Midterm 1
March 15, 2016

Name: _____ **UID:** _____

- **Show your work to receive full credit.**
- **Calculators are allowed.**
- **Time: 75 minutes.**

Problem	Score	Points
1		9
2		10
3		8
4		7
5		8
Total		42

Problem 1. (3 pts each) Consider the following system of equations

$$x + 4y - 2z = 1$$
$$x + 7y - 6z = 6$$
$$3y + qz = t$$

a) Which number q makes the corresponding coefficient matrix noninvertible?

b) For which value t will the system have infinitely many solutions?

c) Find the solution that has $z = 1$.

Problem 2. (5 pts each)

a) Solve for A.

$$(A^{-1} - 2I)^T = -2 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

b) Prove that if A, B, and C are $n \times n$ matrices and $ABC = I$, then B is invertible and $B^{-1} = CA$

Problem 3. (4 pts each)

a) If the Gaussian elimination leads to $x + y = 1$ and $2y = 3$. Find two possible original problems whose solution set is equivalent to the latter.

b) For which three numbers a will elimination fail to give three pivots?

$$A = \begin{pmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{pmatrix}.$$

Problem 4. Given $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ with $det(A) = -3$. Find

a) (2 pts) $det(4A^{-1}(A^T)^2)$,

b) (2 pts) $det((-A^4)^{-1} \times det(A))$,

c) (3 pts) $det \begin{pmatrix} 5d & -a & 4g - 7a \\ 5e & -b & 4h - 7b \\ 5f & -c & 4i - 7c \end{pmatrix}$.

Problem 5.

a) (4 pts) Represent $A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix}$ as a product of elementary matrices.

b) (2 pts)Is A an invertible matrix? Explain your answer.

c) (2 pts) Is the LU factorization of A unique? Explain your answer.

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