Formula Sheet PHYS305 Sem211

$$\overrightarrow{\nabla T} = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi}$$

$$\overrightarrow{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \ v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\overrightarrow{\nabla} \times \vec{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta \ v_\phi) - \frac{\partial v_\phi}{\partial \phi} \right] \hat{r}$$

$$+ \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta}$$

$$+ \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\nabla^2 T = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2}$$

$$\overrightarrow{\nabla} \cdot \vec{V} = \left(\frac{\partial T}{\partial s} \right) \hat{s} + \frac{1}{s} \left(\frac{\partial T}{\partial \phi} \right) \hat{\phi} + \left(\frac{\partial T}{\partial z} \right) \hat{z}$$

$$\overrightarrow{\nabla} \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial}{\partial \phi} (v_\phi) + \frac{\partial v_z}{\partial z}$$

$$\overrightarrow{\nabla} \times \vec{v} = \left(\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right) \hat{\phi}$$

$$+ \frac{1}{s} \left(\frac{\partial}{\partial s} (s v_s) - \frac{\partial v_s}{\partial \phi} \right) \hat{z}$$

$$\nabla^2 T = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial T}{\partial s} \right) + \frac{1}{s^2} \left(\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{\partial^2 T}{\partial z^2}$$

$$\overrightarrow{\nabla} \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3 (\vec{r})$$

$$e^{kx} + e^{-kx} = 2 \cosh(kx)$$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$V(r, \phi) = a_o \ln(r) + b_o$$

$$+ \sum_{m=1}^{\infty} \left[\left(A_m r^m + \frac{B_m}{r^m} \right) (C_m \cos(m\phi) + D_m \sin(m\phi)) \right]$$

$$V(P) = \frac{1}{4\pi \varepsilon_o} \left[\frac{1}{r} \int \rho(r') d\tau' + \frac{1}{r^2} \int \rho(r') r' \cos \theta' d\tau' + \frac{1}{r^3} \int \rho(r') r'^2 \left(\frac{3}{2} \cos^2 \theta' - \frac{1}{2} \right) d\tau' + \cdots \right]$$

$$P_0(x) = 1$$

$$\begin{split} P_{2}(x) &= (3x^{2} - 1)/2 \\ P_{3}(x) &= (5x^{3} - 3x)/2 \\ P_{4}(x) &= (35x^{4} - 30x^{2} + 3)/8 \\ P_{5}(x) &= (63x^{5} - 70x^{3} + 15x)/8 \\ \vec{p} &= \int \rho(r') \vec{r}' d\tau' \\ \vec{D} &= \varepsilon_{o} \vec{E} + \vec{P} \; ; \; \vec{P} = \varepsilon_{o} \chi_{e} \vec{E} \; ; \; \sigma_{b} = \vec{P}.\hat{n} \; ; \; \rho_{b} = \vec{\nabla}.\vec{P} \\ \rho_{f} &= \vec{\nabla}.\vec{D} \; ; \; W = \frac{\varepsilon_{o}}{2} \int E^{2} d\tau = \frac{1}{2} \int \vec{D}.\vec{E} \; d\tau \\ \vec{F}_{mag} &= \int I(d\vec{l} \times \vec{B}) \; ; \; \vec{\nabla}.\vec{J} = -\frac{d\rho}{dt} \; ; \\ \vec{B}(\vec{r}) &= \frac{\mu_{o}I}{4\pi} \int \frac{d\vec{l} \times \Delta \hat{r}}{\Delta r^{2}} \\ \vec{B}(P) &= \frac{\mu_{o}I}{4\pi} \int \frac{\vec{K} \times \Delta \hat{r}}{\Delta r^{2}} da \; ; \; \vec{B}(P) = \frac{\mu_{o}I}{4\pi} \int \frac{\vec{J} \times \Delta \hat{r}}{\Delta r^{2}} d\tau \\ \vec{\Phi}\vec{B} \cdot d\vec{l} &= \mu_{o}i_{enclosed} \; ; \\ \vec{A}(\vec{r}) &= \frac{\mu_{o}I}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\Delta r} d\tau' \; ; \; \vec{A} = \frac{\mu_{o}}{4\pi} \int \frac{\vec{K}(\vec{r}')}{\Delta r} da' \; ; \\ \vec{B}_{above} &- \vec{B}_{below} = \mu_{o}\vec{K} \times \hat{n} \\ \frac{1}{\Delta r} &= \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{r'}{r} \right)^{n} P_{n}(\cos\theta') \\ \vec{A} &= \frac{\mu_{o}I}{4\pi} \left[\frac{1}{r} \oint d\vec{l}' + \frac{1}{r^{2}} \oint r' \cos\theta' d\vec{l}' \\ &+ \frac{1}{r^{3}} \oint r'^{2} \left(\frac{3}{2} \cos^{2}\theta' - \frac{1}{2} \right) d\vec{l}' + \cdots \right] \\ \vec{A}(\vec{r}) &= \frac{\mu_{o}}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \Delta \hat{r}}{\Delta r^{2}} d\tau' \\ \vec{H} &= \frac{1}{\mu_{o}} \vec{B} - \vec{M} \end{split}$$