Math 2090 Numerical Solution of ODEs
Final Project, Due December 4, 2020

Consider the advection-diffusion boundary-value problem

$$-\epsilon u''(t) + au'(t) = 0, \quad 0 < t < 1,$$

$$u(0) = 0, \quad u(1) = 1,$$

where $0 < \epsilon << 1$ and a are given constants.

- Write a code for solving the above problem numerically using the shooting method, the symmetric finite difference method, and the upwind finite difference method. Describe the methods used in the code.
- Take $\epsilon = 0.01$, $a = 1$, $1/h = 20, 40, 80, 160$. Plot the solutions from the three methods in these cases. Explain the results in relation to the stability and convergence properties of the methods.
- The true solution of the problem is

  $$u(t) = \frac{1 - e^{Rt}}{1 - e^{R}},$$

  where $R = a/\epsilon$ (R is called the Peclet number). Establish numerically the order of convergence of the three methods.
- For the finite difference methods, employ error estimation and adaptive mesh refinement as described in class. Comment on the quality of the nonuniform mesh and the solution obtained with the method.