Exam 2
April 24, 2021

Name: _____ **UID:** _____

- **The exam consists of FOUR problems.**
- **Unsupported answers will receive little or no credit.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Upload your answers to Gradescope as a pdf only. Make sure to allocate your work to the appropriate question.**
- **Missing or blank pages will result in an automatic zero for the question.**
- **Time: 90 minutes.**

Problem	Score	Points
1		8
2		12
3		8
4		24
Total		52

Problem 1. (8 points) For what value(s) of k is

$$S = \{(1, 1, 2), (1, 0, -1), (1, -1, k)\}$$

a basis for $\mathbb{R}^3$? Justify your answer.

Problem 2. Let A and its reduced row echelon form B be given by

$$A = \begin{bmatrix} 1 & 0 & a_{13} & 1 & 3 \\ -1 & 1 & a_{23} & -1 & -3 \\ 0 & 2 & a_{33} & 0 & 1 \\ 1 & 1 & a_{43} & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

i) (2 points) Find the rank and the nullity of A.

ii) (4 points) Find a basis for $\mathrm{Nul}(A)$, the null space of A.

iii) (3 points) Find the 3 rd column of A.

iv) (3 points) Is $\text{Col}(A) = \text{Col}(B)$? Why ?

Problem 3. (8 points) Let $\mathcal{C}[-1, 1]$ be the space of all continuous functions on the closed interval $[-1, 1]$, and consider the inner product function

$$\langle f , g \rangle = \int_{-1}^{1} f(x)g(x)\, dx.$$

Apply Gram-Schmidt process to transform the set

$$\{1,\ 1 + 3x,\ \cos(\pi x)\}$$

into an orthonormal set.

Problem 4. (4 points each) True or False (Circle one and state your reason):

i) The set $W = \{(x, y, z) : x + y - z \leq 1\}$ is a subspace of $\mathbb{R}^3$.

Reason: True False

ii) If two unit vectors $\underline{\mathbf{u}}$ and $\underline{\mathbf{v}}$ are orthogonal, then so are $\underline{\mathbf{u}} + \underline{\mathbf{v}}$ and $\underline{\mathbf{u}} - \underline{\mathbf{v}}$.

Reason: True False

iii) The set $\{1,\ 3x,\ 1 - x^2,\ 1 - 7x\}$ is a basis for $\mathcal{P}_2$ (the space of all polynomials of degree ≤ 2).

Reason: True False

iv) There exists a 3×6 matrix A such that $\text{Nullity}(A) = 2$.

Reason: True False

v) For $\underline{\mathbf{u}} = (u_1, u_2, u_3)$, $\underline{\mathbf{v}} = (v_1, v_2, v_3) \in \mathbb{R}^3$, the product

$$\langle \underline{\mathbf{u}}, \underline{\mathbf{v}} \rangle = u_1 v_1 + u_3 v_3$$

defines an inner product on $\mathbb{R}^3$.

Reason: True False

vi) The set $\{1, 1 - \sin^2(x), 1 - \cos^2(x)\}$ is linearly independent in $\mathcal{C}[-1, 1]$ (the space of all continuous functions on the closed interval $[-1, 1]$).

Reason: True False

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