جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

Intro. to JW Solution to Kitaev Honeycomb Model

A Summary of PHYS497 Progress

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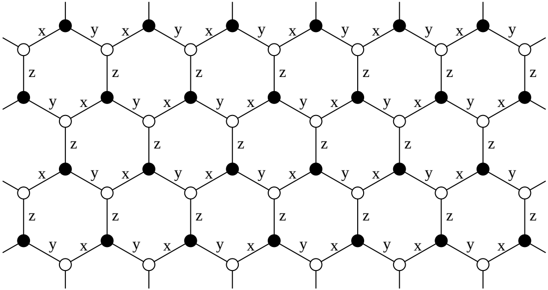
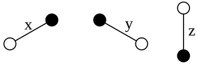
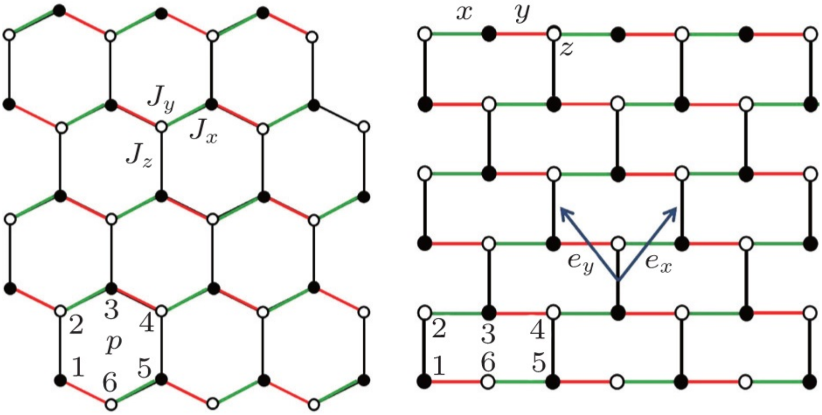
Kitaev's Honeycomb Hamiltonian

$$H = -\left(J_x \sum_{x-links} \sigma_j^x \sigma_k^x + J_y \sum_{y-links} \sigma_j^y \sigma_k^y + J_z \sum_{z-links} \sigma_j^z \sigma_k^z \right)$$

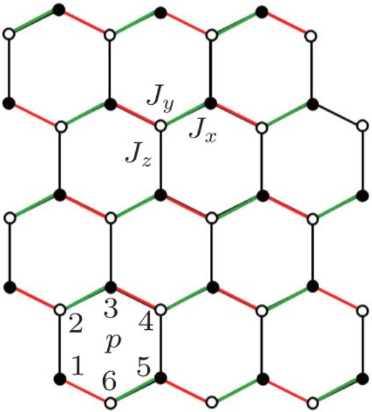


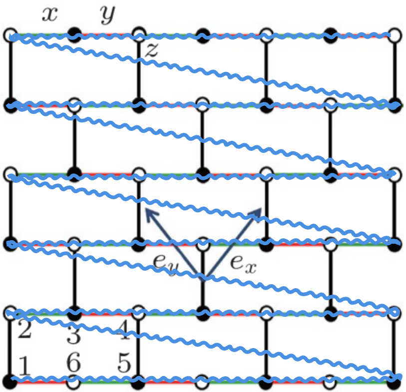


Deforming The Hamiltonian



Threading The Lattice





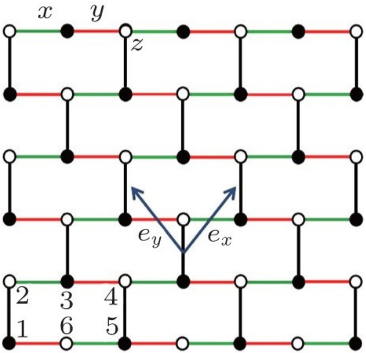
Jordan-Wigner Definition

$$\sigma_{ij}^{+} = 2\left[\prod_{j'<j}\prod_{i'}\sigma_{i'j'}^{z}\right]\underbrace{\left[\prod_{i'<i}\sigma_{i'j}^{z}\right]}_{1D\ String} c_{ij}^{\dagger}$$

$$\sigma_{ij}^{z} = 2c_{ij}^{\dagger}c_{ij} - 1$$

$$\sigma_{ij}^{x} = \frac{1}{2}\left(\sigma_{ij}^{+} + \sigma_{ij}^{-}\right)$$

$$\sigma_{ij}^{y} = \frac{i}{2}\left(\sigma_{ij}^{-} - \sigma_{ij}^{+}\right)$$



Example

$$\sigma^x_{i,j}\sigma^x_{i+1,j} = \frac{1}{4}\left(\sigma^+_{i,j}\sigma^+_{i+1,j} + \sigma^+_{i,j}\sigma^-_{i+1,j} + \sigma^-_{i,j}\sigma^+_{i+1,j} + \sigma^-_{i,j}\sigma^-_{i+1,j}\right)$$

$$\Longrightarrow\; c^\dagger_{i,j}c^\dagger_{i+1,j} + c^\dagger_{i,j}c_{i+1,j} - c_{i,j}c^\dagger_{i+1,j} - c_{i,j}c_{i+1,j}$$

$$\Longrightarrow\; \left(c^\dagger_{i,j} - c_{i,j}\right)\left(c^\dagger_{i+1,j} + c_{i+1,j}\right)$$

$$A_w = \frac{\left(c - c^\dagger\right)_w}{i}; \quad B_w = \left(c^\dagger + c\right)_w$$

$$A_b = \left(c^\dagger + c\right)_b; \quad B_b = \frac{\left(c - c^\dagger\right)_b}{i}$$

$$H = -iJ_x \sum_{x-links} A_w A_b + iJ_y \sum_{y-links} A_b A_w + J_z \sum_{z-links} B_b B_w A_b A_w$$

New 1-Fermion Operator

$$H = -iJ_x \sum_{x-links} A_w A_b + iJ_y \sum_{y-links} A_b A_w + J_z \sum_{z-links} B_b B_w A_b A_w$$

$$d = \frac{A_w + iA_b}{2}; \qquad d^\dagger = \frac{A_w - iA_b}{2}$$

$$\begin{aligned} H = J_x \sum_r \left(d_r^\dagger + d_r\right)\left(d_{r+\hat{e}_x}^\dagger + d_{r+\hat{e}_x}\right) \\ + J_y \sum_r \left(d_r^\dagger + d_r\right)\left(d_{r+\hat{e}_y}^\dagger + d_{r+\hat{e}_y}\right) \\ + J_z \sum_r \pm 1 \left(2d_r^\dagger d_r - 1\right) \end{aligned}$$

Thank you!