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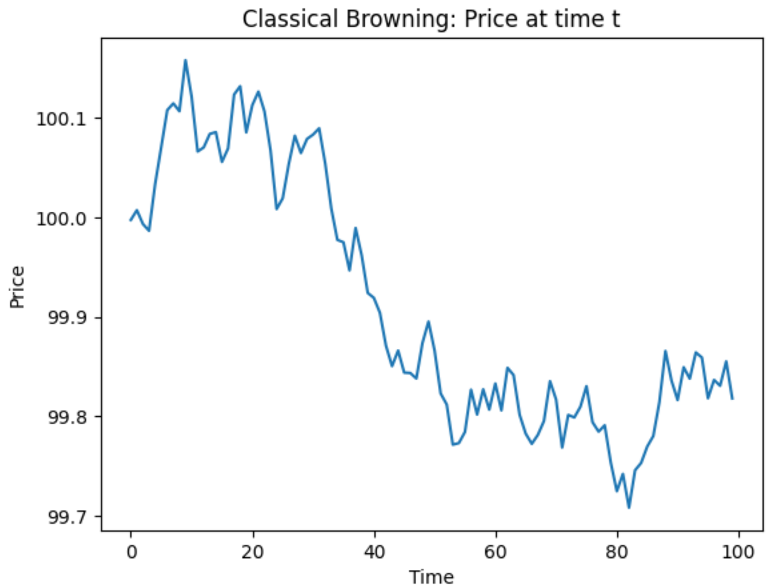
Week 04 Answers

Problem 1

a) Classical Brownian Motion



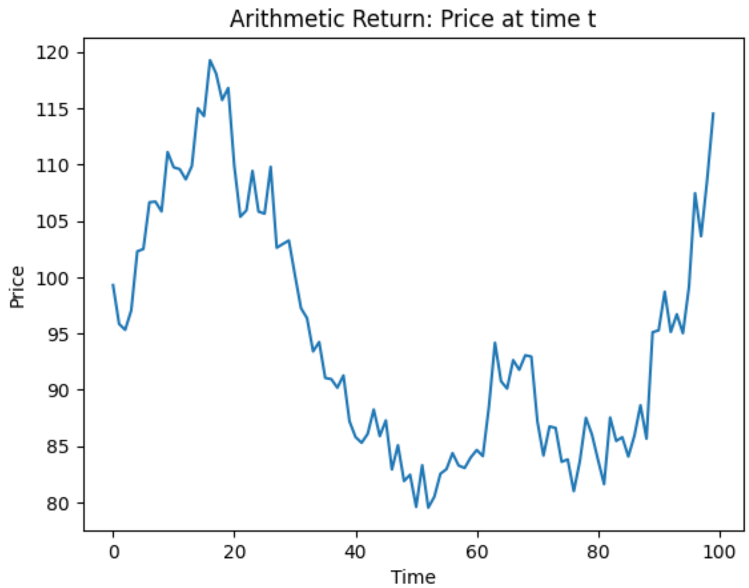
The assumption given was returns, rt is normally distributed with mean 0 and standard deviation of sigma. For this simulation problem, I set the initial price to 100, number of simulations to 1000, time period of 100 and standard deviation of 0.3. From the simulation, I found the expected value and standard deviation of p_t as :
Expected Value of P_t: 100.00510924547586
Standard Deviation of P_t: 0.3000607121422593

Here, since the mean of r is 0, the expected value of p_t will be the initial price, p_0, which is set to be 100. The theoretical standard deviation is calculated by the formula sigma * square root of time period. We found the theoretical values to be :

Theoretical Mean of P_t: 100
Theoretical Standard Deviation of P_t: 0.3

Comparing two sets of simulated values and theoretical values, we can see that the simulated values match with the theoretical mean and standard deviations very closely.

b) Arithmetic Return System



The mean for this simulation has also been set at 0, with standard deviation at 0.03, number of periods as 100, number of simulations at 1000, and p_0 at 100. Simulated mean and standard deviations values are as follows:

Expected Value of P_t: 100.28114699617191
Standard Deviation of P_t: 32.00549162589664

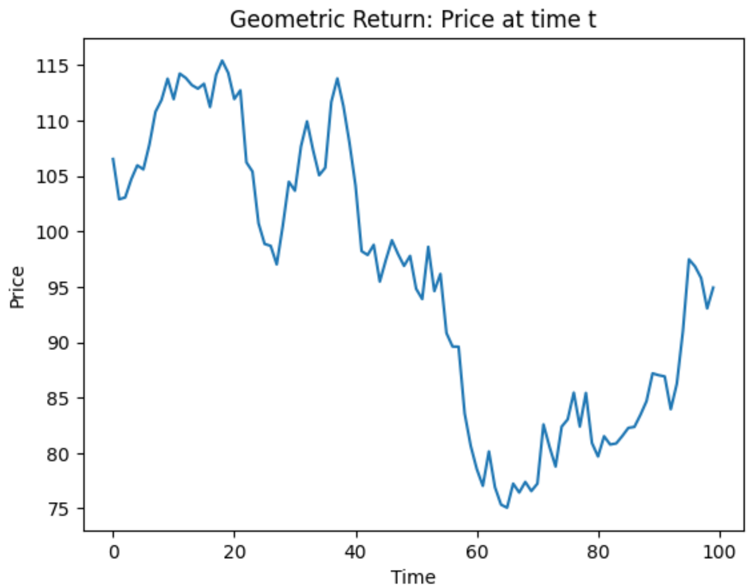
The arithmetic return system assumes that there is a constant mean return of mu. Here, we assumed that the mean return is 0. Therefore, the mean of p_t and all additional P_t, P_t+1 and so will be equal to the initial price of P_0, which is set $100. Therefore, the theoretical mean of the price at time t is $100.

However, the theoretic standard deviation for arithmetic return is calculated using the formula: p_0(initial price) * sigma (given standard deviation) * square root of time periods. We found the value to be :

Theoretical Standard Deviation of P_t: 30.0

Here also, we noticed that the simulated values closely match with the theoretical values for arithmetic return system.

c) Geometric Brownian Motion



The mean for this simulation has also been set at 0, with standard deviation at 0.03, number of periods as 100, number of simulations at 1000, and p_0 at 100. Simulated mean and standard deviations values are as follows:

Expected Value of P_t: 100.28114699617191

Standard Deviation of P_t: 31.902029036602144

The theoretical expected value of P_t is calculated using the formula p_0 times the exponent of (mu*T(, where mu is (sigma^2/2). The expected standard deviation is the product of p_0 and the square root of a function related to mu and sigma, shown below:

Theoretical Mean of P_t: 104.6027859908717
Theoretical Standard Deviation of P_t: 33.57783310629757

Although the theoretical values for geometric Brownian motions are more than the simulated values for those, however, the theoretical values are still somewhat close to the expected values found from the simulation.

```python
mu = (sigma**2)/2

theo_mean_3 = p_0 * np.exp(mu*t_period)
print(f"Theoretical Mean of P_t: {theo_mean_3}")

theo_sd_3 = p_0 * np.exp(2*mu*t_period) * np.sqrt(np.exp(sigma**2*t_period)-1)
print(f"Theoretical Standard Deviation of P_t: {theo_sd_3}")
```
✓ 0.0s Python

In terms of performance and accuracy, the Classical Brownian motion is the fasters and easiest to simulate, followed by Arithmetic Return System and Geometric Brownian Motion methods. They require additional calculations and complexity which provides a more accurate stock price predictions over long periods of time. As a result, while classical Brownian motion is simplest and fastest to model, it might not provide accurate estimates of stock prices compared to the other pricing methods.

Problem 2

Calculate VAR using the following ways and compare their values:

1. Calculate VaR using a normal distribution:

Calculating Value at Risk (VaR) using a normal distribution involves assuming that returns follow a normal distribution. The VaR is then determined by finding the z-score corresponding to the desired confidence level and multiplying it by the standard deviation of the returns. It is simple and widely used but might not be useful during extreme market conditions or high volatility. I calculated the VaR at 95% confidence interval using META and found the value to be -0.05418. This implies that there is a 5% chance that the portfolio could lose more than this value over the specified time period.

2. Calculate VaR using a normal distribution with an exponentially weighted variance (lambda = 0.94):

   This method is like the first one, except it uses an EWMA variance to estimate volatility instead of fixed historical standard deviation method. The EWMA method gives more weight to recent observations, reflecting the belief that recent data is more relevant for estimating future volatility. The estimate I found in my simulation -0.03014, suggesting the loss calculated in this method provides less risk than the normal distribution, reflecting a much accurate picture of risk assessment of our portfolio

3. Using a MLE fitted T distribution:

   Using a Maximum Likelihood Estimation (MLE) fitted t distribution for calculating Value at Risk (VaR) has its advantages ranging from being more robust to outliers, better at accommodating fat tails, incorporates degrees of freedom. These advantages make it a better choice when dealing with financial data that may have heavy tails or extreme events. The degrees of freedom allow for modelling of different levels of kurtosis. This gives more flexibility in modelling the underlying data.

   In short, using an MLE fitted t distribution for VaR calculation can be a useful approach, especially in situations where the data exhibits heavy tails and potential outliers. However, it's important to carefully select the degrees of freedom parameter and understand the assumptions and limitations of the t distribution. Additionally, VaR should be used in conjunction with other risk measures and stress testing for a comprehensive risk assessment.

   Using the MLE fitted T distribution, we found our VAR to be -0.10762.

4. Using a fitted AR(1) model

   using a fitted AR(1) model for VaR calculation can be a useful approach, especially for capturing short-term dependencies and mean reversion. However, it's important to carefully select the lag order, and understand the assumptions and limitations of the AR(1) model. Additionally, VaR should be used in conjunction with other risk measures and stress testing for a comprehensive risk assessment.
   In my simulation, my VaR using fitted AR(1) model was -0.05426

5. Using historic simulation

historical simulation is a straightforward and intuitive method for estimating VaR, especially when dealing with non-normal and complex market conditions. However, it should be used in conjunction with other risk measures and stress testing for a comprehensive risk assessment. Additionally, it is important to be aware of its limitations and potential sensitivities to the choice of historical data. The VaR using historic simulation was -0.03948.

List of VAR values:

VaR using normal distribution at 95% confidence level: -0.10762
VaR using normal distribution with EWM variance: -0.03014
VaR using MLE Fitted T distribution: -0.10762
VaR using fitted AR(1) model: -0.05426
VaR using a historic simulation: -0.03948

From our calculations, we can see that VAR using normal distribution with EWM variance has the highest value while the historic simulation provided the lowest VaR of -0.03948 at the same confidence interval of 95%

Problem 3:

For problem 3, I calculated the VaR for each portfolio and as well as the total VaR comprising of the total holdings using exponentially weighted covariance with lambda = 0.94 and also using fitted AR(1) model. The results for each methods are provided below:

Exponentially weighted covariance

Portfolio A VAR: $15189.50331
Portfolio B VAR: $7423.58090
Portfolio C VAR: $25128.04557
Total Portfolio VAR: $47576.11536

Fitted AR(1) model:

Portfolio A VAR: $611.37705
Portfolio B VAR: $344.45046
Portfolio C VAR: $776.32527
Total Portfolio VAR: $47576.11536

For the exponentially weighted covariance model, I calculated the stocks, holding and daily prices for each stock within each portfolio. Using those results, I calculated the daily returns for each portfolio, and also its exponentially weighted covariance using the provided lambda value. The VaR is then calculated using 95% confidence interval. The VaR is the produce of the z-score and the square root of the product of the portfolio value and the covariance matrix for that portfolio.

The AR(1) model provides a time series analysis and trend of the stock prices which is why I chose this method. Additionally, it provides a way to capture serial correlation, modelling mean reversion and its relatively easy interpretability.

The VaR values above show that the two models have provided different values for the VAR. This could arise due to various reasons in the way the covariance is calculated. Additionally, the exponentially weighted method might capture details that the fitted AR(1) does not and vice versa. For instance, since

EWMA uses more weights to recent returns, any recent volatility would lead to high VaR estimates using EWMA compared to fitted AR(1) model.