The American University in Cairo | **MACT 2132**
Mathematics and Actuarial Science | **Spring 2020**
Linear Algebra | **Exam 1, V1**
March 19, 2020 | **Time Limit: 75 Minutes**

Name: ______________________________ **UID:** ____________

- This exam contains 8 pages (including this cover page and the draft page).
- Answer all the problems.
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		7
2		11
3		14
4		20
Total		52

Problem 1. The coefficient matrix of a homogenous system of linear equations is given by

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 3 & 1 & k \\ 1 & 0 & 1 \end{bmatrix}$$

a) (1 pts) Determine the number of equations and the number of variables.

b) (6 pts) Find the value(s) of k that make the system consistent, and then solve the system.

Problem 2. Let s and t be fixed real numbers. Consider the matrix

$$B = \begin{bmatrix} s+t & s & s \\ s & s+t & s \\ s & s & s+t \end{bmatrix}.$$

a) (4 pts) Show that $\det(B) = t^2(3s+t)$.

b) (5 pts) Find the inverse of B when $s = 1$ and $t = -1$ or show that it does not exist.

c) (2 pts) Determine the condition(s) on s and t for which B is invertible.

Problem 3. Let A be the 3×3 matrix

$$\begin{bmatrix} 4 & -1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

a) (8 pts) Determine the eigenvalues for A and the corresponding eigenvectors.

b) (6 pts) Compute the adjoint matrix adj(A) and show that $A \cdot \operatorname{adj}(A) = \det(A)\, I_3$.

$$A = \begin{bmatrix} 4 & -1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

Problem 4. (5 pts each) Prove or disprove four of the following.

a) A system of linear equations with fewer equations than variables always has a solution.

Reason: True False

b) If A and B are $n \times n$ matrices such that A^3B is singular, then A or B is singular.

Reason: True False

c) A square matrix is invertible if it is row-equivalent to the identity matrix.

Reason: True False

d) If A is a 2×3 matrix, then the only vector $\underline{\mathbf{u}}$ in $\mathbb{R}^3$ such that $A\,\underline{\mathbf{u}} = \underline{\mathbf{0}}$ is $\underline{\mathbf{u}} = \underline{\mathbf{0}}$.

Reason: True False

e) If A is a square matrix such that $A^2 = A$, then $I - 2A = (I - 2A)^{-1}$.

Reason: True False

f) If A and B are $n \times n$ matrices such that $AB = O_{n\times n}$, then $A = O_{n\times n}$ or $B = O_{n\times n}$.

Reason: True False

Draft: