

Name: \_\_\_\_\_ UID: \_\_\_\_\_

- This exam contains 5 pages (including this cover page).
- Answer **ALL** the problems (total of points is 40).
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		12
2		16
3		12
Total		40

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**Problem 1.** (5 pts + 7 pts) Consider the following linear system of equations in  $x_1, x_2, x_3$ .

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 1, \\ -3x_1 + tx_2 + 9x_3 &= -3, \\ 2x_2 + 7x_3 &= -2q.\end{aligned}$$

(a) Which number  $t$  makes the corresponding coefficient matrix not invertible?

(b) For which values of  $q$ , the associated system will have infinitely many solutions?

**Problem 2.** (6 pts + 7 pts + 3 pts) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & -3 \\ -3 & 1 & 9 \\ 0 & 2 & 7 \end{pmatrix}.$$

(a) Find an  $LU$ -decomposition for  $A$ .

(b) Solve the linear system of equations  $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$ .

(c) Does  $A$  have a unique  $LU$ -decomposition? Why?

**Problem 3.** (4 pts + 4 pts + 4 pts)

(a) Find a  $2 \times 2$  matrix  $A$  that satisfies the equation:

$$\begin{pmatrix} 1 & 0 \\ 5 & -2 \end{pmatrix} A \begin{pmatrix} 4 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

(b) Suppose that  $B$  is an  $n \times n$  matrix such that  $B^2 - 2B - 5I = O$ . Conclude that  $B$  is invertible with  $B^{-1} = \frac{1}{5}(B - 2I)$ .

(c) Evaluate  $\det(7(-A^5)^{-1}A^T)$  if  $A$  is a  $4 \times 4$  matrix whose  $\det(A) = 7$ .

**Draft:**