- $T_1 = O(T_3)$

$T_1 = 3n^4 + 3n^3 + 1$, $T_3 = (n-2)!$

$$\lim_{n\to\infty} \frac{3n^4 + 3n^3 + 1}{(n-2)!}$$

choose the one with the biggest power, from T_1

$$\lim_{n\to\infty} \frac{3n^4}{(n-2)!} = 3 \lim_{n\to\infty} \frac{n^4}{(n-2)!}$$

let's apply Ratio Test

$$\boxed{\begin{array}{ll} L<1 & \text{, convergent} \\ L>1 & \text{, divergent} \\ L=1 & \text{, may be both} \end{array}}$$

$$L = \lim_{n\to\infty} \frac{(n+1)^4}{(n-1)!} \cdot \frac{(n-2)!}{n^4}$$

$$= \lim_{n\to\infty} \frac{(n+1)^4 \cdot (n-2)!}{(n-1)(n-2)!\, n^4} = 0$$

$$T_1 = O(T_3)$$

- $T_3 = O(T_6)$

$T_3 = (n-2)!$, $T_6 = \sqrt[3]{n}$

$$\lim_{n\to\infty} \frac{(n-2)!}{n^{1/3}}$$

Ratio test

$$L = \lim_{n\to\infty} \frac{(n-1)!}{(n+1)^{1/3}} \cdot \frac{n^{1/3}}{(n-2)!} = \lim_{n\to\infty} \frac{\cancel{(n-1)!}}{\cancel{(n+1)^{1/3}}} \cdot \frac{n^{1/3}}{(n-2)\cancel{(n-1)!}}$$

$$\lim_{n\to\infty} \frac{1}{n-2} = 0$$

$$T_3 \neq O(T_6)$$

- $T_2 = O(T_4)$

$T_2 = 3^n$, $T_4 = \ln^2 n$

$\lim_{n\to\infty} \frac{3^n}{\ln^2 n}$ $\frac{\infty}{\infty}$, we have an indeterminate form

Applying L'Hospital

$\lim_{n\to\infty} \frac{\ln 3 . 3^n}{2.\ln n . \frac{1}{n}} = \lim_{n\to\infty} \frac{\ln 3 . 3^n . n}{2.\ln n}$ still $\frac{\infty}{\infty}$

Applying L'Hospital again

$\lim_{n\to\infty} \frac{\ln 3 . \ln 3 . 3^n . n + \ln 3 . 3^n}{2 . \frac{1}{n}} = \lim_{n\to\infty} \frac{(\ln^2 3 . 3^n . n + \ln 3 . 3^n) n}{2} = \infty$

$T_2 \neq O(T_4)$

- $T_4 = O(T_1)$

$T_4 = \ln^2 n$, $T_1 = 3n^4 + 3n^3 + 1$

$\lim_{n\to\infty} \frac{\ln^2 n}{3n^4 + 3n^3 + 1}$ $\frac{\infty}{\infty}$

L'Hospital

$\lim_{n\to\infty} \frac{2 . \ln n . \frac{1}{n}}{12n^3 + 9n^2}$ $\frac{\infty}{\infty}$ $= \lim_{n\to\infty} \frac{2 . \ln n}{12n^4 + 9n^3}$

L'Hospital

$\lim_{n\to\infty} \frac{2 . \frac{1}{n}}{48n^3 + 27n^2} = \lim_{n\to\infty} \frac{2}{48n^4 + 27n^2} = 0$

$T_4 = O(T_1)$

- $T_6 = O(T_5)$

$T_6 = \sqrt[3]{n}, \quad T_5 = 2^{2n}$

$$\lim_{n\to\infty} \frac{\sqrt[3]{n}}{2^{2n}} = \lim_{n\to\infty} \frac{n^{1/3}}{2^{2n}}$$

Ratio test

$$L = \lim_{n\to\infty} \frac{(n+1)^{1/3}}{2^{2+2n}} \cdot \frac{2^{2n}}{(n)^{1/3}} = \lim_{n\to\infty} \frac{\cancel{2^{2n}}}{\cancel{2^{2n}} \cdot 2^2} = \frac{1}{4}$$

$T_6 = O(T_5)$

- we need to know this one as well

$T_2 = O(T_5)$

$T_2 = 3^n, \quad T_5 = 2^{2n}$

$$\lim_{n\to\infty} \frac{3^n}{2^{2n}}$$

$$\lim_{n\to\infty} \left(\frac{3}{4}\right)^n = 0$$

$T_2 = O(T_5)$

- we need to know also

$T_1 = O(T_6)$

$T_1 = 3n^4 + 3n^3 + 1, \quad T_6 = \sqrt[3]{n}$

$$\lim_{n\to\infty} \frac{3n^4+3n^3+1}{\sqrt[3]{n}} = \lim_{n\to\infty} \frac{\cancel{n^{1/3}}\left(3n^{11/3} + 3n^{8/3} + \frac{1}{n^{1/3}}\right)}{\cancel{n^{1/3}}\,(1)} = \infty$$

$T_1 \neq O(T_6)$

• Last one

$T_4 = O(T_6)$

$T_4 = \ln^2 n$, $T_6 = \sqrt[3]{n}$

$$\lim_{n\to\infty} \frac{\ln^2 n}{n^{1/3}} \qquad \frac{\infty}{\infty}$$

L'hospital

$$\lim_{n\to\infty} \frac{2 \cdot \ln n \cdot \frac{1}{n}}{\frac{1}{3} n^{-2/3}} = \lim_{n\to\infty} \frac{2 \cdot \ln n}{\frac{1}{3} \cdot n^{1/3}}$$

L'hospital again

$$\lim_{n\to\infty} \frac{2 \cdot \frac{1}{n}}{\frac{1}{3} \cdot \frac{1}{3} n^{-2/3}} = \lim_{n\to\infty} \frac{2}{\frac{1}{9} \cdot n^{1/3}} = 0$$

$T_4 \neq O(T_6)$

$T_5 > T_2 > T_3 > T_1 > T_6 > T_4$

Q2

(a) This algorithms does first find the largest and the smallest elements in array. Then stores them in watermelon and plum respectively.
(largest) (smallest)

And then fruit iterates through elements in the array until the condition is met. OrangeTime controls the while loop.

After we find out the largest and the smallest element in the array, we look for the element whose value is the closest to the average of watermelon and plum (largest and smallest). finally we return this element in orange.

(*) I explained role of all variables above.

(b) — Best case: When the array has only one element
— array is sorted —
: O(1) (constant time

— Worst case: when in while loop, the for loop iterates through all elements. for example if the array = [1, 2, 3, 4, 5], it must iterate through all elements to find watermelon O(n). And the for loop below also executed n times as maximum.

So it is : O(n) (lineer time)

Q3

a) $\sum_{i=0}^{n-1} (i^2+1)^2$

$(i^2+1)^2 = (i^4+2i^2+1)$ → formula of the sum

$\sum_{i=0}^{n} 1 = 1+2+3+ \dots +n$

$\frac{n(n+1)}{2} = \theta(n^2)$

$\sum_{i=0}^{n-1} i^4 + 2\sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 1$

$\theta(n^5) + \theta(2n^3) + \theta(n)$

$\theta(n^3)$

$\boxed{\in \theta(n^5)}$

b) $\sum_{i=2}^{n-1} \log i^2$

$\log i^2 = 2\log i$

$= 2\sum_{i=2}^{n-1} \log i$ → To apply to formula I want to change the summation like this form ($\sum_{i=1}^{n}$)

$2\sum_{i=2}^{n} \log i - 2\log n = 2\sum_{i=1}^{n} \log i - \log 1 - 2\log n$

$\log 1 = 0$

$= 2\sum_{i=1}^{n} \log i - 2\log n$

$2\log n \rightarrow \theta(\log n)$ (2)

$2[\log 1 + \log 2 + \log 3 + \dots + \log n]$

$2\log\left(\frac{n(n+1)}{2}\right) \rightarrow \theta(\log(n^2)) \rightarrow \theta(n\log n)$ (1)

(1) − (2)

$\theta(n\log n) - \theta(\log n)$

$\boxed{\in \theta(n\log n)}$

(c) $\sum_{i=1}^{n} (i+1)\, 2^{i-1}$

$(i+1)\, 2^{i-1} = i\, 2^{i-1} + 2^{i-1} = \frac{i\, 2^i}{2} + \frac{2^i}{2}$

$\Theta(n)$

$\frac{1}{2} \sum_{i=1}^{n} 2^i = \frac{1}{2}[2 + 2^2 + 2^3 + \dots + 2^n]$

$= \frac{1}{2}[2^{n^2}] \in \Theta(2^n)$

we already found

$\Theta(2^n)$

$\Theta(n 2^n) + \Theta(2^n)$

$\boxed{\in \Theta(n 2^n)}$

(d) $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$ let's do this first

we have $\sum_{j=0}^{i-1} i + \sum_{j=0}^{i-1} j$

$\frac{i^2}{1} + \frac{i(i-1)}{2} = \frac{2i^2 + i^2 - i}{2} = \frac{3i^2 - i}{2}$

(2)

now we solve this

$\frac{3}{2} \sum_{i=0}^{n-1} i^2 - \frac{1}{2} \sum_{i=0}^{n-1} i$

$\in \Theta(n^3) - \in \Theta(n)$

$\boxed{= \Theta(n^3)}$

```c
#include <stdio.h>

int main() {
    /*hw1 q3 a*/
    int sum = 0, i, n = 10;
    for(i = 0; i < n; ++i){
        sum += (i * i + 1) * (i * i + 1);
    }
    printf("a-) Sum: %d\n", sum);

    /*hw1 q3 d*/
    sum = 0;
    int j;
    for(i = 0; i < n; ++i){
        for(j = 0; j < i; ++j){
            sum += (i + j);
        }
    }
    printf("d-) Sum: %d\n", sum);

    return 0;
}
```

Q4

complexity

-box2-

• outer for loop is $O(\log n)$ because the loop gets divided by 2 as it runs.

it executed from 0 to $\log n$ times.

for example for $n = 32$, the outer loop runs for

$i=32$, $i=16$, $i=8$, $i=4$, $i=2$ (5 times) $= \log_2 32$

• $O(\log n)$

• and the inner for loop, we consider both loops

$(n-1) + (\frac{n}{2} - 1) + (\frac{n}{4} - 1) + \dots + 1$

we sum up terms with n

$n + \frac{n}{2} + \frac{n}{4} + \dots \quad n\left(\frac{1}{1-r}\right) \quad r = \frac{1}{2}$

$n\left(\frac{1}{1-\frac{1}{2}}\right) = 2n$ $\quad 2n \quad$ -ve $\log n$ done -1 for

$\boxed{O(n)}$ (I eliminated smaller terms)

Q5

$n=0$ $1-3$
$n=1$ $8-81$
$n=2$

(a) $n^3 \in O(3^{2n})$

$$L = \lim_{n\to\infty} \frac{n^3}{3^{2n}}$$ ratio test

$$\lim_{n\to\infty} \frac{(n+1)^3}{3^{2n}\cdot 3^2} \cdot \frac{\cancel{3^{2n}}}{\cancel{n^3}} = \frac{1}{9} < 1 \quad 3^{2n} \quad n^3$$

✓

(b) $n \in o(\log\log n)$

$$\lim_{n\to\infty} \frac{n}{\log\log n}$$ L'Hospital $\quad n > \log\log n$

$$\lim_{n\to\infty} \frac{1}{\frac{1}{n\log n}} = \infty$$

✗

(c) $n^2\log^2 n \in O(n!)$

$$L = \lim_{n\to\infty} \frac{n^2\log^2 n}{n!}$$ ratio test

$$\lim_{n\to\infty} \frac{(n+1)^2\log^2(n+1)}{(n+1)!} \cdot \frac{n!}{n^2\log^2 n} = \lim_{n\to\infty} \frac{\cancel{2}\log(n+1)\cdot \cancel{n!}}{\cancel{2}\log n\cdot(n+1)\cancel{n!}} = 0$$

$$n^2\log^2 n \le n!$$

✓

(d) $\sqrt{10n^2+7n+3} \in \Theta(n)$

$$\lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n} = \lim_{n\to\infty} \frac{\sqrt{n^2(10+\frac{7}{n}+\frac{3}{n^2})}}{n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\sqrt{10+\frac{7}{n}+\frac{3}{n^2}}}{\cancel{n}} = \sqrt{10}$$

✓