The American University in Cairo | **MACT 2132**
Mathematics and Actuarial Science | **Spring 2020**
Linear Algebra | **Exam 1, V2**
March 19, 2020 | **Time Limit: 75 Minutes**

Name: _____ **UID:**_____

- This exam contains 8 pages (including this cover page and the draft page).
- Answer all the problems.
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		7
2		11
3		14
4		20
Total		52

Problem 1. The augmented matrix of a system of linear equations is given by

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & k & k \\ -4 & 3 & 1 \end{bmatrix}$$

a) (1 pts) Determine the number of equations and the number of variables.

b) (6 pts) Find the value(s) of k that make the system consistent, and then solve the system.

Problem 2. Let s, t and u be fixed real numbers. Consider the matrix

$$B = \begin{bmatrix} 1 & s & s^2 \\ 1 & t & t^2 \\ 1 & u & u^2 \end{bmatrix}.$$

a) (4 pts) Show that $|B| = (t-s)(u-s)(u-t)$.

b) (5 pts) Find the inverse of B when $s = 0$, $t = 1$ and $u = 2$ or show that it does not exist.

c) (2 pts) Determine the condition(s) on s, t and u for which B is invertible.

Problem 3. Let A be the 3×3 matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}.$$

a) (8 pts) Determine the eigenvalues for A and the corresponding eigenvectors.

b) (6 pts) Use Cayley-Hamilton Theorem to express A^5 in terms of A^2, A, and I.

Problem 4. (5 pts each) Prove or disprove four of the following.

a) A homogeneous system of three linear equations in two variables has infinitely many solutions.

Reason: True False

b) If A and B are $n \times n$ matrices and A is invertible, then $|\,(A\,B\,A^{-1})^2\,| = |B^2|$.

Reason: True False

c) A square matrix can have two different row-echelon forms.

Reason: True False

d) If $\underline{\mathbf{x}}$ is an eigenvector for A relative to an eigenvalue λ, then so is $c\,\underline{\mathbf{x}}$ for any scaler $c \neq 0$.

Reason: True False

e) If A and B are 2×2 matrices such that $A + B$ is invertible, then A or B is invertible.

Reason: True False

f) If A is a symmetric invertible matrix, then $5A^{-1}$ is symmetric.

Reason: True False

Draft: