Stochastic Processes Calculator

CIE 327 - University of Science and Technology- Zewail City

Ibrahim Hamada Ibrahim – 201800739

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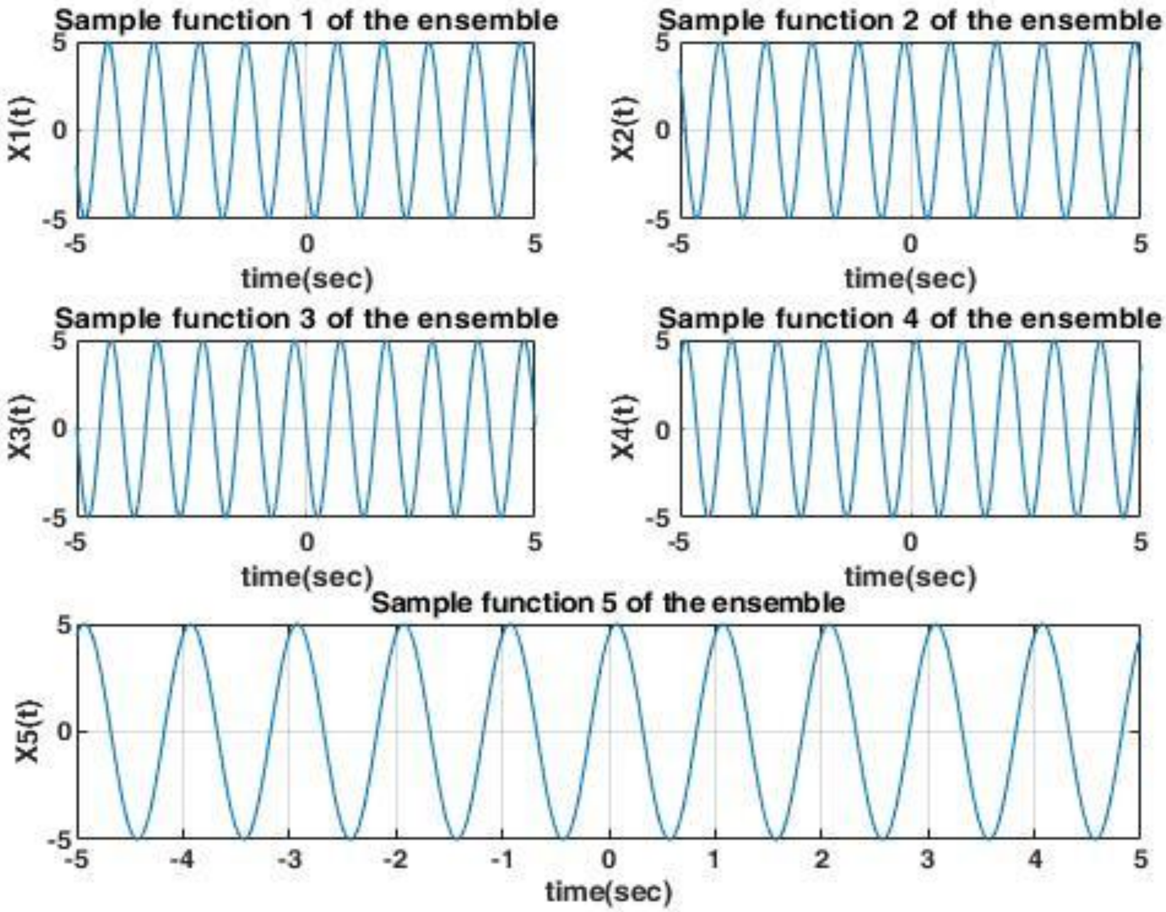
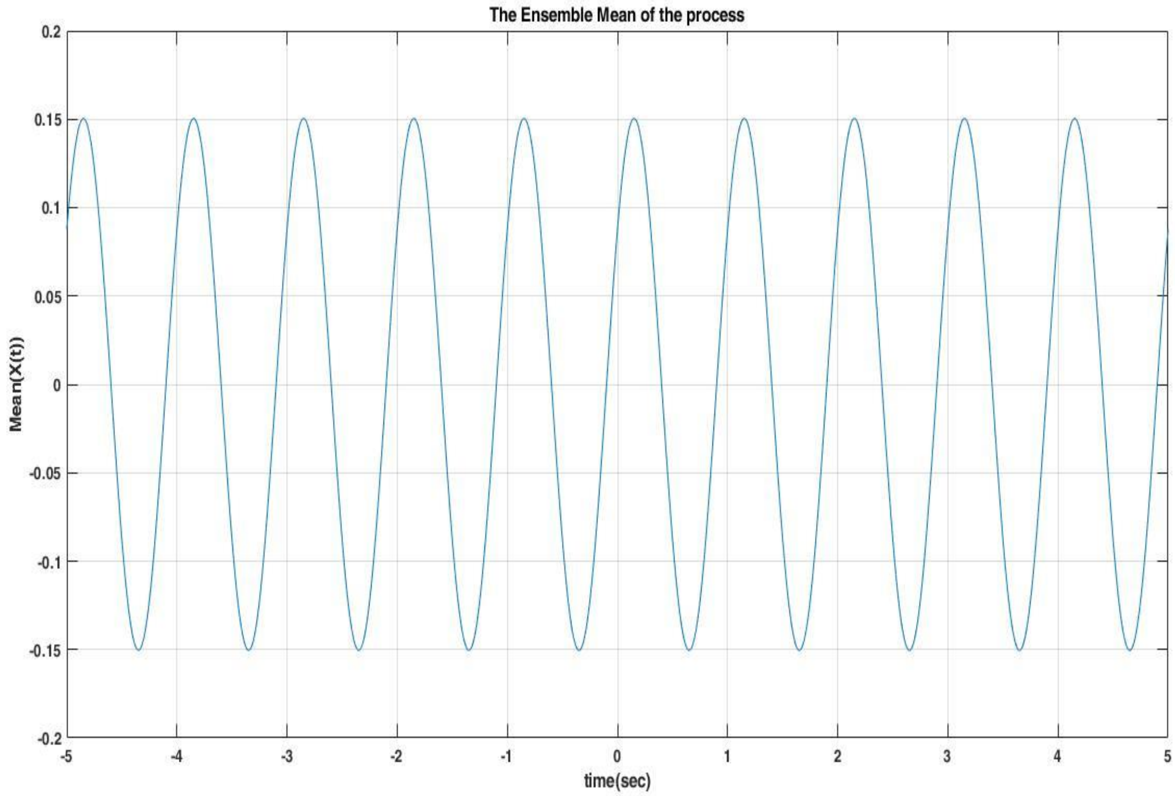
1) Part I:

➢ Plot of 5 random sample functions:



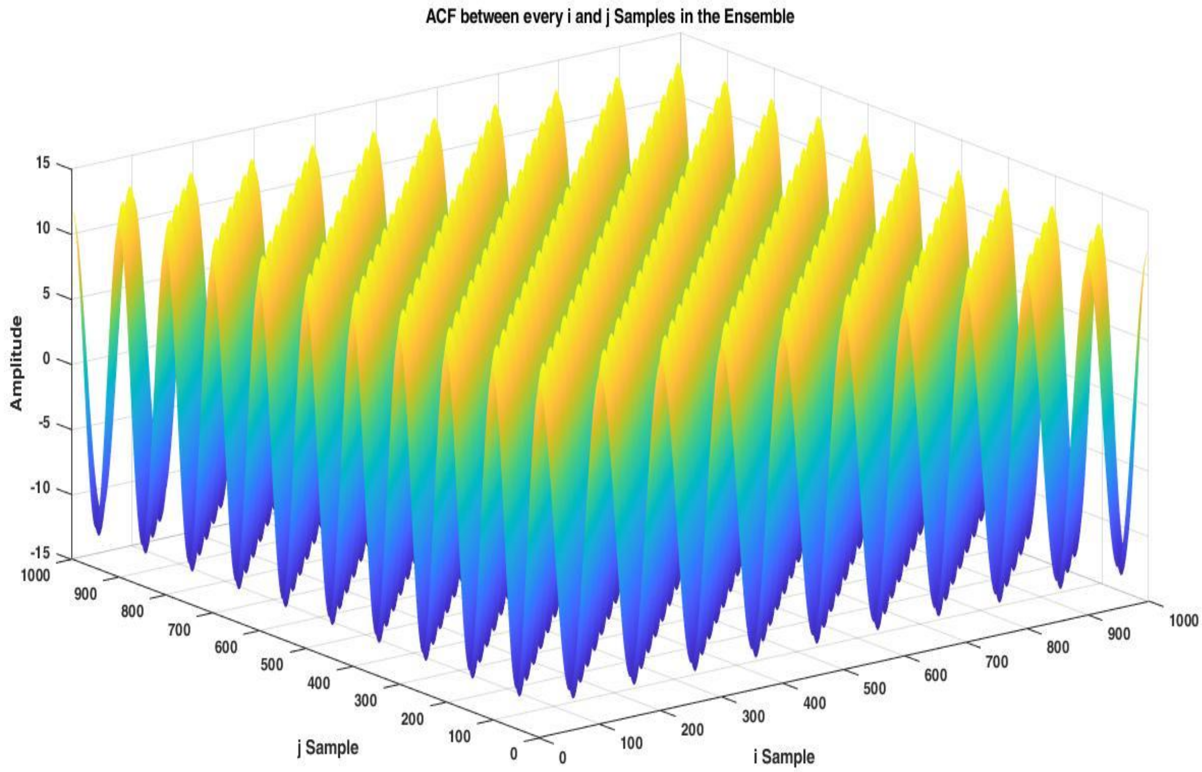
➢ Plot of the ensemble mean:



- **Comment:**

The ensemble mean is oscillating between (-0.15) and (0.15) as we are not dealing with infinite sample functions, but the test case I used has only 1000 sample functions and hence the mean value is closer to (0) [Theoretical Value]. The accuracy will increase by increasing the number of sample functions.

➢ 3D plot of the ACF between i^{th} and j^{th} samples for all i and j:



- **Comment:**

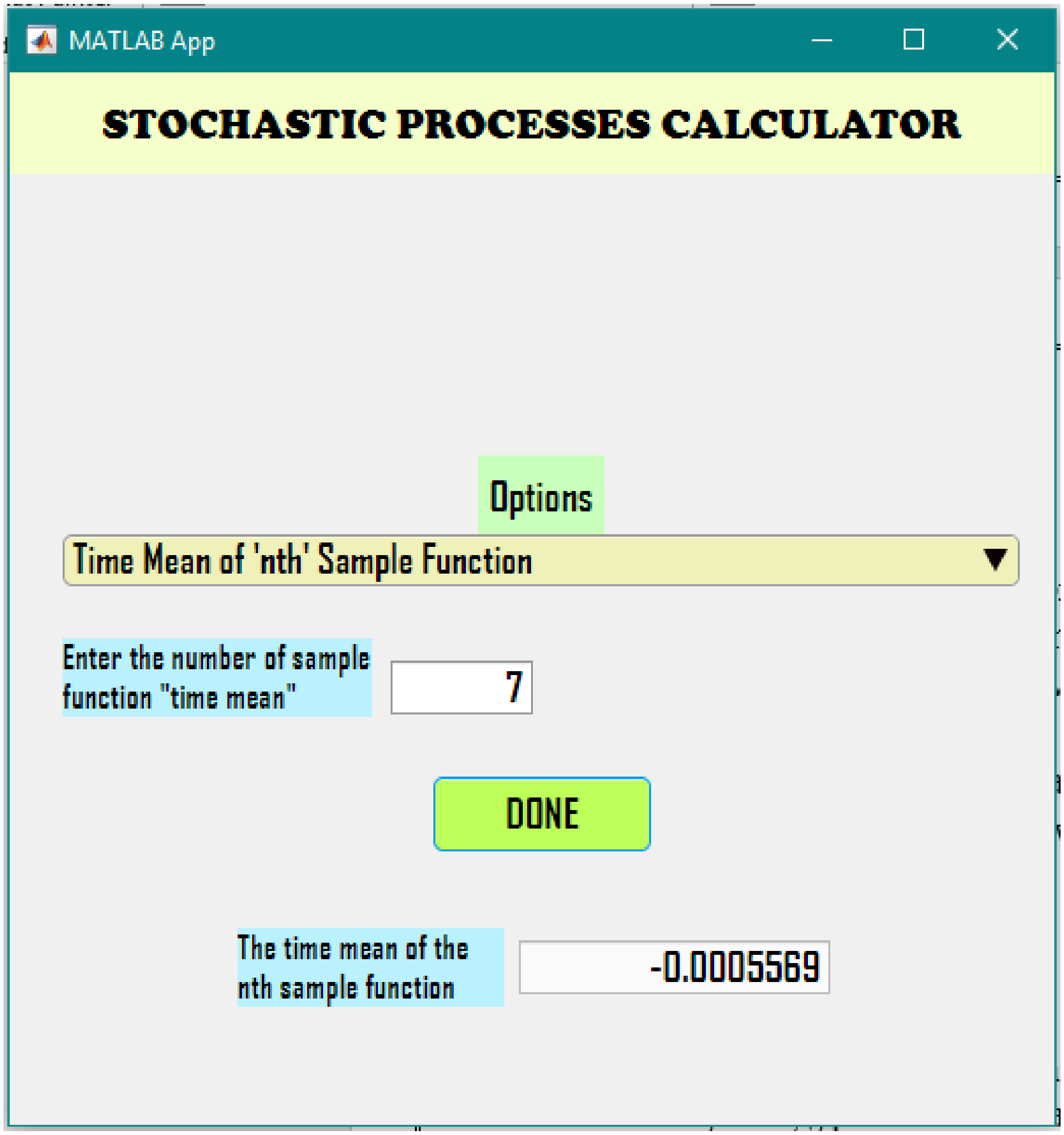
The above 3D plot shows the Auto-correlation Function between every i and j in the ensemble of the random process. As the theoretical ACF of the process is

$$\boldsymbol{R(\tau) = \frac{A^2}{2}\cos(\omega_c\tau)}$$

That's why the AFC is oscillating between (-12.5, 12.5).

For each value of i and j, there exist a value of the statistical Auto-correlation function that follow the cosine function mentioned above.

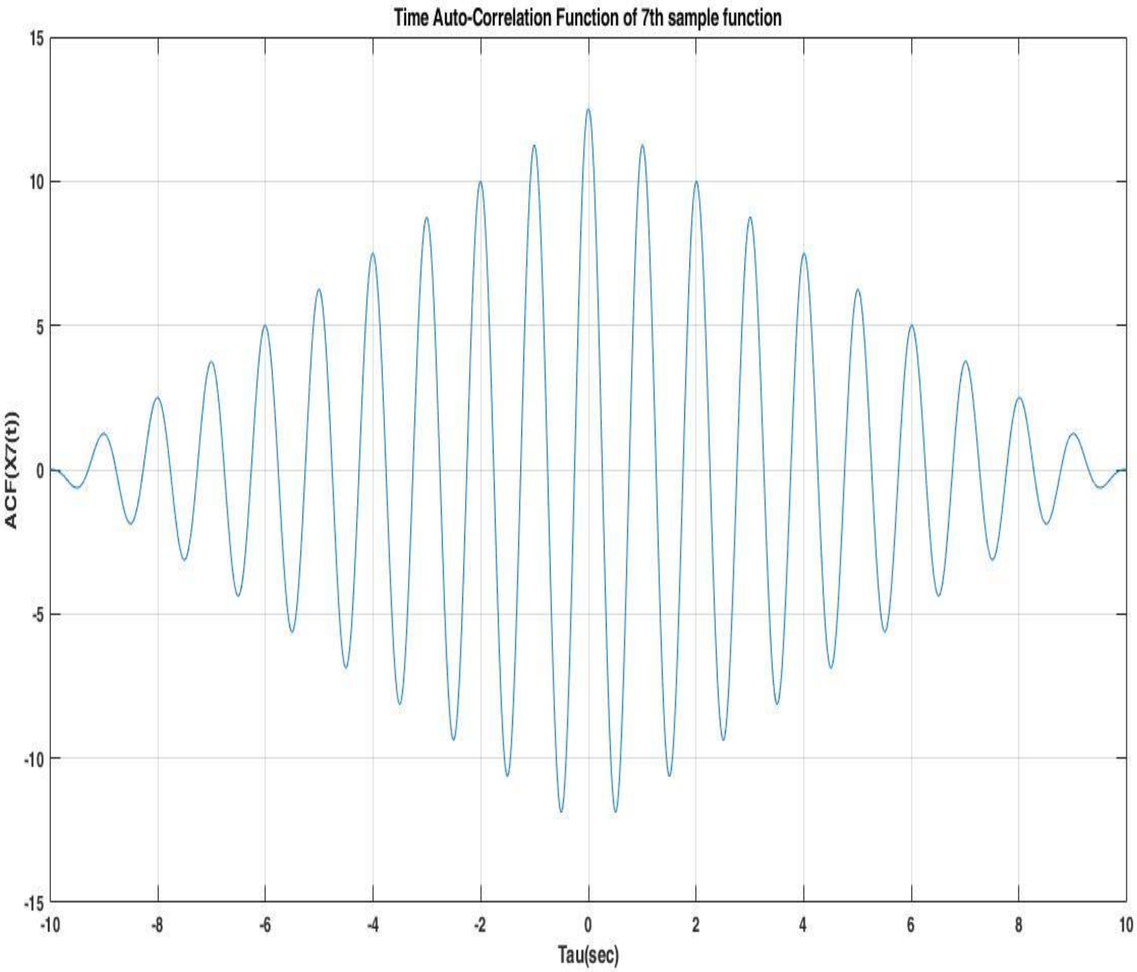
- **Value of the time average of a sample function (7):**



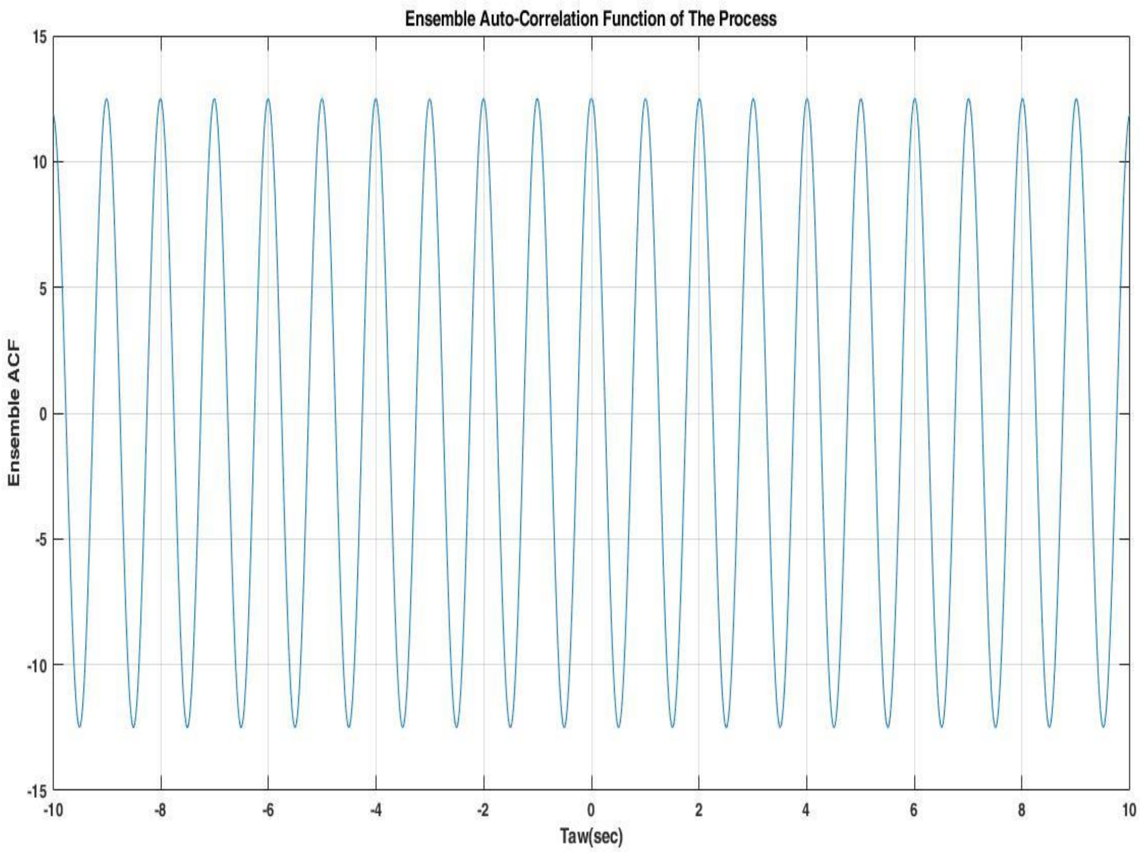
- **Relation between statistical and time mean (Comment):**

There is a relation between the statistical mean and the time mean as they are both nearly equal (0), as this random process is a wide sense stationary (WSS).

➢ The time ACF of a random sample function:



➢ The statistical ACF of the process:



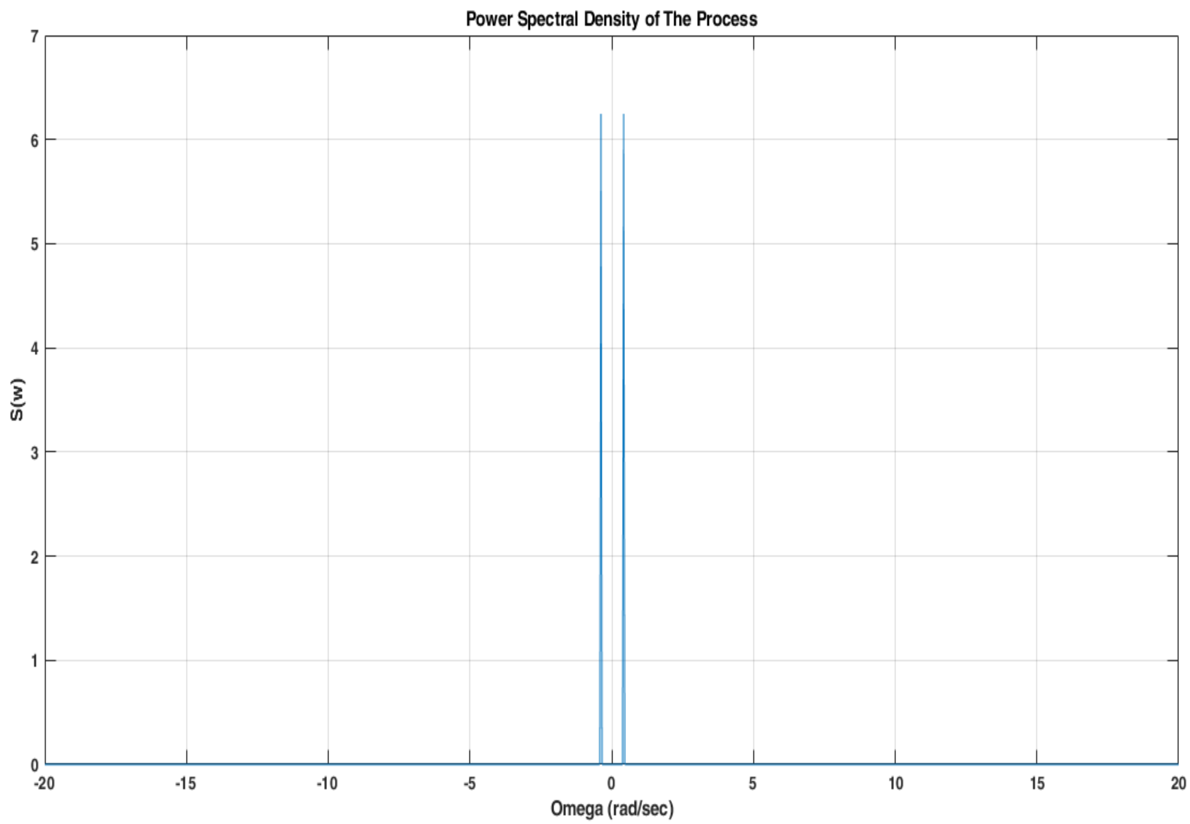
▪ Relation between statistical and time ACF (Comment):

The time ACF looks like a damped sinusoidal wave, because when (**τ**) increases, the number of the multiplied added terms will decrease and hence the value of time ACF decreases. This is also because of dealing with only one sample function.

The statistical ACF is a sinusoidal wave as it results from all the sample functions of the ensemble.

There is a relation between the statistical ACF and the time ACF as they are both are forming a sinusoidal wave, with slight differences due to the reasons mentioned above. We can say that this random process is an Ergodic Process.

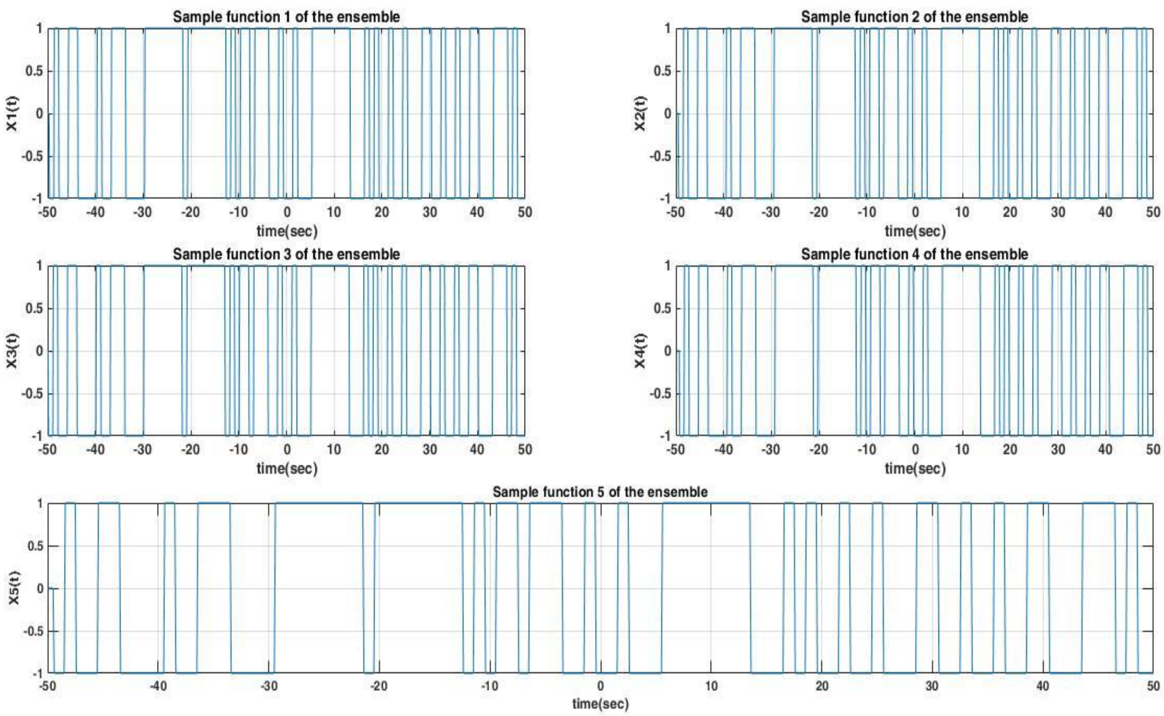
➢ Plot the PSD of the process:



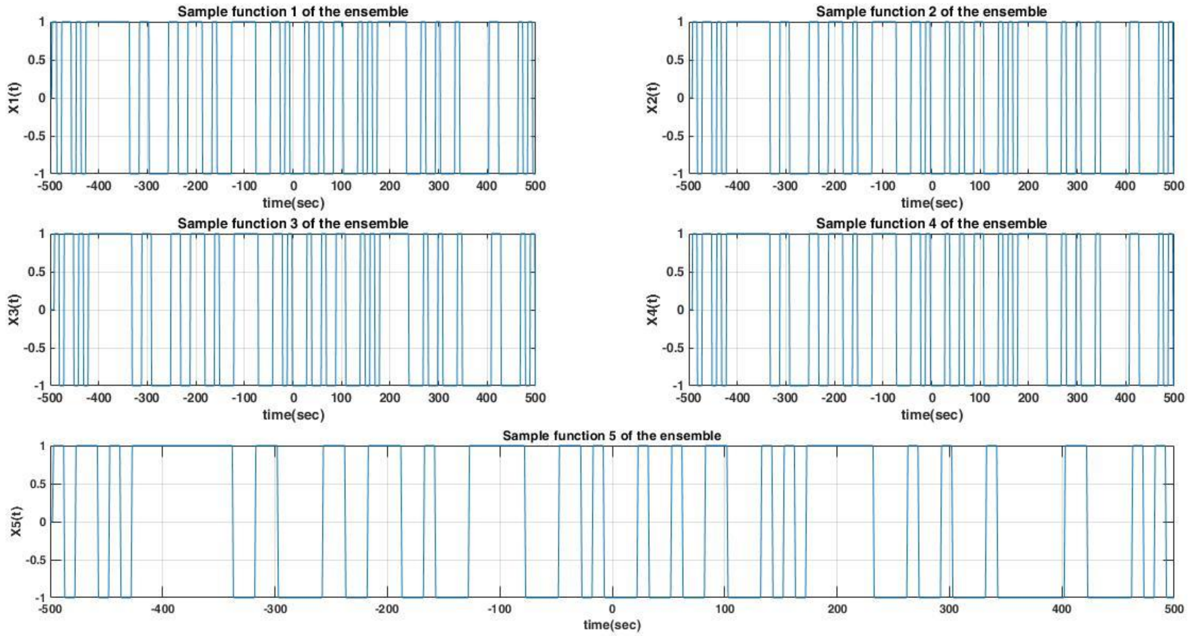
Part II:

- **Plot of 5 random sample functions:**

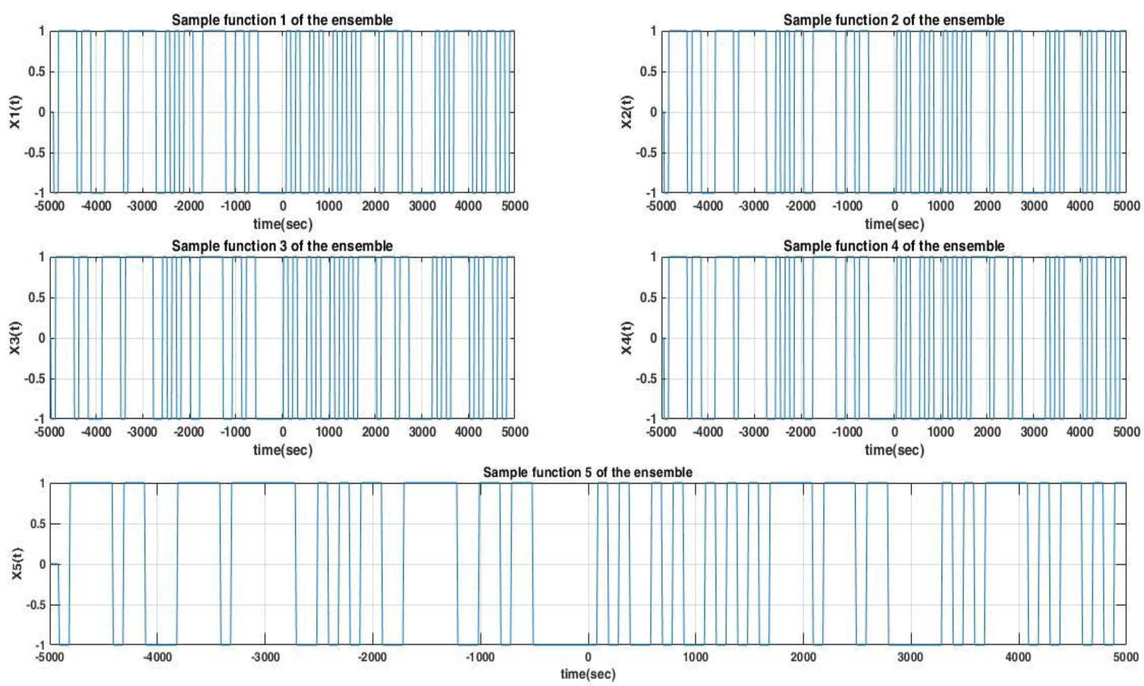
  - **Plot of PNRZ Sample functions at Tb = 1 sec:**



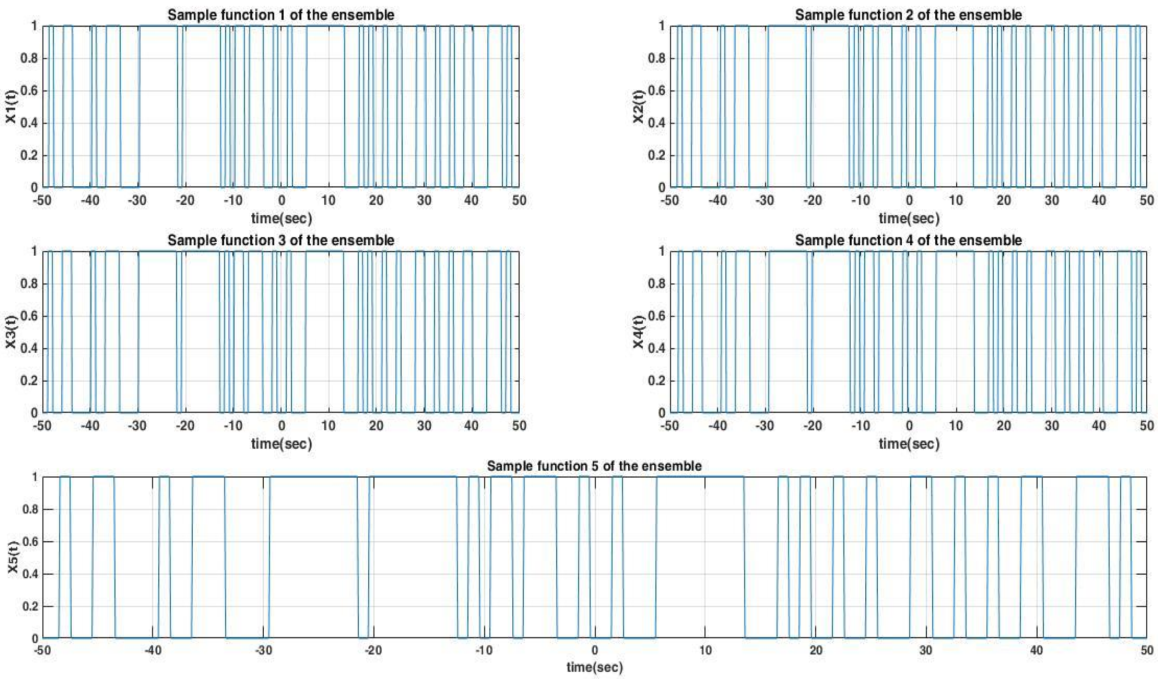
- **Plot of PNRZ Sample functions at Tb = 10 sec:**



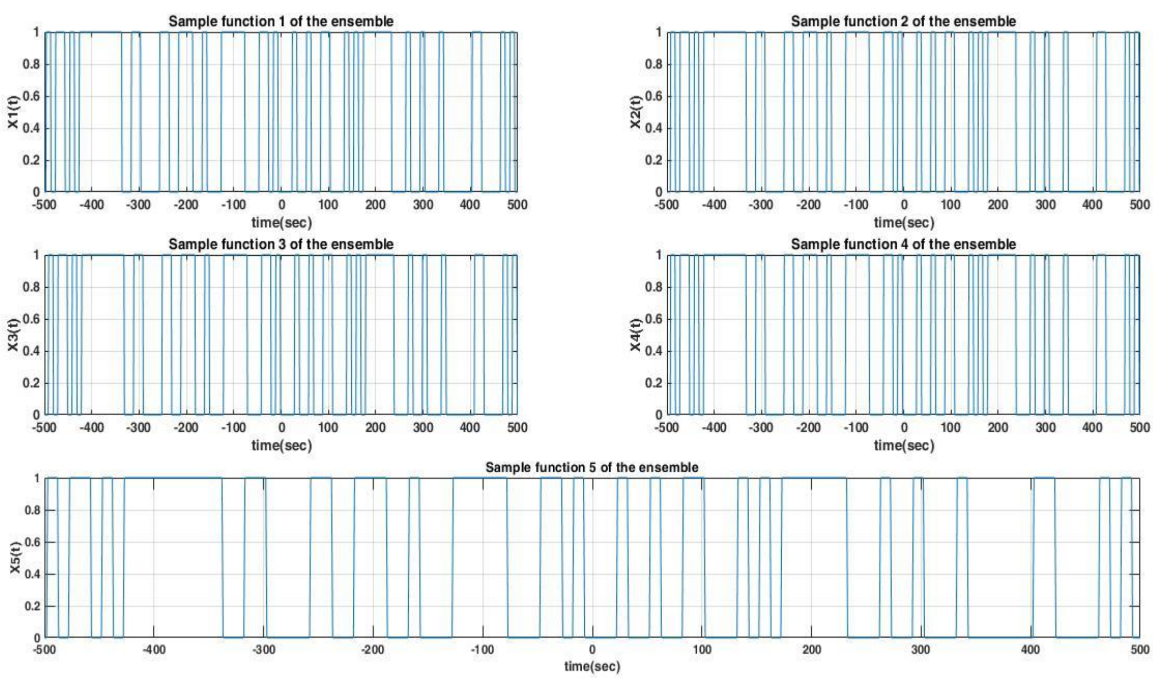
- **Plot of PNRZ Sample functions at Tb = 100 sec:**



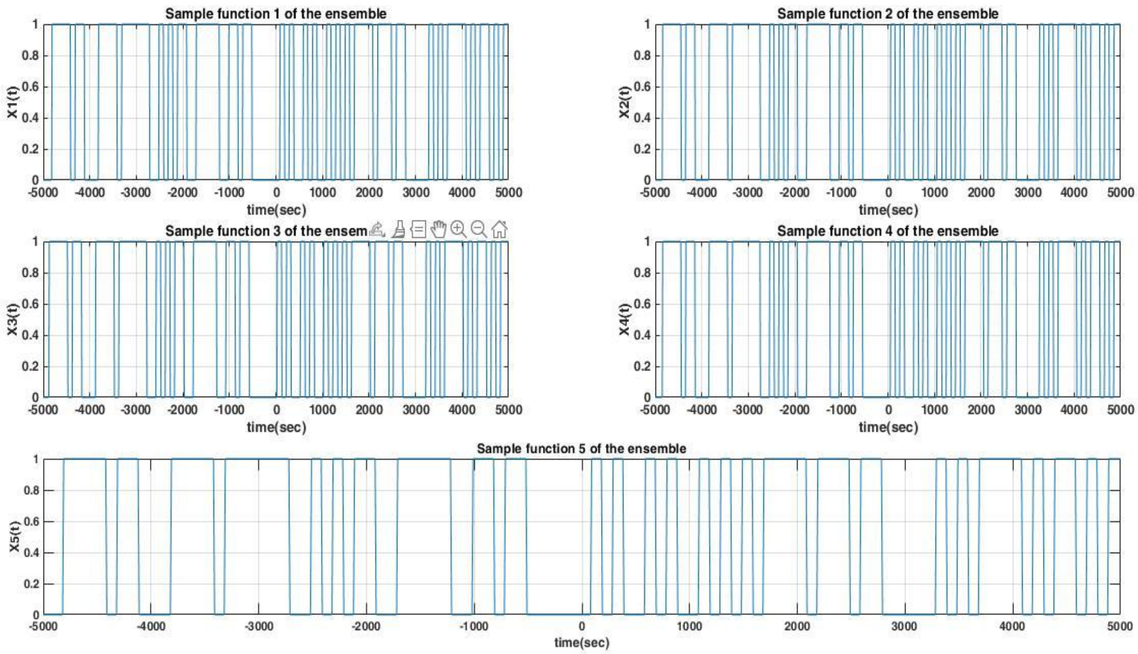
- **Plot of UNRZ Sample functions at Tb = 1 sec:**



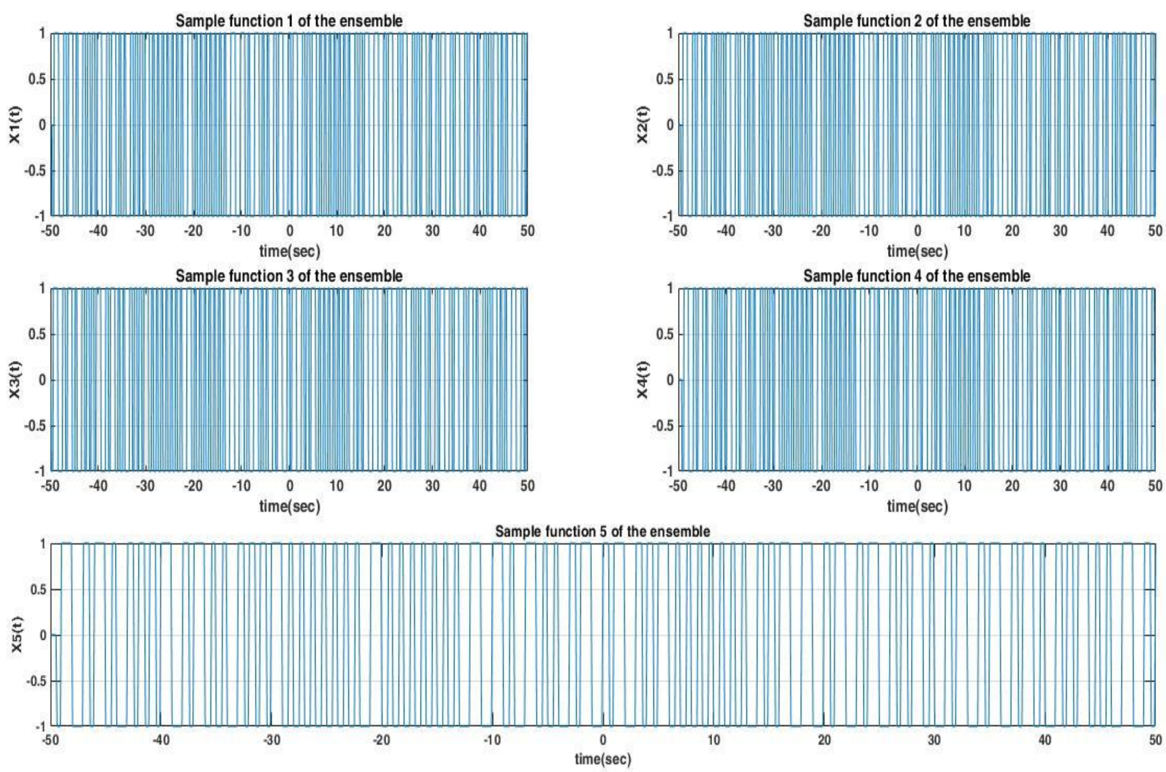
- **Plot of UNRZ Sample functions at Tb = 10 sec:**



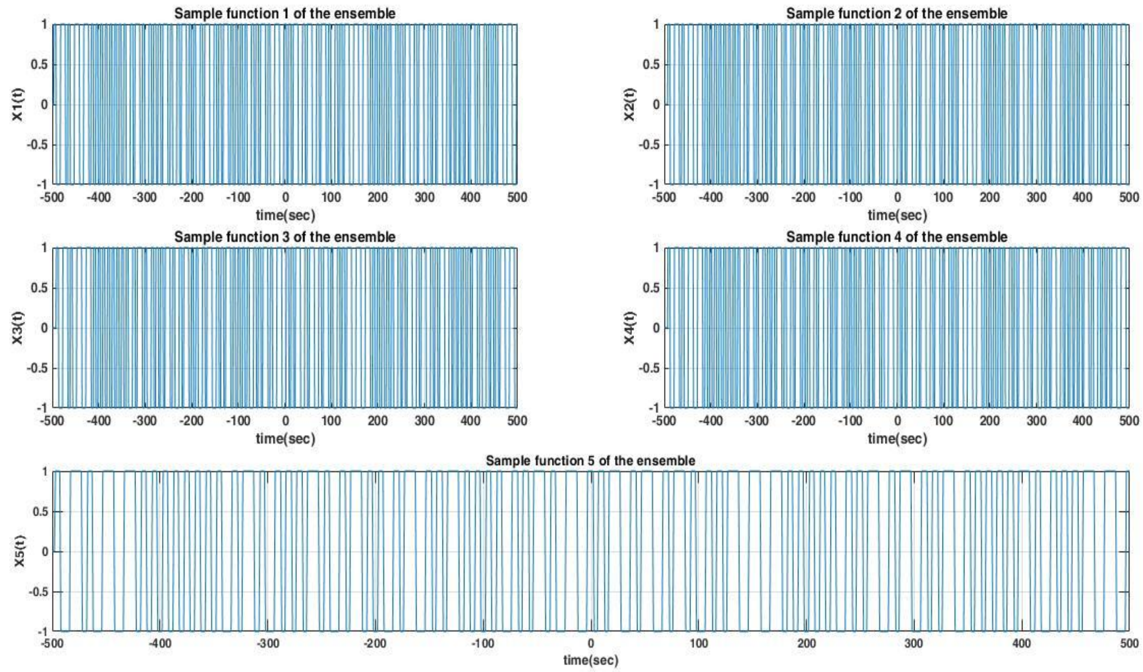
- **Plot of UNRZ Sample functions at Tb = 100 sec:**



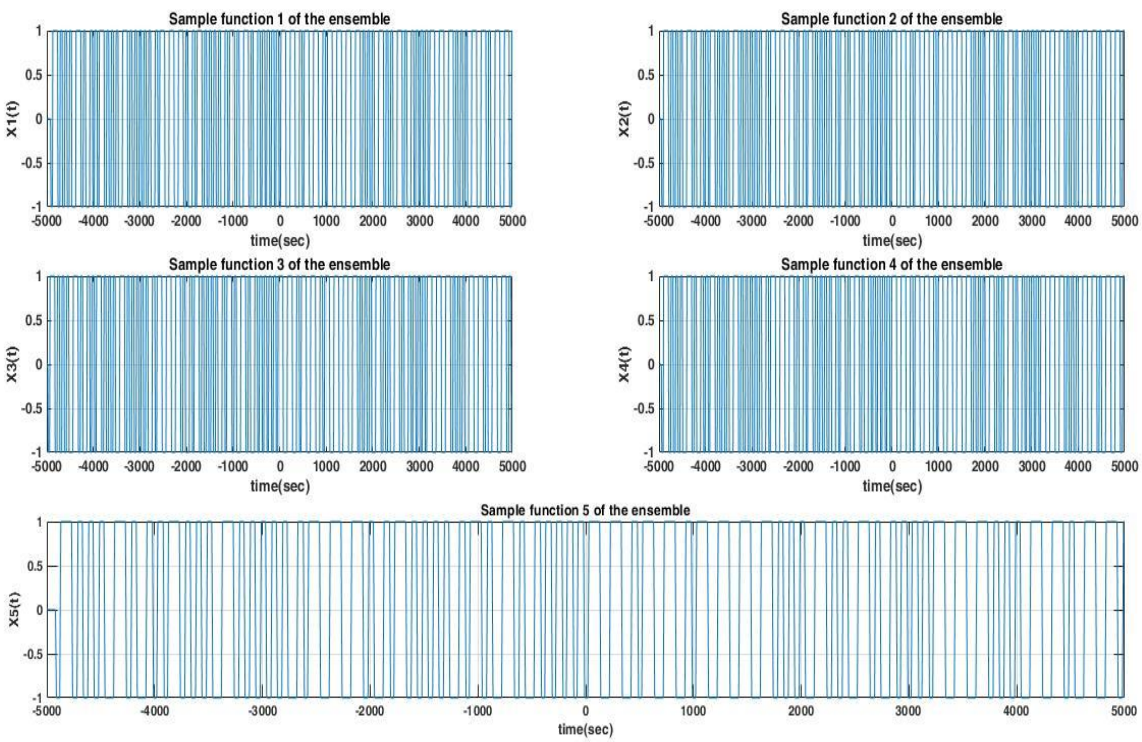
- **Plot of MAN Sample functions at Tb = 1 sec:**



- **Plot of MAN Sample functions at Tb = 10 sec:**

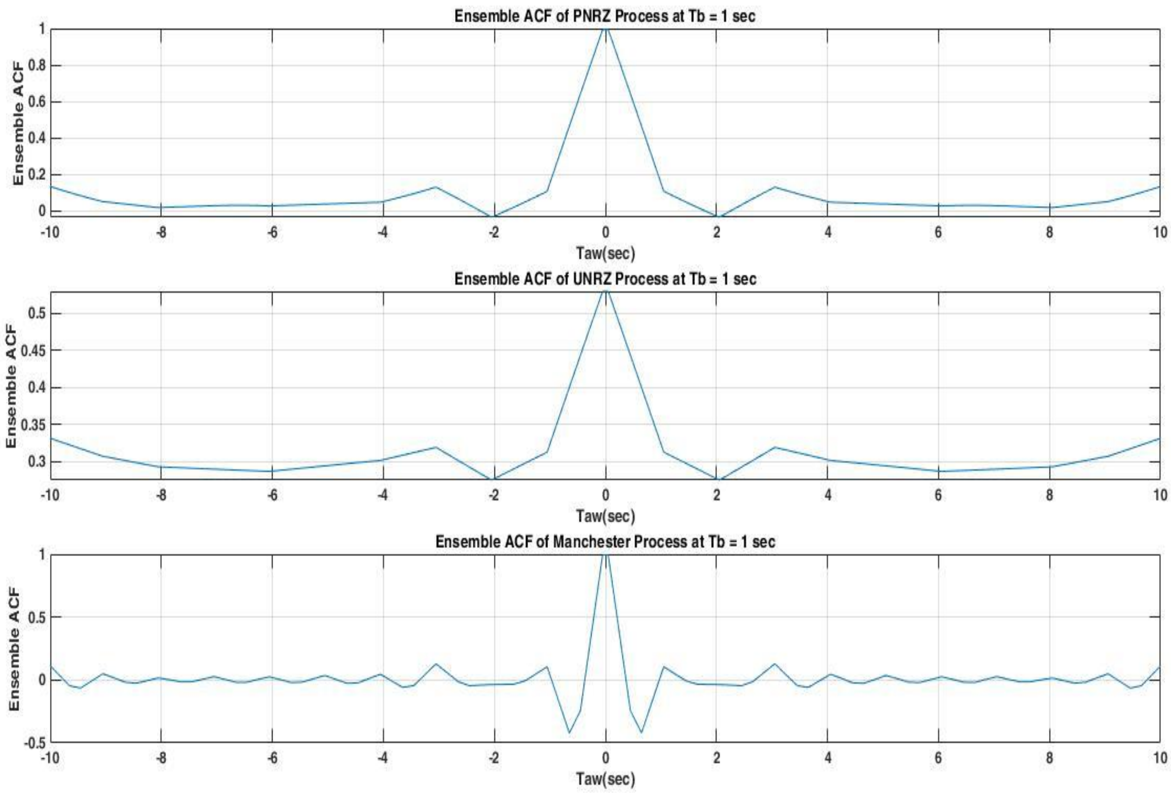


- **Plot of MAN Sample functions at Tb = 100 sec:**

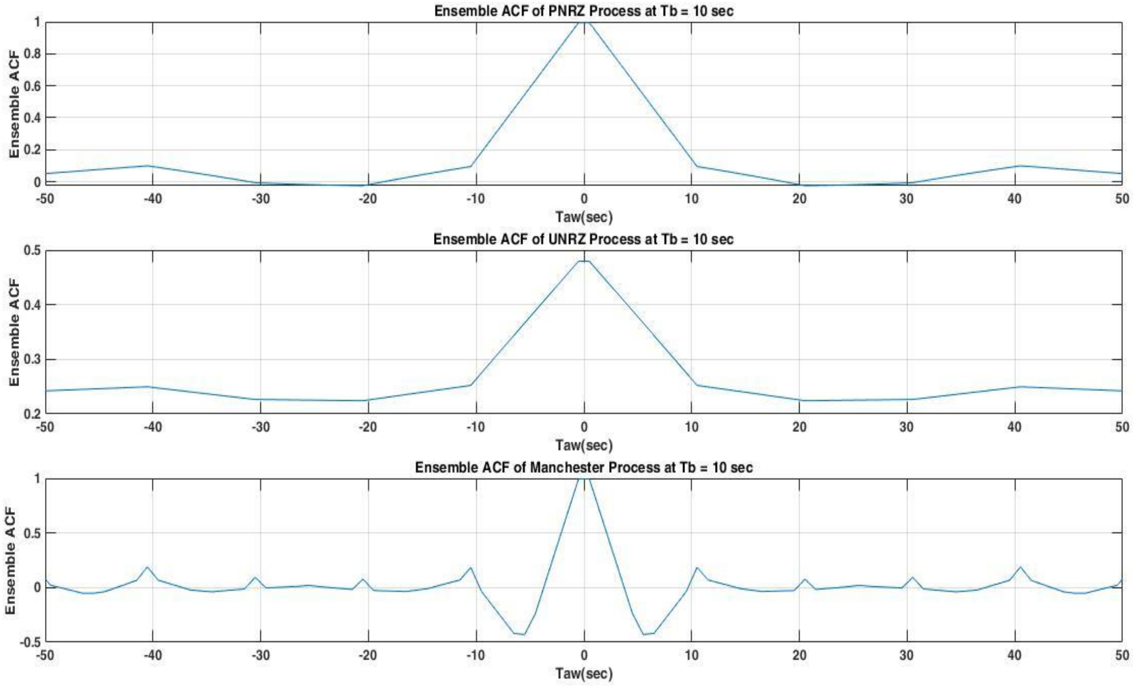


➢ Plot of the Ensemble ACF:

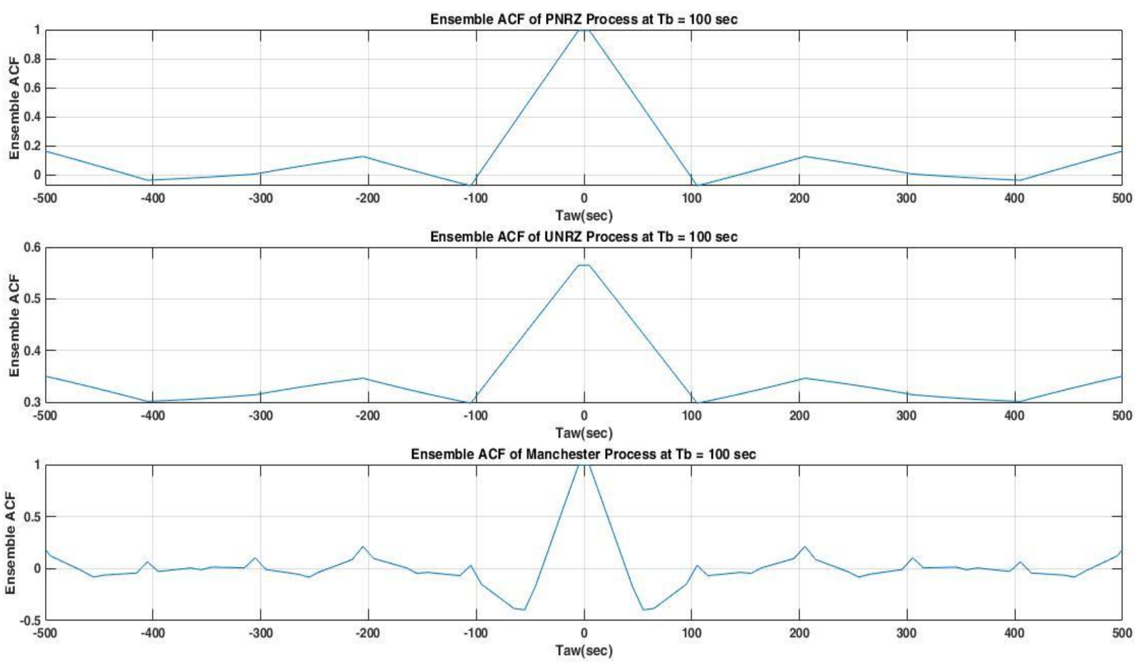
▪ Plot of ACF of PNRZ, UNRZ, MAN at Tb = 1 sec:



- **Plot of ACF of PNRZ, UNRZ, MAN at Tb = 10 sec:**



- **Plot of ACF of PNRZ, UNRZ, MAN at Tb = 100 sec:**



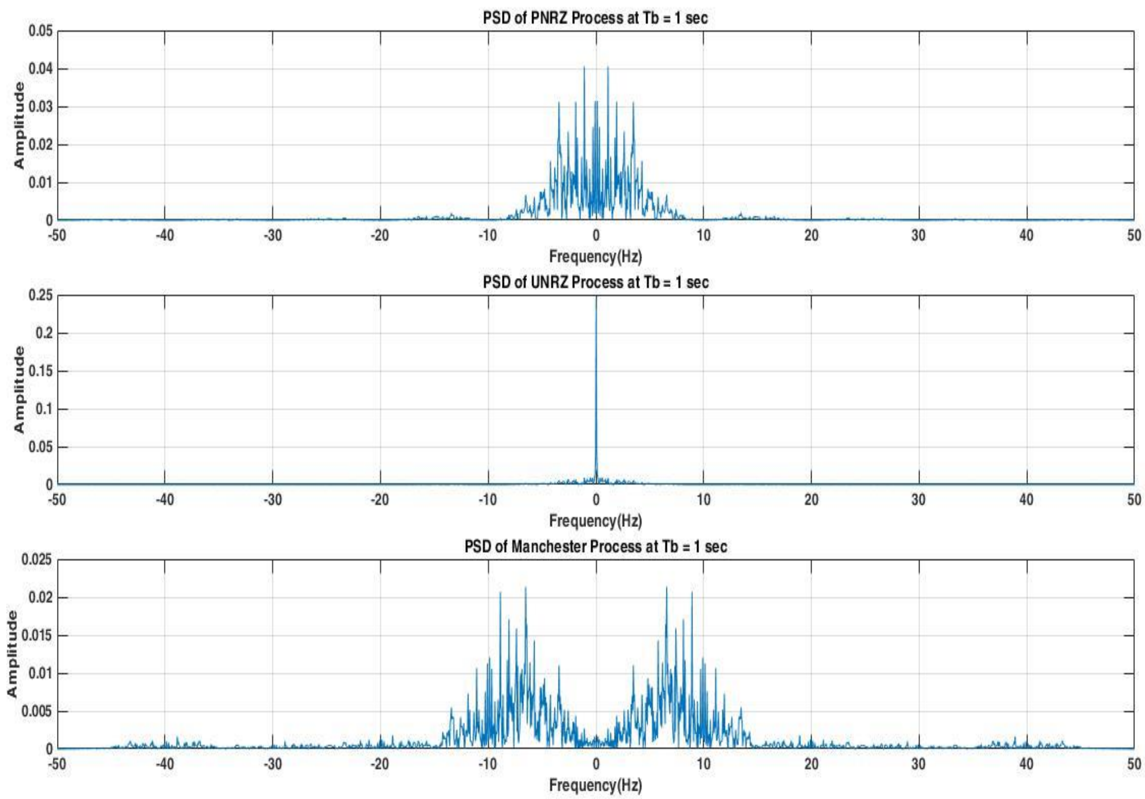
- ## Comment:

The figures of the statistical Auto-correlation function are accurate as they are similar to the theoretical figures, but with some noise that results from dealing with finite sample functions. The amount of existing noise is dependent upon the number of sample functions and also the size of the time vector or the sample function itself.

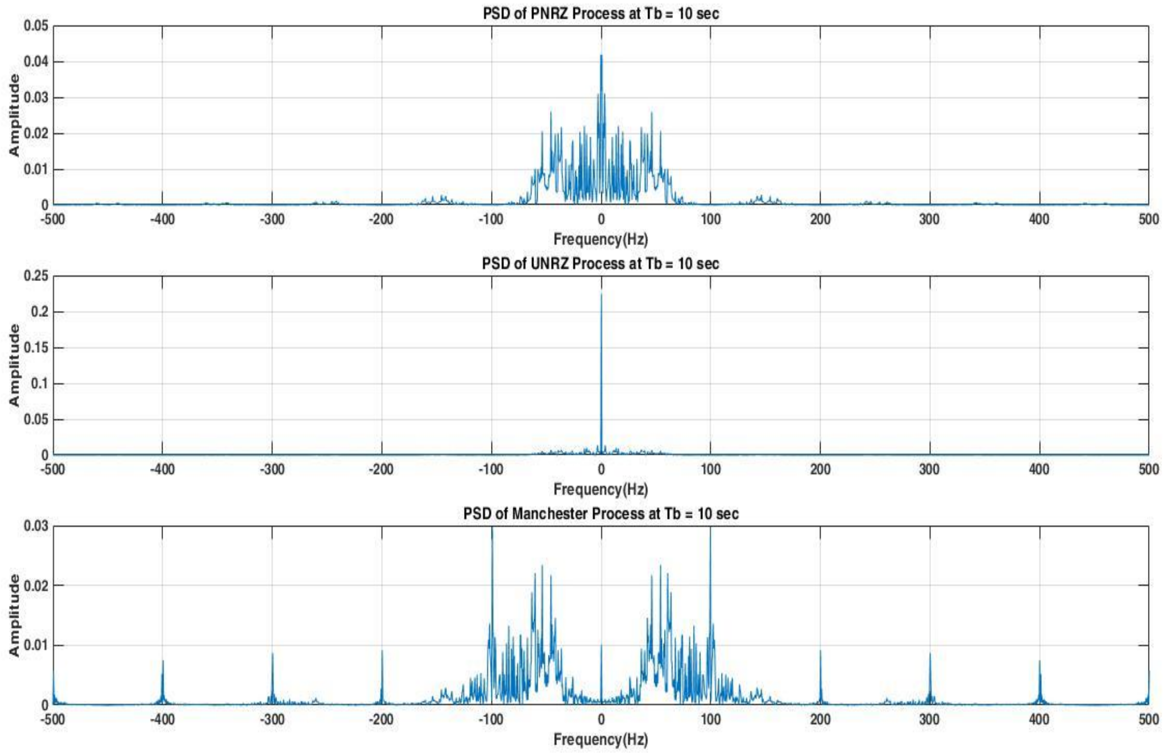
The (tri) function of Maximum Amplitude of ($A^2 = 1^2 = 1$) is defined on the period **[-Tb Tb]** for all the three cases (Tb = 1 sec, Tb = 10 sec, Tb = 100 sec) as stated in the theoretical analysis.

➢ Plot of the Power Spectral Density:

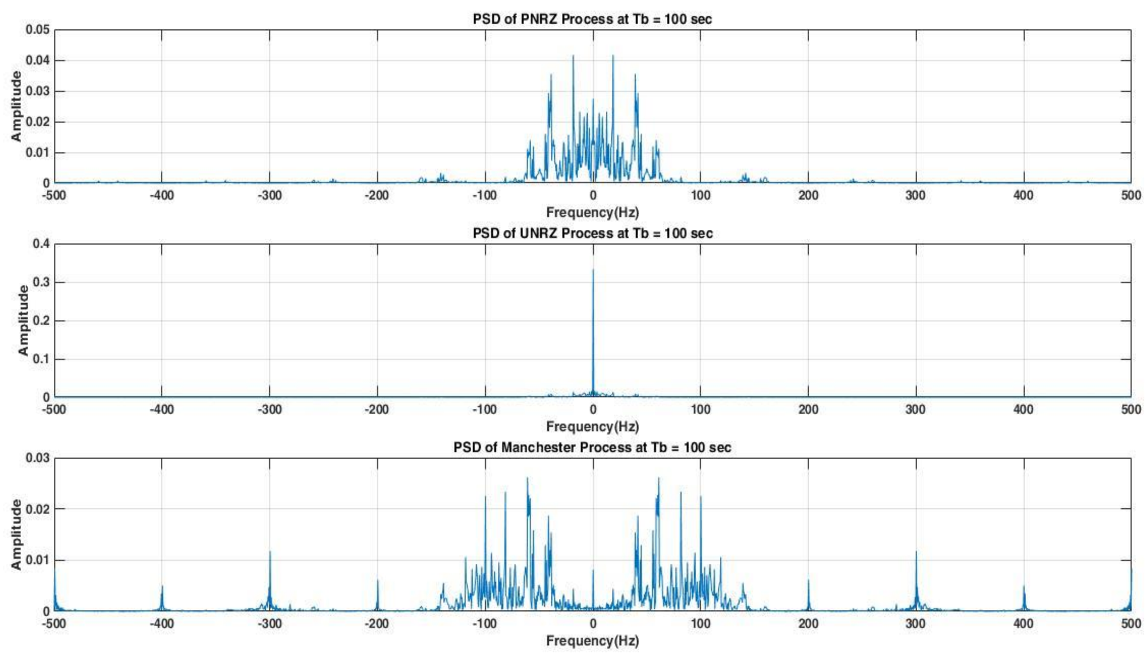
- ### Plot of PSD of PNRZ, UNRZ, MAN at Tb = 1 sec:



- **Plot of PSD of PNRZ, UNRZ, MAN at Tb = 10 sec:**



- **Plot of PSD of PNRZ, UNRZ, MAN at Tb = 100 sec:**



▪ Comment:

The figures of power spectral density of the three processes (PNRZ, UNRZ, Manchester) at the three different cases (Tb = 1 sec, Tb = 10 sec, Tb = 100 sec) are similar to the theoretical figures which are as follows:

- **PNRZ**

$$\boldsymbol{S(\omega) = A^2\, T_b\, sinc^2(\pi f T_b)}$$

- **UNRZ**

$$\boldsymbol{S(\omega) = \frac{1}{4} A^2\, T_b\, sinc^2(2\pi T_b) + \frac{1}{4} A^2 \delta(2\pi f)}$$

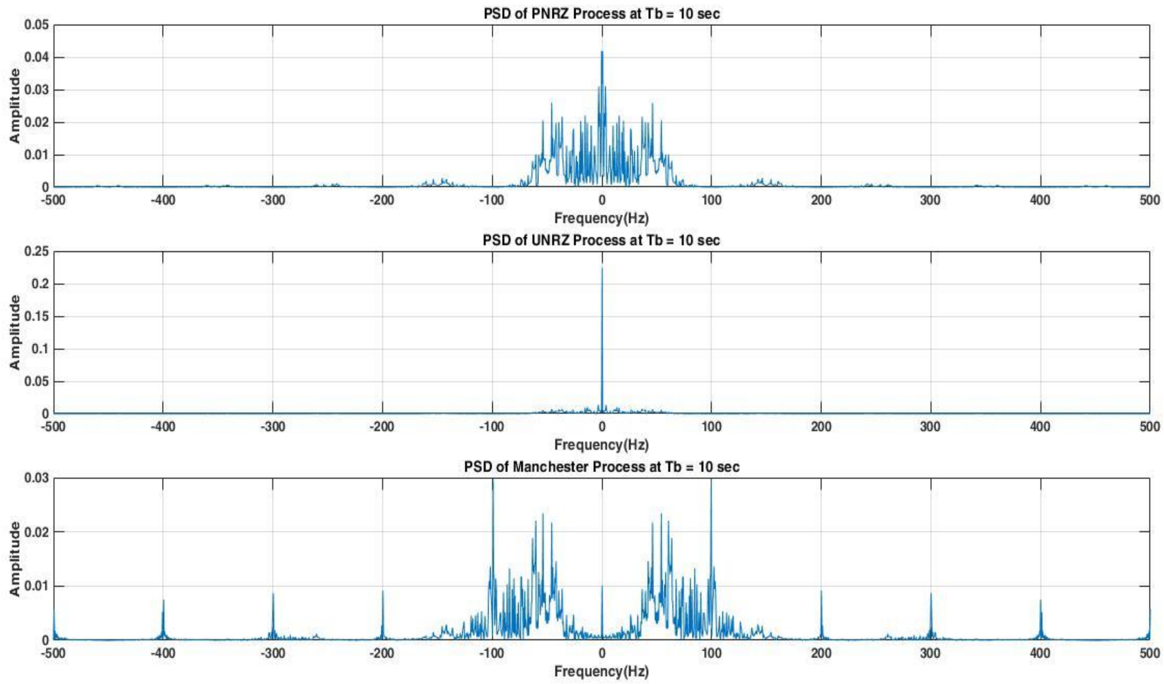
- **Manchester**

$$\boldsymbol{S(\omega) = A^2\, T_b\, sinc^2\left(\frac{f T_b}{2}\right) sin(\frac{\pi f T_b}{2})}$$

The power spectral density is dependent upon the value of the bit duration (Tb). This dependency is not shown in the figures above due to the following reason. In my code, the time step in the three cases is not the same. It was a must for me not to hold the time step to be the same as if I did so, I will not be able to run my code. To give more Clarification, I used a time step of (**0.1**) in the processes with (Tb = 1 sec), and hence each bit is represented by (**10 numbers**) in the vector of the signal. On the other hand, I used a time step of (**1**) in the processes with (Tb = 10 sec), and hence each bit is represented by (**10 numbers**) in the vector of the signal. For the processes with (Tb = 100 sec), I used a time step of **(10)** and hence each bit is represented by (**10 numbers**) in the vector of the signal. This was the reason behind having the shown amplitude in my figures.

Again, I had to do this as it I held the time step to be constant **(0.1)**, The length of the signal at **(Tb = 100)** will be 100000 and this would take hours to run on my Laptop.

- **The process with largest bandwidth at Tb = 10 sec:**



The process with the largest bandwidth at (Tb = 10 sec) is the **Manchester** Process Line Code. The Manchester Line Code has an average DC value of (0) as it spends half of its bit duration in the positive side, while the other half of bit duration is spent in the negative side. It also has twice the bandwidth as in the UNRZ or PNRZ line codes.

The following figure shows the PSD of each of the three Line codes.