UNIVERSITY OF GHANA
MATH 223-CALCULUS II
Quick Revison chapters 1,2,3

1- a) By using the definition of the natural logarithmic function, prove that for any $x > 0$,

$$\frac{1}{x+1} < \ln\left(\frac{x+1}{x}\right) < \frac{1}{x}.$$

Deduce that

$$\frac{1}{3} + \ln 2 < \ln 3 < \frac{1}{2} + \ln 2.$$

b) Let $f(x) = 2\log_2^2 x - \frac{3}{\ln 2}\ln x + 1$, $x > 0$.

(i) Find the intervals where f is stricly increasing or strictly decreasing and explain why f is a one-to-one function from these intervals to their corresponding ranges to be specified.

(ii) Find the value of $\left(f^{-1}\right)'(-\frac{1}{8})$.

c) Find the values of x that satisfy the equation

$$12\cosh^2 x + 7\sinh x = 24.$$

2- a) Find the following limits:

$$\lim_{x\to 0} x^{\ln(1+x)};\ \lim_{x\to\infty} \frac{x-\sqrt{x}}{coshx - \sinh x};\ \lim_{x\to 0}\frac{\sin(2x)}{\sinh(7x)};\ \lim_{h\to 0}\left(1+\frac{h}{3}\right)^{1/h}.$$

b) By using the MVT, prove that for any $x > 0$,

$$x < \sinh x < x\cosh x.$$

Deduce that

$$\ln 2 < \frac{3}{4} < \frac{5}{4}\ln 2.$$

c) i) Find the derivative of the function $f(x) = (\sin x)^{\tan x}$ defined for $x \in (0,\pi)\setminus\{\frac{\pi}{2}\}$.
ii) Given that $\sinh x = \frac{5}{12}$, find the value of $\cosh x$, $\tanh x$, $\sinh(2x)$, $\cosh(2x)$, $\coth x$.