Formula Sheet PHYS305 2nd Major Exam Sem211

$$\begin{split} \overrightarrow{\nabla T} &= \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi} \\ \overrightarrow{\nabla} \cdot \overrightarrow{v} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \ v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \\ \overrightarrow{\nabla} \times \overrightarrow{v} &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta \ v_\phi) - \frac{\partial v_\phi}{\partial \phi} \right] \hat{r} \\ &\quad + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} \\ &\quad + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \\ \overrightarrow{\nabla}^2 T &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \\ \overrightarrow{\nabla} \overrightarrow{V} &= \left(\frac{\partial T}{\partial s} \right) \hat{s} + \frac{1}{s} \left(\frac{\partial T}{\partial \phi} \right) \hat{\phi} + \left(\frac{\partial T}{\partial z} \right) \hat{z} \\ \overrightarrow{\nabla} \cdot \overrightarrow{v} &= \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial}{\partial \phi} (v_\phi) + \frac{\partial v_z}{\partial z} \\ \overrightarrow{\nabla} \times \overrightarrow{v} &= \left(\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right) \hat{\phi} \\ &\quad + \frac{1}{s} \left(\frac{\partial}{\partial s} (s v_s) - \frac{\partial v_s}{\partial \phi} \right) \hat{z} \\ \overrightarrow{\nabla}^2 T &= \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial T}{\partial s} \right) + \frac{1}{s^2} \left(\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{\partial^2 T}{\partial z^2} \\ \overrightarrow{\nabla} \cdot \frac{\hat{r}}{r^2} &= 4\pi \delta^3 (\vec{r}) \\ e^{kx} &+ e^{-kx} &= 2 \cosh(kx) \\ V(r, \theta) &= \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l (\cos \theta) \\ V(r, \phi) &= a_o \ln(r) + b_o \\ &\quad + \sum_{m=1}^{\infty} \left[\left(A_m r^m + \frac{B_m}{r^m} \right) (C_m \cos(m\phi) \\ &\quad + D_m \sin(m\phi)) \right] \end{split}$$

$$\begin{split} &\int_0^a \sin\left(\frac{n'\pi}{a}y\right) \sin\left(\frac{n\pi}{a}y\right) dy = \begin{cases} 0 & for \quad n' \neq n \\ \frac{a}{2} & for \quad n' = n \end{cases} \\ &V(P) = \frac{1}{4\pi\varepsilon_o} \left[\frac{1}{r} \int \rho(r') \, d\tau' + \frac{1}{r^2} \int \rho(r') \, r' \cos\theta' \, d\tau' \right. \\ & \left. + \frac{1}{r^3} \int \rho(r') r'^2 \left(\frac{3}{2}\cos^2\theta' - \frac{1}{2}\right) \, d\tau' + \cdots \right] \\ &P_0(x) = 1 \\ &P_1(x) = x \\ &P_2(x) = (3x^2 - 1)/2 \\ &P_3(x) = (5x^3 - 3x)/2 \\ &P_4(x) = (35x^4 - 30x^2 + 3)/8 \\ &P_5(x) = (63x^5 - 70x^3 + 15x)/8 \\ &\vec{p} = \int \rho(r') \, \vec{r}' d\tau' \\ &\vec{D} = \varepsilon_o \vec{E} + \vec{P} \; ; \; \vec{P} = \varepsilon_o \chi_e \vec{E} \; ; \; \sigma_b = \vec{P}. \, \hat{n} \; ; \; \rho_b = \vec{\nabla}. \, \vec{P} \\ &\rho_f = \vec{\nabla}. \, \vec{D} \; ; \; W = \frac{\varepsilon_o}{2} \int E^2 d\tau = \frac{1}{2} \int \vec{D}. \, \vec{E} \, d\tau \end{split}$$