Exam 1
March 13, 2021

Name: ____________________________ **UID:** ______________________

- **The exam consists of FOUR problems.**
- **Unsupported answers will receive little or no credit.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Upload your answers to Gradescope as a pdf only. Make sure to allocate your work to the appropriate question.**
- **Missing or blank pages will result in an automatic zero for the question.**
- **Time: 90 minutes.**

Problem	Score	Points
1		10
2		12
3		10
4		20
Total		52

Problem 1. (5 points each) Consider the system of equations given by

$$\begin{array}{rcrcrcrcr} x_1 & + & 2\,x_2 & + & x_3 & - & 4\,x_4 & = & 1 \\ x_1 & + & 3\,x_2 & + & 7\,x_3 & + & 2\,x_4 & = & 2 \\ x_1 & & & - & 11\,x_3 & - & 16\,x_4 & = & -1 \end{array}$$

1) Express the system in *augmented* matrix form, and perform row operations on it to get it in reduced row echelon form (RREF).

2) Find **all** solutions to the above system of equations.

Problem 2. (5 + 7 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & c \\ 0 & a & -b \\ -1/a & x & x^2 \end{bmatrix},$$

where a, b, c are given constants with $a \neq 0$.

1) Find **all** values of x in terms of a, b, c such that the matrix A is invertible.

2) Find **all** possible LU-Factorizations of A when $a = b = c = x = 1$.

Problem 3. (5 points each) Consider the matrix

$$B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

1) Find an invertible 3×3 matrix A satisfying $A^2 + A\,B = 3\,A$.

2) Find the inverse of A.

Problem 4. (5 points each) True or False (Circle one and state your reason):

1) If $x_1 = 0$, $x_2 = 0$, $x_3 = 1$ is a solution to a homogeneous system of linear equation in x_1, x_2, x_3, then the system has infinitely many solutions.

Reason: True False

2) Let A, B, C be $n \times n$ invertible matrices. Then,

$$\det\left(C^{-1}\,(A\,B^{-1})^{-1}\,(C\,A^{-1})^{-1}\,C^2\right) = \det(B\,C)\,.$$

Reason: True False

3) Let A, B, and $A + B$ be invertible matrices. Then,

$$\left(A^{-1} + B^{-1}\right)^{-1} = A\,(A + B)^{-1}\,B.$$

Reason: True False

4) Let A be a 3×3 matrix such that

$$A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Then, there exists a 3×3 non-zero matrix B such that $A\,B = O$.

Reason: True False

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