Verification of the wave equation

```
HoldForm[v^2 D[y[x, v t], {x, 2}] "=?=" D[y[x, v t], {t, 2}]] // TraditionalForm(*We want to verify this*)
```

$$v^2 \frac{\partial^2 y(x, v\,t)}{\partial x^2} \stackrel{?}{=} \frac{\partial^2 y(x, v\,t)}{\partial t^2}$$

```
y[x_, t_] := Exp[-(x + t)^2] // Quiet;(*The equation of the wave, has to have this form: y(x,t)=y(x±vt). I have put four examples, try whatever comes to your mind*)

y[x_, t_] := 1/((x + t)^2 + 1) // Quiet ;

y[x_, t_] := Exp[-Abs[x + t]] // Quiet ;

y[x_, t_] := Sin[x + t] // Quiet ;

y[x_, t_] := t/(x + t) (*Not a wave!*)

SameQ[FullSimplify[v^2 D[y[x, v t], {x, 2}]], FullSimplify[D[y[x, v t], {t, 2}]]](*You can test this with different difinitions of y(x,t)*)
```

```
True
```

```
v^2 D[y[x, v t], {x, 2}] // FullSimplify(*If you want to check the derivatives yourself*)
D[y[x, v t], {t, 2}] // FullSimplify
```

$$2\, e^{-(t\,v+x)^2}\, v^2 \left(-1 + 2\,(t\,v + x)^2\right)$$

$$2\, e^{-(t\,v+x)^2}\, v^2 \left(-1 + 2\,(t\,v + x)^2\right)$$

Interactive plot

```
prt = Table[{i, y[i, c q]}, {i, -5, 5, 1.5}];(*Contact me if u want explaining to the rest :P*)
ll = Table[Arrow[{{prt[[j]][[1]], 0}, {prt[[j]][[1]], 1}}], {j, 1, prt // Length}];
lk = Table[Arrow[Reverse@{{prt[[j]][[1]], -1}, {prt[[j]][[1]], 0}}], {j, 1, prt // Length}];
Manipulate[
  Grid[{{"y(x,t)=" Block[{Plus, Times}, With[{result = Evaluate[α /. {0 → y[x, +c V t], 1 → y[x, +c V t] - y[2 x, c V t]}]}, HoldForm[result] // TraditionalForm]]},
    {Legended[Show[Plot[y[x, c q] - α y[2 x, c q], {x, -10, 10}, ImageSize → Large, Prolog → {Directive[{Thick, Black}], ll, lk}, PlotStyle → Black,
        PlotRange → {{-10, 10}, {-2, 2}}, Axes → False, Frame → True, FrameLabel → {"x", "y(x,t)"}],
       Graphics[{Red, PointSize[0.035], Point /@ Table[{i, y[i, c q] - α *y[2 i, c q]}, {i, -5, 5, 1.5}]}, PlotRange → {{-10, 10}, {-2, 2}}]],
      {LineLegend[{Black}, {"Wave"}], PointLegend[{Directive[Red, PointSize[0.035]]}, {"Particle"}]}]}}, ItemSize → {{Full}, {Full}}],
  {{q, -10, "t"}, -10, 10, AnimationRate → 2 V, Appearance → "Open", ControlType → Animator}, {{V, 1, "v="}, {1, 2}}
  , {{c, -1, "Direction: "}, {-1 → "To Right", 1 → "To Left"}}, {{α, 0, "Type: "}, {0 → "One Wave", (1 // FullSimplify) → "Two Waves"}}] // Quiet
```