## Exam 1 March 13, 2021

Name:	UID:

- The exam consists of FOUR problems.
- Unsupported answers will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.
- Upload your answers to Gradescope as a pdf only. Make sure to allocate your work to the appropriate question.
- Missing or blank pages will result in an automatic zero for the question.

• Time: 90 minutes.

Problem	Score	Points
1		10
2		12
3		10
4		20
Total		52

**Problem 1.** (5 points each) Consider the system of equations given by

$$x_1 + 2x_2 + x_3 - 4x_4 = 1$$
  
 $x_1 + 3x_2 + 7x_3 + 2x_4 = 2$   
 $x_1 - 11x_3 - 16x_4 = -1$ 

1) Express the system in *augmented* matrix form, and perform row operations on it to get it in reduced row echelon form (RREF).

2) Find all solutions to the above system of equations.

**Problem 2.** (5 + 7 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & c \\ 0 & a & -b \\ -1/a & x & x^2 \end{bmatrix},$$

where a, b, c are given constants with  $a \neq 0$ .

1) Find all values of x in terms of a, b, c such that the matrix A is invertible.

2) Find all possible LU-Factorizations of A when a = b = c = x = 1.

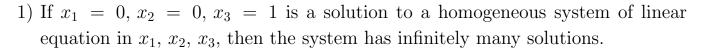
**Problem 3.** (5 points each) Consider the matrix

$$B = \left[ \begin{array}{rrr} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 0 & 1 \end{array} \right].$$

1) Find an invertible  $3 \times 3$  matrix A satisfying  $A^2 + AB = 3A$ .

2) Find the inverse of A.

**Problem 4.** (5 points each) True or False (Circle one and state your reason):



Reason: True False

2) Let A, B, C be  $n \times n$  invertible matrices. Then,

$$\det \left( C^{-1} (A B^{-1})^{-1} (C A^{-1})^{-1} C^{2} \right) = \det (B C).$$

Reason: True False

3) Let A, B, and A + B be invertible matrices. Then,

$$(A^{-1} + B^{-1})^{-1} = A (A + B)^{-1} B.$$

Reason: True False

4) Let A be a  $3 \times 3$  matrix such that

$$A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Then, there exists a  $3 \times 3$  non-zero matrix B such that AB = O.

Reason: True False

Draft: