Can They Meet?

Let's try to formulate this problem in a mathematical way:

Firstly, if both a, b are 0, then none of A, B can make a move.

Wlog let A be on the left of B. Suppose A performed a_l left moves and a_r right moves. Now, B will move $a_l - k$ left and $a_r + k$ right moves ($a_l - k, a_r + k \geq 0$).

Hence, $a(a_r - a_l) + b(a_l - a_r + 2k) = x$.

Replace $a_r - a_l$ with t and $a_l - a_r + 2k$ with u.

And we will get, $at + bu = x$ such that $t + u$ is even.

It's easy to see that if a solution exists, we can get a (possibly distinct) solution where all $a_l, a_l - k, a_r, a_r + k$ are sufficiently large, and hence there is a solution to the problem in this case.

A solution of the above equation exists iff x is divisible by $g = gcd(a, b)$ and t and u have the same parity.

(`https://cp-algorithms.com/algebra/linear-diophantine-equation.html`)

Now, if t and u are even, then x should be divisible by $2g$.

Otherwise, if t and u are odd, then, $x + a + b$ should be divisible by $2g$ (adding $a + b$ on both sides of equation reduces it to the previous case).

Hence, they will meet if x or $x + a + b$ is divisible by $2g$.

Another way of solving this problem is as follows:

Note that the distance between A and B changes by $\pm(a - b)$ or $\pm(a + b)$. Hence we merely need to check if there is a solution of the equation $m(a + b) + n(a - b) = x$, which is equivalent to checking if x is divisible by $\gcd(a + b, a - b)$.

Additional task - Find the minimum number of moves in which they can meet.