Linear Algebra
Exam 1
October 15, 2022

Name: ______________________ **UID:** ____________________

- **The exam consists of FOUR problems.**
- **Unsupported answers will receive little or no credit.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Upload your answers to Gradescope as a pdf only. Make sure to allocate your work to the appropriate question.**
- **Missing or blank pages will result in an automatic zero for the question.**
- **Time: 75 minutes.**

Problem	Score	Points
1		12
2		8
3		6
4		16
Total		42

Best wishes!

Dr. Isabel and Dr. Eslam

Problem 1, Part 1 (6 points) Find the inverse of A or show that it does not exist.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 6 & 2 & 3 \\ 2 & 5 & 2 & -2 \\ 3 & 8 & 4 & -3 \end{bmatrix}.$$

Problem 1, Part 2 (6 points) Let A and B be 2×2 matrices such that $\det(A) = 3$ and $\det(B) = 1/3$.

(i) Find the value of $\det\big(4\,A\,(A\,B)^T\,(BA)^{-1}\,B\big)$.

(ii) Does it follow that B is the inverse of A? **Explain your answer**.

Problem 2. (8 points)

(i) Find the value(s) of λ for which the system of linear equations in x and y

$$\begin{aligned} x + \lambda\, y &= 1 \\ \lambda\, x + y &= 1 \\ x - y &= \lambda \end{aligned}$$

has **(a)** no solutions, **(b)** a unique solution, **(c)** infinitely many solutions.

(ii) For all λ such that there is a unique solution, state the solution!

Problem 3. (6 points) In this question, you are asked to prove that

"For any square matrix $\boldsymbol{A}$, $\boldsymbol{A}$ is invertible iff $\mathbf{det}(\boldsymbol{A}) \neq \mathbf{0}$"

(i) First, assume that A is invertible and deduce that $\det(A) \neq 0$.

(ii) Second, assume that $\det(A) \neq 0$ and deduce that A has inverse.

Problem 4. (4 points each) True or False (circle one and state your reason):

(i) If A and B are symmetric $n \times n$ matrices, then AB is also symmetric.

Reason: True False

(ii) Let A, B and C be square matrices of the same size such that A is invertible and

$$B = A^{-1}CA.$$

Then, B is invertible if and only if C is invertible.

Reason: True False

(iii) If a square matrix B is obtained from A by swapping two rows and $\det(A) > \det(B)$, then A must be invertible.

Reason: True False

(iv) For any invertible matrix A, we have

$$\operatorname{adj}\left(A^T\right) = (\operatorname{adj}(A))^T.$$

Reason: True False

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