Linear Algebra

MACT2132 Midterm 2 Spring 2019

Name: ______________________________ ID: ____________________

QUESTION 1. [2 marks] Let $A = \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$.

Is $x = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ an eigenvector to A corresponding to the eigenvalue $\lambda = -3$? Explain your answer.

Is $x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ an eigenvector to A corresponding to the eigenvalue $\lambda = -3$? Explain your answer.

QUESTION 2. [2 marks] Determine whether the set $\{\, (x, y) \mid x \geq 0 \text{ and } y \geq 0 \,\}$ is a subspace of the vector space $\mathbb{R}^2$ with standard addition and scalar multiplication.

QUESTION 3. [8 marks] Prove the following. State which axiom of vector spaces is used at each step.

Let V be a vector space, and $\vec{v}$ be a vector in V.

1. $0\vec{v} = \vec{0}$ (Hint: $0 + 0 = 0$)
2. $\vec{v} + (-1)\vec{v} = \vec{0}$
3. The inverse of $\vec{v}$ is unique.
4. $(-1)\vec{v} = -\vec{v}$

QUESTION 4. [4 marks]

Show that the set of all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a + b + c + d = 0$ is a subspace of M_2.

QUESTION 5. [5 marks] Let V be a vector space, and $S = \{\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_n}\}$ a subset of V.
Define the following terms.

1. S is a spanning set of V.

2. S is linearly independent.

3. S is a basis for V.

4. The dimension of V.

QUESTION 6. [6 marks] Let $S = \{(4,3,2),(0,3,2),(0,0,2)\}$, a subset of the vector space $\mathbb{R}^3$.

1. Show that S is a basis for $\mathbb{R}^3$.
2. Find the coordinates of $\vec{u} = (8,3,8)$ relative to S. That is, find $[\vec{u}]_S$.

QUESTION 7. [5 marks] Let V be a vector space with $\dim(V) = n$. We know that every subset of V containing more than n vectors is linearly dependent. Use this fact to prove the following.

(1) Let W be a subspace of V. Then $\dim(W) \leq \dim(V)$.

(2) Let S be the set of the row vectors of the matrix B below. Explain why S is a linearly dependent subset of $\mathbb{R}^3$.

$$B = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 6 & 5 \\ 7 & 1 & 5 \\ 1 & 6 & 3 \end{bmatrix}.$$

QUESTION 8. [8 marks] Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 4 & 3 & -2 \end{bmatrix}$.

1. Write down a vector which belongs to the rowspace of A.
2. Find a basis for the rowspace of A.
3. Find a basis for the columnspace of A.
4. Find rank(A).
5. Find the nullspace of A.
6. Find a basis for the nullspace of A.
7. Find the nullity of A.

Best of luck ☺ *Daoud Siniora*