Linear Algebra I, Midterm 2, Fall 2020

Problem 1:

$$W = \text{span}\{\underbrace{(4,3,0,0)}_{v_1}, \underbrace{(0,5,0,4)}_{v_2}, \underbrace{(3,1,0,-1)}_{v_3}, \underbrace{(0,0,6,0)}_{v_4}\}$$

(i) Orthogonal basis for W:

Gram-Schmidt

$$w_1 = v_1 = (4,3,0,0)$$

$$w_2 = v_2 - \frac{v_2 \cdot w_1}{w_1 \cdot w_1} w_1 = (0,5,0,4) - \frac{15}{25}(4,3,0,0)$$

$$= \left(-\frac{12}{5}, \frac{16}{5}, 0, 4\right)$$

$$w_3 = v_3 - \frac{v_3 \cdot w_1}{w_1 \cdot w_1} w_1 - \frac{v_3 \cdot w_2}{w_2 \cdot w_2} w_2$$

$$= (3,1,0,-1) - \frac{15}{25}(4,3,0,0) - \frac{-8}{32}\left(-\frac{12}{5}, \frac{16}{5}, 0, 4\right)$$

$$= (3,1,0,-1) - \left(\frac{12}{5}, \frac{9}{5}, 0, 0\right) + \left(\frac{3}{5}, \frac{4}{5}, 0, 1\right) = (0,0,0,0)$$

So w_3 is <u>not</u> an element of the basis since v_3 is a lin. comb. of v_1 & v_2.

$$w_4 = v_4 - \frac{v_4 \cdot w_1}{w_1 \cdot w_1} w_1 - \frac{v_4 \cdot w_2}{w_2 \cdot w_2} w_2 = v_4 - \vec{0} - \vec{0} = v_4 = (0,0,6,0)$$

$$\boxed{\text{orth. Basis} = \left\{(4,3,0,0), \left(-\frac{12}{5}, \frac{16}{5}, 0, 4\right), (0,0,6,0)\right\}}$$

(ii) Is $W = \mathbb{R}^4$: **No**, since $\dim(W) = 3 < \dim(\mathbb{R}^4)$

(iii) $\vec{v} = (8,1,3,-4)$, $\frac{v \cdot w_1}{w_1 \cdot w_1} = \frac{35}{25} = \frac{7}{5}$

$$\frac{v \cdot w_2}{w_2 \cdot w_2} = \frac{1}{32}\left(\frac{-96+16}{5} - 16\right) = -1$$

$$\frac{v \cdot w_4}{w_4 \cdot w_4} = \frac{15}{36} = 1/2$$

So $\boxed{v = \frac{7}{5} w_1 - w_2 + 1/2\, w_4}$