Deserted Freshers

Consider the usual graph formulation, where cities are vertices and roads are edges.

Note that two vertices u, v are reachable iff they are in the same component in the graph. Hence, we merely need to assign freshers to components in such a way that the following quantity is maximized:

value $= \sum_{\text{component } C} \left(\sum_{u \in C} f(u)\right)^2$

Here, $f(u)$ is the number of freshers in city u.

Now we claim that if we move some freshers from a component with lesser members to a component with more members, value will increase. Indeed, suppose there are a freshers in component A and b in component B, and $a < b$, and we shift c freshers from a to b. Then the difference in value is $(a-c)^2 + (b+c)^2 - a^2 - b^2 = 2c^2 - 2ac + 2bc \geq 2c^2 > 0$.

So it suffices to greedily assign the groups with the highest number of freshers to the component with the highest number of vertices in it (else, we can shift some freshers by swapping two groups and get a better value, which would be a contradiction).

To do this, we can find all components by BFS/DFS, or even using DSU, and simply sort the freshers as well as the components in decreasing order and keep on assigning freshers greedily.

The time complexity of this approach using BFS/DFS is $O(n + m + n \log n)$ which is $O(m + n \log n)$, which passes, and the time complexity of the DSU approach is $O(m\alpha(m, n) + n \log n)$, which also passes (here α is the inverse Ackermann function).