The American University in Cairo | **MACT 2132**
Mathematics and Actuarial Science | **Fall 2019**
Linear Algebra | **Exam 1**
October 8, 2019 | **Time Limit: 75 Minutes**

Name: _____ **UID:**_____

- This exam contains 6 pages (including this cover page).
- Answer all the problems (total of points is 45).
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		10
2		10
3		10
4		15
Total		45

Problem 1. (10 pts) Find the value(s) of k such that the associated system of linear equations

$$\begin{bmatrix} 1 & 1 & k \\ 1 & k & 1 \\ k & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

has **a)** exactly one solution **b)** an infinite number of solutions **c)** no solution.

Problem 2. (10 pts) Let A be the 4×4 matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 5 & -2 & 9 & 0 \\ 0 & 2 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) Find an LU-Factorization for A.

b) Does A have a unique LU-Factorization? Justify your answer.

Problem 3. (5 pts each)

a) Show that the matrix equation has no solution.

$$\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

b) Let A be a square matrix such that $A^2 - 2A + I = O$. Show that A is invertible and then find A^{-1}.

Problem 4. (5 pts each) True or False (Circle one and state your reason):

a) A system of two linear equations in three variables always has infinitely many solutions.

Reason: True False

b) An $n \times n$ matrix can have only one eigenvalue.

Reason: True False

c) If A is an invertible matrix with $A^3 = A$, then $\det(A^8) = 1$.

Reason: True False

Draft: