Math 2602 Computer Project 3

Due March 25, 2022

The goal of this project is to simulate coupled flow and transport using the mixed finite element method with Raviart-Thomas elements in Freefem++.

- Write a user code for solving

$$\alpha p - \nabla \cdot K \nabla p = f \quad \text{in } \Omega \subset \boldsymbol{R}^2, \qquad p = g \quad \text{on } \partial\Omega,$$

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0, \qquad c(x, y, 0) = c_0(x, y),$$

  where $\mathbf{u} = -K\nabla p$ is the Darcy velocity computed in the flow equation. Use the mixed finite element method with RT0 spaces for the flow equation. Use the Freefem++ function `convect` for the transport equation (see Section 9.5.2 for information on `convect`).

  Note: You can use the velocity vector $[u_1, u_2]$ directly in the `convect` function.

- Run the following problem.

  $\Omega = (0,1) \times (0,1)$, $\alpha = 1$, $f = 0$. Use $g(x, y) = 1 - x$ to impose pressure boundary conditions (this will give flow from left to right).

$$K = \begin{cases} 0.0001, & 0.4 < x, y < 0.6, \\ 1, & \text{otherwise}, \end{cases}$$

  The initial condition for the transport equation is

$$c_0(x, y) = \exp(-100 * ((x - 0.2)^2 + (y - 0.5)^2)).$$

  This corresponds to a plume of contaminant.

  Take $h = 1/40$, $\Delta t = .01$ and final time $t = 1$.

- You can see the moving front by plotting the solution c on every time step and using `wait=0`.

- Next run a modified simulation, using velocity computed with the Galerkin continuous P1-finite element method. You will need to define the elements of the velocity vector `[vel1,vel2]`, where

$$\texttt{vel1} = -\texttt{K} * \texttt{dx(p)}; \texttt{vel2} = -\texttt{K} * \texttt{dy(p)};$$

  to be in the continuous P1 space, in order to work with `convect`.

- For both cases, plot and submit the computed pressure and velocity, as well as the contaminant concentration at the final time. Compare the quality of the solutions with the mixed FEM and the Galerkin FEM methods.

- Submit your code.