Problem 1. (1 pt each) Circle either True or False. No justification is needed.

a) If the columns of a 5×4 matrix are linearly independent, then the columns of A span $\mathbb{R}^5$.

True False

4 vectors cannot span $\mathbb{R}^5$

b) If $\mathbf{b}$ is in the column space of A, then the matrix equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

True False

c) If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are vectors in $\mathbb{R}^2$ such that $span(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k) = \mathbb{R}^2$, then $k = 2$.

they may be linearly dependent but $k \geq 2$

True False

d) A linearly independent set in a subspace H is a basis for H.

True False

it's a basis if it's span is H

e) If matrices A and B have the same reduced row echelon form then Row(A)=Row(B).

True False

f) If H is a subspace of $\mathbb{R}^3$, then there is a 3×4 matrix A such that $H = C(A)$.

True False

A's columns are the basis of H which will include at most 3 vectors, then we can repeat vectors.

Problem 2. Consider the matrix

$$A = \begin{pmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{pmatrix}.$$

You may use the fact that

$$rref(A) = \begin{pmatrix} 1 & 0 & -6 & 0 & 6 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) (3 pts) Find a basis for the nullspace of A.

Let $x_3 = t$ $x_5 = s$

$x_4 + 2x_5 = 0$ $x_4 = -2s$

$x_2 + 2t - 2s = 0$ $x_2 = 2s - 2t$

$x_1 - 6t + 6s = 0$ $x_1 = 6t - 6s$

$$x = \begin{pmatrix} -6 \\ 2 \\ 0 \\ -2 \\ 1 \end{pmatrix} s + \begin{pmatrix} 6 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} t$$

Basis for $NS(A) = \{ (-6, 2, 0, -2, 1), (6, -2, 1, 0, 0) \}$

b) (3 pts) Find a nonzero vector that is in the column space of A, **but not one of the columns of** A. Explain your answer.

The pivot columns are C1, C2 and C4, and they form a basis for the column space of A.

$\therefore$ $C1 + C2 \in CS(A)$

$$\begin{pmatrix} 6 \\ 7 \\ 8 \\ 7 \end{pmatrix} \in CS(A).$$

Problem 3. (4 pts) Suppose that A is a 5×8 matrix such that

$$\left\{ \begin{pmatrix} 7 \\ 0 \\ 4 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 2 \\ -6 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 3 \\ 1 \end{pmatrix} \right\}$$

is a basis for the column space of A. Find p and q so that the following statement is true: The nullspace of A is a p-dimensional subspace of $\mathbb{R}^q$.

$\text{rank}(A) = 3$

$\therefore \text{nullity}(A) = \text{no. of columns} - \text{rank}(A)$
$= 8 - 3$
$= 5$

$\therefore p = 5$

$q = 8$ = no. of entries in a row vector of A
= no. of columns of A

Problem 4. In each of the following questions, find a set S of vectors in $\mathbb{R}^3$ that satisfy the given condition(s) or explain why there is no such set (three different problems).

a) The set S is linearly independent, and contains exactly two different non-zero vectors.

$S = \{ (1,0,0), (0,1,0) \}$

b) The set S is linearly dependent, and contains exactly three different non-zero vectors.

$S = \{ (1,0,0), (2,0,0), (3,0,0) \}$

c) The set S has five vectors and is linearly independent.

impossible, any set of $n > 3$ vectors in $\mathbb{R}^3$ is linearly dependent.