

Numerical Homework 1

Prob.1 Range of a Projectile Under Air Drag

In class we got the following equation that need to be solved numerically to get the maximum range of a projectile (R) under the action of a frictional air drag force (Example 2.7)

- Knowing $T = kt$, solve for

$$T = (K + 1) (1 - e^{-T})$$

- After finding T , find $X(T) = R$ using

$$\frac{R}{R_0} = \frac{T}{2K(K + 1)}$$

where $R_0 = 2UV/g$ is the range without air resistance (i.e. $k = 0$) and $K = kV/g$, $V = v_0 \sin \theta$, $U = v_0 \cos \theta$.

- Then check that the limit $k \rightarrow 0$ gives you the expected result $R = R_0$.
- Plot of R/R_0 vs. K and attach the code for the numerical computation.

Prob.2 Quadratic Drag Force

For a particle of mass m vertically thrown upward with initial speed v_0 . The air resistance experienced by the particle is proportional to v^2 : $f_R = -\alpha v^2$.

- Show that:

$$m\dot{v} = -mg - \alpha v^2$$

and integrate this equation to find $v(t)$ knowing $v(t = 0) = v_0$

- Find the time needed to reach maximum height H and study the limit $\alpha \rightarrow 0$ (i.e. no air resistance).
- Plot $H(\alpha)$ generated from numerical computations. Given that:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right)$$

$$\arctan(x) \equiv x - \frac{x^3}{2} \quad \text{as } x \rightarrow 0$$

$$\int \tan(ax) dx = \frac{-1}{a} \ln(\cos(ax))$$

- Then plot of H vs. α from the numerical computation and attach the numerical code.