

Wave Equations & Verification of $v^2 \frac{\partial^2 y(x,v t)}{\partial x^2} = \frac{\partial^2 y(x,v t)}{\partial t^2}$

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y[x_, t_] := Exp[-(x + t)^2] // Quiet; (*The equation of the wave, has to have this form: y(x,t)=y(x±vt). I have put four examples, try whatever comes to your mind*)

y[x_, t_] := 1/(x + t)^2 + 1 // Quiet ;

y[x_, t_] := Exp[-Abs[x + t]] // Quiet ;

y[x_, t_] := Sin[x + t] // Quiet ;

HoldForm[v^2 D[y[x, v t], {x, 2}] - D[y[x, v t], {t, 2}]] // TraditionalForm (*You can quickly verify this relation through the next line*)

v^2 \frac{\partial^2 y(x, v t)}{\partial x^2} - \frac{\partial^2 y(x, v t)}{\partial t^2}

SameQ[FullSimplify[v^2 D[y[x, v t], {x, 2}]], FullSimplify[D[y[x, v t], {t, 2}]]] (*You can test this with different difinitions of y(x,t)*)

True
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v^2 D[y[x, v t], {x, 2}] // FullSimplify (*If you want to check yourself*)
D[y[x, v t], {t, 2}] // FullSimplify

2 e^{-(t v + x)^2} v^2 (-1 + 2 (t v + x)^2)

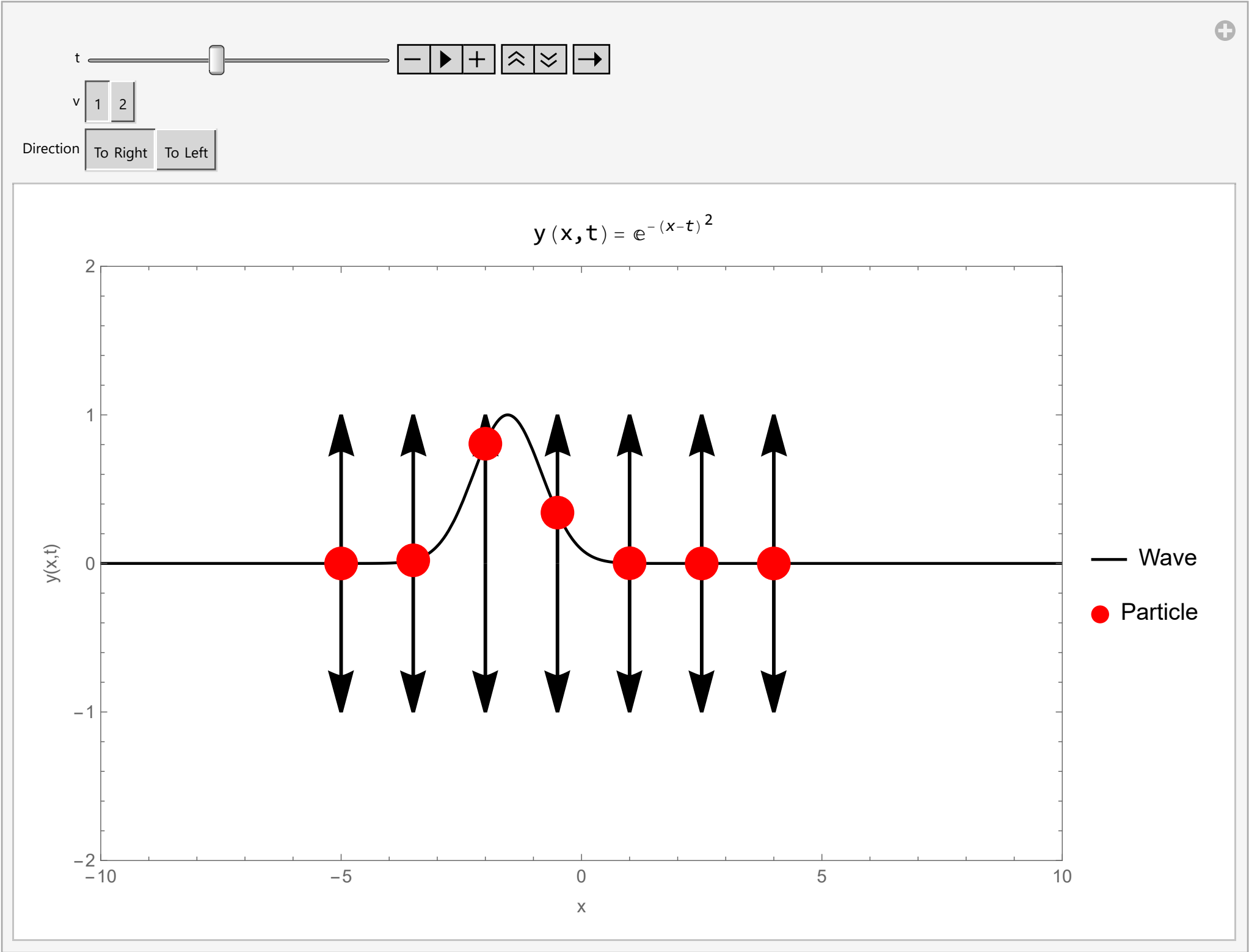
2 e^{-(t v + x)^2} v^2 (-1 + 2 (t v + x)^2)
```

Animations

```
prt = Table[{i, y[i, c q]}, {i, -5, 5, 1.5}]; (*Contact me if u want explaining to the rest :P*)

l1 = Table[Arrow[{prt[[j]][1], 0}, {prt[[j]][1], 1}], {j, 1, prt // Length}];
l1k = Table[Arrow[Reverse@{prt[[j]][1], -1}, {prt[[j]][1], 0}], {j, 1, prt // Length}];

Manipulate[Grid[{{"y(x,t)=" Block[{Plus, Times}, With[{result = y[x, c v t]}, HoldForm[result] // TraditionalForm]}},
  {Legended[Show[Plot[y[x, c q], {x, -10, 10}, ImageSize -> Large, Prolog -> {Directive[{Thick, Black}], l1, l1k}, PlotStyle -> Black,
    PlotRange -> {{-10, 10}, {-2, 2}}, Axes -> False, Frame -> True, FrameLabel -> {"x", "y(x,t)"}],
    Graphics[{Red, PointSize[0.035], Point /@ Table[{i, y[i, c q]}, {i, -5, 5, 1.5}], PlotRange -> {{-10, 10}, {-2, 2}}]},
    {LineLegend[{Black}, {"Wave"}], PointLegend[{Directive[Red, PointSize[0.035]}], {"Particle"}]}]}], ItemSize -> {{Full}, {Full}}},
  {{q, -10, "t"}, -10, 10, AnimationRate -> 4 v, Appearance -> "Open", ControlType -> Animator}, {{v, 1, "v"}, {1, 2}}
, {{c, -1, "Direction"}, {-1 -> "To Right", 1 -> "To Left"}}, AutorunSequencing -> {0, 0}] // Quiet
```



```
Manipulate[Grid[{{"y(x,t)=" Block[{Plus, Times}, With[{result = y[x, + c v t] - y[x + 1, c v t]}, HoldForm[result] // TraditionalForm]}},
  {Legended[Show[Plot[y[x, c q] - y[x + 1, c q], {x, -10, 10}, ImageSize -> Large, Prolog -> {Directive[{Thick, Black}], l1, l1k}, PlotStyle -> Black,
    PlotRange -> {{-10, 10}, {-2, 2}}, Axes -> False, Frame -> True, FrameLabel -> {"x", "y(x,t)"}],
    Graphics[{Red, PointSize[0.035], Point /@ Table[{i, y[i, c q] - y[i + 1, c q]}, {i, -5, 5, 1.5}], PlotRange -> {{-10, 10}, {-2, 2}}]},
    {LineLegend[{Black}, {"Wave"}], PointLegend[{Directive[Red, PointSize[0.035]}], {"Particle"}]}]}], ItemSize -> {{Full}, {Full}}},
  {{q, -10, "t"}, -10, 10, AnimationRate -> 4 v, Appearance -> "Open", ControlType -> Animator}, {{v, 1, "v"}, {1, 2}}
, {{c, -1, "Direction"}, {-1 -> "To Right", 1 -> "To Left"}}] // Quiet
```

