

Q 1:

$$\begin{aligned}
 P_A^\mu + P_B^\mu &= P_C^\mu + P_D^\mu \\
 \text{Transform using } \Lambda_\mu^\nu & \\
 \Lambda_\mu^\nu(P_A^\mu + P_B^\mu) &= \Lambda_\mu^\nu(P_C^\mu + P_D^\mu) \\
 P_A^\nu + P_B^\nu &= P_C^\nu + P_D^\nu
 \end{aligned}$$

Which is also conserved.

Q 2:

- a) $p + p \rightarrow p + p + \pi^0$
- b) $p + p \rightarrow p + p + \pi^+ + \pi^-$
- c) $\pi^- + p \rightarrow p + \bar{p} + n$
- d) $\pi^- + p \rightarrow K^0 + \Sigma^0$
- e) $p + p \rightarrow p + \Sigma^+ + K^0$

Problem 3.16 result:

$$E_{min} = \frac{M^2 - m_A^2 - m_B^2}{2m_B} c^2; \quad M = m_1 + m_2 + \dots + m_n$$

a):

$$E_{min} = \frac{(2m_p + m_{\pi^0})^2 - 2m_p^2}{2m_p} c^2 = 1218 \text{ MeV}$$

b):

$$E_{min} = \frac{(2m_p + m_{\pi^+} + m_{\pi^-})^2 - 2m_p^2}{2m_p} c^2 = 1538 \text{ MeV}$$

c):

$$E_{min} = \frac{(2m_p + m_n)^2 - m_{\pi^-}^2 - m_p^2}{2m_p} c^2 = 3747 \text{ MeV}$$

d):

$$E_{min} = \frac{(m_{K^0} + m_{\Sigma^0})^2 - m_{\pi^-}^2 - m_p^2}{2m_p} c^2 = 1043 \text{ MeV}$$

e):

$$E_{min} = \frac{(2m_p + m_{K^0} + m_{\Sigma^+})^2 - 2m_p^2}{2m_p} c^2 = 21734 \text{ MeV}$$

Q 3:

a) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

b) $\pi^0 \rightarrow \gamma + \gamma$

c) $K^+ \rightarrow \pi^+ + \pi^0$

d) $\Lambda \rightarrow p + \pi^-$

e) $\Omega^- \rightarrow \Lambda + K^-$

Problem 3.19 result:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2; \quad E_C = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A} c^2$$

a):

$$E_{\mu^-} = \frac{m_{\pi^-}^2 + m_{\mu^-}^2 - m_{\bar{\nu}_\mu}^2}{2m_{\pi^-}} c^2 = 109.8 \text{ MeV}; \quad E_{\bar{\nu}_\mu} = \frac{m_{\pi^-}^2 + m_{\bar{\nu}_\mu}^2 - m_{\mu^-}^2}{2m_{\pi^-}} c^2 = 29.8 \text{ MeV}$$

b):

$$E_\gamma = \frac{m_{\pi^0}^2 + m_\gamma^2 - m_\gamma^2}{2m_{\pi^0}} c^2 = 67.5 \text{ MeV}$$

c):

$$E_{\pi^+} = \frac{m_{K^+}^2 + m_{\pi^+}^2 - m_{\pi^0}^2}{2m_{K^+}} c^2 = 248.1 \text{ MeV}; \quad E_{\pi^0} = \frac{m_{K^+}^2 + m_{\pi^0}^2 - m_{\pi^+}^2}{2m_{K^+}} c^2 = 245.6 \text{ MeV}$$

d):

$$E_p = \frac{m_\Lambda^2 + m_p^2 - m_{\pi^-}^2}{2m_\Lambda} c^2 = 943.6 \text{ MeV}; \quad E_{\pi^-} = \frac{m_\Lambda^2 + m_{\pi^-}^2 - m_p^2}{2m_\Lambda} c^2 = 172.1 \text{ MeV}$$

e):

$$E_\Lambda = \frac{m_{\Omega^-}^2 + m_\Lambda^2 - m_{K^-}^2}{2m_{\Omega^-}} c^2 = 1135 \text{ MeV}; \quad E_{K^-} = \frac{m_{\Omega^-}^2 + m_{K^-}^2 - m_\Lambda^2}{2m_{\Omega^-}} c^2 = 536.6 \text{ MeV}$$

Q 4 :

First, we need to find the velocity of the muon using:

$$P_\mu = -P_\nu; \quad P_\mu, P_\nu \text{ are 4-momentum vector}$$

After working out the 4-momentum conservation, we will find this value of the muon velocity (Example 3.3):

$$v = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} c = 0.27c$$

$$\gamma = \frac{1}{\sqrt{1 - 0.27^2}} = 1.039$$

$$d = \gamma v \tau = 1.039 \times (0.27 \times 3 \times 10^8) \times (2.20 \times 10^{-6}) = 184 \text{ m}$$

Q 5:

$$s = \frac{(P_A + P_B)^2}{c^2} = \frac{(\frac{E_A + E_B}{c})^2 - (p_A + p_B)^2}{c^2}$$

Since $p_A = -p_B$ & $E_A = E_B = E$ and $E = \sqrt{p^2 c^2 + m^2 c^4}$:

$$s = \frac{\frac{4E^2}{c^2} - 0}{c^2} = \frac{4(p^2 c^2 + m^2 c^4)}{c^4} = \frac{4(p^2 + m^2 c^2)}{c^2} \checkmark$$

$$t = \frac{(P_A - P_C)^2}{c^2} = \frac{(\frac{E_A - E_C}{c})^2 - (p_A - p_C)^2}{c^2}$$

Since $E_A = E_C = E$ and $p_A^2 = p_C^2 = p^2$:

$$t = \frac{(\frac{E-E}{c})^2 - (p_A - p_C)^2}{c^2} = -\frac{p^2 + p^2 - 2p_A \cdot p_C}{c^2} = -\frac{2p^2(1 - \cos \theta)}{c^2} \checkmark$$

For u, same as t excepts the cosine of the angle between p_A and p_D will yield negative that of p_A and p_c , so:

$$u = \frac{(\frac{E-E}{c})^2 - (p_A - p_D)^2}{c^2} = -\frac{p^2 + p^2 - 2p_A \cdot p_D}{c^2} = -\frac{2p^2(1 + \cos \theta)}{c^2} \checkmark$$

$$m_{\pi^0} = 139.57 \text{ MeV}/c^2$$

$$m_{\Sigma^0} = 1192.64 \text{ MeV}/c^2$$

$$m_{K^0} = 497.61 \text{ MeV}/c^2$$

$$m_{\Omega^-} = 1672.45 \text{ MeV}/c^2$$

$$m_{\pi^\pm} = 139.57 \text{ MeV}/c^2$$

$$m_{\Sigma^+} = 1189.37 \text{ MeV}/c^2$$

$$m_{K^\pm} = 493.67 \text{ MeV}/c^2$$

$$m_\Lambda = 1115.68 \text{ MeV}/c^2$$