

Name:

ID:

**PHYS 373 – Introduction to Computational Physics
Spring 2023 (Term 222)**

Quiz 2

Date: Tuesday, Feb 14th, 2023

Consider the function $f(x) = e^{-0.5x} + 1 - 0.5x^2$

- A) [2.5 Points] Prove that there is a root in the interval [1, 2]
B) [2.5 Points] Use Newton method to find the zero of the function correct to four decimal points. Start with $x_0 = 1.5$.
C) [2.5 Points] Perform 2 iterations of the Secant method, use $x_0=1$, $x_1=2$.
D) [2.5 Points] How many steps of the Bisection method would be needed to find the root with 4 decimal points rounded for interval [1, 2]?

Solution:

A) $a=1$; $b=2$

$$f(a) = 1.106531, f(b) = -0.6321206$$

$$f(a)f(b) = -0.699460 < 0 \text{ there is at least one root between 1 and 2}$$

B)

$$f(x) = e^{-0.5x} + 1 - 0.5x^2$$

$$f'(x) = -0.5e^{-0.5x} - x$$

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.700074817830975$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.690811316857495$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.690791209504067$$

$$|x_3 - x_2| = \dots$$

x_3 is correct to at least 3 DP rounded

C) Secant method

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{(f(x_1) - f(x_0))} = 1.689035455565734$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{(f(x_2) - f(x_1))} = 1.690787397629161$$

D) $n > (\log(b-a) - \log(10^{-4})) / \log(2) = 13.28 \rightarrow n=14$