```
n = 150;
U[Nup_] := n - 2 Nup // N;
M[Nup_] := -U[Nup]/n // N;
OmegaN[Nup_] := Binomial[n, Nup] // N;
S[Nup_] := Log[OmegaN[Nup]] // N;
T[Nup_] := (U[Nup + 1] - U[Nup - 1])/(S[Nup + 1] - S[Nup - 1]) // N;
Cv[Nup_] := ((S[Nup+1] - S[Nup-1])/(T[Nup+1] - T[Nup-1]))/n * T[Nup] // N;
OmegaA1[Nup_] := h!/(k! b!) /. h_! -> h^h Exp[-h] /. h -> n /. k -> Nup /. b -> (n - Nup) // FullSimplify;
OmegaA2[Nup_] := h!/(k! b!) /. h_! -> h^h Exp[-h] Sqrt[2 π h] /. h -> n /. k -> Nup /. b -> (n - Nup) // FullSimplify;
SA1[Nup_] := Log[OmegaA1[Nup]] // N;
SA2[Nup_] := Log[OmegaA2[Nup]] // N;
TA1[Nup_] := (U[Nup + 1] - U[Nup - 1])/(SA1[Nup + 1] - SA1[Nup - 1]) // N;
```

Figure 1: Mathematica code used for this homework

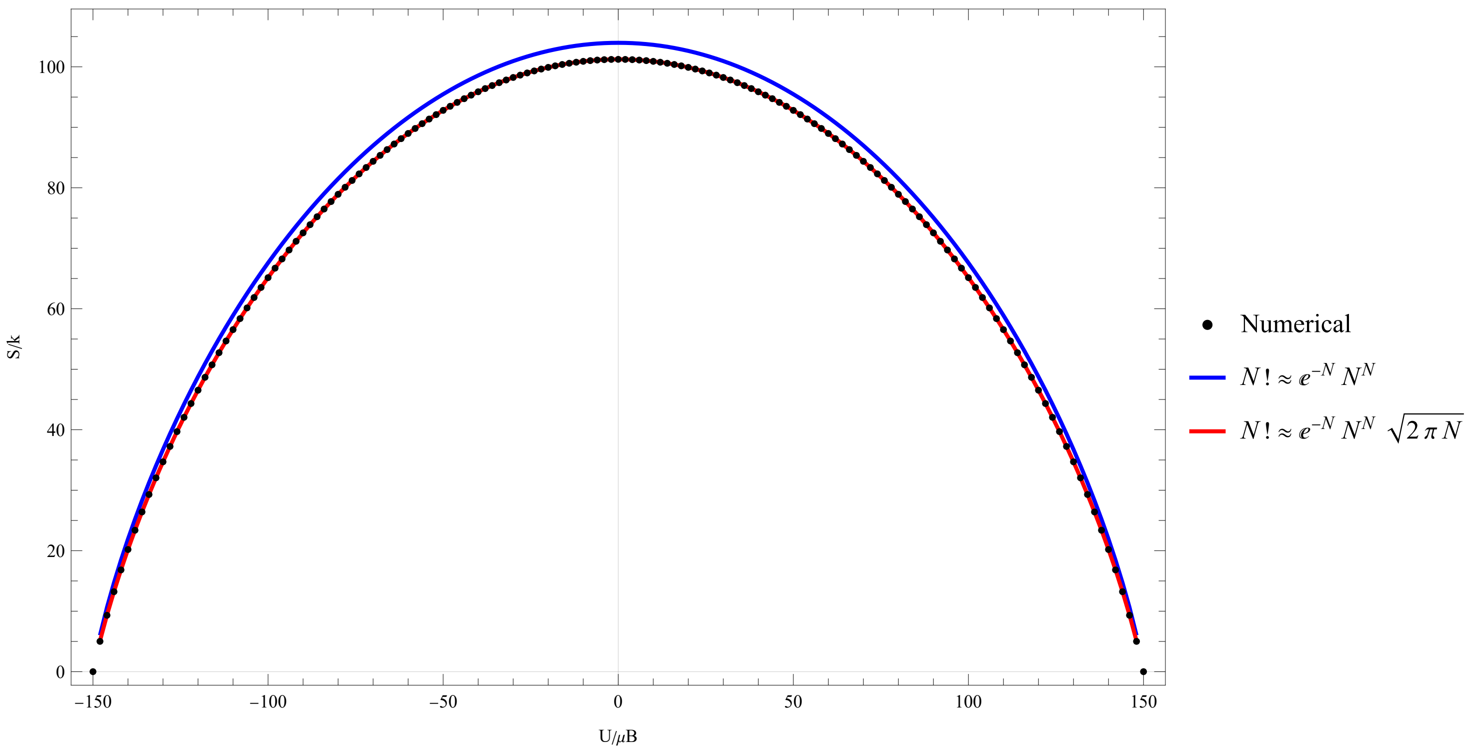

Figure 2: Q1) Entropy as a function of energy for a two-state paramagnet consisting of 150 elementary dipoles.

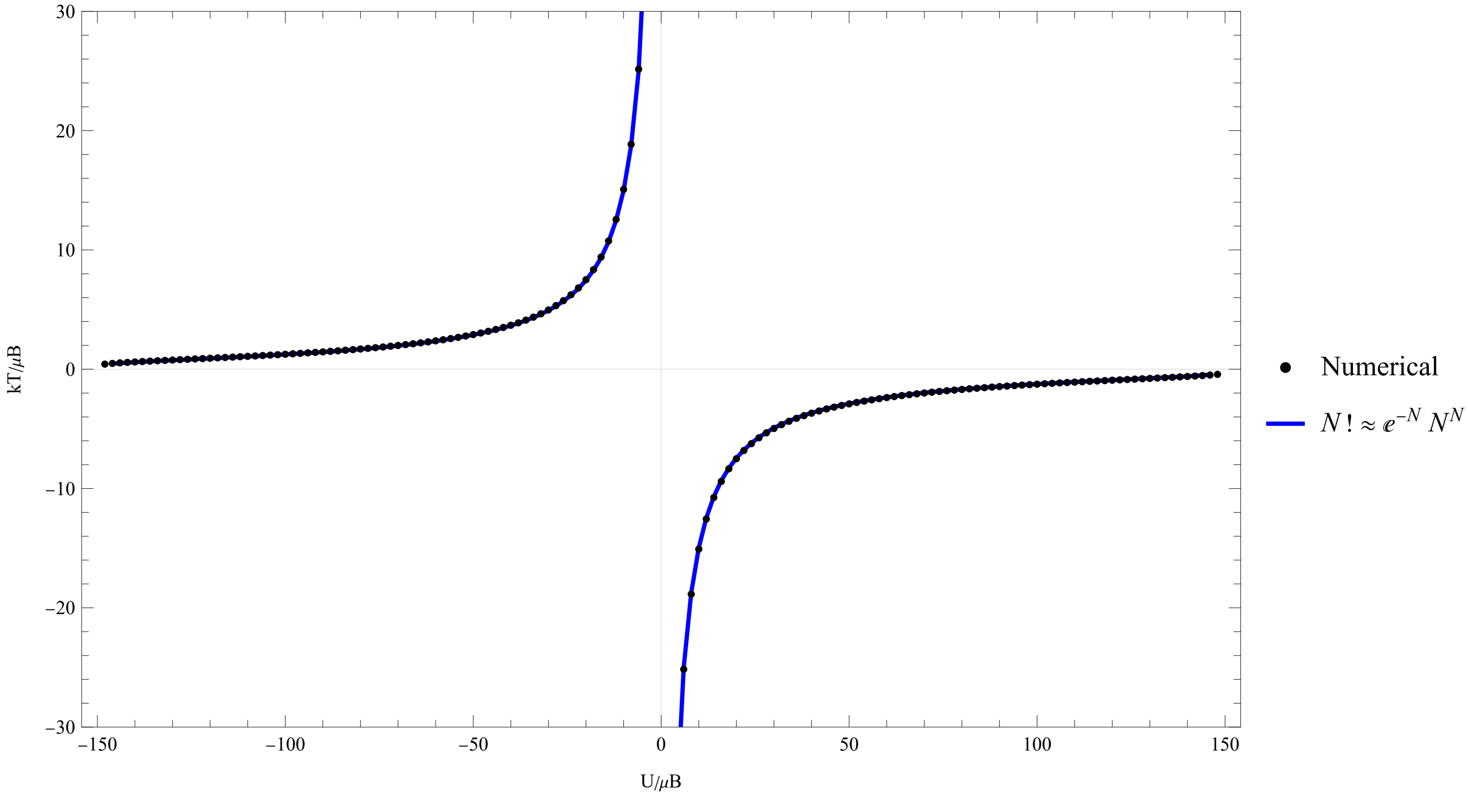

Figure 3: Q2) Temperature as a function of energy for a two-state paramagnet consisting of 150 elementary dipoles.

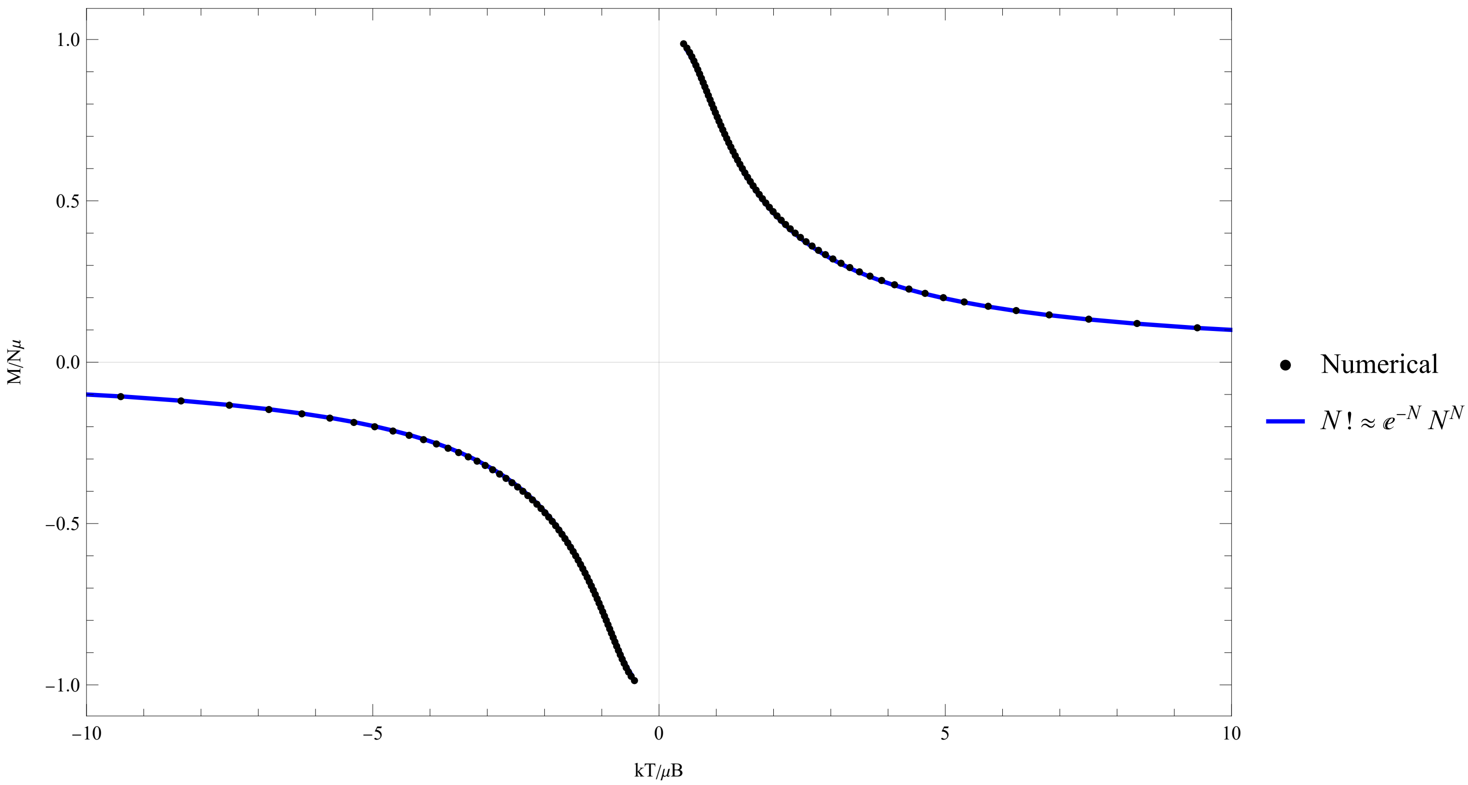

Figure 4: Q3) Magnetization for a two-state paramagnet consisting of 150 elementary dipoles.

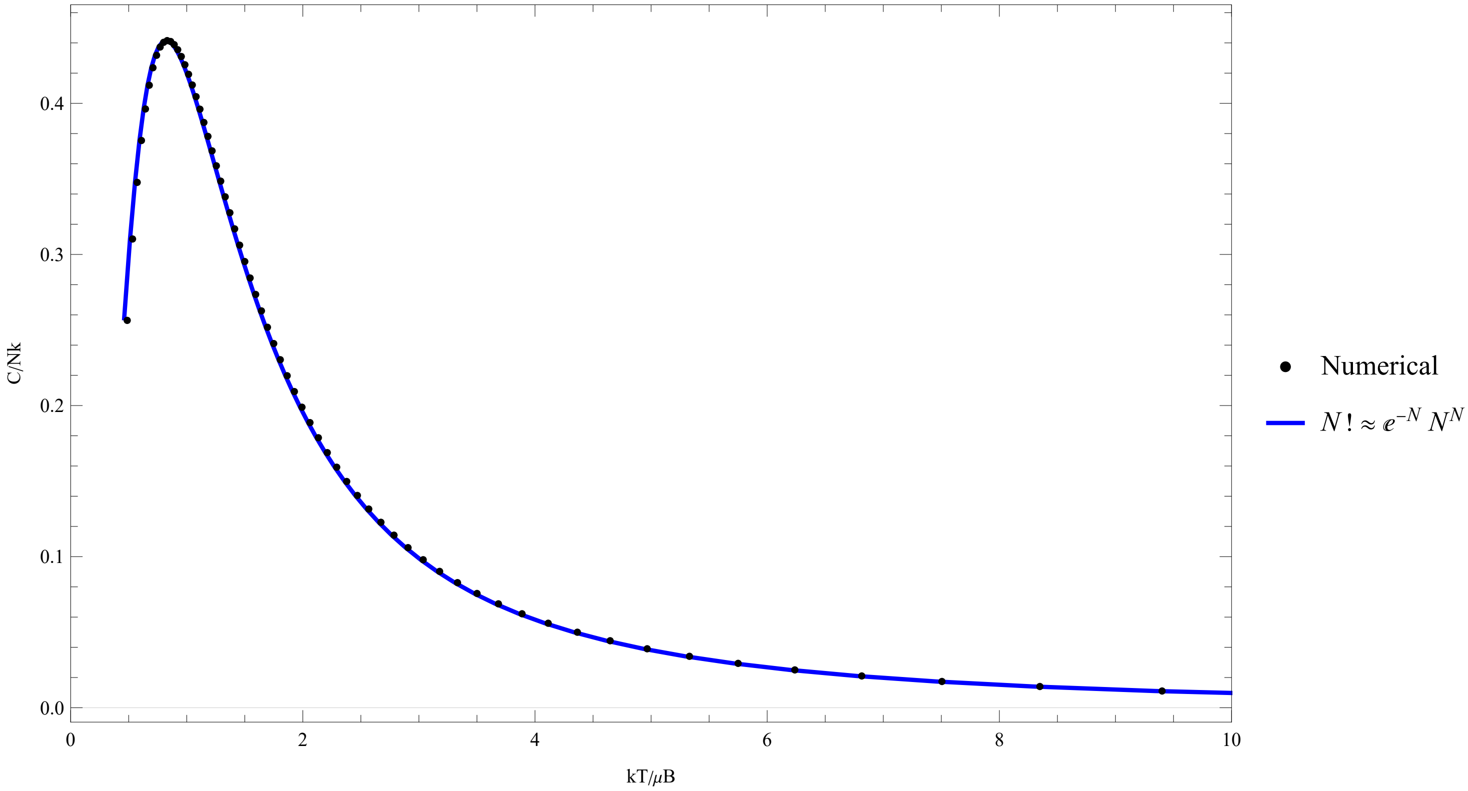

Figure 5: Q4) Heat capacity for a two-state paramagnet consisting of 150 elementary dipoles.

Q5): Using $dU = TdS - PdV$ & $H \equiv U + PV$:

$$C_V \equiv \left(\frac{\partial U}{\partial T}\right)_V ; \qquad C_P \equiv \left(\frac{\partial H}{\partial T}\right)_P$$

For constant volume:

$$\left(\frac{\partial U}{\partial T}\right)_V = T\frac{\partial S}{\partial T} - \cancelto{0}{P\frac{\partial V}{\partial T}} \implies C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

For constant pressure:

$$\left(\frac{\partial H}{\partial T}\right)_P = \frac{\partial U}{\partial T} + P\frac{\partial V}{\partial T} = T\frac{\partial S}{\partial T} \cancelto{0}{-P\frac{\partial V}{\partial T} + P\frac{\partial V}{\partial T}} \implies C_P = T\left(\frac{\partial S}{\partial T}\right)_P$$