Math 2602 Computer Project 5

Due April 27, 2022

The goal of this project is to test overlapping and non-overlapping domain decomposition algorithms for mixed finite element discretizations of elliptic problems in Freefem++.

Part I: getting familiar

- Read Section 5.8 Domain Decomposition from the manual. Make sure you understand the methods and the codes for all three examples.
- Run the three examples in examples/tutorial/ : schwarz-overlap.edp, schwarz-no-overlap.edp, and schwarz-gc.edp.
- Add a convergence criteria to schwarz-overlap.edp. Use the code in examples/mpi/schwarz-b.edp as an example of how to do this. Note: schwarz-b.edp runs in parallel with mpi and uses broadcast. You do not need to use broadcast, since you are running on one processor. Increase the maximum number of iterations to 500. Change the convergence tolerance to 1e-2:

  if (err < 1e-2) break;

  The reason for choosing this tolerance is that the numerical error due to the grids being non-matching in the overlap region cannot be reduced any further.
- Run the code for n = 4,8,16,32. Report the number of iterations versus n and discuss the results.
- Submit your modified code.
- In schwarz-gc.edp, change the LinearCG tolerance to 1e-6 and max number of iterations to 500:

  LinearCG(BoundaryProblem,p[],q[],eps=1.e-6,nbiter=500);
- Run the code for n = 4,8,16,32. Report the number of CG iterations versus n and discuss the results.
- Change the convergence tolerance to 1e-2. Check that you get the same solution as in schwarz-overlap.edp.
- Run the code for n = 4,8,16,32. Report the number of iterations versus n. These should be the same as in schwarz-overlap.edp.

Part II: DD for MFE

- Following the templates from the modified schwarz-overlap.edp and schwarz-gc.edp from Part I, write two-subdomain overlapping Schwartz and non-overlapping Shur-complement based codes for solving

  $$\alpha p - \nabla \cdot K \nabla p = f \quad \text{in } \Omega \subset \boldsymbol{R}^2, \qquad p = g \quad \text{on } \partial\Omega,$$

  using the mixed finite element method with RT0 spaces.

  Note: schwarz-gc.edp solves local Neumann problems and iterates for matching pressures. For the MFE implementation need to modify to solve local Dirichlet problems and iterate to match fluxes. Therefore the funcion BoundaryProblem needs to be modified.

- In schwarz-overlap.edp, change the convergence tolerance to 1e-2 and the maximum number of iterations to 500.

- In schwarz-gc.edp, change the LinearCG tolerance to 1e-6 and max number of iterations to 500:

  LinearCG(BoundaryProblem,p[],q[],eps=1.e-6,nbiter=500);

- Run the problems from Project 2:

  - True solution $p(x, y) = x^3 + y^3$, $\Omega = (0, 1) \times (0, 1)$, $\alpha = 1$, $K = 1$.
  - Same as above, except $K(x, y) = 1/(1 + 10(x^2 + y^2))$.
  - Ω is the L-shaped domain obtained by removing the upper-right quarter from the unit square; $\alpha = 1$, $K = 1$, $f = 1$, $g = 0$.
  - $\Omega = (0, 1) \times (0, 1)$, $\alpha = 1$,

    $$K = \left\{ \begin{array}{ll} 100, & 0 < x, y < 1/2 \text{ and } 1/2 < x, y < 1, \\ 1, & \text{otherwise}, \end{array} \right\}$$

    $f = 0$, $g = (1 - x)|_{\partial\Omega}$.

- In all cases take the interface to be along $x = .5$. For the overlapping algorithm choose overlap $1/10$.

- For all examples run the code for $h = 1/10, 1/20, 1/40, 1/80$. Report the number of iterations versus h and discuss the results.

- Submit your code.

- **Bonus:** Using the last test case, check the dependence of the convergence on the jump in K by keeping h fixed and taking $K = 10, 100, 1000, 10000$ in $0 < x, y < 1/2$ and $1/2 < x, y < 1$. Report and discuss your results.