Math 2602 Computer Project 2

Due February 21, 2022

The goal of this project is to test the convergence of the mixed finite element method with Raviart-Thomas elements using the software Freefem++.

- Run the demo LaplaceRT.edp in examples/tutorial. Make sure you understand what it does. Information on the finite element space RT0 can be found in the manual.
- Replace the solver option GMRES with LU and remove all GMRES-related parameters from the calling sequence. GMRES does not work well for larger problems. I recommend using LU for all your tests.
- You can also look at examples/misc/aaRT.edp for another example using RT0.
- Write a user code for solving

$$\alpha p - \nabla \cdot K \nabla p = f \quad \text{in } \Omega \subset \boldsymbol{R}^2, \qquad p = g \quad \text{on } \partial\Omega,$$

  using the mixed finite element method with RT0 spaces.
- Run the following problems.
  - True solution $p(x,y) = x^3 + y^3$, $\Omega = (0,1) \times (0,1)$, $\alpha = 1$, $K = 1$.
  - Same as above, except $K(x,y) = 1/(1 + 10(x^2 + y^2))$.
    In these two cases compute $f(x,y)$ and $g(x,y)$ by substituting the true solution into the differential equation.
  - Ω is the L-shaped domain obtained by removing the upper-right quarter from the unit square; $\alpha = 1$, $K = 1$, $f = 1$, $g = 0$.
  - $\Omega = (0,1) \times (0,1)$, $\alpha = 1$,

$$K = \left\{ \begin{array}{ll} 100, & 0 < x, y < 1/2 \text{ and } 1/2 < x, y < 1, \\ 1, & \text{otherwise,} \end{array} \right\}$$

    $f = 0$, $g = (1 - x)|_{\partial\Omega}$. This case corresponds to flow from left to right through porous media with discontinuous permeability.
- For all cases:
  - Estimate the convergence for the finite element errors $\|p - p_h\|_{L^2(\Omega)}$, $\|\mathbf{u} - \mathbf{u}_h\|_{L^2(\Omega)}$, and $\|\nabla \cdot (\mathbf{u} - \mathbf{u}_h)\|_{L^2(\Omega)}$. To do this run the code for four levels of refinement starting with $h = 1/10$. In the last two cases, use the computed solution with $h = 1/160$ as a true solution. Discuss your results.
  - Plot and submit the computed pressure and velocity, as well as the pressure error for $h = 1/40$. Comment on the distribution of the error.
  - Submit your code.