=c$ then show that $z_{xy}=-(x\log ex)^{-1}$ at $x=y=z.$ **6**

b) If $z=z(x,y)$ and $x=e^u+e^{-v}$, $y=e^{-u}+e^v$ then prove that $\dfrac{\partial z}{\partial u}-\dfrac{\partial z}{\partial v}=x\dfrac{\partial z}{\partial x}-y\dfrac{\partial z}{\partial y}$. **7**

c) Expand $f(x,y)=\cos x\cos y$ in powers of x and y upto fourth degree terms. **7**

OR

3 a) If $u=e^{a\theta}\cos(a\log r)$ then show that $\dfrac{\partial^2 u}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial\theta^2}=0$ **6**

b) Verify $J\,J'=1$ for the functions $u=x+\dfrac{y^2}{x}$, $v=\dfrac{y^2}{x}$, where $J=\dfrac{\partial(u,v)}{\partial(x,y)}$ and $J'=\dfrac{\partial(x,y)}{\partial(u,v)}$. **7**

c) The temperature T at any point (x, y, z) in space is $T=400\,x\,y\,z^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. **7**

UNIT 3

4 a) Solve $\dfrac{dz}{dx}+\dfrac{z}{x}\log z=\dfrac{z}{x}(\log z)^2$. **6**

b) Solve $\left(x^2y^2+x\,y+1\right)\,y\,dx+\left(x^2y^2-x\,y+1\right)\,x\,dy=0.$ **7**

c) Show that the family of curves $\dfrac{x^2}{a^2+\lambda}+\dfrac{y^2}{b^2+\lambda}=1$ are self-orthogonal, λ being a parameter. **7**

UNIT 4

5 a) Solve $\left(D^6-D^4\right)y=x^2,$ where $D=\dfrac{d}{dx}.$ **6**

b) Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+5y=x^2\sin(\log x).$ **7**

c) Find the current in the RLC circuit connected in series given $R=200\Omega,\ L=0.1H,$ $C=0.006F,\ E=te^{-t}$ volts. Assume zero initial current and charge. **7**

OR

6 a) Solve $\left(D^2-4D+3\right)y=e^x\cos 2x,$ where $D=\dfrac{d}{dx}.$ **6**

b) Solve $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=e^x\log x,$ by the method of variation of parameters. **7**

c) If a weight 6 lbs hangs from a spring with constant k =12 and no damping force exists, find the motion of weight when an external force 3 cos8t acts. Initially x = 0, and $\dfrac{dx}{dt}=0.$ **7**

UNIT 5

7 a) Evaluate $\displaystyle\int_0^4 x^3\sqrt{4x-x^2}\,dx.$ **6**

b) Find the perimeter of the curve $x^{2/3}+y^{2/3}=a^{2/3}.$ **7**

c) Obtain the series solution of Legendre's differential equation. **7**

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