Linear Algebra

First Exam

Name: .... ID number: ....

Question 1 (4 marks)**.** Find the solution set of the system of linear equations below using Gaussian or Gauss–Jordan elimination. State which elementary row operations are used.

$$\begin{aligned} x + y - z &= 0 \\ x + 2y - z &= 2 \\ 2x + 4y - 2z &= 4 \end{aligned}$$

Circle your answer. The system above is (i) consistent (ii) inconsistent.

Question 2 (3 marks)**.** Consider the matrix A below.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & 9 \\ 0 & 1 & 1 \end{bmatrix}.$$

(i) Find the determinant of A.

(ii) As A is invertible, find the inverse of A.

Question 3 (6 marks)**.** Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be 2×2 matrices where their entries are given by $a_{ij} = i + j$ and $b_{ij} = i \cdot j$.

(i) Write down the matrices A and B.

(ii) Find the matrix C where $C = (3A + AB)^2$.

Question 4. Answer the following.

(i) (1 mark) Define what it means for a square matrix to be invertible.

(ii) (1 mark) Suppose A is invertible. Prove that $\det(A^{-1}) = 1/\det(A)$.

(iii) (3 marks) Suppose that A is a 3×3 matrix. The matrix B is obtained from A by multiplying the second row of A by a nonzero real number c. Prove that $\det(B) = c\ \det(A)$.

Question 5 (12 marks)**.** Determine whether each of the following statements is **true** or **false**. Moreover, **explain** your answer.

1. Suppose that A is an invertible matrix. If matrix B is row equivalent to A, then B must be invertible too.

   True **False**

2. If A is a singular square matrix, then A^2 must be singular too.

   True **False**

3. Any square matrix in reduced row echelon form (RREF) contains only 0s and 1s.

   True **False**

4. For any $n \times n$ matrices A and B, it is true that $\det(A + B) = \det(A) + \det(B)$.

True **False**

5. Let A and B be $n \times n$ matrices. If $\det(A) = \det(B)$, then $A = B$.

True **False**

6. Choose any real number $c \in \mathbb{R}$. Then one can find a square matrix A such that $\det(A) = c$.

True **False**

Best wishes!

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