Accelerated derivative-free projection method with restart technique for pseudo-monotone nonlinear equations

Abdulkarim Hassan Ibrahim[1,*]

[1]*Center for Smart Mobility and Logistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia*

*Corresponding Author

APPENDIX

Table 1: *Numerical results for Modified Exponential Function [1, Example 4.1]*

		MDFPM1			MDFPM2			MDFPM3			WDFPM		
Dm	Inip	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	43	174	4.1328e-11	43	174	4.1328e-11	43	174	4.099e-11	80	635	3.1964e-11
	$x_0^{(2)}$	39	155	2.7221e-11	39	155	2.7221e-11	39	155	2.8907e-11	79	704	1.1665e-11
	$x_0^{(3)}$	40	159	3.357e-11	40	159	3.357e-11	40	159	4.5256e-11	71	609	1.4859e-11
	$x_0^{(4)}$	44	178	3.1487e-11	44	178	3.1487e-11	44	178	3.1477e-11	80	682	1.4709e-11
	$x_0^{(5)}$	43	174	3.5088e-11	43	174	3.5088e-11	43	174	3.5068e-11	77	631	1.6918e-11
	$x_0^{(6)}$	43	174	4.1328e-11	43	174	4.1328e-11	43	174	4.099e-11	80	635	3.1964e-11
	$x_0^{(7)}$	42	168	3.8508e-11	41	164	4.9708e-11	41	164	4.0591e-11	69	576	2.9236e-11
15000	$x_0^{(1)}$	48	197	2.6813e-11	48	197	2.6813e-11	48	197	2.6813e-11	149	1037	1.1336e-11
	$x_0^{(2)}$	41	163	3.6113e-11	41	163	3.6113e-11	41	163	3.6222e-11	88	789	1.5576e-11
	$x_0^{(3)}$	42	167	3.905e-11	42	167	3.905e-11	42	167	3.9649e-11	82	607	3.0311e-11
	$x_0^{(4)}$	107	408	2.6106e-11	107	408	2.6106e-11	107	408	2.5725e-11	122	822	3.5846e-11
	$x_0^{(5)}$	45	182	4.1607e-11	45	182	4.1607e-11	45	182	4.1552e-11	131	883	1.4736e-11
	$x_0^{(6)}$	48	197	2.6813e-11	48	197	2.6813e-11	48	197	2.6813e-11	149	1037	1.1336e-11
	$x_0^{(7)}$	65	284	4.254e-11	44	176	3.6946e-11	44	176	2.6179e-11	96	737	1.4344e-11
75000	$x_0^{(1)}$	55	244	3.454e-11	55	244	3.454e-11	55	244	3.454e-11	314	2000	1.1034e-11
	$x_0^{(2)}$	42	167	4.0256e-11	42	167	4.0256e-11	42	167	4.0377e-11	90	689	1.1783e-11
	$x_0^{(3)}$	44	176	3.2837e-11	44	176	3.2837e-11	44	176	3.2715e-11	140	989	1.0729e-11
	$x_0^{(4)}$	49	204	3.3931e-11	49	204	3.3931e-11	49	204	3.381e-11	169	1114	1.7987e-11
	$x_0^{(5)}$	51	216	3.3567e-11	51	216	3.3567e-11	51	216	3.3567e-11	232	1480	1.1482e-11
	$x_0^{(6)}$	55	244	3.454e-11	55	244	3.454e-11	55	244	3.454e-11	314	2000	1.1034e-11
	$x_0^{(7)}$	46	186	2.4982e-11	45	182	4.9952e-11	45	182	4.9897e-11	131	892	2.1189e-11
250000	$x_0^{(1)}$	66	319	3.6415e-11	66	319	3.6415e-11	66	319	3.6415e-11	-	-	-
	$x_0^{(2)}$	43	171	3.6859e-11	43	171	3.6859e-11	43	171	3.6859e-11	103	730	1.2082e-11
	$x_0^{(3)}$	47	191	3.3307e-11	47	191	3.3307e-11	47	191	3.3307e-11	149	1001	1.4946e-11
	$x_0^{(4)}$	56	249	4.2855e-11	56	249	4.2855e-11	56	249	4.3077e-11	-	-	-
	$x_0^{(5)}$	60	277	4.3743e-11	60	277	4.3743e-11	60	277	4.3743e-11	-	-	-
	$x_0^{(6)}$	66	319	3.6415e-11	66	319	3.6415e-11	66	319	3.6415e-11	-	-	-
	$x_0^{(7)}$	48	198	3.7664e-11	48	198	3.7732e-11	48	198	3.7646e-11	155	977	4.2526e-11

*ibrahimkarym@gmail.com

Table 2: *Numerical results for Logarithmic [2, Problem 10]*

		MDFPM1			MDFPM2			MDFPM3			WDFPM		
Dm	Inip	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	28	84	2.205e-09	28	84	2.205e-09	28	84	2.205e-09	45	252	1.5931e-11
	$x_0^{(2)}$	26	81	4.1281e-09	26	81	4.1281e-09	26	81	4.1281e-09	40	343	1.2764e-11
	$x_0^{(3)}$	25	75	4.2448e-09	25	75	4.2448e-09	25	75	4.2448e-09	48	393	1.5331e-11
	$x_0^{(4)}$	27	81	2.2936e-09	27	81	2.2936e-09	27	81	2.2936e-09	52	372	1.1367e-11
	$x_0^{(5)}$	27	81	3.392e-09	27	81	3.392e-09	27	81	3.392e-09	54	392	1.1996e-11
	$x_0^{(6)}$	28	84	2.205e-09	28	84	2.205e-09	28	84	2.205e-09	45	252	1.5931e-11
	$x_0^{(7)}$	29	89	3.1361e-09	29	89	2.8303e-09	29	89	2.2822e-09	56	436	1.5102e-11
15000	$x_0^{(1)}$	28	87	6.0836e-08	28	87	6.0836e-08	28	87	6.0836e-08	167	932	1.259e-11
	$x_0^{(2)}$	24	75	1.4943e-07	24	75	1.4943e-07	24	75	1.4943e-07	51	272	1.2373e-11
	$x_0^{(3)}$	24	72	5.7676e-08	24	72	5.7676e-08	24	72	5.7676e-08	72	461	1.2482e-11
	$x_0^{(4)}$	25	75	8.2206e-08	25	75	8.2206e-08	25	75	8.2206e-08	83	482	8.2721e-11
	$x_0^{(5)}$	26	80	1.2259e-07	26	80	1.2259e-07	26	80	1.2259e-07	99	465	1.0306e-11
	$x_0^{(6)}$	28	87	6.0836e-08	28	87	6.0836e-08	28	87	6.0836e-08	167	932	1.259e-11
	$x_0^{(7)}$	27	83	1.2754e-07	27	83	1.2791e-07	28	86	5.7411e-08	82	528	4.2106e-11
75000	$x_0^{(1)}$	33	116	4.6596e-07	33	116	4.6596e-07	33	116	4.6596e-07	297	1571	1.6175e-11
	$x_0^{(2)}$	24	75	3.3362e-07	24	75	3.3362e-07	24	75	3.3362e-07	75	472	1.0581e-11
	$x_0^{(3)}$	25	79	6.1719e-07	25	79	6.1719e-07	25	79	6.1719e-07	98	593	1.1675e-11
	$x_0^{(4)}$	27	85	3.028e-07	27	85	3.028e-07	27	85	3.028e-07	202	1017	4.0328e-10
	$x_0^{(5)}$	28	91	4.4926e-07	28	91	4.4926e-07	28	91	4.4926e-07	241	1246	5.6491e-11
	$x_0^{(6)}$	33	116	4.6596e-07	33	116	4.6596e-07	33	116	4.6596e-07	297	1571	1.6175e-11
	$x_0^{(7)}$	28	88	5.1583e-07	28	88	5.1357e-07	28	88	5.0612e-07	118	725	2.3418e-11
250000	$x_0^{(1)}$	40	155	2.4199e-06	40	155	2.4199e-06	40	155	2.4199e-06	-	-	-
	$x_0^{(2)}$	23	72	1.616e-06	23	72	1.616e-06	23	72	1.616e-06	72	420	3.6526e-11
	$x_0^{(3)}$	25	79	1.1265e-06	25	79	1.1265e-06	25	79	1.1265e-06	173	950	1.0214e-11
	$x_0^{(4)}$	33	119	1.0375e-06	33	119	1.0375e-06	33	119	1.0375e-06	286	1474	5.9619e-11
	$x_0^{(5)}$	36	134	1.1111e-06	36	134	1.1111e-06	36	134	1.1111e-06	332	1785	2.3426e-11
	$x_0^{(6)}$	40	155	2.4199e-06	40	155	2.4199e-06	40	155	2.4199e-06	-	-	-
	$x_0^{(7)}$	28	88	9.6319e-07	28	88	9.6043e-07	27	85	2.4794e-06	200	1135	2.8226e-11

Table 3: *Numerical results for Problem 2 [3]*

Dm	Inip	MDFPM1			MDFPM2			MDFPM3			WDFPM		
		Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	63	249	4.7982e-07	63	249	4.7982e-07	63	249	4.7982e-07	65	422	1.4341e-11
	$x_0^{(2)}$	64	253	4.723e-08	64	253	4.723e-08	64	253	4.723e-08	43	287	1.895e-10
	$x_0^{(3)}$	64	253	1.1623e-07	64	253	1.1623e-07	64	253	1.1623e-07	54	359	1.6392e-11
	$x_0^{(4)}$	63	249	3.4092e-07	63	249	3.4092e-07	63	249	3.4092e-07	63	415	1.0826e-10
	$x_0^{(5)}$	63	249	4.024e-07	63	249	4.024e-07	63	249	4.024e-07	67	438	1.3061e-11
	$x_0^{(6)}$	63	249	4.7982e-07	63	249	4.7982e-07	63	249	4.7982e-07	65	422	1.4341e-11
	$x_0^{(7)}$	64	253	1.3466e-07	64	253	1.2886e-07	64	253	1.2542e-07	63	475	1.2461e-11
15000	$x_0^{(1)}$	65	260	2.6006e-06	65	260	2.6006e-06	65	260	2.6006e-06	154	872	2.0524e-11
	$x_0^{(2)}$	64	253	2.5869e-07	64	253	2.5869e-07	64	253	2.5869e-07	60	390	1.1109e-11
	$x_0^{(3)}$	64	253	6.3661e-07	64	253	6.3661e-07	64	253	6.3661e-07	69	420	2.625e-11
	$x_0^{(4)}$	63	249	1.8673e-06	63	249	1.8673e-06	63	249	1.8673e-06	109	572	2.5091e-11
	$x_0^{(5)}$	63	249	2.204e-06	63	249	2.204e-06	63	249	2.204e-06	129	696	1.0624e-09
	$x_0^{(6)}$	65	260	2.6006e-06	65	260	2.6006e-06	65	260	2.6006e-06	154	872	2.0524e-11
	$x_0^{(7)}$	64	253	7.152e-07	64	253	7.1582e-07	64	253	7.1845e-07	75	485	1.0672e-11
75000	$x_0^{(1)}$	70	290	5.6195e-06	70	290	5.6195e-06	70	290	5.6195e-06	218	1137	2.2381e-11
	$x_0^{(2)}$	64	253	5.7845e-07	64	253	5.7845e-07	64	253	5.7845e-07	69	416	1.0304e-11
	$x_0^{(3)}$	64	253	1.4235e-06	64	253	1.4235e-06	64	253	1.4235e-06	109	612	7.035e-11
	$x_0^{(4)}$	65	261	4.8965e-06	65	261	4.8965e-06	65	261	4.8965e-06	183	976	1.3785e-11
	$x_0^{(5)}$	66	266	5.3307e-06	66	266	5.3307e-06	66	266	5.3307e-06	200	1061	1.4496e-11
	$x_0^{(6)}$	70	290	5.6195e-06	70	290	5.6195e-06	70	290	5.6195e-06	218	1137	2.2381e-11
	$x_0^{(7)}$	64	253	1.597e-06	64	253	1.6003e-06	64	253	1.6027e-06	123	716	2.1714e-11
250000	$x_0^{(1)}$	79	344	8.3051e-06	79	344	8.3051e-06	79	344	8.3051e-06	-	-	-
	$x_0^{(2)}$	64	253	1.0561e-06	64	253	1.0561e-06	64	253	1.0561e-06	71	407	1.0269e-10
	$x_0^{(3)}$	64	254	2.957e-06	64	254	2.957e-06	64	254	2.957e-06	166	912	3.0792e-11
	$x_0^{(4)}$	70	291	8.5024e-06	70	291	8.5024e-06	70	291	8.5024e-06	242	1282	4.8521e-11
	$x_0^{(5)}$	74	314	8.223e-06	74	314	8.223e-06	74	314	8.223e-06	260	1379	1.1709e-11
	$x_0^{(6)}$	79	344	8.3051e-06	79	344	8.3051e-06	79	344	8.3051e-06	-	-	-
	$x_0^{(7)}$	66	264	2.7654e-06	66	264	2.7707e-06	66	264	2.7639e-06	180	1019	1.3425e-11

Table 4: *Numerical results for Discrete boundary value problem [4]*

Dm	Inip	MDFPM1			MDFPM2			MDFPM3			WDFPM		
		Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	-	-	-	-	-	-	-	-	-	126	1132	1.1105e-11
	$x_0^{(2)}$	-	-	-	-	-	-	-	-	-	144	1343	1.4376e-11
	$x_0^{(3)}$	-	-	-	-	-	-	-	-	-	158	1535	1.0043e-11
	$x_0^{(4)}$	-	-	-	-	-	-	-	-	-	110	990	1.1712e-11
	$x_0^{(5)}$	-	-	-	-	-	-	-	-	-	133	1318	1.5904e-11
	$x_0^{(6)}$	-	-	-	-	-	-	-	-	-	126	1132	1.1105e-11
	$x_0^{(7)}$	-	-	-	440	1853	5.2284e-11	-	-	-	124	1140	1.528e-11
15000	$x_0^{(1)}$	196	880	5.9101e-11	196	880	5.9101e-11	197	884	5.7378e-11	194	1429	1.3741e-11
	$x_0^{(2)}$	188	842	7.4937e-11	188	842	7.4937e-11	191	856	6.4991e-11	94	817	1.5943e-11
	$x_0^{(3)}$	193	865	8.1391e-11	193	865	8.1391e-11	196	879	7.1809e-11	111	939	1.0335e-11
	$x_0^{(4)}$	194	870	6.5105e-11	194	870	6.5105e-11	195	872	6.7821e-11	163	1241	1.5541e-11
	$x_0^{(5)}$	195	875	6.463e-11	195	875	6.463e-11	195	875	6.2734e-11	161	1137	1.4602e-11
	$x_0^{(6)}$	196	880	5.9101e-11	196	880	5.9101e-11	197	884	5.7378e-11	194	1429	1.3741e-11
	$x_0^{(7)}$	193	861	7.0582e-11	322	1381	6.0147e-11	338	1446	5.9438e-11	142	1202	1.1863e-11
75000	$x_0^{(1)}$	200	907	7.4595e-11	200	907	7.4595e-11	200	907	7.4094e-11	278	1910	1.4863e-11
	$x_0^{(2)}$	191	855	6.7056e-11	191	855	6.7056e-11	192	860	7.8108e-11	107	898	3.3344e-11
	$x_0^{(3)}$	193	864	5.8632e-11	193	864	5.8632e-11	192	858	6.0116e-11	162	1241	1.4194e-11
	$x_0^{(4)}$	195	876	6.6576e-11	195	876	6.6576e-11	196	880	6.2007e-11	204	1449	1.44e-11
	$x_0^{(5)}$	199	898	5.8639e-11	199	898	5.8639e-11	199	898	5.8243e-11	220	1566	1.4285e-11
	$x_0^{(6)}$	200	907	7.4595e-11	200	907	7.4595e-11	200	907	7.4094e-11	278	1910	1.4863e-11
	$x_0^{(7)}$	198	885	6.5861e-11	193	860	7.1162e-11	196	876	7.2705e-11	174	1289	1.3796e-11
250000	$x_0^{(1)}$	209	973	7.3157e-11	209	973	7.3157e-11	209	973	7.3082e-11	-	-	-
	$x_0^{(2)}$	193	864	5.9111e-11	193	864	5.9111e-11	194	869	6.6491e-11	136	1088	1.503e-11
	$x_0^{(3)}$	192	860	6.7085e-11	192	860	6.7085e-11	192	860	6.5271e-11	209	1649	1.4233e-11
	$x_0^{(4)}$	200	908	6.3016e-11	200	908	6.3016e-11	200	908	6.2589e-11	-	-	-
	$x_0^{(5)}$	203	926	5.737e-11	203	926	5.737e-11	201	917	7.5534e-11	-	-	-
	$x_0^{(6)}$	209	973	7.3157e-11	209	973	7.3157e-11	209	973	7.3082e-11	-	-	-
	$x_0^{(7)}$	198	885	7.3115e-11	200	896	6.3658e-11	200	897	6.3323e-11	203	1403	1.5414e-11

4

Table 5: *Numerical results for Exponential Function [1]*

Dm	Inip	MDFPM1			MDFPM2			MDFPM3			WDFPM		
		Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	63	250	2.6741e-07	63	250	2.6741e-07	63	250	2.6741e-07	65	405	1.63e-11
	$x_0^{(2)}$	58	229	2.4207e-07	58	229	2.4207e-07	58	229	2.4207e-07	40	274	4.3444e-11
	$x_0^{(3)}$	61	241	2.1637e-07	61	241	2.1637e-07	61	241	2.1637e-07	51	341	1.1266e-11
	$x_0^{(4)}$	61	241	3.4822e-07	61	241	3.4822e-07	61	241	3.4822e-07	67	503	1.6305e-11
	$x_0^{(5)}$	61	241	3.5497e-07	61	241	3.5497e-07	61	241	3.5497e-07	56	370	1.4622e-11
	$x_0^{(6)}$	63	250	2.6741e-07	63	250	2.6741e-07	63	250	2.6741e-07	65	405	1.63e-11
	$x_0^{(7)}$	94	375	1.5016e-11	94	375	1.5254e-11	94	375	1.5481e-11	60	428	1.2024e-11
15000	$x_0^{(1)}$	66	265	1.02e-06	66	265	1.02e-06	66	265	1.02e-06	135	771	1.0089e-11
	$x_0^{(2)}$	58	229	1.3259e-06	58	229	1.3259e-06	58	229	1.3259e-06	63	437	1.4114e-11
	$x_0^{(3)}$	59	233	2.1141e-06	59	233	2.1141e-06	61	241	1.1851e-06	66	392	1.8656e-11
	$x_0^{(4)}$	63	250	1.2991e-06	63	250	1.2991e-06	63	250	1.2991e-06	117	684	1.2482e-11
	$x_0^{(5)}$	64	255	1.2935e-06	64	255	1.2935e-06	64	255	1.2935e-06	127	744	1.5338e-11
	$x_0^{(6)}$	66	265	1.02e-06	66	265	1.02e-06	66	265	1.02e-06	135	771	1.0089e-11
	$x_0^{(7)}$	100	399	1.4752e-11	100	399	1.4759e-11	100	399	1.505e-11	80	540	1.0683e-11
75000	$x_0^{(1)}$	73	306	1.7036e-06	73	306	1.7036e-06	73	306	1.7036e-06	269	1489	1.7331e-11
	$x_0^{(2)}$	58	229	2.9648e-06	58	229	2.9648e-06	58	229	2.9648e-06	63	367	4.506e-11
	$x_0^{(3)}$	61	241	2.65e-06	61	241	2.65e-06	61	241	2.65e-06	103	549	2.6695e-11
	$x_0^{(4)}$	64	258	4.2442e-06	64	258	4.2442e-06	67	270	1.7813e-06	159	846	1.1736e-11
	$x_0^{(5)}$	67	275	4.0072e-06	67	275	4.0072e-06	67	275	4.0072e-06	180	1017	1.2284e-11
	$x_0^{(6)}$	73	306	1.7036e-06	73	306	1.7036e-06	73	306	1.7036e-06	269	1489	1.7331e-11
	$x_0^{(7)}$	103	411	1.3873e-11	103	411	1.3896e-11	103	411	1.4069e-11	128	748	1.3143e-11
250000	$x_0^{(1)}$	80	362	7.3749e-06	80	362	7.3749e-06	82	370	4.1341e-06	-	-	-
	$x_0^{(2)}$	58	229	5.4129e-06	58	229	5.4129e-06	58	229	5.4129e-06	80	460	3.2307e-11
	$x_0^{(3)}$	62	248	6.1942e-06	62	248	6.1942e-06	62	248	6.1942e-06	159	884	1.3878e-11
	$x_0^{(4)}$	71	298	5.4613e-06	71	298	5.4613e-06	70	294	7.2943e-06	264	1471	1.0103e-11
	$x_0^{(5)}$	74	318	5.6461e-06	74	318	5.6461e-06	74	318	5.6461e-06	-	-	-
	$x_0^{(6)}$	80	362	7.3749e-06	80	362	7.3749e-06	82	370	4.1341e-06	-	-	-
	$x_0^{(7)}$	107	430	1.4008e-11	107	430	1.4037e-11	107	430	1.4071e-11	170	948	6.9913e-11

Table 6: *Numerical results for Modified Strictly Convex Function 2 [5]*

		MDFPM1			MDFPM2			MDFPM3			WDFPM		
Dm	Inip	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	80	321	7.244e-09	80	321	7.244e-09	101	406	1.8885e-11	78	528	2.9591e-11
	$x_0^{(2)}$	99	359	1.1387e-07	99	359	1.1387e-07	99	359	1.2701e-07	83	557	1.8505e-11
	$x_0^{(3)}$	109	392	8.8574e-08	109	392	8.8574e-08	96	355	4.1035e-08	80	524	2.6518e-11
	$x_0^{(4)}$	88	313	9.7088e-06	88	313	9.7088e-06	102	370	1.3216e-07	68	446	3.0159e-11
	$x_0^{(5)}$	78	312	1.3792e-08	78	312	1.3792e-08	69	274	5.4523e-08	80	503	1.3494e-11
	$x_0^{(6)}$	80	321	7.244e-09	80	321	7.244e-09	101	406	1.8885e-11	78	528	2.9591e-11
	$x_0^{(7)}$	83	334	1.6757e-08	89	355	1.229e-09	78	310	7.2433e-09	68	460	1.0992e-11
15000	$x_0^{(1)}$	118	460	1.3225e-11	118	460	1.3225e-11	74	282	4.4884e-06	135	779	1.2263e-11
	$x_0^{(2)}$	81	305	5.8226e-07	81	305	5.8226e-07	80	303	5.5577e-07	117	723	1.5251e-11
	$x_0^{(3)}$	83	314	5.3373e-07	83	314	5.3373e-07	83	315	4.9962e-07	110	661	1.3754e-11
	$x_0^{(4)}$	79	299	1.3853e-06	79	299	1.3853e-06	118	458	1.3783e-11	114	668	1.2213e-11
	$x_0^{(5)}$	117	456	1.2773e-11	117	456	1.2773e-11	114	446	1.6038e-11	129	791	1.2001e-11
	$x_0^{(6)}$	118	460	1.3225e-11	118	460	1.3225e-11	74	282	4.4884e-06	135	779	1.2263e-11
	$x_0^{(7)}$	84	318	5.8409e-07	85	321	5.9406e-07	84	317	5.7065e-07	109	664	1.3654e-11
75000	$x_0^{(1)}$	117	470	1.6354e-11	117	470	1.6354e-11	118	473	1.6342e-11	199	1093	2.7085e-11
	$x_0^{(2)}$	131	555	1.4908e-11	131	555	1.4908e-11	133	568	1.3232e-11	244	1350	1.2778e-11
	$x_0^{(3)}$	75	290	1.3515e-06	75	290	1.3515e-06	74	286	1.197e-06	200	1091	3.1341e-11
	$x_0^{(4)}$	115	455	1.3815e-11	115	455	1.3815e-11	115	455	1.3426e-11	189	1044	2.5237e-11
	$x_0^{(5)}$	115	456	1.5226e-11	115	456	1.5226e-11	115	456	1.515e-11	199	1128	1.0491e-11
	$x_0^{(6)}$	117	470	1.6354e-11	117	470	1.6354e-11	118	473	1.6342e-11	199	1093	2.7085e-11
	$x_0^{(7)}$	75	291	1.3413e-06	77	297	1.3389e-06	74	286	1.6307e-06	227	1243	1.4303e-11
250000	$x_0^{(1)}$	126	520	1.3349e-11	126	520	1.3349e-11	126	520	1.3343e-11	-	-	-
	$x_0^{(2)}$	87	348	1.641e-07	87	348	1.641e-07	87	348	1.6317e-07	-	-	-
	$x_0^{(3)}$	120	504	4.3592e-10	120	504	4.3592e-10	107	454	3.3098e-08	356	1874	2.8065e-11
	$x_0^{(4)}$	120	479	1.3487e-11	120	479	1.3487e-11	119	474	1.474e-11	288	1545	1.1388e-11
	$x_0^{(5)}$	122	494	1.6457e-11	122	494	1.6457e-11	122	494	1.6424e-11	274	1460	2.1511e-11
	$x_0^{(6)}$	126	520	1.3349e-11	126	520	1.3349e-11	126	520	1.3343e-11	-	-	-
	$x_0^{(7)}$	80	317	1.014e-06	80	317	1.0136e-06	81	320	5.6223e-07	362	1921	1.1494e-11

Table 7: *Numerical results for Modified Tridiagonal Function [6, Problem 4.4]*

Dm	Inip	MDFPM1			MDFPM2			MDFPM3			WDFPM		
		Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	96	383	1.6047e-11	96	383	1.6047e-11	96	383	1.6047e-11	72	565	2.6555e-11
	$x_0^{(2)}$	101	403	1.3247e-11	101	403	1.3247e-11	101	403	1.3247e-11	72	464	1.9206e-11
	$x_0^{(3)}$	100	399	1.558e-11	100	399	1.558e-11	100	399	1.558e-11	71	441	1.5132e-11
	$x_0^{(4)}$	99	395	1.424e-11	99	395	1.424e-11	99	395	1.424e-11	68	437	1.1792e-11
	$x_0^{(5)}$	98	391	1.5263e-11	98	391	1.5263e-11	98	391	1.5263e-11	62	374	1.0777e-10
	$x_0^{(6)}$	96	383	1.6047e-11	96	383	1.6047e-11	96	383	1.6047e-11	72	565	2.6555e-11
	$x_0^{(7)}$	100	399	1.5499e-11	100	399	1.5751e-11	100	399	1.5655e-11	75	493	2.2735e-11
15000	$x_0^{(1)}$	70	277	1.6414e-07	70	277	1.6414e-07	70	277	1.6414e-07	71	423	2.5247e-11
	$x_0^{(2)}$	76	304	1.6931e-07	76	304	1.6931e-07	76	304	1.6931e-07	196	1115	3.3613e-11
	$x_0^{(3)}$	74	294	1.8071e-07	74	294	1.8071e-07	74	294	1.8071e-07	177	981	3.1546e-11
	$x_0^{(4)}$	73	289	1.4562e-07	73	289	1.4562e-07	73	289	1.4562e-07	140	802	1.4305e-11
	$x_0^{(5)}$	72	285	1.5606e-07	72	285	1.5606e-07	72	285	1.5606e-07	121	732	1.1041e-11
	$x_0^{(6)}$	70	277	1.6414e-07	70	277	1.6414e-07	70	277	1.6414e-07	71	423	2.5247e-11
	$x_0^{(7)}$	75	298	1.3658e-07	75	298	1.3662e-07	75	298	1.3606e-07	184	1069	1.0547e-11
75000	$x_0^{(1)}$	63	249	2.783e-06	63	249	2.783e-06	63	249	2.783e-06	143	787	4.3296e-11
	$x_0^{(2)}$	76	317	2.7503e-06	76	317	2.7503e-06	76	317	2.7503e-06	265	1387	3.3202e-11
	$x_0^{(3)}$	73	298	2.5665e-06	73	298	2.5665e-06	73	298	2.5665e-06	256	1366	1.2527e-11
	$x_0^{(4)}$	69	279	3.1507e-06	69	279	3.1507e-06	69	279	3.1507e-06	217	1170	1.3378e-11
	$x_0^{(5)}$	67	269	3.0378e-06	67	269	3.0378e-06	67	269	3.0378e-06	201	1116	2.5905e-11
	$x_0^{(6)}$	63	249	2.783e-06	63	249	2.783e-06	63	249	2.783e-06	143	787	4.3296e-11
	$x_0^{(7)}$	73	298	2.5898e-06	73	298	2.5892e-06	73	298	2.5882e-06	257	1376	1.3494e-11
250000	$x_0^{(1)}$	53	213	0.000188	53	213	0.000188	53	213	0.000188	208	1163	1.3323e-11
	$x_0^{(2)}$	75	340	0.00021199	75	340	0.00021199	75	340	0.00021199	-	-	-
	$x_0^{(3)}$	72	322	0.00021009	72	322	0.00021009	72	322	0.00021009	-	-	-
	$x_0^{(4)}$	63	269	0.00021617	63	269	0.00021617	63	269	0.00021617	278	1472	1.0214e-11
	$x_0^{(5)}$	58	238	0.00019762	58	238	0.00019762	58	238	0.00019762	246	1275	2.931e-11
	$x_0^{(6)}$	53	213	0.000188	53	213	0.000188	53	213	0.000188	208	1163	1.3323e-11
	$x_0^{(7)}$	74	332	0.00015254	74	332	0.00015255	74	332	0.00015252	-	-	-

Table 8: *Numerical results for Problem 2 [3]*

Dm	Inip	MDFPM1			MDFPM2			MDFPM3			WDFPM		
		Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm	Itr	Fvl	Norm
500	$x_0^{(1)}$	45	178	3.6851e-11	45	178	3.6851e-11	45	178	3.6851e-11	44	266	1.1538e-11
	$x_0^{(2)}$	43	171	3.4078e-11	43	171	3.4078e-11	43	171	3.4078e-11	48	388	1.5703e-11
	$x_0^{(3)}$	38	151	2.8586e-11	38	151	2.8586e-11	38	151	2.8586e-11	42	389	1.6462e-11
	$x_0^{(4)}$	45	179	2.1717e-11	45	179	2.1717e-11	45	179	2.1717e-11	50	405	1.0116e-11
	$x_0^{(5)}$	46	184	3.1335e-11	46	184	3.1335e-11	46	184	3.1335e-11	47	358	1.1602e-11
	$x_0^{(6)}$	45	178	3.6851e-11	45	178	3.6851e-11	45	178	3.6851e-11	44	266	1.1538e-11
	$x_0^{(7)}$	43	171	3.7593e-11	43	171	3.825e-11	44	175	2.4334e-11	50	379	2.2394e-11
15000	$x_0^{(1)}$	49	195	3.5782e-11	49	195	3.5782e-11	49	195	3.5782e-11	159	913	2.0709e-11
	$x_0^{(2)}$	46	183	2.7963e-11	46	183	2.7963e-11	46	183	2.7963e-11	48	313	2.3891e-11
	$x_0^{(3)}$	41	163	2.3435e-11	41	163	2.3435e-11	41	163	2.3435e-11	33	296	1.835e-11
	$x_0^{(4)}$	47	187	3.3565e-11	47	187	3.3565e-11	47	187	3.3565e-11	77	462	1.6412e-11
	$x_0^{(5)}$	49	196	2.5685e-11	49	196	2.5685e-11	49	196	2.5685e-11	111	666	1.5943e-11
	$x_0^{(6)}$	49	195	3.5782e-11	49	195	3.5782e-11	49	195	3.5782e-11	159	913	2.0709e-11
	$x_0^{(7)}$	46	183	3.1293e-11	46	183	3.1298e-11	46	183	3.8943e-11	56	403	5.7148e-11
75000	$x_0^{(1)}$	57	237	3.2715e-11	57	237	3.2715e-11	57	237	3.2715e-11	196	1052	1.0626e-11
	$x_0^{(2)}$	47	187	3.3232e-11	47	187	3.3232e-11	47	187	3.3232e-11	71	447	4.2886e-11
	$x_0^{(3)}$	42	167	2.7805e-11	42	167	2.7805e-11	42	167	2.7805e-11	40	319	1.7391e-11
	$x_0^{(4)}$	50	202	3.2047e-11	50	202	3.2047e-11	50	202	3.2047e-11	131	807	1.7893e-11
	$x_0^{(5)}$	53	217	3.0648e-11	53	217	3.0648e-11	53	217	3.0648e-11	145	800	1.7118e-11
	$x_0^{(6)}$	57	237	3.2715e-11	57	237	3.2715e-11	57	237	3.2715e-11	196	1052	1.0626e-11
	$x_0^{(7)}$	47	187	3.728e-11	47	187	3.7206e-11	48	191	2.4767e-11	84	580	1.6475e-11
250000	$x_0^{(1)}$	60	251	2.2538e-11	60	251	2.2538e-11	60	251	2.2538e-11	233	1309	1.335e-11
	$x_0^{(2)}$	49	196	2.3453e-11	49	196	2.3453e-11	49	196	2.3453e-11	113	710	3.9746e-11
	$x_0^{(3)}$	43	171	2.684e-11	43	171	2.684e-11	43	171	2.684e-11	42	348	1.3878e-11
	$x_0^{(4)}$	53	216	2.3315e-11	53	216	2.3315e-11	53	216	2.3315e-11	151	826	1.7486e-11
	$x_0^{(5)}$	56	231	2.1566e-11	56	231	2.1566e-11	56	231	2.1566e-11	183	1039	2.3453e-11
	$x_0^{(6)}$	60	251	2.2538e-11	60	251	2.2538e-11	60	251	2.2538e-11	233	1309	1.335e-11
	$x_0^{(7)}$	49	196	2.2261e-11	49	196	2.2234e-11	49	196	2.4508e-11	116	757	1.1531e-11

References

[1] Wang C, Wang Y, Xu C. A projection method for a system of nonlinear monotone equations with convex constraints. Mathematical Methods of Operations Research. 2007;66:33-46.

[2] Cruz WL, Raydan M. Nonmonotone spectral methods for large-scale nonlinear systems. Optimization Methods and software. 2003;18(5):583-99. Available from: `https://doi.org/10.1080/10556780310001610493`.

[3] Zhou W, Li D. LIMITED MEMORY BFGS METHOD FOR NONLINEAR MONOTONE EQUATIONS. Journal of Computational Mathematics. 2007 2023/05/25/;25(1):89-96.

[4] Moré JJ, Garbow BS, Hillstrom KE. Testing unconstrained optimization software. ACM Transactions on Mathematical Software (TOMS). 1981;7(1):17-41.

[5] Raydan M. The Barzilai and Borwein gradient method for the large scale unconstrained minimization problem. SIAM Journal on Optimization. 1997;7(1):26-33.

[6] Bing Y, Lin G. An efficient implementation of Merrills method for sparse or partially separable systems of nonlinear equations. SIAM Journal on Optimization. 1991;1(2):206-21.