$$\rightarrow \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \cdot \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x'} \cos\theta - \frac{\partial f}{\partial y'} \sin\theta \right)$$

$$\frac{\partial}{\partial x'} \frac{\partial f}{\partial x} \cos\theta - \frac{\partial}{\partial y'} \frac{\partial f}{\partial x} \sin\theta$$

$$\frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x'} \cos\theta - \frac{\partial f}{\partial y'} \sin\theta \right)\cos\theta - \frac{\partial}{\partial y'}\left(\frac{\partial f}{\partial x'} \cos\theta - \frac{\partial f}{\partial y'} \sin\theta \right)\sin\theta$$

$$\textcircled{1} \quad \frac{\partial^2 f}{\partial x'^2} \cos^2\theta - \frac{\partial^2 f}{\partial x' \partial y'} \sin\theta\cos\theta - \frac{\partial^2 f}{\partial x' \partial y'} \sin\theta\cos\theta + \frac{\partial^2 f}{\partial y'^2} \sin^2\theta$$

$$\rightarrow \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \cdot \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x'} \sin\theta + \frac{\partial f}{\partial y'} \cos\theta \right)$$

$$\frac{\partial}{\partial x'} \frac{\partial f}{\partial y} \sin\theta + \frac{\partial}{\partial y'} \frac{\partial f}{\partial y} \cos\theta$$

$$\frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x'} \sin\theta + \frac{\partial f}{\partial y'} \cos\theta \right)\sin\theta + \frac{\partial}{\partial y'}\left(\frac{\partial f}{\partial x'} \sin\theta + \frac{\partial f}{\partial y'} \cos\theta \right)\cos\theta$$

$$\textcircled{2} \quad \frac{\partial^2 f}{\partial x'^2} \sin^2\theta + \frac{\partial^2 f}{\partial x' \partial y'} \sin\theta\cos\theta + \frac{\partial^2 f}{\partial x' \partial y'} \sin\theta\cos\theta + \frac{\partial^2 f}{\partial y'^2} \cos^2\theta$$

Now add ① and ②

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \underbrace{(\cos^2\theta + \sin^2\theta)}_{1}\left(\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} \right)$$

so they are equal...