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Math 2602

COMPUTER PROJECT-2

We are given that

$$\alpha p - \nabla \cdot K \nabla p = f \quad \text{in } \Omega \in \mathbb{R}^2, \quad p = g \text{ on } \partial\Omega$$

$\Rightarrow$ The variational formulation can be obtained as:

$$(K^{-1}u, v) - (p, \nabla \cdot v) = -\int_{\partial\Omega} g\, v \cdot n, \quad \forall v \in H(\mathrm{div}, \Omega)$$

$$(\alpha p, w) + (\nabla \cdot u, w) = (f, w), \quad \forall w \in L^2(\Omega)$$

- We solved the problem using the mixed finite element method with RT0 spaces by using the FreeFem++ software.
- In the submission the 4 .edp files containing the codes for the codes for the problems.
- The plots are attached to the end of the report.

Question 1

- In this question we have
  - $\to p(x,y) = x^3 + y^3$
  - $\to \Omega = (0,1) \times (0,1)$
  - $\to \alpha = 1$
  - $\to K = 1$
  - $\to f = x^3 + y^3 - (6x + 6y)$
  - $\to g = x^3 + y^3$
- The errors can be obtained as:

h	$\lVert p - p_h \rVert$	order	$\lVert u - u_h \rVert$	order	$\lVert \nabla \cdot (u - u_h) \rVert$	order
$\frac{1}{10}$	0.050461		0.068492		0.244953	
$\frac{1}{20}$	0.025265	0.9980	0.035002	0.9685	0.122475	1
$\frac{1}{40}$	0.012636	0.9996	0.017623	0.9900	0.061237	1
$\frac{1}{80}$	0.006318	1	0.008830	0.9970	0.030618	1

- In theory we know that for RT^k spaces we expect $O(h^{k+1})$ convergence. So, for RT^0, we expect $O(h)$ convergence. In the error table, we observe that the numerical results are displaying the expected convergence.
- The plots for the computed pressure and velocity, as well as the pressure error for $h=1/40$ is attached to the end of this report.

Question 2

- In this question we have
  - $\rightarrow p(x,y) = x^3+y^3$
  - $\rightarrow \Omega = (0,1)\times(0,1)$
  - $\rightarrow \alpha = 1$
  - $\rightarrow K = 1/(1+10(x^2+y^2))$
  - $\rightarrow f = x^3+y^3 - \dfrac{[6x[1+10(x^2+y^2)] - 3x^2(20x) + 6y(1+10(x^2+y^2)] - 3y^2 20y]}{[1+10(x^2+y^2)]^2}$
  - $\rightarrow g = x^3+y^3$

- The errors can be obtained as:

h	$\lVert p-p_h\rVert$	order	$\lVert u-u_h\rVert$	order	$\lVert\nabla\cdot(u-u_h)\rVert$	order
$\frac{1}{10}$	0.050444		0.006415		0.033972	
$\frac{1}{20}$	0.025262	0.9977	0.003229	0.9904	0.017150	0.9861
$\frac{1}{40}$	0.012636	0.9994	0.001617	0.9978	0.008595	0.9966
$\frac{1}{80}$	0.006318	1	0.000809	0.9991	0.004300	0.9992

- The numerical results are $O(h)$ as expected. The plots are attached.

Question 3

- In this question, we have
  - $\rightarrow p(x,y) =$ (the computed solution with $h=1/160$ for error analysis)
  - $\rightarrow \Omega =$ the L-shaped domain obtained by removing the upper-right quart of unit square.
  - $\rightarrow \alpha = 1$
  - $\rightarrow K = 1$
  - $\rightarrow f = 1$
  - $\rightarrow g = 0$

- The errors can be obtained as:

h	$\|p-p_h\|$	order	$\|u-u_h\|$	order	$\|\nabla\cdot(u-u_h)\|$	order
$\frac{1}{10}$	0.002679		0.025077		0.002679	
$\frac{1}{20}$	0.001299	1.0443	0.014598	0.7806	0.001299	1.0443
$\frac{1}{40}$	0.000632	1.0394	0.008090	0.8516	0.000632	1.0394
$\frac{1}{80}$	0.000282	1.1642	0.004840	0.7411	0.000282	1.1642

- The numerical results are close to $O(h)$ as expected. The pressure error is larger at the corner point $(\frac{1}{2},\frac{1}{2})$ (which can be observed in the plot), and the order $\|u-u_h\|$ is less than 1, these can be because of the kinked structure of Ω at $(\frac{1}{2},\frac{1}{2})$. All the plots are attached

$\boxed{\text{Question 4}}$

- In this question, we have
  - $\rightarrow$ $p(x,y) =$ (the computed solution with $h=1/60$)
  - $\rightarrow$ $\Omega = (0,1)\times(0,1)$
  - $\rightarrow$ $\alpha = 1$
  - $\rightarrow$ $K = \begin{cases} 100, & 0<x,y<1/2 \text{ and } 1/2<x,y<1 \\ 1, & \text{otherwise} \end{cases}$
  - $\rightarrow$ $f = 0$
  - $\rightarrow$ $g = 1-x$

- The errors can be obtained as:

h	$\|p-p_h\|$	order	$\|u-u_h\|$	order	$\|\nabla\cdot(u-u_h)\|$	order
$\frac{1}{10}$	0.035640		10.9462		0.035640	
$\frac{1}{20}$	0.019593	0.8632	6.82945	0.6806	0.019593	0.8632
$\frac{1}{40}$	0.009833	0.9946	3.72358	0.8751	0.009833	0.9946
$\frac{1}{80}$	0.003385	1.5385	1.97495	0.9149	0.003385	1.5385

- The numerical results are close to $O(h)$ as expected.
- In the plot for pressure error, it can be observed that the error is bigger in the bottom left and upper right quarter of the region, this is because of the discontinuous structure of the permeability function K.
- The plots for all cases are attached in the following 4 pages.