Question 2:

1-) $x_1(n) = 0.5x_1\left(\frac{n}{2}\right) + \frac{1}{n}$

As the master theorem dictates, a >= 1. But in this question, it is 0.5. So it cannot be applied

2-) $x_2\,(n) = 3x_2\left(\frac{n}{4}\right) + nlogn$

3-) $x_3(n) = 3x_3\left(\frac{n}{3}\right) + \frac{n}{2}$

$\mathrm{a} = 3\ \mathrm{b} = 3\ \mathrm{d} = 1$

$\mathrm{a} = b^d$

$3 = 3$

So, it is

$x_3(n) = \mathrm{Theta}(n^d\mathrm{logn})$

$x_3(n) = \mathrm{Theta}(\mathrm{nlogn})$

4-) $x_4(n) = \ 6x_4\left(\frac{n}{3}\right) + n^2logn$

$\mathrm{a} = 6\ \mathrm{b} = 3$

$\mathrm{f(n)} = \mathrm{Omega}(n^{\log_b a + epsilon})$

$\mathrm{epsilon} > 0$

$1 < \log_3 6 < 2$ -> we got n^2 here

$\mathrm{a.f}(\frac{n}{b}) < \mathrm{cf(n)}$ $\qquad$ $\mathrm{c} < 1$

$x_4(n) = Theta(n^2logn)$

5-) $x_5(n) = 4x_5\left(\frac{n}{2}\right) + \frac{n}{logn}$

6-) $x_6(n) = 2^n x_6\left(\frac{n}{2}\right) + n^n$

2^n is not constant. Master theorem cannot be applied

Question 3:

```
def chocolate Algorithm(n):
    if n==1:
        return 1
    else:
        return chocolate Algorithm(n-1) + 2 * n - 1
```

a-)

This function returns the square of a positive integer n

F(n) = F(n-1) + 2n -1

F(1) = 1
F(2) = 1 + 4 - 1 = 4
F(3) = 4 + 6 – 1 = 9
F(4) = 9 + 8 – 1 = 16
Goes on…
It returns square of input n
We have this
F(n) = n^2
And this
F(n-1) + 2n – 1 = $(n-1)^2 + 2n - 1 = n^2 - 2n + 1 + 2n - 1 = n^2$
Which are the same results.

b-)

There is one multiplication in the function.

G(n) = G(n-1) + 1

= (G(n-2) +1) +1
= (G(n-3) +1) + 1*2
…
…
…
= G(n-i) + 1*i = G(n-i) + i

$G(1) = 0$

Let $i = n-1$ and write $G(n-i) + i$ again

$G(n - n + 1) + n - 1 = G(1) + n - 1 = n - 1$

$G(n) = n - 1$

c-)
There are three addition and substraction in the function

$$
\begin{aligned}
T(n) &= T(n-1) + 3 \\
&= T(n-2) + 3 + 3 \\
&= T(n-3) + 3 + 3.2 \\
&\dots \\
&\dots \\
&\dots \\
&= T(n-i) + 3.i
\end{aligned}
$$

Let $i = n - 1$ again like we did above
Then we have $T(n-n+1) + 3n - 3$
Therefore it is $T(1) + 3n - 3$ and $T(1) = 0$

$T(n) = 3n - 3$