Ahmed Ibrahim
Hw 6, week 7

Problem 1

I implemented a function called **closed_form_GBSM** that takes in 6 inputs, i.e. (current stock price, Strike price, Time in decimals, coupon rate, implied volatility, option_type) and uses the Black Scholes Merton model to calculate the Greeks. Depending on the option type(call/put), the function then calculates delta, gamma, theta, vega and rho and returns the values.

Then, I implemented another function named **"option_price"** that uses Black schole's equation to calculate the option price at call or put option. The "**option_price"** function is then used inside the functon "**finite_diff"** to calculate the greeks using the finite difference derivative method. Using the option price function, we calculate the option's price using the BSM formula based on the shifted parameters. The resulting option prices are then used to calculate the finite difference approximations for the delta, gamma, vega, theta, and rho by computing their derivatives and the results are then returned as tuple.

Below, are the results from the two methods:

Compare two methods: Closed Form vs Finite Difference					
	Delta	Gamma	Theta	Vega	Rho
Call Closed Form	0.184413	0.019551	-21.048149	12.095776	2.399128
Put Closed Form	-0.815108	0.019551	-14.862618	12.095776	-12.461469
Call Finite Difference	0.184413	0.019551	-21.027531	12.091556	2.399178
Put Finite Difference	-1.077401	0.023543	26.119862	-11.775928	-15.700182

The following code implements a binomial tree valuation for American call and put options with and without discrete dividends.

The binomial tree method is a numerical approach for pricing options by modeling the possible future paths of the underlying asset's price. It considers the option's payoff at each node and works backward through the tree, evaluating the expected value and comparing it with the early exercise value to determine the optimal decision at each node. The adjustments for dividends involve subtracting the present value of dividends from the stock price. I implemented the code that calculates the option values at the initial node with and without dividends, considering the possibility of early exercise.

The final printed values represent the estimated prices of American call and put options under different scenarios (with and without dividends) based on the binomial tree model. Below are the results for call and put options for the underlying asset with and without dividends.

Call with dividend:	1.16
Put with dividend:	15.62
Call without dividend:	2.04
Put without dividend:	14.74

Following on, I implemented a function called **"binomial_tree"** that takes in 10 parameters, including the current stock price (`S`), strike price (`K`), time to maturity (`T`), risk-free rate (`r`), volatility (`sigma`), number of time steps (`n`), option type (`'call'` or `'put'`), exercise type (`'american'` or `'european'`), dividend date (`div_date`), and dividend amount (`div_amt`). It is then used to construct a binomial tree for the option price considering the specified parameters and adjusts stock prices for dividends if applicable. The function reutns a matrix representing the option prices at different nodes in the binomial tree.

Another function called "**binomial_tree_greeks"** uses the output of the binomial tree function to calculate option prices with perturbations in stock prices(i.e. shift in volatility and delta) and computes the greeks by observing the impact on these option prices. The results are displayed in two DataFrames (`call_greeks` and `put_greeks`), showing the calculated Greeks for call and put options with and without dividends.

Call Options:		
	Call without div	Call with div
Delta	0.187140	0.168456
Gamma	0.010242	0.014499
Theta	-21.275140	-21.133863
Vega	11.830702	11.892077
Rho	2.374598	2.176596
Put Options:		
	Put without div	Put with div
Delta	-0.829161	-0.847762
Gamma	0.012821	0.017393
Theta	-15.461561	-14.910113
Vega	11.095237	10.886199
Rho	-7.151379	-7.040558

Lastly, I calculated the sensitivity of the European put and call to a change in the dividend amount. Delta represents the rate of change of the option price concerning a change in the underlying asset price. In this case, it's specifically focused on how the option price changes in response to a change in the dividend amount. A positive Delta indicates that the option price increases with an increase in the dividend amount, and a negative Delta indicates that the option price decreases with an increase in the dividend amount. Overall, this analysis provides insights into how sensitive the option prices are to changes in dividend amounts, which is crucial information for option traders and investors making decisions based on their expectations of future dividend payments.

Please find the results of the delta call and put options below:

Delta Call with Dividend	-0.18440240717524875
Delta Put with Dividend:	0.8150604768492478

Problem 2:

For this problem, I begin by extracting the daily closing prices of AAPL and computing the corresponding daily returns. A normal distribution is then fitted to these returns, on the basis of mean of zero and a standard deviation derived from the returns.

Following this, I conduct simulations for the next 10 days of AAPL returns by drawing random samples from the previously fitted normal distribution. The number of simulations and days can was set to 100 simulations and 10 days, respectively. These simulated returns serve as the basis for calculating the simulated prices for the next 10 days, starting from the initial price of $151.03.

Subsequently, I focus on the simulated prices for the 10th day, storing them in a list named `future_prices`. This collection of simulated prices provides valuable insights into estimating the probability distribution of future stock prices, aiding in making well-informed investment decisions.

To assess the risk associated with various portfolios, I utilize functions such as `black_scholes`, `implied_volatility`, and `portfolio_value`, which were defined earlier. Iterating through each portfolio, I calculate key metrics such as the Mean, Value at Risk (VaR), and Expected Shortfall (ES). This comprehensive analysis enables a thorough understanding of the potential outcomes and risks associated with the considered investment portfolios. Please find the results below:
I was not able to calculate AR(1) in the previous assignment, and therefore did not have the results for comparison.

	Mean	VaR	ES
Straddle	3.122391	0.005287	0.016168
SynLong	-0.033441	18.86814	21.791675
CallSpread	0.07758	3.660869	3.93076
PutSpread	0.453594	2.726307	2.862628
Stock	0.163082	18.091523	20.847488
Call	1.544475	5.755278	6.073702
Put	1.577916	4.446387	4.641181
CoveredCall	-1.433026	14.410403	17.03144
ProtectedPut	1.569352	7.508777	7.862674

Problem 3:

For this problem, I implemented a code that performs a financial optimization task to find the weight of a portfolio for a selected set of stocks based on the Fama-french 3 factor return time series and Carhart Momentum time series. The objective is to construct a superefficient portfolio that maximizes the Sharpe ratio, a measure of risk adjusted return.

I begin by reading the two csv files containing financial factor data (F_Research_Data_Factors_daily.csv` and `F-F_Momentum_Factor_daily.csv`). The data frames are joined based on the 'Date' column, providing a consolidated dataset. The data is filtered to include only the last 10 years. The values in the resulting Data Frame, `factors`, are normalized by dividing by 100. Synthetic stock returns are generated for the given set of stocks and the returns are calculated based on a normal distribution with a mean of 0 and a standard deviation of 0.01. The resulting DataFrame, `stock_returns`, has columns representing the returns for each stock. Following on, a 4-factor model is fitted for each stock using Ordinary Least Squares (OLS) regression. The factors considered are 'Mkt-RF', 'SMB', 'HML', and 'Mom' (momentum). The annual expected returns for each stock are calculated based on the fitted model. The annual covariance matrix is calculated based on the synthetic stock returns, scaled by 252 (the number of trading days in a year) and is displayed in the appendix. The resulting weights of the super-efficient portfolio are printed, indicating the optimal allocation of assets to maximize the Sharpe ratio.

In summary, the code integrates historical financial factor data, generates synthetic stock returns, fits a model to estimate expected returns, calculates the covariance matrix, and optimizes the portfolio weights to construct a super-efficient portfolio that maximizes the Sharpe ratio.

Super efficient Portfolio Weights	
AAPL	-0.0000
FB	0.0156
UNH	0.0725
MA	0.0000
MSFT	-0.0000
NVDA	0.0274
HD	-0.0000
PFE	0.0045
AMZN	0.1689
BRK-B	-0.0000
PG	0.1419
XOM	0.0029
TSLA	0.0448
JPM	-0.0000
V	0.0675

DIS	0.1110
GOOGL	0.1546
JNJ	0.0000
BAC	0.0158
CSCO	0.1727

Appendix:

Problem 1

```
Compare two methods: Closed Form vs Finite Difference
                            Delta     Gamma      Theta       Vega         Rho
Call Closed Form         0.184413  0.019551 -21.048149  12.095776   2.399128
Put Closed Form         -0.815108  0.019551 -14.862618  12.095776 -12.461469
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```

```
Call with dividend: 1.16
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```
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Theta        -21.275140     -21.133863
Vega          11.830702      11.892077
Rho            2.374598       2.176596

Put Options:
       Put without div  Put with div
Delta        -0.829161     -0.847762
Gamma         0.012821      0.017393
Theta       -15.461561    -14.910113
Vega         11.095237     10.886199
Rho          -7.151379     -7.040558
```

```
Delta Call with Dividend:  -0.18440240717524875
Delta Put with Dividend:  0.8150604768492478
```

Problem 2

	Mean	VaR	ES
Straddle	3.122391	0.005287	0.016168
SynLong	-0.033441	18.86814	21.791675
CallSpread	0.07758	3.660869	3.93076
PutSpread	0.453594	2.726307	2.862628
Stock	0.163082	18.091523	20.847488
Call	1.544475	5.755278	6.073702
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Problem 3:

```
Super-efficient Portfolio Weights:
 AAPL    -0.0000
FB        0.0156
UNH       0.0725
MA        0.0000
MSFT     -0.0000
NVDA      0.0274
HD       -0.0000
PFE       0.0045
AMZN      0.1689
BRK-B    -0.0000
PG        0.1419
XOM       0.0029
TSLA      0.0448
JPM      -0.0000
V         0.0675
DIS       0.1110
GOOGL     0.1546
JNJ       0.0000
BAC       0.0158
CSCO      0.1727
dtype: float64
```

```
Annual Covariance Matrix:
               AAPL        FB       UNH        MA      MSFT          NVDA  \
AAPL   2.540198e-02  0.000909  0.000237  0.001332  0.000540  4.736077e-07
FB     9.090429e-04  0.024775  0.000776  0.000375  0.000584  6.120711e-04
UNH    2.373840e-04  0.000776  0.025639 -0.000446  0.000447 -6.105530e-04
MA     1.332454e-03  0.000375 -0.000446  0.025799 -0.000400  2.469505e-04
MSFT   5.397247e-04  0.000584  0.000447 -0.000400  0.025557 -2.414327e-04
NVDA   4.736077e-07  0.000612 -0.000611  0.000247 -0.000241  2.423939e-02
HD    -9.694325e-05  0.001028 -0.000109  0.000151 -0.000291 -2.038146e-04
PFE   -9.158532e-04 -0.000413 -0.000580  0.000541  0.000164  8.012190e-05
AMZN   2.634226e-04  0.000142  0.000759  0.000449 -0.000470 -6.729659e-05
BRK-B  7.182287e-04  0.000180 -0.000340 -0.000812  0.000665  1.399480e-04
PG    -5.012590e-04 -0.000119 -0.000547 -0.000631 -0.000431  7.090678e-04
XOM   -2.328060e-04  0.000307  0.000953  0.000026 -0.000165 -1.052433e-03
TSLA  -3.894219e-04 -0.000231 -0.000177 -0.000574  0.000043 -6.294013e-04
JPM   -2.541714e-04  0.000399  0.000371 -0.000475  0.000326 -6.407462e-05
V      7.846951e-04  0.000829 -0.000048 -0.000541 -0.000085 -4.094981e-04
DIS    2.611921e-04  0.000008 -0.000923  0.000583  0.000795 -3.020400e-04
GOOGL -1.434067e-04 -0.000589  0.000179 -0.000788 -0.000622 -5.953800e-04
JNJ   -5.066375e-04  0.000428 -0.001149 -0.000114 -0.000305 -1.625184e-04
BAC   -2.475413e-04 -0.000070  0.000820 -0.000202  0.000510  4.214579e-04
CSCO   3.627356e-04  0.000326 -0.000061  0.000381 -0.000544  2.246743e-04
```

```
             HD           PFE          AMZN     BRK-B        PG       XOM  \
AAPL  -0.000097 -9.158532e-04  2.634226e-04  0.000718 -0.000501 -0.000233
FB     0.001028 -4.134428e-04  1.421927e-04  0.000180 -0.000119  0.000307
UNH   -0.000109 -5.795082e-04  7.585450e-04 -0.000340 -0.000547  0.000953
MA     0.000151  5.411395e-04  4.491569e-04 -0.000812 -0.000631  0.000026
MSFT  -0.000291  1.644540e-04 -4.702920e-04  0.000665 -0.000431 -0.000165
NVDA  -0.000204  8.012190e-05 -6.729659e-05  0.000140  0.000709 -0.001052
HD     0.025861 -9.392723e-05  8.607554e-04 -0.000015  0.000138 -0.000984
PFE   -0.000094  2.491332e-02  7.525784e-07 -0.000362 -0.000476  0.000022
AMZN   0.000861  7.525784e-07  2.564555e-02  0.000845 -0.000158 -0.000935
BRK-B -0.000015 -3.623885e-04  8.454961e-04  0.026331  0.000239  0.000523
PG     0.000138 -4.762005e-04 -1.583136e-04  0.000239  0.025035 -0.000233
XOM   -0.000984  2.211149e-05 -9.354254e-04  0.000523 -0.000233  0.024162
TSLA  -0.000023 -7.275685e-04 -7.758376e-04  0.000409  0.000995 -0.000205
JPM   -0.000366 -3.175123e-05 -4.817856e-04 -0.000845  0.000040 -0.000737
V     -0.000899  3.581677e-04 -7.859876e-04  0.001225 -0.000764 -0.000575
DIS   -0.000027 -3.734288e-04 -6.293669e-04  0.000332 -0.000077  0.000961
GOOGL -0.000244 -3.891371e-04  1.213202e-04 -0.000240  0.000259  0.000263
JNJ   -0.000291 -3.484020e-04  1.909777e-04  0.001040  0.000103 -0.000413
BAC   -0.000252  2.554970e-04 -2.803908e-05 -0.001027  0.001062 -0.000689
CSCO  -0.000596  4.489008e-04  3.007267e-05 -0.000252 -0.000683 -0.000484
```

```
            TSLA       JPM         V       DIS     GOOGL       JNJ       BAC  \
AAPL  -0.000389 -0.000254  0.000785  0.000261 -0.000143 -0.000507 -0.000248
FB    -0.000231  0.000399  0.000829  0.000008 -0.000589  0.000428 -0.000070
UNH   -0.000177  0.000371 -0.000048 -0.000923  0.000179 -0.001149  0.000820
MA    -0.000574 -0.000475 -0.000541  0.000583 -0.000788 -0.000114 -0.000202
MSFT   0.000043  0.000326 -0.000085  0.000795 -0.000622 -0.000305  0.000510
NVDA  -0.000629 -0.000064 -0.000409 -0.000302 -0.000595 -0.000163  0.000421
HD    -0.000023 -0.000366 -0.000899 -0.000027 -0.000244 -0.000291 -0.000252
PFE   -0.000728 -0.000032  0.000358 -0.000373 -0.000389 -0.000348  0.000255
AMZN  -0.000776 -0.000482 -0.000786 -0.000629  0.000121  0.000191 -0.000028
BRK-B  0.000409 -0.000845  0.001225  0.000332 -0.000240  0.001040 -0.001027
PG     0.000995  0.000040 -0.000764 -0.000077  0.000259  0.000103  0.001062
XOM   -0.000205 -0.000737 -0.000575  0.000961  0.000263 -0.000413 -0.000689
TSLA   0.025716  0.000470 -0.000009 -0.000407 -0.000030 -0.000223  0.000669
JPM    0.000470  0.026742  0.000708 -0.000496 -0.000591  0.000618  0.000070
V     -0.000009  0.000708  0.025084 -0.000008  0.000312  0.000424 -0.000605
DIS   -0.000407 -0.000496 -0.000008  0.025306 -0.000059 -0.000572 -0.000300
GOOGL -0.000030 -0.000591  0.000312 -0.000059  0.025792 -0.000627 -0.000458
JNJ   -0.000223  0.000618  0.000424 -0.000572 -0.000627  0.026044 -0.000095
BAC    0.000669  0.000070 -0.000605 -0.000300 -0.000458 -0.000095  0.025486
CSCO  -0.000767 -0.000578 -0.000398 -0.000204 -0.000281  0.000623  0.000099
```

```
            CSCO
AAPL    0.000363
FB      0.000326
UNH    -0.000061
MA      0.000381
MSFT   -0.000544
NVDA    0.000225
HD     -0.000596
PFE     0.000449
AMZN    0.000030
BRK-B  -0.000252
PG     -0.000683
XOM    -0.000484
TSLA   -0.000767
JPM    -0.000578
V      -0.000398
DIS    -0.000204
GOOGL  -0.000281
JNJ     0.000623
BAC     0.000099
CSCO    0.026171
```