Cool Numbers

This problem can be easily solved using dynamic programming.

We maintain a 2-Dimensional dp matrix $dp[n+1][10]$, where each entry $dp[i][j]$ represents the number of *cool* numbers starting at index i from the left and with first digit as j (here we allow j to be 0, and we deal with the case of first digit to be non-zero while computing the final answer).

Since difference between two consecutive digits is atmost c, the dp relations are as follows:

$dp[i][j] = \sum_{k=max(0,j-c)}^{min(9,j+c)} dp[i+1][k]$.

For the base case, we set $dp[n][j] = 1$ for each $9 \geq j \geq 0$. The proof of this can be argued inductively, with the induction claim being that for each first digit j, $dp[i][j]$ gives the number of *cool* numbers (where 0 can also be the first digit) beginning at index i. Now, for any number beginning at index $i-1$, if it is *cool*, then the next digit will differ by atmost c, and the the number from index i till n must also be *cool*, hence the above relation covers all such *cool* numbers and counts them exactly once. Hence our transition is valid.

To compute the final answer ans, simply do : $ans = \sum_{k=1}^{k=9} dp[1][k]$ (here we exclude 0 as the starting digit).

Time Complexity: $O(bnc)$

Space Complexity: $O(bn)$

Here $b = 10$ is the base of the number system chosen.