1 Calculate, using the rate equation approach, the in- and out-degree distribution of the resulting network.

Let us denote with N(k,t) the number of nodes with degree k at time t. Then the rate equation is

$$m[\prod(k-1)N(k-1,t) - \prod(k)N(k,t)]$$

Substitutiong in the given probability, we can simplify the rate equation to

$$m[N(k-1,t)\cdot\frac{(k-1)+A}{mt+At} - N(k,t)\cdot\frac{k+A}{mt+At}]$$

where mt is the sum of all the in-degrees.When t is large, we assume that N(k,t)=t*p__k such that

$$p_k = m[p_{k-1}\cdot\frac{(k-1)+A}{m+A} - p_k\cdot\frac{k+A}{m+A}]$$

which simplifies to

$$p_k = p_{k-1}\cdot\frac{m(k-1+A)}{k+m+2A}$$

2 By using the properties of the Gamma and Beta functions, can you find a power-law scaling for the in-degree distribution?

Using a recursive approach we see that

$$p_k = p_1\prod_{j=1}^{k-1}\frac{m(j+A)}{j+1+m+2A}$$

Using the properties of the Gamma and Beta functions, we get

$$p_k = k^{-2+\frac{A}{m+A}}$$

when k is sufficiently large.

3 For A = 0 the scaling exponent of the in-degree distribution is different from gamma = 3, the exponent of the Barabási-Albert model. Why?

The Barabási-Albert model is undirected so whe attaching new nodes to the graph, it considers the total number of degrees in the graph rather than just the in-degrees. This affects the degree distribution, and as a result, the scaling exponent.