

### Math 3071 Computer Project 3

Due December 15, 2021

1. Write a finite element code (using a programming language of your choice) for solving

$$\begin{aligned} -\nabla \cdot (a(x, y) \nabla u) + b(x, y)u &= f(x, y) \quad \text{in } \Omega = (0, 1)^2 \\ u &= g(x, y) \quad \text{on } \partial\Omega \end{aligned}$$

where  $a$ ,  $b$ ,  $f$ , and  $g$  are given functions. Use continuous piecewise linears on a grid of right triangles obtained by first partitioning  $\Omega$  with a rectangular grid and then dividing all rectangles with diagonals in the same direction.

2. Apply your code to the following problems:

A) True solution  $p(x, y) = \sin(\pi x) \sin(\pi y)$

i)  $a(x, y) = 1$ ,  $b(x, y) = 1$

ii)  $a(x, y) = 1/(1 + 10(x^2 + y^2))$ ,  $b(x, y) = 0$

B) True solution

$$p(x, y) = \begin{cases} x^2 y^3 + \cos(xy), & 0 \leq x \leq 1/2 \\ ((2x + 9)/20)^2 y^3 + \cos(((2x + 9)/20)y), & 1/2 \leq x \leq 1 \end{cases}$$

$$a(x, y) = \begin{cases} 1, & 0 \leq x < 1/2 \\ 10, & 1/2 < x \leq 1 \end{cases} \quad b(x, y) = 0$$

In all cases compute  $f(x, y)$  and  $g(x, y)$  by plugging the true solution into the differential equation.

3. Estimate the convergence of finite element error in  $H^1$  and  $L^2$  for the above cases. To do this you need to run the code for several levels of refinement of the finite element grid. Plot the computed solutions for all cases for  $h = 1/4$  and  $h = 1/32$ . Submit your code and a discussion on its implementation and the results.