Special Triangles

Consider a triangle at $A(2,0), B(2,p), C(3,0)$ with area $p/2$. Now complete the parallelogram to get a point $D(1,p)$. Reflect A over D to get $E(0,2p)$. Since $ACBD$ is a parallelogram, the area of BCD is half of that of the parallelogram, which is the area of ABC, which is $p/2$. Now since DE is parallel to BC, area of ECB and DCB must be the same(same height, common base).

Note that since p is non-zero, all three have distinct x and y coordinates. Hence BCE is a valid triangle, and we can always print $(0,2p),(2,p),(3,0)$ for a triangle with area $p/2$ where $p \in \mathbb{Z}$, and in all other cases, no solution exists as above.

Here p can be found by dividing a and b by $GCD(a,b)$.

Finding the GCD has time complexity $= \mathcal{O}(\log(ab))$ and all other operations are of $\mathcal{O}(1)$.

Hence the time complexity is $\mathcal{O}(\log(ab))$.

Another possible solution is the following family of triangles: $(0,0),(1,2),(p,p)$ when $p > 2$ and the triangles $(0,4),(2,2),(3,0)$ for $p = 2$ and $(0,2),(2,1),(3,0)$ for $p = 1$.