

Numerical Homework 2

Please solve the following questions and include both the analytical part and computational details along with programs and plots.

Q1. Obtain the Fourier Series expansion of the function:

$$F(t) = \begin{cases} -1, & -\pi/\omega \leq t \leq 0 \\ +1, & 0 \leq t \leq \pi/\omega \end{cases}$$

in the interval $-\pi/\omega \leq t \leq \pi/\omega$. Take $\omega = 1 \text{ rad/s}$. In the periodical interval, calculate and plot sums of the first two terms, the first three terms, then $N = 5, 10, \dots$ etc to demonstrate the convergence of the series for an optimal value of $N = \text{number of terms in the series}$. Identify the value of N at which you believe full convergence has been achieved, also make your own remarks about the interval boundary effect.

Q2. Obtain the Fourier Series expansion of the function

$$F(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2 \\ 0, & \text{elsewhere } (t < -T/2) \cup (t > T/2) \end{cases}$$

i.e. $F(t)=1$ in the interval $[-T/2, T/2]$, recall that $T = 2\pi/\omega$ and take $T = 2$ for numerical computations. In the periodical interval, calculate the plot sums of the first two terms, the first three terms, then $N = 5, 10, \dots$ etc to demonstrate the convergence of the series for an optimal value of $N = \text{number of terms in the series}$. Identify the value of N at which you believe full convergence has been achieved, also make your own remarks about the interval boundary effect.