

- 1 Find the normalization factor  $A$ , assuming that the network has a power law degree distribution with  $2 < \gamma < 3$ , with minimum degree  $k_{\min}$  and maximum degree  $k_{\max}$ .

From the textbook, we know that

$$C = \frac{1 - \gamma}{k_{\max}^{1-\gamma} - k_{\min}^{1-\gamma}}$$

Also applying the normalization condition to  $q_k$ , and substituting  $p_k$  into the equation, we obtain

$$\begin{aligned} \int_{k_{\min}}^{k_{\max}} AkCk^{-\gamma} dk &= 1 \\ AC \int_{k_{\min}}^{k_{\max}} k^{1-\gamma} dk &= 1 \\ AC \left( \frac{k_{\max}^{2-\gamma} - k_{\min}^{2-\gamma}}{2-\gamma} \right) &= 1 \\ AC \int_{k_{\min}}^{k_{\max}} k^{1-\gamma} dk &= 1 \end{aligned}$$

Solve for  $A$ , making sure to substitute the  $C$  value and simplify

$$A = \frac{(2-\gamma)(k_{\max}^{1-\gamma} - k_{\min}^{1-\gamma})}{(1-\gamma)(k_{\max}^{2-\gamma} - k_{\min}^{2-\gamma})}$$

- 2 In the configuration model  $q_k$  is also the probability that a randomly chosen node has a neighbor with degree  $k$ . What is the average degree of the neighbors of a randomly chosen node?

We want to find the average, or expected, value of  $k$  in respect to  $q_k$ . We denote this  $k$  as  $\langle k_q \rangle$  and substitute the values for  $q_k$  and then  $p_k$  in the continuous expectation formula

$$\begin{aligned} \langle k_q \rangle &= \int_{k_{\min}}^{k_{\max}} kAkCk^{-\gamma} dk \\ \langle k_q \rangle &= AC \int_{k_{\min}}^{k_{\max}} k^{2-\gamma} dk \end{aligned}$$

Substitute in the values for A and for C and solve the integral

$$\langle k_q \rangle = \frac{(2 - \gamma)(k_{max}^{3-\gamma} - k_{min}^{3-\gamma})}{(3 - \gamma)(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}$$

**3 Calculate the average degree of the neighbors of a randomly chosen node in a network with  $N = 10^4$ ,  $\gamma = 2.3$ ,  $k_{min} = 1$  and  $k_{max} = 1000$ . Compare the result with the average degree of the network,  $\langle k \rangle$ .**

Substitute the given values into the equation for  $\langle k_q \rangle$

$$61.23431879119234$$

Use the textbook equation and substitute the value for C to find the equation for  $\langle k \rangle$

$$\langle k \rangle = \frac{1 - \gamma}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}} \cdot \frac{(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}{(2 - \gamma)}$$

Substitute the given values into the equation for  $\langle k \rangle$

$$3.78827590390276$$