Linear Algebra
Exam 2
November 23, 2022

Name: ____________________ **UID:** ____________________

- **The exam consists of FIVE problems.**
- **Unsupported answers will receive little or no credit.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Time: 75 minutes.**

Problem	Score	Points
1		6
2		6
3		8
4		10
5		12
Total		42

Best wishes!

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Problem 1. (6 points) Determine if each of the following sets is a subspace.

(i) Is $W = \left\{(x, y, z) \in \mathbb{R}^3 : 2\,x\,z = y\right\}$ a subspace of $(\mathbb{R}^3, +, \cdot)$ with the standard addition and multiplication?

(ii) Is $W = \{f(x) \in \mathcal{F}(-\infty, \infty) \,:\, f(-x) = -f(x)\}$ a subspace of $(\mathcal{F}(-\infty, \infty), +, \cdot)$ with the standard addition and multiplication?

Problem 2. Consider the set of vectors

$$S = \left\{1 - x,\, x - x^3,\, 1 + x - kx^2 + x^3\right\}$$

in $\mathcal{P}_{\leq 3}$, the space of polynomials in x of degree ≤ 3.

(i) (4 points) Determine the value(s) of k for which S is linearly independent.

(ii) (2 points) Can S be a basis for $\mathcal{P}_{\leq 3}$? Justify your answer.

Problem 3. (8 points) Consider the set

$$\mathbb{R}^2 = \{(x, y) \,:\, x, y \text{ real numbers}\},$$

equipped with the following addition and scalar multiplication:

$$\begin{aligned}(x, y) + (x', y') &= (x\,x', y\,y')\\ \alpha \cdot (x, y) &= (\alpha\,x + 1, \alpha\,y + 1)\end{aligned}$$

The triple $(\mathbb{R}^2, +, \cdot)$ is **not** a vector space because ...

Choose all answers that apply and justify why the property fails.

(i) it does not satisfy the additive commutativity axiom,

(ii) it does not satisfy the additive identity axiom,

(iii) it does not satisfy the scalar multiplicative identity axiom,

(iv) it does not satisfy the compatibility axiom for scalar multiplication,

(v) it does not satisfy the additive inverse axiom.

Problem 4. Suppose that the following two matrices are row-equivalent

$$A = \begin{bmatrix} 1 & 2 & a \\ 1 & 1 & b \\ 1 & 0 & c \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

(i) (6 points) Describe NullSpace(A) and RowSpace(A) as vector spaces by finding a basis and the dimension of each of them.

(ii) (2 points) Find the column vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

(iii) (2 points) For A and B as given above, are ColSpace(A) and ColSpace(B) equal? Justify.

Problem 5. (4 points each) True or False? Circle one and prove your answer.

(i) The map $\langle\ ,\ \rangle$ given by

$$\langle (x, y, z)\,,\,(x', y', z')\rangle = x\,x' + y\,y' + (z + z')^2$$

is an inner product on $\mathbb{R}^3$.

Reason: True False

(ii) Let $(V, +, \cdot)$ be a vector space such that $\dim(V) = n < \infty$. For any set S containing exactly n many vectors, if S is linearly independent, then S must span V.

Reason: True False

(iii) Let $A \in M_{m \times n}$. If $m > n$, then the m-many rows of A cannot form a basis for $\mathbb{R}^m$.

Reason: True False

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