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Smooth Extensions of Feedback Motion Planners via Reference Governors

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Abstract— **In robotics, it is often practically and theoretically convenient to design motion planners for approximate low-order (e.g., position- or velocity-controlled) robot models first, and then adapt such reference planners to more accurate high-order (e.g., force/torque-controlled) robot models. In this paper, we introduce a novel provably correct approach to extend the applicability of low-order feedback motion planners to high-order robot models, while retaining stability and collision avoidance properties, as well as enforcing additional constraints that are specific to the high-order models. Our smooth extension framework leverages the idea of reference governors to separate the issues of stability and constraint satisfaction, affording a bidirectionally coupled robot-governor system where the robot ensures stability with respect to the governor and the governor enforces state (e.g., collision avoidance) and control (e.g., actuator limits) constraints. We demonstrate example applications of our framework for augmenting path planners and vector field planners to the second-order robot dynamics.**

I. Introduction

A long-standing open challenge of robotics is the design of provably correct computationally efficient feedback motion planners that can simultaneously handle kinematic (e.g., collision avoidance) and dynamics (e.g., velocity and acceleration saturation) constraints, and guarantee global navigation, if possible [1]. The traditional, theoretically sound, and practically feasible approach partially addresses this problem in two steps: first design a motion plan for an approximate low-order robot model, and then extend this reference plan to a more accurate high-order robot model [2]. It is the latter that motivates the present paper. Given a feedback motion planner that solves the collision-free global navigation problem for a low-order (e.g., position- or velocity-controlled) system model, we propose a new provably correct computationally efficient approach to extend the given reference planner to a high-order (e.g., force-controlled) system model, while maintaining stability and invariance properties.

A. Motivation and Prior Literature

That the "natural motion" of dissipative mechanical systems causes the system energy to decay — an observation made by Lord Kelvin [3] — motivates the idea of programming reference dynamics in mechanical systems using total energy [2], [4], [5], which is summarized in details in Section II-A. In motion planning, the negated gradient field of navigation functions that solves the collision-free (almost) global navigation problem for the first-order (fully-actuated single-integrator) robot model are further extended to the second-order (fully-actuated double-integrator) robot dynamics, using the total energy of Lagrangian systems with dissipative external forces, while retaining global convergence and collision avoidance guarantees [6]. In general, smooth extensions of rather generic vector fields, with known Lyapunov functions, can be constructed using total energy, but the resulting policies can only ensure local stability and offer no assurance of collision avoidance [7]. In [8], instead of total energy, a similar approach based on angular momentum is utilized to design a locally stable reorientation controller for a second-order tailed biped robot that tracks reference dynamics constructed for a simplified kinematic model.

Although the limit behavior of its gradient field can be exactly embedded in the second-order robot dynamics with guaranteed collision avoidance, finding or constructing a navigation function for an arbitrary environment is known to be very hard. This issue of navigation functions is mitigated in [9] by using sequential composition [10] of smooth extensions of local feedback rules. Approximation simulation [11] is another smooth extension method that aims to keep the spatial distance between the low-order and the high-order models bounded, but is too restrictive and computationally costly. Also, backstepping is applied to extend only stability properties of kinematic unicycles to dynamic unicycles [12].

B. Contributions and Organization of the Paper

This paper proposes a new approach to extend stability and invariance properties of low-order feedback motion planners to high-order robot models in a provably correct and computationally efficient way. Like the total energy based smooth extensions [4], our construction uses sublevel sets of total energy to ensure stability and to guard against collisions, but, instead of simultaneously tackling stability and collision avoidance requirements, it separates the problems of stability and constraint enforcement via reference governors [13]. We introduce a new concept of a bidirectionally coupled robot-governor system, where the second-order robot asymptotically chases the governor irrespective of state and control constraints and the first-order governor enforces constraint satisfaction while following the flow of the reference dynamics as closely as possible. A significant property of our smooth extension framework is that it only requires the local knowledge of the environment.

This paper is organized as follows. Section II presents a formal statement of the smooth extension problem, and provides an overview of total energy based smooth extensions of gradient fields and reference governors. Section III,

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comprising the central contribution of the paper, constructs and analyzes our reference governor based smooth extension framework. Section IV presents example applications of the proposed framework for augmenting path and vector field planners to the second-order robot. Section V concludes with a summary of our contributions and future work.

II. Problem Formulation

For ease of exposition, we consider a disk-shaped robot, of radius $r \in \mathbb{R}_{>0}$ centered at $\mathrm{x} \in \mathcal{W}$, moving in a closed compact environment $\mathcal{W}$ in the n-dimensional Euclidean space $\mathbb{R}^n$, where $n \geq 2$, possibly punctured with $m \in \mathbb{N}$ open sets $\mathcal{O} := \{O_1, O_2, \ldots, O_m\}$, representing obstacles. Therefore, the free space $\mathcal{F}$ of the robot is given by

$$\mathcal{F} := \left\{ \mathrm{x} \in \mathcal{W} \,\middle|\, B(\mathrm{x}, r) \subseteq \mathcal{W} \setminus \bigcup_{i=1}^{m} O_i \right\}, \qquad (1)$$

where $B(\mathrm{x}, r) := \left\{ \mathrm{q} \in \mathbb{R}^n \middle| \, \|\mathrm{q} - \mathrm{x}\| \leq r \right\}$ is the closed ball centered at x with radius r, and $\|.\|$ denotes the standard Euclidean norm. In this paper, we assume that the free space is path-connected to ensure that global navigation is possible.

Smooth Extensions of Vector Field Planners. *Suppose* $\mathrm{v} : \mathcal{F} \rightarrow \mathbb{R}^n$ *is a Lipschitz continuous vector field planner for the first-order (fully-actuated single-integrator) robot dynamics,*

$$\dot{\mathrm{x}} = \mathrm{v}(\mathrm{x}), \qquad (2)$$

that leaves the robot's free space $\mathcal{F}$ *positively invariant and asymptotically steers almost all robot configurations*[1] *in* $\mathcal{F}$ *to any given goal location* $\mathrm{x}^* \in \mathcal{F}$.

A smooth extension of vector field v *is a construction of a Lipschitz continuous vector field planner* $\mathrm{u}_{\mathrm{v}} : \mathcal{F} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ *that embeds (the limit behavior of)* v *in the the second-order (fully-actuated double-integrator) robot dynamics,*

$$\ddot{\mathrm{x}} = \mathrm{u}_{\mathrm{v}}(\mathrm{x}, \dot{\mathrm{x}}), \qquad (3)$$

such that u_{v} *asymptotically steers almost all zero velocity*[2] *initial configurations*[3] $\mathcal{F} \times \{\mathbf{0}\}$ *to the goal location* x^* *while avoiding collisions along the way.*

In brief, smooth extensions of low-order vector field planners to a high-order dynamical system aims to augment the stability and invariance properties to the high-order system.

A. Smooth Extensions of Gradient Dynamics via Total Energy

A standard example of smooth extensions of dynamical systems is the embedding of an artificial potential field, that is constructed as the negated gradient of a scalar valued function, in second-order dynamics using the total energy of the system [2], [4]. More precisely, let $V : \mathcal{F} \rightarrow \mathbb{R}$ be an artificial potential function that is

i) twice differentiable on $\mathcal{F}$,
ii) polar at x^*, i.e., has a unique local minimum at x^*,
iii) is a Morse function [16], i.e., has no degenerate critical points,
iv) is admissible [16] on $\mathcal{F}$, i.e., takes its maximum value uniformly on the boundary $\partial\mathcal{F}$ of $\mathcal{F}$.

Such a potential function is referred to as a *navigation function* [6], [17], because its negated gradient field

$$\dot{\mathrm{x}} = -\nabla V(\mathrm{x}), \qquad (4)$$

is asymptotically stable at x^* whose domain of attraction includes all $\mathcal{F}$, possibly excluding a set of measure zero [1].

A natural way of embedding such first-order gradient dynamics in second-order robot dynamics is via the total energy of Lagrangian systems [2], [4]. For instance, define the total energy and the Lagrangian of the robot, resp., to be

$$E(\mathrm{x}, \dot{\mathrm{x}}) := T(\dot{\mathrm{x}}) + V(\mathrm{x}), \qquad (5)$$

$$L(\mathrm{x}, \dot{\mathrm{x}}) := T(\dot{\mathrm{x}}) - V(\mathrm{x}), \qquad (6)$$

where $T(\dot{\mathrm{x}}) := \frac{1}{2}\|\dot{\mathrm{x}}\|^2$ and $V(\mathrm{x})$ are the robot's kinetic and potential energies, respectively. If the robot obeys Lagrangian dynamics with no external input,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L(\mathrm{x}, \dot{\mathrm{x}})}{\partial \dot{\mathrm{x}}}\right) - \frac{\partial L(\mathrm{x}, \dot{\mathrm{x}})}{\partial \mathrm{x}} = 0 \implies \ddot{\mathrm{x}} = -\nabla V(\mathrm{x}), \qquad (7)$$

then one can readily verify that the total energy E is preserved during the motion of the robot, i.e., $\dot{E}(\mathrm{x}, \dot{\mathrm{x}}) = 0$ [18]. Moreover, in the presence of a dissipative external input, we know from Lord Kelvin [3] that such Lagrangian dynamics decay toward and stabilize at a local minimum of E [4]. Hence, the following smooth extension of the gradient field

$$\ddot{\mathrm{x}} = -\nabla V(\mathrm{x}) - \zeta\dot{\mathrm{x}}, \qquad (8)$$

solves the collision-free navigation problem for all zero velocity initial configurations of the second-order robot model, because $\dot{E}(\mathrm{x}, \dot{\mathrm{x}}) = -\zeta\|\dot{\mathrm{x}}\|^2 \leq 0$, where $\zeta > 0$ is a fixed artificial damping coefficient [4].

Finally, we find it useful to emphasize that the reason why a smooth extension of an artificial potential field inherits the invariance (i.e., collision avoidance) properties of the original gradient dynamics is the admissibility property of the potential function. In general, one can ensure the positive invariance of the free space for the first-order robot dynamics by having an inward-pointing vector field along the boundary of the free space, and a similar approach can be used to embed such a rather general (perhaps non-gradient) vector field planner, with a known Lyapunov function, in second-order systems. However, such an embedding only guarantees the local stability of the system around the goal location, and generally cannot ensure the invariance of the free space (i.e, collision avoidance) [7]. Fortunately, control theory offers a simple, yet practical approach to augment locally-stable feedback motion planners for enforcing desired state and control constraints via *reference governors*.

[1]It is known both in topology [14] and dynamical systems theory [15] that a continuous global motion planner in a configuration space X exists if and only if X is contractible. Since the free space of a robotic system is generally non-contractible, the domain of a continuous navigation planner must exclude at least a set of measure zero.

[2]This requirement on initial configurations can be relaxed to include configurations that can be brought to a halt before colliding with an obstacle or configurations with bounded total energy relative to the free space boundary, as in Definition 3 and Proposition 1, respectively.

[3]Here, $\mathbf{0}$ is a vector of all zeros with the appropriate size.

B. Reference Governors

Reference governors are add-on control schemes for closed-loop dynamical systems to enforce pointwise-in-time state and control constraints while maintaining stability properties [13], [19]–[21]. The fundamental idea of reference governors is based on the separation of the issues of stability and constraint satisfaction. Given a closed-loop system that performs satisfactorily in the absence of constraints, a reference governor modifies the desired reference command, whenever necessary, to the closed-loop system in order to avoid constraint violation for all future time while ensuring system stability. To demonstrate an application of reference governors in motion planning, we now present a reinterpretation of our recently introduced provably correct reactive robot navigation algorithm [22], for a first-order disk-shaped robot operating in a "sphere world" [6], in the reference governor framework.[4] Later in Section IV-B, we shall also provide its reference governor based smooth extension.

In [22], we consider the collision-free navigation problem of a velocity-controlled disk-shaped robot, centered at $\mathrm{x} \in \mathcal{W}$ with radius $r \in \mathbb{R}$, in a closed compact convex environment $\mathcal{W} \in \mathbb{R}^n$ populated with $m \in \mathbb{N}$ open disk-shaped obstacles, centered at $\mathbf{p} := (\mathrm{p}_1, \mathrm{p}_2, \ldots, \mathrm{p}_m) \in \mathcal{W}^m$ with a tuple of positive radii $\boldsymbol{\rho} := (\rho_1, \rho_2, \ldots, \rho_m) \in (\mathbb{R}_{>0})^n$.

Using the power diagram — a generalized Voronoi diagram [25] — of the robot's workspace $\mathcal{W}$, generated by disks representing the robot and obstacles, in [22] we define the robot's local workspace and local free space, respectively, as

$$\mathcal{LW}(\mathrm{x}) := \left\{ \mathrm{q} \in \mathcal{W} \,\middle|\, \|\mathrm{q}-\mathrm{x}\|^2 - r^2 \leq \|\mathrm{q}-\mathrm{p}_i\|^2 - \rho_i^2 \quad \forall i \right\}, \tag{9}$$

$$\mathcal{LF}(\mathrm{x}) := \left\{ \mathrm{q} \in \mathcal{LW}(\mathrm{x}) \,\middle|\, B(\mathrm{q}, r) \subseteq \mathcal{LW}(\mathrm{x}) \right\}. \tag{10}$$

Note that $\mathcal{LW}(\mathrm{x})$ and $\mathcal{LF}(\mathrm{x})$ are both nonempty closed convex sets for any $\mathrm{x} \in \mathcal{F}$, and we have $\mathcal{LF}(\mathrm{x}) \subseteq \mathcal{F}$ [22].

Accordingly, for the fully-actuated single-integrator robot dynamics in (2), we propose in [22] a simple reactive navigation strategy, called "move-to-projected-goal" law, $\mathrm{v} : \mathcal{F} \to \mathbb{R}^n$ that drives the robot at location $\mathrm{x} \in \mathcal{F}$ toward a designated global goal $\mathrm{x}^* \in \mathcal{F}$ through a safe local goal, $\overline{\mathrm{x}}^* := \Pi_{\mathcal{LF}(\mathrm{x})}(\mathrm{x}^*)$, called "projected-goal", as follows:

$$\mathrm{v}(\mathrm{x}) = -\frac{k}{2} \nabla_{\mathrm{x}} \|\mathrm{x} - \overline{\mathrm{x}}^*\|^2 \Big|_{\overline{\mathrm{x}}^* \text{is fixed}} = -k(\mathrm{x} - \overline{\mathrm{x}}^*), \tag{11}$$

where $\Pi_A(\mathrm{q}) := \arg\min_{\mathrm{a} \in A} \|\mathrm{a}-\mathrm{q}\|$ is the *metric projection* of $\mathrm{q} \in \mathbb{R}^n$ onto a close convex set $A \subseteq \mathbb{R}^n$, and $k > 0$ is a fixed control gain.

In brief, the metric projection here plays the role of a reference governor, as depicted in Fig. 1, that continuously modifies the target position to the closed-loop motion planner to avoid collisions while preserving the system stability. Note that, the closed-loop motion planner is (a positive constant times) the negated gradient of the squared Euclidean distance

[4]In [23] we present a further extension of [22] to respect sensory limits, and in [24] we design a collision-free coverage control algorithm for heterogeneous disk-shaped multiple robots. Both of these studies have a similar interpretation in the reference governor framework.

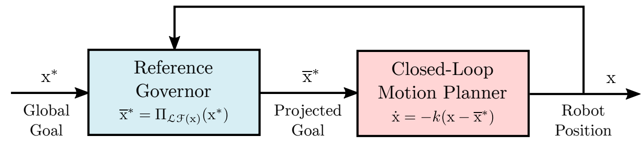

Fig. 1. A reference governor interpretation of the "move-to-projected goal" law of [22], where the reference governor ensures collision avoidance and the feedback motion planner guarantees stability.

to the (projected) goal, and, in the absence of obstacles, it is known to be stable around any given fixed goal location [26]. Lastly, we find it useful to summarize some important qualitative properties of the "move-to-projected-goal" law as:

Theorem 1 ([22]) *If obstacles are separated*[5] *from each other by clearance of at least* $\|\mathrm{p}_i - \mathrm{p}_j\| > (\rho_i + \rho_j + 2r)$ *for all* $i \neq j$, *and from the boundary* $\partial\mathcal{W}$ *of the workspace* $\mathcal{W}$ *as* $\min_{\mathrm{q} \in \partial\mathcal{W}} \|\mathrm{q} - \mathrm{p}_i\| > (\rho_i + 2r)$ *for all* i, *then the piecewise continuously differentiable "move-to-projected-goal" law in (11) asymptotically drives almost all configurations in the free space* $\mathcal{F}$ *to the goal* x^* *with no collisions along the way.*

III. Smooth Extensions via Reference Governors: General Framework

In this section, we present a novel application of reference governors for augmenting the stability and invariable properties of a generic first-order vector field planner to the second-order robot dynamics. We first introduce the concept of a robot-governor system, and then proceed with the construction of our smooth extension algorithm and its important qualitative properties.

A. Robot-Governor System

Definition 1 A *robot-governor system* is a dynamical system that consists of a *second-order robot*, represented by state $(\mathrm{x}, \dot{\mathrm{x}}) \in \mathcal{F} \times \mathbb{R}^n$, and a *first-order governor* — a virtual low-order copy of the robot —, represented by state $\mathrm{x}_g \in \mathcal{F}$. Accordingly, the robot-governor system is described by concatenated robot-governor state $\mathbf{x} := (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \mathcal{F} \times \mathbb{R}^n \times \mathcal{F}$, and its motion is determined by

$$\ddot{\mathrm{x}} = \mathrm{f}(\mathbf{x}), \quad \text{and} \quad \dot{\mathrm{x}}_g = \mathrm{g}(\mathbf{x}), \tag{12}$$

where $\mathrm{f} : \mathcal{F} \times \mathbb{R}^n \times \mathcal{F} \to \mathbb{R}^n$ and $\mathrm{g} : \mathcal{F} \times \mathbb{R}^n \times \mathcal{F} \to \mathbb{R}^n$ are the Lipschitz continuous evolution rules for the robot and the governor, respectively.

Definition 2 The potential energy V of the robot in a robot–governor system, $\mathbf{x} = (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g)$, is defined relative to the governor as

$$V(\mathrm{x}, \mathrm{x}_g) := \kappa \|\mathrm{x} - \mathrm{x}_g\|^2, \tag{13}$$

where $\kappa > 0$ is the potential energy coefficient. Hence, the total energy E of the robot-governor system is given by

$$E(\mathbf{x}) := T(\dot{\mathrm{x}}) + V(\mathrm{x}, \mathrm{x}_g), \tag{14}$$

where $T(\dot{\mathrm{x}}) = \frac{1}{2}\|\dot{\mathrm{x}}\|^2$ is the robot's kinetic energy.

[5]This assumption is equivalent to the "isolated obstacle" assumption of [6], and it guarantees that the free space obstacles do not intersect with each other and with the free space boundary.

Definition 3 A robot-governor state $(\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \mathcal{F} \times \mathbb{R}^n \times \mathcal{F}$ is collision free if and only if there exist Lipschitz continuous control laws f and g, possibly respecting certain control limits, such that both the robot and the governor stay in the free space for all future times, i.e., $\mathrm{x}^t, \mathrm{x}_g^t \in \mathcal{F}$ for all $t \geq 0$, where $\mathrm{x}^t, \mathrm{x}_g^t$ denote the unique solution of the robot-governor dynamics in (12) starting at $(\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g)$.

Since the exact determination of collision-free configurations in such kinodynamic planning settings is hard [1], alternatively, we introduce a conservative, but simple condition to check for collisions using total energy, which does not require the explicit knowledge of the system trajectory.

Proposition 1 *A robot-governor state* $\mathbf{x} = (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \mathcal{F} \times \mathbb{R}^n \times \mathcal{F}$ *is collision-free if*

$$E(\mathbf{x}) \leq \kappa\, d(\mathrm{x}_g, \partial\mathcal{F})^2, \tag{15}$$

where $d(\mathrm{x}_g, \partial\mathcal{F}) := \min\limits_{\mathrm{p} \in \partial\mathcal{F}} \|\mathrm{p} - \mathrm{x}_g\|$ *is the governor's distance to the boundary* $\partial\mathcal{F}$ *of the free space* $\mathcal{F}$.

Proof. To bring the robot-governor system to a safe stop, consider the following Lipschitz continuous evolution rules,

$$\mathrm{f}(\mathbf{x}) = -\nabla_{\mathrm{x}} V(\mathrm{x}, \mathrm{x}_g) - \zeta\dot{\mathrm{x}} = -2\kappa(\mathrm{x} - \mathrm{x}_g) - \zeta\dot{\mathrm{x}}, \tag{16}$$

$$\mathrm{g}(\mathbf{x}) = 0, \tag{17}$$

where V is the system's potential energy, defined as in (13), and $\zeta > 0$ is a fixed artificial damping coefficient. Further, denote by $\mathbf{x}^t = \left(\mathrm{x}^t, \dot{\mathrm{x}}^t, \mathrm{x}_g^t\right)$ the unique solution of the robot-governor dynamics in (12) starting at $(\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g)$.

Since $\mathrm{g}(\mathbf{x}) = 0$, the governor remains constant, i.e., $\mathrm{x}_g^t = \mathrm{x}_g$, and one can readily verify that $\dot{E}(\mathbf{x}^t) = -\zeta \|\dot{\mathrm{x}}^t\|^2 \leq 0$. Hence, we have for all $t \geq 0$ that

$$\kappa\, d(\mathrm{x}_g, \partial\mathcal{F})^2 \geq E(\mathbf{x}) \geq E\left(\mathbf{x}^t\right) \geq V\left(\mathrm{x}^t, \mathrm{x}_g^t\right), \tag{18}$$

$$\geq \kappa \left\|\mathrm{x}^t - \mathrm{x}_g^t\right\|^2 = \kappa \left\|\mathrm{x}^t - \mathrm{x}_g\right\|^2. \tag{19}$$

Thus, since $\|\mathrm{x}^t - \mathrm{x}_g\| \leq d(\mathrm{x}_g, \partial\mathcal{F})$, the result follows. ∎

Accordingly, we define the *energy-safe configuration space* of the robot-governor system to be

$$\mathrm{Conf}(\mathcal{F}) := \left\{\mathbf{x} \in \mathcal{F} \times \mathbb{R}^n \times \mathcal{F} \,\middle|\, E(\mathbf{x}) \leq \kappa d(\mathrm{x}_g, \partial\mathcal{F})^2\right\}. \tag{20}$$

It is also convenient to define the *local energy zone* $\mathcal{LE}(\mathbf{x})$ of a robot-governor configuration $\mathbf{x} = (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \mathrm{Conf}(\mathcal{F})$ as

$$\mathcal{LE}(\mathbf{x}) := \left\{\mathrm{q} \in \mathbb{R}^n \,\middle|\, \|\mathrm{q} - \mathrm{x}_g\| \leq \sqrt{\triangle E(\mathbf{x})/\kappa}\right\}, \tag{21}$$

where $\triangle E(\mathbf{x}) := \kappa d(\mathrm{x}_g, \partial\mathcal{F})^2 - E(\mathbf{x})$ is the amount of extra energy that can be safely added to the system. Note that for any $\mathbf{x} \in \mathrm{Conf}(\mathcal{F})$, $\mathcal{LE}(\mathbf{x})$ is a closed spherical subset of the free space $\mathcal{F}$, because $\sqrt{\triangle E(\mathbf{x})/\kappa} \leq d(\mathrm{x}_g, \partial\mathcal{F})$.

B. Smooth Extensions of Vector Field Planners

Given a vector field planner for the first-order (velocity-controlled) robot model, which we shall refer to as the *reference planner*, we now present a construction that extends its stability and invariance properties to the second-order (force-controlled) robot dynamics via reference governors.

Suppose $\mathrm{r} : \mathcal{F} \to \mathbb{R}^n$ is a vector field planner that

i) is Lipschitz continuous,
ii) has a unique stable point at x^*,
iii) has no degenerate critical points,
iv) is inward pointing on the boundary of free space, i.e., $\mathrm{r}(\mathrm{x}) \cdot \mathrm{n}_{\mathrm{x}} > 0$ for all $\mathrm{x} \in \mathcal{F}$, where n_{x} is the inward pointing normal of $\partial\mathcal{F}$ at x.

Such a construction has the following qualitative properties, which can be readily verified and so the proof is omitted:

Proposition 2 *The vector field planner* r *leaves the robot's free space* $\mathcal{F}$ *positive invariant, and its unique continuous flow asymptotically reaches the goal location* x^* *from almost any configuration in* $\mathcal{F}$*, while strictly decreasing a smooth Lyapunov function along the way.*

To embed the limit behavior of the reference planner r in the second-order robot dynamics, we propose the following robot-governor law: for any $\mathbf{x} \in \mathrm{Conf}(\mathcal{F})$,

$$\mathrm{f}(\mathbf{x}) = -\nabla_{\mathrm{x}} V(\mathrm{x}, \mathrm{x}_g) - \zeta\dot{\mathrm{x}} = -2\kappa\,(\mathrm{x} - \mathrm{x}_g) - \zeta\dot{\mathrm{x}}, \tag{22a}$$

$$\mathrm{g}(\mathbf{x}) = -k_g\left(\mathrm{x}_g - \Pi_{\mathcal{LE}(\mathbf{x})}\left(\mathrm{x}_g + \mathrm{r}(\mathrm{x}_g)\right)\right), \tag{22b}$$

$$= k_g \frac{\mathrm{r}(\mathrm{x}_g)}{\|\mathrm{r}(\mathrm{x}_g)\|} \min\left(\|\mathrm{r}(\mathrm{x}_g)\|, \sqrt{\triangle E(\mathbf{x})/\kappa}\right), \tag{22c}$$

where $V(\mathrm{x}, \mathrm{x}_g)$ (13) is the robot's potential energy relative to the governor, $\kappa > 0$ is the potential energy coefficient, $\zeta > 0$ is the artificial damping coefficient, $k_g > 0$ is a fixed control gain, and $\Pi_{\mathcal{LE}(\mathbf{x})}$ denotes the metric projection onto the local energy zone $\mathcal{LE}(\mathbf{x})$ (21).

In summary, our smooth extension framework comprises a reference motion planner, a first-order reference governor, and a second-order closed-loop robot motion planner, as illustrated in Fig. 2. The reference motion planner generates a (continuously varying) reference goal $\mathrm{r}^* := \mathrm{x}_g + \mathrm{r}(\mathrm{x}_g)$ for the reference governor to guide the governor's (internal) state x_g toward the designated global goal x^*. The reference governor uses both the reference goal r^* and the robot's state $(\mathrm{x}, \dot{\mathrm{x}})$ to continuously update its internal state x_g in such a way that the robot-governor system avoids collisions in the sense of Proposition 1 while the governor's state x_g stays as close as possible to the reference goal r^*; and commands its internal state x_g to the closed-loop robot motion planner as the modified target position. The closed-loop motion planner is the dissipative smooth extension of the negated gradient of the robot's potential energy V (relative to the governor), and

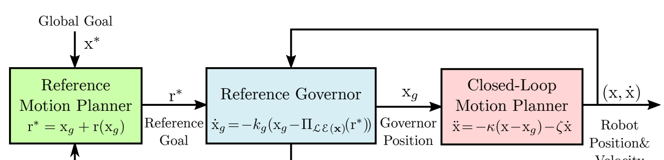

Fig. 2. A reference governor interpretation of smooth extensions of feedback motion planners, where the reference motion planner offers a collision-free navigation solution for the first-order robot model, the reference governor asymptotically achieves the reference dynamics while ensuring collision avoidance for the robot-governor system, and the close-loop motion planner stabilizes the robot around the governor.

so it asymptotically reaches a halt and stabilizes the robot at the governor's position if the governor is stationary. Finally, it is important to observe that our reference governor has a memory and it is actually equivalent to a closed-loop system with a memoryless reference governor, as depicted in Fig. 1.

C. Qualitative Properties

Proposition 3 *The robot-governor law in (22) is Lipschitz continuous.*

Proof. The result follows because metric projection onto a (moving) convex set with piecewise differentiable boundaries is piecewise continuously differentiable and so is Lipschitz continuous [27]–[29], and the composition of Lipschitz continuous functions are again Lipschitz continuous [30]. ■

Proposition 4 *The energy-safe configuration space* $\mathrm{Conf}(\mathcal{F})$ *(20) is positive invariant under the robot-governor law (22).*

Proof. Let $\mathbf{x} \in \partial \mathrm{Conf}(\mathcal{F})$ be a robot-governor configuration on the boundary $\partial \mathrm{Conf}(\mathcal{F})$ of $\mathrm{Conf}(\mathcal{F})$. Since $\triangle E(\mathbf{x}) = 0$ and $\mathcal{LE}(\mathrm{x}_g) = \{\mathrm{x}_g\}$, we have $\mathrm{g}(\mathbf{x}) = 0$, i.e., the governor remains constant. Further, since $\mathrm{g}(\mathbf{x}) = 0$, the total energy of the system is non-increasing, i.e., $\dot{E}(\mathbf{x}) = -\zeta \|\dot{\mathrm{x}}\|^2 \leq 0$, implying that $E(\mathbf{x})$ stays bounded above by $\kappa d(\mathrm{x}_g, \partial\mathcal{F})^2$. Thus, the result follows. ■

Proposition 5 *For any initial configuration in* $\mathrm{Conf}(\mathcal{F})$*, the robot-governor law (22) has a unique continuously differentiable flow in* $\mathrm{Conf}(\mathcal{F})$ *(20) defined for all future time.*

Proof. The existence, uniqueness and continuous differentiability of the flow of the robot-governor law follow from its Lipschitz continuity in $\mathrm{Conf}(\mathcal{F})$ (Proposition 3) [31]. ■

Proposition 6 *The robot-governor law in (22) asymptotically steers almost all energy-safe configurations*[1] *in* $\mathrm{Conf}(\mathcal{F})$ *(20) to the goal configuration* $(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)$.

Proof. It is straightforward to observe from (22) that $(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)$ is a critical point of the robot-governor law.

Let $U : \mathcal{F} \to \mathbb{R}$ be a smooth Lyapunov function associated with the reference vector field r (see Proposition 2). Recall that r has a unique stable point at x^*, whose domain of attraction, denoted by $\mathcal{D}$, includes all $\mathcal{F}$, possibly excludes a set of measure zero. Since x^* is the only critical point of r in $\mathcal{D}$, we have $\mathrm{r}(\mathrm{x}) \neq \mathbf{0}$ and $U(\mathrm{x}) \cdot \mathrm{r}(\mathrm{x}) < 0$ for all $\mathrm{x} \in \mathcal{D} \setminus \{\mathrm{x}^*\}$.

In the rest of the proof, we shall only consider robot-governor configurations $\mathbf{x} = (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \mathrm{Conf}(\mathcal{F})$ with $\mathrm{x}_g \in \mathcal{D}$, because all other configurations in $\mathrm{Conf}(\mathcal{F})$ are not contained in the stable manifold of $(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)$ and has measure zero.

If the governor is on the free space boundary, i.e., $\mathrm{x}_g \in \partial\mathcal{F}$, then, by definition of $\mathrm{Conf}(\mathcal{F})$ (20), we have $\mathrm{x} = \mathrm{x}_g$, $\dot{\mathrm{x}} = \mathbf{0}$, and $\triangle E(\mathbf{x}) = 0$. Hence, the robot-governor system stays stationary on the boundary for all future time. Fortunately, the set of such initial conditions also has a measure zero and are also not included in the stable manifold of $(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)$.

Otherwise, the governor is in the interior of the free space, i.e., $\mathrm{x}_g \in \mathring{\mathcal{F}}$, and remains in $\mathring{\mathcal{F}}$ for all future time, because the reference field r is inward pointing along the free space boundary $\partial\mathcal{F}$. Moreover, since the governor's evaluation rule g in (22b) is a scaled version of the reference field r with a nonnegative factor, we have $\dot{U}(\mathrm{x}_g) \leq 0$. Accordingly, define

$$S = \left\{ \mathbf{x} \in \mathrm{Conf}(\mathcal{F}) \,\middle|\, \mathrm{x}_g \in D \setminus \partial\mathcal{F},\, \dot{U}(\mathrm{x}_g) = 0 \right\}. \quad (23)$$

Note that $\dot{U}(\mathrm{x}_g) = 0$ if and only if $\mathrm{g}(\mathbf{x}) = 0$. Hence, S contains only robot-governor configurations where the governor is stationary. Further, it follows from $\mathrm{g}(\mathbf{x}) = 0$ that $\dot{E}(\mathbf{x}) = -\zeta \|\dot{\mathrm{x}}\|^2 \leq 0$. Thus, the energy of the robot-governor system asymptotically decays to zero and the robot becomes a stop at the governor's position. Thus, the maximum positive invariant set in S contains robot-governor configurations where $E(\mathbf{x}) = 0$, i.e., $\mathrm{x}_g = \mathrm{x}$ and $\dot{\mathrm{x}} = 0$. Further, if $\mathrm{x}_g \in \mathring{\mathcal{F}}$ and $E(\mathbf{x}) = 0$, then we have $\triangle E(\mathbf{x}) > 0$; and $\mathrm{g}(\mathbf{x}) = 0$ and $\triangle E(\mathbf{x}) > 0$ implies that $\mathrm{r}(\mathrm{x}_g) = 0$ and so $\mathrm{x}_g = \mathrm{x}^*$. Hence, the largest positive invariant set in S is $\{(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)\}$. Therefore, we have from LaSalle's Invariance Principle [31] that the robot-governor law asymptotically reaches $(\mathrm{x}^*, 0, \mathrm{x}^*)$, whose domain of attraction is $\left\{\mathbf{x} \in \mathrm{Conf}(\mathcal{F}) \,\middle|\, \mathrm{x}_g \in \mathcal{D} \setminus \partial\mathcal{F}\right\}$ and contains all $\mathrm{Conf}(\mathcal{F})$, except a measure zero set. ■

Therefore, important qualitative properties of the robot-governor law can be summarized as:

Theorem 2 *The Lipschitz continuous robot-governor law in (22) asymptotically drives almost all configurations*[1] *in its positively invariance domain* $\mathrm{Conf}(\mathcal{F})$ *(20) to the goal configuration* $(\mathrm{x}^*, \mathbf{0}, \mathrm{x}^*)$*, with no collisions along the way.*

D. Enforcing Control Constraints

In the interest of greater practicability, we now present an extension of our smooth extension framework to respect control constraints by limiting the total energy of the system.

Define the *energy-bounded configuration space* of the robot-governor system to be

$$\widehat{\mathrm{Conf}}(\mathcal{F}) := \left\{ \mathbf{x} \in \mathrm{Conf}(\mathcal{F}) \,\middle|\, E(\mathbf{x}) \leq E_{\max} \right\} \quad (24)$$

where $E_{\max} > 0$ is the maximum allowable total energy of the system, and $\mathrm{Conf}(\mathcal{F})$ is the energy-safe configuration space in (20). Consequently, for any $\mathbf{x} \in \widehat{\mathrm{Conf}}(\mathcal{F})$, the *local energy-bounded zone* of the robot-governor is constructed from its local energy zone $\mathcal{LE}(\mathbf{x})$ as

$$\widehat{\mathcal{LE}}(\mathbf{x}) := \left\{ \mathrm{q} \in \mathcal{LE}(\mathbf{x}) \,\middle|\, \|\mathrm{q} - \mathrm{x}_g\| \leq \sqrt{\widehat{\triangle E}(\mathbf{x}) / \kappa} \right\}, \quad (25)$$

where $\widehat{\triangle E}(\mathbf{x}) := E_{\max} - E(\mathbf{x})$ is the maximum amount of extra energy that can be injected to the system while ensuring the total energy limit.

Following the same line of the proof procedure in Section III-C, one can verify that the robot-governor law

$$\mathrm{f}(\mathbf{x}) = -2\kappa(\mathrm{x} - \mathrm{x}_g) - \zeta\dot{\mathrm{x}}, \quad (26a)$$

$$\mathrm{g}(\mathbf{x}) = -k_g \left(\mathrm{x}_g - \Pi_{\widehat{\mathcal{LE}}(\mathbf{x})}(\mathrm{x}_g + \mathrm{r}(\mathrm{x}_g)) \right), \quad (26b)$$

ensures the positive invariance of $\widehat{\mathrm{Conf}}(\mathcal{F})$ and its unique flow, starting at almost any configuration[1] in $\widehat{\mathrm{Conf}}(\mathcal{F})$, asymptotically reaches $(\mathrm{x}^*, 0, \mathrm{x}^*)$.

An important consequence of putting an explicit upper bound on the total energy of the system is:

Proposition 7 *For any* $\mathbf{x} \in \widehat{\mathrm{Conf}}(\mathcal{F})$*, the robot-governor law in (26) satisfies the following control bounds*

$$\mathrm{f}(\mathbf{x}) \leq (2\sqrt{\kappa}+\zeta\sqrt{2})\sqrt{E(\mathbf{x})} \leq (2\sqrt{\kappa}+\zeta\sqrt{2})\sqrt{E_{\max}}\,, \quad (27)$$

$$\mathrm{g}(\mathbf{x}) \leq k_g\sqrt{\widehat{\triangle E}(\mathbf{x})/\kappa} \leq k_g\sqrt{E_{\max}/\kappa}. \quad (28)$$

Proof. The proof directly follows from (24), (25) (26), so it is omitted for the sake of brevity. ■

Note that one also has that $\dot{\mathrm{x}} \leq \sqrt{2E_{\max}}$ for any $\mathbf{x} = (\mathrm{x}, \dot{\mathrm{x}}, \mathrm{x}_g) \in \widehat{\mathrm{Conf}}(\mathcal{F})$, because $T(\dot{\mathrm{x}}) \leq E(\mathbf{x}) \leq E_{\max}$.

IV. Smooth Extensions via Reference Governors: Examples

In this section, we provide some example applications of our smooth lifting framework for extending low-order reference planners to the second-order robot dynamics, and illustrate and compare the navigation trajectories of reference dynamics and their smooth embeddings.

A. Navigation in a Convex Workspace

Although it is very straightforward to solve, navigation in a convex workspace $\mathcal{W} \in \mathbb{R}^n$ with no obstacles offers a simple setting to demonstrate the strength of our smooth extension framework over the standard total energy based embedding of gradient dynamics, summarized in Section II-A. Since convex sets can be homeomorphically mapped to Euclidean balls with certain degrees of smoothness, for instance, see [32], for the sake of simplicity, we consider navigation in a closed Euclidean ball $\mathcal{W} = B(\mathrm{p}_{\mathcal{W}}, r_{\mathcal{W}})$, centered at $\mathrm{p}_{\mathcal{W}} \in \mathbb{R}^n$ with radius $r_{\mathcal{W}} > r$, toward a given goal location $\mathrm{x}^* \in \mathring{\mathcal{F}}$ in the interior $\mathring{\mathcal{F}}$ of the free space $\mathcal{F}$ of our disk-shaped robot, centered at $\mathrm{x} \in \mathcal{F}$ and of radius $r > 0$, by using the negated gradient of the following well established artificial potential functions

$$V_1(\mathrm{x}) = \|\mathrm{x} - \mathrm{x}^*\|^2, \quad (29)$$

$$V_2(\mathrm{x}) = \frac{10\,\|\mathrm{x}-\mathrm{x}^*\|^2}{\|\mathrm{x}-\mathrm{x}^*\|^2 + (r_{\mathcal{W}} - r)^2 - \|\mathrm{x} - \mathrm{p}_{\mathcal{W}}\|^2}. \quad (30)$$

Note that while V_1 fails to be admissible on $\mathcal{F}$ unless $\mathrm{x}^* = \mathrm{p}_{\mathcal{W}}$, V_2 is admissible and so a navigation function [6]. Nonetheless, the negated gradients of both potential functions guarantee collision-free global navigation for the first-order robot model in $\mathcal{W}$, because they are both inward pointing along the free space boundary and has a unique global minimum at the goal position.

In Fig. 3, we present the resulting navigation trajectories of both total energy based and reference governor based smooth extensions of gradient dynamics. In our simulation studies[6], we consider two different sets of parameters that yield underdamped and overdamped second-order navigation planners. Irrespective of the admissibility property of their generating potential functions, both overdamped embeddings of the gradient dynamics guarantee collision avoidance with the workspace boundary for all zero velocity initial conditions. However, we observe that the total energy based underdamped embedding of the negated gradient of the non-admissible potential function V_1 does not ensure the invariance of the robot's workspace, whereas, by construction, the

[6]For all simulations, we set $\kappa = k = k_g = 1$, and $\zeta = 1$ for underdamped embedding and $\zeta = 2\sqrt{2}$ for overdamped embedding. All simulations are obtained through the numerical simulation of the associated robot dynamics using the `ode45` function of MATLAB.

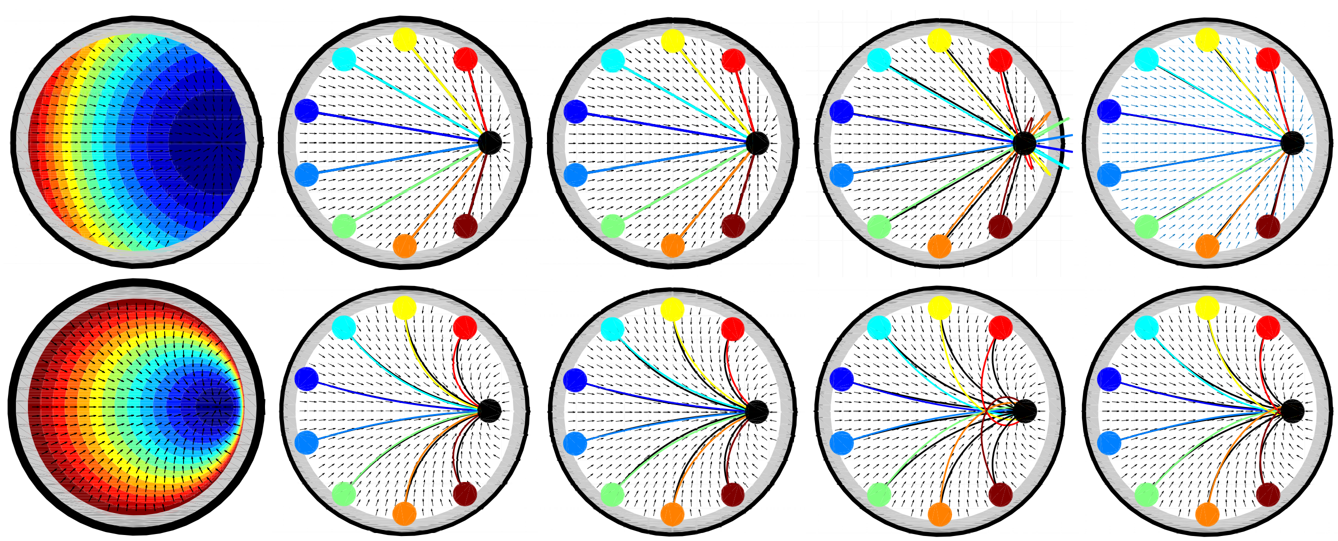

Fig. 3. Example navigation trajectories of smooth extensions of negated gradients of (top) nonadmissible and (bottom) admissible potential functions, V_1 (29) and V_2 (30), respectively, in a circular workspace: (a) Level curves and gradient directions of potential functions, (b) Total energy based overdamped embedding of gradient dynamics, (c) Reference governor based overdamped embedding of gradient dynamics, (d) Total energy based underdamped embedding of gradient dynamics, (e) Reference governor based underdamped embedding of gradient dynamics. Here, the goal is specified by the black disk, and other colored disks show the start locations. Navigation paths in black are for the first-order robot model and all other colored paths are for the second order robot model. Please see the accompanying video submission for the illustration of the navigation motion.

proposed reference governor based smooth extension always guarantees constraint satisfaction (i.e., collision avoidance). Here, to clearly observe the oscillations of underdamped embeddings, we use energy-safe nonzero velocity initial conditions, where the robot has a speed of 0.5 units/sec and initially moves toward the center of the workspace.

B. Navigation in Sphere Worlds

We now consider the extension of the "move-to-projected-goal" navigation strategy [22], summarized in Section II-B, to the second-order robot dynamics via reference governors. Note that such a generic (non-gradient) vector field planner cannot be adapted to the second-order systems using the standard total energy based extension, summarized in Section II-A, while retaining its invariance properties. Also recall that the original "move-to-projected-goal" law can be modeled as a closed-loop motion planner with a memoryless reference governor, see Fig. 1, and so its smooth extension via an additional reference governor with memory, see Fig. 2, has actually a cascade reference governor structure where the outer (primary) reference governor guarantees collision avoidance for the first-order governor (virtual robot) model and the inner (secondary) reference governor guarantees collision avoidance for the second-order (actual) robot model.

In Fig. 4, we illustrate the resulting navigation trajectories of the original first-order and the extended second-order "move-to-projected-goal" laws. Since the environment is very cluttered and the total energy of the robot-governor system is limited by the (squared) clearance between obstacles, we observe a significant spatial consistency between the navigation trajectories of the first-order and the second order "move-to-projected-goal" laws. Remark that the original "move-to-projected-goal" law is either tangent or inward pointing along the boundary of free space, and to have a inward pointing vector field along the free space boundary, one can simply enlarge the robot body or obstacles with a positive safety margin, as we do here.

C. Smooth Extensions of Navigation Paths

As a final example, we consider smooth extensions of path planners of position-controlled (zero-order) robots to the velocity-controlled (first-order) and force-controlled (second-order) robot models via reference governors.

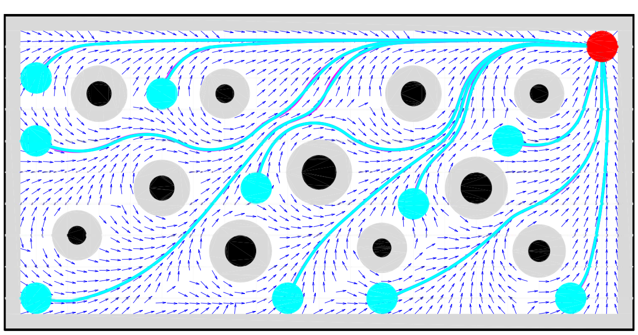

Fig. 4. Example navigation trajectories of the original first-order (magenta) and the extended second-order (cyan) "move-to-projected-goal" laws [22], which spatially overlap significantly. Please see the accompanying video for the resultant motion.

Let $\mathcal{W}$ be a generic (possibly nonconvex) workspace populated with arbitrary shaped obstacles and associated with a path-connected free space $\mathcal{F}$ for a disk-shaped robot. Suppose $\mathcal{P} : [0,1] \to \mathcal{F}$ is a navigation path, for instance, obtained by using a standard path planner [33], [34] or directly specified by the user, that joins a given pair of initial and goal positions $\mathrm{x}^0, \mathrm{x}^* \in \mathring{\mathcal{F}}$ and lies in the interior $\mathring{\mathcal{F}}$ of the free space $\mathcal{F}$, i.e., $\mathcal{P}(0) = \mathrm{x}^0$, $\mathcal{P}(1) = \mathrm{x}^*$, and $\mathcal{P}(\alpha) \in \mathring{\mathcal{F}}$ for all $\alpha \in [0,1]$.

Here, we interpret $\mathcal{P}$ as a high-level, flexible navigation plan toward the goal, and accordingly construct a first-order vector field, called the "move-to-projected-path-goal" law, $\mathrm{u}_{\mathcal{P}} : \mathcal{D}_{\mathcal{P}} \to \mathbb{R}^n$ as follows: for any $\mathrm{x} \in \mathcal{D}_{\mathcal{P}}$,

$$\dot{\mathrm{x}} = \mathrm{u}_{\mathcal{P}}(\mathrm{x}) = -k\left(\mathrm{x} - \mathrm{x}^*_{\mathcal{P}}\right) \tag{31}$$

where the domain $\mathcal{D}_{\mathcal{P}}$ of the vector field planner is defined as the generalized Voronoi cell of the path $\mathcal{P}$ in $\mathcal{F}$,

$$\mathcal{D}_{\mathcal{P}} := \left\{ \mathrm{q} \in \mathcal{F} \middle| d(\mathrm{q}, \mathcal{P}) \leq d(\mathrm{q}, \partial\mathcal{F}) \right\}, \tag{32}$$

and $\mathrm{x}^*_{\mathcal{P}}$ is the "projected path goal" determined as

$$\alpha^* := \max\left(\left\{ \alpha \in [0,1] \middle| \mathcal{P}(\alpha) \in B\big(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F})\big) \right\} \right), \tag{33}$$

$$\mathrm{x}^*_{\mathcal{P}} := \mathcal{P}(\alpha^*). \tag{34}$$

Here, we abuse the notation and write $d(\mathrm{q},\mathcal{P}) := d(\mathrm{q},\mathcal{P}([0,1]))$, and $B(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F}))$ is the largest closed ball centered at x and contained in $\mathcal{F}$. Observe that for any $\mathrm{x} \in \mathcal{D}_{\mathcal{P}}$, we have, by construction, that $\mathcal{P}([0,1]) \cap B(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F})) \neq \varnothing$. Thus, the "projected path goal" $\mathrm{x}^*_{\mathcal{P}}$ is well defined in $\mathcal{D}_{\mathcal{P}}$, and is the closest point along $\mathcal{P}$ in $B(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F}))$ to $\mathrm{x}^* = \mathcal{P}(1)$, and α^* is the associated path parameter.

Although, a comprehensive formal analysis of this new construction is now work in progress and left to a future paper, we still find it useful to highlight its important qualitative properties without proof:

Proposition 8 *If $\mathcal{P}$ is piecewise continuously differentiable and $\mathcal{P}([0,1]) \cap B(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F}))$ is path-connected for all $\mathrm{x} \in \mathcal{D}_{\mathcal{P}}$, then the "move-to-project-path-goal" law $\mathrm{u}_{\mathcal{P}}$ in (31) is piecewise continuously differentiable, and is inward pointing along the boundary $\partial\mathcal{D}_{\mathcal{P}}$ of its positively invariant domain $\mathcal{D}_{\mathcal{P}}$, and asymptotically steer all configuration $\mathrm{x} \in \mathcal{D}_{\mathcal{P}}$ to x^* while strictly decreasing $(1 - \alpha^*)$ along the way.*

Note that the requirement $\mathcal{P}([0,1]) \cap B(\mathrm{x}, d(\mathrm{x}, \partial\mathcal{F}))$ being path-connected is an admissible assumption that often holds in practice for the output of many standard path planner [33], [34]. Further, the failure of this requirement only affects the continuity properties of the "move-to-projected-path-goal", but its existence, uniqueness and stability properties are generally retained, which will be discussed in a future paper.

In Fig. 5, we illustrate the resulting navigation trajectories of the original first-order "move-to-projected-path-goal" law and its smooth extension to the second-order robot dynamics via reference governors. As expected, the resulting trajectories of the first-order and second-order navigation planners differ around abrupt changes along the input navigation path, otherwise they show significant spatially consistency. Finally, we would like to emphasize that the smooth extension of the

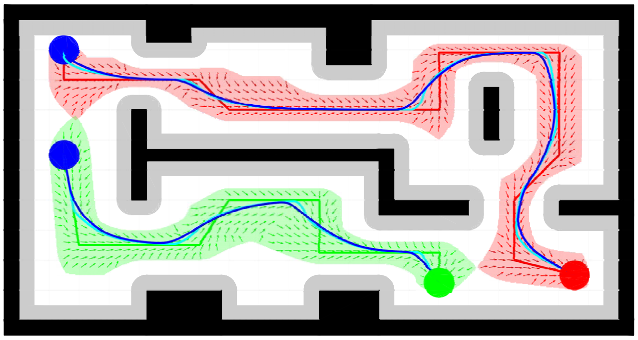

Fig. 5. Example navigation trajectories of the original first-order (cyan) and the extend second-order (blue) "move-to-projected-path-goal' laws. The domain (red/green) of the first-order "move-project-path-goal" law are colored according the associated path color(red/green). Please see the accompanying video for the full motion.

"move-to-projected-path-goal" also has a cascade reference-governor structure, as the "move-to-projected-goal" law discussed in Section IV-B.

V. Conclusions

In this paper, we present a novel application of reference governors to extend low-order feedback motion planners to high-order robot dynamics while preserving stability and invariance properties. To the best of our knowledge, this is the first time a smooth extension framework can augment global navigation and collision avoidance properties of a generic path planner or a vector field planner to the second-order systems. We demonstrate the effectiveness of the proposed smooth extension algorithm in numerical simulations.

Work now in progress targets a comprehensive analysis of the "move-to-projected-path-goal" law, presented in Section IV-C, and its application to smooth trajectory planning. We are also investigating extensions of these ideas to a generalized smooth extension theorem for Lagrangian dynamical systems, to nonholonomically constrained dynamical systems, and to a more generic potential energy definition with ellipsoidal or polygonal level sets. In the near term, we also plan to perform empirical validation of the proposed algorithms for safe, high-speed robot navigation.

References

[1] B. Donald, P. Xavier, J. Canny, and J. Reif, "Kinodynamic motion planning," *Journal of the ACM*, vol. 40, no. 5, pp. 1048–1066, 1993.

[2] D. E. Koditschek, "The control of natural motion in mechanical systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 113, no. 4, pp. 547–551, 1991.

[3] S. W. Thomson and P. G. Tait, *Treatise on natural philosophy*. Cambridge University Press, 1888.

[4] D. E. Koditschek, "The application of total energy as a Lyapunov function for mechanical control systems," *Contemporary Mathematics*, vol. 97, pp. 131–158, 1989.

[5] D. Koditschek, "Robot planning and control via potential functions," *The Robotics Review*, pp. 349–367, 1989.

[6] E. Rimon and D. Koditschek, "Exact robot navigation using artificial potential functions," *IEEE Transactions on Robotics and Automation*, vol. 8, no. 5, pp. 501–518, 1992.

[7] D. E. Koditschek, "Adaptive techniques for mechanical systems," in *Fifth Yale Workshop on Applications of Adaptive Systems Theory*, 1987, pp. 259–265.

[8] G. Wenger, A. De, and D. E. Koditschek, "Frontal plane stabilization and hopping with a 2DOF tail," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2016, pp. 567–573.

[9] A. A. Rizzi, "Hybrid control as a method for robot motion programming," in *IEEE International Conference on Robotics and Automation*, vol. 1, 1998, pp. 832–837.

[10] R. R. Burridge, A. A. Rizzi, and D. E. Koditschek, "Sequential composition of dynamically dexterous robot behaviors," *The International Journal of Robotics Research*, vol. 18, no. 6, pp. 535–555, 1999.

[11] G. E. Fainekos, A. Girard, H. Kress-Gazit, and G. J. Pappas, "Temporal logic motion planning for dynamic robots," *Automatica*, vol. 45, no. 2, pp. 343–352, 2009.

[12] R. Fierro and F. L. Lewis, "Control of a nonholomic mobile robot: Backstepping kinematics into dynamics," *Journal of Robotic Systems*, vol. 14, no. 3, pp. 149–163, 1997.

[13] I. Kolmanovsky, E. Garone, and S. D. Cairano, "Reference and command governors: A tutorial on their theory and automotive applications," in *American Control Conference*, 2014, pp. 226–241.

[14] M. Farber, "Topological complexity of motion planning," *Discrete and Computational Geometry*, vol. 29, no. 2, pp. 211–221, 2003.

[15] N. P. Bhatia and G. P. Szegö, *Stability Theory of Dynamical Systems*. Springer Science & Business Media, 2002, vol. 161.

[16] M. W. Hirsch, *Differential Topology*. Springer Science & Business Media, 2012, vol. 33.

[17] D. E. Koditschek and E. Rimon, "Robot navigation functions on manifolds with boundary," *Advances in Applied Mathematics*, vol. 11, no. 4, pp. 412 – 442, 1990.

[18] V. I. Arnold, *Mathematical Methods of Classical Mechanics*. Springer Science & Business Media, 1989, vol. 60.

[19] P. Kapasouris, M. Athans, and G. Stein, "Design of feedback control systems for stable plants with saturating actuators," in *IEEE Conference on Decision and Control*, vol. 1, 1988, pp. 469–479.

[20] E. G. Gilbert, I. Kolmanovsky, and K. T. Tan, "Discrete-time reference governors and the nonlinear control of systems with state and control constraints," *International Journal of Robust and Nonlinear Control*, vol. 5, no. 5, pp. 487–504, 1995.

[21] A. Bemporad, "Reference governor for constrained nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 3, pp. 415–419, 1998.

[22] O. Arslan and D. E. Koditschek, "Exact robot navigation using power diagrams," in *IEEE International Conference on Robotics and Automation*, 2016, pp. 1–8.

[23] ——, "Sensor-based reactive navigation in unknown convex sphere worlds," in *The Twelveth International Workshop on the Algorithmic Foundations of Robotics*, 2016.

[24] ——, "Voronoi-based coverage control of heterogeneous disk-shaped robots," in *IEEE International Conference on Robotics and Automation*, 2016, pp. 4259–4266.

[25] F. Aurenhammer, "Power diagrams: Properties, algorithms and applications," *SIAM Journal on Computing*, vol. 16, no. 1, pp. 78–96, 1987.

[26] M. W. Hirsch, S. Smale, and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, 2nd ed. Academic Press, 2003.

[27] L. Kuntz and S. Scholtes, "Structural analysis of nonsmooth mappings, inverse functions, and metric projections," *Journal of Mathematical Analysis and Applications*, vol. 188, no. 2, pp. 346 – 386, 1994.

[28] A. Shapiro, "Sensitivity analysis of nonlinear programs and differentiability properties of metric projections," *SIAM Journal on Control and Optimization*, vol. 26, no. 3, pp. 628–645, 1988.

[29] J. Liu, "Sensitivity analysis in nonlinear programs and variational inequalities via continuous selections," *SIAM Journal on Control and Optimization*, vol. 33, no. 4, pp. 1040–1060, 1995.

[30] K. Eriksson, D. Estep, and C. Johnson, *Applied Mathematics: Body and Soul: Volume 1: Derivatives and Geometry in IR^3*. Springer Science & Business Media, 2013.

[31] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2001.

[32] D. Conner, H. Choset, and A. Rizzi, "Flow-through policies for hybrid controller synthesis applied to fully actuated systems," *IEEE Transactions on Robotics*, vol. 25, no. 1, pp. 136–146, 2009.

[33] H. Choset, K. M. Lynch, S. Hutchinson, G. A. Kantor, W. Burgard, L. E. Kavraki, and S. Thrun, *Principles of Robot Motion: Theory, Algorithms, and Implementations*. MIT Press, 2005.

[34] S. M. LaValle, *Planning Algorithms*. Cambridge University Press, 2006.