# Shifting

Let $p_i = a_1 + \cdots + a_i$, with $p_0 = 0$. Note that by Euclid's lemma, we have

$$\gcd(p_0, p_1, \ldots, p_{n-1}, p_n) = \gcd(p_0, p_1, \ldots, p_{n-1}, p_n - p_{n-1}) = \cdots = \gcd(p_0, p_1 - p_0, p_2 - p_1, \ldots, p_n - p_{n-1})$$
$$= \gcd(0, a_1, \ldots, a_n) = \gcd(a_1, \ldots, a_n)$$

Hence, we can shift our focus to the $p_i$s. Note that when we do the operation, we replace either $p_i$ by $p_{i+1}$ or $p_i$ by $p_{i-1}$. Hence an operation is equivalent to deleting one of the $p_i$s, since duplicate $p_i$s in the expression for gcd don't matter. We can't delete either of $p_0$ or $p_n$, and can delete any of the remaining $p_i$s.

Note that one deletion corresponds to precisely one move as well. Hence the problem reduces to finding the maximum number $s$ of indices $1 \leq i \leq n$ such that $p_i$ shares a prime factor with $p_n$. Then the answer will be $n - s$.

To find this, we simply need to factorize $p_n$, and for each prime factor $p$ of $p_n$, find the number of indices $i$ such that $p_i$ is divisible by $p$. Note that in the case when $p_n = 1$, we have no solution.

Note that there are $O(\log p_n)$ distinct prime divisors of $p_n$*, and finding them takes $O(\sqrt{p_n})$ time. Since we do a pass for each prime divisor of $p_n$, the total time complexity becomes $O(\sqrt{p_n} + n \log p_n)$. Note that the maximum value for $p_n$ is $10^{11}$, so this fits within the time limit.

* This bound can be substantially improved, however, it suffices for our purposes.