

1. Denklemler Sistemi

$$a_{11} + 2a_{12} + a_{13} = 2$$

$$2a_{11} + a_{12} + a_{13} = -1$$

$$3a_{11} + a_{12} + a_{13} = -4$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -1 \\ 3 & 1 & 1 & -4 \end{bmatrix}$$

Yukardaki son rowu augmented olan matrixi gauss elimination ile aşağıdaki hale getiriyoruz.

$$E_2 - 2E_1 \rightarrow E_2$$

$$E_3 - 3E_1 \rightarrow E_3$$

$$\frac{E_2}{-3} \rightarrow E_2$$

$$E_3 + 5E_2 \rightarrow E_3$$

$$E_1 - 2E_2 \rightarrow E_1$$

$$E_1 + E_3 \rightarrow E_1$$

$$E_2 + E_3 \rightarrow E_2$$

$$-3E_3 \rightarrow E_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$a_{11} = -3$$

$$a_{12} = 0$$

$$a_{13} = 5 \quad \text{Buluruz.}$$

2. denklem sistemi

$$a_{21} + 2a_{22} + a_{23} = 2$$

$$2a_{21} + a_{22} + a_{23} = 4$$

$$3a_{21} + a_{22} + a_{23} = 4$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \end{bmatrix}$$

Yukardaki son rowu augmented olan matrixi gauss elimination ile asagidaki hala getiriyoruz.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$a_{21} = 0$$

$$a_{22} = -2$$

$$a_{23} = 6$$

Buluruz.

Bunlari yerine yazdigimizda

$$A = \begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

Tersini almak icin 3x3 birim matris ile arttiriyoruz

Son 3 row birim matrixe denk geliyor

$$\begin{bmatrix} -3 & 0 & 5 & 1 & 0 & 0 \\ 0 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{E_2}{-2} \rightarrow E_2$$

$$\frac{E_3}{-3} \rightarrow E_3$$

$$E_2 + 3E_3 \rightarrow E_2$$

$$E_1 + \frac{5}{3}E_3 \rightarrow E_1$$

Yukarıdaki adımları uyguladığımızda aşağıdaki matrisi elde ederiz. Son 3 rowumuz A matrisinin tersi

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 & -5/3 \\ 0 & 1 & 0 & 0 & -1/2 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & 0 & -5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$