KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF PHYSICS
PHYS 422-222
Nuclear and Partcile Physics
HW-3 Solutions
Due on Saturday 11 Feb. 2022
Problems are from Ch.3 of Textbook and are equally weighted (10 pts each)
Attempt the problems by yourself first, and then seek help if needed.
If you use a reference/solution manual, mention it and you will get full credit for a correct answer.
Please submit good PDF copy by email to khiari@kfupm.edu.sa

Q1. Pb # 17

17. The spin-parity of ^{9}Be and ^{9}B are both $\frac{3}{2}^-$. Assuming in both cases that the spin and parity are characteristic only of the odd nucleon, show how it is possible to obtain the observed spin-parity of ^{10}B (3^+). What other spin-parity combinations could also appear? (These are observed as excited states of ^{10}B.)

Solution

$\boxed{3\text{-}17}$ The structure of $^{10}_{5}B_5$ can be analyzed in terms of one valence neutron and one valence proton. By comparison with $^9_4Be_5$, we expect the valence neutron to have $j_n = 3/2$ and $\pi_n = -$. Comparing with $^9_5B_4$, we expect the valence proton also to have $j_p = 3/2$ and $\pi_p = -$. The vector coupling of j_p to j_n can give values 0, 1, 2, or 3 for the total J. The combined parity $\pi_p \pi_n$ is positive. The 3^+ state can be formed in this way. Other possible states would be 0^+, 1^+, and 2^+.

Q2. Pb # 18

18. Let's suppose we can form ^{3}He or ^{3}H by adding a proton or a neutron to ^{2}H, which has spin equal to 1 and even parity. Let ℓ be the orbital angular momentum of the added nucleon relative to the ^{2}H center of mass: What are the possible values of the total angular momentum of ^{3}H or ^{3}He? Given that the ground-state parity of ^{3}H and ^{3}He is even, which of these can be eliminated? What is the most likely value of the ground-state angular momentum of ^{3}H or ^{3}He? Can you make a similar argument based on removing a proton or a neutron from ^{4}He? (What is the ground-state spin-parity of ^{4}He?) How would you account for the spin-parity of ^{5}Li and ^{5}He ($\frac{3}{2}^-$)?

Solution

$\boxed{3\text{-}18}$ To form ^{3}He or ^{3}H, out of a proton or neutron added to ^{2}H, we must do a vector coupling of the spin of ^{2}H (1), the spin of the added nucleon ($\frac{1}{2}$), and the orbital angular momentum of the added nucleon (ℓ):

$$\vec{J} = \vec{\tfrac{1}{2}} + \vec{1} + \vec{\ell}$$

Possible values are:

$\ell = 0$: $j = \frac{1}{2}$ or $\frac{3}{2}$ (even parity)

$\ell = 1$: $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ (odd parity)

$\ell = 2$: $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ (even parity)

etc.

Because ^{2}H and the added nucleon both have even parity, the parity of the A=3 nucleus is determined by the orbital parity of the nucleon. Because ^{3}He and ^{3}H have even parity, only ℓ=even is permitted and $\ell=0$ is the best guess. Furthermore, the pairing effect suggests that the two like nucleons in ^{3}He or ^{3}H will couple to 0, leaving the odd nucleon to determine the net j. Since there is no orbital contribution, we must have $j = \frac{1}{2}$.

A similar argument based on removing a proton or neutron from ^{4}He (0^+) gives the same result.

The $\frac{3}{2}^-$ assignment of ^{5}Li and ^{5}He suggests a single $\ell=1$ nucleon ($j = \frac{3}{2}, \pi = -$) coupled to a 0^+ ^{4}He core.