1 Find the normalization factor A, assuming that the network has a power law degree distribution with 2 < \gamma < 3, with minimum degree k_min and maximum degree k_max.

From the textbook, we know that

$$C = \frac{1-\gamma}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}}$$

Also applying the normalization condition to q_k, and substituting p_k into the equation, we obtain

$$\int_{k_{min}}^{k_{max}} AkCk^{-\gamma} dk = 1$$
$$AC \int_{k_{min}}^{k_{max}} k^{1-\gamma} dk = 1$$
$$AC\left(\frac{k_{max}^{2-\gamma} - k_{min}^{2-\gamma}}{2-\gamma}\right) = 1$$
$$AC \int_{k_{min}}^{k_{max}} k^{1-\gamma} dk = 1$$

Solve for A, making sure to substitute the C value and simplify

$$A = \frac{(2-\gamma)(k_{max}^{1-\gamma} - k_{min}^{1-\gamma})}{(1-\gamma)(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}$$

2 In the configuration model q_k is also the probability that a randomly chosen node has a neighbor with degree k. What is the average degree of the neighbors of a randomly chosen node?

We want to find the average, or expected, value of k in respect to q_k. We denote this k as <k_q> and substitute the values for q_k and then p_k in the continuous expectation formula

$$< k_q > = \int_{k_{min}}^{k_{max}} kAkCk^{-\gamma} dk$$
$$< k_q > = AC \int_{k_{min}}^{k_{max}} k^{2-\gamma} dk$$

Substitute in the values for A and for C and solve the integral

$$< k_q > = \frac{(2-\gamma)(k_{max}^{3-\gamma} - k_{min}^{3-\gamma})}{(3-\gamma)(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}$$

3 Calculate the average degree of the neighbors of a randomly chosen node in a network with N = 10ˆ4, \gamma= 2.3, k_min= 1 and k_max= 1000. Compare the result with the average degree of the network,⟨k⟩.

Substitute the given values into the equation for <k_q>

61.23431879119234

Use the textbook equation and substitute the value for C to find the equation for <k>

$$< k > = \frac{1-\gamma}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}} \cdot \frac{(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}{(2-\gamma)}$$

Substitute the given values into the equation for <k>

3.78827590390276