Name: KEY

UID: FALL 2020

Problem 1. (5 points each) Consider the system of equations given by

$$x + 2z = 4$$

$$y + z = -1$$

$$2x - 3y + z = 11$$

1) Express the system in augmented matrix form, and perform row operations on it to get it in reduced row echelon form (RREF).

$$\begin{bmatrix} 0 & 2 & 1 & 4 \\ 0 & 1 & 1 & 1 \\ 2 & -3 & 1 & 11 \end{bmatrix} \xrightarrow{R_3 - 2R_1 \Rightarrow R_3} \begin{bmatrix} 1 & 0 & 2 & 1 & 4 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & 3 \end{bmatrix}$$

2) Find all solutions to the system of equations, and describe the geometric nature of the solution set in \mathbb{R}^3 (i.e. the solution set forms a point, line or plane?) Justify your answer.

The Solution set is a line given by the parametric equations above.

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Problem 2. (6 points + 4 points + 4 points) Consider the matrix

$$A = \left[\begin{array}{ccc} a & b & c \\ 3 & 3 & 5 \\ 1 & 0 & 0 \end{array} \right].$$

1) Compute all the cofactors of A that are independent of the values of a, b, c.

$$C = C = 1 \rightarrow b = 3/5$$
 $C = 2 \rightarrow b = 6/5 = 3$
 $C = 3 \rightarrow b = 6/5$
 C

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 4 \\ 7 \end{bmatrix}$$

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Problem 3. (4 points + 8 points)

1) Let A and B be two 3×3 matrices such that det(A) = -3 and det(B) = 4.

2) Find the inverse of the following matrix or show that it is not invertible.

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 0 & -3 & 3 \\ 2 & 5 & -1 \end{bmatrix}$$

$$del(A) = 1(-12) + 6(-6)$$

$$= > clet(A) = 148 \neq 0$$

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 0 & -3 & 3 \\ 0 & -3 & 3 \end{bmatrix}$$

$$= > clet(A) = 148 \neq 0$$

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 0 & -3 & 3 \\ 0 & -3 & 3 \end{bmatrix}$$

$$C_{11} = -n$$
, $C_{12} = -(-6)=6$, $C_{21} = -6$

$$Cof(A) = \begin{bmatrix} -12 & 6 & 6 \\ -6 & -1 & -17 \end{bmatrix}$$

$$Cof(A) = \begin{bmatrix} -12 & -6 & -18 \\ -12 & -6 & -18 \end{bmatrix}$$

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$$Cof(A) = \begin{bmatrix} -12 & -12 \\ -12 & -6 & -18 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -12 & -6 - -187 \\ -1 & 6 & -1 & -3 \\ 6 & -17 & -3 \end{bmatrix}$$

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Problem 4. (6 points each)

1) Prove that if $A^3 - \frac{3}{2}A^2 + \frac{3}{4}A = O$, (where O denotes the zero matrix) then (I - 2A) is invertible and $(I - 2A)^{-1} = (I - 2A)^{2}$.

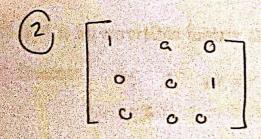
The strong and
$$(I-2A)^{-1} = (I-2A)^{2}$$
.

 $\Rightarrow \times 8$; $8A^{3} - 12A^{2} - 6A = 0$
 $\Rightarrow -T$; $8A^{3} - 12A^{2} + 6A - T = -T$
 $\Rightarrow -T = -T$

" I-2-As muer tible CI - 2A) (I - 2A)2 = I => (I-241"= (+2A)2

| * |)c=- | 24)(| I-2A | בונ | -221 |
|---|------------|------|-------|------|------|
| | | | +4 12 | | |
| ŧ | ± - | - GA | +12, | 12 - | 8A3 |
| | | | | | * |

2) List all possible 3 × 3 matrices in RREF with exactly one zero row.



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Problem 5. (5 points each) True or False (Circle one and state your reason):

1) There is a 2×2 invertible matrix A such that:

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} A = A \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}.$$

Reason:

det
$$([23]A) = [23]A1 = A1 [23]$$
 $=> 0 = SIA1$

iff $|A| = 0$

iff $A \leq ngular$.

satisfy the

2) If A is invertible and AB = BA, then B must be invertible as well.

True False

Reason: Let
$$A = F$$

$$B = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

3) If A is an invertible matrix then $adj(A^2) = |A|^{n-1} adj(A)^2$.

Reason:
$$1G^{-1} \in A^{-1} \in A^{1} \in A^{-1} \in A^$$