1 Write the probability that there is a link between i and j, e_{ij} and the probability that there is a link between i and j conditional on the existence of a link between l and s.

$$P(e_{ij}) = \frac{L}{\binom{N}{2}}$$
$$P(e_{ij}) = \frac{L}{\frac{N(N-1)}{2}}$$
$$P(e_{ij}) = \frac{2L}{N(N-1)}$$

$$P(e_{ij}|e_{ls}) = \frac{2(L-1)}{N(N-1)-2}$$

2 What is the ratio of such two probabilities for small networks? And for large networks?

Small Networks

$$\frac{P(e_{ij}|e_{ls})}{P(e_{ij})} = \frac{(L-1)N(N-1)}{L[N(N-1)-2]}$$

Large Networks

$$\frac{P(e_{ij}|e_{ls})}{P(e_{ij})} = 1$$

since N goes to infinity.

3 What do you obtain for the quantities discussed in (a) and (b) if you use the Erdős-Rényi G(N,p) model?

$$P(e_{ij} = p)$$

$$P(e_{ij}|e_{ls}) = p$$

$$\frac{P(e_{ij}|e_{ls})}{P(e_{ij})} = 1$$

4 Based on the results found for (a)-(c) discuss the implications of using the G(N,L) model instead of the G(N,p) model for generating random networks with small number of nodes.

In small networks, the dependent nature between links is significant since the probability of a link is affected by other existing links. This makes the analysis for small networks more complex in the G(N,L) model compared to the G(N,p) model.