

Formula Sheet PHYS305 Mid-term Exam Sem211

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\begin{pmatrix} \vec{A}_x \\ \vec{A}_y \\ \vec{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix};$$

$$\vec{A}_i = \sum_{j=1}^3 R_{ij} A_j ; \vec{\nabla} = \left(\frac{\partial}{\partial x}\right) \hat{i} + \left(\frac{\partial}{\partial y}\right) \hat{j} + \left(\frac{\partial}{\partial z}\right) \hat{k}$$

$$\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$$

$$\vec{\nabla}(\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$$

$$\vec{\nabla} \cdot (f\vec{A}) = f(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla} f)$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) + \vec{A} \times (\vec{\nabla} f)$$

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A})$$

$$\nabla^2 T = \left(\frac{\partial^2 T}{\partial x^2}\right) + \left(\frac{\partial^2 T}{\partial y^2}\right) + \left(\frac{\partial^2 T}{\partial z^2}\right)$$

$$\nabla^2 \vec{v} = \left(\frac{\partial^2 v_x}{\partial x^2}\right) \hat{i} + \left(\frac{\partial^2 v_y}{\partial y^2}\right) \hat{j} + \left(\frac{\partial^2 v_z}{\partial z^2}\right) \hat{k}$$

$$\vec{\nabla} \times (\vec{\nabla} T) = 0 ; \vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \nabla^2 \vec{v}$$

$$\int_a^b (\vec{\nabla} T) \cdot d\vec{l} = T(b) - T(a)$$

$$\int_a^b (\vec{\nabla} \cdot \vec{v}) d\tau = \oint \vec{v} \cdot d\vec{a}$$

$$\int_a^b (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l}$$

$$x = r \sin \theta \cos \phi ; y = r \sin \theta \sin \phi ; z = r \cos \theta$$

$$dl_r = dr ; dl_\theta = r d\theta ; dl_\phi = r \sin \theta d\phi$$

$$\vec{\nabla} T = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\phi}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\nabla^2 T = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2}$$

$$x = s \cos \phi ; y = s \sin \phi ; z = z$$

$$dl_s = ds ; dl_\phi = s d\phi ; dl_z = dz$$

$$\vec{\nabla} T = \left(\frac{\partial T}{\partial s}\right) \hat{s} + \frac{1}{s} \left(\frac{\partial T}{\partial \phi}\right) \hat{\phi} + \left(\frac{\partial T}{\partial z}\right) \hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial}{\partial \phi} (v_\phi) + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial}{\partial s} (s v_s) - \frac{\partial v_s}{\partial \phi} \right) \hat{z}$$

$$\nabla^2 T = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial T}{\partial s} \right) + \frac{1}{s^2} \left(\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{\partial^2 T}{\partial z^2}$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3(\vec{r}) ; \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} ; V(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} ;$$

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \sum_{j>i}^n \frac{q_i q_j}{\xi_{ij}} ; W = \frac{\epsilon_0}{2} \int E^2 d\tau$$