

Pair Making

The brute-force solution will include iterating through all pairs of numbers $\leq n$. This makes the solution $O(n^2)$ and will result in a TLE. Here is a faster solution:

For all positive integers i such that $i < p$, calculate $j = \text{inverse}(i^2) \times k$.

Now, we have 2 integers i, j such that $0 < i, j < p$ and $i^2 j \equiv k \pmod{p}$.

For all such i, j , find the number of ordered pairs (a, b) such that $a \equiv i \pmod{p}$ and $b \equiv j \pmod{p}$, $1 \leq a, b \leq n$ as $\left(\frac{n+p-i}{p}\right) \times \left(\frac{n+p-j}{p}\right)$.

Final answer:

$$\sum_{i=1}^{p-1} \left(\frac{n+p-i}{p}\right) \times \left(\frac{n+p-j}{p}\right) \text{ where } j = \text{inverse}(i^2) \times k$$

Time Complexity : $O(p \log p)$ (There are p iterations of i and each *inverse* function is $O(\log p)$)

Space Complexity: $O(1)$

Note : *inverse*(x) stands for "modular inverse" of x with respect to p .