Math 2602

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COMPUTER PROJECT-3

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(*) We are given that

$$\alpha p - \nabla \cdot k \nabla p = f \quad \text{in } \Omega \subset \mathbb{R}^2, \quad p = g \text{ on } \partial\Omega$$

$$\frac{\partial c}{\partial t} + u \cdot \nabla c = 0, \quad c(x,y,0) = c_0(x,y)$$

Question 1

(*) The mixed variational formulation of the flow equation can be obtained as:

$$(k^{-1}u, v) - (p, \nabla \cdot v) = -\int_{\partial\Omega} g\, v \cdot n, \quad \forall v \in H(\mathrm{div}, \Omega)$$

$$(\alpha p, w) + (\nabla \cdot u, w) = (f, w), \quad \forall w \in L^2(\Omega)$$

(*) In this question we have

-> $\Omega = (0,1) \times (0,1)$

-> $\alpha = 1$

-> $k = \begin{cases} 0.0001, & 0.4 < x,y < 0.6 \\ 1, & \text{otherwise} \end{cases}$

-> $f = 0$

-> $g = 1 - x$

-> The initial condition for the transport equation:

$$c_0(x,y) = \exp(-100 * ((x-0.2)^2 + (y-0.5)^2))$$

-> $h = 1/40$, $\Delta t = 0.01$ and final time $t = 1$.

Question 2

(*) The following is the expression that is used for the finite element variational formulation of the flow equation

$$(k \nabla p, \nabla v) + (\alpha p, v) = (f, v)$$

$$p = g \text{ on } \partial\Omega$$

(*) We use the same parameters as in Question 1 described above.

(*) We used RTO spaces and P1 spaces to solve Question 1 and 2, respectively.

(*) We used the Freefem++ function convect for the transport equation

(*) Two graphs for contaminant concentration, one for pressure and one for velocity are attached for each question to the end of this report.

(*) Since k is discontinuous, we do not expect velocity vectors to enter the small square in the middle of the domain. We can observe that this happens for the mixed method, which shows that mixed FEM gives a better solution

(*) The 2 .edp files in the submission contain the codes for the problems.