The American University in Cairo | **MACT 2132**
Mathematics and Actuarial Science | **Fall 2018**
Linear Algebra | **Exam 2**
November 8, 2018 | **Time Limit: 75 Minutes**

Name: ______________________________ **UID:**______________

- This exam contains 7 pages (including this cover page).
- Answer **ALL** the questions (total of points is 50).
- Unsupported answers are considered miracles and will receive little or no credit.
- Anyone caught writing after time has expired will be given a mark of zero.

Problem	Score	Points
1		12
2		14
3		12
4		12
Total		50

Problem 1. Consider the set of vectors

$$W := \{(2s - t + 4r, s + r, t - 2r) \,:\, s,\, t,\, r \in \mathbb{R}\}.$$

a) (5 pts) Show that W is a vector subspace of $\mathbb{R}^3$.

b) (5 pts) Find a basis for W.

c) (2 pts) What is the dimension of W? Justify your answer.

Problem 2. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 3 & 0 & -12 \\ 2 & 1 & 1 & -3 \end{pmatrix}.$$

a) (5 pts) Find a basis for the row space $\mathrm{RS}(A)$ of A.

b) (2 pts) Is the linear system $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -12 \\ -3 \end{pmatrix}$ consistent? Justify your answer.

c) (5 pts) Find a basis for the null space $\mathrm{NS}(A)$ of A.

d) (2 pts) Determine the rank and the nullity of A? Justify your answer.

Problem 3. Let A be the 3×3 matrix

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$$

a) (5 pts) Find the adjoint matrix $\mathrm{adj}(A)$ of A.

b) (5 pts) Find the eigenvalues of A.

c) (2 pts) Does Cramer's Rule apply for solving $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$? Justify your answer.

Problem 4. True/False (only **Three** items are required). Justify your answer.

a) (4 pts) If A is an $m \times n$ matrix, then $\text{Rank}(A) = \text{Rank}(A^T)$

b) (4 pts) The set of all third-degree polynomials is a vector subspace of $C(-\infty, \infty)$.

c) (4 pts) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent set of vectors, then so does $\{\mathbf{v}_1 + \mathbf{v}_2,\ \mathbf{v}_2 - \mathbf{v}_3\}$.

d) (4 pts) If $\lambda = 0$ is an eigenvalue for an $n \times n$ matrix, then A is not invertible.

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