Phys. 373 Formula Sheet

$$f(x) = \underbrace{\sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k}_{P_n(x)} + \underbrace{\frac{f^{(n+1)}(\xi(x))}{(n+1)!} (x - x_0)^{n+1}}_{R_n(x)}$$

$$c = \frac{a+b}{2}; \quad |error| = |r - c_n| \le E_a^n = \frac{b-a}{2^n}; \quad n \ge \left\lceil \frac{\log(b-a) - \log(\varepsilon)}{\log(2)} \right\rceil$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}; \quad X_{k+1} = X_k - [F'(X_k)]^{-1} F(X_k)$$

$$F(X) = \begin{bmatrix} f_1(x_1, x_2, \dots) \\ f_1(x_1, x_2, \dots) \\ \vdots \end{bmatrix}; \quad F'(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$x_{i+1} = x_i - f(x_i) \frac{(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$$y = mx + b;$$
 $m = \frac{N\Sigma(xy) - \Sigma x \Sigma y}{N\Sigma(x^2) - (\Sigma x)^2};$ $b = \frac{\Sigma y - m(\Sigma x)}{N}$

$$\Phi(a_0, a_1, \dots) = \sum_{i=0}^{N} |y_i - f(x_i)|^2; \quad \frac{\partial \Phi(a_0, a_1, \dots)}{\partial a_i} = 0 \quad ; \forall a_i \implies Normal \ Equations$$

$$P_n(x) = \sum_{i=0}^{n} b_i \prod_{k=0}^{i-1} (x - x_k); \quad b_i : The \ i^{th} \ DD$$

$$P_n(x) = \sum_{i=0}^n f(x_i)\ell_i(x); \quad \ell_i(x) = \prod_{j=0, \ j\neq i}^n \frac{(x-x_j)}{(x_i-x_j)}; \quad |f(x)-P_n(x)| \le \frac{M}{4(n+1)} \left(\frac{b-a}{n}\right)^{n+1}$$

$$\int \approx \frac{L+U}{2}; \quad Error \leq \frac{U_L}{2}; \quad L(f,P) = \sum_{i=0}^{n-1} m_i(x_{i+1} - x_i); \quad U(f,P) = \sum_{i=0}^{n-1} M_i(x_{i+1} - x_i)$$

$$I_T = (b-a)\frac{f(b) + f(a)}{2}; \quad I_T = \sum_{i=0}^{n-1} \frac{1}{2} (x_{i+1} - x_i) (f(x_{i+1}) + f(x_i))$$

$$I_T = h\left[\frac{1}{2}\left[f(x_0) + f(x_n)\right] + \sum_{i=1}^{n-1} f(x_i)\right]; \quad |Error| \le \frac{b-a}{12}h^2 \max|f''(x)_{x \in [a,b]}|$$

$$S_{1/3} = \frac{h}{3} \sum_{i=1}^{n} \Delta f(x_i); \quad \Delta \equiv 1 \overline{42} 1$$

$$S_{3/8} = \frac{3h}{8} \sum_{i=0}^{n} \Delta f(x_i); \quad \Delta \equiv 1 \ \overline{3} \ \overline{3} \ \overline{2} \ 1$$

$$S_{3/8} = \frac{3h}{8} \left\{ f(x_0) + 3 \sum_{i=1,4,7,\dots}^{n-2} f(x_i) + 3 \sum_{i=2,5,8,\dots}^{n-1} f(x_i) + 2 \sum_{i=3,6,9,\dots}^{n-2} f(x_i) + f(x_n) \right\}$$