The American University in Cairo | **MACT 2132**
Mathematics and Actuarial Science | **Fall 2019**
Linear Algebra | **Exam 2**
November 26, 2019 | **Time Limit: 75 Minutes**

Name: ______________________________ **UID:** ____________

- This exam contains 6 pages (including this cover page).
- Answer **ALL** the questions (total of points is 45).
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		12
2		12
3		12
4		9
Total		45

Problem 1. Let $\mathcal{D}$ be the set of all 3×3 matrices of the shape

$$\begin{bmatrix} t & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & r \end{bmatrix},$$

where t, s and r are real numbers.

a) (6 pts) Show that $\mathcal{D}$ is a vector subspace of $\mathcal{M}_{3\times 3}$.

b) (4 pts) Find a basis for $\mathcal{D}$.

c) (2 pts) What is the dimension of $\mathcal{D}$? Justify your answer.

Problem 2. (12 pts) Let $\mathcal{P}_3$ be the vector space of polynomials of degree at most <u>three</u> in the variable x. Consider the following inner product on $\mathcal{P}_3$:

$$< \mathbf{p}(x), \mathbf{q}(x) >:= \int_{-1}^{1} \mathbf{p}(x)\mathbf{q}(x)\, dx.$$

Determine an orthonormal basis for $\mathcal{P}_3$, relative to the above inner product function.

Problem 3. (3 pts each) Consider the matrix

$$A = \begin{bmatrix} 3 & -6 & 21 \\ -2 & 4 & -14 \\ 1 & -2 & 7 \end{bmatrix}.$$

a) Find a basis for the row space of A.

b) Find a basis for the column space of A.

c) Find a basis for the null space of A.

d) What is the rank of A? What is the nullity of A? Justify your answer.

Problem 4. (3 pts each) Prove or disprove **Three** of the following.

a) The subset of $\mathbb{R}^2$ consisting of all points (x, y) on the ellipse $2x^2 + 3y^2 = 1$ is a subspace.

b) Let $\underline{\mathbf{u}} = (u_1, u_2)$, $\underline{\mathbf{v}} = (v_1, v_2) \in \mathbb{R}^2$. The function $< \underline{\mathbf{u}}\,, \underline{\mathbf{v}} >= 2u_1v_2 - u_2v_2$ is an inner product on $\mathbb{R}^2$.

c) If A is an $m \times n$ matrix, then $\text{Nullity}(A) = \text{Nullity}(A^T)$

d) Let V be an an inner product space. Then, for any $\mathbf{u}, \mathbf{v} \in V$,

$$||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||.$$

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