MATLAB Exercises (7%)

These MATLAB selected exercises correspond to the chapters in the textbook and are designed to enhance your understanding of concepts. You will first need to set an online calculator account at https://www.mathworks.com/mwaccount/register, using your AUC email. You then need to activate your account through an email message you will receive. Once you have an active account, you will get your private online licence that gives you an access to MATLAB Online.

Throughout your journey to work with MATLAB, you may like to use this Guide Book.

MATLAB 1: Matrices.

1. (8 marks) Enter the three matrices

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}, \; B = \begin{bmatrix} 4 & -6 & 3 & 5 \\ 0 & -3 & -6 & 8 \\ 3 & 5 & 0 & 7 \\ -1 & 0 & -7 & 9 \end{bmatrix}, \; C = \begin{bmatrix} 16 & -1 & 4 & -1 \\ -3 & 12 & -7 & 8 \\ 4 & -5 & 0 & 0 \\ -14 & 3 & 2 & 8 \end{bmatrix}.$$

   Use MATLAB to calculate

   (i) $A(B - C)$ and $AB - AC$. What do you observe?

   (ii) $3(AC)$, $A(3C)$, and $(3A)C$. What do you observe?

   (iii) Use the command **inv** to find the inverse of the matrix A.

2. (3 marks) Use the MATLAB command **diag** to form the 5×5 diagonal matrix D with diagonal entries $0, -1, -2, -3$, and -4. Find the products D^3 and D^6.

   If D is any $k \times k$ diagonal matrix, describe how to find the product D^n for any positive integer n.

3. (3 marks) Compute A^2, A^3, A^8, A^{100} and A^{10000} where

$$A = \begin{bmatrix} 1 & 1/3 \\ 0 & 1/4 \end{bmatrix}.$$

Describe the matrix A^n for large n.

4. (4 marks) Use the MATLAB command **inv** to find the inverse of the matrix A below. Then adjoin the identity 3×3 matrix $I = \mathbf{eye(3)}$ to A to form the 3×6 matrix $B = [A\ I]$. Row-reduce B to compute the inverse of A again. What do you observe?

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -2 \end{bmatrix}.$$

5. (5 marks) Let A and B be the following 3×3 matrices.

$$A = \begin{bmatrix} 2 & 4 & 5/2 \\ -3/4 & 2 & 1/4 \\ 1/4 & 1/2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1/2 & 3/4 \\ 3/2 & 1/2 & -2 \\ 1/4 & 1 & 1/2 \end{bmatrix}.$$

(i) Calculate $C_1 = (AB)^{-1}$ and $C_2 = B^{-1}A^{-1}$. What do you observe?

(ii) Calculate $(A^{-1})^T$ and $(A^T)^{-1}$. What do you observe?

The MATLAB command for the transpose of a matrix A is A'.

MATLAB 2: System of Linear Equations.

1. (7 marks) Consider the linear system of equations:

$$\begin{aligned} 3x + 3y + 12z &= 6 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4 \\ x + y + 4z &= 2. \end{aligned}$$

Let A be the coefficient matrix of the system, and B the constants column vector.

(i) Form the augmented matrix C for this system by using the MATLAB command $\mathbf{C} = [\mathbf{A}\ \mathbf{B}]$. Then, solve the system using **rref**.

(ii) Use the MATLAB command $\mathbf{A} \setminus \mathbf{B}$ to solve the system.

2. (4 marks) The MATLAB command **polyfit** allows you to fit a polynomial of degree $n-1$ to a set of n data points

$$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$$

in the plane.

Find a fourth-degree polynomial that fits the five data points

$$(-2, 3),\ (-1, 5),\ (0, 1),\ (1, 4),\ (2, 10)$$

by letting

$$\mathbf{x} = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{y} = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 4 \\ 10 \end{bmatrix}$$

and entering the MATLAB command $\mathbf{polyfit}(\mathbf{x}, \mathbf{y}, \mathbf{4})$.

MATLAB 3: Determinants.

1. (8 marks) Use the determinant command **det** to calculate the determinants of the following matrices:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 2 & -1 & 4 \\ 0 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}.$$

(i) Verify that $\det(AB) = \det(A)\det(B)$.

(ii) Verify that $\det(A^t) = \det(A)$.

(iii) Verify that $\det(A^{-1}) = 1/\det(A)$.

2. (2 marks) Choose an arbitrary real number t. Form the matrix

$$\begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$$

and calculate its determinant. Does the value of the determinant depend on t?

MATLAB 4: Vector Spaces

1. (3 marks) Use MATLAB to determine whether the set spans $\mathbb{R}^4$ or not.

$$\{(0,1,-1,1),(2,-2,3,1),(7,0,1,0),(5,2,-2,1)\}.$$

2. (3 marks) Use MATLAB to determine whether the set is linearly independent or dependent.

$$\{(0,0,1,2,3),(0,0,2,3,1),(1,2,3,4,5),(2,1,0,0,0),(-1,-3,-5,0,0)\}.$$

3. (6 marks) Let

$$A = \begin{bmatrix} -1 & 2 & 0 & 0 & 3 \\ 0 & 2 & 3 & -1 & 2 \\ -1 & 4 & 3 & -1 & 5 \\ 2 & -4 & 0 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(i) Find a basis for the row space of A.

(ii) Find a basis for the column space of A.

(iii) Use the MATLAB command **rank** to find the rank of A.

4. (12 marks) Find a basis for the nullspace of the given matrix A. Then verify that the sum of the rank and nullity of A equals the number of columns.

   (i) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$.

   (ii) $A = \text{hilb}(5)$.

   (iii) $A = \text{pascal}(5)$.

   (iv) $A = \text{magic}(6)$.

MATLAB 5: Eigenvalues and Eigenvectors

The MATLAB command **poly(A)** produces the coefficients of the characteristic polynomial of the square matrix A, beginning with the highest degree term.

If we set **p=poly(A)**, then the command **roots(p)** calculates the roots of the characteristic polynomial of the matrix A.

The MATLAB command **[V, D]=eig(A)** produces a diagonal matrix D containing the eigenvalues of A on the diagonal, and a matrix V whose columns are the corresponding eigenvectors.

1. (12 marks) Use this sequence of commands to find the eigenvalues and eigenvectors of the matrices:

   (i) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.

   (ii) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$.

(iii) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$.

2. (12 marks) Use MATLAB to find the eigenvalues and corresponding eigenvectors of A, A^T, and A^{-1}, where

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

What do you observe?

MATLAB 6: Linear Transformations

1. (12 marks) Use MATLAB to help you in finding the kernel and range of the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ for the matrices:

(i) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \\ -2 & 0 & -2 \end{bmatrix}$.

(ii) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 9 & 10 & 11 & 12 \\ 5 & 6 & 7 & 8 \\ -13 & -14 & -15 & -16 \end{bmatrix}$.

(iii) $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}$.

2. (5 marks) Let B be the upper Δ matrix $\mathbf{B} = \mathbf{triu}(\mathbf{ones}(\mathbf{6}))$. Determine the rank and nullity of the linear transformation

$$T : \mathbb{R}^6 \to \mathbb{R}^6 : \mathbf{x} \mapsto A\mathbf{x},$$

where $A = BB^t - B$.

3. (12 marks) Which of these linear transformations defined by $T(\mathbf{x}) = A\mathbf{x}$ are one-to-one? Which are onto?

   (i) $A = \text{magic}(6)$.

   (ii) $A = \text{hilb}(6)$.

   (iii) $A = \text{tri}(\text{ones})(6)$.

MATLAB 7: Inner Product Spaces.

1. (6 marks) Use the MATLAB command **norm(v)** to find

   (i) the length of the vector $\mathbf{v} = (0, -2, 1, 4, -2)$.

   (ii) a unit vector in the direction of $\mathbf{v} = (-3, 2, 4, -5, 0, 1)$.

   (iii) the distance between $\mathbf{u} = (0, 2, 2, -3)$ and $\mathbf{v} = (-4, 7, 10, 1)$.

2. (6 marks) The standard dot product of the vectors $\mathbf{u}$ and $\mathbf{v}$ (written as columns) can be computed simply by multiplying the transpose of $\mathbf{u}$ times $\mathbf{v}$.

   Let $\mathbf{u} = (2, -5, 0, 4, 8)$, $\mathbf{v} = (0, -3, 2, -1, 1)$ and $\mathbf{w} = (1, -1, 0, 0, 7)$. Use MATLAB to find the following.

   (i) $\mathbf{u} \cdot \mathbf{v}$

   (ii) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

   (iii) $\mathbf{u} \cdot (2\mathbf{v} - 3\mathbf{w})$

3. (4 marks) Use the built-in inverse cosine function **acos** to find the angle between $\mathbf{u} = (-3, 4, 0)$ and $\mathbf{v} = (1, 1, 4)$.

4. (6 marks) Use MATLAB to find the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

   (i) $\mathbf{u} = (3, 1, 2)$ and $\mathbf{v} = (7, 1, -2)$.

   (ii) $\mathbf{u} = (0, 1, 3, -3)$ and $\mathbf{v} = (4, 0, 0, 1)$.

Programming with MATLAB (3%)

Write a programming code using MATLAB, Python, C++ or any programming language of your preferences to solve a system of n linear equations in n variables using **Cramer's Rule**.

1. Your main reference is Section 3.4 "Applications of Determinants" on pages 134 – 137 . Of course, you are encouraged to use other references.
2. Your code should allow the user to choose the size nn of the system.
3. Make sure that your program first checks that the coefficient matrix is invertible.
4. The output of your program must be the unique solution of the system.
5. Use your code to solve exercises 25, 26 of Section 3.4, page 142.
6. Save your code together with the inputs and outputs of exercise 25 or 26 of section 3.4 as a PDF file, then submit it to its assigned slot on Gradescope.
7. Those who chose MATLAB and might need some tips may consult your undegraduate TAs.
8. Only for this part, we encourage you to work in groups of two students among all the sections. We will ask for an interview in case a particular team has copied their work.

Best wishes,

Dr. Isabel Müller and Dr. Eslam Badr