Fundamentals of Computational Mathematics - Optimization

Project: Yield Curve Model Calibration

Overview:

The Nelson-Siegel model (1987) is one of the first used by central banks as a model for the term structure of interest rates. The model relies on four parameters that need to be properly set. The aim of this project is employing unconstrained optimization techniques to find optimal parameters that make the Nelson-Siegel model fit the yield curve of some bonds or other securities.

The model is expressed as:

$$R(t) = \beta_0 + \beta_1 \left(\frac{1 - e^{-t/\tau}}{t/\tau} \right) + \beta_2 \left(\frac{1 - e^{-t/\tau}}{t/\tau} - e^{-t/\tau} \right)$$

where:

- $R(t)$: Yield at time t.
- $\beta_0, \beta_1, \beta_2$: Parameters associated to the long-run yield level, slope of the yield, and curvature of the yield respectively.
- τ: Parameter influencing the rate at which yields converge to the long-term level.

Now, suppose we are given a data set $\{(t_i, \bar{R}(t_i)) : i = 1, \dots T\}$ with the yield of our bond of interest at several time stamps. In order to assess how well our model fits the data, we compute the sum of squared differences between observed and model-predicted yields:

$$f(\beta_0, \beta_1, \beta_2, \tau) = \sum_{i=1}^{T} \left(\bar{R}(t_i) - R(t_i) \right)^2 .$$

Tasks:

- **Data Collection:** Obtain historical yield data for various securities (short-term and medium-term). For instance, data on many Europeans bond can be found on the website investing.com.
- **Nelson-Siegel Model:** Investigate the Nelson-Siegel model and the interpretation of its parameters. Write Python code to compute $R(t)$ and $f(\beta_0, \beta_1, \beta_2, \tau)$.
- **Parameter Estimation:** adapt the code from the lab sessions for the gradient descent method with approximate line search and for the Newton's method and run experiments in order to minimize f. Try several starting points and step sizes. For each instance and each algorithm report the optimal solutions, optimal values and the number of iterations or, if significant, the running time. Plot how well the curve $(t, R(t))$ fits your data.
- **Extension to the Svensson model (optional):** look up the Svensson model, an extension of the Nelson-Siegel model that uses two additional parameters. Repeat the same experiments above on the Svensson model.

- **Comment on the results:** compare the performances of the different algorithms considered. Optional: search the literature for values (or ranges) of parameters that have actually been used for this model and compare them with yours; search the literature for other methods to do model calibration, and comment on why they might be preferable to gradient-based methods.

Report:

Your report should include:

- Your names and identification numbers.
- A description of the problem, of your approach and the algorithms used, focusing on the methodology and justifying the choices you made.
- A comment and analysis of the results as indicated above.
- All the code and the data used.
- Citations to any relevant source.
- (Optional) suddivision of the work within the team.

The report should be submitted via email (`francesco.marchetti@unipd.it`**,** `manuel.aprile@unipd.it` **) as a single PDF file, no later than February 16$^{\mathbf{th}}$**.

Presentation:

Prepare a concise presentation with slides summarizing the key aspects of the project, focusing on the problem statement, methodology, results, and insights. Each member of the team should take part in the presentation. Presentations should take around 20 minutes, plus extra time for questions.