

- 1 Find the normalization factor A , assuming that the network has a power law degree distribution with $2 < \gamma < 3$, with minimum degree k_{\min} and maximum degree k_{\max} .

From the textbook, we know that

$$C = \frac{1 - \gamma}{k_{\max}^{1-\gamma} - k_{\min}^{1-\gamma}}$$

Also applying the normalization condition to q_k , and substituting p_k into the equation, we obtain

$$\begin{aligned} \int_{k_{\min}}^{k_{\max}} A k C k^{-\gamma} dk &= 1 \\ A C \int_{k_{\min}}^{k_{\max}} k^{1-\gamma} dk &= 1 \\ A C \left(\frac{k_{\max}^{2-\gamma} - k_{\min}^{2-\gamma}}{2-\gamma} \right) &= 1 \\ A C \int_{k_{\min}}^{k_{\max}} k^{1-\gamma} dk &= 1 \end{aligned}$$

Solve for A , making sure to substitute the C value and simplify

$$A = \frac{(2-\gamma)(k_{\max}^{1-\gamma} - k_{\min}^{1-\gamma})}{(1-\gamma)(k_{\max}^{2-\gamma} - k_{\min}^{2-\gamma})}$$

- 2 In the configuration model q_k is also the probability that a randomly chosen node has a neighbor with degree k . What is the average degree of the neighbors of a randomly chosen node?

We want to find the average, or expected, value of k in respect to q_k . We denote this k as $\langle k_q \rangle$ and substitute the values for q_k and then p_k in the continuous expectation formula

$$\begin{aligned} \langle k_q \rangle &= \int_{k_{\min}}^{k_{\max}} k A k C k^{-\gamma} dk \\ \langle k_q \rangle &= A C \int_{k_{\min}}^{k_{\max}} k^{2-\gamma} dk \end{aligned}$$

Substitute in the values for A and for C and solve the integral

$$\langle k_q \rangle = \frac{(2 - \gamma)(k_{max}^{3-\gamma} - k_{min}^{3-\gamma})}{(3 - \gamma)(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}$$

3 Calculate the average degree of the neighbors of a randomly chosen node in a network with $N = 10^4$, $\gamma = 2.3$, $k_{min} = 1$ and $k_{max} = 1000$. Compare the result with the average degree of the network, $\langle k \rangle$.

Substitute the given values into the equation for $\langle k_q \rangle$

$$61.23431879119234$$

Use the textbook equation and substitute the value for C to find the equation for $\langle k \rangle$

$$\langle k \rangle = \frac{1 - \gamma}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}} \cdot \frac{(k_{max}^{2-\gamma} - k_{min}^{2-\gamma})}{(2 - \gamma)}$$

Substitute the given values into the equation for $\langle k \rangle$

$$3.78827590390276$$