1 Use the rate equation approach to show that the directed copying model leads to a scale-free network with incoming degree exponent gamma_i=(2-p)/(1-p).

Let us denote with N(k,t) the number of nodes with degree k at time t and the degree distribution

$$p_k(t) = \frac{N(k,t)}{N(t)}$$

The expected change in N(k,t) after one timestep is

$$N(k,t+1) - N(k,t))$$
$$= p[(k-1)\frac{N(k-1,t)}{N(t)} - k\frac{N(k,t)}{N(t)}] + (1-p)(\frac{N(k-1,t)}{N(t)})$$
$$= p[\frac{(k-1)N(k-1,t) - kN(k,t)}{t}] + (1-p)(\frac{N(k-1,t) - N(k,t)}{t})$$

When t is large, we assume that N(k,t)=t*p_k such that the equation now becomes

$$p_k = p[(k-1)p_{k-1} - kp_k] + (1-p)[p_k - p_{k-1}]$$

Rewriting the equation we get

$$p_k = p_{k-1} \cdot \frac{k-1+(1/p)}{k+1+(1/p)}$$

Using a recursive approach we see that

$$p_k = C\prod_{j=i}^{k} \frac{j-1+(1/p)}{j+1+(1/p)}$$

which is about

$$k^{\frac{-(2-p)}{(1-p)}}$$

when k is sufficiently large.